THREE ESSAYS ON VOLATILITY

A Dissertation

Presented to the Faculty of the Graduate School

of Cornell University

in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

by

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May 2013

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Cornell University 2013

My dissertation focuses on economic studying of volatility issues. Three essays are contained in my dissertation. Essay 1 extends a microstructure model to explain the change of volatility and thus links traders' belief to the volatility change. Our model shows that when market is more uncertain about the value of the stock, the higher the (return) volatility. Essay 2 turns to explore more economic factors that could cause volatility regime switch. We find that US stock return processes, including drift, diffusion, and jump, differ along with US political cycle. Our results imply that the presidency in different parties has distinct policy making processes and thus influence the way information flows into the market, altering the return processes.

In the final essay, we document and explain a volatility Bid-Ask spread pattern that increases as time to maturity decreases. Our research develops a model that explains the volatility spread pattern. We show that, as time passes, the required hedging uncertainty premium charged by the liquidity providers decays more slowly while the premium contained in the quoted options price decays at an increasingly higher rate which is determined by the option pricing model. Therefore, liquidity providers need to increase asking and decrease bidding volatility to maintain the profit necessary to compensate slowly decaying hedging uncertainty premium. Our results strongly suggest that studies on volatility spread should detrend the data to make the estimation models correct as well as the series stationary. Without adjusting the trend and autocorrelation problems,

statistical results are inaccurate and misleading. More importantly, based on our theoretical model, we also find that: (a) the implied volatility spread does not increase in proportion to the increase of implied volatility, and (b) the increase of volatility uncertainty is not a sufficient condition for an increase in the percentage spread. Finally, to augment the validity of our claims, we provide rigorous econometric tests which support our propositions.

BIOGRAPHICAL SKETCH

PeiLin Hsieh served as an equity derivatives trader before he entered the Economics Ph.D. program at Cornell University in 2009. He had the Master in Finance degree from University of Maryland, College Park, as well as Bachelor of Science degree in Management Science from National Chiao Tung University, Taiwan. His interests include financial economics and econometrics, and his current research focuses on theoretical and empirical works of volatility issues.

To my wife, Lan-Chu, and my daughter, Ho-Ying.

ACKNOWLEDGEMENTS

I would like to thank my graduate committee members, Professors Robert Jarrow, YongMiao Hong, and David Easley for their teaching and advice during my doctoral study. Especially for my committee co-chairmen Professors Jarrow and Hong, I have been greatly inspired by the conversation and meeting with them and have been feeling myself strongly supported in every progress of Ph.D. program because of their encouragements. Finally, I would like to express my gratitude and thanks to my wife, LanChu Yang, for her understanding. This dissertation can not be complete without her dedication to our family.

TABLE OF CONTENTS

CHAPTER 1
INTRODUCTION

There has been a huge development gap between time series volatility models, which is capable of capturing volatility patterns, and economics models in volatility, which are able to explain volatility changes. Since the ARCH (Auto-Regression on Conditional Heteroskedasticity) model was proposed in the 1970s to analyze the time varying volatility, extended and related econometrics models have been developed with extreme rapidity. However, in contrast to econometrics, the financial economics literature exploring the insight about what economic factors would affect return volatility is relatively sparse. My dissertation focuses on enriching volatility issues with structure modeling and with economics reasoning, filling the gap mentioned previously. Three essays are contained in my dissertation, and they are illustrated with details in Chapter 2, Chapter 3 and Chapter 4 respectively.

In the first essay, I extend the structure of Probability of Information Base Trade Model to explain the volatility change. By extrapolating Bayesian quoting strategy of market makers, a tree with all possible closed prices is formed, and I derive the general form for Bayesian base expected volatility, under the assumption that buy and sell orders follow a mixed Poisson distribution. I show that the return volatility increases as news arrival rate increases, peaking when the probability of bad news, conditional on occurrence of the news, equals 50%, reaching global maximum when market makers expect an event with certainty but do not know whether the news will be bad or good.

The return volatility has two prominent stylized facts: volatility asymmetry and volatility clustering. However only a little literature addresses economics reasons for these stylized facts. In 1976 Black Fisher used the leverage effect to illustrate the volatility asymmetry. Not until 2006 did Clive Granger and Mark Machina used the concept of structure change to account for volatility clustering, but no empirical work so far has tested this argument. While Bollerslev, Sizova, and Tauchen (2010) developed a model incorporating prospects of future economic growth and the current uncertainty about the future economic conditions to explain stylized facts, this research approaches the topic differently. In the research, I link Bayesian updating process, describing trader's quoting behavior, to volatility, thus providing a new economic explanation for the volatility change.

As predicted by our theoretical model, volatility increases as information arrival rate increases. Thus we investigate if index return volatility increases during the weeks of quarterly earnings announcement. During this period, higher information arrival rate should induce higher volatility. We apply Spectral Density analysis, which is the model independent mythology, and find that volatility has 3 month cycle, corresponding to the cycle of quarterly earnings announcement. Furthermore, we also apply the Maximum Likelihood Estimation method to estimate parameters and calculate expected volatility. Our empirical results show that, out of 46 randomly picked stocks, 29 show their estimated model volatility to be significant in explaining their historical volatility.

My second essay turns to explore political cycle effect on return. This topic was been studied extensively before, but the literature all focuses on mean rather than higher order movements. To investigate the impact of political cycle on higher order movements of return, this study takes the perspective from time series models and proposes numerous GARCH with jump models incorporating exogenous economic factors to examine the political cycle effect on return volatility. We also reexamine the return puzzle studied by Santa-Clara and Valkanov (2003).

A great deal of literature documents the obvious difference in stock return under Republican and Democratic presidents. The paper of Santa-Clara and Valkanov (2003) is the first one formally testing it. They used monthly return to regress on drift term, presidential dummy and other control variables. Applying the method of Newy-West variance and covariance estimation to overcome the heteroskedasticity problem, they concluded positive excessive return in Democratic presidential periods. To test if the abnormal return can be explained by the return risk, they used monthly volatility (computed from within-month daily returns) as a measure for return variation, and their result surprisingly showed that volatility is significantly higher for Republican presidencies since 1956. In other words, in the long run, the higher return in Democratic presidencies was not due to the higher risk (volatility) that requires higher return for compensation.

Considering this finding, it is of particular interest to analyze the question using time series models, which decompose the daily return processes into drift, slow diffusion and jump. Thus we expand volatility with jump models, based on GARCH(1,1) family models, allowing us to endogenize the risk problem and to test volatility and return jointly. In our research, we find that the presidency effect does affect the jump process and that the jump arrival rate is significantly higher during Republican administrations. Additionally, the conditional daily volatility is significant in explaining daily return, while enduring 1% daily deviation requires 0.1% to 0.14% index daily return to compensate. Finally, after controlling volatility and business cycles, we also find that daily index return is still lower by 0.0274%, approximately 6.9% annually, during Republican presidencies. This result is consistent with conclusion of Santa-Clara and Valkanov (2003).

My third essay advances to investigate the volatility implied by the trading price of the options, given that volatility is the most prominent factor that prices options and given that implied volatility thus contains the information of future volatility. I first document the stylized fact that the Bid-Ask spread widens at increasing rate as options contracts approach the expiration date. Distinct from other research on this topic, the Bid-Ask spread calculated here is based on model free implied volatility.

Chong, Ding and Tan (2003) wrote the first paper addressing maturity effect on the volatility Bid-Ask spread. They claimed there are two risks related to maturity. First, market risk, higher gamma for short term options, drives the market to demand larger Bid-Ask spread for shorter time to maturity options. Second, because their data is the trading record of OTC currency options, the trading involves credit/default risk of opponents. Longer time to maturity implies higher credit risk and thus needs larger Bid-Ask spread for compensation. Through their empirical work they found that volatility spread, measured by Black-Shoes Merton Model, increases as time to maturity decreases. Hence they concluded that market risk dominates credit risk. Other empirical works, like Wei and Zheng (2010), used Ivy DB's OptionMetrics data to inspect the impact of trading activities on the liquidity of stock options. Instead of using the volatility measure, those papers defined spread in terms of ratio of dollar measure and also concluded existence of the maturity effect. However, the spread measurement and reasoning mentioned in those papers are not indisputable.

With a focus on the disparity between the decay rate of hedging portfolios of derivatives and the decay of required premium for hedging risk, our research develops a model which explains why the spread pattern occurs. Our results strongly suggest two important issues that need to be considered in empirical works on related topics. First, studies on volatility spread should detrend the data to make the series stationary. Second, research should also consider the heteroskedasity issue of error terms. Without adjusting trend and autocorrelation problems, statistical results are inaccurate and misleading. More importantly, based on our theoretical model, we also find: (1) volatility spread measured in percentage of implied volatility decreases as implied volatility increases. (2) an increase in volatility uncertainty is not a sufficient condition leading to an increase in volatility percentage spread. Finally, to augment the validity of our claims, we provide rigorous econometric tests which support our findings.

CHAPTER 2

LINKING THE BELIEF BASE QUOTING STRATEGY TO VOLATILITY

2.1 Introduction

Volatility has been an important issue in derivatives pricing and time series research. This is because volatility decides the possible range of future price, and thus it has been the most important factor in pricing of derivatives. In the 1970s, the most influential finance model, the Black Scholes Merton model, was created, and has since been utilized for options pricing. However, it was not until the early 2000s began that most closed form models of derivatives pricing allowed volatility to vary over the duration of derivatives. So, after the 1970s, how to forecast volatility accurately became an important issue. Moreover, the oil shock, high inflation rate, and implementation of a floating exchange rate in seventies made the finance market so volatile that econometricians started to use time series analysis to model volatility. In 1982, Robert Engle developed the ARCH (Autoregressive Conditional Heteroskedasticity) model for analyzing time varying volatility, and the model was quickly expanded to create numerous versions of the General Autoregressive Conditional Heteroskedasticity model.

GARCH related models can forecast the volatility more accurately because time series models can capture the stylized facts. Volatility has a number of important stylized facts, two of which are volatility clustering and volatility asymmetry. Volatility clustering refers to the high auto-correlated movement of the return. In other words, one dramatic jump or drop of return tends to be followed by other large movements of the return. As for volatility asymmetry, it was first documented in 1963 by Mandelbrot. He found the return volatility tends to be higher when stock price is decreasing and lower when stock price is increasing. In time series, the GARCH and EGARCH models are designed to respectively capture the stylized facts of volatility clustering and volatility asymmetry and to further forecast the volatility.

However, the literature on reasoning stylized facts of volatility is very sparse. In 1976 Black Fisher used the leverage effect to explain the volatility asymmetry. Basically, the leverage effect refers to the operation risk of a company. A decrease in the company's stock price would increase the debt ratio and leverage ratio of the company, so in this case the company would run the business in a more risky environment. The more risky a company, the more volatile its stock. Though the leverage effect has been used extensively to explain the volatility asymmetry, empirical works, for example Duffee (1995) and Bekaert and Wu (2000), have not demonstrated strong support for this argument. Furthermore, the literature addressing reasons for volatility clustering is even less. In 2006, Clive Granger and Mark Machina used the concept of structure change to explain volatility clustering, but no empirical work so far has tested this argument.

Recently Bollerslev, Sizova, and Tauchen are dealing with the same issues identified above. Their research, whcih is the finest research addressing those issues so far, incoporates prospects of future economic growth and the current uncertainty about the future economic conditions to explain stylized facts. This paper approaches the related topics differently. This research extend the market microstructure model to derive the theoretical volatility and tests it. The author builds up a link between the market makers' quoting strategy and volatility, and thus provides a new economic explanation for the change in volatility. The author is going to extend the model to explain the stylized facts in future work. Additionally, empirical evidence demonstrates significance of theoretical volatility, based on quoting strategy, in explaining the historical volatility.

In this chapter, we link the concept of Bayesian based beliefs to volatility. It not only provides new economic reasoning for change in volatility but also sheds light on improving volatility forecasting. Compared with the GARCH related models, which primarily use information of price and return, the microstructure model extracts the information from the buy and sell numbers.[1] So, by incorporating the threshold model or regime switch model, it may improve the forecasting ability of GARCH model.

This Chapter is organized as follows. In section 2.2, the author discusses the market microstructure theoretical ground for this research and the extension of theoretical work in the research. Section 2.3 describes the data set and the empirical results. Section 2.4 shows an advanced theoretical model analysis by simulation method. Finally, the conclusion and possible future development are provided in section 2.5.

2.2 Market Microstructure Model

This section first addresses the main structure and assumptions of the PIN model. Then the author discusses how the original structure is extended to link

[1] There are many researches, which analyze joint dynamics between trades and prices or show evidence supporting information is contained in the order of trading. Examples are Chordia, Roll, and Subrahmanyam (2000, 2001a, 2001b, 2002, 2005), Chordia and Subrahmanyam (2004).

the volatility.

2.2.1 Trade Process

The research is based on the structure created in the series of papers by David Easley and Maureen O'Hara. These papers constructed the model that is now known as the PIN model. In the model, everyday an information event occurs with probability, α, and δ is the probability of an information event that leads to the lowest daily price $\underline{V}$. In previous papers, $\underline{V}$ is defined more generally and, for each day i, $\underline{V}$ can be $\underline{V}_i$, which could vary over time. Here the close price of yesterday is scaled onto 1, and the research assumes $\underline{V}$ fixed, for example $\underline{V}$ = 0.9. Similarly, $1 - \delta$ is the probability leading to the daily highest price, $\bar{V}$.

After the information event arrives, the market starts to trade. The transactions arise from informed and uninformed traders. If information event occurs, the expected number of total informed trades is μ, and, no matter whether the event occurs or not, uninformed sell and buy take place everyday at expected total trading number ε. Given the occurrence of an information event that supplies bad news, the number of sells is $\mu + \varepsilon$, and the number of buys is ε. Similarly, if the event is good, the number of buy orders is $\mu + \varepsilon$ and the number of sell orders is ε. For days when no information event occurs, the number of sell orders and buy orders are ε respectively. So each day the expected total number of trades is $\alpha\mu + 2\varepsilon$.

Figure 2-1-1 shows how the trade process is in the model. For each coming trade, the result could be a buy or sell transaction in bad news, good news or no news. There are six possible results; 2 of the 6 results, initiated by informed and uninformed traders, occur at a probability of $\tilde{\varepsilon} + \tilde{\mu} = (\varepsilon + \mu)/(\alpha\mu + 2\varepsilon)$. The portion $\tilde{\mu}$ in the numerator is initiated by informed traders if the information event occurs. The other 4 results are initiated by uninformed traders at a probability of $\tilde{\varepsilon} = \varepsilon/(\alpha\mu + 2\varepsilon)$. So for each coming trade, the sum of the probabilities of the 6 possible results is equal to 1.

2.2.2 Model Assumption

In addition to the trade process, there are four assumptions made in the structure of the PIN Model. These assumptions are:

(1) Buy and sell orders follow the Poisson distribution

(2) The market has market makers to provide liquidity

(3) Trade size is not considered in the Model

(4) The market makers apply Bayes rule in their quoting strategy

From the equilibrium point of view, market makers suffer a loss when an information event occurs, whereas they, based on their beliefs, earn from spread of selling at higher offer price and buying at lower bid price. Competitive market makers should able to earn the opportunity cost by quoting at the best suitable price.

Buy and sell orders follow the Poisson distribution.

The Poisson distribution is characterized by the expected trading number. So given certain expected number of informed and uninformed trades, people know the probability that the certain number of buy orders and sell orders happens. In another way, people can collect the observations of buy and sell orders each day. The expected number of informed and uninformed trades can be derived by applying the maximum likelihood function incorporated with Poisson distribution assumption. In the game tree, 2-1-1, there are three main scenarios, including the occurrence of good news, bad news and no news. Each scenario has its Poisson process on each day, and the joint Poisson distribution, that seeing number of buy, B_t, and sell, S_t, is

$$\Pr[y_t = (B_t, S_t)|I_{t=1}]$$

$$=\alpha(1-\delta)e^{-(\mu_{t-1}+2\varepsilon_{t-1})}\frac{(\mu_{t-1}+\varepsilon_{t-1})^{B_t}(\varepsilon_{t-1})^{S_t}}{B_t!S_t!} + \alpha\delta e^{-(\mu_{t-1}+2\varepsilon_{t-1})}\frac{(\mu_{t-1}+\varepsilon_{t-1})^{S_t}(\varepsilon_{t-1})^{B_t}}{B_t!S_t!}$$

$$+(1-\alpha)e^{-2\varepsilon_{t-1}}\frac{{\varepsilon_{t-1}}^{S_t+B_t}}{B_t!S_t!}, \text{ (2-1)}$$

The maximum likelihood function is

$$\pounds(\{y_t\}_{t=1}^{T}|\Theta) = \sum_{t=1}^{T} \ln \Pr\{y_t = (B_t, S_t)|I_{t-1}\}, \Theta = (\alpha, \delta, \mu, \varepsilon),$$

This research uses the maximum likelihood function proposed by Easley, Engle, O'Hara and Wu in 2008.

$$\pounds(\{y_t\}_{t=1}^{T}|\Theta) = \sum_{t=1}^{T}[-2\varepsilon_{t-1} + (B_t + S_t)\ln(\mu_{t-1} + \varepsilon_{t-1})]$$

$$+\sum_{t=1}^{T}\ln[\alpha(1-\delta)e^{=\mu_{t-1}}x_{t-1}^{St} + \alpha\delta e^{-u_{t-1}}x_{t-1}^{B_t} + (1-\alpha)x_{t-1}^{B_t+S_t}], \text{ (2-2)}$$

$x_t = \frac{\varepsilon_t}{\varepsilon_t + \mu_t}$,

The market has market makers to provide liquidity.

The model assumes the existence of market makers to provide liquidity for the informed and uninformed trades. Market makers do not know in which scenario they stand but they know the whole structure. Informed traders and uninformed traders buy from, and sell stocks to, market makers. Market makers earn the profit, when no information arrives, by selling stock at a higher price and buying at a lower price. They lose the money once the information event happens, but in the long run equilibrium, the profit earned when there is no news should cover the loss, caused by the arrival of information events.

Though this assumption is a bit strong, the market maker mechanism did prevail in previous times and, in market microstructure, it is the key that drives the whole information learning process. This process is characterized by Bayes rule, which is believed how investors learn the information of trades in the market. The role and behavior of market makers have been extensively studied in the literature (Hendershott and Seasholes, 2007; Comerton-Forde, Hendershott, Jones, Seasholes and Moulton, 2010). Moreover, market makers are still playing important roles to provide liquidity in some other markets, such as options and ETF markets, so the structure of this assumption may be completely valid in other finance markets.

Trade size is not considered in the model.

The model assumes that every transaction, no matter what size it is, carries equally weighted information. Trade is classified by the Lee-Ready algorithm into sell or buy. Using the Lee-Ready algorithm, people define trades above the previous midpoint of the bid-ask spread as buy orders, and trades below the midpoint as sell orders. If the trade is at the mid point, then we compare the trade price with the previous dissimilar transaction price which is nearest. The trade is a buy order if the price moves up and a sell order if price moves down.

The market makers apply Bayes rule in quoting strategy.

Everyday market makers revise their beliefs, which are $(\alpha, \delta, \mu, \varepsilon)$. This can be derived by using the maximum likelihood function estimation method. Using Bayes rule, for each coming intraday trade the expected value of stock price, conditional on one buy(sell) observation, can be derived, and market makers use the conditional expected stock price as the asking(offering) price. The conditional expected value is also called the "regret free" price, because, once the transaction happens, the market makers sell(buy) the stock at their expected price.

2.2.3 Extension of Theoretical Work

In the extended structure, market makers keep revising their beliefs according to past transaction data. For the trading day t, the market makers set up the quoting strategy based on beliefs, $(\alpha_{t-1}, \delta_{t-1}, \mu_{t-1}, \varepsilon_{t-1})$. Those beliefs are derived from the maximum likelihood function. Figure 2-2-1 shows the Bayes quoting strategy scheme for intraday, given beliefs, $(\alpha, \delta, \mu, \varepsilon)$, which are revised after close of market at day, $t-1$.[2] Under the market maker's assumption, all transactions happen implicitly on the asking and bidding price, so the quoting strategy scheme contains all possible price paths. Each price on the node of the tree can be expressed in the generalized form below. H and L are the daily highest price and lowest price respectively.[3]

$$\text{Price}(\{B_t,S_t\}|I_{t-1}) = \frac{H(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}(\tilde{\varepsilon}_{t-1}^{S_t})\alpha_{t-1}(1-\delta_{t-1})+(\tilde{\varepsilon}_{t-1}^{B_t})(\tilde{\varepsilon}_{t-1}^{S_t})[1-\alpha_{t-1}]+L(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}(\tilde{\varepsilon}_{t-1}^{B_t})\alpha_{t-1}\delta_{t-1}}{(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}(\tilde{\varepsilon}_{t-1}^{S_t})\alpha_{t-1}(1-\delta_{t-1})+(\tilde{\varepsilon}_{t-1}^{B_t})(\tilde{\varepsilon}_{t-1}^{S_t})[1-\alpha_{t-1}]+(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}(\tilde{\varepsilon}_{t-1}^{B_t})\alpha_{t-1}\delta_{t-1}}, \quad (2\text{-}3)$$

Given the beliefs forming the quoting strategy, the denominator is the probability of seeing number of buy, B_t, and sell, S_t. In the probability space $\{B_t, S_t\}$, there are three possible subsets, in which stock value is equal to H, 1 or L, and the probability for each subset is:

$$\Pr(\text{price}=H|\{(B_t,S_t),(\alpha_{t-1},\delta_{t-1},\mu_{t-1},\varepsilon_{t-1})\}) = \frac{(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}(\tilde{\varepsilon}_{t-1}^{S_t})[\alpha_{t-1}(1-\delta_{t-1})]}{(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}(\tilde{\varepsilon}_{t-1}^{S_t})[\alpha_{t-1}(1-\delta_{t-1})]+(\tilde{\varepsilon}_{t-1}^{B_t})(\tilde{\varepsilon}_{t-1}^{S_t})[1-\alpha_{t-1}]+(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}(\tilde{\varepsilon}_{t-1}^{B_t})[\alpha_{t-1}\delta_{t-1}]},$$

$$\Pr(\text{price}=1|\{(B_t,S_t),(\alpha_{t-1},\delta_{t-1},\mu_{t-1},\varepsilon_{t-1})\}) = \frac{(\tilde{\varepsilon}_{t-1}^{B_t})(\tilde{\varepsilon}_{t-1}^{S_t})[1-\alpha_{t-1}]}{(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}(\tilde{\varepsilon}_{t-1}^{S_t})[\alpha_{t-1}(1-\delta_{t-1})]+(\tilde{\varepsilon}_{t-1}^{B_t})(\tilde{\varepsilon}_{t-1}^{S_t})[1-\alpha_{t-1}]+(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}(\tilde{\varepsilon}_{t-1}^{B_t})[\alpha_{t-1}\delta_{t-1}]},$$

$$\Pr(\text{price}=L|\{(B_t,S_t),(\alpha_{t-1},\delta_{t-1},\mu_{t-1},\varepsilon_{t-1})\}) = \frac{(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}(\tilde{\varepsilon}_{t-1}^{B_t})[\alpha_{t-1}\delta_{t-1}]}{(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}(\tilde{\varepsilon}_{t-1}^{S_t})[\alpha_{t-1}(1-\delta_{t-1})]+(\tilde{\varepsilon}_{t-1}^{B_t})(\tilde{\varepsilon}_{t-1}^{S_t})[1-\alpha_{t-1}]+(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}(\tilde{\varepsilon}_{t-1}^{B_t})[\alpha_{t-1}\delta_{t-1}]},$$

Therefore, the equation (2-3) is conditional expected price, given B_t and S_t observed. Moreover, based on the assumption that buys and sells follow the

[2] This research calibrates the dynamic beliefs, α, δ, μ and ε by past transaction data. So in the equation we use α_{t-1}, δ_{t-1}, μ_{t-1} and ε_{t-1} to denote the beliefs.

[3] Easley, Kiefer, O'Hara, and Paperman (1996) have the most well regarded discussion on Bayes learning process in quoting strategy.

Poisson distribution, each price in equation (2-3) can be assigned a probability by using the joint Poisson distribution function. In other words, we use equation (2-1) to describe the distribution of the price,which is expressed in equation (2-3). Thus for each day, incorporating price and probability, we can calculate the theoretical model volatility, which is based on quoting strategy. This paper accordingly links the market quoting strategy to the volatility and deems quoting strategy an important factor affecting volatility. The theoretical model variance can be written in its general form as,

$$E(R_t|I_{t-1})$$

$$=\sum_{B_t=0}^{\infty}\sum_{S_t=0}^{\infty}\{[\frac{H(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}\tilde{\varepsilon}_{t-1}^{S_t}[\alpha_{t-1}(1-\delta_{t-1})]+\tilde{\varepsilon}_{t-1}^{B_t}\tilde{\varepsilon}_{t-1}^{S_t}[1-\alpha_{t-1}]+L(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}\tilde{\varepsilon}_{t-1}^{B_t}[\alpha_{t-1}\delta_{t-1}]}{(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}\tilde{\varepsilon}_{t-1}^{S_t}[\alpha_{t-1}(1-\delta_{t-1})]+\tilde{\varepsilon}_{t-1}^{B_t}\tilde{\varepsilon}_{t-1}^{S_t}[1-\alpha_{t-1}]+(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}\tilde{\varepsilon}_{t-1}^{B_t}[\alpha_{t-1}\delta_{t-1}]}-1]$$

$$*[\alpha(1-\delta)e^{-(\mu_{t-1}+2\varepsilon_{t-1})}\frac{(\mu_{t-1}+\varepsilon_{t-1})^{B_t}(\varepsilon_{t-1})^{S_t}}{B_t!S_t!}+\alpha\delta e^{-(\mu_{t-1}+2\varepsilon_{t-1})}\frac{(\mu_{t-1}+\varepsilon_{t-1})^{S_t}(\varepsilon_{t-1})^{B_t}}{B_t!S_t!}$$

$$+(1-\alpha)e^{-2\varepsilon_{t-1}}\frac{{\varepsilon_{t-1}}^{S_t+B_t}}{B_t!S_t!}]\}, \quad \text{(2-4)}$$

$$E(R_t^2|I_{t-1})$$

$$=\sum_{B_t=0}^{\infty}\sum_{S_t=0}^{\infty}\{[\frac{H(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}\tilde{\varepsilon}_{t-1}^{S_t}[\alpha_{t-1}(1-\delta_{t-1})]+\tilde{\varepsilon}_{t-1}^{B_t}\tilde{\varepsilon}_{t-1}^{S_t}[1-\alpha_{t-1}]+L(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}\tilde{\varepsilon}_{t-1}^{B_t}[\alpha_{t-1}\delta_{t-1}]}{(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{B_t}\tilde{\varepsilon}_{t-1}^{S_t}[\alpha_{t-1}(1-\delta_{t-1})]+\tilde{\varepsilon}_{t-1}^{B_t}\tilde{\varepsilon}_{t-1}^{S_t}[1-\alpha_{t-1}]+(\tilde{\varepsilon}_{t-1}+\tilde{\mu}_{t-1})^{S_t}\tilde{\varepsilon}_{t-1}^{B_t}[\alpha_{t-1}\delta_{t-1}]}-1]^2$$

$$*[\alpha(1-\delta)e^{-(\mu_{t-1}+2\varepsilon_{t-1})}\frac{(\mu_{t-1}+\varepsilon_{t-1})^{B_t}(\varepsilon_{t-1})^{S_t}}{B_t!S_t!}+\alpha\delta e^{-(\mu_{t-1}+2\varepsilon_{t-1})}\frac{(\mu_{t-1}+\varepsilon_{t-1})^{S_t}(\varepsilon_{t-1})^{B_t}}{B_t!S_t!}$$

$$+(1-\alpha)e^{-2\varepsilon_{t-1}}\frac{{\varepsilon_{t-1}}^{S_t+B_t}}{B_t!S_t!}]\}, \quad \text{(2-5)}$$

$$Var(R_t|I_{t-1}) = E(R_t^2|I_{t-1}) - E(R_t|I_{t-1})^2,$$

Theoretical Model Standard Deviation=$\sqrt{Var(R_t|I_{t-1})}$,

It hereby needs to be emphasized that the volatility we measure is interday volatility, because the price in equation (2-3) refers to the close price in day t, and has the corresponding joint Poisson probability, so the theoretical model standard deviation can be interpreted as the expected deviation of daily return from the expected mean of daily return. The equation (2-4) is the expected daily return, given the market makers beliefs and quoting strategy. The equation (2-5) is the expected value of the square of the daily return and is the function which captures the second movement of the daily return. Having equation (2-4) and (2-5), we can apply the transformed formula of variance and take the square root of variance to get the standard deviation.

In the empirical study of this paper, the beliefs of market makers are calibrated through MLE from the data set of day $t-1$ to $t-T$, so it is plausible to consider the calibrated beliefs the same as the approximated beliefs held by market makers throughout the data set period. Compared to the historical standard deviation, which was interpreted as the sample averaged deviation of daily return from the sample averaged daily return, this research considers the theoretical volatility model a natural data generation process and the historical volatility one of its possible realizations.

If the number of observations is large enough, the regression result can show if theoretical volatility, based on quoting strategy, is significant enough to affect historical volatility. In the empirical work, in order to derive the application of theoretical volatility, I impose a stronger assumption. I assume the parameters of H and L equal to 1.07 and 0.93. As long as the price of yesterday is scaled to 1, and the H and L are fixed over time, the statistics results show no difference. The level of H and L would affect the theoretical volatility and coefficient of regression in the scale.

2.3 Data and Empirical Work

The research randomly selects 46 medium or small cap company stocks to test if the theoretical model volatility is a significant factor in explaining the historical volatility. The stocks include Kraft Foods, GAP, GARMIN, MOVADO, Chesapeake Utilities, Hospitality Properties and another 40 stocks. This research selects mid and small cap company stocks ,which generally have less than 2000 trades each day, not only because each trade carries more meaningful information, but also because they allow for easier computation of model parameters and theoretical volatility. The sample period ranges from the beginning of 2001 to the end of 2006, and all the data are taken from the TAQ database. The data shows the intraday price of quoting and transaction for every second. In order to get the number of buy and sell orders each day, the Lee-Ready algorithm is used to classify transactions into buy or sell orders.

I select a data window of 60 days from $t-1$ to $t-60$ and keep rolling over to the next day to get estimators of $(\alpha_{t-1}, \delta_{t-1}, \mu_{t-1}, \varepsilon_{t-1})$ for each day. Given daily estimators, the research calculates the theoretical volatility for day t and compares it with the stock return volatility of 60 days, from t to $t-59$.

The following features can be observed in Figure 2-3-1, which graphs the relation between historical return standard deviation and theoretical return standard deviation for 46 stocks. First, for more than 25 stocks out of 46, the theoretical volatility demonstrates a significantly positive correlation with historical return volatility, but the relation patterns of each stock looks very different. Secondly, the variation of theoretical volatility and historical volatility tends to be larger when theoretical volatility is higher. The model seems capable of capturing correct beliefs when historical volatility is low. However, when theoretical model volatility is high, data shows more noises. The research runs the following regression model to investigate the significance of the theoretical volatility in explaining historical volatility.

$$\sigma_{Historical} = C + B_1 * \sigma_{Model} + B_2 * P_{Averaged} + \varepsilon_t \quad , (2\text{-}6)$$

Where $\sigma_{Historical}$ is the standard deviation of historical return, and σ_{Model} is the theoretical standard deviation, generated by the model. Since the leverage effect is a popular factor in explaining the volatility asymmetry, the research puts averaged price, $P_{averaged}$, into the regression model as a benchmark comparison. The author chooses the 60-day data window, from $t-1$ to $t-60$, to derive parameters of theoretical model volatility and assumes that the estimated parameters from maxima likelihood function can give the best explanation for the 60-day historical standard deviation, based on t to $t-59$. Considering observations are highly auto-correlated, the paper uses the generalized method of moment to estimate the regressions. The weighting matrix is calculated according to Newey and West (1987) with lags, equal to the cube root of observations.

Table 2-3-1 and Table 2-3-2 report the regression results. If a 10% critical value, which implies t=1.646, is used as the criteria the theoretical volatility shows significance in explaining historical volatility in 29 out of the 46 stocks. Though there are cases where the coefficient of theoretical volatility is insignificant and negative, the coefficient tends to be positive for most stocks. The R square ranges from 1.8% to 63%. The graphs of relation between theoretical volatility and historical volatility are shown in Figure 2-3-1, the order of which corresponds to the order of Table 2-3-1 and 2-3-2.

2.4 Advanced Analysis by Simulation

In the theoretical model, four parameters, $(\alpha, \delta, \mu, \varepsilon)$, play the main role in determining the theoretical volatility. In this paper, how parameters,(α, δ), affect the volatility is of particular interest. However, the equation for the theoretical model standard deviation is very complicated and not in closed form, so instead I use simulation for advanced partial analysis. The parameters for simulation are cross sectional and time averaged statistics from Easley, Hvidkjaer, and O'Hara (2002). Table 2-4-1 shows the statistic data, where $\alpha = 0.283$, $\delta = 0.331$, $\mu = 31.08$ and $\varepsilon = 23.1$.

I first fix the parameters, $(\delta, \mu, \varepsilon)$, and calculate the theoretical standard deviation at different levels of α, probability of event occurrence. Our simulation finds that, given $(\delta, \mu, \varepsilon)$ fixed, theoretical volatility monotonically increases over α, which is the probability of event occurrence. If only δ varies, while other $(\alpha, \mu, \varepsilon)$ parameters remain constant, the theoretical volatility peaks when δ equals 0.5. The results above indicate that when the market makers are more uncertain about the true value of the stock, the volatility will increase. If we take a closer look at bidding and asking strategy, when the market makers are more uncertain about the true value of the stock, market makers enlarge the quoting spread in comparison to when market makers are more certain about the true value. The graphs with partial single variable simulation are demonstrated in Figure 2-4-1 and Figure 2-4-2.

If α and δ are variables, while μ and ε are constants, we can derive a three dimensional graph to investigate how theoretical volatility changes at different levels of α and δ. From the 3D graph, Figure 2-4-3, the theoretical volatility has global maximization when α and δ are equal to 1 and 0.5 respectively. Again, the graph shows that the more uncertain the market about the true stock value, the more volatile the stock return will be. Theoretical volatility gradually increases to maximization, where parameters α and δ are equal to 1 and 0.5. In this case, the market believes an information event certainly will occur but is not sure whether the event will be good or bad.

It is worthwhile to note that there is an interesting pattern when δ is close to 0 or 1. In this case, an increase in α, the probability of event occurrence, decreases the volatility. Actually given that the information event is certainly good or bad, an increase in α, the probability of event occurrence, synonymously raises the certainty of the stock value. In this scenario, the theoretical volatility peaks when $\alpha = 0.5$. Based on the model, I further conclude that in financial crises, the real factor driving volatility up may not be the bad news itself, but rather the uncertainty surrounding the arrival of bad news. Once markets become more sure about if the information event will occur, the volatility will decrease, even if the news is bad.

2.5 Conclusion

This research extends the structure of the PIN model and links the belief base quoting strategy to volatility. By doing this, the research provides a new economic explanation and model for volatility, and additionally the empirical evidence shows the significance of theoretical model volatility, which was based on the quoting strategy. In 29 out of 46 stocks, the theoretical model volatility is significantly correlated with historical volatility.

Moreover, by the partial analysis, this research shows how market beliefs, the probability of event occurrence and the probability that the event is bad news, affect the theoretical volatility. These beliefs are key factors in the quoting scheme of the market, and when the market is more uncertain about the true value of stock, the spread of quoting prices is generally wider in every intraday tick. Hence, the quoting scheme tree is more extensive, and the theoretical model volatility increases.

The research at this point has not expanded to explain volatility clustering and volatility asymmetry. This research is going to allow the heterogeneity of uninformed sell orders and uninformed buy orders, so the volatility asymmetry can be explained by the model. Additionally, to explain volatility clustering, we need to show that the beliefs demonstrate the pattern of high auto-correlation which is consistent to volatility clustering.

This research also sheds light on developing the following research. First of all, the orthodox time series volatility forecasting models mainly use information of return. Incorporating our parameters, containing information from buy and sell orders, into the time series threshold model or the regime switching model could improve the forecasting ability. Secondly, in this research the author has not formally discussed the return distribution, generated by the Poisson distribution and Bayes rule. According to initial observation, the return distribution is bell shape with heavier tails than in normal distribution. Modifying the Poisson distribution or Bayes rule in advance to better capture the dynamic return distribution is a challenging future topic to explore. Finally, in options markets, the phenomenon of the implied volatility smile (skew) has been famous since the late 1990s. The research structure of this paper may possibly be extended to explain the volatility smile.

CHAPTER 3

UNITED STATES POLITICAL CYCLE IMPACT ON DRIFT, VOLATILITY AND JUMP PROCESS OF STOCK MARKET RETURN

3.1 Introduction

The relation between stock market performance and political or economic events started to gain attention since 1970s. Previous related discussion centers around explaining stock returns by factors such as election cycle, presidency cycle and business cycle. Some examples are Niederhoffer, Gibbs, and Bullock (1970), Riley and Luksetich (1980), Reilly and Drzycimski (1976), and Siegel (1998). In addition to the relationship between short run performance of stock return and presidential election, the literature also documented other interesting return patterns. Allvine and O'Neil (1980) found significantly lower returns in the first half of presidential administration than in the second half, and Stovall (1992) claimed it is because the party in office usually implement more accommodating and easier (money) policies to prepare the next election. While some work focused on monetary policies to explain return patterns, Bizer and Durlauf (1990; henceforth, BD) discovered that an eight year tax cycle period, and taxes were reduced two year prior to successful presidential reelection attempt.

Along with topics of election cycle, many researchers also addressed the relation between presidency cycle and stock returns Smith (1992), Stovall (1992), Hensel and Ziemba (1995), and Johnson, Chittenden and Jensen (1999) showed that stock return is much higher during Democratic administration than Republican administration. In Table 3-1-1, we briefly summarize the conclusions of previous literature. However, most of the work did not proceed with a rigorous statistical approach. The paper of Santa-Clara and Valkanov (2003; henceforth, SCV) is the first attempts which formally test for the relation of return to presidency effect, while BD utilizes the methodology of spectral analysis and focuses on tax cycle.

In SCV, the authors regressed monthly return on drift term, presidential dummy and other control variables. Applying Newey-West variance and covariance estimator to overcome heteroskedasticity problem, they concluded positive excessive returns were usually associated with democratic presidential period. To test if abnormal returns can be explained by return risk, they used monthly volatility (computed from within-month daily returns) as measure for return variation, and their result surprisingly showed that volatility is significantly higher for Republican presidency since 1956. In other words, over the 50 year time horizon, higher return in democratic presidency was not due to higher risk (volatility) for compensation.

Considering this anomaly, it is of particular interest to analyze the question in a time series model, decomposing the daily return dynamic process into drift, slow diffusion and jump terms. Lots of time series researches have been devoted to analysis of the process of return,[1] but only limited number of work tried to combine time series volatility and jump models with economic and political factors. However, from the economics and finance point of views, it is interesting to understand what components of return process are affected by inflow of economic factors and information, and hybrid models help us to answer

[1]Those models include ARCH, GARCH, stochastic volatility and jump diffusion models, each of which can be widely extended. Additionally, instead of adding jumps in the return, there were literatures working on modeling new distributions as well, for example skew t distribution, to capture stylized facts of higher order moments in return.

this question. In this paper, because of the discrete nature of observed economic data, we will focus on discrete time GARCH models. Papers which did survey and comparison among ARH typed models, include but are not limited to Bollerslev (2010), Poon and Granger (2005), and Hensen and Lunde (2005). In contrast to that literature, our work aims at proposing and comparing different hybrid models that incorporate presidency anomaly.

In addition to pure GARCH typed models, we also introduce jumps and investigate political cycle effect on volatility, jump and return. The diffusion process can be considered as slow movement in volatility, while the jump process represents sudden and big volatility changes. Hence adding jumps empowers GARCH models to capture sudden large changes of volatility. Das (2002) proposed an ARCH model with daily jump following a Bernoulli distribution, and he allowed observed economic factors to enter the parameters of jump intensity to investigate week effect and Fed Reserve meeting effect. He concluded that the Fed Fund rate does not follow a martingale process owing to market's overreaction to large jumps. His model is a good paragon paralleling to our research, while retaining the Bernoulli daily jump process assumption but we extend his model to GARCH diffusion process. On the other hand, Macheu and McCurdy (2003; henceforth, MM) proposed ARJI (Autoregressive conditional jump intensity) model to capture jump clustering and modeled the intraday jumps by a Poisson process. In this paper, we emt-stat economic factors into the parameters of Das' model to inspect how this will affect the return process.

The Chapter is organized as follows. In section 3.2, we discuss 18 proposed models with their properties. Basing on those properties, we can have deeper insight about what economic factor is suitable for the proposed model. We use section 3.3 to discuss the unconditional moments of each model and give impulsive function analysis of the model. Descriptive data statistics and empirical estimations results are covered in section 3.4 and 3.5. In section 3.6 we use TGARCH in mean with jump as example to to explore the insight of exogenous effect on jump and difussion process. Section 3.7 and 3.8 are robustness analysis and conclusion.

3.2 Model Specification

We proposed 18 GARCH-typed models to explore the relationship between presidency and business cycle effect on volatility and jump process. In addition to Threshold GARCH type model applied often to explore the economic effect on volatility, we also extend standard GARCH(1,1) model to incorporate political and economic factors to become prototype models, while those factors are presumed to be exogenous. This implies that we assume no feedback effect from daily index return to presidency and contraction dummy variables. The first 6 models are pure GARCH-typed and EGARCH-typed difussion models with exogenous factors, and then we include conditional mean into return process of previous models for controlling the required compensation on return uncertainty and label those models from (7) to (13). Finally we include the jump process into model (7) to model (13) and test presidency and business cycle effect on return, diffusion and jump process together.

In the models followed, X_t is the vector containing the constant 1, the Republican presidency dummy and contraction dummy at time t, and Y_t contains only Republican presidency dummy and contraction dummy at time t. H_t denotes the vector containing the daily conditional standard deviation, $\sqrt{h_t}$, from the diffusion process and daily conditional standard deviation, $\sqrt{q\gamma^2}$, from the pure jump process. For simplicity, we let innovations, ε_t, be iid normal with mean 0 and variance 1. Jump process follows Bernoulli distribution, $\mathbf{B}$, with intensity q and jump size γ^2.

$$\begin{cases} X_t = [1, d_{(R,t)}, d_{(C,t)}]' \\ Y_t = [d_{(R,t)}, d_{(C,t)}]' \\ H_t = [\sqrt{h_t}, \sqrt{q\gamma^2}] \\ \varepsilon_t \tilde{} IID\ N(0,1),\ J \tilde{} N(0,\gamma^2) \\ \mathbf{B} \tilde{} Bernoulli(q),\ q = LX_{t-1} \end{cases}$$

$d_{(R,t)}$	:dummy variable for Repubicant presidency
$d_{(C,t)}$	:dummy variable for business cycle
h_t	:conditional volatiltiy from difussion movement
$q\gamma^2$	:conditional volatility from pure jump process
q	:jump arrival rate
L	:coefficients of economic factors affecting jump arrival rate

$$\begin{cases} X_t = [1, d_{(R,t)}, d_{(C,t)}]' \\ Y_t = [d_{(R,t)}, d_{(C,t)}]' \\ H_t = [\sqrt{h_t}, \sqrt{q\gamma^2}] \\ \varepsilon_t \tilde{} IID\ N(0,1),\ J \tilde{} N(0,\gamma^2) \\ \mathbf{B} \tilde{} Bernoulli(q),\ q = LX_{t-1} \end{cases}$$

The first model is the Threshold GARCH(1,1) model.(hereafter called TGARCH(1,1)) The return process is decomposed into the drift term, AX_t, and the diffusion term, $h_t^{1/2}\varepsilon_t$. According to previous literature, it is expected that the coefficient vector A, $[A_0, A_R, A_C]$, is significant such that constant drift term, presidency and contraction dummies are able to explain the observed level return pattern. On the other hand, including dummy variables, X_t, in the conditional volatility allows us to analyze the effects of political and business cycle on conditional volatility. The coefficient vector C, $[C_{0,}C_R, C_C]$, represents the effect of lag 1 squared innovation, and the corresponding incremental effects due to political and business cycles. D_0 in the vector D, $[D_{0,}D_R, D_C]$, is the persistence parameter for conditional volatility, h_t. D_R and D_C are incremental effect of political and business cycle indicators on the persistence of h_t.

Model(1): TGARCH

$$\begin{cases} r_t = AX_t + h_t^{1/2}\varepsilon_t \\ h_t = B_0 + CX_{t-1}(r_{t-1} - AX_{t-1})^2 + DX_{t-1}h_{t-1} \end{cases}$$

The second model is the pure GARCH(1,1) model with linear addition of political and economic dummy factors in to volatility of difussion process. (hereafter called GARCH(1,1)-LX) W is the coefficient vector, $[W_1, W_2]$, representing incremental effect on conditional volatility brought by dummy factors. In this model the factors play a drift term effect on the volatility, compared to TGARCH model in which exogenous factors take incremental effect through innovation and lag conditional volatility.

Model(2): GARCH-LX

$$\begin{cases} r_t = AX_t + h_t^{1/2}\varepsilon_t \\ h_t = B_0 + C_0(r_{t-1} - AX_{t-1})^2 + D_0h_{t-1} + WY_{t-1} \end{cases}$$

This model is also modified model from GARCH(1,1), but political and economic factors affect volatility through exponential form, which imposes no additional constraint other than GARCH(1,1). That is to say we don't need to put any additional constraints other than $B_0 > 0$, $C_0 > 0$ and $D_0 > 0$. The model is denoted as GARCH(1,1)-E-X.

Model(3): GARCH-E-X

$$\begin{cases} r_t = AX_t + h_t^{1/2}\varepsilon_t \\ h_t = (B_0 + C_0(r_{t-1} - AX_{t-1})^2 + D_0 h_{t-1}) \exp(WY_{t-1}) \end{cases}$$

EGARCH is proposed by Nelson (1991) to capture volatility asymmetric stylized fact. This fact indicates that volatility tends to be lower while return is positive but higher when return is negative. Therefore parameter θ_0 should be negative so that if $r_t - AX_t < 0$, $g(r_t - AX_t)$ and h_t are comparatively higher than the case when $r_t - AX_t > 0$. Additionally, another appealing advantage for EGARCH model is no need to consider conditions for insuring $h_t > 0$, and the economic factors can be naturally enclosed in original EGARCH setting. Paralleling to GARCH(1,1), we also incorporate political and economic factors addictively into volatility process, and name extended EGARCH model as EGARCH(1,1)-LX.

Model(4): EGARCH-LX

$$\begin{cases} r_t = AX_t + h_t^{1/2}\varepsilon_t \\ \ln(h_t) = B_0 + C_0 g(r_{t-1} - AX_{t-1}) + D_0 \ln(h_{t-1}) + WY_{t-1} \\ g(r_t - AX_t) = \theta_0(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

Following previous model, we furtherly decompose $g(r_t - AX_t)$ to explore effect of exogenous factors on the degree of assymetry. θ represents coefficient vector, $[\theta_0, \theta_1, \theta_2]$. θ_0 is constant asymmetric effect from return innovation, and θ_1 and θ_2 correpond to incremental effect brought by exogenous factors on volatility asymmetry. We use E-X-GARCH(1,1)-LX to denote model (5).

Model(5): E-X-GARCH-LX

$$\begin{cases} r_t = AX_t + h_t^{1/2}\varepsilon_t \\ \ln(h_t) = B_0 + C_0 g(r_{t-1} - AX_{t-1}) + D_0 \ln(h_{t-1}) + WY_{t-1} \\ g(r_t - AX_t) = \theta X_{t-1}(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

In addition to applying linear addition form for incorporating exogenous factors into log conditional volatility process, we also apply threshold model concept into EGARCH model to investigate if political and economic factors have incremental effect on volatility asymmetry and volatility clustering together. The model is denoted as E-X-TGARCH(1,1).

Model(6): E-X-TGARCH

$$\begin{cases} r_t = AX_t + h_t^{1/2}\varepsilon_t \\ \ln(h_t) = B_0 + CX_{t-1} g(r_{t-1} - AX_{t-1}) + DX_{t-1} \ln(h_{t-1}) \\ g(r_t - AX_t) = \theta X_{t-1}(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

Model (7) to (12) were modified from model 1 to 6. Those models are called GARCH-typed in Mean models and allow conditional volatility to explain the return. We put $h_t^{1/2}$ as explanatory factor in return drift process. Alternatively speaking, the return process is now under the control of exogenous factors and risk uncertainty.

In Model (7), ϕ represents the risk premium on conditional volatility. By economic intuition, ϕ is expected to be positive because bearing higher risk requires higher return for compensation. Model (7) is TGARCH in Mean model. (hereafter called TGARCH-M).

Model(7): TGARCH-M

$$\begin{cases} r_t = AX_t + \phi h_t^{1/2} + h_t^{1/2}\varepsilon_t \\ h_t = B_0 + CX_{t-1}(r_{t-1} - AX_{t-1} - \phi h_{t-1}^{1/2})^2 + DX_{t-1} h_{t-1} \end{cases}$$

Paralleling to what we do to model (7), we modify prototype model (2) to (6) into model (8) to (12) and list in the following. Those models are denoted with letter M at the end.

Model(8): GARCH-LX-M

$$\begin{cases} r_t = AX_t + \phi h_t^{1/2} + h_t^{1/2}\varepsilon_t \\ h_t = B_0 + C_0(r_{t-1} - AX_{t-1})^2 + D_0 h_{t-1} + WY_{t-1} \end{cases}$$

Model(9): GARCH-EY-M

$$\begin{cases} r_t = AX_t + \phi h_t^{1/2} + h_t^{1/2}\varepsilon_t \\ h_t = (B_0 + C_0(r_{t-1} - AX_{t-1})^2 + D_0 h_{t-1}) \exp(WY_{t-1}) \end{cases}$$

Model(10): EGARCH-LX-M

$$\begin{cases} r_t = AX_t + \phi h_t^{1/2} + h_t^{1/2}\varepsilon_t \\ \ln(h_t) = B_0 + C_0 g(r_{t-1} - AX_{t-1}) + D_0 \ln(h_{t-1}) + WY_{t-1} \\ g(r_t - AX_t) = \theta_0(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

Model(11): E-X-GARCH-LX-M

$$\begin{cases} r_t = AX_t + \phi h_t^{1/2} + h_t^{1/2}\varepsilon_t \\ \ln(h_t) = B_0 + Cg(r_{t-1} - AX_{t-1}) + D \ln(h_{t-1}) + WY_{t-1} \\ g(r_t - AX_t) = \theta X_{t-1}(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

Model(12): E-X-TGARCH-M Model

$$\begin{cases} r_t = AX_t + \phi h_t^{1/2} + h_t^{1/2}\varepsilon_t \\ \ln(h_t) = B_0 + CX_{t-1}g(r_{t-1} - AX_{t-1}) + DX_{t-1}\ln(h_{t-1}) \\ g(r_t - AX_t) = \theta X_{t-1}(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

Starting from model (13), we add Bernoulli type jump into model (7), (8), (9), (10), (11) and (12) and label new model as (13), (14), (15), (16), (17) and (18). The Bernoulli type jump with ARCH difussion process is proposed by Das (2002), and here we extend it to garch type jump process. Take Model (13) for example, the notation, TGARCH-BJX-M, represents the model is TGARCH difussion model with conditional volatility to explain mean return and with Bernoulli jump incorporating exogenous factors, X_t, into jump pprobability, q. We assume jump size, γ^2, is constant over time, but jump arrival probability, q, is the function of exogenous factors. Alternatively speaking, models from (13) to (18) are able to test the presidency effect and business cycle effect on return processes of mean, diffusion volatility and jump together, while EGARCH-typed models with jump also include the test of exogenous facotrs' effect on volatility asymmetry. The advantage of adding jump process will be discussed in the next section.

Model(13): TGARCH-BJ-X-M

$$\begin{cases} r_t = AX_t + \phi H_t + h_t^{1/2}\varepsilon_t + \mathbf{B}J \\ h_t = B_0 + CX_{t-1}(r_{t-1} - AX_{t-1} - \phi H_t)^2 + DX_{t-1}h_{t-1} \end{cases}$$

Model(14): GARCH-LX Model

$$\begin{cases} r_t = AX_t + \phi H_t^{1/2} + h_t^{1/2}\varepsilon_t + \mathbf{B}J \\ h_t = B_0 + C_0(r_{t-1} - AX_{t-1})^2 + D_0 h_{t-1} + WY_{t-1} \end{cases}$$

Model(15): GARCH-EY-BJ-X-M

$$\begin{cases} r_t = AX_t + \phi H_t + h_t^{1/2}\varepsilon_t + \mathbf{B}J \\ h_t = (B_0 + C_0(r_{t-1} - AX_{t-1} - \phi H_t))^2 + D_0 h_{t-1}) * \exp(WY_{t-1}) \end{cases}$$

Model(16): E-X-GARCH-LX-BJ-X-M model

$$\begin{cases} r_t = AX_t + \phi H_t + h_t^{1/2}\varepsilon_t + \mathbf{B}J \\ \ln(h_t) = B_0 + CX_t g(r_{t-1} - AX_{t-1})^2 + DX_t \ln(h_{t-1}) + WY_{t-1} \\ g(r_t - AX_t) = (r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

Model(17): E-X-GARCH-LX-BJ-X-M

$$\begin{cases} r_t = AX_t + \phi H_t + h_t^{1/2}\varepsilon_t + \mathbf{B}J \\ \ln(h_t) = B_0 + Cg(r_{t-1} - AX_{t-1})^2 + D\ln(h_{t-1}) + WY_{t-1} \\ g(r_t - AX_t) = \theta Y_t(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

Model(18): E-X-TGARCH-BJ-X-M

$$\begin{cases} r_t = AX_t + \phi H_t + h_t^{1/2}\varepsilon_t + \mathbf{B}J \\ \ln(h_t) = B_0 + CX_{t-1} g(r_{t-1} - AX_{t-1})^2 + DX_{t-1}\ln(h_{t-1}) \\ g(r_t - AX_t) = \theta Y_t(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \end{cases}$$

3.3 Unconditional Moment and Impulsive Function Analysis

Unconditional moments can be considered as characteristics of distribution in the long run. Checking unconditional movements generated by each dynamic process enable us to understand linkage between distribution and model parameters. Moreover, going through impulsive function analysis let us have clear insight about how volatility is affected by exogenous factors in each model and have comprehensive unerstanding what kind of economic factors can fit into models. The first, second and fourth center moments and impulsive function for each model follow.

(1)Model 1:T-GARCH(1,1)

$$\begin{cases} Var(r_t - AX_t) = \frac{B_0}{(1-CX_{t-1}-DX_{t-1})} \\ E((r_t - AX_t)^4) = \frac{3B_0^2(1+CX_t+DX_t)}{(1-CX_{t-1}-DX_{t-1})(1-(DX_{t-1})^2-2(CX_{t-1})(DX_{t-1})-3(CX_{t-1})^2)} \\ \\ \frac{\Delta h_t}{\Delta D_{(R,t-1)}} = C_1(r_{t-1} - AX_{t-1})^2 + 2CX_{t-1}(-A_1)(r_{t-1} - A_1X_{t-1}) + D_1h_{t-1} \\ \frac{\Delta^2 h_t}{\Delta D^2_{(R,t-1)}} = C_1 2(-A_1)(r_{t-1} - AX_{t-1}) - 2C_1A_1(r_{t-1} - A_1X_{t-1}) + 2CX_{t-1}A_1^2 \\ \frac{\Delta h_{t+n}}{\Delta D_{(R,t-1)}} = D_1^n[C_1(r_{t-1} - AX_{t-1})^2 + 2CX_{t-1}(-A_1)(r_{t-1} - A_1X_{t-1}) + D_1h_{t-1}] \end{cases}$$

(2)Model 2:GARCH(1,1)-LX

$$\begin{cases} Var(r_t - AX_t) = \frac{B_0+WY_{t-1}}{(1-C_0-D_0)} \\ E((r_t - AX_t)^4) = \frac{3(B_0+W)^2(1+C_0+D_0)}{(1-C_0-D_0)(1-D_0^2-2C_0D_0-3C_0{}^2)} \\ \\ \frac{\Delta h_t}{\Delta D_{(R,t-1)}} = W_1 \\ \frac{\Delta^2 h_t}{\Delta D^2_{(R,t-1)}} = 0 \\ \frac{\Delta h_{t+n}}{\Delta D_{(R,t-1)}} = D_1^n W_1 \end{cases}$$

(3)Model 3:GARCH(1,1)-EX

$$
\begin{cases}
Var(r_t - AX_t) = \frac{B_0 + WY_{t-1}}{1-(C_0+D_0)\exp(WY_{t-1})} \\
E((r_t - AX_t)^4) = \frac{3(B_0+F)^2(1+C_0+D_0)}{[1-(C_0+D_0)\exp(WY_{t-1})][1-(D_0^2+2C_0D_0+3C_0{}^2)\exp(2WY_{t-1})]} \\
\\
\\
\frac{\Delta h_t}{\Delta D_{(R,t-1)}} = W_1 h_t \\
\frac{\Delta^2 h_t}{\Delta D^2_{(R,t-1)}} = W_1^2 h_t \\
\frac{\Delta h_{t+n}}{\Delta D_{(R,t-1)}} = W_1 D_0^n \exp(WY_{t-1}) h_t
\end{cases}
$$

(4)Model 4:EGARCH(1,1)-LX

$$
\begin{cases}
Var(r_t - AX_t) = \exp[\frac{B_0}{1-D_0}]E(\exp[\frac{WY_t}{1-D_0}])E(\exp[\frac{C_0 g(r_{t-1}-AX_{t-1})}{1-D_0}]) \\
E((r_t - AX_t)^4) = \exp[\frac{2B_0}{1-D_0}]E(\exp[\frac{2WY_{t-1}}{1-D_0}])E(\exp[\frac{2C_0 g(r_{t-1}-AX_{t-1})}{1-D_0}]) \\
\\
\\
\frac{\Delta h_t}{\Delta D_{(R,t-1)}} = (-C_0 g' + W_1) h_t \\
\frac{\Delta^2 h_t}{\Delta D^2_{(R,t-1)}} = (-C_0 g' + W_1) \frac{\Delta h_t}{\Delta D_{(R,t-1)}} = (-C_0 g' + W_1)^2 h_t \\
\frac{\Delta h_{t+n}}{\Delta D_{(R,t-1)}} = (-C_0 g' + W_1) D_0^n (WY_{t-1})^n h_t \\
g' = \frac{\Delta[\theta(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|)]}{\Delta D_{(R,t-1)}}
\end{cases}
$$

(5)Model 5:E-X-GARCH(1,1)-LX

$$
\begin{cases}
Var(r_t - AX_t) = \exp[\frac{B_0}{1-D_0}]E(\exp[\frac{WY_t}{1-D_0}])E(\exp[\frac{C_0 g(r_{t-1}-AX_{t-1})}{1-D_0}]) \\
E((r_t - AX_t)^4) = \exp[\frac{2B_0}{1-D_0}]E(\exp[\frac{2WY_{t-1}}{1-D_0}])E(\exp[\frac{2C_0 g(r_{t-1}-AX_{t-1})}{1-D_0}]) \\
\\
\\
\frac{\Delta h_t}{\Delta D_{(R,t-1)}} = (-C_0 g' + W_1) h_t \\
\frac{\Delta^2 h_t}{\Delta D^2_{(R,t-1)}} = (-C_0 g' + W_1) \frac{\Delta h_t}{\Delta D_{(R,t-1)}} = (-C_0 g' + W_1)^2 h_t \\
\frac{\Delta h_{t+n}}{\Delta D_{(R,t-1)}} = (-C_0 g' + W_1) D_0^n (WY_{t-1})^n h_t \\
g' = \frac{\Delta[\theta X_{t-1}(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|)]}{\Delta D_{(R,t-1)}}
\end{cases}
$$

(6)Model 6:E-X-TGARCH(1,1)

$$
\begin{cases}
Var(r_t - AX_t) = \exp[\frac{B_0}{1-DX_t}]E(\exp[\frac{CX_t g(r_{t-1}-AX_{t-1})}{1-DX_t}]) \\
E((r_t - AX_t)^4) = \exp[\frac{2B_0}{1-DX_t}]E(\exp[\frac{2CX_t g(r_{t-1}-AX_{t-1})}{1-DX_t}]) \\
\\
\\
\frac{\Delta h_t}{\Delta D_{(R,t-1)}} = (C_1 g + CX_{t-1}g^{'} + D_1 \ln h_{t-1})h_t \\
\frac{\Delta^2 h_t}{\Delta D^2_{(R,t-1)}} = (C_1 g + CX_{t-1}g^{'} + D_1 \ln h_{t-1})\frac{\Delta h_t}{\Delta D_{(R,t-1)}} = (C_1 g + CX_{t-1}g^{'} + D_1 \ln h_{t-1})^2 h_t \\
\frac{\Delta h_{t+n}}{\Delta D_{(R,t-1)}} = (C_1 g + CX_{t-1}g^{'} + D_1 \ln h_{t-1})D_0^n h_t \\
g = \theta X_{t-1}(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|) \\
g' = \frac{\Delta[\theta X_{t-1}(r_t - AX_t) + (|r_t - AX_t| - E|r_t - AX_t|)]}{\Delta D_{(R,t-1)}}
\end{cases}
$$

For the GARCH-family in mean models, including Model(7), (8), (9), (10), (11) and (12), the center higher order movements are the same as model (1) to (6).

As for the GARCH-typed and EGARCH-typed in Mean with jump models, the form of unconditional second order and fourth order center movement are very comlicated and hard to gain intuition inside so we will list those complicated equation in formal paper. However, we still can gain some intuition from a general form of unconditional variance, unconditional kurtosis and impulsive function. For model(13), (14), (15), (16), (17) and (18), their second center movement, fourth center order movement and impulsive function are,[2]

$$
\begin{cases}
Var(UR_t) = E[(h_t^{1/2}\varepsilon_t + BJ)^2] = E(h_t) + q\gamma^2, UR_t = r_t - AX_t - \phi H_t \\
E(UR_t^4) = E[(h_t^{1/2}\varepsilon_t + BJ)^4] = 3E(h_t^2) + 3\gamma^2 q[2E(h_t) + \gamma^2] \\
\\
\\
\frac{\Delta Var(r_t|I_{t-1})}{\Delta D_{(R,t-1)}} = \frac{\Delta h_t}{\Delta D_{R,t-1}} + \frac{\Delta q r^2}{\Delta D_{R,t-1}} \\
\frac{\Delta^2 Var(r_t|I_{t-1})}{\Delta D^2_{(R,t-1)}} = \frac{\Delta^2 h_t}{\Delta D^2_{R,t-1}} \\
\frac{\Delta Var(r_{t+n}|I_{t-1})}{\Delta D_{(R,t-1)}} = \frac{\Delta Var(r_{t+n}|I_{t-1})}{\Delta h_{t+n}} \frac{\Delta h_{t+n}}{\Delta h_{t+n-1}} \cdots \frac{\Delta h_{t+n}}{\Delta D_{(R,t-1)}} = (DX_{t-1})^n \frac{\Delta Var(r_t|I_{t-1})}{\Delta D_{(R,t-1)}}
\end{cases}
$$

[2]More details are included in Chapter 3 Appendix.

$E(h_t)$ and $q\gamma^2$ are the unconditional volatility brought by difusion process and jump process respectively. For pure difussion model like GARCH-typed model, the conditional volatility is $h_{t,garch}$ and conditional kurtosis is $3h^2_{t,garch}$, totally bounded by $h_{t,garch}$. However, for GARCH-typed model with jump, the conditional volatility is $h_{t,garch\_J} + q\gamma^2$ and kurtosis is $3h^2_{t,garch\_J} + 3q^2r^4$. Becuase we do not know the parameters before calibration, we can't not compare conditional volatility and conditional kurtosis directly. But the jump arrival probability, q, is the function of exogenous factors, so jump models have more parameters and flexibility to capture the dynamic conditional kurtosis.

The unconditional volatility for pure difussion and difussion with jump models are $\widehat{Var}(UR_t) = E(h_{t,garch})$ and $\widetilde{Var}(UR_t) = E(h_{t,garch\_J}) + q\gamma^2$ and the unconditional kurtosis of GARGH(1,1) is

$$E_{GARCH(1,1)}(UR_t^4) = 3\widehat{Var}(UR_t)^2[\frac{(1-(\widehat{D}X_{t-1})^2-2\widehat{D}X_{t-1}\widehat{C}X_{t-1}-(\widehat{C}X_{t-1})^2}{(1-(\widehat{D}X_{t-1})^2-2\widehat{D}X_{t-1}\widehat{C}X_{t-1}-(3\widehat{C}X_{t-1})^2}] > 3\widehat{Var}(UR_t)^2,$$

and kurtosis of models with jump is

$$E_{Jump}(UR_t^4) = 3E(h^2_{t,garch\_J}) + 3\gamma^2 q[2E(h_{t,garch\_J}) + \gamma^2] = 3\widetilde{Var}(UR_t)^2 + 3q\gamma^4(1+q) > 3(\widetilde{Var}(UR_t)^2).$$[3].

Thus we can set up the sufficient and necessary conditions for jump model to have fatter tails given the unconditional volatility is less, equal or greater than that of pure diffusion model.

In summary, from the model analysis, we conclude that GARCH-typed and EGARCH-typed in Mean with jump models have stronger power and flexibility in describing the conditional and unconditional volatility and kurtosis such that it is not suprised if performance of difussion with jump models is better than

[3]Assume $B_0, C, D > 0$ to insure $E(h)^2 = E(h^2)$.

pure difussion models. The estimation results in section 5 will tell us how mcuh difussion with jump models outperform pure difussion models.

3.4 Data

Our data sample starts from 1/21/1957, the inauguration date of President Eisenhower, to 12/31/2010, and the underlying data are CRSP value weighted daily returns with dividend included. There are eleven presidents, six of whom are Republican and five are Democratic. To control the effect of business cycle, we include the contraction indicator, defined by NBER, as one of explaining factors. Panel A of Table 3-4-1 contains the presidency information and descriptive statistics for each presidency period. The averaged daily return is 0.042%, and the daily standard deviation is 0.953% for the total sample size of 50 years. Panel B compares the return statistics for different parties in White House. Democratic presidencies have higher averaged daily return than Republican presidencies (0.06% vs. 0.03%), and the standard deviation of daily return during Democratic administration is a bit lower than that during Republican administration (0.886% vs. 0.995%). The result is robust if the sample was divided into subsample of (1957-2001, 1957-1989 and 1989-2010). Another interesting stylised fact in Panel B is that the kurtosis is much larger during Republican presidencies both in the whole sample and sub-samples.

3.5 Empirical Results

The estimation results of pure difussion models, difussion models in mean, and difussion model in mean with jump are demonstrated in Table 3-5-1, Table 3-5-2 and Table 3-5-3. First, all of 18 models show that presidency factor affects conditional volatility, as we can observe either $C_1, D_{1,}, W_1$ or L_1 is significant in explaining the conditional volatility. Moreover, the conditional volatiltiy on difussion process does play significant role to explain the return in GARCH-tpyed models and marginal significant role in EGARCH-typed models, while we don't find volatility brought by jump has significant power to explain return except. If we focus on jump models, there are strong evidences showing presidency effect increases jump probability while it decrease the persistence of volatility clustering. Conversely, contraction generally has no incremental effect on jump probability or on volatility of difussion process except GARCH(1,1)-LX-BJ-M model. As for the presidency and business cycle effect on mean return, after controlling the volatility risk premium, the political factor has negative and marginal significant effect on return. We also have no evidence showing that presidency and contraction have effect on volatility asymmetry.

From the perspective of model performance, GARCH-typed in Mean with jump models outperforms GARCH-typed in Mean models and GARCH-typed models. The log-likelihood increase at least more than 100 for jump models. After adjusting the number of parameters, the best models among 18 models is E-GARCH(1,1)-LX-BJ-M model. If we only compare within groups of difussion, difussion in mean and difussion with jump, EGACH-typed models performe better than GARCH-typed models by higher loglikelihood. On the other hand, using T-GARCH model to incorporate political and business cycle factors beats the GARCH-typed models with linear addition or with expoential form which incorporates exogenous factors.

One of the interesting results among models is risk premium on difussion volatility. It can be observaed that the premium on GARCH-typed in mean and GARCH-typed in mean with jump is about 0.12 and 0.1, while the premium is about 0.04 or 0.05 for EGARCH-typed in mean and EGARCH-typed in mean with jump models. Alternatively speaking, in grach typed model enduring 1% conditional volatilty requires 0.1% or 0.12% return and 0.04% or 0.05% return for EGARCH-typed models. The premium, 0.1, on conditional volatility can be considered as the premium bearing short run risk while the Sharpe ratio on US indexes, based on unconditional volatility, is generally about 0.4 and can be considered as the premium to bear long run risk. However, estimated political and economic effect delivered by different models to explain mean return movement seems not consistent. For pure difussion in mean model contraction has significant negative effect on mean return, but significance ceases when jump is included into the models. On the other hand, the political factors are significant in GARCH-typed difussion models, but it is not the case with EGARCH difussion models. After jump is incorporated, presidency effect is marginally significant for all models with jump except E-X-GARCH-LX-BJ-M model.

The constant jump arrival probability also differs among models. Daily constant arrival probability ranges from 0.05% to 3.3% and political factor increase arrival probability by from 0.15% to 0.42% with t-stat greater than 2, while in most models business cycle factor demonstrates insignificant effect on jump arrival probability. After integrating jump process into the model, we also observe a decrease in the persistence of conditional variance of return in GARCH equation. However, it is amazing to find that risk premium on the jumps is insignificant. The jump risk premium, ϕ_2, ranges from -2.38 to 46.21 with t-stat lower than 1.64 for most models. If we check the arrival probability conditional on political factors, the probability is from 0.31% to 3.63%, which implies that expected jump frequency about 0.8 to 9.1 times a year. Additionally, estimated jump size is from 0.000331 to 0.00049, which implies jump size is from 1.8% to 2.21% in return. Becuase of the nature of the data, if jump size is defined at higher threshold, the arrival probability of the jump is certainly lower. We can observe this negative pattern in arrival probability and jump sized estimated in the model. Though we conclude that political factor affects conditional volatility, the presidency effect does not necessarily increase the conditional volatility, and we will discuss this issue in the next section, model analysis.

3.6 Model analysis

Here, we pick TGARCH(1,1)-BJ-X for advanced model analysis, which can also be applied to other models. The conditional return of TGARCH(1,1) follows a normal mixture distribution:

$$f(r_t|I_{t-1}) = (1-q)\exp(\tfrac{-(r_t-AX_t-\phi H_t)^2}{2h_t})\tfrac{1}{\sqrt{2\pi h_t}} + q\exp(\tfrac{-(r_t-AX_t-\phi H_t)^2}{2(h_t+\gamma^2)})\tfrac{1}{\sqrt{2\pi(h_t+\gamma^2)}}$$

Model estimation can then be based on MLE(Maximum Likelihood Estimation) with the following log-likelihood function:

$$\underset{\theta=\{A,B_0,C,D,\phi,\gamma\}}{Max}\sum_{t=1}^{T}\log(f(r_t|I_{t-1}))$$

The whole conditional variance, which can be decomposed into combinations from diffusion and jump processes, is $Var(r_t|I_{t-1}) = h_t + q\gamma^2$. The effect of presidency dummy variable on conditional variance is

$$\begin{cases} \frac{\Delta Var(r_t|I_{t-1})}{\Delta D_{R,t-1}} = \frac{\Delta h_t}{\Delta D_{R,t-1}} + \frac{\Delta qr^2}{\Delta D_{R,t-1}} = C_1(r_{t-1} - AX_{t-1} - \phi H_{t-1})^2 + D_1 h_{t-1} + L_1\gamma^2 \\ = -0.0168(UR_{t-1})^2 + 0.0021 * (2.21\%)^2, (3-1) \\ \frac{\Delta Var(r_{t+k}|I_{t-1})}{\Delta D_{R,t-1}} = \frac{\Delta h_{t+k}}{\Delta h_{t+k-1}} \frac{\Delta h_{t+k-1}}{\Delta h_{t+k-2}} .. \frac{\Delta h_t}{\Delta D_{R,t-1}} = (D_0 + D_1 + D_2)^k [C_1(r_{t-1} - AX_{t-1} - \phi H_{t-1})^2] \\ = 0.89^k[-0.0168 * (UR_{t-1})^2], (3-2) \end{cases},$$

By estimation result, $D_1 = 0, D_2 = 0,$ and we let $UR_{t-1} = r_{t-1} - AX_{t-1} - \phi H_{t-1}$.

The term $(r_{t-1} - AX_{t-1} - \phi H_{t-1})$ can be considered as unexpected return and is denoted by UR_{t-1}. Based on the model, we find that if the Republicant is in presidency, the conditional volatility, h_t, increases because of jump intensity raised by presidency effect, while the effect by past unexpected innovation is negative. In other words, if the unexpected innovation falls out of interval $(-\sqrt{\frac{0.0021*(2.21\%)^2}{0.0168}}, +\sqrt{\frac{0.0021*(2.21\%)^2}{0.0168}})$, the Republicant presidency lowers the conditional volatility. Conversely, if unexpected innovation lies inside the interval, the Republicant presidency raises the conditional volatility.

We also derive the unconditional variance and unconditional kurtosis and are able to compare them with those of other models.

$$E[(r_{t-1} - AX_{t-1} - \phi H_{t-1})^2] = \frac{B_0 + qr^2}{1 - B_1 - B_2}$$
$$E[(r_{t-1} - AX_{t-1} - \phi H_{t-1})^4] = \frac{3B_0(1 + B_1 + B_2) + 6B_0 qr^2(1 + B_1) + 3qr^4(1 + 2q - B_0)}{(1 - B_1 - B_2)(1 - B_2^2 - 2B_1 B_2 - 3B_1^2)}$$

3.7 Robustness analysis

In this section, we alter presidency periods artificially and use TGARCH(1,1)-BJ-X model to test the robustness of the significance of the political factor. First, we set the political dummy variable 1 in odd dates and 0 in even dates. (hereafter called R1 experimental test.) Second, political dummy is set to 1 in odd months and 0 in even months (hereafter called R2 experimental test.) Finally we extend the real setting each of the actual the Republican presidency so that it overlaps with the following Democratic presidency and check if the significance of the coefficient decreases. Specifically, we first extend each Republican presidency by 1 year longer than actual period and then gradually extend it to a total of 6 years longer. If political parties really can explain returns, we will expect a diminishing significance of presidency effect as we expand the Republican presidency artificially.

Table 3-7-1 shows the estimation results. In experimental test R1 and R2, the political effects in drift, diffusion and jump processes are all insignificant. If we try to extend the Republican presidency artificially to over 4 years, the presidency dummy becames insignificant in each process.

3.8 Conclusion

From economic perspective, the diffusion and jump processes represent very different inflow of information. While diffusion process represents a relatively slow shift of return dynamics, incorporating jumps can induce a sudden big change in return. Our preliminary result shows that Republican presidency effect on volatility mainly goes through jump process, but at the same time it also decreases the effect of unexpected innovation on conditional volatility. Presidency effect only demonstrates its significant to have negative impact on mean return after risk premium is incorporate into return. However, the result shows that the political effect is significant at marginal level or insignificant in some models. Our TGARCH(1,1)-BJ-X model analysis conclude that if unexpected innovation is huge, the conditional volatility is relatively lower comparing to Democratic presidency. As the previous literature documented an excessive negative return during Republican presidency, our results confirm the previous conclusion after further controlling for business cycle, diffusion volatility risk premium and jump risk premium.

The paper include the preliminary results of 18 models and we are still working on related issues. First we plan to use simulation to draw the return distribution diagram for each model and compare the return distributions among those models. Secondly, we will draw the impulsive function of each model to check political effect on conditional volatility for multiple periods. Third, we will use CRSP 10 decimal capital size index returns to investigate politicla cycle effect on stocks in different cap. Finally, as volatility risk premium is incorporated into return, we derive a risk-neutral process and thus are able to price the political uncertainty.

CHAPTER 4

VOLATILITY UNCERTAINTY, TIME DECAY, AND OPTION BID-ASK SPREADS

4.1 Introduction

While much of the literature is focused on investigating trading prices in the options market, few studies have examined option quoting prices. Similarly, the behavior of market makers in the stock market has been studied extensively, but only a few works have specifically examined market making behavior in the derivatives market. The sparsity of related studies can be attributed to the low availability of high frequency data that includes quoting information; therefore, compared to the studies about market making behavior in the stock market, research related to the bid-ask volatility spread in the option market has yet to be well explored.

Among the studies investigating quotation prices in the options market, the majority focus on price discovery of the underlying asset. For example, Muravyev, Pearson and Broussard (2012), Hsieh, Lee and Yuan (2008), and Holowczak, Simaan, and Wu (2007) employed put-call parity to examine underlying price information contained in options trading. On the other hand, Chakravarty, Huseying, and Mayhew (2004) extended the work of Hasbrouck (1995) to examine the information contribution from the options market price discovery in the stock market. They also concluded that the volume ratio and spread ratio are significant in explaining information shares from the options market to the stock market.[1] In addition, Easley, O'Hara and Srinivas (1998) found that option

[1] Chakravarty, Huseying, and Mayhew (2004) defined a volume ratio as the ratio of option

volumes lead stock price changes, and Pan and Poteshman (2006) showed that a put-call ratio constructed using option volume has predictive power for the weekly return of its underlying asset.

In contrast to the aforementioned research, our paper focuses on the volatility spread *per se* and constructs a micro-structure model to explain the occurrence of a periodic pattern. The pattern refers to an increasing spread in terms of implied volatility, which was previously attributed to a maturity effect. Chong, Ding and Tan (2003), henceforth, CDT (2003), wrote the first paper addressing the maturity effect in the bid-ask volatility spread, the calculation of which is based on the Black-Scholes-Merton model. They claim there are two risks related to maturity. First, higher gamma risk for short term options drives the market to demand a higher volatility spread for compensation.[1] Second, because they investigated OTC currency options, the trading involves counter party credit (default) risk which increases when the contract duration increases. Through their empirical work, CDT (2003) concluded that the implied volatility spread increases as maturity decreases, suggesting gamma risk dominates counterparty risk.

Following CDT (2003), Cao and Wei (2010) and Wei and Zheng (2010) used Ivy DB's OptionMetrics data to investigate the impact of trading activities on the liquidity of stock options. Instead of adopting a volatility measure, those two papers utilized the spread defined in terms of a dollar ratio, dividing the dollar spread by the midpoint of the bid and ask prices. Furthermore, although CW (2010) did not investigate maturity effect in their regression models, WZ (2010)

volume to stock volume and defined a spread ratio as the ratio of effective option spread to stock spread.

[1] CDT (2003) considered gamma risk as market risk, and they did not provide an explanation for the source of gamma risk.

also found the existence of the maturity effect for dollar ratio measure. However, this liquidity measure has problems. The first shortcoming is its weakness to link the spread to options pricing factors, such as volatility. Therefore, interpreting the quoted spread is difficult. Second, the pattern of this ratio measure can be the natural result of pricing models. For example, given fixed parameters and using the Black-Scholes-Merton model, the dollar ratio spread exhibits an upward trend as time approaches to expiration. This trend is generated naturally by the non-linear relationship between the pricing factors and options prices so that any empirical study, investigating options quotation spread in dollar ratio measure, needs to control upward increasing trend in regression models.[2]

In this Chapter, we provide empirical evidence confirming the "maturity effect" by using two different model-free IMV volatility estimations. Furthermore, we explain the observed pattern, after introducing the concept of disparity between the time decay of derivatives and time decay of required hedging risk premium. Along with our theoretical work, we provide empirical works testing our theoretical findings. The paper is organized as follows. In section 4.2, the volatility spread pattern in the option market is presented. Section 4.3 contains details of the model setup, equilibrium, and propositions. We separate an analysis of volatility uncertainty in section 4.4, because volatility uncertainty needs to be defined before we connect it to quoting spread. In section 4.5, descriptive statistics are summarized. Section 4.6 provides econometric tests on propositions listed in the previous sections. Finally conclusions as well as important implications of this research are presented in section 4.7.

[2] An numerical example is given in Figure 4-1-1.

4.2 Stylized Facts

Using implied volatility is usually considered the most valid way to study the price of options. Until 2003, the CBOE still used the Black-Scholes-Merton model to derive IMV. However, criticism had risen as more and more pricing models, which assume more realistic assumptions, were developed. The CBOE thus employed a model-free IMV method for VIX index calculation in 2003. Now there are two methods published for model-free IMV calculation. The first estimation method was proposed by Carr and Madan (1998), henceforth, CM.[3] It utilized only out of money (OTM) call and put options for implied volatility calculation and was later slightly modified by the CBOE to become the VIX index formula. This paper follows the CBOE modification to calculate CM IMV. While the CBOE further weight-averages the first and second nearest term contracts to construct the VIX index,[4] we calculate implied volatility for only the nearest month contract because our research is investigating the volatility spread pattern over the time period of a contract. The second method is suggested by Jiang and Tian (2005), henceforth, JT, who used all call options to estimate model-free implied volatility.

In Figure 4-2-1, we plot CM IMV, JT IMV and high frequency intraday realized volatility (RV) together.[5] The underlying asset of those 3 volatility measures is the Taiwan weight averaged index. The index option is traded at the Taiwan Futures Exchange and is the European-style option. Our data sample period ranges from December 20, 2007 to July 21, 2010. During the sample pe-

[3]The formulas of CM IMV, JT IMV and original CM(1998) estimation are listed in Appendix B.

[4]Becuase the calculation of CBOE VIX index is weight-averaged implied volatility of two nearest term contracts, the VIX index is commonly considered an expected volatility for the next 30 days.

[5]The calculation of RV is defined in the section 6, Empirical Work.

riod, the number of daily observations is 630. For a given time, t, IMV can be considered a forward looking and expected volatility for future time period $T-t$, where T is the expiration date of the option, but RV is realized volatility occurring for the past data window $[t-(T-t), t]$. If volatility follows the Markov and diffusion process, then RV defined over $[t-(T-t), t]$ is an unbiased estimator for future volatility over time period $T-t$. As can be seen, three volatility indexes move closely with each other and reached the peak, above 70%, during the 2008 financial crisis. The related descriptive statistics are listed in Table 4-5-1 and will be discussed in a later section.

Figure 4-2-2 and 4-2-3 each contains two charts. The top and bottom charts represent the volatility spread based on the CM and JT IMV measures respectively. After demonstrating volatility spread in Figure 4-2-2, we divide volatility spread by implied volatility and show spread in percentage of IMV in Figure 4-2-3. In both figures, the vertical dashed lines represent expiration dates, and the circle sign denotes the volatility spread in Figure 4-2-2 and percentage spread in Figure 4-2-3 for 30 consecutive nearest-term contracts which have 18 to 25 days to maturity. We also plot IMV with the plus symbol for reader's reference. As shown in Figure 4-2-2, volatility spreads of both IMV measures enlarge with increasing magnitude when time approaches expiration date. Though quotations in different IMV measures display the same pattern, spread widths are quite different for two measures. The spread of CM IMV measure starts approximately at 0.2% and finally moves up to more than 0.5% a few days before expiration.

However, spread based on JT IMV measure generally starts above 1% and reaches more than 10% at the last few trading days of a contract.[6] No matter how

[6]CDT(2003) investigated ATM options of different currencies. They show that the volatility spread starts from $2\%\tilde{}6\%$ for options with 1 year TTM, increases gradually untill the last month, enlarges rapidly during the last month and finally reaches 8-16%.

implied volatility changes, the mentioned volatility pattern occurs for every contract and obviously dominates any movement which other factors could result in. Following Figure 4-2-2, we further investigate if the pattern remains for the percentage spread in Figure 4-2-3. The percentage spread, which employs CM IMV measure, in the top chart starts commonly about 0.7% when time to maturity is still a month and increases to over 10% at the final few days of a contract, while the percentage spread by JT IMV measurement in the next chart begins at about 2% and finally goes up to more than 40%. Moreover, we also can observe convexity for pattern of both volatility and percentage spreads.[7]

Because calculation of model-free IMV requires information on contracts over numerous strikes, we are not able to show spread pattern for a specific strike. Given that JT IMV calculation requires prices of all call options over all different strikes and that CM IMV calculation includes prices of all OTM options, the pattern must happen extensively over contracts of different strikes. In Appendix C-35, we also modify CM formula so that we can extract information only from all ITM options. Subsequently we can show that volatility spread based on the prices of ITM options also demonstrates a similar pattern in Figure 4-2-4 and 4-2-5. Thus, this observed pattern is not limited to a specific strike contract. Moreover, because CDT (2003) used currency options traded over the counter in Singapore and documented the maturity effect, it is quite implausible that the previously mentioned patterns are due to regulations; (e.g., minimum tick). We also do not think the pattern is due to the decreasing participation of traders, for it cannot account for why spread starts to enlarge in weeks with heavy transactions.

[7] We also display the spread cross all contracts in the form of panel data in Figure 4-2-6 and 4-2-7.

Volatility has been considered the most important factor in options pricing; therefore, many researchers think of volatility bid-ask spread as either a good indication containing volatility information or an indication containing the information of underlying return. As our graphs strongly imply that the maturity effect dominates factors that in any other case could affect quoting spread, the observed stylized facts point out the challenge for studies on related topics. To verify the patterns thoroughly, our research validates the stylized facts through constructing a theoretical model which explains the sources of the maturity effect and uses econometric methods to test our theoretical findings.

4.3 Model

In this section, we present a model describing the optimization problem for homogenous market makers, who give quotes on the options market and face two possible estimated volatility states. Once a transaction occurs, market makers immediately implement the delta hedge to remove price risk. If market makers sell an option at the asking price, they buy the delta portion of the underlying asset for hedging. Conversely, if they buy at the bidding price, they short the delta portion of the underlying asset. With two volatility states, a 4-branch tree can be constructed. On this tree, the inner and outer pairs are determined by low and high volatility estimates. Unlike in a 2-branch tree, market makers cannot perfectly hedge an option using its underlying asset. Therefore, when providing the liquidity in the options market, market makers need extra profit in addition to fair expected option value to compensate for the hedging variance.

Before addressing the model setup, we first explain the idea of the hedging uncertainty under the structure where traders face volatility uncertainty in an incomplete market. The Black-Scholes-Merton partial differential equation is applied here for illustration.[8] Next, we discuss arbitrage pricing when traders face two possible volatility states and later connect it to the model equilibrium. Equilibrium implications, including maturity effect, are provided in the final subsection.

4.3.1 Uncertainty of Hedging

The Black-Scholes-Merton differential equation follows

$$\frac{\partial C}{\partial \tau} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

$$\Rightarrow [\frac{\partial C}{\partial \tau}]\Delta t = -\frac{1}{2}\sigma^2 \Delta t S^2 \frac{\partial^2 C}{\partial S^2} - r\Delta t S\frac{\partial C}{\partial S} + r\Delta t C \quad , (4-1)$$

C is the options price, which is a function of the parameters, (σ^2, τ, S, r). S represents the spot price, and parameters, (σ^2, τ, r), are annualized return volatility, time to maturity, and interest rate respectively. In this subsection, we take the perspective of traders who write (sell) ATM options for illustrating the concept of hedging uncertainty. As shown in Figure 4, the solid bold line stands for the prices of call options with strike, X, and TTM, τ_1, over different spot prices. After the transaction occurs, market makers buy the delta, $\frac{\partial C}{\partial S}$, portion of the unit underlying asset for hedging and will rewind it at expiration. With the cash inflow from selling an option and with the position for hedging, market makers hedge the linear price movement of an option, while the curvature

[8]The stochastic volatility model also can be applied for illustration because it assumes constant variance of volatility, and traders still face uncertainty of this parameter. For simplicity, we use the Black-Scholes-Merton differential equation to explain the concept of hedging uncertainty caused by volatility uncertainty.

change of an options price, $\frac{1}{2}\sigma^2 \Delta t S^2 \frac{\partial^2 C}{\partial S^2}$, cannot be hedged by the underlying asset. Moreover, the cost of constructing a hedging portfolio should be considered. This cost includes interest received from cash inflow, C, for selling options and interest paid for buying the delta-hedging position. On the other side, the probability that the final price will go further above X decreases as time passes. Hence, the options price decreases, and writers have the advantage of time value decay, $\frac{\partial C}{\partial \tau}$, *ceteris paribus*. In equation (4-1), this value decay of an option, $[\frac{\partial C}{\partial \tau}]\Delta t$, should equal the sum of hedging loss as well as cost of constructing a hedge portfolio.

Considering time change from $\tau 1$ to $\tau 2$, new options prices on the solid line shift to the dotted line in Figure 4-3-1. If an option was sold at a volatility higher than the real volatility, the market makers earn from selling the options. This case corresponds to the segment where a dotted-dashed line is above the dotted line in Figure 4-3-1. Conversely, buying the options with hedging suffers from time decay but benefits from large stock price movement. If real volatility is higher than buying volatility, market makers earn from buying options. If the options contract is short term and the interest rate is low, the time (decay) value of options is mainly determined by the curvature, gamma; thus, the assumption of return distribution decides the decay rate of an option.

In reality, market makers encounter volatility (model) uncertainty such that they would charge extra profit over the fair options price to compensate for hedging risk caused by volatility uncertainty.[9] However, the decay of charged premium in the option price is determined by return distribution and may be different from the decay of required premium decided by utility function. The

[9]The analysis here assumes that traders cannot hedge continuously and the market is incomplete.

disparity between two decay rates is the key ingredient in this paper. By using mean-variance utility function, we show that the required markup for volatility uncertainty decays at a relatively lower speed.[10] Therefore, as time passes, market makers need to increase asking volatility and decrease bidding volatility to maintain quotation prices that satisfy the conditions allowing liquidity providers to engage in quoting. Further details will be discussed in the following subsections.

4.3.2 Arbitrage Pricing

In our model, market makers serve as risk-averse liquidity providers of bidding and asking prices in the market. They apply the static delta hedge if an option transaction occurs. Market makers are given information from two volatility forecasting models, which could be ARCH/GARCH-in-Mean family typed models or realized volatility forecasting models. Under no arbitrage, given that our market is incomplete, there exist non-unique risk neutral probabilities such that the price of an option is equal to a expected price discounted directly by risk-free rate.

There are two different ways to explain the role of the binomial options pricing model in our setup. First, traders can use different econometric methods, either parametric or non-parametric, to estimate parameters of an option pricing model and then plug estimated parameters into a pricing model. And we use the binomial model as a representative model, for that binomial tree can

[10]Mean-variance utility function is based on exponential utility function and normal distribution assumption on future payoffs. It is extensively applied in the literature, and it helps us to deliver the analytical results. Our results should be robust as long as exponential utility function well characterizes risk-adverse behavior.

intuitively illustrate the ideas and that the binomial model possesses every important characteristic that most option pricing models have. Alternatively, the binomial model can be regarded as a simplification for the numerical method. Because volatility models are also numerical methods for pricing options, plugging volatility delivered by a certain volatility model into the binomial model makes pricing of the binomial model close to the pricing of that specified volatility model.[11] Here we should emphasize that the volatility forecasting models need to be estimated directly from return data rather than calibrated by option trading prices such that the estimated volatility does not necessarily deliver risk neutral probability discounting fair option values to a trading (observed) price.

The observed IMV is σ ,and σL and σH are volatility estimated by forecasting models, in which $L < 1 < H$. While L and H are latent and exogenously given, σ can be thought of as IMV of the previous day. Conditional on one volatility state, the tree is complete and the σ-conditional risk neutral probabilities are uniquely determined. Let $\pi_{\sigma L}$ and $\pi_{\sigma H}$ be these unique σ-conditional risk neutral probabilities. However, given that a 4-branch tree is contructed by two possible volatility states, the market is incomplete. Let $(\Phi, 1 - \Phi)$ be the non-unique risk-neutral probabilities over the two volatility states $(\sigma L, \sigma H)$. In a subsequent section, it will be shown that market makers acting optimally, in equilibrium, determine these risk neutral probabilities. Figure 4-3-2 demonstrates the probabilistic structure and price movement of the underlying asset. Following equations need to be satisfied by the conditional risk neutral probabilities, $(\pi_{\sigma L}, \pi_{\sigma H})$, of two volatility states. Without loss of generality, we assume zero risk-free rate in our model.

[11] If the volatility model has the assumption that return follows normal distribution, then the mean and variance can completely characterize the distribution.

$$\begin{cases} \pi_{\sigma L} \cdot u_{\sigma L} + (1 - \pi_{\sigma L}) \cdot d_{\sigma L} = 1 \\ \pi_{\sigma L} \cdot u_{\sigma L}^2 + (1 - \pi_{\sigma L}) \cdot d_{\sigma L}^2 - [\pi_{\sigma L} \cdot u_{\sigma L} + (1 - \pi_{\sigma L}) \cdot d_{\sigma L}]^2 = (\sigma L)^2 \tau \end{cases}, (4-2)$$

$$\begin{cases} \pi_{\sigma H} \cdot u_{\sigma H} + (1 - \pi_{\sigma H}) \cdot d_{\sigma H} = 1 \\ \pi_{\sigma H} \cdot u_{\sigma H}^2 + (1 - \pi_{\sigma H}) \cdot d_{\sigma H}^2 - [\pi_{\sigma H} \cdot u_{\sigma H} + (1 - \pi_{\sigma H}) \cdot d_{\sigma H}]^2 = (\sigma H)^2 \tau \end{cases}, (4-3)$$

$u_{\sigma L}$ and $u_{\sigma H}$ are the returns corresponding to two upper nodes among four branches, while $d_{\sigma L}$ and $d_{\sigma H}$ are the returns at two lower nodes. τ is TTM on yearly basis, so that, if time to maturity is 1 year, τ equals 1. The initial price, P_0, can possibly end up with the prices and probabilities listed as follows.

$$\begin{cases} P_0 \cdot u_{\sigma H} \text{ with probability } p_{u_{\sigma H}} = (1 - \Phi) \cdot \pi_{\sigma H} \\ P_0 \cdot u_{\sigma L} \text{ with probability } p_{u_{\sigma L}} = \Phi \cdot \pi_{\sigma L} \\ P_0 \cdot d_{\sigma L} \text{ with probability } p_{d_{\sigma L}} = \Phi \cdot (1 - \pi_{\sigma L}) \\ P_0 \cdot d_{\sigma H} \text{ with probability } p_{d_{\sigma H}} = (1 - \Phi) \cdot \pi_{\sigma H} \end{cases}$$

The arbitrage-free values for the call options are $C(\tau, V\sigma) = \Phi_\omega C(\tau, \sigma L) + (1 - \Phi_\omega) C(\tau, \sigma H)$, where $V \in \{a, b\}$ and $\omega \in \{A, B\}$. a and b are asking and bidding volatility multipliers such that $V\sigma$ is the quoting volatility. $C(\tau, \sigma L)$ and $C(\tau, \sigma H)$ are fair values of call options over two possible volatility states and will be explained in detail at next subsection. Φ_A and Φ_B denote the risk neutral probabilities which discount fair options values to observed quoting call prices valued at asking volatility and bidding volatility. The next section determins Φ_ω such that equilibrium quoting price equals $\Phi_\omega C(\tau, \sigma L) + (1 - \Phi_\omega) C(\tau, \sigma H)$. This setup allows us to focus on the volatility (model) uncertainty, while it sets aside the issue of premium on equity return risk by assuming that liquidity providers adopt estimations delivered by volatility forecasting models.

4.3.3 Market Maker's Problem

This section elucidates the problem faced by market makers. Based on their subjective probabilities, $(\phi, 1-\phi)$, over $(\sigma L, \sigma H)$, market makers maximize their utility by choosing q and Δ at a given call price, valued at volatility, $V\sigma$, where $V \in \{a, b\}$, and a and b are volatility multipliers for asking and bidding option prices. q is the option quoting amount submitted by the market maker, and Δ_A and Δ_B are the delta hedge ratios for writing and buying a unit of ATM option. If a transaction happens, trader's final wealth, W_ω, $\omega \in \{A, B\}$, falls into four outcomes, $(W_{\omega,u_{\sigma H}}, W_{\omega,u_{\sigma L}}, W_{\omega,d_{\sigma L}}, W_{\omega,d_{\sigma H}})$, at expiration date. W_A and W_B are the final outcomes corresponding to writing (selling) and buying a ATM call option with hedging. For a hedging portfolio containing a ATM call option that was written, the outcomes are

$$\begin{cases} W_{A,u_{\sigma H}} = C(\tau, a\sigma) - C_N(u_{\sigma H}) + \Delta_A P_0(u_{\sigma H} - 1) & = C_{\tau,a} - C_1 + \Delta_A S_1 \\ W_{A,u_{\sigma L}} = C(\tau, a\sigma) - C_N(u_{\sigma L}) + \Delta_A P_0(u_{\sigma L} - 1) & = C_{\tau,a} - C_2 + \Delta_A S_2 \\ W_{A,d_{\sigma L}} = C(\tau, a\sigma) - C_N(d_{\sigma L}) + \Delta_A P_0(d_{\sigma L} - 1) & = C_{\tau,a} - C_3 + \Delta_A S_3 \\ W_{A,d_{\sigma H}} = C(\tau, a\sigma) - C_N(d_{\sigma H}) + \Delta_A P_0(d_{\sigma H} - 1) & = C_{\tau,a} - C_4 + \Delta_A S_4 \end{cases}$$

If market makers buy the ATM call option at a price valued at $b\sigma$, the hedging portfolio outcomes are

$$\begin{cases} W_{B,u_{\sigma H}} = C_N(u_{\sigma H}) - C(\tau, b\sigma) + \Delta_B P_0(u_{\sigma H} - 1) & = C_1 - C_{\tau,b} + \Delta_B S_1 \\ W_{B,u_{\sigma L}} = C_N(u_{\sigma L}) - C(\tau, b\sigma) + \Delta_B P_0(u_{\sigma L} - 1) & = C_2 - C_{\tau,b} + \Delta_B S_2 \\ W_{B,d_{\sigma L}} = C_N(d_{\sigma L}) - C(\tau, b\sigma) + \Delta_B P_0(d_{\sigma L} - 1) & = C_3 - C_{\tau,b} + \Delta_B S_3 \\ W_{B,d_{\sigma H}} = C_N(d_{\sigma H}) - C(\tau, b\sigma) + \Delta_B P_0(d_{\sigma H} - 1) & = C_4 - C_{\tau,b} + \Delta_B S_4 \end{cases}$$

where

$$C_N(u_{\sigma H}) = C_1, C_N(u_{\sigma L}) = C_2, C_N(d_{\sigma L}) = C_3, C_N(d_{\sigma H}) = C_4$$

$$P_0(u_{\sigma H} - 1) = S_1, P_0(u_{\sigma L} - 1) = S_2, P_0(d_{\sigma L} - 1) = S_3, P_0(d_{\sigma H} - 1) = S_4$$

$$C_i = Max(S_i + P_0 - X, 0), i = \{1, 2, 3, 4\}, X \text{ :exercise price}$$

$C(\tau, V\sigma)$ refers to the price of a call option with maturity τ and priced at volatility, $V * \sigma$. $C_N(m)$ is the intrinsic value of options at expiration, where $m \in \{u_{\sigma H}, u_{\sigma L}, d_{\sigma L}, d_{\sigma H}\}$, and C_i and S_i, $i \in \{1, 2, 3, 4\}$, are used to denote final call prices and stock dollar payoffs for four end nodes. In summary, W_A (W_B) is the portfolio payoff at expiration, and the portfolio contains selling (buying) one call options to get (pay) the amount of $C_{\tau,a}$ $(C_{\tau,b})$ as well as buying $\Delta_A(\Delta_B)$ shares of stock, which requires cash outflow (inflow) $\Delta_A P_0$ $(\Delta_B P_0)$ at the beginning. At the end, the total payoff includes the possible cash outflow (inflow), $C_N(m)$, due to the obligation (right) of writing (buying) a call, and inflow(outflow) from unwinding $\Delta_A(\Delta_B)$ shares of stock.

In the following sections, our discussion primarily focuses on the maximization problem of writing a call option. The problem of buying derivatives is an analogy, and the results are both listed in the appendix. The optimization problem for liquidity providers is as follows:[12]

$$\arg\max_{q,\Delta_A} qE(W_A) - \tfrac{1}{\gamma}Var(qW_A) - cq \quad , (4-4)$$

where $E(.)$ and $V(.)$ are expected profit and hedging variance of writing an option with hedging. c is the transaction or opportunity cost for trading, and W_A is the profit for a portfolio at expiration. Each unit of the portfolio includes

[12]Constructing the optimization problem of giving the bidding price is analogous to giving the asking price; however, the expected profit of buying options is not the same as writing a call options. Expected profit is $C(\tau, a\sigma) - \phi C(\tau, \sigma L) - (1-\phi)C(\tau, \sigma H)$ for writing a call option, while it is $\phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H) - C(\tau, b\sigma)$ for buying a call.

shorting an option and buying Δ_A shares of underlying asset. γ is the risk aversion coefficient, and q is the quoting amount. The model can also be refined in an alternative way. We can use $\bar{q}$ to represent inventory (open interest), such that traders will take open interest into consideration. Equation (4-5) refines this idea.

$$\arg\max_{q,\Delta_A} qE(W_A) - \tfrac{1}{\gamma}[Var((q+\bar{q})W_A) - Var(\bar{q}W_A)] \quad , (4-5)$$
$$\Rightarrow \arg\max_{q,\Delta_A} qE(W_A) - \tfrac{q^2}{\gamma}Var(W_A) - 2q\tfrac{\bar{q}}{\gamma}Var(W_A)$$

Given that traders had open interest position $\bar{q}$, the hedging variance of the new integrated position becomes $Var((q+\bar{q})W_A)$ as a whole. While $Var(\bar{q}W_A)$ is generated by the old position, $\bar{q}$, and is compensated by the previous transaction, we subtract it from total variance. After we set $\frac{2\bar{q}}{\gamma}Var(W_A)$ equals c,[13] the maximization problem (4-5) now becomes (4-4). Hence, c can be related to the inventory effect of existing open interest.

Because equation (4-2) and (4-3) need to be satisfied, the expected payoff from stock is always zero.[14] So, intuitively, expected payoff, $E(W_A)$, is the selling price minus expected fair values of options in two different volatility states. After rearrangement, the expected payoff follows, and it is exactly the difference between the selling (buying) price and expected intrinsic values of $C_N(m)$.

[13]We will later show that the optimal solution for Δ does not depend on q.

[14]Because the expected return is discounted by conditional risk neutral probability, it means expected return is 0 after adjustment by equity return risk. Equations (2) and (3) allow us to focus on analyzing volatility (model) uncertainty and setting the issue of equity return risk aside.

$$E(W_A) = C(\tau, a\sigma) - \phi C(\tau, \sigma L) - (1-\phi)C(\tau, \sigma H)$$
$$E(W_B) = -C(\tau, b\sigma) + \phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H)$$
and $C(\tau, \sigma L) = \pi_{\sigma L} C_N(u_{\sigma L}) + (1-\pi_{\sigma L})C_N(d_{\sigma L})$
$$C(\tau, \sigma H) = \pi_{\sigma H} C_N(u_{\sigma H}) + (1-\pi_{\sigma H})C_N(d_{\sigma H})$$

(Deduction is shown in Proof 4-1)

The expected price using the subjective probabilities is

$\phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H)$. Unless liquidity providers are not risk averse, subjectively expected price is supposed to be below the asking price because the asking price, $C(\tau, a\sigma)$, should contain the premium for volatility (model) risk. The optimal solution needs to satisfy the following first order conditions, and the solution refers to the optimal individual supply of writing the call options.

F.O.C of q : $E(W_A) - \frac{2q^*}{\gamma} Var(W_A) - c = 0$, $(4-6)$

F.O.C of Δ_A : $\frac{\partial Var(W_A)}{\partial \Delta_A} = 0$, $(4-7)$

$$\begin{cases} q^* = \frac{\gamma\{C(\tau,a\sigma) - \phi C(\tau,\sigma L) - (1-\phi)C(\tau,\sigma H) - c\}}{2Var(W_A)} \\ \text{if } C(\tau, a\sigma) > \phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H) - \frac{q^*}{2\gamma} Var(W_A) - c \\ q^* = 0 \\ \text{if } C(\tau, a\sigma) \leq \phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H) - \frac{q^*}{2\gamma} Var(W_A) - c \end{cases}$$

Moreover, Δ^* is independent of q^*, because expected payoff of holding underlying is always zero; therefore, stock for hedging only affects the variance of hedging results rather than expected payoff. The optimal delta position minimizing hedging variance is mainly determined by volatility estimations, and its formula is,

$$\Delta_A^* = \frac{\sum_{i=1}^{4} P_i S_i C_i}{\sum_{i=1}^{4} P_i S_i^2} \quad \text{and} \quad \Delta_B^* = -\frac{\sum_{i=1}^{4} P_i S_i C_i}{\sum_{i=1}^{4} P_i S_i^2}$$

(Detail is shown in Proof 4-2)

Δ_A^* and Δ_B^* are the delta hedging positions for writing and buying a call option, and (P_1, P_2, P_3, P_4) are the subjective probabilities, $((1-\phi)\pi_{\sigma H}, \phi\pi_{\sigma L}, \phi(1-\pi_{\sigma L}), (1-\phi)(1-\pi_{\sigma H}))$, for each branch. While (c_1, c_2, c_3, c_4) are four possible intrinsic values of a call option, (S_1, S_2, S_3, S_4) are the stock payoffs at expiration. We further check if delta, Δ_ω^* , makes sense for two particular strike prices. It can be shown that $\Delta_A^*(\Delta_B^*)$ is approximately $0.5(-0.5)$ for ATM call options. On the other hand, $\Delta_A^*(\Delta_B^*)$ is approaching $1(-1)$ if the strike price drops to 0.[15]

There are other ways to construct the decision making problem for market makers. For example, market makers may try to minimize the hedging risk, while requiring minimum expected return.

$$\text{Alternative Problem (1)} \begin{cases} \arg\min\limits_{q,\Delta_A} Var(qW_A) \\ \text{s.t. } E(qW_A) \geq \theta \end{cases}$$

Additionally, market makers may set their priorities on pursuing minimized variance. Then, after the portfolio that minimizes hedging variance is constructed, market makers can try to optimize profit.

$$\text{Alternative Problem (2)} \begin{cases} \arg\min\limits_{\Delta_A} Var(qW_A) \\ \arg\max\limits_{q,\Delta_A=\Delta_A^*} qE(W_A) - \frac{1}{\gamma}Var(qW_A) - cq \end{cases}$$

[15]The related deduction of optimal delta for the ATM option is included in Appendix D-4.

Intuitively, because expected pay off of stock is zero, solutions of optimized delta portion are the same for those optimization problems. Quoting amount increases variance through the quadratic term so that optimal quoting amount is bounded by asking price and coefficient of risk averse. In Proof 4-3 and 4-4, we show that newly proposed decision making problems have the same solution as the optimization problem (4) has.

4.3.4 Equilibrium

To understand the disparity between derivatives decay and decay for volatility risk premium, a static equilibrium rather than dynamic equilibrium is applied for our analysis here. As the Exchange commonly regulates liquidity providers to quote minimum amount everyday and then allows eligible market makers to enjoy discounted fee, we set up a regulated quoting amount, Q, that M homogeneous market makers are required to quote each day.

It is noteworthy that the regulated quoting amount is not the same as market demand, in which quantity is the amount that demanders commit to buy at given prices. Hence, our model always refers to the equilibrium of quoting amount and quoting price instead of trading volume and trading price. In our model, liquidity providers mainly care about hedging risk resulting from estimation uncertainty.[16] In a competitive environment, they provide the supply of writing and buying derivatives given that selling or bidding price can compensate for required risk premium and cost. This set up should be close to the reality of the index options market, and private information is not as large a con-

[16]The optimization problems of bidding and quoting behavior are separated in this paper. This assumption implies that market makers do not consider the possibility that delta hedging positions for selling and buying options could offsets with each other.

cern as it is in other markets. In the equilibrium, M homogenous traders will finally compete until utility equals zero in a non-cooperative game. Therefore we have the following equilibrium condition for the asking price quotation.

$$E(W_A) - \frac{Q}{\gamma}Var(W_A) - c = 0 \qquad ,(4-8)$$
$$\Rightarrow C(\tau, a^*\sigma) = \phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H) + \frac{Q}{\gamma}Var(W_A) + c \qquad ,(4-9)$$

Similarly, the equilibrium condition for the bidding price is

$$E(W_B) - \frac{Q}{\gamma}Var(W_B) - c = 0$$
$$\Rightarrow C(\tau, b^*\sigma) = \phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H) - \frac{Q}{\gamma}Var(W_B) - c \qquad (4-10)$$

According to Equation (9), price should cover three parts, including expected fair price, $\phi C(\tau, L) + (1-\phi)C(\tau, H)$, required premium for hedging variance, $Var(W_A)$, and transaction cost, c. That is, $E(W_A)$, which equals $C(\tau, a^*\sigma) - (\phi C(\tau, L) + (1-\phi)C(\tau, H))$, is extra profit, sold at volatility $a^*\sigma$, and it should cover the premium of hedging uncertainty and transaction cost. In the following section, we utilize comparative static analysis based on equilibrium condition (4-8) to derive our propositions. After setting exercise price equal to spot price, we derive the options price, expected extra profit, and the variance of hedging results for writing and buying an ATM call option through the binomial pricing model. The equations of options price, expected extra profit, and the variance of hedging results follow.

$$C(\tau, V*\sigma) = \quad P_0\pi_{V\sigma}(u_{\tau,V\sigma} - 1) = \frac{P_0(1-e^{-V\sigma\sqrt{\tau}})}{e^{V\sigma\sqrt{\tau}} - e^{-V\sigma\sqrt{\tau}}}(e^{V\sigma\sqrt{\tau}} - 1) = \frac{P_0(e^{V\sigma\sqrt{\tau}} + e^{-V\sigma\sqrt{\tau}} - 2)}{e^{V\sigma\sqrt{\tau}} - e^{-V\sigma\sqrt{\tau}}}$$

$$E(W_A) = \quad P_0\left(\frac{e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}} - \phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} - (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}}\right)$$

$$\begin{aligned} Var(W_A) = \quad & P_0^2\{\phi\frac{\Delta_A^2(e^{2\sigma L\sqrt{\tau}}-2e^{\sigma L\sqrt{\tau}}+2e^{-\sigma L\sqrt{\tau}}-e^{-2\sigma L\sqrt{\tau}})+(1-2\Delta_A)(e^{2\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3-e^{-\sigma L\sqrt{\tau}})}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} \\ & +(1-\phi)\frac{\Delta_A^2(e^{2\sigma H\sqrt{\tau}}-2e^{\sigma H\sqrt{\tau}}+2e^{-\sigma H\sqrt{\tau}}-e^{-2\sigma H\sqrt{\tau}})+(1-2\Delta_A)(e^{2\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3-e^{-\sigma H\sqrt{\tau}})}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}} \\ & -(\phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} + (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}})^2\} \end{aligned}$$

$$E(W_B) = \quad P_0\left(-\frac{e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2}{e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}}} + \phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} + (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}}\right)$$

$$\begin{aligned} Var(W_B) = \quad & P_0^2\{\phi\frac{\Delta_B^2(e^{2\sigma L\sqrt{\tau}}-2e^{\sigma L\sqrt{\tau}}+2e^{-\sigma L\sqrt{\tau}}-e^{-2\sigma L\sqrt{\tau}})+(1+2\Delta_B)(e^{2\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3-e^{-\sigma L\sqrt{\tau}})}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} \\ & +(1-\phi)\frac{\Delta_B^2(e^{2\sigma H\sqrt{\tau}}-2e^{\sigma H\sqrt{\tau}}+2e^{-\sigma H\sqrt{\tau}}-e^{-2\sigma H\sqrt{\tau}})+(1+2\Delta_B)(e^{2\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3-e^{-\sigma H\sqrt{\tau}})}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}} \\ & -(\phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} + (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}})^2\} \end{aligned}$$

4.3.5 Equilibrium and Risk Neutral Probability

Here we discuss the link between the subjective probability constituting equilibrium and risk neutral probability. Given that the volatility state is either σL or σH, the risk neutral probability, Φ_A , is determined by the equilibrium value of the options, $C(\tau, a^*\sigma)$, i.e.

$$\begin{aligned} C(\tau, a^*\sigma) \quad & = \Phi_A C(\tau, L) + (1-\Phi_A)C(\tau, H) \\ & = P_0\left\{\Phi_A * \pi_{\sigma L}(u_{\sigma H}-1) + (1-\Phi_A) * \pi_{\sigma H}(u_{\sigma H}-1)\right\} \\ \Rightarrow \quad & \Phi_A = \frac{C(\tau,H)-C(\tau,a^*\sigma)}{C(\tau,H)-C(\tau,L)} = \frac{\pi_{\sigma L}(u_{\sigma H}-1)-\pi_{\sigma a}(u_{\sigma a}-1)}{\pi_{\sigma L}(u_{\sigma H}-1)-\pi_{\sigma L}(u_{\sigma L}-1)} \end{aligned}$$

On the other hand given the subjective probability, set $\left\{\frac{Q}{\gamma}Var(W_A)+c\right\} = P_0\rho > 0$ and plug it into the equilibrium condition.

$$\begin{aligned} C(\tau, a^*\sigma) \quad & = \phi C(\tau, L) + (1-\phi)C(\tau, H) + P_0\rho \\ & = P_0\left\{\phi\pi_{\sigma L}(u_L-1) + (1-\phi)\pi_{\sigma H}(u_{\sigma H}-1) + \rho\right\} \end{aligned}$$

This yields the following relation.

$$\phi \quad = \frac{C(\tau,H\sigma)+P_0\rho-C(\tau,a^*\sigma)}{C(\tau,H\sigma)-C(\tau,L\sigma)} = \frac{\pi_{\sigma H}(u_{\sigma H}-1)+\rho-\pi_{\sigma a}(u_{\sigma a}-1)}{\pi_{\sigma L}(u_{\sigma H}-1)-\pi_{\sigma L}(u_{\sigma L}-1)}$$

Therefore, Φ_A is smaller than ϕ, where ϕ and Φ_A are the subjective and risk neutral probability assigned to lower estimated volatility state. We can find a multiplier, RN_A, such that $\Phi_A = \phi * RN_A$ and

$$RN_A = \frac{\Phi_A}{\phi} \quad = \frac{C(\tau,H)-C(\tau,a^*)}{C(\tau,H)+P_0\rho-C(\tau,a^*)} = \frac{\pi_{\sigma L}(u_{\sigma H}-1)-\pi_{\sigma a}(u_{\sigma a}-1)}{\pi_{\sigma H}(u_{\sigma H}-1)+\rho-\pi_{\sigma a}(u_{\sigma a}-1)} < 1$$

In the model, equilibrium asking price is higher than fair value because risk averse traders require risk premium compensation. Therefore, intuitively, given that the risk neutral probability directly discounts fair values at the options of two volatility states, the risk neutral probability assigned to lower estimated volatility should be lower in that it needs to discount two fair options value to a higher price. Similarly, for the equilibrium of buying options, we derive risk neutral probability for bidding price in the following and show $\Phi_B = \phi * RN_B$, where $RN_B > 1$. The risk neutral probability for the lower estimated volatility is higher in that it discounts two fair option values into a price that is lower than expected fair option price.

$$\begin{aligned}
\Phi_B \quad &= \frac{C(\tau,\sigma H)-C(\tau,b^*\sigma)}{C(\tau,\sigma H)-C(\tau,\sigma L)} = \frac{\pi_{\sigma L}(u_{\sigma H}-1)-\pi_{\sigma b}(u_{\sigma b}-1)}{\pi_{\sigma L}(u_{\sigma H}-1)-\pi_{\sigma L}(u_{\sigma L}-1)} \\
\phi \quad &= \frac{C(\tau,\sigma H)-P_0\rho-C(\tau,b^*\sigma)}{C(\tau,\sigma H)-C(\tau,\sigma L)} = \frac{\pi_{\sigma H}(u_{\sigma H}-1)-\rho-\pi_{\sigma a}(u_{\sigma b}-1)}{\pi_{\sigma L}(u_{\sigma H}-1)-\pi_{\sigma L}(u_{\sigma L}-1)} \\
RN_B \quad &= \frac{C(\tau,\sigma H)-C(\tau,b^*\sigma)}{C(\tau,\sigma H)-P_0\rho-C(\tau,b^*\sigma)} = \frac{\pi_{\sigma L}(u_{\sigma H}-1)-\pi_{\sigma b}(u_{\sigma b}-1)}{\pi_{\sigma H}(u_{\sigma H}-1)-\rho-\pi_{\sigma b}(u_{\sigma b}-1)} > 1
\end{aligned}$$

4.3.6 Equilibrium Implications

Applying comparative static analysis, we can explore the quoting strategies of market makers in a static environment. In this subsection, our model concludes two major effects: (a) the maturity effect and (b) the level effect. In the following section are advanced discussions about volatility uncertainty and volatility spread.

Proposition 1 *The spread increases as maturity decreases (*$\frac{\partial a}{\partial \tau} < 0, \frac{\partial b}{\partial \tau} > 0$ *and* $\frac{\partial (a-b)}{\partial \tau} < 0$*)*

The proposition refers to the maturity effect and can explain the major puzzle. An implicit function, $f(\tau, \sigma, H, L, V)$, $V = \{a, b\}$, is defined over equilibrium condition in which a is the function of exogenous variables, (τ, σ, H, L).

$$f(\tau, \sigma, H, L, V) = E(W_\omega) - \frac{Q}{\gamma} Var(W_\omega) - c = 0 \quad , (4-11)$$

How the asking multiplier, a, changes given a small change of maturity, τ, is

$$\frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial a} da = 0 \Rightarrow \frac{da}{d\tau} = \frac{-\frac{\partial f}{\partial \tau}}{\frac{\partial f}{\partial a}} \text{ similarly } \frac{db}{d\tau} = \frac{-\frac{\partial f}{\partial \tau}}{\frac{\partial f}{\partial b}}$$

The following are first derivative equations of $f(\tau, \sigma, H, L, V)$ on a and b respectively.

$$\frac{\partial f}{\partial a} = \quad P_0 \frac{2\sigma\sqrt{\tau}(e^{a\sigma\sqrt{\tau}} + e^{-a\sigma\sqrt{\tau}} - 2)}{(e^{a\sigma\sqrt{\tau}} - e^{-a\sigma\sqrt{\tau}})^2} > 0, \because e^{a\sigma\sqrt{\tau}} + e^{-a\sigma\sqrt{\tau}} - 2 > 0$$

$$\frac{\partial f}{\partial b} = \quad -P_0 \frac{2\sigma\sqrt{\tau}(e^{b\sigma\sqrt{\tau}} + e^{-b\sigma\sqrt{\tau}} - 2)}{(e^{b\sigma\sqrt{\tau}} - e^{-b\sigma\sqrt{\tau}})^2} < 0, \because e^{b\sigma\sqrt{\tau}} + e^{-b\sigma\sqrt{\tau}} - 2 > 0$$

Intuitively, the last equation tells us that selling options price at higher or buying at lower volatility increases the value of implicit function. After taking derivetives, we use the Taylor expansion to get equations for approximating the derivatives of $f(\tau, \sigma, H, L, V)$ on τ.

$$\begin{aligned} \frac{\partial f(\tau,\sigma,H,L,a)}{\partial \tau} &= \frac{\partial E(W_A)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \tau} \\ &\cong \frac{P_0}{4\sqrt{\tau}}\left\{a\sigma - \phi\sigma L - (1-\phi)\sigma H\right\} - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left[(1-\phi)(\sigma H)^2 + \phi(\sigma L)^2\right]\right\} \\ &+ \frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L + (1-\phi)\sigma H\right]^2\right\} \end{aligned}$$

$$\begin{aligned} \frac{\partial f(\tau,\sigma,H,L,b)}{\partial \tau} &= \frac{\partial E(W_B)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_B)}{\partial \tau} \\ &\cong \frac{P_0}{4\sqrt{\tau}}\left\{-b\sigma + \phi\sigma L + (1-\phi)\sigma H\right\} - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_B^2 + 2\Delta_B + 1)}{2}\left[(1-\phi)(\sigma H)^2 + \phi(\sigma L)^2\right]\right\} \\ &+ \frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L + (1-\phi)\sigma H\right]^2\right\} \end{aligned}$$

(Deduction in Proof 4-5)

The derivative of $f(\tau, \sigma, H, L, V)$ on τ can be separated into two parts. The first part is time decay of expected profit, $\frac{\partial E(W_\omega)}{\partial \tau}$, and the second part is $\frac{Q}{\gamma}\frac{\partial Var(W_\omega)}{\partial \tau}$, which is the decay rate of premium for hedging uncertainty over time. If the required premium of hedging risk decays at a rate slower than decay rate of extra profit of derivatives portfolio, then $\frac{\partial f}{\partial \tau} > 0$ leads to $\frac{da}{d\tau} < 0$. According to the last equation, expected extra profit decays dramatically at a rate of inverse $\sqrt{\tau}$, while the volatility uncertainty premium decays at a fixed rate. As τ gradually decreases, $\frac{\partial f}{\partial \tau}$ necessarily turns out be positive; thus, $\frac{da}{dt} < 0$ will certainly happen at some time before expiration. Because the value decay of extra profit is extremely large in final few days where τ is very small, the equation above also explains why outrageous quotation is commonly observed a few days before expiration.

Figure 4-4-1 illustrates the dynamic between the change of expected profit for writing options at volatility, $a\sigma$, and the change of required premium for hedging risk. Point E_1 indicates the first equilibrium where expected profit, $E(W_{A,E_1})$ valued at $a_1\sigma$, equals the required premium with transaction costs (i.e., $\frac{Q}{\gamma}Var(W_{A,E_1}) + c$). However, as time moves to maturity date, $E(W_A)$ valued at shorter TTM decreases to a level on which the newly required premium as well as transaction costs cannot be affoded. Therefore, to engage in quoting, traders need to increase selling volatility such that, for shorter TTM, $E(W_{A,E_2})$, which is priced with $a_2\sigma$, can cover newly required model risk premium and transaction costs at equilibrium point E_2. Here it should be emphasized that a change of asking volatility doesn't alter hedging variance so there is no line shifting for hedging variance. Briefly, bid-ask spread quoting pattern is the result of the disparity between daily value decay of a hedging portfolio and daily decay of required premium for hedging uncertainty. A more general condition can be derived to insure $\frac{\partial a}{\partial \tau} < 0$ for every τ.

$$\frac{\partial f(\tau,\sigma,H,L,a)}{\partial \tau} = \frac{\partial E(W_A)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \tau} > 0 \text{ if and only if}$$
$$c > \frac{P_0\sqrt{\tau}(a\sigma - \phi\sigma L - (1-\phi)\sigma H)}{4} = \frac{E(W_A)}{2}$$
$$\Rightarrow c > \frac{\frac{Q}{\gamma}Var(W_A)+c}{2} \Rightarrow c > \frac{Q}{\gamma}Var(W_A)$$

(Details are shown in Proof 4-6)

A general condition allowing $\frac{\partial b}{\partial \tau} > 0$ for every τ is similar.

$$\frac{\partial f(\tau,\sigma,H,L,b)}{\partial \tau} = \frac{\partial E(W_B)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_B)}{\partial \tau} > 0 \text{ if and only if}$$
$$c > \frac{P_0\sqrt{\tau}(-b\sigma + \phi\sigma L + (1-\phi)\sigma H)}{4} = \frac{E(W_B)}{2}$$
$$\Rightarrow c > \frac{\frac{Q}{\gamma}Var(W_B)+c}{2} \Rightarrow c > \frac{Q}{\gamma}Var(W_B)$$
and $Var(W_A) = Var(W_B)$

Because $E(W_\omega)$ shrinks as time passes, volatility spread is expected to widen from a certain TTM, τ. Although we are not able to test when this condition starts to take hold, the condition seems to happen easily, as documented by CDT (2003), in that the pattern starts months before the expiration date. Moreover, c in the model can be defined more generally in terms of opportunity cost, (e.g., the profit that can be earned from devotion to other projects); therefore, c could be large, allowing patterns to happen couple months before expiration.

Proposition 2 *The spread enlarges with increasing magnitude. (*$\frac{\partial^2 a}{\partial \tau^2} > 0, \frac{\partial^2 b}{\partial \tau^2} < 0$ *and* $\frac{\partial^2 (a-b)}{\partial \tau^2} > 0$*)*

Another phenomenon worthy of our attention is that spread seems to widen with increasing speed. We check second derivatives further based on equation $\frac{\partial a}{\partial \tau}$.

$$\begin{cases} \frac{\partial \frac{\partial a}{\partial \tau}}{\partial \tau} = \frac{-\partial \frac{\frac{\partial f(\tau,\sigma,H,L,a)}{\partial \tau}}{\frac{\partial f(\tau,\sigma,H,L,a)}{\partial a}}}{\partial \tau} = \frac{-\left\{\frac{\partial^2 f}{\partial \tau^2}\frac{\partial f}{\partial a} - \frac{\partial f}{\partial \tau}\frac{\partial^2 f}{\partial a \partial \tau}\right\}}{(\frac{\partial f}{\partial a})^2} > 0 \\ \because \frac{\partial^2 f}{\partial \tau^2} < 0, \frac{\partial f}{\partial a} > 0, \frac{\partial f}{\partial \tau} > 0, \frac{\partial^2 f}{\partial a \partial \tau} > 0 \end{cases}$$

$$\begin{cases} \frac{\partial \frac{\partial b}{\partial \tau}}{\partial \tau} = \frac{-\partial \frac{\frac{\partial f(\tau,\sigma,H,L,b)}{\partial \tau}}{\frac{\partial f(\tau,\sigma,H,L,b)}{\partial a}}}{\partial \tau} = \frac{-\left\{\frac{\partial^2 f}{\partial \tau^2}\frac{\partial f}{\partial b} - \frac{\partial f}{\partial \tau}\frac{\partial^2 f}{\partial b \partial \tau}\right\}}{(\frac{\partial f}{\partial a})^2} < 0 \\ \because \frac{\partial^2 f}{\partial \tau^2} < 0, \frac{\partial f}{\partial b} < 0, \frac{\partial f}{\partial \tau} > 0, \frac{\partial^2 f}{\partial b \partial \tau} < 0 \end{cases}$$

(Details are shown in Proof 4-7.)

Because required premium for hedging risk demonstrates no curvature over TTM, the concavity of $f(\tau, \sigma, H, L, a)$ is determined by extra profit, $E(W_A)$, and we derive $\frac{\partial^2 f(\tau,\sigma,H,L,a)}{\partial \tau^2} \doteq \frac{\partial^2 E(W_A)}{\partial \tau^2}$ in Proof 4-7. This result can also be attributed to the characteristics of the pricing model. For ATM options, the value decays at an increasing speed as time approaches the expiration date, so extra profit also decays with increasing speed as TTM decreases. While the inequations $\frac{\partial f(\tau,\sigma,H,L,a)}{\partial \tau} > 0$ and $\frac{\partial f(\tau,\sigma,H,L,a)}{\partial a} > 0$ have been discussed previously, the inequation $\frac{\partial^2 f(\tau,\sigma,H,L,a)}{\partial a \partial \tau} > 0$ is another natural result of options pricing characteristics. The effect of volatility on options price, known as $Vega$, increases (decreases) as TTM increases (decreases). In other words, compared with selling 1% volatility higher for shorter TTM, increasing selling volatility 1% for longer term options has a stronger positive impact on expected profit. It should be noted that $\frac{\partial^2 f(\tau,\sigma,H,L,b)}{\partial b \partial \tau}$ is negative, because the extra profit for buying an option is the value of fair options minus bidding price.

Proposition 3 *The spread decreases as IMV level increases ($\frac{\partial a}{\partial \sigma} < 0, \frac{\partial b}{\partial \sigma} > 0$ and $\frac{\partial (a-b)}{\partial \sigma} < 0$)*

A phenomenon not as obvious as the previous finding is termed "level effect" in this paper. To get rid of the noises of the maturity effect, we average the percentage spread by CM IMV measures for each contract and plot averaged percentage spread in Figure 4-4-2. A negative relationship between percentage spread and IMV is thus depicted, implying that liquidity providers do not enlarge volatility quoting spread in proportion to an increase of IMV.

In the model, σ refers to trading volatility level so that we derive the first derivative of $f(\tau, \sigma, H, L, a)$ on σ. After approximating by the Taylor expansion, we rearrange the equation in the following and will discuss further in proposition 5.

$$
\begin{aligned}
&\frac{\partial f}{\partial \sigma}=\frac{\partial E(W_A)}{\partial \sigma}-\frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \sigma}\\
\cong\quad & \frac{2}{\sigma}\Big(\frac{P_0\sqrt{\tau}}{4}[a\sigma-\phi\sigma L-(1-\phi)\sigma H]-\frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2-2\Delta_A+1)}{2}\left[(1-\phi)(\sigma H\sqrt{\tau})^2+\phi(\sigma L\sqrt{\tau})^2\right]\right\}\\
&+\frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau}\right]^2\right\}\Big)>0
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial f}{\partial \sigma}=\frac{\partial E(W_B)}{\partial \sigma}-\frac{Q}{\gamma}\frac{\partial Var(W_B)}{\partial \sigma}\\
\cong\quad & \frac{2}{\sigma}\Big(\frac{P_0\sqrt{\tau}}{4}[-b\sigma+\phi\sigma L+(1-\phi)\sigma H]-\frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_B^2-2\Delta_B+1)}{2}\left[(1-\phi)(\sigma H\sqrt{\tau})^2+\phi(\sigma L\sqrt{\tau})^2\right]\right\}\\
&+\frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau}\right]^2\right\}\Big)>0
\end{aligned}
$$

(Details are shown in Proof 4-8.)

According to the equation, we find that the condition resulting in $\frac{\partial f}{\partial \sigma}>0$ is the same as the sufficient and necessary condition for maturity effect. Hence, as long as, we observe the occurrence of the maturity effect, the level effect exists. That is; if the following conditions are held,

$$
\begin{aligned}
c &> \frac{P_0\sqrt{\tau}(a\sigma-\phi\sigma L-(1-\phi)\sigma H)}{4}=\frac{E(W_A)}{2}\\
c &> \frac{P_0\sqrt{\tau}(-b\sigma+\phi\sigma L+(1-\phi)\sigma H)}{4}=\frac{E(W_B)}{2}
\end{aligned}
$$

then level effect happens.

$$
\frac{\partial a}{\partial \sigma}=-\frac{\frac{\partial f}{\partial \sigma}}{\frac{\partial f}{\partial a}}<0,\ \frac{\partial b}{\partial \sigma}=-\frac{\frac{\partial f}{\partial \sigma}}{\frac{\partial f}{\partial b}}>0 \text{ and } \frac{\partial (a-b)}{\partial \sigma}<0
$$

4.4 Volatility Uncertainty and Volatility Spread

We provide some insights here about the relationship between volatility percentage spread and volatility uncertainty. First of all, given variables

$(\phi, \tau, \sigma, L, H)$, we define volatility uncertainty as a variance of possible future realization, and the equation is given as:

$$Var(v) = \phi(\sigma L)^2\tau + (1-\phi)(\sigma H)^2\tau - E(v)^2$$
$$v : \{\sigma L, \sigma H\} \text{ and } E(v) = \phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau}$$

Also, a change of parameters, $\{\sigma, L, H\}$, has the following effect on volatility uncertainty.

$$\frac{\partial Var(v)}{\partial H} = 2\phi(1-\phi)\sigma^2\tau(H-L) > 0$$
$$\frac{\partial Var(v)}{\partial L} = 2\phi(1-\phi)\sigma^2\tau(L-H) < 0$$
$$\frac{\partial Var(v)}{\partial \sigma} = 2\sigma\tau\phi(1-\phi)(L-H)^2 > 0$$

Volatility uncertainty increases as higher estimated volatility increases or lower estimated volatility decreases. Additionally, an increase in trading volatility level also increases volatility uncertainty given that L and H are fixed. Because uncertainty is based on three parameters, (H, L, σ),[17] we can explore the relationship between volatility uncertainty and volatility spread.

Proposition 4 *The increase in volatility uncertainty by the change of volatility estimation, H or L, results in larger volatility spread. If parameters, (H, L), move in the same direction with same magnitude, the volatility spread and volatility uncertainty remain the same. ($\frac{\partial(a-b)}{\partial H} > 0$, $\frac{\partial(a-b)}{\partial L} < 0$ and $\frac{\partial(a-b)}{\partial H} + \frac{\partial(a-b)}{\partial L} = 0$)*

The change in volatility estimation affects expected fair options price as well as expected hedging risk. Based on implicit function, we can derive $\frac{\partial a}{\partial H}$ to understand how asking volatility adjusts to new estimated volatility. First, the effect

[17] Here we do not discuss volatility uncertainty over ϕ, becuase it is similar to the case where H and L shift to opposite direction.

from an increase of H results in a higher fair options value, and expected extra profit decreases by $\frac{P_0\sigma\sqrt{\tau}(1-\phi)}{4}$, if an option is sold at original volatility a. Second, a rise to higher estimated volatility also leads to higher hedging variance by $\frac{(P_0\sigma\sqrt{\tau})^2(1-\phi)\{H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}}{2}$. Given a decreasing expected profit and an increasing hedging risk, traders should increase the selling volatility to restore the value function, engaging themselves in giving quotations again. Because a small change of a increases $f(\tau,\sigma,H,L,a)$ by $\frac{P_0\sigma\sqrt{\tau}}{2}$, how much a needs to be increased in response to a change of H is

$$\begin{aligned}\frac{\partial a}{\partial H} &= -\frac{\frac{\partial f(\tau,H,L,a)}{\partial H}}{\frac{\partial f(\tau,H,L,a)}{\partial a}} \cong -\frac{\frac{P_0\sigma\sqrt{\tau}}{2}\Big\{-\frac{(1-\phi)}{2}-\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\Big\}}{\frac{P_0\sigma\sqrt{\tau}}{2}} \\ &\cong \Big\{\frac{(1-\phi)}{2}+\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\Big\} > 0 \\ &\because 2\Delta_A^2-2\Delta_A+1 \geq 0.5\end{aligned}$$

The same argument applies to $\frac{\partial b}{\partial H}$. However, the integrated effect on bidding volatility from change of fair value together with hedging risk is unclear because an increase in expected profit and an increase in hedging variance by higher H move bidding volatility in opposite directions.

$$\frac{\partial b}{\partial H} \cong \frac{(1-\phi)}{2}-\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_B^2+2\Delta_B+1)-(H+\phi(L-H)\}$$

Still, the total spread change is positive, because the effects from the change of fair values cancel out, and hedging variance doubles.

$$\begin{aligned}\frac{\partial a}{\partial H}-\frac{\partial b}{\partial H} &\cong \Big\{\frac{(1-\phi)}{2}+\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\Big\} \\ &-\Big\{\frac{(1-\phi)}{2}-\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_B^2+2\Delta_B+1)-(H+\phi(L-H)\}\Big\} \\ &\because \Delta_B = -\Delta_A \\ &= 2\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\} > 0\end{aligned}$$

In contrast to the previous case, how spread changes over lower estimated volatility requires more assumptions on parameters, $(H,\ L)$. The following equations explain how the spread changes over lower volatility estimation.

$$\begin{aligned}
\frac{\partial a}{\partial L} - \frac{\partial b}{\partial L} \quad & \cong \left\{ \tfrac{\phi}{2} + \tfrac{Q\phi P_0 \sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_A^2 - 2\Delta_A + 1) - (H + \phi(L-H)\}\right\} \\
& - \left\{ \tfrac{\phi}{2} - \tfrac{Q\phi P_0 \sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_B^2 + 2\Delta_B + 1) - (H + \phi(L-H)\}\right\} \\
& = 2\tfrac{QP_0\phi\sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_A^2 - 2\Delta_A + 1) - (H + \phi(L-H))\} \\
& \begin{cases} \frac{\partial a}{\partial L} - \frac{\partial b}{\partial L} > 0 & \text{if } 2L(2\Delta_A^2 - 2\Delta_A + 1) > (1-\phi)H + \phi L \\ \frac{\partial a}{\partial L} - \frac{\partial b}{\partial L} \le 0 & \text{if } 2L(2\Delta_A^2 - 2\Delta_A + 1) \le (1-\phi)H + \phi L \end{cases}
\end{aligned}$$

(Details are shown in Proof 4-9.)

Because we focus on ATM and set $X = 1$ in model analysis, here we consider that Δ_A is very close to 0.5, and change of spread, $\frac{\partial a}{\partial L} - \frac{\partial b}{\partial L}$ tends to be negative in response to an increase of L. The spread movement induced by either H or L is consistent with the volatility uncertainty change. If (H, L) move together in the same direction, the change of volatility uncertainty is 0 and is also 0 for the change of volatility spread.

$$\begin{aligned}
&(\tfrac{\partial a}{\partial H} - \tfrac{\partial b}{\partial H}) + (\tfrac{\partial a}{\partial L} - \tfrac{\partial b}{\partial L}) \\
&\cong \tfrac{2QP_0\sigma\sqrt{\tau}}{\gamma}\{2(2\Delta_A^2 - 2\Delta_A + 1)((1-\phi)H + \phi L) - ((1-\phi)H + \phi L)\} = 0 \\
&\because 2(2\Delta_A^2 - 2\Delta_A + 1) \doteqdot 1 \text{ for ATM options.}
\end{aligned}$$

However if the magnitudes for the changes of H and L are different, then the change of volatility spread and volatility uncertainty is positively correlated.

Proposition 5 *The increase in volatility uncertainty by an increase of trading volatility level results in the lower volatility percentage spread, while volatility uncertainty increases.* ($\frac{\partial(a-b)}{\partial\sigma} < 0$ *and* $\frac{\partial Var(v)}{\partial\sigma} > 0$)

Here we provide a very interesting example where an increase of volatility uncertainty does not lead to an increase of difference between volatility multipliers. In other words, volatility uncertainty is apparently not a sufficient condition for the percentage spread. To understand this proposition, we first decompose the effect on trading volatility level, σ, into two parts. The level effect increases expected fair value as well as hedging risk, pushing asking volatility to move in the opposite direction. By using condition leading spread pattern in proposition 1, we find $\frac{\partial f}{\partial \sigma} > 0$. The volatility level effect produces more expected fair value than adverse value which results from an increase in hedging variance.

$$
\begin{aligned}
&\frac{\partial f(\tau,H,L,a)}{\partial \sigma} \\
\cong \quad & \frac{2}{\sigma}\{\frac{P_0\sqrt{\tau}}{4}[a\sigma - \sigma H - \sigma L] - \frac{Q}{\gamma}[\frac{P_0^2(\Delta_A^2 - 2\Delta_A + 1)}{2}((1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})^2)] \\
& + \frac{Q}{\gamma}[\frac{P_0^2}{4}(\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau})^2] > 0
\end{aligned}
$$

Similarly, for the change of value function of optimal bidding volatility over the change of trading volatility level, the equation follows.

$$
\begin{aligned}
&\frac{\partial f(\tau,H,L,b)}{\partial \sigma} \\
\cong \quad & \frac{2}{\sigma}\{\frac{P_0\sqrt{\tau}}{4}[-b\sigma + \sigma H + \sigma L] - \frac{Q}{\gamma}[\frac{P_0^2(\Delta_B^2 + 2\Delta_B + 1)}{2}((1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})^2)] \\
& + \frac{Q}{\gamma}[\frac{P_0^2}{4}(\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau})^2] > 0
\end{aligned}
$$

Because an rise of σ would increases value function, in equilibrium, traders would lower asking and increase bidding volatility multipliers. The effect of volatility level on multipliers of asking and bidding volatility follows.

$\frac{\partial a}{\partial \sigma} = \frac{-\frac{\partial f}{\partial \sigma}}{\frac{\partial f}{\partial a}} < 0$
$\cong \frac{-1}{\sigma^2}\{[a\sigma - \sigma H - \sigma L] - \frac{Q}{\gamma}[2P_0\sqrt{\tau}(\Delta_A^2 - 2\Delta_A + 1)((1-\phi)(\sigma H)^2 + \phi(\sigma L)^2)]$
$+\frac{Q}{\gamma}[P_0\sqrt{\tau}(\phi\sigma L + (1-\phi)\sigma H)^2]$

$\frac{\partial b}{\partial \sigma} = \frac{-\frac{\partial f}{\partial \sigma}}{\frac{\partial f}{\partial a}} > 0$
$\cong \frac{1}{\sigma^2}\{[-b\sigma + \sigma H + \sigma L] - \frac{Q}{\gamma}[2P_0\sqrt{\tau}(\Delta_A^2 - 2\Delta_A + 1)((1-\phi)(\sigma H)^2 + \phi(\sigma L)^2)]$
$+\frac{Q}{\gamma}[P_0\sqrt{\tau}(\phi\sigma L + (1-\phi)\sigma H)^2]$

The bid-ask spread change over the change of trading volatility level is

$\frac{\partial a}{\partial \sigma} - \frac{\partial b}{\partial \sigma}$
$\cong \frac{1}{\sigma^2}\{-[a\sigma + b\sigma]$
$+\frac{2Q}{\gamma}[2P_0\sqrt{\tau}(\Delta_A^2 - 2\Delta_A + 1)((1-\phi)(\sigma H)^2 + \phi(\sigma L)^2)] - \frac{2Q}{\gamma}[P_0\sqrt{\tau}(\phi\sigma L + (1-\phi)\sigma H)^2]\} < 0$

(Details are shown in Proof 4-9.)

In summary, we find that the change of volatility spread in percentage of IMV is correlated with volatility uncertainty. However, we rule out that an increase (decrease) of volatility uncertainty is a sufficient condition to increase (decrease) percentage volatility spread.

4.5 Data

To test our propositions, we use trading and quoting data from the Taiwan Index options traded in the Taiwan Futures Exchange. This data base contains intraday high frequency trading and quoting information as well as daily open interests, and the sample period ranges from December 20, 2007 to July 21, 2010. Because contracts with the shortest maturity are traded most intensively, this research focuses on 30 consecutive nearest term contracts. Each contact becomes nearest term on Thursday of the third week each month and expires on Wednesday of the third week in the following month. For those contracts, averaged daily options quoting frequency and quoting volume are 93,845 and 686,087. On the other hand, the averaged number of daily transaction and trading volume are 133,856 and 314,281, while the averaged daily open interests are 248,960. All of those figures indicate that this index option is heavily traded.

Because our data involve high frequency trading records, the best price quotation is commonly updated once every few seconds. For each minute we average best quotations to calculate IMV and then average bidding and asking IMV of each minute to get daily observations. If no ask or bid price is quoted at a certain minute, the observations in that minute are dropped. If the best bidding price is lower than intrinsic value, we use intrinsic value for IMV calculation; otherwise, we could derive negative volatility, which cannot occur. Additionally because the CBOE uses the midpoint of the asking and bidding option price to derive a VIX index, the paper follows this rule to calculate effective IMV. Finally, we divide volatility spread by effective IMV to calculate volatility spread in percentage of IMV and list descriptive statistics in Table 4-5-1 through Table 4-5-6. In each table, the statistics in sub-table (a) are derived from all data sets, and statistics which exclude observations of every last 5 trading days are listed in sub-table (b).

Table 4-5-1(a) contains the simple statistics for CM IMV, JT IMV and RV. After deleting 8 days in which data are damaged or incomplete, we can derive 630 daily IMV observations. The mean for both IMV measures is about 31.562% and 37.249% for CM and JT IMV, while the mean for RV is 29.225%. To have a more concrete idea about volatility, we can transform annual implied volatility into expected one day return deviation by multiplying it by $\sqrt{\frac{1}{365}}$. Hence the mean of those volatility measures indicates 1.652%, 1.9497% and 1.5297% daily return deviation.[18] The volatility index is apparently very volatile, for three volatility measures have quite high standard deviations. The maximum of JT IMV can even reach 245.29%. When we delete observations of every final 5 trading days, the statistics of JT IMV approach those of CM IMV. It is obvious that, in the final week, the prices of ITM options are relatively high and boost the JT IMV. Although the maximum of RV also drops greatly from 100.595% to 65.097%, the situation in which RV reaches over 80% occurs only a few times but not regularly. Compared to a 4.807% drop for JT IMV, the mean of RV only decreases by 0.724% if every 5 trading days are dropped.[19] Correlations among three volatility measures are very high, especially for data excluding the final 5 trading days. The coefficient between two IMV measures is 0.9757, and it is 0.89574 (0.86907) between RV and CM (JT) IMV.

Table 4-5-3 and 4-5-4 are statistics and correlation matrices for volatility spread based on two IMV measures. As demonstrated in Table 4-5-3(a) and (b), the means of volatility spread defined by CM IMV are 0.569% and 0.337% for two different sample sizes, while the means of volatility spread are about 9.892% and 3.718% for the JT IMV measure. Hence, the volatility spread does increase dramatically, if we include ITM options into calculation of IMV measures.

[18] According to the RV formula, this measure implies that the expected return is 0. However, CM and JT IMV use a forward index, derived from put-call parity equation by ATM options, as the expected return to expiration.

[19] The number of times that RV climbs above 80% are rare. In our sample period, this rise occurs at 10/14/2008, 12/17/2008 and 5/20/2009.

The correlation of two volatility spreads is not high and is only marginally significant, as Table 4-5-4(b) shows the coefficient is 0.07648 with p-value at 0.0945. Combining the information from Table 4-5-1(b), we expected that, for the CM IMV measure, volatility spread is about 1% of CM IMV and 10% for JT IMV measure. The percentage measurement is particularly of interest in our research, for the difference between the asking and bidding multipliers in our model corresponds to volatility spread in percentage of IMV. The statistics and correlation of two percentage spreads are shown in Table 4-5-5 and 4-5-6. In the tables, two percentage spreads have a low but still significant positive correlation of 0.15837. In the following empirical work, the dependent variable is volatility spread in percentage of IMV based on CM and JT IMV calculation, and we test the maturity effect, level effect and volatility uncertainty effect on two measures separately.

4.6 Empirical Work

4.6.1 Estimation Model

Based on Propositions 1 and 2, we consider models $\mathcal{A}$ and $\mathcal{B}$ to be our estimation models, which are able to capture an increasing and convex spread pattern.

Model ($\mathcal{A}$):

$$SpPct_t = \alpha + \theta\frac{1}{TM_t} + \beta_1 * IMV_{t-1} + \beta_2 * Volatility\,Uncertainty_t + \varepsilon_t$$

Model ($\mathcal{B}$):

$$SpPct_t = \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 * IMV_{t-1} + \beta_2 * Volatility\,Uncertainty_t + \varepsilon_t$$

One of them uses a reciprocal form of TTM, and the other uses dummy variables, D_i, $i \in \{2, 3, 4, 5\}$, indicating the i-th week before expiration. Again, the dependent variable, $SpPct_t$, in the model is defined as percentage spread, which divides volatility spread by IMV and then multiples by 100.

Because of an increasing and convex spread pattern, the coefficients θ and θ_i, $i \in \{2, 3, 4, 5\}$,are expected to be positive. In addition to the maturity effect, we use one lag implied volatility, IMV_{t-1}, to test the trading level effect. Our theory shows that the spread in percentage decreases as trading level increases, meaning volatility spread does not increase in proportion to the rise of implied volatility. Hence, the coefficient of lag1 IMV is expected to be negative.

Additionally, Proposition 5 demonstrates that, due to level effect, volatility uncertainty is not a sufficient condition for the change in percentage spread, but proposition 4 shows that they should be correlated. Therefore, after controlling for level effect, we would like to test their correlation. However, testing volatility uncertainty effect on the spread without an advanced assumption is difficult, especially when we are not able to observe ΔH and ΔL. Therefore, we find a proxy for volatility uncertainty, which is defined as

$$Volatility\,Uncetainty_t = \{IMV_{t-1} - Volatility\ Premium_{t-2} - Realized\,Volatility_t\}^2$$

The high frequency realized volatility over time $(t-(T-t), t)$ is calculated as

$$Realized\,Volatility_t = \frac{1}{\tau}\sum_{i=1}^{n} R_i^2 + \frac{1}{\tau}\sum_{h=1}^{4}(\frac{n}{n-h})R_i R_{i-h}$$

While $T-t$ is the number of days to maturity, n is the number of total trading minutes during this period. R_i is the index return at i-th minute interval over time period $[t-(T-t), t]$. Our paper uses 1-minute returns, which is highly auto-correlated, so that we include the lags till 4 into RV calculation. Then, we first assume that the volatility premium is zero such that IMV_{t-1} is the market expectation for future volatility at time $t-1$, and realized volatility can be considered an unbiased estimator for future volatility if the volatility process is assumed to be a Markov process. Any deviation of realized volatility occurring at t from implied volatility at $t-1$ can be considered as a shock to the market and would increase the uncertainty of volatility.

After assuming zero volatility premium, we relax this assumption and allow it to become dynamic. While JT (2006) shows that sample mean of IMV for S&P index options is $1\tilde{}2\%$ higher than RV, the mean of IMV in our data set is also $2\tilde{}3\%$ higher than RV. Dufour, Garcia and Taamouti (2011) found that the volatility risk premium, considered as anticipated increase of volatility, does exist and has an impact on returns. Therefore, in our paper we use the moving average of difference between implied volatility and realized volatility to capture the dynamic movement of volatility premium. It is defined as,

$$Volatility\ Premium_t = \tfrac{1}{T-t}\left\{\sum_{i=1}^{T-t} IMV_{t-i} - Realized\, Volatility_{t-i}\right\}$$

T is option expiration date; thus, the number of days to expiration is $T-t$. We then use a moving average of daily volatility premium for the past $T-t$ days as a volatility premium estimator for options with days to maturity, $T-t$. Given Proposition 4, the volatility uncertainty and percentage volatility spread move in the same direction if magnitudes for change of variables (H, L) are different.

Hence, we expect positive β_2. Our models are semi-nonparametric regressions, because on the one hand we need to specify the form that incorporates structure factors, including TTM, IMV and volatility uncertainty, while on the other hand we impose no assumption on error terms and use the Newy-West nonparametric estimation to adjust statistical values.

Table 4-6-1 to 4-6-4 present the estimation results for models adopting two different volatility premium assumptions and two different IMV measures. Each table contains estimation results of Model (A) and Model (B) together.[20]

4.6.2 Estimation Results

Our results consistently agree on the existence of maturity effect and level effect, regardless of what volatility uncertainty proxy, IMV measure and regression model are used for estimation. Additionally, given JT IMV measure, both volatility uncertainty proxies are significant in explaining the percentage spread no matter what the regression model is, but, for CM volatility measures, the volatility uncertainty effect needs to be investeigated further. Maturity effect is a more prominent effect than the other two effects. Given CM IMV measure, volatility uncertainty is significant when Model $\mathcal{B}$ and dynamic volatility risk premium are applied for estimations. Given the proxy of volatility uncertainty and the IMV measure, the coefficients for level effect and volatility uncertainty

[20]Our conclusion is based on a data set excluding the final 5 trading days. Additionally, we also try to use RV^2 as factor of volatility uncertainty and list estimation results in 4-6-5 and 4-6-6. RV^2 does perform better in explaining change of percentage spread. Estimation results for total daily observations are demonstrated in 4-6-1-a, 4-6-2-a, 4-6-3-a, 4-6-4-a, 4-6-5-a and 4-6-6-a. Factors of maturity effect and level effect are significant in those tables, but estimation results do not broadly support our claim on volatility uncertainty factor. The empirical works based on S&P 500 index options are also supplemented.The data is daily closing quoted prices, the sample period ranges from January 2, 2001 to May 20, 2010. The estimations are listed in tables 4-6-7, 4-6-8 and 4-6-9.

effect are quite close for both regression models.

Table 4-6-1 presents estimation results for Model $\mathcal{A}$ and $\mathcal{B}$ based on CM IMV, in which we assume zero volatility premium for volatility risk measures. The percentage spread, implied volatility and realized volatility are all in terms of percentages. Both models have adjusted R square values of 0.0386 and 0.0405 and show that a 1% increase in IMV results in decrease of percentage spread by approximately 0.015. While percentage spread steadily increases at least by 0.5 percent of IMV from week 5 to week 2, a 10% increase in implied volatility only drops the percentage spread by about 0.15. Given that the occurrence of a 10% increase in IMV happens quite rarely, a maturity effect mainly determines the pattern of spread. In comparison to the spread based on CM IMV, the spread based on JT method is wider, and models $\mathcal{A}$ and $\mathcal{B}$ are able to explain the movement better with an adjusted R-square at 0.1012 and 0.0968. Additionally, volatility uncertainty becomes a significant factor affecting the spread, as shown in Table 4-6-2. The coefficient of the volatility uncertainty is about 0.0006 in both models, meaning 10% of realized volatility deviation from IMV results in a percentage spread increase of 0.06. The magnitude combining both effects of volatility uncertainty and IMV level is generally not as large as the maturity effect.

Table 4-6-3 and 4-6-4 list the estimation results adopting the proxy with the MA dynamic premium. R-square decreases to 0.0386 and 0.0402 for CM IMV measure and increases to 0.1045 and 0.0994 for JT IMV. Overall, the new proxy does not improve the ability of the model to explain the movement of percentage spread. Whereas volatility uncertainty becomes a marginally significant factor, based on CM IMV, with a coefficient of 0.000093 in Model $\mathcal{B}$. After adopting a new proxy, for the JT IMV measure, the magnitude of level effect greatly decreases from $|-0.09021|$ to $|-0.04073|$ in Model $\mathcal{A}$ and $|-0.08892|$ to $|-0.03645|$ in Model $\mathcal{B}$, and volatility uncertainty effect increases from about 0.00065 to 0.0057. Similar to Table 4-6-1 and 4-6-2, the maturity effect obscures the level effect and volatility uncertainty effect.

For each regression model, we first estimate under an assumption presuming independence of innovation, and then we release the i.i.d. assumption by using NW variance and co-variance estimation with different bandwidth. As suggested in Newey and West (1987), we use cube root through quintic root of total observation for ideal bandwidth and make our conclusion based on statistics adjusted by the NW estimation. Meanwhile, we do find that one model does not dominate another. Because model $\mathcal{B}$ always has a higher adjusted R-square for the CM IMV measure, while Model $\mathcal{A}$ performs better if the JT IMV measure is employed. Additionally, the volatility uncertainty proxy assuming dynamic risk premium does not improve the model, because using different volatility uncertainty proxy doesn't greatly increase R-square.

4.7 Conclusion

In this paper, based on model-free implied volatility, we document the monthly pattern of volatility bid-ask spread in the option market. We then introduce a model using the disparity between the time decay of premium contained in an option and the decay of the required premium of hedging risk to explain the increasing and convex volatility spread pattern. Because the premium contained in the derivatives decays faster, especially for options with short maturity, than the decay of required volatility risk premium, market makers need to keep raising the spread such that utility is maintained at a level on which market makers want to engage in quoting. Additionally, our model shows that volatility spread does not increase in proportion to IMV level, and we term this finding as "level effect". Finally, we show that the volatility uncertainty is highly correlated with volatility spread but is not a sufficient condition to result in an increase of percentage volatility spread.

Along with a theoretical justification, we provide empirical evidence supporting the existence of maturity effect and level effect. Although we do not have generalized evidence that volatility uncertainty is a significant cause to the increase of percentage volatility spread, we leave this effect for future research because finding a proper proxy for volatility uncertainty is still an issue that remains to be explored. As a whole, our results strongly suggest that detrending is vital for studying volatility spread in the options market. Without proper trend adjustment, statistical results can be misleading. This is because researchers tend to exaggerate the significance value by ignoring the high autocorrelation of residuals, which is the result of ignoring the trend.

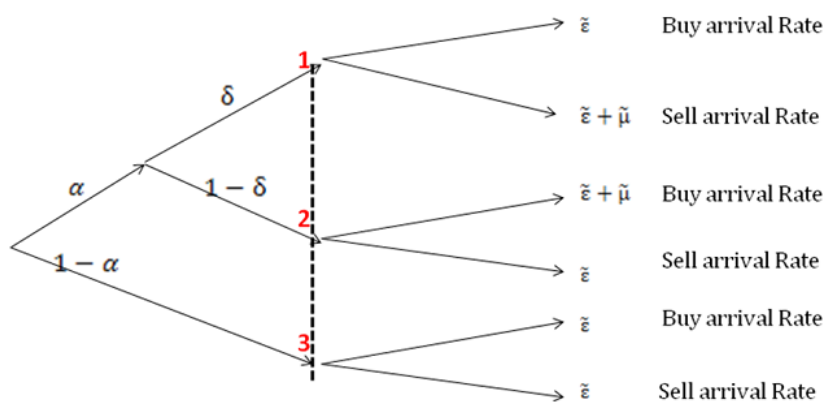
Our research also has strong implications for the following matters, some of which need to be investigated further. First, using the transaction price to derive model free implied volatility is certainly biased in forecasting future volatility, given that the premium of volatility uncertainty is included in the options price. Meanwhile, our model also suggests that implied volatility estimation, which is derived from the midpoint of bid and ask quotes, could also be biased, because a midpoint method implicitly assigns bidding and asking prices with equal weights which may not be true. Secondly, our paper also sheds light on the issues of pricing volatility (model) uncertainty for that we provide an application showing volatility (model) uncertainty premium increases in a linear way over time. We look forward to any advanced study and reexamination of how people price volatility (model) uncertainty.

APPENDIX A

CHAPTER 2 OF APPENDIX

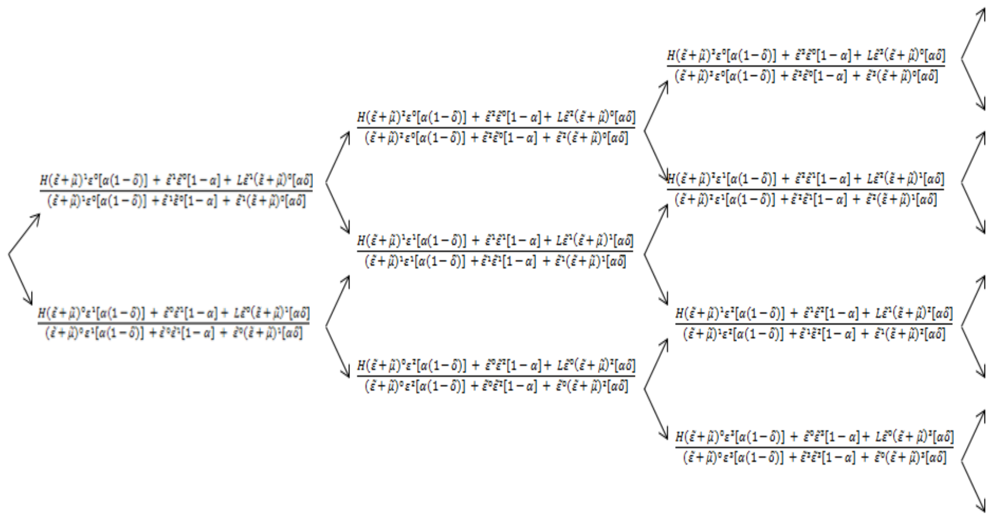
A.1 Figure 2-1-1

This figure illustrates the structure of the trade process in the PIN Model. Each day, an information event arrives with probability α, and δ is the probability that leads to the lowest price, $\underline{V}$. There are three possible scenarios, including (1) the occurrence of a bad event; (2) the occurrence of a good event; (3) no information event. Based on past transaction data, Market makers have the expectation on total number of informed trades, uninformed buys and uninformed sells, which are μ, ε and ε respectively. However market makers do not know what scenario they stand at any given point in time. The informed traders will certainly arrive when an event occurs, and whether event occurs or not the uninformed traders will trade. So the total expected number of trades is $\alpha\mu + 2\varepsilon$. For each coming intraday trade, buy(sell) arrival rates of 6 possible outcomes are defined as the expected number of buy(sell) in each scenario divided by the expected total number of trades. The sum of probabilities of 6 possible outcomes equals 1.



Trade Process of PIN Model

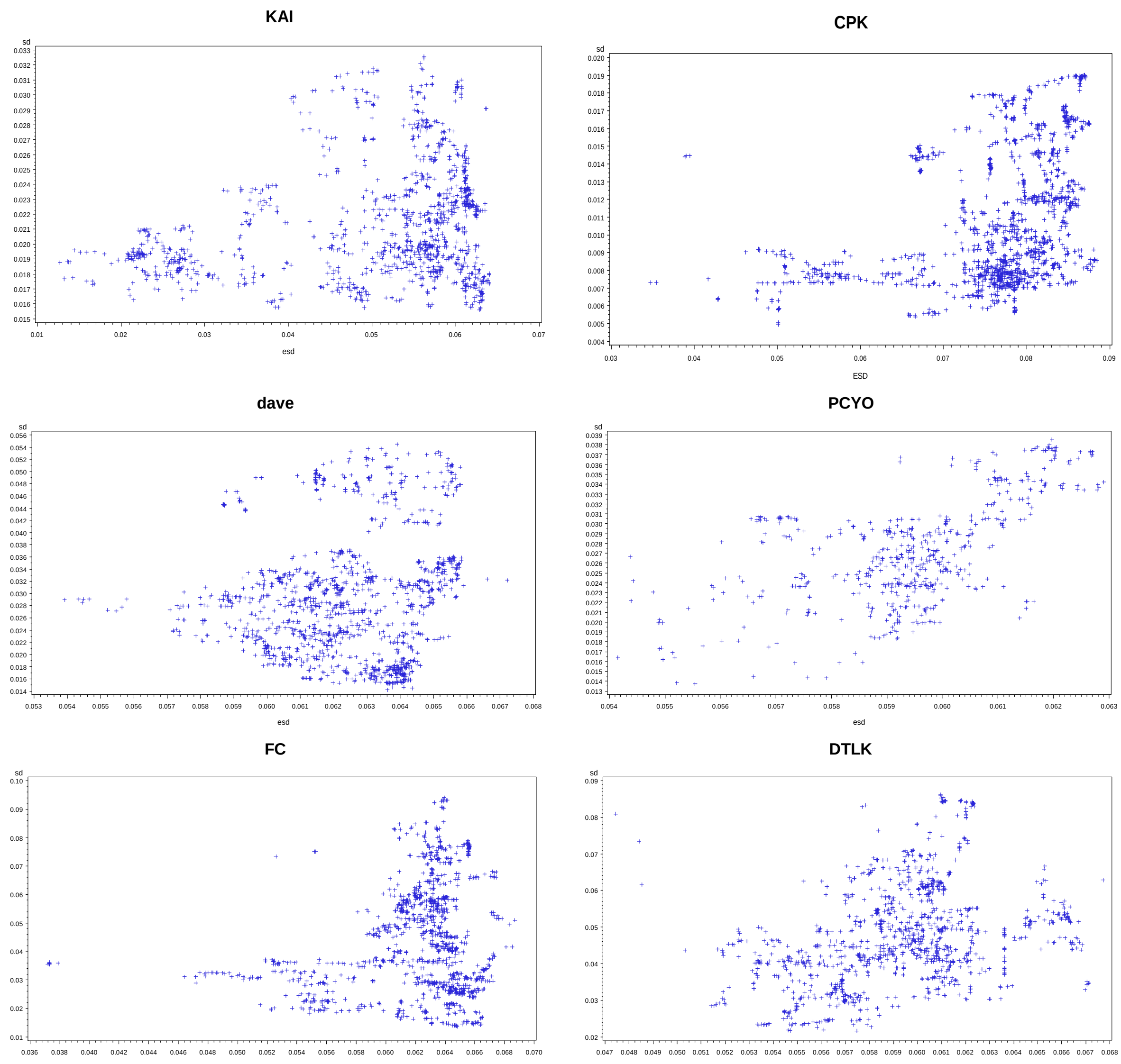
A.2 Figure 2-2-2



Tree of Trades

Figure 2-2-1 demonstrates the Tree of Trades. The market makers have the beliefs of $(\alpha, \delta, \mu, \varepsilon)$ and use the beliefs to form the quoting scheme, based on Bayes rule. In the model, market makers are assumed to provide the liquidity for the market, so the transaction price exclusively occurs on the quoting price. The general form for each note on the tree follows equation (2-3). Yesterday's close price is scaled to 1, and H and L are the daily highest price and lowest price. The Poisson probability distribution can assign probability to each note of the tree, such that we can calculate the theoretical model volatility by equation (2-4) and (2-5).

A.3 Figure 2-3-1



Relation Between Historical and Theoretical Volatility for each stock

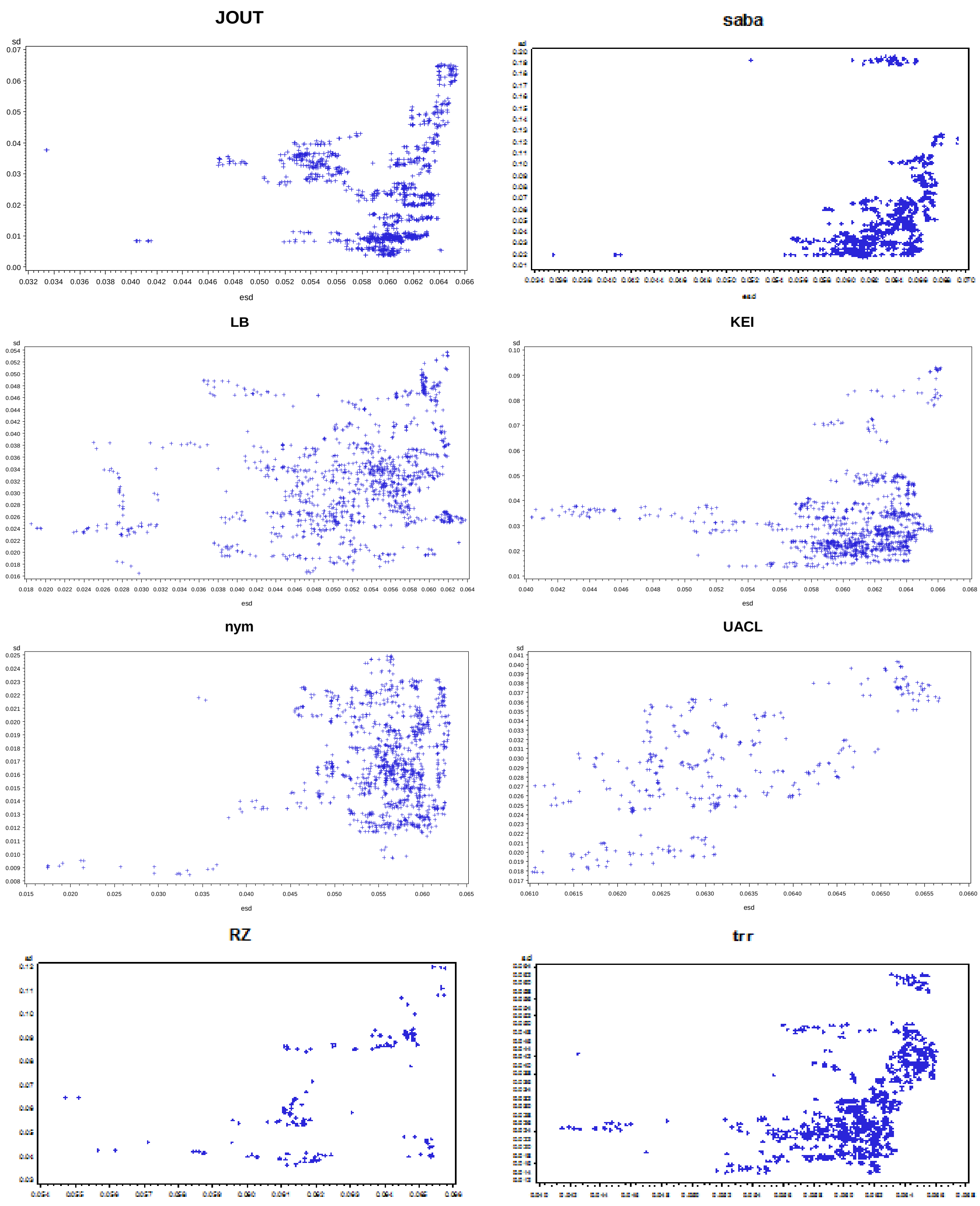


Figure 2-3-1 (Continued)

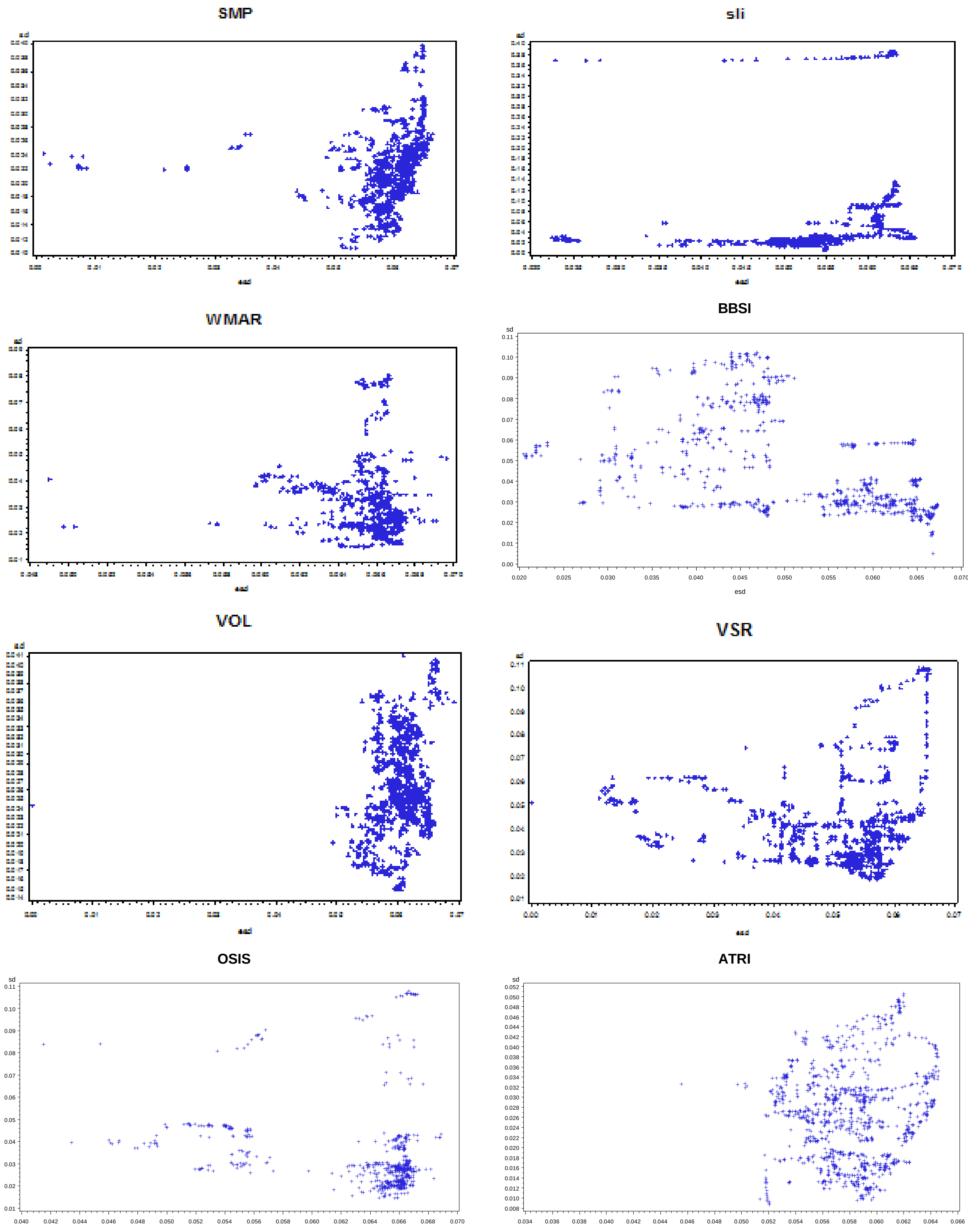


Figure 2-3-1 (Continued)

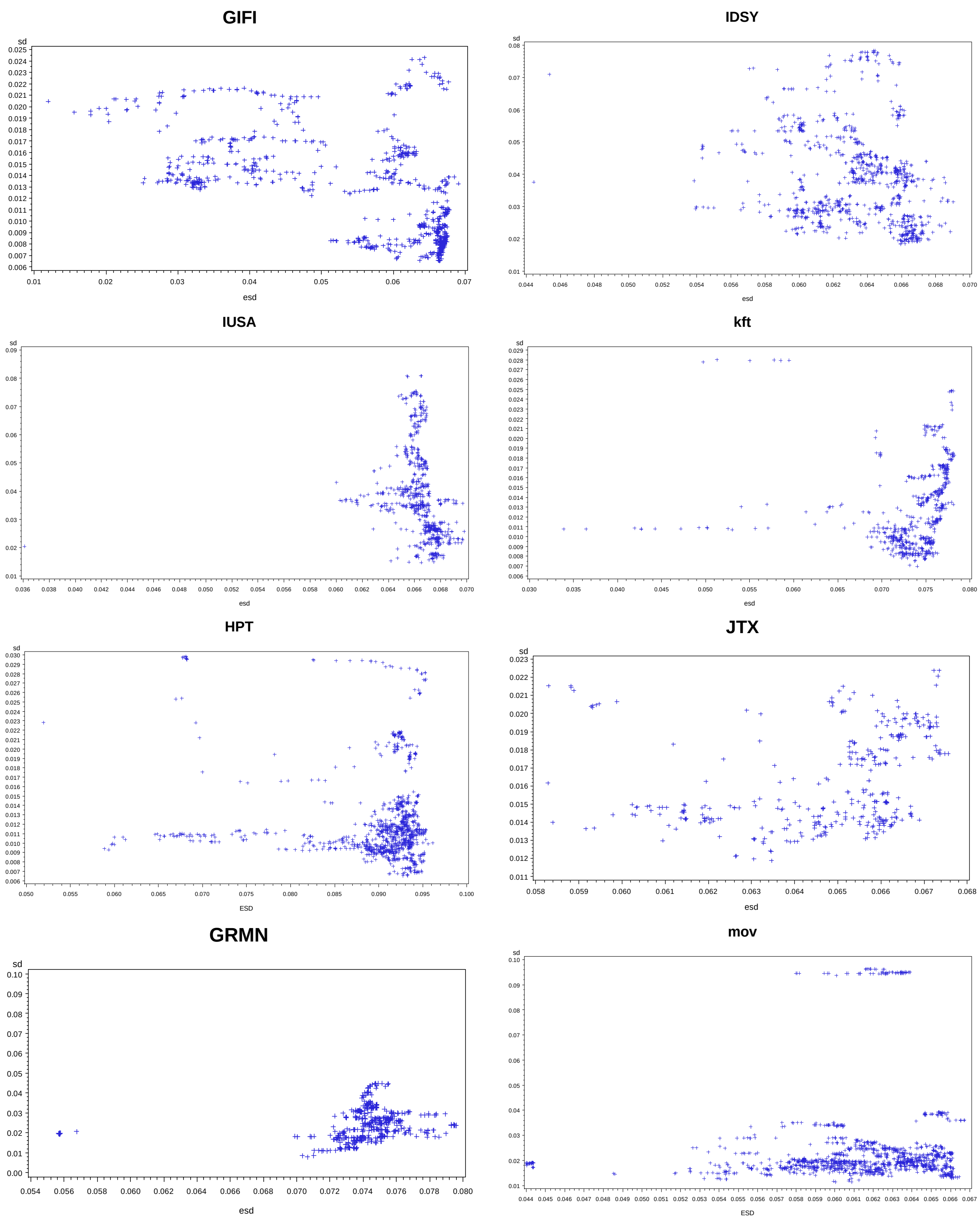


Figure 2-3-1 (Continued)

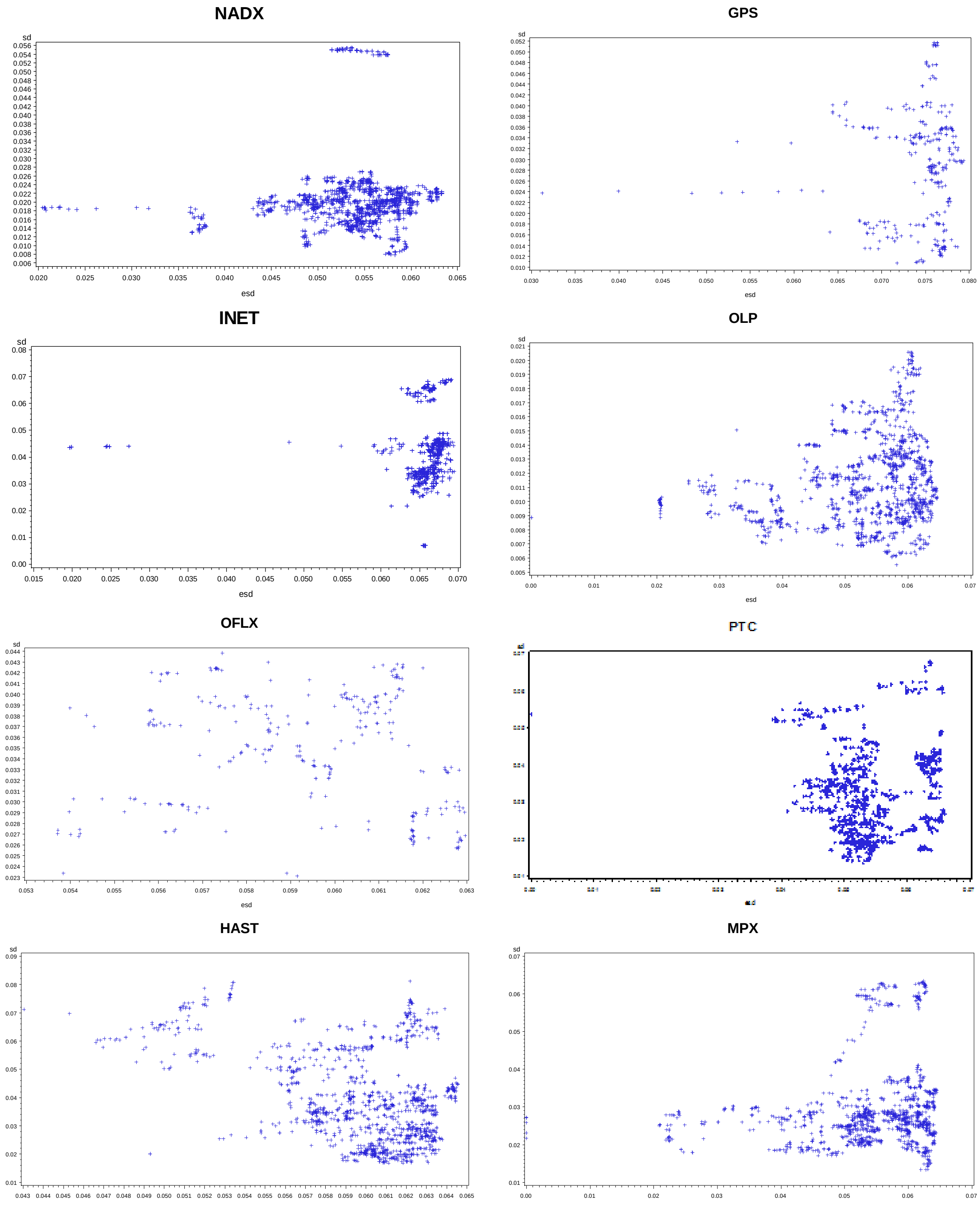


Figure 2-3-1 (Continued)

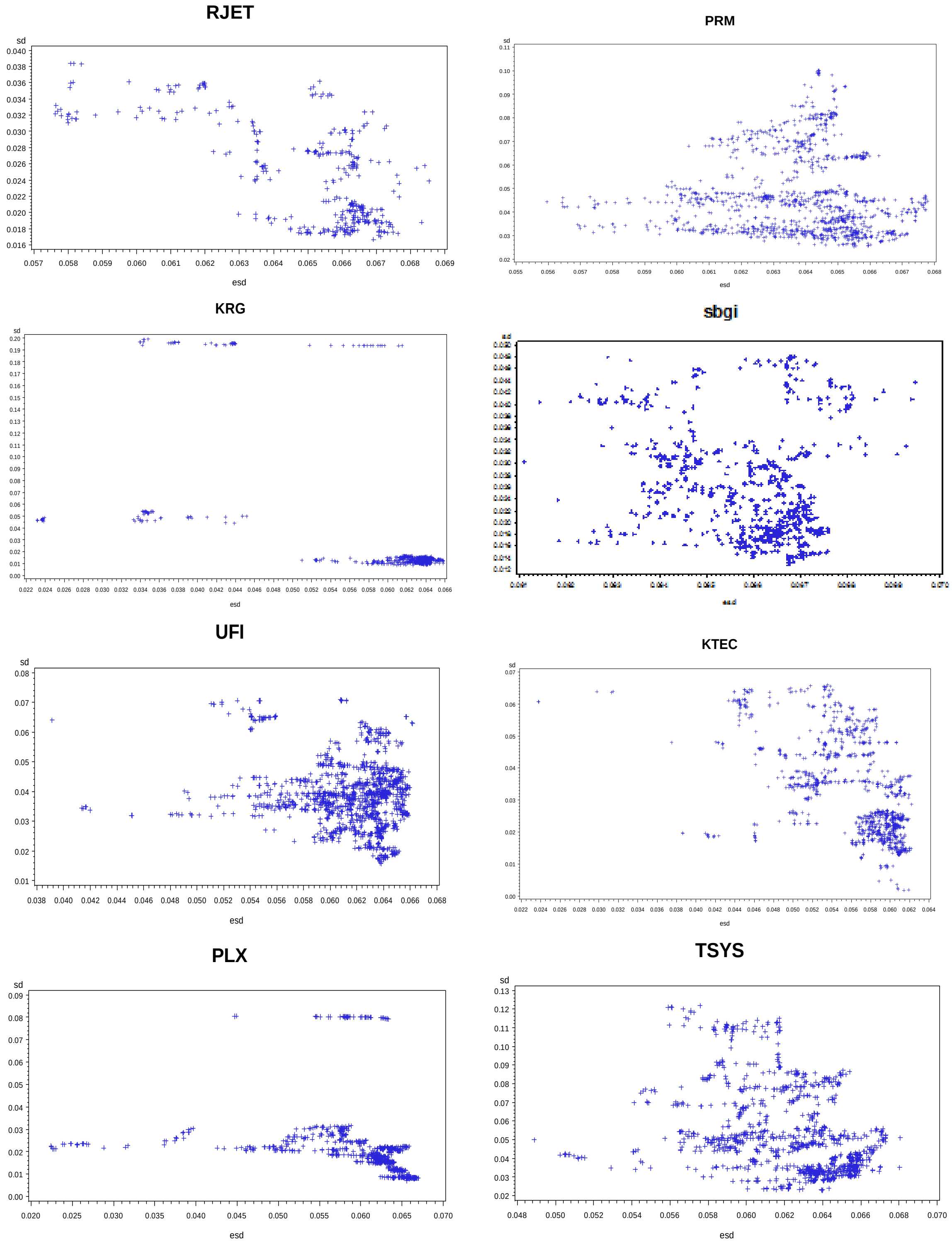
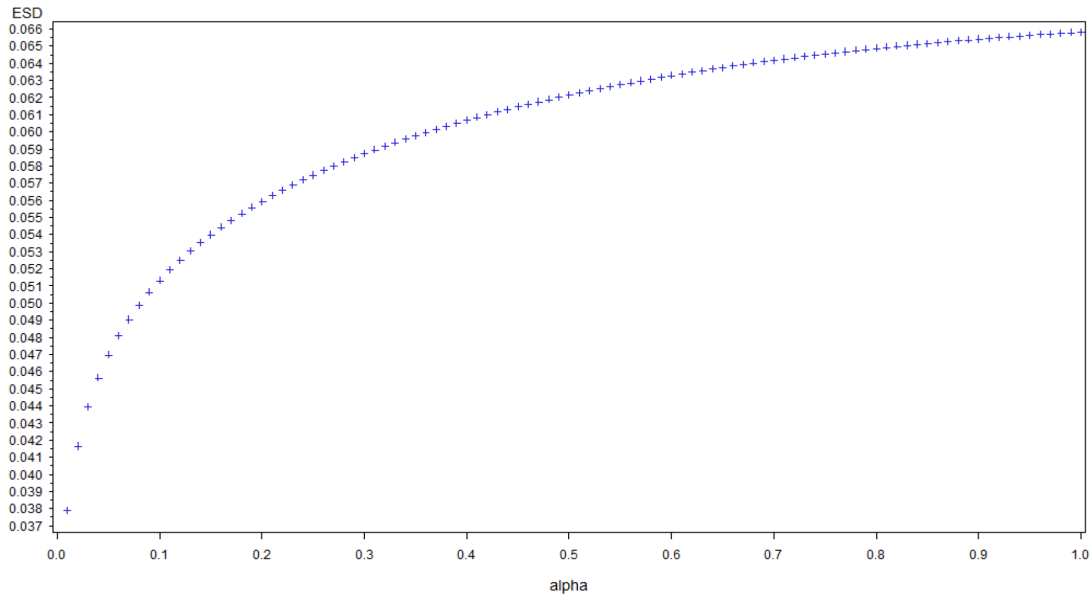


Figure 2-3-1 (Continued)

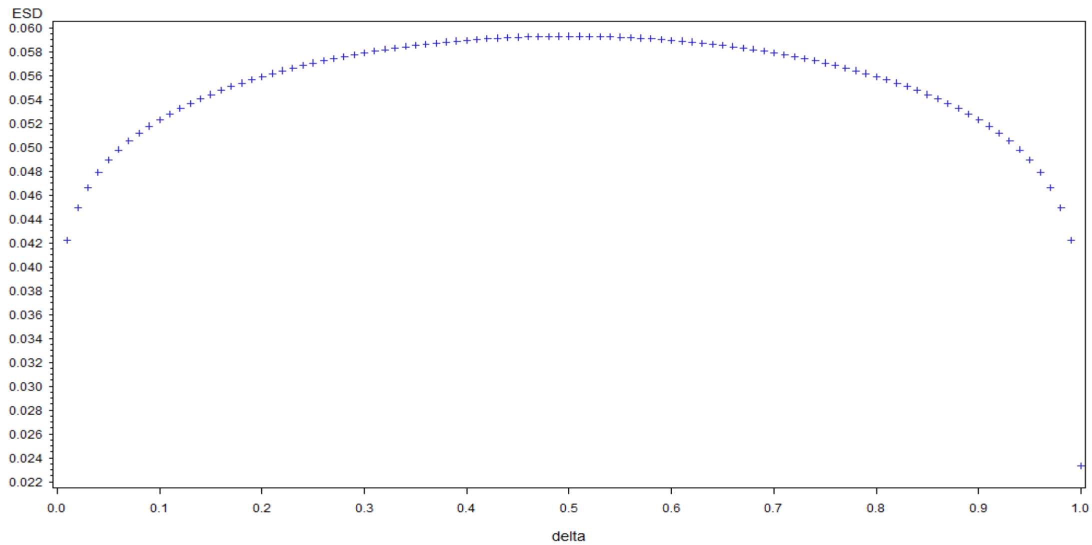
A.4 Figure 2-4-1



Theoretical Partial Analysis on α by Simulation

Figure 2-4-1 shows how theoretical volatility is affected by the α, probability of event occurrence. In the simulation, parameters,(δ,μ,ε), are assumed equal to 0.331, 31.08, and 23.1 respectively. These numbers are the cross-sectional statistic result in, “Is information Risk a Determinant of Asset Return?” by Easley, D., S. Hvidkjaer, and O’Hara. The theoretical volatility monotonically increases as α increases. In other words, given $\delta = 0.331$, $\mu = 31.8$ and $\varepsilon = 23.1$, the higher probability of event occurrence, the higher the theoretical volatility.

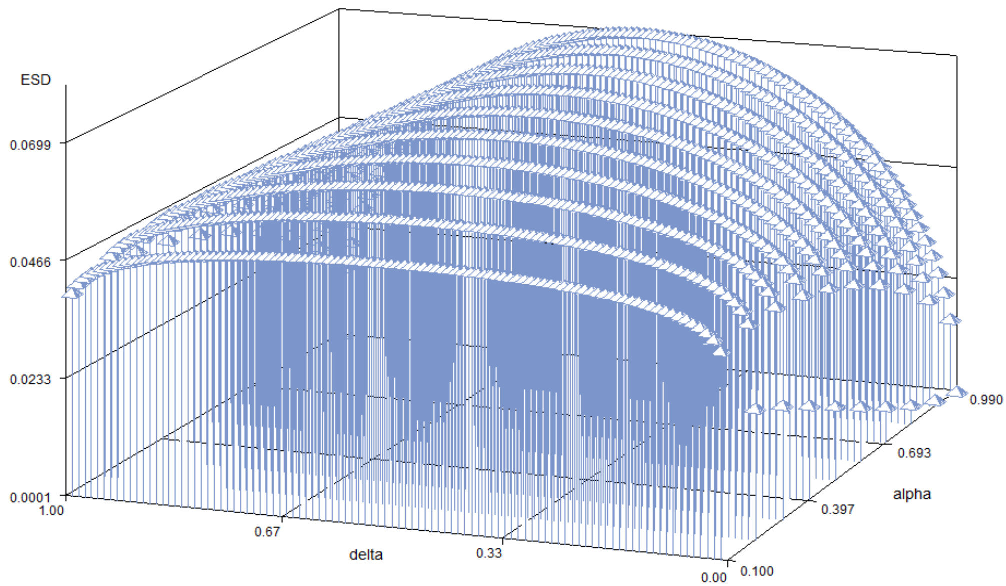
A.5 Figure 2-4-2



Theoretical Partial Analysis on δ by Simulation

This figure shows how theoretical volatility is affected by δ, the probability of the event leading stock price to the lowest daily price, $\underline{V}$. In the simulation, parameters,(α,μ,ε), are assumed equal to 0.283, 31.08, and 23.1 respectively. The theoretical volatility peaks at $\delta = 0.5$ and decreases gradually on both sides. That is, given $\alpha = 0.283$, $\mu = 31.08$ and $\varepsilon = 23.1$, the theoretical volatility reaches its maximum when market makers are not sure if the information event is good or bad news.

A.6 Figure 2-4-3



Theoretical Partial Analysis on α and δ by Simulation

Figure 2-4-3 shows how α and δ affect the theoretical volatility together. In the simulation, parameters,(μ, ε), are assumed equal to 31.08, and 23.1 respectively. The global maximum happens when $\alpha = 1$ and $\delta = 0.5$, in which the market makers know with certainty that information event is going to occur but don't know whether it will be good news or bad news. Additionally, another interesting pattern can be observed when δ closes to 0 or 1. When news is good or bad for certain, the theoretical volatility peaks as probability of event occurrence, α, is equal to 0.5.

A.7 test.

Stock with Regression Showing Factor of Theoretical Volatility is Significant.

$$\sigma_{Historical} = C + B_1 * \sigma_{Model} + B_2 * P_{Averaged} + \varepsilon_t$$

STOCK ID	C	B_1	B_2	R^2	Stock Name
KAI	0.0052	0.1114	0.000573	0.259	KADANT INC
t-stat	2.57	8.65	6.15		
CPK	-0.0016	0.1539	-0.000024	0.158	CHESAPEAKE UTILITIES
t-stat	-0.77	6.39	0.43		
DAVE	-0.0891	2.151	-0.0019	0.334	FAMOUS DAVES OF AMERICA
t-stat	-4.71	6.67	-9.68		
PCYO	-0.0738	1.5263	0.0013	0.455	PURECYCLE CORP
t-stat	-4.81	5.6	5.11		
FC	-0.0004	1.0200	-0.0048	0.398	FRANKLIN COVEY CO
t-stat	-0.03	5.32	-13.03		
DTLK	-0.0034	1.2520	0.0016	0.178	DATALINK CORP
t-stat	-2.17	4.70	3.65		
JOUT	-0.0283	1.5101	-0.0027	0.555	JOHNSON OUTDOORS INC
t-stat	-1.58	4.64	-20.08		
SABA	-0.0249	4.8873	-0.0016	0.127	SABA SOFTWARE INC
t-stat	-3.60	4.19	-1.05		
LB	0.0213	0.2912	-0.0005	0.178	LABARGE INC
t-stat	6.67	4.12	-2.96		
KEI	-0.0496	0.7063	-0.0022	0.401	KEITHLEY INSTRS INC
t-stat	-3.14	3.17	7.72		
NYM	-0.0111	0.1669	-0.00015	0.046	NYMAGIC INC
t-stat	4.4	3.15	-2.65		
UACL	-0.0693	1.3184	0.0006	0.609	UNIVERSAL TRUCKLOAD SVC
t-stat	-2.69	3.11	6.11		
RZ	-0.1962	4.3906	-0.0009	0.319	RASER TECHNOLOGIES IN
t-stat	-2.09	3.09	0.95		
TRR	-0.0258	0.7418	0.0005	0.158	TRC COS INC
t-stat	-1.76	3.00	7.72		
SMP	0.0231	0.1881	-0.0010	0.259	STANDARD MOTOR PRODUCTS
t-stat	5.69	2.96	-7.24		
SLI	0.0454	1.4933	-0.0070	0.293	SL INDUSTRIES INC
t-stat	1.68	2.79	-5.44		
WMAR	-0.0377	1.4059	-0.0014	0.398	WEST MARINE INC
t-stat	-1.19	2.6	-6.36′		
BBSI	0.0715	0.2301	-0.0026	0.638	BARRETT BUSINESS SERVIC
t-stat	17.75	2.39	-14.28		
VOL	0.0059	0.3367	0.00002	0.334	VOLT INFORMATION SCI
t-stat	0.72	2.39	0.23		
VSR	0.0452	0.4467	-0.0078	0.455	VERSAR INC
t-stat	5.84	2.3	-3.87		
OSIS	0.0069	0.0283	-0.0002	0.052	OSI SYSTEMS INC
t-stat	-1.12	2.26	-3.56		
ATRI	0.0028	0.0695	-0.0006	0.553	ATRION CORP
t-stat	3.59	2.03	-1.16		
GIFI	0.0135	0.7200	-0.0015	0.38	GULF ISLAND FABRICATION
t-stat	0.66	2.02	-9.88		
IDSY	-0.0134	1.0448	-0.0015	0.248	I D SYSTEMS INC
t-stat	-0.42	1.98	-6.7		
IUSA	0.0078	1.0019	-0.0052	0.598	INFOGROUP INC
t-stat	0.24	1.95	-12.36		
KFT	-0.0030	0.2002	-0.0009	0.45	KRAFT FOODS INC KFT
t-stat	-0.42	1.82	8.91		
JTX	-0.0120	0.3883	-0.0001	0.095	JACKSON HEWITT TAX SVC
t-stat	-0.76	1.79	1.3		
HPT	0.0114	0.1804	-0.0004	0.24	HOSPITALITY PPTYS TRUST HPT
t-stat	1.43	1.83	-5.87		
GRMN	-0.0138	0.5349	-0.00006	0.055	GARMIN LTD GRMN
t-stat	-0.62	1.82	-0.98		

A.8 Table 2-3-2 & Table 2-4-1

Table 2-3-2 Stock with Regression Showing Factor of Theo. Vol. is Not Significant.

$$\sigma_{Historical} = C + B_1 * \sigma_{Model} + B_2 * P_{Averaged} + \varepsilon_t$$

STOCK ID	C	B_1	B_2	R^2	Stock Name
MOV	-0.0043	0.3184	0.00004	0.018	MOVADO GROUP INC
t-stat	-0.39	1.64	0.87		
NADX	0.0023	0.0695	0.0006	0.061	NATIONAL DENTEX CORP
t-stat	0.75	1.35	3.2		
GPS	0.0284	0.1516	-0.0006	0.100	GAP INC GPS
t-stat	2.96	1.22	-2.83		
INET	0.037	0.1094	-0.0002	0.038	INTERNET BRANDS INC
t-stat	6.89	1.21	-0.91		
OLP	0.0069	0.0283	0.0002	0.052	ONE LIBERTY PROPERTIES
t-stat	6.58	1.13	1.57		
OFLX	0.0681	-0.0283	-0.0016	0.352	OMEGA FLEX INC
t-stat	3.45	-0.27	-4.62		
PTC	0.0365	-0.1642	0.0004	0.058	PAR TECHNOLOGY CORP
t-stat	3.42	-0.77	2.38		
HAST	0.0875	-0.4198	-0.0042	0.237	HASTINGS ENTERTAINMENT
t-stat	4.49	-1.1	-5.08		
MPX	0.0204	-0.0635	-0.0010	0.194	MARINE PRODUCTS CORP
t-stat	9.94	-1.14	4.42		
RJET	0.0798	0.3752	-0.023	0.621	REPUBLIC AIRWAYS HLD
t-stat	6.06	-1.41	-5.99		
PRM	0.0978	-0.6569	-0.0027	0.063	PRIMEDIA INC
t-stat	3.23	-1.42	-4.23		
KRG	0.1892	-2.3517	-0.0015	0.335	KITE REALTY GROUP TRUST
t-stat	3.28	-1.66	-0.57		
SBGI	0.1203	-1.6198	0.0012	0.091	SINCLAIR BROADCAST GROU
t-stat	2.31	-2.08	2.03		
UFI	0.0716	-0.6735	0.0016	0.138	UNIFI INC
t-Stat	3.91	-2.29	4.21		
KTEC	0.0817	-0.4587	-0.00246	0.589	KEY TECHNOLOGY INC
t-Stat	7.84	-2.31	-11.52		
PLX	0.0590	-0.5223	-0.0003	0.417	PROTALIX BIOTHERAPEUTIC
t-Stat	3.56	-2.64	-0.85		
TSYS	0.2465	-3.2570	0.0029	0.156	TELECOMMUNICATION SYSTE
t-Stat	5.09	-4.12	2.04		

Table 2-4-1: Statistic Data for Simulation

Variable	Mean
α(probability of event occurrence)	0.283
δ(probability of that event leads to Daily Lowest price)	0.331
μ (expected number of informed trade)	31.08
ε(expected number of uninformed trade)	23.1

APPENDIX B
CHAPTER 3 OF APPENDIX

B.1 Table 3-1-1

Summary of Previous Political Cycle Research

Literature	Conclusions and Findings
Niederhoffer, Gibbs, and Bullock. (1970)	Stock market performed far better in days or weeks following Republican presidential victories than following Democratic.
Riley and Luksetich (1980)	Stock market demonstrates consistently positive cumulative average residuals for the election of a Republican president and negative cumulative averaged residuals for a Democratic victory.
Reilly and Drzycimski (1976)	Similar finding as Riley and Luksetich (1980).
Siegel (1998)	Similar finding as Riley and Luksetich (1980).
Stovall (1992)	Fed was more accommodating and played easier money policy prior to an election.
Allvine and O'Neil (1980)	From 1948 to 1978, S&P 400 returns average 0.6% and 0.7% for first and second years of a presidential administration, but 22.1% and 9.2% for third and fourth years.
Huang (1985)	Similar patterns, as Allvine and O'Neil (1980) dcument, within sample period of 1933-1979.
Smith (1992)	S&P500 returns were 2.5% higher during Democratic administration than Republican administration, but not statistically significant.
Johnson, Chittenden and Jensen (1999)	For large-cap, there was no statistically difference in Republican or Democratic presidencies. For small-cap there was substantially 20% higher return during Democratic administrations. Additionally, stock return was significantly higher in the second half of a presidency.
Stovall (1992)	Averaged return in Dow Jones Industry during a Republican term was 30.5% versus 34.9% for a Democratic term for more than 100 years data sample.

Table 3-1-1 (Continued)

Literature	Conclusions and Findings
Hensel and Ziemba (1995)	From 1937 to 1993, investing small-cap in Democratic and large-cap in Republican administrations has higher mean return with higher standard deviation.
Bizer and Durlauf (1990)	This paper uses frequency domain examination to find that there is a tax cycle with a period of 8 years. It was suggested by regression analysis that taxes were reduced 2 years prior to successful presidential reelection attempts.
Santa-Clara and Valkanov (2003)	This paper was the first one formally testing the excessive return claim. Excess return in Democratic administrations was 6-20% higher than in Republican administrations. Monthly volatility was higher in Republican administrations as well.

B.2 Table 3-4-1

Descriptive Statistics for each Presidency

Panel A President	In Office Period	P.	N	A.R. X10^{-4}	St. X10^{-3}	Sk. X10^{-2}	Kur.
Dwight D. Eisenhower	Jan./21/1957–Jan./20/1961	R	1010	4.76	6.46	-3.89	3.74
John F. Kennedy	Jan./21/1961–Nov./22/1963	D	715	3.24	7.29	-100.04	16.93
Lyndon B. Johnson	Nov./23/1963–Jan./20/1969	D	1271	5.10	5.41	-2.35	4.58
Richard M. Nixon	Jan./21/1969–Aug./9/1974	R	1401	-0.91	8.17	24.34	2.65
Gerald Ford	Aug./10/1974–Jan./20/1977	R	619	6.57	9.72	27.15	1.46
Jimmy Carter	Jan./21/1977–Jan./20/1981	D	1009	5.84	7.76	-21.34	2.30
Ronald Reagan	Jan./21/1981–Jan./20/1989	R	2024	5.60	9.73	-280.00	54.84
George H. W. Bush	Jan./21/1989–Jan./20/1993	R	1011	5.58	7.51	-50.65	4.30
Bill Clinton	Jan./21/1993–Jan./20/2001	D	2020	6.40	9.69	-33.69	5.64
George W. Bush	Jan./21/2001–Jan./20/2009	R	2010	-0.40	13.36	-7.22	9.82
Barack Obama	Jan./21/2009-(data ends at Dec./31/2010)	D	492	12.21	14.72	6.80	2.50
Panel B	**Sub-Sample periods**	.	N	A.R.	St.	Sk.	Kur.
Republican	Jan./21/1957–Dec./31/2010		8073	2.99	9.95	-68.99	21.61
Democratic	Jan./21/1957–Dec./31/2010		5509	6.03	8.86	-23.28	7.40
Republican	Jan./21/1957–Jan./20/2001		6064	4.03	8.55	-134.04	32.15
Democratic	Jan./21/1957–Jan./20/2001		5016	5.53	8.05	-37.36	7.48
Republican	Jan./21/1957–Jan./20/1989		5054	3.71	8.75	-143.73	34.77
Democratic	Jan./21/1957–Jan./20/1989		2995	4.96	6.73	-42.59	8.00
Republican	Jan./21/1989–Dec./31/2010		3019	1.79	11.69	-11.68	11.77
Democratic	Jan./21/1989–Dec./31/2010		2514	7.30	10.91	-17.82	5.14
Total	Jan./21/1957–Dec./31/2010		13582	4.23	9.53	-54.76	17.56

This table reports the descriptive statistics for each presidency. The columns with abbreviations (P, N, A.R., St., Sk., Kur.) stand for political party, sample size, averaged daily return, standard deviation, skewness and kurtosis

B.3 Table 3-5-1

Estimation of Model (1) -Model (6)

	(1)TGARCH(1,1)	(2)GARCH(1,1) -LX	(3)GARCH(1,1) -EX	(4)E-GARCH(1,1) -LX	(5)E-X-GARCH(1,1) -LX	(6)E-X-TGARCH(1,1)
$A_0 X10^{-4}$	7.67 (9.78)***	7.39 (9.49)***	7.41 (9.37)***	5.30 (6.81)***	5.13 (6.32)***	5.41 (6.09)***
$A_1 X10^{-4}$	-1.40 (-1.24)	-1.00 (-0.86)	-1.1 (-0.95)	-0.7 (-0.64)	-0.5 (-0.44)	-0.9 (-0.70)
$A_2 X10^{-4}$	-3.1 (-1.32)	-3.3 (-1.42)	-3.0 (-1.28)	-4.9 $(-2.18)^{**}$	-4.7 $(-2.05)^{*}$	-4.5 $(-1.98)^{*}$
ϕ_1						
ϕ_2						
B_0	$1.03{*}10^{-6}$ (9.50)***	$6.90{*}10^{-7}$ (7.58)***	$9.56{*}10^{-7}$ (9.39)***	-0.24 $(-13.40)^{***}$	-0.25 $(-12.1)^{***}$	-0.25 (-10.03)
C_0	$1.29{*}10^{-1}$ (13.69)***	$0.97{*}10^{-1}$ (18.85)***	$0.95{*}10^{-1}$ (19.15)***	0.18 (23.29)***	0.18 (23.02)***	0.20 (15.93)***
$C_1 X10^{-2}$	-5.48 $(-5.20)^{***}$					-0.04 $(-2.12)^{*}$
$C_2 X10^{-2}$	1.09 (0.85)					-0.02 (-0.67)
$D_0 X10^{-1}$	0.86 (102.27)***	0.891 (163.25)***	0.89 (156.87)***	0.97 (527.92)***	0.97 (469.40)***	0.97 (373.64)***
$D_1 X10^{-2}$	5.13 (5.81)***					$-0.4{*}10^{-3}$ (-1.4)
$D_2 X10^{-3}$	-1.72 (-0.16)					$-1.1{*}10^{-3}$ $(-2.6)^{**}$
W_1		$6.67{*}10^{-7}$ (5.89)***	$8.89{*}10^{-3}$ (3.43)**	$6.06{*}10^{-3}$ (2.24)*	$5.09{*}10^{-3}$ (1.83)*	
W_2		$6.30{*}10^{-7}$ (2.06)**	$6.61{*}10^{-3}$ (1.83)	$12.16{*}10^{-3}$ (2.83)***	$12.24{*}10^{-3}$ (2.90)**	
θ				-0.47 $(-15.51)^{***}$	-0.54 $(-12.02)^{***}$	-0.49 $(-9.75)^{***}$
θ_1					0.10 (1.90)*	$0.3{*}10^{-2}$ (0.05)
θ_2					0.02 (0.35)	$0.2{*}10^{-2}$ (0.02)
log-likelihood	46907	46908	46891	46980	46983	46988

*In the parenthesis are t-stat, and $(^{***}, ^{**}, ^{*})$ denotes the p-value falling in the,$(0, 0.01)$, $(0.01, 0.05)$, $(0.05, 0.1)$ respectively.

B.4 Table 3-5-2

Estimation of Model (7) -Model (12)

	(7)TGARCH(1,1) -M	(8)GARCH(1,1) -LX-M	(9)GARCH(1,1) -EX-M	(10)E-GARCH(1,1) -LX-M	(11)E-X-GARCH(1,1) -LX-M	(12)E-X-TGARCH(1,1) -M
A_0X10^{-4}	0.93 (0.56)	0.85 (0.50)	0.94 (0.56)	2.87 (1.78)	2.29 (1.30)	3.12 $(1.91)^{*}$
A_1X10^{-4}	-3.00 $(-2.43)^{**}$	-2.80 $(-2.29)^{**}$	-2.50 $(-2.09)^{**}$	-1.40 (-1.19)	-1.20 (-0.98)	-1.60 (-1.21)
A_2X10^{-4}	-5.80 $(-2.38)^{**}$	-6.00 $(-2.45)^{**}$	-5.40 $(-2.25)^{**}$	-5.50 $(-2.40)^{**}$	-5.4 $(-2.34)^{**}$	-5.2 $(-2.21)^{**}$
ϕ_1	0.12 $(4.62)^{***}$	0.12 $(4.46)^{***}$	0.12 $(4.42)^{***}$	0.05 $(1.79)^{*}$	0.05 $(1.81)^{*}$	0.04 (1.51)
ϕ_2						
B_0	$10.27{*}10^{-7}$ $(9.61)^{***}$	$6.90{*}10^{-7}$ $(7.58)^{***}$	$9.44{*}10^{-7}$ $(9.07)^{***}$	-0.26 $(-12.04)^{***}$	-0.26 $(-11.06)^{***}$	-0.26 (-10.06)
C_0	$1.29{*}10^{-1}$ $(11.94)^{***}$	$0.96{*}10^{-1}$ $(18.52)^{***}$	$0.95{*}10^{-1}$ $(19.23)^{***}$	0.18 $(22.78)^{***}$	0.18 $(22.43)^{***}$	0.21 $(15.39)^{***}$
C_1X10^{-2}	-5.50 $(-4.23)^{***}$					-3.78 $(-2.37)^{**}$
C_2X10^{-2}	1.02 (0.65)					-1.40 (-0.59)
D_0X10^{-1}	0.86 $(90.61)^{***}$	0.89 $(163.25)^{***}$	0.89 $(156.85)^{***}$	0.97 $(456.92)^{***}$	0.97 $(411.01)^{***}$	0.97 $(372.07)^{***}$
D_1	$0.51{*}10^{-1}$ $(4.87)^{***}$					$-0.5{*}10^{-3}$ (-1.56)
D_2	$-0.92{*}10^{-3}$ (-0.07)					$-1.26{*}10^{-3}$ $(-2.76)^{**}$
W_1		$6.52{*}10^{-7}$ $(5.78)^{***}$	$8.80{*}10^{-3}$ $(3.40)^{***}$	$6.73{*}10^{-3}$ $(2.36)^{**}$	$5.90{*}10^{-3}$ $(2.03)^{**}$	
W_2		$6.74{*}10^{-7}$ $(2.20)^{***}$	$6.69{*}10^{-3}$ $(1.85)^{*}$	$12.70{*}10^{-3}$ $(2.91)^{***}$	$12.85{*}10^{-3}$ $(2.93)^{***}$	
θ				-0.47 $(-15.43)^{***}$	-0.54 $(-12.35)^{***}$	-0.48 $(-11.39)^{***}$
θ_1					0.11 $(2.19)^{**}$	$0.46{*}10^{-2}$ (0.07)
θ_2					0.02 (0.38)	$0.60{*}10^{-2}$ (0.07)
log-likelihood	46917	46918	46901	46982	46985	46989

*In the parenthesis are t-stat, and $(^{***}, ^{**}, ^{*})$ denotes the p-value falling in the,$(0, 0.01)$, $(0.01, 0.05)$, $(0.05, 0.1)$ respectively.

B.5 Table 3-5-3

Estimation of Model (13) -Model (18)

	(13)TGARCH(1,1) -BJ-M	(14)GARCH(1,1) -LX-BJ-M	(15)GARCH(1,1) -EX-BJ-M	(16)E- GARCH(1,1) -LX–BJ-M	(17)E-X- GARCH(1,1) -LX–BJ-M	(18)E-X- TGARCH(1,1) -BJ-M
$A_0 X10^{-2}$	-1.45 (-1.17)	-4.98 (-0.63)	-1.17 (-1.16)	-0.22 (1.71)	0.17 (1.76)	-0.18 (-1.29)
$A_1 X10^{-2}$	-0.59 $(-1.74)^{*}$	-0.28 $(-2.16)^{**}$	-0.74 $(-1.66)^{*}$	-0.34 $(-2.30)^{**}$	0.16 (1.32)	-0.27 $(-1.79)^{*}$
$A_2 X10^{-2}$	-0.95 (-1.50)	-9.38 (-0.66)	-0.57 (-1.26)	-0.25 $(-2.11)^{**}$	-0.10 (-1.42)	-0.18 (-1.59)
ϕ_1	0.10 $(3.55)^{***}$	0.09 $(3.35)^{***}$	0.10 $(3.59)^{***}$	0.04 $(1.74)^{*}$	0.04 (1.51)	0.05 $(1.96)^{*}$
ϕ_2	17.11 (1.29)	46.21 (0.72)	14.76 (1.25)	4.46 $(2.08)^{**}$	-2.38 (-1.38)	3.57 (1.67)
B_0	$2.92{*}10^{-12}$ (0.01)	$-4.09{*}10^{-7}$ $(-1.02)^{***}$	$3.25{*}10^{-10}$ (0.01)	-0.14 $(-6.61)^{***}$	-0.14 $(-6.47)^{***}$	-0.15 $(-6.22)^{***}$
C_0	$1.07{*}10^{-1}$ $(11.50)^{***}$	$0.88{*}10^{-1}$ $(15.90)^{***}$	$0.89{*}10^{-1}$ $(16.91)^{***}$	$1.88{*}10^{-1}$ $(20.83)^{***}$	0.18 $(20.35)^{***}$	0.18 $(13.95)^{***}$
$C_1 X10^{-2}$	-3.44 $(-3.06)^{***}$					1.04 (0.56)
$C_2 X10^{-2}$	1.68 (1.13)					-2.40 (-0.89)
D_0	0.88 $(90.14)^{***}$	0.89 $(130.98)^{***}$	0.90 $(137.77)^{***}$	0.99 $(447.52)^{***}$	0.99 $(421.17)^{***}$	0.99 $(373.94)^{***}$
D_1	0.02 (1.68)					$2.07{*}10^{-3}$ $(2.85)^{***}$
D_2	-0.02 (-0.97)					$-1.16{*}10^{-3}$ (-1.25)
W_1		$5.18{*}10^{-7}$ $(2.16)^{**}$	$-1.36{*}10^{-2}$ (-1.8)	$-2.36{*}10^{-2}$ $(-3.81)^{***}$	$-2.34{*}10^{-2}$ $(-3.64)^{***}$	
W_2		$-7.42{*}10^{-6}$ $(2.3)^{**}$	$0.30{*}10^{-2}$ (0.38)	$1.01{*}10^{-2}$ (1.22)	$2.76{*}10^{-2}$ $(3.77)^{***}$	
L_0	$1.65{*}10^{-3}$ $(2.71)^{***}$	$3.3{*}10^{-2}$ $(2.18)^{**}$	$1.48{*}10^{-3}$ $(2.65)^{**}$	$1.00{*}10.^{-3}$ $(2.40)^{**}$	$0.58{*}10.^{-3}$ $(2.20)^{**}$	$0.94{*}10.^{-3}$ $(2.46)^{**}$
L_1	$1.52{*}10^{-3}$ $(2.31)^{**}$	$0.33{*}10^{-2}$ (0.77)	$2.36{*}10^{-3}$ $(2.38)^{**}$	$4.21{*}10^{-3}$ $(2.93)^{***}$	$2.48{*}10^{-3}$ $(2.43)^{**}$	$3.67{*}10^{-3}$ $(2.81)^{***}$
L_2	$3.26{*}10^{-3}$ (1.56)	$2.43{*}10^{-2}$ $(2.21)^{**}$	$2.19{*}10^{-3}$ (1.47)	$3.6{*}10^{-3}$ $(1.90)^{*}$	$-0.58{*}10^{-3}$ (-0.8)	$2.62{*}10^{-3}$ (1.52)
θ				-0.51 $(-15.03)^{***}$	-0.54 $(-11.05)^{***}$	-0.54 $(-10.54)^{***}$
θ_1					0.03 (0.52)	0.08 (1.07)
θ_2					0.08 (1.18)	-0.02 (-0.23)
$\gamma^2_{x10^{-4}}$	4.49 $(3.74)^{***}$	3.55 $(3.87)^{***}$	4.40 $(3.46)^{***}$	3.31 $(3.98)^{***}$	5.49 $(3.09)^{***}$	3.57 $(4.45)^{***}$
log-likelihood	47075	47075	47070	47128	47126	47126

*In the parenthesis are t-stat, and (***,**,*) denotes the p-value falling in the,$(0, 0.01)$, $(0.01, 0.05)$, $(0.05, 0.1)$ respectively.

B.6 Table 3-7-1

Robustness Test on TGARCH(1,1)-EX-BJ-X-M for Artificial Republican Presidency Period

			Artificial extension of Republican presidency period				
	R1 test	R2 test	0	3	4	5	6
A_0X10^4							
A_1X10^4	-1.05	-1.60	-3.40	-4.70	-1.80	0.54	-0.80
	(**0.99**)	(**−1.08**)	(**−2.29**)**	(**−3.39**)**	(**−0.93**)	(**0.29**)	(**−0.29**)
A_2X10^4	-8.50	-9.30	-7.90	-9.40	-9.30	-8.60	-9.20
	$(-3.51)^{***}$	$(-3.69)^{***}$	$(-3.18)^{***}$	$(-3.56)^{***}$	$(-3.61)^{***}$	$(-3.45)^{***}$	$(-3.32)^{***}$
ϕ_1	0.09	0.09	0.10	0.09	0.09	0.09	0.09
	$(3.36)^{***}$	$(2.97)^{***}$	$(3.63)^{***}$	$(3.57)^{***}$	$(3.44)^{***}$	$(3.43)^{***}$	$(3.38)^{***}$
ϕ_2	0.15	0.31	0.31	0.52	0.37	0.16	0.29
	(0.86)	(1.27)	(1.59)	$(2.01)^{**}$	(1.45)	(0.62)	(0.89)
B_0X10^7	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
C_0X10	0.83	0.88	1.07	1.19	1.30	1.20	1.04
	$(11.46)^{***}$	$(14.87)^{***}$	$(11.46)^{***}$	$(8.85)^{***}$	$(7.80)^{***}$	$(4.53)^{***}$	$(6.29)^{***}$
C_1X10^2	1.04	0.02	-3.17	-3.85	-4.85	-3.67	-1.73
	(**1.18**)	(**0.01**)	(**−2.84**)**	(**−2.63**)***	(**−2.80**)***	(**−1.39**)	(**−1.07**)
C_2X10^2	-0.40	-0.35	0.57	0.34	0.10	-0.10	-0.22
	(-0.58)	(-0.47)	(0.84)	(0.5)	(0.13)	(-0.20)	(-0.33)
D_0X10	8.98	9.02	8.82	8.81	8.70	8.70	8.91
	$(80.77)^{***}$	$(92.10)^{***}$	$(94.74)^{***}$	$(73.03)^{***}$	$(59.03)^{***}$	$(30.74)^{***}$	$(57.90)^{***}$
D_1X10^2	-0.45	-1.03	1.78	1.62	2.95	2.60	0.55
	(-0.16)	(-1.28)	(1.47)	(1.16)	$(1.86)^{*}$	(0.90)	(0.37)
D_2X10^3							
W_1X10^3							
W_2X10^3							
L_0X10^3	2.44	1.85	1.62	1.07	1.50	2.52	1.64
	$(2.50)^{**}$	(1.54)	$(3.20)^{***}$	$(2.80)^{***}$	$(2.41)^{**}$	(1.53)	$(2.14)^{**}$
L_1X10^3	-0.69	0.51	1.65	1.71	0.82	-0.42	0.58
	(**−0.72**)	(**0.75**)	(**2.38**)**	(**2.77**)**	(**1.41**)	(**−0.30**)	(**0.84**)
L_2X10^3	3.04	2.99	0.46	2.18	2.52	2.84	2.97
	$(2.01)^{**}$	(1.44)	(1.39)	(1.75)	(1.70)	$(1.85)^{*}$	$(2.15)^{**}$
γ^2X10^4	4.43	4.42	4.58	4.55	4.43	4.38	4.33
	$(3.68)^{***}$	$(2.58)^{**}$	$(3.96)^{***}$	$(4.13)^{***}$	$(3.94)^{***}$	$(3.58)^{***}$	$(4.12)^{***}$
log-likelihood	47052	47052	47072	47071	47059	47054	47052

*R1 test is the experimental test that sets odd dates to dummy 1 as Republican presidency and even dates to 0 as Democratic presidency. For R 2 test, days in odd months are set as Republican dummy 1, and days in even months are set as dummy 0. Column 4 titled with 0 is the estimation based on real presidency data. Column 5,6,7 and 8 are results on 3,4,5,6 years experimental extension on each Republican period.

B.7 Higher order movement for GARCH Jump models

(13)Model 13:T-GARCH(1,1)-BJX-M

$$\begin{cases} Var(UR_t) = \frac{B_0+CX_{t-1}q\gamma^2}{(1-CX_{t-1}-DX_{t-1})} + q\gamma^2 \\ E(UR_t^4) = \frac{3[B_0+2B_0CX_{t-1}E(UR_{t-1}^2)+2B_0DX_{t-1}E(h_{t-1})+2CX_{t-1}DX_{t-1}E(UR_{t-1}^2h_{t-1})]}{1-D^2X_{t-1}-3C^2X_{t-1}} \\ UR_t = r_t - AX_t - \phi H_t, E(h_t) = \frac{B_0+CX_{t-1}q\gamma^2}{1-CX_{t-1}-DX_{t-1}}, E(UR_{t-1}^2) = Var(UR_t) \\ E(UR_{t-1}^2h_{t-1}) = (1-q+q\gamma^2)E(h_{t-1}) + qE(h_{t-1}^2) \end{cases}$$

$$\begin{cases} \frac{\Delta Var(r_t|I_{t-1})}{\Delta D_{(R,t-1)}} = \frac{\Delta h_t}{\Delta D_{R,t-1}} + \frac{\Delta qr^2}{\Delta D_{R,t-1}} = C_1(UR_{t-1})^2 - 2CX_{t-1}(UR_{t-1})A_1 + D_1h_{t-1} + L_1\gamma^2 \\ \frac{\Delta^2 Var(r_t|I_{t-1})}{\Delta D_{(R,t-1)}^2} = -2C_1(UR_{t-1})A_1 - 2C_1(UR_{t-1})A_1 + 2CX_{t-1}A_1^2 \\ \frac{\Delta Var(r_{t+n}|I_{t-1})}{\Delta D_{(R,t-1)}} = (DX_{t-1})^n[C_1(UR_{t-1})^2 - 2CX_{t-1}(UR_{t-1})A_1 + D_1h_{t-1} + L_1\gamma^2] \end{cases}$$

(14)Model 14: GARCH-LX-BJX-M

$$\begin{cases} Var(UR_t) = \frac{B_0+CX_{t-1}q\gamma^2}{(1-CX_{t-1}-DX_{t-1})} + q\gamma^2 \\ E(UR_t^4) = \frac{3[B_0+2B_0CX_{t-1}E(UR_{t-1}^2)+2B_0DX_{t-1}E(h_{t-1})+2CX_{t-1}DX_{t-1}E(UR_{t-1}^2h_{t-1})]}{1-D^2X_{t-1}-3C^2X_{t-1}} \\ UR_t = r_t - AX_t - \phi H_t, E(h_t) = \frac{B_0+CX_{t-1}q\gamma^2}{1-CX_{t-1}-DX_{t-1}}, E(UR_{t-1}^2) = Var(UR_t) \\ E(UR_{t-1}^2h_{t-1}) = (1-q+q\gamma^2)E(h_{t-1}) + qE(h_{t-1}^2) \end{cases}$$

$$\begin{cases} \frac{\Delta Var(r_t|I_{t-1})}{\Delta D_{(R,t-1)}} = \frac{\Delta h_t}{\Delta D_{R,t-1}} + \frac{\Delta qr^2}{\Delta D_{R,t-1}} = C_1(UR_{t-1})^2 - 2CX_{t-1}(UR_{t-1})A_1 + D_1h_{t-1} + L_1\gamma^2 \\ \frac{\Delta^2 Var(r_t|I_{t-1})}{\Delta D_{(R,t-1)}^2} = -2C_1(UR_{t-1})A_1 - 2C_1(UR_{t-1})A_1 + 2CX_{t-1}A_1^2 \\ \frac{\Delta Var(r_{t+n}|I_{t-1})}{\Delta D_{(R,t-1)}} = (DX_{t-1})^n[C_1(UR_{t-1})^2 - 2CX_{t-1}(UR_{t-1})A_1 + D_1h_{t-1} + L_1\gamma^2] \end{cases}$$

APPENDIX C
CHAPTER 4 OF APPENDIX

C.1 Figure 4-1-1

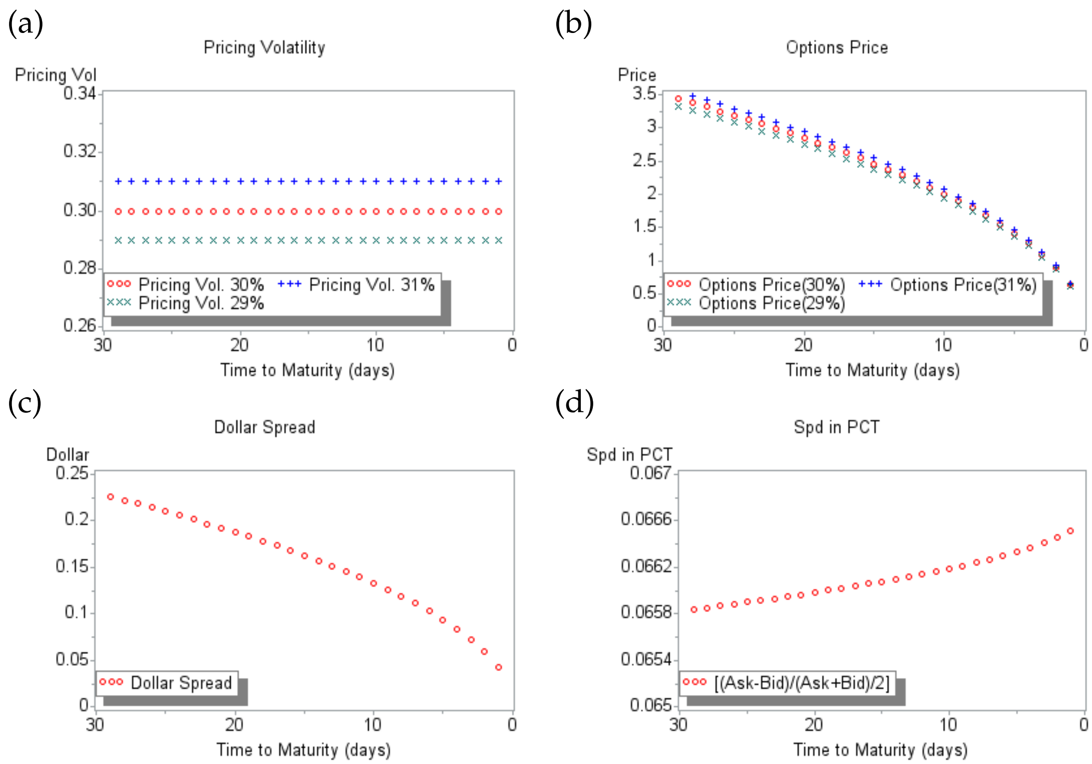
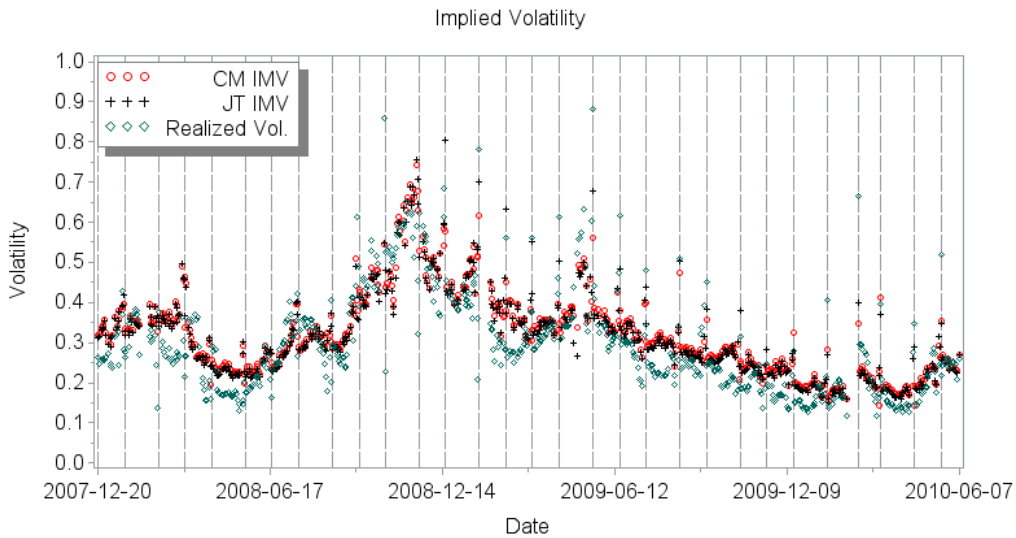


Figure 4-1-1: Example of Dollar Measure Spread. (Black-Schole-Merton Model)

Figure 4-1-1 illustrates an example that the pattern of dollar measure spread can be the natural result of pricing models. We assume a 1% risk free interest rate and 30% volatility. Market makers use 29% and 31% volatility for quoting asking and bidding prices. As seen in chart (c), for ATM options, the dollar spread naturally decays to 0 given that all other factors are fixed, including spot price and volatility. More importantly, the percentage spread in dollar measurement demonstrates an upward trend with convexity. The example shows that the increasing percentage spread based on dollar measure can be the natural result of pricing models.

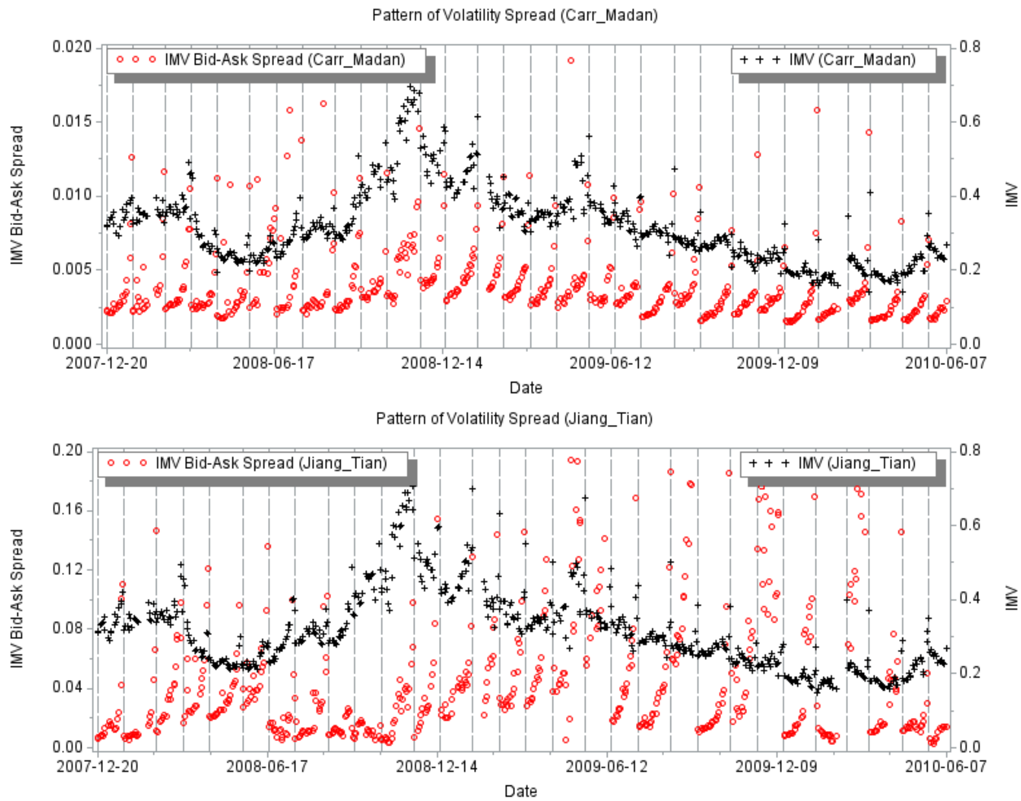
C.2 Figure 4-2-1



Implied Volatility

Figure 4-2-1 demonstrates two model-free IMV indexes, proposed by CM and JT, as well as historical daily high frequency realized volatility (RV). For any given time, t, IMV is forward looking expected volatility for the period of time to maturity, $T - t$, where T is the expiration date. We adopt data window, $[t - (T - t), t]$ for RV calculation and consider it a good reference for future volatility. As shown in the graph, those three measures are highly correlated, and they all reached the highest volatility, approximately 80%, during the 2008 financial crisis. Moreover, IMV is frequently higher than RV in our data sample.

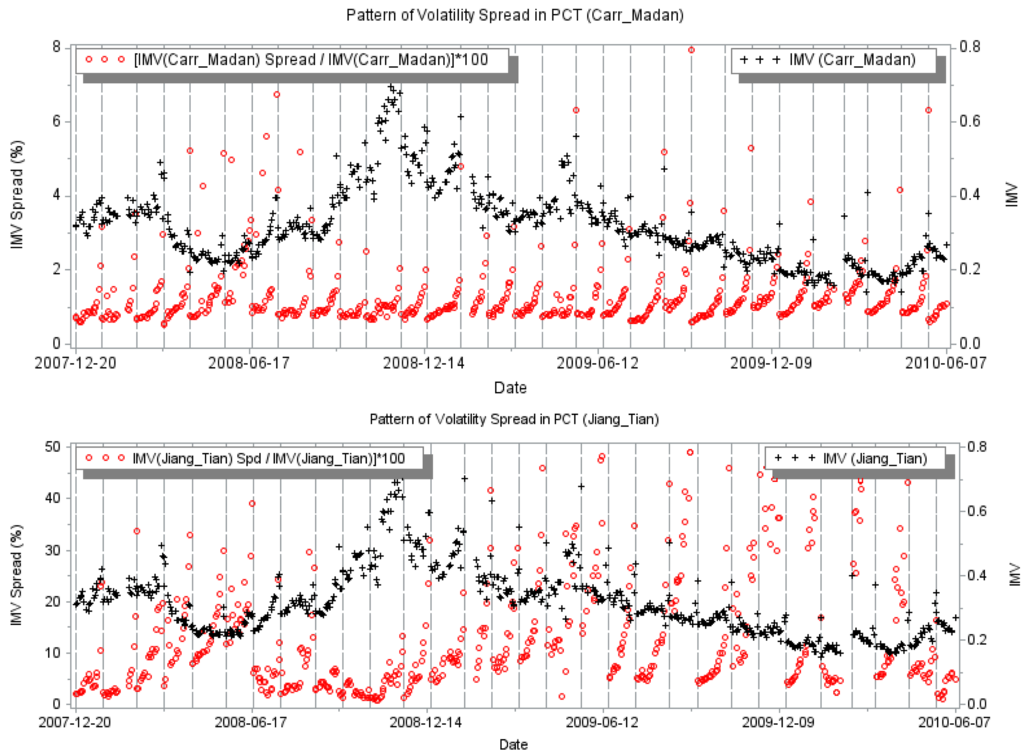
C.3 Figure 4-2-2



Implied Volatility Spread

Figure 4-2-2 shows the pattern of implied volatility spread over the sample period. The implied volatility in the first chart is calculated by the CM method, and in the second chart it is calculated by JT method. As shown in each chart, the volatility spread enlarges with increasing magnitude when the contract progresses toward the expiration date, denoted by dashed lines.

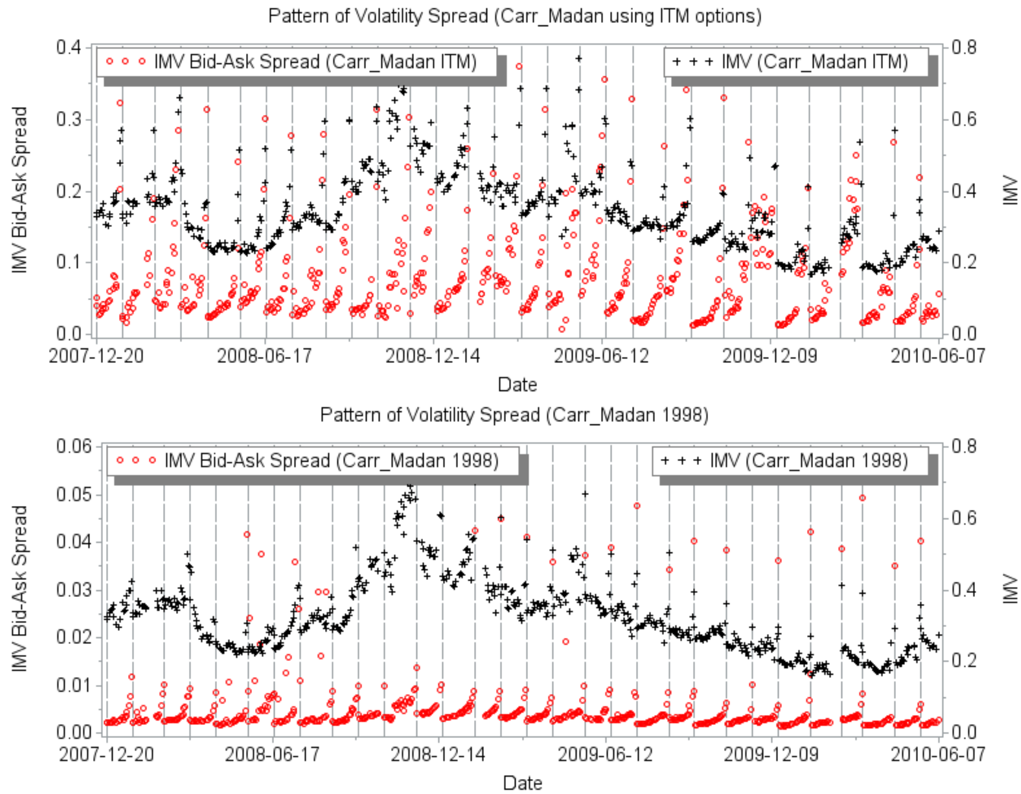
C.4 Figure 4-2-3



Implied Volatility Spread in the PCT of Implied Volatility

Figure 4-2-3 shows the pattern of the percentage spread, dividing volatility by IMV. The pattern is similar to implied volatility spread demonstrated in Figure 4-2-2, but it is very obvious that the width of the spread is very different for two different IMV measures.

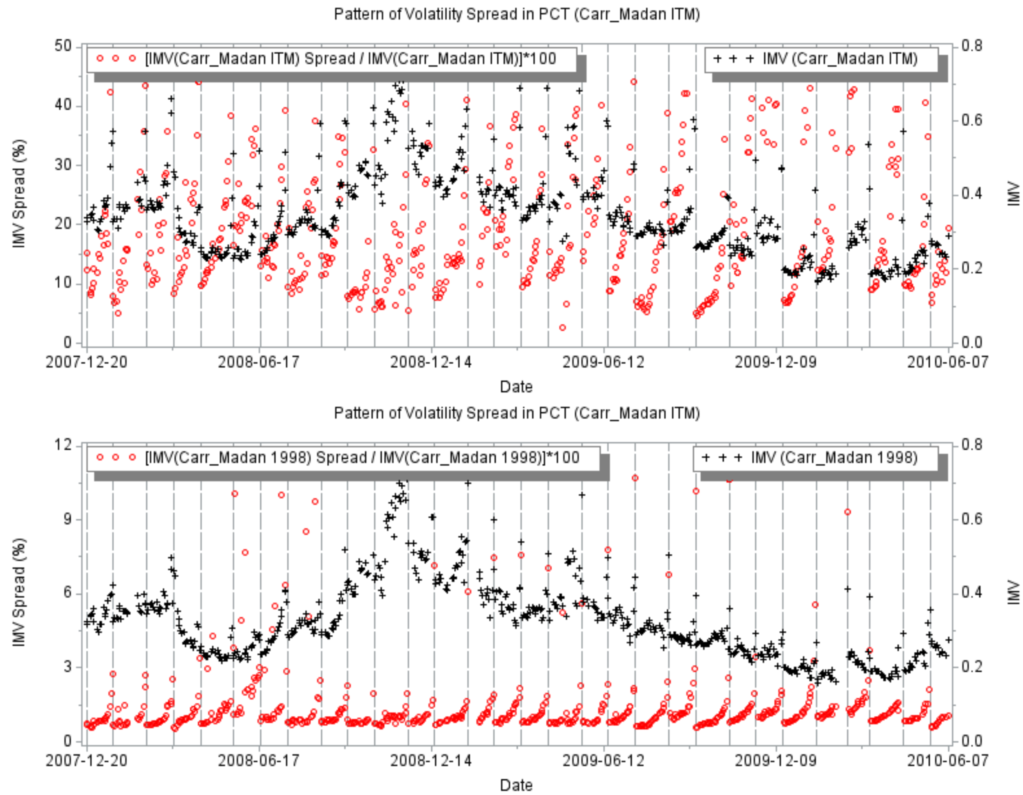
C.5 Figure 4-2-4



Implied Volatility Spread by CMITM & CM (1998) IMV

Graphs above show the pattern of volatility spread over sample period. In comparison with Figure 4-2-2, estimations of implied volatility here are CMITM and CM (1998) IMV. The implied volatility in the first chart is calculated by the CMITM method and by CM (1998) in the second chart. As shown in first chart, the volatility spread based on the ITM options prices also enlarges with increasing magnitude when the contract progresses toward expiration date, denoted by dashed lines.

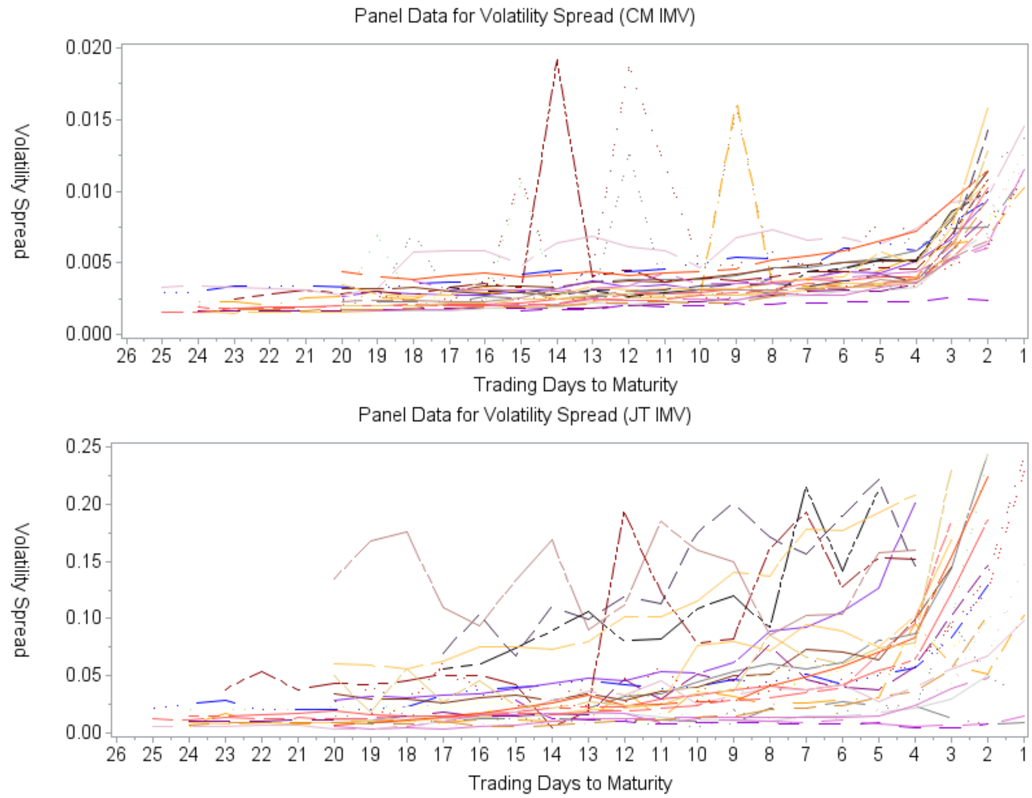
C.6 Figure 4-2-5



Implied Volatility Spread in the PCT of Implied Volatility by CMITM & CM (1998)

Figure 4-2-5 shows the pattern of the percentage spread, dividing volatility spread by IMV. In comparison with Figure 4-2-3, the calculations of implied volatility here are CMITM and CM (1998) methods. As shown, the volatility percentage spreads based on CMITM IMV and CM (1998) also demonstrate same pattern.

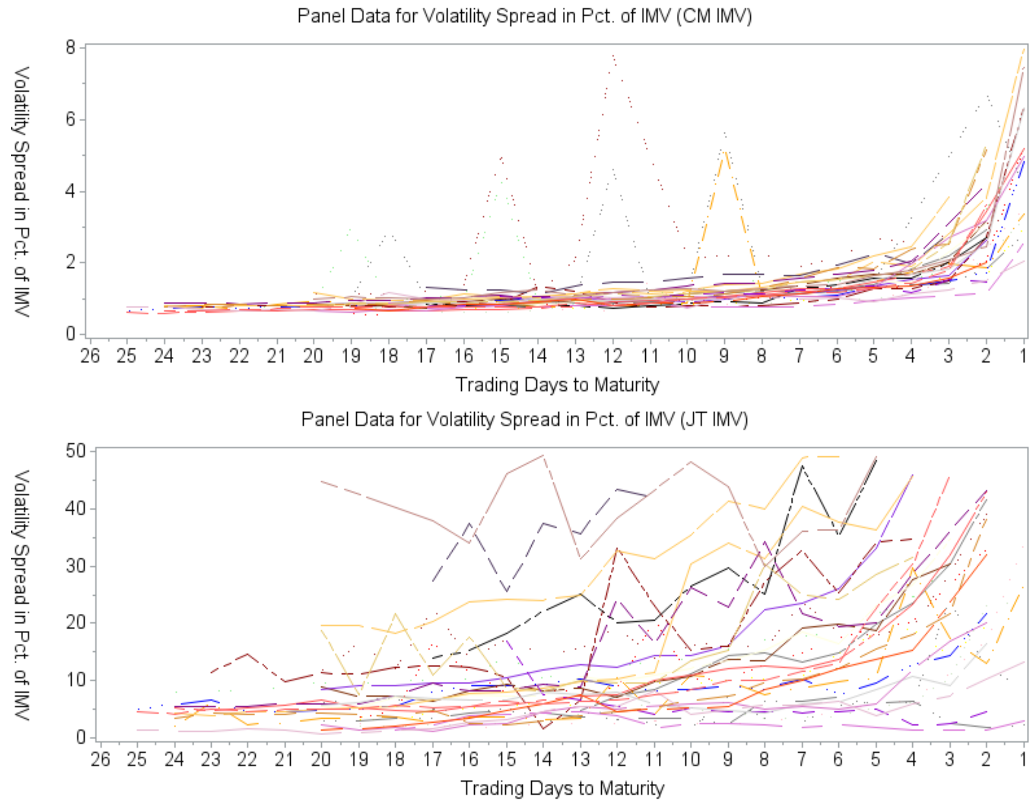
C.7 Figure 4-2-6



Implied Volatility Spread over days to maturity

Figure 4-2-6 displays panel data of volatility spread calculated by CM and JT IMV. Instead of showing 30 consecutive nearest month contracts over the sample period, the charts lay out spread data cross all 30 contracts over different days to maturity. The Y axis is volatility spread, and the X axis is day to maturity. The days to maturity for a nearest term contract generally starts from 20 to 25 days. As can be seen, quoting spread increases with convexity when time to maturity decreases.

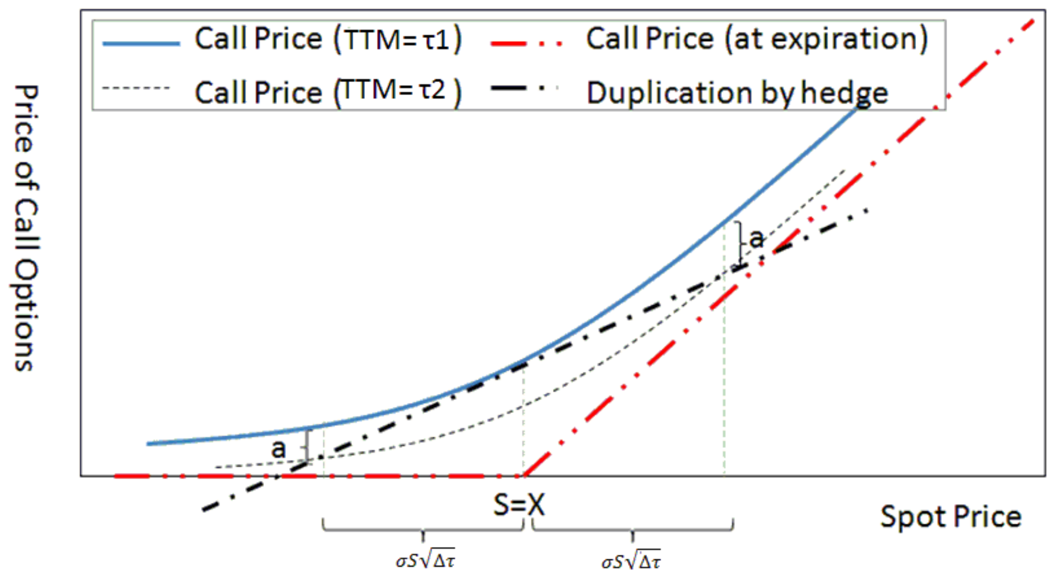
C.8 Figure 4-2-7



Pct Volatility Spread over days to maturity

Here we display panel data of volatility spread in percentage of IMV over days to maturity. Again we find that the volatility spread in percentage enlarges with increasing magnitude as time approaches to maturity.

C.9 Figure 4-3-1



Delta Hedging and Hedging Uncertainty

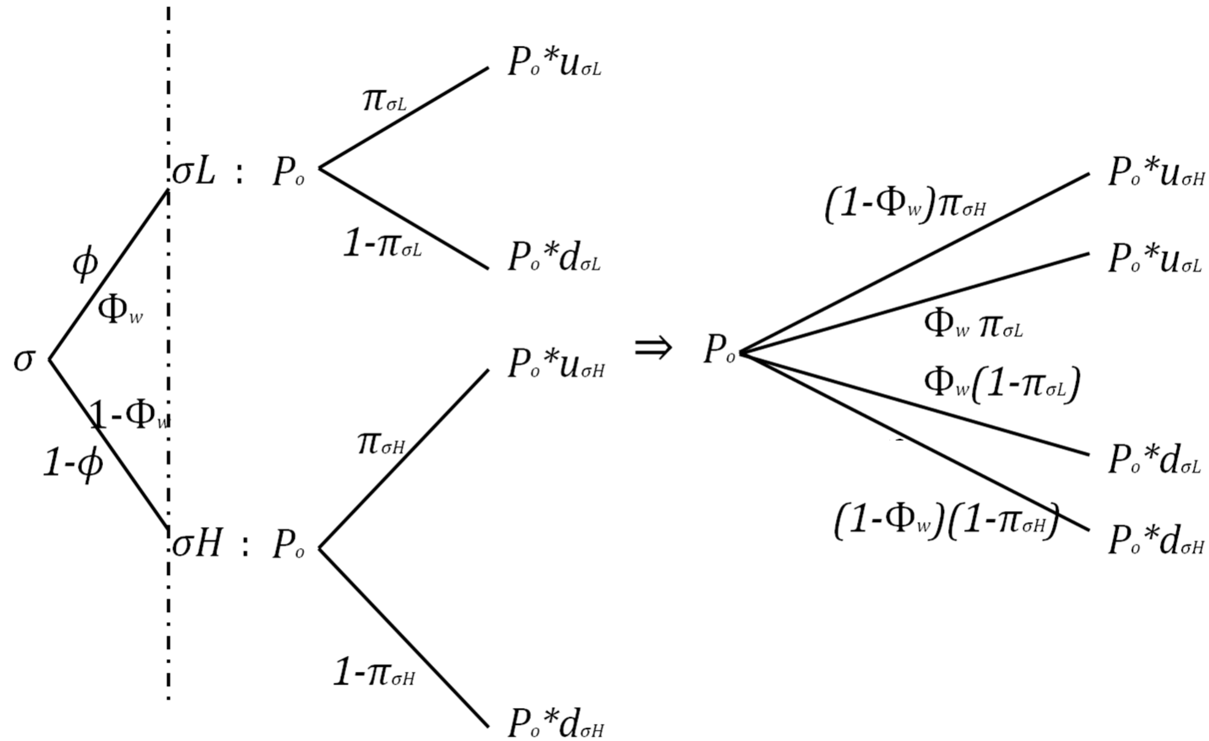
Figure 4-3-1 depicts the example of Delta hedging for ATM call options under the model of Black-Scholes-Merton which assumes the following differential equation:

$$\frac{\partial C}{\partial \tau} = -\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rS\frac{\partial C}{\partial S} + rC$$

$$\Rightarrow [\frac{\partial C}{\partial \tau}]\Delta t = -\frac{1}{2}\sigma^2 \Delta t S^2 \frac{\partial^2 C}{\partial S^2} - r\Delta t S\frac{\partial C}{\partial S} + r\Delta t C$$

The solid line denotes the prices of an ATM call options, which has strike, X, and TTM, $\tau 1$, over various spot prices. Instantaneous spot price movement drives call price to move along the solid line. An option is a contract in which writers (sellers) sell the right to buyer and allow options purchasers to buy stock at price X or to enjoy payoff $Max(0, S_T - X)$ at expiration. As time passes, the probability of asset price goes above exercise price decreases; therefore, call prices shift to the dashed line when TTM is $\tau 2$. Delta hedge is the liner duplication of call options, which hedges the linear movement of options price. For the writer, the cost of buying the underlying asset and cash inflow from receiving the call price is $(-rS\frac{\partial C}{\partial S} + rC)\Delta t$. On the other side, the curvature denoted as a, which equals $\frac{1}{2}\sigma^2 \Delta t S^2 \frac{\partial^2 C}{\partial S^2}$, can not be hedged. Time decay, $\frac{\partial C}{\partial \tau}\Delta t$, should be able to cover curvature and cost of hedging portfolio if volatility is certain, meaning the decay rate is completely determined by the model and the assumption of distribution of return. However, in an incomplete market, traders add a markup for volatility uncertainty which is not considered by the pricing models. Because the possible range of underlying asset price movement shrinks as TTM decreases, the markup decreases because of the reduced hedging variance. However, this decay of required markup does not necessarily follow with the the decay of chraged premium which is contained in the quoted option price. The decay of the charged premium contained in the quoted option price is determined by assumed return distribution.

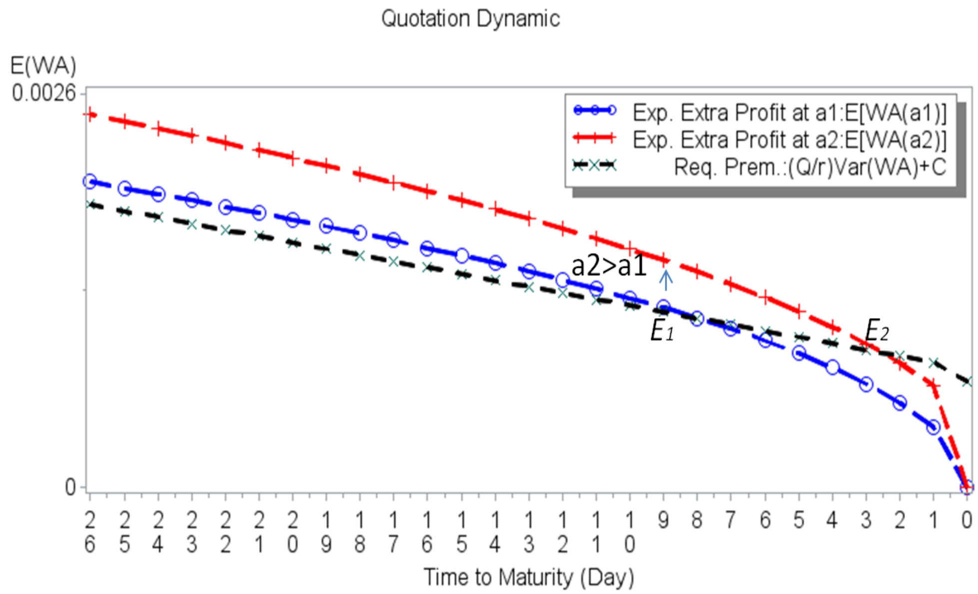
C.10 Figure 4-3-2



The Structure of Underlying Asset Price Movement and Probability.

The diagram shows the probabilistic structure of an option trading. Given two volatility estimations, $(\sigma L, \sigma H)$, we have conditional risk neutral probabilities $(\pi_{\sigma L}, \pi_{\sigma H})$ over two volatility states. ϕ is the subjective probability that liquidity providers assign to two volatility models, and Φ_w is the risk neutral probability which discounts fair options value into equilibrium (observed) quoting prices. Combining two 2-branch trees, we can form a 4-branch tree in which the prices of each node are $(P_0 * u_{\sigma H}, P_0 * u_{\sigma L}, P_0 * d_{\sigma L}, P_0 * d_{\sigma H})$.

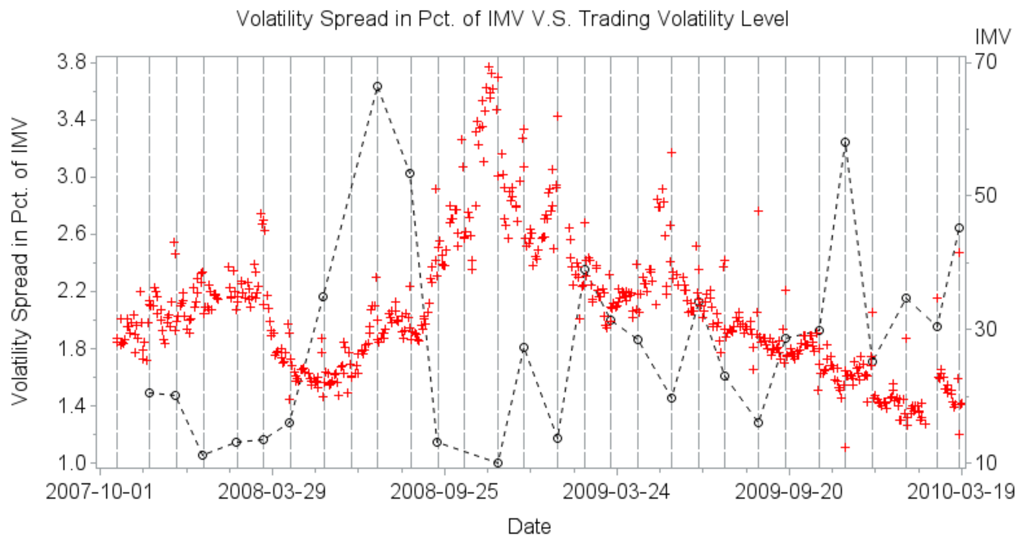
C.11 Figure 4-4-1



Quotation Dynamic

Figure 4-4-1 illustrates the dynamics of asking price quotations for selling options. When expected extra profit equals risk premium, traders engage in quoting. As shown, intersection E_1 is the first equilibrium where selling the options at volatility $a1$ can have extra profit to afford risk premium. However, the expected profit decays faster than required premium so that expected profit falls below risk premium as time to maturity decreases. To compensate the required premium for quotation at the next round, traders need to increase selling volatility to $a2$, shifting the expected extra profit curve upward, while increasing selling volatility doesn't shift the hedging variance. Even though our result is based on a binomial options pricing model, the characteristic of profit decay should remain the same for all other options pricing models.

C.12 Figure 4-4-2



Averaged Volatility Spread in Pct. of IMV V.S. Trading Volatility

In Figure 4-4-2, we demonstrate a negative relationship between volatility percentage spread and implied volatility. Because the increasing pattern obscures the level effect, this negative relationship is not clearly observed if every data point is displayed. Here we averaged daily data and plot averaged daily volatility percentage spread for each contract at expiration date. As can be seen, when implied volatility is high, spread in percentage of implied volatility tends to be low. The pattern implies that volatility spread does not increase in proportion to trading volatility.

C.13 Table 4-5-1 & 4-5-2

Table 4-5-1(a): Statistics of IMV and RV

	Obs.	Mean	Std. Dev.	Minimum	Maximum
CM IMV	630	0.31562	0.10231	0.14217	0.74136
JT IMV	630	0.37249	0.22794	0.16408	2.45290
RV	630	0.29225	0.12702	0.11678	1.00595

Table 4-5-1(b): Statistics of IMV and RV (excl. days to maturity<6 trading days)

	Obs.	Mean	Std. Dev.	Minimum	Maximum
CM IMV	479	0.31117	0.09952	0.15753	0.69426
JT IMV	479	0.32442	0.09901	0.16908	0.69747
RV	479	0.28501	0.11269	0.12788	0.65097

Table 4-5-2(a): Correlation Coefficients

	CM IMV	JT IMV	RV
CM IMV P-value	1.0000	0.53582 <0.001	0.83573 <0.001
JT IMV P-value		1.0000	0.63947 <0.001
RV P-value			1.0000

Table 4-5-2(b): Correlation Coefficients
(excl. days to maturity<6 trading days)

	CM IMV	JT IMV	RV
CM IMV P-value	1.000	0.97570 <0.001	0.89574 <0.001
JT IMV P-value		1.000	0.86907 <0.001
RV P-value			1.000

We exclude the data of following dates becuase the files in these dates are either damaged or incomplete: 2008/10/13, 2008/11/21, 2008/12/11, 2008/12/25, 2009/11/18, 2010/4/19, 2010/5/7, 2010/6/2

C.14 Table 4-5-3 & 4-5-4

Table 4-5-3(a): Statistics of Volatility Spreads

	Obs.	Mean	Std. Dev.	Minimum	Maximum
Vol. Spd (CM)	630	0.00569	0.00937	0.00151	0.08908
Vol. Spd (JT)	630	0.09892	0.27746	0.00270	2.7508

Table 4-5-3(b): Statistics of Vol. Spread
(excl. days to maturity<6 trading days)

	Obs.	Mean	Std. Dev.	Minimum	Maximum
Vol. Spd (CM)	479	0.00337	0.00284	0.00151	0.03758
Vol. Spd (JT)	479	0.03718	0.03847	0.00298	0.21567

Table 4-5-4(a): Correlation Coefficients of Volatility Spreads

	Vol. Spd (CM)	Vol.Spd (JT)
Vol. Spd (CM) P-value	1.000	0.69542 $<$0.0001
Vol. Spd (JT) P-value		1.000

Table 4-5-4(b): Correlation Coefficients of Volatility Spreads
(excl. days to maturity<6 trading days)

	Vol. Spd (CM)	Vol.Spd (JT)
Vol Spd (CM) P-value	1.000	0.07895 0.0843
Vol.Spd (JT) P-value		1.000

C.15 Table 4-5-5 & 4-5-6

Table 4-5-5(a): Statistics of Volatility Spreads in Percentage of IMV

	Obs.	Mean(%)	Std. Dev(%)	Minimum(%)	Maximum(%)
Vol. Spd in Pct of IMV (CM)	630	1.71856	2.37930	0.53511	26.32054
Vol. Spd in Pct of IMV (JT)	630	18.17716	21.9723	0.76191	125.62653

Table 4-5-5(b): Statistics of Volatility Spreads in Percentage of IMV (excl. days to maturity<6 trading days)

	Obs.	Mean(%)	Std. Dev(%)	Minimum(%)	Maximum(%)
Vol. Spd in Pct of IMV (CM)	479	1.11808	1.02286	0.53511	15.14102
Vol. Spd in Pct of IMV (JT)	479	11.36921	10.62312	0.76191	63.07412

Table 4-5-6(a): Correlation Coefficients for Vol. Spds in Pct of IMV

	CM	JT
CM P-value	1.000	0.57476 <0.0001
JT P-value		1.000

Table 4-5-6(b): Correlation Coefficients for Vol. Spds in Pct of IMV (excl. days to maturity<6 trading days)

	CM	JT
CM P-value	1.000	0.15837 0.0015
JT P-value		1.000

C.16 Table 4–6-1

Estimation Based on CM IMV and 0 Volatility Risk Premium.
(Data sample excludes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta \frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (T-stat)	1.252475 (6.77)***	0.011438 (3.18)***	-0.01516 (-3.18)***	-0.00036 (-0.52)	0.0386	i.i.d.
Estimation (T-stat)	1.252475 (6.55)***	0.011438 (5.52)***	-0.01516 (-3.97)***	-0.00036 (-1.03)	0.0386	heteroscedasticity[1]
Estimation (T-stat)	1.252475 (5.87)***	0.011438 (5.23)***	-0.01516 (-3.47)***	-0.00036 (-0.97)	0.0386	heteroscedasticity[2]
Estimation (T-stat)	1.252475 (5.26)***	0.011438 (5.02)***	-0.01516 (-3.07)***	-0.00036 (-0.93)	0.0386	heteroscedasticity[3]

$$Model: \begin{cases} SpPct_t = \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	1.735036 (10.42)***	1.708992 (10.37)***	1.44112 (8.67)***	1.247435 (5.89)***	-0.01498 (-3.14)***	-0.00035 (-0.51)	0.0405	i.i.d.
Estimation (t-stat)	1.735036 (12.37)***	1.708992 (6.94)***	1.44112 (8.58)***	1.247435 (9.62)***	-0.01498 (-3.94)***	-0.00035 (-1.01)	0.0405	hete.[1]
Estimation (t-stat)	1.735036 (10.83)***	1.708992 (6.25)***	1.44112 (7.82)***	1.247435 (8.47)***	-0.01498 (-3.46)***	-0.00035 (-0.96)	0.0405	hete.[2]
Estimation (t-stat)	1.735036 (9.61)***	1.708992 (5.7)***	1.44112 (7.08)***	1.247435 (7.55)***	-0.01498 (-3.07)***	-0.00035 (-0.92)	0.0405	hete.[3]

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.17 Table 4–6-2

Estimation Based on JT IMV and 0 Volatility Risk Premium.
(Data sample excludes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta \frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	5.612757	0.28526	-0.09021	0.000646	0.1012	i.i.d.
(t-stat)	(2.88)***	(7.29)***	(-2.05)**	(2.14)**		
Estimation	5.612757	0.28526	-0.09021	0.000646	0.1012	heteroscedasticity[1]
(t-stat)	(2.36)**	(4.46)***	(-1.93)*	(2.29)**		
Estimation	5.612757	0.28526	-0.09021	0.000646	0.1012	heteroscedasticity[2]
(t-stat)	(2.04)**	(3.94)***	(-1.66)*	(1.95)*		
Estimation	5.612757	0.28526	-0.09021	0.000646	0.1012	heteroscedasticity[3]
(t-stat)	(1.87)*	(3.74)***	(-1.48)	(1.72)*		

$$Model: \begin{cases} SpPct_t = \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	19.1538	13.96775	11.6982	7.616757	-0.08892	0.000673	0.0968	i.i.d.
(t-stat)	(11.16)***	(8.33)***	(6.69)***	(3.38)***	(-2.01)**	(2.22)**		
Estimation	19.1538	13.96775	11.6982	7.616757	-0.08892	0.000673	0.0968	hete.[1]
(t-stat)	(7.76)***	(6.50)***	(6.13)***	(4.64)***	(-1.91)*	(2.36)**		
Estimation	19.1538	13.96775	11.6982	7.616757	-0.08892	0.000673	0.0968	hete.[2]
(t-stat)	(6.64)***	(5.53)***	(5.28)***	(4.08)***	(-1.64)*	(2.00)**		
Estimation	19.1538	13.96775	11.6982	7.616757	-0.08892	0.000673	0.0968	hete.[3]
(t-stat)	(6.02)***	(5.06)***	(4.68)***	(3.67)***	(-1.45)**	(1.76)*		

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.18 Table 4-6-3

Estimation Based on CM IMV and MA Dynamic Volatility Risk Premium.
(Data sample excludes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta\frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{Volatility Premium})_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	1.264867 (6.83)***	0.01163 (3.19)***	-0.0163 (-3.44)***	0.000057 (0.27)	0.0381	i.i.d.
Estimation (t-stat)	1.264867 (6.5)***	0.01163 (5.49)***	-0.0163 (-3.97)***	0.000057 (1.29)	0.0381	heteroscedasticity[1]
Estimation (t-stat)	1.264867 (5.81)***	0.01163 (5.2)***	-0.0163 (-3.46)***	0.000057 (1.24)	0.0381	heteroscedasticity[2]
Estimation (t-stat)	1.264867 (5.21)***	0.01163 (5)***	-0.0163 (-3.05)***	0.000057 (1.19)	0.0381	heteroscedasticity[3]

$$Model: \begin{cases} SpPct_t = \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{Volatility Premium})_{t-1} - (\text{Hight Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	1.76228 (10.39)***	1.732693 (10.35)***	1.463301 (8.75)***	1.255782 (5.93)***	-0.01633 (-3.45)***	0.000093 (0.44)	0.0402	i.i.d.
Estimation (t-stat)	1.76228 (11.98)***	1.732693 (6.83)***	1.463301 (8.51)***	1.255782 (9.47)***	-0.01633 (-3.93)***	0.000093 (2.02)*	0.0402	hete.[1]
Estimation (t-stat)	1.76228 (10.44)***	1.732693 (6.14)***	1.463301 (7.72)***	1.255782 (8.31)***	-0.01633 (-3.43)***	0.000093 (1.94)*	0.0402	hete.[2]
Estimation (t-stat)	1.76228 (9.22)***	1.732693 (5.6)***	1.463301 (6.95)***	1.255782 (7.38)***	-0.01633 (-3.03)***	0.000093 (1.84)*	0.0402	hete.[3]

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.19 Table 4-6-4

Estimation Based on JT IMV and MA Dynamic Volatility Risk Premium.
(Data sample excludes last 5 trading days.)

$$Model:\begin{cases} SpPct_t = \alpha + \theta\frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{Volatility Premium})_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	3.493824	0.289991	-0.04073	0.005743	0.1045	i.i.d.
(t-stat)	(2.20)**	(7.38)***	(-1.70)*	(2.60)***		
Estimation	3.493824	0.289991	-0.04073	0.005743	0.1045	heteroscedasticity[1]
(t-stat)	(1.69)*	(4.59)***	(-2.17)**	(2.08)**		
Estimation	3.493824	0.289991	-0.04073	0.005743	0.1045	heteroscedasticity[2]
(t-stat)	(1.47)	(4.06)***	(-1.97)**	(2.15)**		
Estimation	3.493824	0.289991	-0.04073	0.005743	0.1045	heteroscedasticity[3]
(t-stat)	(1.38)	(3.88)***	(-1.79)*	(2.25)**		

$$Model:\begin{cases} SpPct_t = \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{Volatility Premium})_{t-1} - (\text{Hight Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	17.21005	11.80949	9.552206	5.755409	-0.03645	0.005709	0.0994	i.i.d.
(t-stat)	(14.74)***	(10.43)***	(7.62)***	(2.89)***	(-1.50)	(2.58)**		
Estimation	17.21005	11.80949	9.552206	5.755409	-0.03645	0.005709	0.0994	hete.[1]
(t-stat)	(9.36)***	(8.57)***	(6.72)***	(6.46)***	(-1.93)*	(2.10)**		
Estimation	17.21005	11.80949	9.552206	5.755409	-0.03645	0.005709	0.0994	hete.[2]
(t-stat)	(8.10)***	(7.32)***	(5.89)***	(5.78)***	(-1.77)*	(2.18)**		
Estimation	17.21005	11.80949	9.552206	5.755409	-0.03645	0.005709	0.0994	hete.[3]
(t-stat)	(7.57)***	(6.68)***	(5.45)***	(5.33)***	(-1.64)*	(2.29)**		

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.20 Table 4-6-5

Estimation Based on CM IMV and Squared RV Volatility Uncertainty
(Data sample excludes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta \frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = (\text{High Frequency Volatility})_t^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	1.354496	0.011589	-0.02155	0.000085	0.0394	i.i.d.
(t-stat)	(6.17)***	(3.23)***	(-2.57)***	(0.80)		
Estimation	1.354496	0.011589	-0.02155	0.000085	0.0394	heteroscedasticity[1]
(t-stat)	(6.63)***	(5.58)***	(-3.98)***	(1.87)*		
Estimation	1.354496	0.011589	-0.02155	0.000085	0.0394	heteroscedasticity[2]
(t-stat)	(5.89)***	(5.28)***	(-3.47)***	(1.66)*		
Estimation	1.354496	0.011589	-0.02155	0.000085	0.0394	heteroscedasticity[3]
(t-stat))	(5.29)***	(5.07)***	(-3.12)***	(1.53)		

$$Model: \begin{cases} SpPct_t = \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = (\text{High Frequency Volatility})_t^2 \end{cases}$$

	Parameters θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	1.840913	1.81179	1.54772	1.341013	-0.02121	0.000083	0.0412	i.i.d.
(t-stat)	(8.95)***	(8.90)***	(7.48)***	(5.59)***	(-2.53)***	(0.78)		
Estimation	1.840913	1.81179	1.54772	1.341013	-0.02121	0.000083	0.0412	hete.[1]
(t-stat)	(11.14)***	(6.88)***	(8.36)***	(9.13)***	(-3.87)***	(1.73)*		
Estimation	1.840913	1.81179	1.54772	1.341013	-0.02121	0.000083	0.0412	hete.[2]
(t-stat)	(9.72)***	(6.16)***	(7.55)***	(8.00)***	(-3.38)***	(1.52)		
Estimation	1.840913	1.81179	1.54772	1.341013	-0.02121	0.000083	0.0412	hete.[3]
(t-stat))	(8.67)***	(5.61)***	(6.80)***	(7.14)***	(-3.03)***	(1.39)		

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.21 Table 4-6-6

Estimation Based on JT IMV and Squared RV Volatility Uncertainty
(Data sample excludes last 5 trading days.)

$$Model : \begin{cases} SpPct_t = \alpha + \theta \frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = (\text{High Frequency Volatility})_t^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	5.612757 (2.88)***	0.28526 (7.29)***	-0.09021 (-2.05)**	0.000646 (2.14)**	0.1012	i.i.d.
Estimation (t-stat)	5.612757 (2.36)**	0.28526 (4.46)***	-0.09021 (-1.93)*	0.000646 (2.29)**	0.1012	heteroscedasticity[1]
Estimation (t-stat)	5.612757 (2.04)**	0.28526 (3.94)***	-0.09021 (-1.66)*	0.000646 (1.95)*	0.1012	heteroscedasticity[2]
Estimation (t-stat)	5.612757 (1.87)*	0.28526 (3.74)***	-0.09021 (-1.48)	0.000646 (1.72)*	0.1012	heteroscedasticity[3]

$$Model : \begin{cases} SpPct_t = \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = (\text{High Frequency Volatility})_t^2 \end{cases}$$

	Parameters θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	19.1538 (11.16)***	13.96775 (8.33)***	11.6982 (6.69)***	7.616757 (3.38)***	-0.08892 (-2.01)**	0.000673 (2.22)**	0.0968	i.i.d.
Estimation (t-stat)	19.1538 (7.76)***	13.96775 (6.50)***	11.6982 (6.13)***	7.616757 (4.64)***	-0.08892 (-1.91)*	0.000673 (2.36)**	0.0968	hete.[1]
Estimation (t-stat)	19.1538 (6.64)***	13.96775 (5.53)***	11.6982 (5.28)***	7.616757 (4.08)***	-0.08892 (-1.64)*	0.000673 (2.00)**	0.0968	hete.[2]
Estimation (t-stat)	19.1538 (6.02)***	13.96775 (5.06)***	11.6982 (4.68)***	7.616757 (3.67)***	-0.08892 (-1.45)**	0.000673 (1.76)*	0.0968	hete.[3]

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.22 Table 4-6-1-a

Estimation Based on CM IMV and 0 Volatility Risk Premium.
(Data sample includes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta\frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	1.599353 (6.51)***	0.017825 (15.95)***	-0.03346 (-4.64)***	0.000544 (1.28)	0.3852	i.i.d.
Estimation (t-stat)	1.599353 (7.07)***	0.017825 (8.44)***	-0.03346 (-5.54)***	0.000544 (0.82)	0.3852	heteroscedasticity[1]
Estimation (t-stat)	1.599353 (6.55)***	0.017825 (8.50)***	-0.03346 (-5.26)***	0.000544 (0.81)	0.3852	heteroscedasticity[2]
Estimation (t-stat)	1.599353 (6.20)***	0.017825 (8.52)***	-0.03346 (-5.11)***	0.000544 (1.28)	0.3852	heteroscedasticity[3]

$$Model: \begin{cases} SpPct_t = \theta_1 D_1 + \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters θ_1	θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	4.323302 (14.55)***	2.349584 (7.92)***	2.302003 (7.82)***	2.032064 (6.80)***	1.811517 (4.54)***	-0.03815 (-4.86)***	0.00287 (7.01)***	0.2705	i.i.d.
Estimation (t-stat)	4.323302 (9.58)***	2.349584 (10.06)***	2.302003 (7.75)***	2.032064 (8.39)***	1.811517 (7.66)***	-0.03815 (-5.25)***	0.00287 (3.99)***	0.2705	hete.[1]
Estimation (t-stat)	4.323302 (9.85)***	2.349584 (9.61)***	2.302003 (7.38)***	2.032064 (8.06)***	1.811517 (7.32)***	-0.03815 (-5.11)***	0.00287 (3.93)***	0.2705	hete.[2]
Estimation (t-stat)	4.323302 (10.12)***	2.349584 (9.46)***	2.302003 (7.19)***	2.032064 (8.01)***	1.811517 (7.17)***	-0.03815 (-5.12)***	0.00287 (3.89)***	0.2705	hete.[3]

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.23 Table 4-6-2-a

Estimation Based on JT IMV and 0 Volatility Risk Premium.
(Data sample includes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta\frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	17.49044	0.204591	-0.38425	-0.00306	0.5251	i.i.d.
(t-stat)	(9.01)***	(24.18)***	(-6.59)***	(-0.96)		
Estimation	17.49044	0.204591	-0.38425	-0.00306	0.5251	heteroscedasticity[1]
(t-stat)	(5.84)***	(10.42)***	(-4.92)***	(-0.48)		
Estimation	17.49044	0.204591	-0.38425	-0.00306	0.5251	heteroscedasticity[2]
(t-stat)	(4.97)***	(9.93)***	(-4.20)***	(-0.48)		
Estimation	17.49044	0.204591	-0.38425	-0.00306	0.5251	heteroscedasticity[3]
(t-stat)	(4.35)***	(9.68)***	(-3.71)***	(-0.48)		

$$Model: \begin{cases} SpPct_t = \theta_1 D_1 + \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters θ_1	θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	49.76579	28.63694	23.14229	20.81895	17.20998	-0.41756	0.017193	0.3462	i.i.d.
(t-stat)	(19.41)***	(11.25)***	(9.22)***	(8.04)***	(4.95)***	(-6.10)***	(4.91)***		
Estimation	49.76579	28.63694	23.14229	20.81895	17.20998	-0.41756	0.017193	0.3462	hete.[1]
(t-stat)	(10.85)***	(8.11)***	(7.39)***	(6.71)***	(5.95)***	(-4.59)***	(2.17)**		
Estimation	49.76579	28.63694	23.14229	20.81895	17.20998	-0.41756	0.017193	0.3462	hete.[2]
(t-stat)	(9.81)***	(6.95)***	(6.41)***	(5.85)***	(5.29)***	(-4.09)***	(2.19)**		
Estimation	49.76579	28.63694	23.14229	20.81895	17.20998	-0.41756	0.017193	0.3462	hete.[3]
(t-stat)	(9.09)***	(6.14)***	(5.77)***	(5.26)***	(4.82)***	(-3.72)***	(2.23)**		

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.24 Table 4-6-3-a

Estimation Based on CM IMV and MA Dynamic Volatility Risk Premium.
(Data sample includes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta\frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{Volatility Premium})_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters				Adj.	Innovation
	α	θ	β_1	β_2	R-Sq	assumption
Estimation	1.578849	0.018275	-0.0331	0.000247	0.3842	i.i.d.
(t-stat)	(6.42)***	(18.10)***	(-4.55)***	(0.90)		
Estimation	1.578849	0.018275	-0.0331	0.000247	0.3842	heteroscedasticity[1]
(t-stat)	(6.98)***	(9.41)***	(-5.39)***	(0.68)		
Estimation	1.578849	0.018275	-0.0331	0.000247	0.3842	heteroscedasticity[2]
(t-stat)	(6.49)***	(9.49)***	(0.67)***	(0.67)		
Estimation	1.578849	0.018275	-0.0331	0.000247	0.3842	heteroscedasticity[3]
(t-stat)	(6.16)***	(9.53)***	(-5.03)***	(0.66)		

$$Model: \begin{cases} SpPct_t = \theta_1 D_1 + \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{Volatility Premium})_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters							Adj.	Innovation
	θ_1	θ_2	θ_3	θ_4	θ_5	β_1	β_2	R-Sq	assumption
Estimation	4.545148	2.3717224	2.309912	1.972528	1.671424	-0.03655	0.00145	0.2423	i.i.d.
(t-stat)	(15.04)***	(7.77)***	(7.65)***	(6.46)***	(4.12)***	(-4.53)***	(4.93)***		
Estimation	4.545148	2.3717224	2.309912	1.972528	1.671424	-0.03655	0.00145	0.2423	hete.[1]
(t-stat)	(9.99)***	(10.14)***	(7.82)***	(8.32)***	(7.03)***	(-5.05)***	(3.49)***		
Estimation	4.545148	2.3717224	2.309912	1.972528	1.671424	-0.03655	0.00145	0.2423	hete.[2]
(t-stat)	(10.48)***	(9.93)***	(7.55)***	(8.17)***	(7.07)***	(-5.06)***	(3.47)***		
Estimation	4.545148	2.3717224	2.309912	1.972528	1.671424	-0.03655	0.00145	0.2423	hete.[3]
(t-stat)	(10.90)***	(9.80)***	(7.34)***	(8.12)***	(7.05)***	(-5.09)***	(3.45)***		

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.25 Table 4-6-4-a

Estimation Based on JT IMV and MA Dynamic Volatility Risk Premium.
(Data sample includes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta \frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{Volatility Premium})_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	18.28002 (9.37)**	0.200069 (24.63)***	-0.41039 (-6.99)***	0.001094 (0.48)	0.5243	i.i.d.
Estimation (t-stat)	18.28002 (5.83)**	0.200069 (10.35)***	-0.41039 (-4.85)***	0.001094 (0.22)	0.5243	heteroscedasticity[1]
Estimation (t-stat)	18.28002 (4.98)**	0.200069 (9.86)***	-0.41039 (-4.19)***	0.001094 (0.22)	0.5243	heteroscedasticity[2]
Estimation (t-stat)	18.28002 (4.39)**	0.200069 (9.62)***	-0.41039 (-3.74)***	0.001094 (0.22)	0.5243	heteroscedasticity[3]

$$Model: \begin{cases} SpPct_t = \theta_1 D_1 + \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = \{IMV_{t-1} - (\text{Volatility Premium})_{t-1} - (\text{High Frequency Volatility})_t\}^2 \end{cases}$$

	Parameters θ_1	θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation (t-stat)	50.86253 (19.75)***	29.36456 (11.45)***	23.75971 (9.42)***	21.21307 (8.18)***	16.85814 (4.87)***	-0.43231 (-6.29)***	0.013709 (5.28)***	0.3496	i.i.d.
Estimation (t-stat)	50.86253 (10.98)***	29.36456 (8.35)***	23.75971 (7.68)***	21.21307 (6.93)***	16.85814 (6.03)***	-0.43231 (-4.90)***	0.013709 (2.71)***	0.3496	hete.[1]
Estimation (t-stat)	50.86253 (9.92)***	29.36456 (7.10)***	23.75971 (6.60)***	21.21307 (5.99)***	16.85814 (5.38)***	-0.43231 (-4.29)***	0.013709 (2.69)***	0.3496	hete.[2]
Estimation (t-stat)	50.86253 (9.18)***	29.36456 (6.23)***	23.75971 (5.87)***	21.21307 (5.34)***	16.85814 (4.87)***	-0.43231 (-3.85)***	0.013709 (2.68)***	0.3496	hete.[3]

[1]: Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]: .Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]: Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.26 Table 4-6-5-a

Estimation Based on CM IMV and Squared RV Volatility Uncertainty
(Data sample includes last 5 trading days.)

$$Model:\begin{cases} SpPct_t = \alpha + \theta\frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = (\text{High Frequency Volatility})_t^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	1.951533	0.01751	-0.05376	0.000344	0.3938	i.i.d.
(t-stat)	(7.20)***	(17.52)***	(-5.49)***	(3.25)***		
Estimation	1.951533	0.01751	-0.05376	0.000344	0.3938	heteroscedasticity[1]
(t-stat)	(8.03)***	(9.72)***	(-5.33)***	(2.49)**		
Estimation	1.951533	0.01751	-0.05376	0.000344	0.3938	heteroscedasticity[2]
(t-stat)	(7.83)***	(9.90)***	(-5.49)***	(2.57)**		
Estimation	1.951533	0.01751	-0.05376	0.000344	0.3938	heteroscedasticity[3]
(t-stat)	(7.58)***	(10.01)***	(-5.59)***	(2.61)**		

$$Model:\begin{cases} SpPct_t = \theta_1 D_1 + \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = (\text{High Frequency Volatility})_t^2 \end{cases}$$

	Parameters θ_1	θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	5.101932	2.937483	2.892663	2.648502	2.384066	-0.07536	0.000738	0.2645	i.i.d.
(T-Stat)	(16.38)***	(9.14)***	(9.08)***	(8.16)***	(5.73)***	(-7.09)***	(6.61)***		
Estimation	5.101932	2.937483	2.892663	2.648502	2.384066	-0.07536	0.000738	0.2645	hete.[1]
(T-Stat)	(10.75)***	(9.97)***	(8.35)***	(8.72)***	(8.12)***	(-5.58)***	(3.95)***		
Estimation	5.101932	2.937483	2.892663	2.648502	2.384066	-0.07536	0.000738	0.2645	hete.[2]
(T-Stat)	(11.50)***	(10.29)***	(8.38)***	(9.08)***	(8.38)***	(-5.77)***	(3.91)***		
Estimation	5.101932	2.937483	2.892663	2.648502	2.384066	-0.07536	0.000738	0.2645	hete.[3]
(T-Stat)	(12.16)***	(10.67)***	(8.37)***	(9.39)***	(8.60)***	(-5.97)***	(3.88)***		

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.27 Table 4-6-6-a

Estimation Based on JT IMV and Squared RV Volatility Uncertainty
(Data sample includes last 5 trading days.)

$$Model: \begin{cases} SpPct_t = \alpha + \theta \frac{1}{TTM_t} + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = (\text{High Frequency Volatility})_t^2 \end{cases}$$

	Parameters α	θ	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	20.64115	0.19466	-0.55214	0.002493	0.5311	i.i.d.
(t-stat)	(10.05)***	(24.31)***	(-7.38)***	(2.98)**		
Estimation	20.64115	0.19466	-0.55214	0.002493	0.5311	heteroscedasticity[1]
(t-stat)	(6.13)***	(10.26)***	(-4.82)***	(1.91)*		
Estimation	20.64115	0.19466	-0.55214	0.002493	0.5311	heteroscedasticity[2]
(t-stat)	(5.41)***	(9.82)***	(-4.54)***	(1.94)*		
Estimation	20.64115	0.19466	-0.55214	0.002493	0.5311	heteroscedasticity[3]
(t-stat)	(4.85)***	(9.65)***	(-4.29)***	(1.96)*		

$$Model: \begin{cases} SpPct_t = \theta_1 D_1 + \theta_2 D_2 + \theta_3 D_3 + \theta_4 D_4 + \theta_5 D_5 + \beta_1 IMV_{t-1} + \beta_2 Shock_t + \varepsilon_t \\ Shock_t = (\text{High Frequency Volatility})_t^2 \end{cases}$$

	Parameters θ_1	θ_2	θ_3	θ_4	θ_5	β_1	β_2	Adj. R-Sq	Innovation assumption
Estimation	53.78476	31.9205	26.48654	24.99762	21.17399	-0.68985	0.006117	0.3637	i.i.d.
(t-stat)	(20.48)***	(12.18)***	(10.24)***	(9.25)***	(6.00)***	(-7.96)***	(6.48)***		
Estimation	53.78476	31.9205	26.48654	24.99762	21.17399	-0.68985	0.006117	0.3637	hete.[1]
(t-stat)	(11.21)***	(8.87)***	(8.29)***	(7.73)***	(6.93)***	(-5.96)***	(3.73)***		
Estimation	53.78476	31.9205	26.48654	24.99762	21.17399	-0.68985	0.006117	0.3637	hete.[2]
(t-stat)	(10.26)***	(7.69)***	(7.31)***	(6.93)***	(6.26)***	(-5.70)***	(3.71)***		
Estimation	53.78476	31.9205	26.48654	24.99762	21.17399	-0.68985	0.006117	0.3637	hete.[3]
(t-stat)	(9.53)***	(6.81)***	(6.59)***	(6.34)***	(5.73)***	(-5.48)***	(3.68)***		

[1]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 3.
[2]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 5.
[3]:Variance-covariance matrix is adjusted by NW estimation with Bartlett Kernel and bandwidth parameter of 8.
*** significant at 1% level. ** significant at 5% level. * significant at 10% level.

C.28 Proof 4-1

$$
\begin{aligned}
& E(W_A) \\
= \;& (1-\phi)\pi_{\sigma H}\{C(\tau, a\sigma) - C_N(u_{\sigma H}) + \Delta_A P_0(u_{\sigma H}-1)\} \\
& +\phi\pi_{\sigma L}\{C(\tau, a\sigma) - C_N(u_{\sigma L}) + \Delta_A P_0(u_{\sigma L}-1)\} \\
& +\phi(1-\pi_{\sigma L})\{C(\tau, a\sigma) - C_N(d_{\sigma L}) + \Delta_A P_0(d_{\sigma L}-1)\} \\
& +(1-\phi)\pi_{\sigma H}\{C(\tau, a\sigma) - C_N(d_{\sigma H}) + \Delta_A P_0(d_{\sigma H}-1)\}
\end{aligned}
$$

$\because (1-\phi)\pi_{\sigma H} + (1-\phi)\pi_{\sigma H} + \phi\pi_{\sigma L} + \phi(1-\pi_{\sigma L}) = 1$

$$
\begin{aligned}
= \;& C(\tau, a\sigma) - (1-\phi)\pi_{\sigma H}\{C_N(u_{\sigma H}) - \Delta_A P_0(u_{\sigma H}-1)\} - \phi\pi_{\sigma L}\{C_N(u_{\sigma L}) - \Delta_A P_0(u_{\sigma L}-1)\} \\
& -\phi(1-\pi_{\sigma L})\{C_N(d_{\sigma L}) - \Delta_A P_0(d_{\sigma L}-1)\} - (1-\phi)\pi_{\sigma H}\{C_N(d_{\sigma H}) - \Delta_A P_0(d_{\sigma H}-1)\}
\end{aligned}
$$

Assuming 0 risk free interest rate
Therefore $\pi_{\sigma L}\cdot u_{\sigma L} + (1-\pi_{\sigma L})\cdot d_{\sigma L} = 1, \pi_{\sigma H}\cdot u_{\sigma H} + (1-\pi_{\sigma H})\cdot d_{\sigma H} = 1$

$$
\begin{aligned}
= \;& C(\tau, a\sigma) - (1-\phi)\pi_{\sigma H}C_N(u_{\sigma H}) - \phi\pi_{\sigma L}C_N(u_{\sigma L}) - \phi(1-\pi_{\sigma L})C_N(d_{\sigma L}) - (1-\phi)\pi_{\sigma H}C_N(d_{\sigma H}) \\
= \;& C(\tau, a\sigma) - \phi C(\tau, \sigma L) - (1-\phi)C(\tau, \sigma H)
\end{aligned}
$$

$\because C(\tau, \sigma L) = \pi_{\sigma L}C_N(u_{\sigma L}) + (1-\pi_{\sigma L})C_N(d_{\sigma L})$ and $C(\tau, \sigma H) = \pi_{\sigma L}C_N(u_{\sigma L}) + (1-\pi_{\sigma L})C_N(d_{\sigma L})$

Following the same procedure, we can derive $E(W_B) = -C(\tau, \sigma b) + \phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H)$.

C.29 Proof 4-2

$$
\begin{aligned}
& Var(W_A) \\
= \;& p_1 W^2_{A,u_{\sigma L}} + p_2 W^2_{A,u_{\sigma L}} + p_3 W^2_{A,d_{\sigma L}} + p_4 W^2_{A,d_{\sigma H}} - (p_1 W_{A,u_{\sigma H}} + p_2 W_{A,u_{\sigma L}} + p_3 W_{A,d_{\sigma L}} + p_4 W_{A,d_{\sigma H}})^2 \\
= \;& C^2_{\tau,a}(p1+p2+p3+p4) + 2C_{\tau,a}\{p_1(\Delta_A S_1 - C_1) + p_2(\Delta_A S_2 - C_2) + p_3(\Delta_A S_3 - C_3) + p_4(\Delta_A S_4 - C_4)\} \\
& + \left\{p_1(\Delta_A S_1 - C_1)^2 + p_2(\Delta_A S_2 - C_2)^2 + p_3(\Delta_A S_3 - C_3)^2 + p_4(\Delta_A S_4 - C_4)^2\right\} \\
& - \{C_{\tau,a} - (p_1C_1 + p_2C_2 + p_3C_3 + p_4C_4) + (p_1\Delta_A S_1 + p\Delta_A S_2 + p_3\Delta_A S_3 + p_4\Delta_A S_4)\}^2
\end{aligned}
$$

$\because p_1\Delta_A S_1 + p\Delta_A S_2 + p_3\Delta_A S_3 + p_4\Delta_A S_4 = 0$ (This holds becuase equation (2) and (3) tells that expected return of underlying is 0.)
and $[p_1W_1 + p_2W_2 + p_3W_3 + p_4W_4]^2 = [C_{\tau,a} - \phi C(\tau, L) - (1-\phi)C(\tau, H)]^2$

$$
= \left\{p_1(\Delta_A S_1 - C_1)^2 + p_2(\Delta_A S_2 - C_2)^2 + p_3(\Delta_A S_3 - C_3)^2 + p_4(\Delta_A S_4 - C_4)^2\right\} - \{\phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H)\}^2
$$

For ATM options, $X = 1, C_1 = S_1, C_2 = S_2, C_3 = 0, C_4 = 0$

$$
= \left\{p_1(\Delta_A - 1)^2 S_1^2 + p_2(\Delta_A - 1)^2 S_2^2 + p_3\Delta_A^2 S_3{}^2 + p_4\Delta_A^2 S_4{}^2\right\} - \{\phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H)\}^2
$$

$$
\begin{aligned}
& Var(W_B) \\
= \;& p_1 W^2_{B,u_{\sigma H}} + p_2 W^2_{B,u_{\sigma L}} + p_3 W^2_{B,d_{\sigma L}} + p_4 W^2_{B,d_{\sigma H}} - (p_1 W_{B,u_{\sigma H}} + p_2 W_{B,u_{\sigma L}} + p_3 W_{B,d_{\sigma L}} + p_4 W_{B,d_{\sigma H}})^2 \\
= \;& \left\{p_1(\Delta_B S_1 + C_1)^2 + p_2(\Delta_B S_2 + C_2)^2 + p_3(\Delta_B S_3 + C_3)^2 + p_4(\Delta_B S_4 + C_4)^2\right\} \\
& - \{\phi C(\tau, \sigma L) + (1-\phi)C(\tau, \sigma H)\}^2
\end{aligned}
$$

For ATM options

$$
= \left\{p_1(\Delta_B + 1)^2 S_1^2 + p_2(\Delta_B + 1)^2 S_2^2 + p_3\Delta_B^2 S_3{}^2 + p_4\Delta_B^2 S_4{}^2\right\} - \{\phi C(\tau, L) + (1-\phi)C(\tau, H)\}^2
$$

$$\frac{\partial Var(WA)}{\partial \Delta a} = 0$$

$$\Rightarrow \quad 2p_1 W_{A,u_{\sigma H}} S_1 + 2p_2 W_{A,u_{\sigma L}} S_2 + 2p_3 W_{A,d_{\sigma L}} S_3 + 2p_4 W_{A,d_{\sigma H}} S_4$$
$$-2(p_1 W_{A,u_{\sigma H}} + p_2 W_{A,u_{\sigma L}} + p_3 W_{A,d_{\sigma L}} + p_4 W_{A,d_{\sigma H}}) * \{p_1 S_1 + p_2 S_2 + p_3 S_3 + p_4 S_4\}$$

$$\because p_1 S_1 + p_2 S_2 + p_3 S_3 + p_4 S_4 = 0$$

$$\Rightarrow \quad 2p_1 W_{A,u_{\sigma H}} S_1 + 2p_2 W_{A,u_{\sigma L}} S_2 + 2p_3 W_{A,d_{\sigma L}} S_3 + 2p_4 W_{A,d_{\sigma H}} S_4 = 0$$

$$\Rightarrow \quad C_{\tau,a}(p_1 S_1 + p_2 S_2 + p_3 S_3 + p_4 S_4) + \Delta_A(p_1 S_1^2 + p_2 S_2^2 + p_3 S_3^2 + p_4 S_4^2)$$
$$-(p_1 S_1 C_1 + p_2 S_2 C_2 + p_3 S_3 C_3 + p_4 S_4 C_4) = 0$$

$$\Rightarrow \quad \Delta_A = \frac{p_1 S_1 C_1 + p_2 S_2 C_2 + p_3 S_3 C_3 + p_4 S_4 C_4}{p_1 S_1^2 + p_2 S_2^2 + p_3 S_3^2 + p_4 S_4^2}$$

Similarily

$$\frac{\partial Var(W_B)}{\partial \Delta_B} = 0$$

$$\Rightarrow \quad -C_{\tau,b}(p_1 S_1 + p_2 S_2 + p_3 S_3 + p_4 S_4) + \Delta_B(p_1 S_1^2 + p_2 S_2^2 + p_3 S_3^2 + p_4 S_4^2)$$
$$+(p_1 S_1 C_1 + p_2 S_2 C_2 + p_3 S_3 C_3 + p_4 S_4 C_4) = 0$$

$$\Rightarrow \quad \Delta_B^* = -\frac{p_1 S_1 C_1 + p_2 S_2 C_2 + p_3 S_3 C_3 + p_4 S_4 C_4}{p_1 S_1^2 + p_2 S_2^2 + p_3 S_3^2 + p_4 S_4^2}$$

Therefore, we conclude that $\Delta_A^* = -\Delta_B^*$, and $Var(W_A) = Var(W_B)$

Here we can show for ATM, $\Delta_A^* = -\Delta_B^* \doteqdot 0.5$ by Taylor expansion

$$\Delta_A^* = \frac{p_1 S_1 C_1 + p_2 S_2 C_2 + p_3 S_3 C_3 + p_4 S_4 C_4}{p_1 S_1^2 + p_2 S_2^2 + p_3 S_3^2 + p_4 S_4^2} = \frac{p_1 S_1 C_1 + p_2 S_2 C_2}{p_1 S_1^2 + p_2 S_2^2 + p_3 S_3^2 + p_4 S_4^2} \doteqdot \frac{\frac{1}{2}\left\{\left(\sigma H \sqrt{\tau}\right)^2 + \left(\sigma L \sqrt{\tau}\right)^2\right\}}{\left(\sigma H \sqrt{\tau}\right)^2 + \left(\sigma L \sqrt{\tau}\right)^2}$$

C.30 Proof 4-3 & 4-4

Proof 4-3:

First it can be show that alternative optimization problem (1) has same solution as second alternative optimization model. For first alternative optimization problem, lagrange multiplier condition is

$$\frac{\partial Var(qW_A)}{\partial \Delta_A} + \lambda \frac{\partial qE(W_A)}{\partial \Delta_A} = 0 \Rightarrow \frac{\partial Var(W_A)}{\partial \Delta_A} = 0.$$

Therefore 3 optimization problems have same delta solution.

Proof 4-4:

If θ can be equal or smaller than 0, then the optimal solution to alternative problem (1) is $q^* = 0$. This optimization problem then becomes meaningless.

If $\theta > 0, \theta = \frac{2Var(qW_A)}{\gamma} + cq$, then first alternative problem has the same solution as optimization problem (3) does. Given specified constraint payoff for writing call options, a positive θ implies problem has solution $q^* > 0$ if solution exists. Conditions for optimized solutions are:

$$\begin{cases} \pounds = -Var(qW_A) + \lambda(-E(qW_A) + \frac{2Var(qW_A)}{\gamma} + cq) \\ \frac{\partial \pounds}{\partial \Delta_A} = (-q^{*2} + \frac{\lambda q^{*2}}{\gamma}) \frac{2\partial Var(q^* W_A)}{\partial \Delta_A} = 0 \\ \frac{\partial \pounds}{\partial q} = -2q^* Var(W_A) + \lambda(-E(W_A) + \frac{4q^* Var(W_A)}{\gamma} + c) = 0 \\ \frac{\partial \pounds}{\partial \lambda} = -E(q^* W_A) + \frac{Var(q^* W_A)}{2\gamma} + cq^* = q^*[-E(W_A) + \frac{2q^* Var(W_A)}{\gamma} + c] = 0 \end{cases}$$

Because $\frac{\partial \pounds}{\partial \lambda} = 0$ and $q^* > 0$, condition $E(W_A) = \frac{2qVar(W_A)}{\gamma} + c$ needs to be satisfied for optimal q^*. And we can plug this condition into $\frac{\partial \pounds}{\partial q}$ to have following equation.

$$\frac{\partial \pounds}{\partial q} = Var(W_A)(-2q + \frac{2\lambda q}{\gamma}) = 0 \Rightarrow q(-1 + \frac{\lambda}{\gamma}) = 0 \Rightarrow \lambda = \gamma > 0$$

Then λ can be substituted into $\frac{\partial \pounds}{\partial \Delta}$ such that $\frac{\partial \pounds}{\partial \Delta} = q^2 \frac{\partial Var(qW_A)}{\partial \Delta} = 0$. Condition, $\frac{\partial Var(W_A)}{\partial \Delta} = 0$, needs to be held in maximization, and $E(W_A) = \frac{2q^* Var(W_A)}{\gamma} + c$. All optimal conditions for first alternative optimization are the same as optimization problem (3), if θ is set to be equal to $\frac{2Var(qW_A)}{\gamma} + cq$.

It is trivial to show the second alternative optimization problem has the same optimal condition. Traders first minimize the hedging variance; therefore the condition is $\frac{\partial Var(W_A)}{\partial \Delta} = 0$. The second step is maximizing profit by choosing q, given Δ^*, and the F.O.C. condition is $E(W_A) = \frac{2q^* Var(W_A)}{\gamma} + c$.

C.31 Proof 4-5

By introducing binomial model and assuming $X = 1$, each component in implicit function is:

$$C(\tau, a*\sigma) = P_0\pi_{a\sigma}(u_{\tau,a\sigma}-1) = \frac{P_0(1-e^{-a\sigma\sqrt{\tau}})(e^{a\sigma\sqrt{\tau}}-1)}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}} = \frac{P_0(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}}$$

$$\begin{aligned}
E(W_A) =\ & P_0\left(\frac{e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}} - \phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} - (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}}\right) \\
Var(W_A) =\ & P_0^2\{\phi\frac{\Delta_A^2(e^{2\sigma L\sqrt{\tau}}-2e^{\sigma L\sqrt{\tau}}+2e^{-\sigma L\sqrt{\tau}}-e^{-2\sigma L\sqrt{\tau}})+(1-2\Delta_A)(e^{2\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3-e^{-\sigma L\sqrt{\tau}})}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} \\
& +(1-\phi)\frac{\Delta_A^2(e^{2\sigma H\sqrt{\tau}}-2e^{\sigma H\sqrt{\tau}}+2e^{-\sigma H\sqrt{\tau}}+e^{-2\sigma H\sqrt{\tau}})+(1-2\Delta_A)(e^{2\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3-e^{-\sigma H\sqrt{\tau}})}{e^{aH\sqrt{\tau}}-e^{-aH\sqrt{\tau}}} \\
& -(\phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} + (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}})^2\} \\
E(W_B) =\ & P_0\left(-\frac{e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2}{e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}}} + \phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} + (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}}\right) \\
Var(W_B) =\ & P_0^2\{\phi\frac{\Delta_B^2(e^{2\sigma L\sqrt{\tau}}-2e^{\sigma L\sqrt{\tau}}+2e^{-\sigma L\sqrt{\tau}}-e^{-2\sigma L\sqrt{\tau}})+(1+2\Delta_B)(e^{2\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3-e^{-\sigma L\sqrt{\tau}})}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} \\
& +(1-\phi)\frac{\Delta_B^2(e^{2\sigma H\sqrt{\tau}}-2e^{\sigma H\sqrt{\tau}}+2e^{-\sigma H\sqrt{\tau}}+e^{-2\sigma H\sqrt{\tau}})+(1+2\Delta_B)(e^{2\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3-e^{-\sigma H\sqrt{\tau}})}{e^{aH\sqrt{\tau}}-e^{-aH\sqrt{\tau}}} \\
& -(\phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} + (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}})^2\}
\end{aligned}$$

$$\begin{aligned}
C(\tau, a\sigma) =\ & P_0\frac{e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}} \\
\frac{\partial C(0,a\sigma)}{\partial\tau} =\ & \frac{P_0 a\sigma}{\sqrt{\tau}}\frac{(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^2}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E(W_A)}{\partial\tau} =\ & \frac{P_0}{\sqrt{\tau}}\left[\frac{a\sigma(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^2} - \phi\frac{\sigma L(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2} - (1-\phi)\frac{\sigma H(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right] \\
\frac{\partial E(W_B)}{\partial\tau} =\ & \frac{P_0}{\sqrt{\tau}}\left[\frac{-b\sigma(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2)}{(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})^2} + \phi\frac{\sigma L(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2} + (1-\phi)\frac{\sigma H(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right] \\
\frac{\partial Var(W_A)}{\partial\tau} =\ & \frac{P_0^2\sigma H(1-\phi)}{2\sqrt{\tau}}\left\{\frac{\Delta_A^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})+(1-2\Delta_A)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right\} \\
& +\frac{P_0^2\sigma L\phi}{2\sqrt{\tau}}\left\{\frac{\Delta_A^2(e^{3\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3e^{-\sigma L\sqrt{\tau}}-e^{-3\sigma L\sqrt{\tau}})+(1-2\Delta_A)(e^{3\sigma L\sqrt{\tau}}-6e^{\sigma L\sqrt{\tau}}+8-3e^{-\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2}\right\} \\
& -\frac{\partial(\phi C_{\tau,\sigma L}+(1-\phi)C_{\tau,\sigma H})^2}{\partial\tau} \\
\frac{\partial Var(W_B)}{\partial\tau} =\ & \frac{P_0^2\sigma H(1-\phi)}{2\sqrt{\tau}}\left\{\frac{\Delta_B^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})+(1+2\Delta_B)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right\} \\
& +\frac{P_0^2\sigma L\phi}{2\sqrt{\tau}}\left\{\frac{\Delta_B^2(e^{3\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3e^{-\sigma L\sqrt{\tau}}-e^{-3\sigma L\sqrt{\tau}})+(1+2\Delta_B)(e^{3\sigma L\sqrt{\tau}}-6e^{\sigma L\sqrt{\tau}}+8-3e^{-\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2}\right\} \\
& -\frac{\partial(\phi C_{\tau,\sigma L}+(1-\phi)C_{\tau,\sigma H})^2}{\partial\tau} \\
\because \Delta_A =& -\Delta_B, \frac{\partial Var(W_A)}{\partial\tau} = \frac{\partial Var(W_B)}{\partial\tau}
\end{aligned}$$

We can derive approximation of the equation by Taylor expansion theory. Generally, the annualized volatility ranges from 20% to 40% and τ is always less than 0.1 for nearest month contracts, so $\sigma\sqrt{\tau}$ and $-\sigma\sqrt{\tau}$ is around 0. Because we apply Taylor theory to expand around 0, the higher order terms is dominated lower order term. Hence, we drop the higher order terms if the lower order term is non zero.

Taylor expansion series follows.

$$\because e^{a\sigma\sqrt{\tau}} = e^0 + e^0(a\sigma\sqrt{\tau}-x)|_{x=0} + \frac{e^0}{2!}(a\sigma\sqrt{\tau}-x)^2|_{x=0} + \frac{e^0}{3!}(a\sigma\sqrt{\tau}-x)^3|_{x=0} + \ldots$$

$$\begin{aligned}
&(e^{a\sigma\sqrt{\tau}} - e^{-a\sigma\sqrt{\tau}})^2 \\
&= (2a\sigma\sqrt{\tau} + O((a\sigma\sqrt{\tau})^3))^2 \doteqdot (2a\sigma\sqrt{\tau})^2
\end{aligned}$$

$$\begin{aligned}
&e^{a\sigma\sqrt{\tau}} + e^{-a\sigma\sqrt{\tau}} - 2 \\
&= \left\{(a\sigma\sqrt{\tau})^2 + O((a\sigma\sqrt{\tau})^4)\right\} \doteqdot (a\sigma\sqrt{\tau})^2
\end{aligned}$$

Expanding exponential functions from x equal to 0, we can derive approximation for previous functions, including $E(W_A), E(W_B), V(W_A), V(W_B), \frac{\partial E(W_A)}{\partial \tau}, \frac{\partial E(W_B)}{\partial \tau}, \frac{\partial V(W_A)}{\partial \tau}, \frac{\partial V(W_B)}{\partial \tau}$. In other words, we derive equation above first and then apply Taylor expansion theory on derived equation to get approximation around zero x. The following is the result after approximation.

$$\begin{aligned}
E(W_A) &\doteqdot \frac{P_0\sqrt{\tau}}{2}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] > 0 \\
E(W_B) &\doteqdot \frac{P_0\sqrt{\tau}}{2}[-b\sigma + \phi\sigma L + (1-\phi)\sigma H] > 0 \\
\frac{\partial E(W_A)}{\partial \tau} &\doteqdot \frac{P_0}{4\sqrt{\tau}}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] \\
\frac{\partial E(W_B)}{\partial \tau} &\doteqdot \frac{P_0}{4\sqrt{\tau}}[-b\sigma + \phi\sigma L + (1-\phi)\sigma H]
\end{aligned}$$

$$\begin{aligned}
Var(W_A) &\doteqdot \frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left\{(1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})^2\right\} \\
&-\frac{P_0^2}{4}\left\{\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau}\right\}^2 \\
Var(W_B) &\doteqdot \frac{P_0^2(2\Delta_B^2 + 2\Delta_B + 1)}{2}\left\{(1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})^2\right\} \\
&-\frac{P_0^2}{4}\left\{\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau}\right\}^2 \\
\frac{\partial Var(W_A)}{\partial \tau} &\doteqdot \frac{P_0^2(2\Delta_s^2 - 2\Delta_A + 1)}{2}\left\{(1-\phi)(\sigma H)^2 + \phi(\sigma L)^2\right\} \\
&-\frac{P_0^2}{4}\left\{\phi\sigma L + (1-\phi)\sigma H\right\}^2 \\
\frac{\partial Var(W_B)}{\partial \tau} &\doteqdot \frac{P_0^2(2\Delta_B^2 + 2\Delta_B + 1)}{2}\left\{(1-\phi)(\sigma H)^2 + \phi(\sigma L)^2\right\} \\
&-\frac{P_0^2}{4}\left\{\phi\sigma L + (1-\phi)\sigma H\right\}^2
\end{aligned}$$

$$\begin{aligned}
&E(W_A) - \frac{Q}{\gamma}Var(W_A) - c \\
&\doteqdot \quad \frac{P_0\sqrt{\tau}}{2}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left[(1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})^2\right]\right\} \\
&\qquad + \frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau}\right]^2\right\} - c\} \\
&E(W_B) - \frac{Q}{\gamma}Var(W_B) - c \\
&\doteqdot \quad \frac{P_0\sqrt{\tau}}{2}[-b\sigma + \phi\sigma L + (1-\phi)\sigma H] - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_B^2 + 2\Delta_B + 1)}{2}\left[(1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})^2\right]\right\} \\
&\qquad + \frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau}\right]^2\right\} - c\} \\
&\frac{\partial E(W_A)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \tau} \\
&\doteqdot \quad \frac{P_0}{4\sqrt{\tau}}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left[(1-\phi)(\sigma H)^2 + \phi(\sigma L)^2\right]\right\} \\
&\qquad + \left\{\frac{P_0^2}{4}\left[\phi\sigma L + (1-\phi)\sigma H\right]^2\right\} \\
&\frac{\partial E(W_B)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_B)}{\partial \tau} \\
&\doteqdot \quad \frac{P_0}{4\sqrt{\tau}}[-b\sigma + \phi\sigma L + (1-\phi)\sigma H] - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_B^2 + 2\Delta_B + 1)}{2}\left[(1-\phi)(\sigma H)^2 + \phi(\sigma L)^2\right]\right\} \\
&\qquad + \left\{\frac{P_0^2}{4}\left[\phi\sigma L + (1-\phi)\sigma H\right]^2\right\}
\end{aligned}$$

C.32 Proof 4-6

First, the equilibrium condition is

$$
\begin{aligned}
& E(W_A) - \frac{Q}{\gamma} Var(W_A) - c = 0 \\
\implies \quad & \frac{P_0\sqrt{\tau}}{2}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left[(1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})\right]^2\right\} \\
& + \frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau}\right]^2\right\} - c\} = 0 \\
\implies \quad & \tau\left\{\frac{P_0}{4\sqrt{\tau}}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] - \frac{Q}{\gamma}\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left[(1-\phi)(\sigma H)^2 + \phi(\sigma L)\right]^2\right. \\
& \left. + \frac{Q}{\gamma}\frac{P_0^2}{4}\left[\phi\sigma L + (1-\phi)\sigma H\right]^2\right\} = c - \frac{P_0\sqrt{\tau}}{4}[a\sigma - \phi\sigma L - (1-\phi)\sigma H]
\end{aligned}
$$

Given $\tau > 0$, we can derive sufficient condition for $\frac{\partial E(W_A)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \tau} > 0$

$$
\begin{aligned}
& \frac{\partial E(W_A)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \tau} \\
= \quad & \frac{P_0}{4\sqrt{\tau}}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left[(1-\phi)(\sigma H)^2 + \phi(\sigma L)^2\right]\right\} \\
& + \left\{\frac{P_0^2}{4}\left[\phi\sigma L + (1-\phi)\sigma H\right]^2\right\}
\end{aligned}
$$

$\frac{\partial E(W_A)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \tau} > 0$ iff $c > \frac{P_0\sqrt{\tau}}{4}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] = \frac{E(W_A)}{2}$

or $\quad \frac{\partial E(W_A)}{\partial \tau} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \tau} > 0$ iff $c > \frac{Q}{\gamma} Var(W_A)$

C.33 Proof 4-7

$$\frac{\partial^2 a}{\partial \tau^2} = \frac{\partial \frac{\partial a}{\partial \tau}}{\partial \tau} = \frac{\partial \frac{-\frac{\partial f(\tau,H,L,a)}{\partial \tau}}{\frac{\partial f(\tau,H,L,a)}{\partial a}}}{\partial \tau} = -\frac{\frac{\partial^2 f}{\partial \tau^2}\frac{\partial f}{\partial a} - \frac{\partial f}{\partial \tau}\frac{\partial^2 f}{\partial a \partial \tau}}{(\frac{\partial f}{\partial a})^2} > 0$$

First we claim $\frac{\partial^2 f(\tau,H,L,a)}{\partial a \partial \tau} > 0$

$$\frac{\partial^2 f(\tau,H,L,a)}{\partial a \partial \tau}$$
$$= \frac{P_0\left\{\frac{\sigma}{\sqrt{\tau}}(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)+a\sigma^2(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})\right\}(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})-2P_0a\sigma^2(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}})}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^3}$$
$$= \frac{P_0\left\{\frac{\sigma}{\sqrt{\tau}}(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})+a\sigma^2\left[(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^2-2(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}})\right]\right\}}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^3}$$
$$= \frac{P_0\left\{\frac{\sigma}{\sqrt{\tau}}\left[(e^{2a\sigma\sqrt{\tau}}-e^{-2a\sigma\sqrt{\tau}})-2(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})\right]-a\sigma^2\left[(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}})^2-4(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}})+4\right]\right\}}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^3}$$
$$= \frac{P_0\left\{\frac{\sigma}{\sqrt{\tau}}\left[(e^{2a\sigma\sqrt{\tau}}-e^{-2a\sigma\sqrt{\tau}})-2(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})\right]-a\sigma^2\left[(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)^2\right]\right\}}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^3}$$

To show
$$\frac{\sigma}{\sqrt{\tau}}\left[e^{2a\sigma\sqrt{\tau}} - e^{-2a\sigma\sqrt{\tau}} - 2(e^{a\sigma\sqrt{\tau}} - e^{-a\sigma\sqrt{\tau}})\right] - a\sigma^2\left[(e^{a\sigma\sqrt{\tau}} + e^{-a\sigma\sqrt{\tau}} - 2)^2\right] > 0$$
by approximation
$$\doteqdot \frac{\sigma}{\sqrt{\tau}}(2a\sigma\sqrt{\tau})^3 - a\sigma^2\left[(a\sigma\sqrt{\tau})^2\right] = 7a^3\sigma^4\tau > 0$$

$$\frac{\partial^2 f(\tau,H,L,b)}{\partial b \partial \tau}$$
$$= \frac{-P_0\left\{\frac{\sigma}{\sqrt{\tau}}(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2)+b\sigma^2(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})\right\}(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})-2P_0b\sigma^2(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2)(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}})}{(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})^3}$$
$$= \frac{-P_0\left\{\frac{\sigma}{\sqrt{\tau}}(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2)(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})+b\sigma^2\left[(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})^2-2(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2)(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}})\right]\right\}}{(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})^3}$$
$$= \frac{-P_0\left\{\frac{\sigma}{\sqrt{\tau}}\left[(e^{2b\sigma\sqrt{\tau}}-e^{-2b\sigma\sqrt{\tau}})-2(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})\right]-b\sigma^2\left[(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}})^2-4(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}})+4\right]\right\}}{(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})^3}$$
$$= \frac{-P_0\left\{\frac{\sigma}{\sqrt{\tau}}\left[(e^{2b\sigma\sqrt{\tau}}-e^{-2b\sigma\sqrt{\tau}})-2(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})\right]-b\sigma^2\left[(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2)^2\right]\right\}}{(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})^3}$$

given previous approximation
$$\because \frac{\sigma}{\sqrt{\tau}}(2b\sigma\sqrt{\tau})^3 - b\sigma^2\left[(b\sigma\sqrt{\tau})^2\right] = 7b^3\sigma^4\tau > 0$$
$$\therefore \frac{\partial^2 f}{\partial b \partial \tau} < 0$$

Next we are going to show $\frac{\partial^2 f(\tau,H,L,a)}{\partial \tau^2}\frac{\partial f(\tau,H,L,a)}{\partial a} < 0$

We know $\frac{\partial f(\tau,H,L,a)}{\partial a} > 0$ for sure. To derive $\frac{\partial^2 f(\tau,H,L,a)}{\partial \tau^2}$, we decompose it into 2 parts, including $\frac{\partial \frac{\partial E(W_A)}{\partial \tau}}{\partial \tau}$ and $\frac{\partial \frac{\partial Var(W_A)}{\partial \tau}}{\partial \tau}$.

$$
\begin{aligned}
&\frac{\partial \frac{\partial E(W_A)}{\partial \tau}}{\partial \tau} \\
=\ & \frac{\frac{P_0 a\sigma}{2\tau}\left\{\left[\frac{-1}{\sqrt{\tau}}(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)+a\sigma(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})\right](e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})-2a\sigma(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}})\right\}}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^3} \\
&-\frac{\frac{P_0 \phi\sigma L}{2\tau}\left\{\left[\frac{-1}{\sqrt{\tau}}(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)+\sigma L(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})\right](e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})-2\sigma L(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}})\right\}}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^3} \\
&-\frac{\frac{P_0(1-\phi)\sigma H}{2\tau}\left\{\left[\frac{-1}{\sqrt{\tau}}(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)+\sigma H(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})\right](e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3} \\
&-\frac{\frac{P_0(1-\phi)\sigma H}{2\tau}\left\{-2\sigma H(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3} \\
=\ & \frac{\frac{P_0}{2\tau}\left[-\frac{a\sigma}{\sqrt{\tau}}\left[e^{2a\sigma\sqrt{\tau}}-e^{-2a\sigma\sqrt{\tau}}-2(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})\right]\right]+(a\sigma)^2\left[(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^2-2(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}})\right]}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^3} \\
&-\frac{\frac{P_0\phi}{2\tau}\left[-\frac{\sigma L}{\sqrt{\tau}}\left[e^{2\sigma L\sqrt{\tau}}-e^{-2\sigma L\sqrt{\tau}}-2(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})\right]\right]+(\sigma L)^2\left[(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2-2(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}})\right]}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^3} \\
&-\frac{\frac{P_0(1-\phi)}{2\tau}\left[-\frac{\sigma H}{\sqrt{\tau}}\left[e^{2\sigma H\sqrt{\tau}}-e^{-2\sigma H\sqrt{\tau}}-2(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})\right]\right]+(\sigma H)^2\left[(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2-2(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}})\right]}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}
\end{aligned}
$$

$$
\begin{aligned}
=\ & \frac{P_0}{2\tau}\frac{-\frac{a\sigma}{\sqrt{\tau}}\left[e^{2a\sigma\sqrt{\tau}}-e^{-2a\sigma\sqrt{\tau}}-2(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})\right]-(a\sigma)^2(e^{2a\sigma\sqrt{\tau}}+e^{-2a\sigma\sqrt{\tau}}+2-4(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}})+4)}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^3} \\
&-\frac{P_0\phi}{2\tau}\frac{-\frac{\sigma L}{\sqrt{\tau}}\left[e^{2\sigma L\sqrt{\tau}}-e^{-2\sigma L\sqrt{\tau}}-2(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})\right]-(\sigma L)^2(e^{2\sigma L\sqrt{\tau}}+e^{-2aL\sqrt{\tau}}+2-4(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}})+4)}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^3} \\
&-\frac{P_0(1-\phi)}{2\tau}\frac{-\frac{\sigma H}{\sqrt{\tau}}\left[e^{2\sigma H\sqrt{\tau}}-e^{-2\sigma H\sqrt{\tau}}-2(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})\right]-(\sigma H)^2(e^{2\sigma H\sqrt{\tau}}+e^{-2\sigma H\sqrt{\tau}}+2-4(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}})+4)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}
\end{aligned}
$$

After approximation

$$\doteqdot \quad \frac{2P_0}{16\tau\sqrt{\tau}}\{-a\sigma+\phi\sigma L+(1-\phi)\sigma H\}<0$$

Then we show $\frac{\partial \frac{\partial Var(W_A)}{\partial \tau}}{\partial \tau} > 0$

$$
\begin{aligned}
&\frac{\partial \frac{\partial Var(W_A)}{\partial \tau}}{\partial \tau} \\
=\ & \frac{P_0^2\sigma H(1-\phi)}{2}\frac{\partial\left\{\frac{1}{\sqrt{\tau}}\frac{\Delta_A^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})+(1-2\Delta_A)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right\}}{\partial \tau} \\
&+\frac{P_0^2\sigma L\phi}{2}\frac{\partial\left\{\frac{1}{\sqrt{\tau}}\frac{\Delta_A^2(e^{3\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3e^{-\sigma L\sqrt{\tau}}-e^{-3\sigma L\sqrt{\tau}})+(1-2\Delta_A)(e^{3\sigma L\sqrt{\tau}}-6e^{\sigma L\sqrt{\tau}}+8-3e^{-\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2}\right\}}{\partial \tau} \\
&-\frac{\partial\frac{\partial(\phi C_{\tau,\sigma L}+(1-\phi)C_{\tau,\sigma H})^2}{\partial \tau}}{\partial \tau}
\end{aligned}
$$

$$\frac{\partial\left\{\frac{1}{\sqrt{\tau}}\frac{\Delta_A^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})+(1-2\Delta_A)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right\}}{\partial\tau}$$

$$\begin{aligned}
=\quad & \frac{\frac{1}{2\tau^{3/2}}\left\{-\left[\Delta_A^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})+(1-2\Delta_A)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})\right](e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}\\
&+\frac{\left\{\sigma H\sqrt{\tau}\left[\Delta_A^2(3e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}-3e^{-\sigma H\sqrt{\tau}}+3e^{-3\sigma H\sqrt{\tau}})+(1-2\Delta_A)(3e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}})\right](e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}\\
&-\frac{\left\{2\sigma H\sqrt{\tau}\left[\Delta_A^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})+(1-2\Delta_A)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})\right](e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}
\end{aligned}$$

$$\begin{aligned}
=\quad & \frac{1}{2\tau^{3/2}}\frac{\left\{-\Delta_A^2(e^{4\sigma H\sqrt{\tau}}-4e^{2\sigma H\sqrt{\tau}}+6-4e^{-2\sigma H\sqrt{\tau}}+e^{-4\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}\\
&+\frac{1}{2\tau^{3/2}}\frac{\left\{(1-2\Delta_A)(e^{4\sigma H\sqrt{\tau}}-7e^{2\sigma H\sqrt{\tau}}+8e^{\sigma H\sqrt{\tau}}+3-8e^{-\sigma H\sqrt{\tau}}+3e^{-2\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}\\
&+\frac{\left\{\sigma H\sqrt{\tau}\Delta_A^2(e^{4\sigma H\sqrt{\tau}}-2e^{2\sigma H\sqrt{\tau}}+2e^{-2\sigma H\sqrt{\tau}}-e^{-4\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}\\
&+\frac{\left\{(1-2\Delta_A)(e^{4\sigma H\sqrt{\tau}}+e^{2\sigma H\sqrt{\tau}}-16e^{\sigma H\sqrt{\tau}}+27-16e^{-\sigma H\sqrt{\tau}}+3e^{-2\sigma H\sqrt{\tau}})\right\}}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^3}\\
\doteq\quad & \frac{1}{2\tau^{3/2}}\left\{\frac{\left\{-\left[\Delta_A^2(16(\sigma H\sqrt{\tau})^4)+(1-2\Delta_A)(8(\sigma H\sqrt{\tau})^4\right]\right\}}{(2\sigma H\sqrt{\tau})^3}+\frac{\left\{\left[\Delta_A^2(16(\sigma H\sqrt{\tau})^4)+(1-2\Delta_A)(8(\sigma H\sqrt{\tau})^4)\right]\right\}}{(2\sigma H\sqrt{\tau})^3}\right\}=0
\end{aligned}$$

Similarly

$$\frac{\partial\left\{\frac{1}{\sqrt{\tau}}\frac{\Delta_A^2(e^{3\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3e^{-\sigma L\sqrt{\tau}}-e^{-3\sigma L\sqrt{\tau}})+(1-2\Delta_A)(e^{3\sigma L\sqrt{\tau}}-6e^{\sigma L\sqrt{\tau}}+8-3e^{-\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2}\right\}}{\partial\tau}=0$$

$$\frac{\partial\frac{\partial(\phi C_{\tau,\sigma L}+(1-\phi)C_{\tau,\sigma H})^2}{\partial\tau}}{\partial\tau}$$

$$=\quad 2\left\{\left[\phi\frac{\partial C_{\tau,\sigma L}}{\partial\tau}+(1-\phi)\frac{\partial C_{\tau,\sigma H}}{\partial\tau}\right]^2+\left[\phi C_{\tau,\sigma L}+(1-\phi)C_{\tau,\sigma H}\right]\left[\phi\frac{\partial^2 C_{\tau,\sigma L}}{\partial\tau^2}+(1-\phi)\frac{\partial^2 C_{\tau,\sigma H}}{\partial\tau^2}\right]\right\}$$

by approximation

$$\frac{\partial C_{\tau,\sigma L}}{\partial\tau}\doteq\frac{P_0\sigma L}{\sqrt{\tau}}\frac{(\sigma L\sqrt{\tau})^2}{4(\sigma L\sqrt{\tau})^2}=\frac{P_0\sigma L}{4\sqrt{\tau}}$$

$$C_{\tau,\sigma L}=P_0\frac{e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}}\doteq\frac{P_0(\sigma L\sqrt{\tau})^2}{2\sigma L\sqrt{\tau}}=\frac{P_0\sigma L\sqrt{\tau}}{2}$$

$$\begin{aligned}
&\frac{\partial^2 C_{\tau,\sigma L}}{\partial\tau^2}\\
=\quad & \frac{P_0\sigma L\left\{\frac{-\tau^{-3/2}}{2}(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)+\frac{\sigma L}{2\tau}(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})\right\}(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})-\frac{\sigma L}{2\tau}(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^3}\\
=\quad & \frac{P_0\sigma L\frac{-1}{2\tau^{3/2}}\left\{\left[e^{2\sigma L\sqrt{\tau}}+e^{-2\sigma L\sqrt{\tau}}-2(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})\right]+\sigma L\sqrt{\tau}\left[(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2-2(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}})^2+4(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}})\right]\right\}}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^3}\\
=\quad & \frac{P_0\sigma L\frac{-1}{2\tau^{3/2}}\left\{\left[e^{2\sigma L\sqrt{\tau}}+e^{-2\sigma L\sqrt{\tau}}-2(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})\right]-\sigma L\sqrt{\tau}\left[(e^{2\sigma L\sqrt{\tau}}+e^{-2\sigma L\sqrt{\tau}}+2)-4(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}})+4\right]\right\}}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^3}
\end{aligned}$$

By approximation

$$\begin{aligned}
&\frac{\partial^2 C_{\tau,\sigma L}}{\partial\tau^2}\\
&=\frac{P_0\sigma L\frac{-1}{2\tau^{3/2}}\left\{\left[e^{2\sigma L\sqrt{\tau}}+e^{-2\sigma L\sqrt{\tau}}-2(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})\right]-\sigma L\sqrt{\tau}\left[(e^{2\sigma L\sqrt{\tau}}+e^{-2\sigma L\sqrt{\tau}}+2)-4(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}})+4\right]\right\}}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^3}\\
&\doteq\frac{P_0\sigma L\frac{-1}{2\tau^{3/2}}2(\sigma L\sqrt{\tau})^3}{(2\sigma L\sqrt{\tau})^3}\\
&\doteq\frac{-P_0\sigma L}{8\tau^{3/2}}
\end{aligned}$$

$$\frac{\partial \frac{\partial(\phi C_{\tau,\sigma L}+(1-\phi)C_{\tau,\sigma H})^2}{\partial\tau}}{\partial\tau}$$
$$= 2\left\{\left[\phi\frac{\partial C_{\tau,\sigma L}}{\partial\tau}+(1-\phi)\frac{\partial C_{\tau,\sigma H}}{\partial\tau}\right]^2+\left[\phi C_{\tau,\sigma L}+(1-\phi)C_{\tau,\sigma H}\right]\left[\phi\frac{\partial^2 C_{\tau,\sigma L}}{\partial\tau^2}+(1-\phi)\frac{\partial^2 C_{\tau,\sigma H}}{\partial\tau^2}\right]\right\}$$
$$= 2\left\{\frac{P_0^2[\phi\sigma L+(1-\phi)\sigma H]^2}{16\tau}-\frac{P_0^2[\phi\sigma L+(1-\phi)\sigma H]^2}{16\tau}\right\}=0$$

Therefore, we derive $\frac{\partial^2 f(\tau,H,L,a)}{\partial\tau^2} = \frac{\partial\frac{\partial E(W_A)}{\partial\tau}}{\partial\tau} - \frac{\partial\frac{\partial Var(W_A)}{\partial\tau}}{\partial\tau}$, in which $\frac{\partial\frac{\partial E(W_A)}{\partial\tau}}{\partial\tau} < 0$ and $\frac{\partial\frac{\partial Var(W_A)}{\partial\tau}}{\partial\tau} = 0$, and $\frac{\partial^2 f(\tau,H,L,a)}{\partial\tau^2} < 0$.

And we know $\frac{\partial^2 f(\tau,H,L,a)}{\partial\tau^2} < 0$, $\frac{\partial^2 f(\tau,H,L,a)}{\partial\tau^2}\frac{\partial f(\tau,H,L,a)}{\partial a} < 0$, $\frac{\partial^2 f(\tau,H,L,a)}{\partial a\partial\tau} < 0$ and conclude $\frac{\partial^2 a}{\partial\tau^2} > 0$

$$\frac{\partial^2 a}{\partial\tau^2} = \frac{\partial\frac{\partial a}{\partial\tau}}{\partial\tau} = \frac{\partial\frac{-\frac{\partial f(\tau,H,L,a)}{\partial\tau}}{\frac{\partial f(\tau,H,L,a)}{\partial a}}}{\partial\tau} = -\frac{\frac{\partial^2 f}{\partial\tau^2}\frac{\partial f}{\partial a}-\frac{\partial f}{\partial\tau}\frac{\partial^2 f}{\partial a\partial\tau}}{(\frac{\partial f}{\partial a})^2} > 0$$

Similarly, $\frac{\partial^2 f(\tau,L,H,b)}{\partial\tau^2} < 0$ and $\frac{\partial f(\tau,H,L,b)}{\partial b} < 0$. Moreover, $\frac{\partial f(\tau,H,L,b)}{\partial\tau} > 0$ and $\frac{\partial^2 f(\tau,H,L,b)}{\partial b\partial\tau} < 0$, therefore, we can conclude $\frac{\partial^2 b}{\partial\tau^2} < 0$.

C.34 Proof 4-8

Here we are going to show, as volatility level increases, a decreases. ($\frac{\partial a}{\partial \sigma} = \frac{-\frac{\partial f}{\partial \sigma}}{\frac{\partial f}{\partial a}} < 0$)

$$\begin{aligned} C(0, a\sigma) &= \frac{1-e^{-a\sigma\sqrt{\tau}}}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}} P_0(e^{a\sigma\sqrt{\tau}} - 1) = P_0 \frac{e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}} \\ \frac{\partial C(0,a\sigma)}{\partial \sigma} &= \frac{2P_0 a\sqrt{\tau}(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^2} \end{aligned}$$

$$\begin{aligned} E(W_A) &= P_0\left(\frac{e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2}{e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}}} - \phi\frac{e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}} - (1-\phi)\frac{e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}}\right) \\ \frac{\partial E(W_A)}{\partial \sigma} &= 2P_0\sqrt{\tau}\left[\frac{a(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^2} - \phi\frac{L(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2} - (1-\phi)\frac{H(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right] \end{aligned}$$

$$\begin{aligned} &\frac{\partial Var(W_A)}{\partial \sigma} \\ =\ & H\sqrt{\tau}\frac{(1-\phi)\Delta_A^2 P_0^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2} \\ &+L\sqrt{\tau}\frac{\phi\Delta_A^2 P_0^2(e^{3\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3e^{-\sigma L\sqrt{\tau}}-e^{-3\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2} \\ &+L\sqrt{\tau}\frac{\phi(2\Delta_A-1)P_0^2(e^{3\sigma L\sqrt{\tau}}-6e^{\sigma L\sqrt{\tau}}+8-3e^{-\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2} \\ &+H\sqrt{\tau}\frac{(1-\phi)(2\Delta_A-1)P_0^2(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2} - \frac{\partial(\phi C_{0,\sigma L}+(1-\phi)C_{0,\sigma H})^2}{\partial \sigma} \end{aligned}$$

By approximation with Taylor expansion and omit the higher order terms

$$\begin{aligned} \frac{\partial E(W_A)}{\partial \sigma} &= 2P_0\sqrt{\tau}\left\{\frac{a(a\sigma\sqrt{\tau})^2}{\left[2a\sigma\sqrt{\tau}\right]^2} - \phi\frac{L\left[(\sigma L\sqrt{\tau})^2\right]}{\left[2\sigma L\sqrt{\tau}\right]^2} - (1-\phi)\frac{H\left[(\sigma H\sqrt{\tau})^2\right]}{\left[2\sigma H\sqrt{\tau}\right]^2}\right\} \\ \frac{\partial E(W_A)}{\partial \sigma} &\doteqdot \frac{P_0\sqrt{\tau}}{2}[a - \phi L - (1-\phi)H] \end{aligned}$$

$$\begin{aligned} &\frac{\partial Var(W_A)}{\partial \sigma} \\ \doteqdot\ & H\sqrt{\tau}\frac{(1-\phi)\Delta_A^2 P_0^2 8(\sigma H\sqrt{\tau})^3}{(2\sigma H\sqrt{\tau})^2} + L\sqrt{\tau}\frac{\phi\Delta_A^2 P_0^2 8(\sigma L\sqrt{\tau})^3}{(2\sigma L\sqrt{\tau})^2} \\ &+L\sqrt{\tau}\frac{\phi(1-2\Delta_A)P_0^2 4(\sigma L\sqrt{\tau})^3}{(2\sigma L\sqrt{\tau})^2} + H\sqrt{\tau}\frac{(1-\phi)(1-2\Delta_A)P_0^2 4(\sigma H\sqrt{\tau})^3}{(2\sigma H\sqrt{\tau})^2} - \frac{\partial(\phi C_{0,\sigma L}+(1-\phi)C_{0,\sigma H})^2}{\partial \sigma} \end{aligned}$$

$$\begin{aligned} &\frac{\partial(\phi C_{0,\sigma L}+(1-\phi)C_{0,\sigma H})^2}{\partial \sigma} \\ \doteqdot\ & 2(\phi C_{0,\sigma L}+(1-\phi)C_{0,\sigma H})\left(\phi\frac{\partial C_{0,\sigma L}}{\partial \sigma}+(1-\phi)\frac{\partial C_{0,\sigma H}}{\partial \tau}\right) \\ =\ & \frac{\sigma P_0^2}{2}(\phi L\sqrt{\tau}+(1-\phi)\phi H\sqrt{\tau})^2 \end{aligned}$$

Therefore,

$$\begin{aligned} &\frac{\partial Var(W_A)}{\partial \sigma} \\ \doteqdot\ & H\sqrt{\tau}(1-\phi)\Delta_A^2 P_0^2 2\sigma H\sqrt{\tau}+L\sqrt{\tau}\phi\Delta_A^2 P_0^2 2\sigma L\sqrt{\tau} \\ &+L\sqrt{\tau}\phi(1-2\Delta_A)P_0^2\sigma L\sqrt{\tau}+H\sqrt{\tau}(1-\phi)(1-2\Delta_A)P_0^2\sigma H\sqrt{\tau} - \frac{\sigma P_0^2}{2}(\phi L\sqrt{\tau}+(1-\phi)\phi H\sqrt{\tau})^2 \\ \doteqdot\ & P_0^2\sigma(H\sqrt{\tau})^2(1-\phi)(2\Delta_A^2-2\Delta_A+1) + P_0^2\sigma(L\sqrt{\tau})^2\phi(2\Delta_A^2-2\Delta_A+1) - \frac{\sigma P_0^2}{2}(\phi L\sqrt{\tau}+(1-\phi)\phi H\sqrt{\tau})^2 \end{aligned}$$

and

$$
\begin{aligned}
\frac{\partial f(L,H,\tau,a)}{\partial \sigma} &= \frac{\partial E(W_A)}{\partial \sigma} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \sigma} \\
&= \frac{P_0\sqrt{\tau}}{2\sigma}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] - \frac{2Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)\sigma}{2}\left[(1-\phi)(H\sqrt{\tau})^2 + \phi(L\sqrt{\tau})^2\right]\right\} \\
&\quad + \frac{2Q}{\gamma}\left\{\frac{\sigma P_0^2}{4}\left[\phi L + (1-\phi)H\right]^2\right\} \\
&= \frac{2}{\sigma}\Big(\frac{P_0\sqrt{\tau}}{4}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left[(1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})^2\right]\right\} \\
&\quad + \frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau}\right]^2\right\}\Big)
\end{aligned}
$$

Given equilibrium condition, we derive the condition below making $\frac{\partial f(\tau,L,H,a)}{\partial \sigma} > 0$ and $\frac{\partial a}{\partial \sigma} = \frac{-\frac{\partial f(\tau,L,H,a)}{\partial \sigma}}{\frac{\partial f(\tau,L,H,a)}{\partial a}} < 0$.

$$
\begin{aligned}
&\frac{P_0\sqrt{\tau}}{2}[a\sigma - \phi\sigma L - (1-\phi)\sigma H] - \frac{Q}{\gamma}\left\{\frac{P_0^2(2\Delta_A^2 - 2\Delta_A + 1)}{2}\left[(1-\phi)(\sigma H\sqrt{\tau})^2 + \phi(\sigma L\sqrt{\tau})\right]^2\right\} \\
&+ \frac{Q}{\gamma}\left\{\frac{P_0^2}{4}\left[\phi\sigma L\sqrt{\tau} + (1-\phi)\sigma H\sqrt{\tau}\right]^2\right\} - c\} = 0
\end{aligned}
$$

$$
\frac{\partial f(\tau,L,H,a)}{\partial \sigma} = \frac{\partial E(W_A)}{\partial \sigma} - \frac{Q}{\gamma}\frac{\partial Var(W_A)}{\partial \sigma} > 0 \text{ iff } C > \frac{P_0\sqrt{\tau}}{4}[a\sigma - \phi\sigma L - (1-\phi)\sigma H]
$$

The condition is the same as that making $\frac{\partial a}{\partial \tau} = \frac{-\frac{\partial f(\tau,L,H,a)}{\partial \tau}}{\frac{\partial f(\tau,L,H,a)}{\partial a}} < 0$

C.35 Proof 4-9

Here we are going to show how asking volatility and bidding volatility change over different volatility estimation. ($\frac{\partial a}{\partial H} - \frac{\partial b}{\partial H} > 0$)

$$
\begin{aligned}
&\frac{\partial a}{\partial H} \\
=\quad & \frac{-\frac{\partial f(\tau,L,H,a)}{\partial H}}{\frac{\partial f(\tau,L,H,a)}{\partial a}} \\
&\frac{\partial f(\tau,L,H,a)}{\partial a} \\
=\quad & \frac{2P_0\sigma\sqrt{\tau}(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^2} \\
\doteqdot\quad & \frac{P_0\sigma\sqrt{\tau}}{2} \\
&\frac{\partial f(\tau,L,H,a)}{\partial H} \\
=\quad & \frac{-2P_0(1-\phi)\sigma\sqrt{\tau}(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2} - \frac{Q}{\gamma}\left\{\frac{P_0^2\sigma\sqrt{\tau}(1-\phi)\Delta_A^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right. \\
& +\frac{P_0^2\sigma\sqrt{\tau}(1-\phi)(1-2\Delta_A)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2} \\
& \left.-2\left(\frac{\phi P_0(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}}+\frac{(1-\phi)P_0(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}}\right)\left(\frac{(1-\phi)2P_0\sigma\sqrt{\tau}(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\right)\right\} \\
\doteqdot\quad & \frac{-P_0(1-\phi)\sigma\sqrt{\tau}}{2}-\frac{Q(1-\phi)}{\gamma}\left\{P_0^2\sigma\sqrt{\tau}\Delta_A^2 2\sigma H\sqrt{\tau}+P_0^2\sigma\sqrt{\tau}(1-2\Delta_A)\sigma H\sqrt{\tau}-\frac{P_0^2(\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau})(\sigma\sqrt{\tau})}{2}\right\} \\
=\quad & \frac{P_0\sigma\sqrt{\tau}}{2}\left\{\frac{-(1-\phi)}{2}-\frac{QP_0(1-\phi)}{\gamma}\{2\sigma H\sqrt{\tau}(2\Delta_A^2-2\Delta_A+1)-(\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau})\}\right\} \\
=\quad & \frac{P_0\sigma\sqrt{\tau}}{2}\left\{\frac{-(1-\phi)}{2}-\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\right\} \\
& \because 1\geqslant \Delta_A\geqslant 0, 2\Delta_A^2-2\Delta_A+1\Delta_A\geqslant 0.5 \therefore \frac{\partial f}{\partial H}<0 \\
&\frac{\partial a}{\partial H} \\
=\quad & \frac{-\frac{\partial f(\tau,L,H,a)}{\partial H}}{\frac{\partial f(\tau,L,H,a)}{\partial a}} \\
=\quad & \left\{\frac{(1-\phi)}{2}+\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\right\}>0
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&\frac{\partial f(\tau,L,H,b)}{\partial b} \\
=\quad & \frac{-2P_0\sigma\sqrt{\tau}(e^{b\sigma\sqrt{\tau}}+e^{-b\sigma\sqrt{\tau}}-2)}{(e^{b\sigma\sqrt{\tau}}-e^{-b\sigma\sqrt{\tau}})^2} \\
\doteqdot\quad & \frac{-P_0\sigma\sqrt{\tau}}{2}
\end{aligned}
$$

Becuase $\frac{\partial f(b,L,H,\tau)}{\partial H} = \frac{\partial f(a,L,H,\tau)}{\partial H}$, we directly plug previous result into $\frac{\partial b}{\partial H}$

$$
\begin{aligned}
&\frac{\partial b}{\partial H} \\
=\quad & \frac{-\frac{\partial f(\tau,L,H,b)}{\partial H}}{\frac{\partial f(\tau,L,H,b)}{\partial b}} \\
\doteqdot\quad & \left\{\frac{(1-\phi)}{2}-\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{2H(2\Delta_B^2+2\Delta_B+1)-(H+\phi(L-H)\}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial a}{\partial H}-\frac{\partial b}{\partial H} \\
\doteqdot\quad & \left\{\frac{(1-\phi)}{2}+\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\right\} \\
& -\left\{\frac{(1-\phi)}{2}-\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{H(2\Delta_B^2+2\Delta_B+1)-(H+\phi(L-H)\}\right\} \\
=\quad & 2\frac{QP_0(1-\phi)\sigma\sqrt{\tau}}{\gamma}\{H(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}>0 \because \Delta_B=-\Delta_A
\end{aligned}
$$

On the other hand, we are also interested in $\frac{\partial a}{\partial L} - \frac{\partial b}{\partial L}$

$$
\begin{aligned}
&\frac{\partial f(\tau,L,H,a)}{\partial L} \\
= \quad & \frac{-2P_0\phi\sigma\sqrt{\tau}(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2} - \frac{Q}{\gamma}\Big\{\frac{P_0^2\sigma\sqrt{\tau}(1-\phi)\Delta_A^2(e^{3\sigma L\sqrt{\tau}}-3e^{\sigma L\sqrt{\tau}}+3e^{-\sigma L\sqrt{\tau}}-e^{-3\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2} \\
&+\frac{P_0^2\sigma\sqrt{\tau}(1-\phi)(2\Delta_A-1)(e^{3\sigma L\sqrt{\tau}}-6e^{\sigma L\sqrt{\tau}}+8-3e^{-\sigma L\sqrt{\tau}})}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2} \\
&-2\Big(\frac{\phi P_0(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}}+\frac{(1-\phi)P_0(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}}\Big)\Big(\frac{(1-\phi)2P_0\sigma\sqrt{\tau}(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2}\Big)\Big\} \\
\doteqdot \quad & \frac{-P_0\phi\sigma\sqrt{\tau}}{2}-\frac{Q\phi}{\gamma}\left\{P_0^2\sigma\sqrt{\tau}\Delta_A^2 2\sigma L\sqrt{\tau}+P_0^2\sigma\sqrt{\tau}(1-2\Delta_A)\sigma L\sqrt{\tau} - P_0^2(\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau})(\sigma\sqrt{\tau})\right\} \\
= \quad & \frac{P_0\sigma\sqrt{\tau}}{2}\left\{\frac{-\phi}{2} - \frac{Q\phi P_0}{\gamma}\{2\sigma L\sqrt{\tau}(2\Delta_A^2-2\Delta_A+1)-(\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau})\}\right\} \\
= \quad & \frac{P_0\sigma\sqrt{\tau}}{2}\left\{\frac{-\phi}{2} - \frac{Q\phi P_0\sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial a}{\partial L} \\
= \quad & \frac{-\frac{\partial f(\tau,L,H,a)}{\partial L}}{\frac{\partial f(\tau,L,H,a)}{\partial a}} \\
= \quad & \left\{\frac{\phi}{2} + \frac{Q\phi P_0\sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\right\}
\end{aligned}
$$

We can not claim if $\frac{\partial a}{\partial L}$ is greater than 0 or not, because $2L(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)<0$ for ATM call.

if $2L(2\Delta_A^2{-}2\Delta_A{+}1)-(H+\phi(L-H)\geq 0$ then

$2L(2\Delta_A^2{-}2\Delta_A{+}1) - (H + \phi(L-H))\geq 0 \;\Rightarrow\; L[2(2\Delta_A^2{-}2\Delta_A{+}1)-\phi]\geq(1-\phi)H \;\Rightarrow\; \frac{L}{H}\geq\frac{1-\phi}{2(2\Delta_A^2-2\Delta_A+1)-\phi}$

Given ATM call, $\Delta_A \doteqdot 0.5$, $2(2\Delta_A^2{-}2\Delta_A{+}1)=1$ and $\frac{L}{H}\geqslant\frac{1-\phi}{2(2\Delta_A^2-2\Delta_A+1)-\phi}\doteqdot 1 \iff \frac{L}{H}<1$

Thus we can't make conclusion on $\frac{\partial a}{\partial L}$ or $\frac{\partial b}{\partial L}$.

$$
\begin{aligned}
\frac{\partial b}{\partial L} \quad &= \frac{-\frac{\partial f(\tau,L,H,b)}{\partial L}}{\frac{\partial f(\tau,L,H,b)}{\partial b}} \\
&= \left\{\frac{\phi}{2} - \frac{Q\phi P_0\sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\right\}
\end{aligned}
$$

However, we can derive conclusion for $\frac{\partial a}{\partial L} - \frac{\partial b}{\partial L}$, if X=1.

$$
\begin{aligned}
\frac{\partial a}{\partial L} - \frac{\partial b}{\partial L} \quad &= \left\{\frac{\phi}{2} + \frac{Q\phi P_0\sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\right\} \\
&-\left\{\frac{\phi}{2} - \frac{Q\phi P_0\sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_A^2-2\Delta_A+1)-(H+\phi(L-H)\}\right\} \\
&= 2\frac{QP_0\phi\sigma\sqrt{\tau}}{\gamma}\{2L(2\Delta_A^2{-}2\Delta_A{+}1)-(H+\phi(L-H)\}
\end{aligned}
$$

Becuase $2L(2\Delta_A^2{-}2\Delta_A{+}1)-(H+\phi(L-H)<0$ for ATM call,

We conclude $\frac{\partial a}{\partial L} - \frac{\partial b}{\partial L} < 0$.

Finally, in proposition 5, we want to know effect of H and L together on the spread.

$$
\begin{aligned}
&(\tfrac{\partial a}{\partial H} - \tfrac{\partial b}{\partial H}) + (\tfrac{\partial a}{\partial L} - \tfrac{\partial b}{\partial L}) \\
\doteqdot \quad & 2\frac{QP_0\sigma\sqrt{\tau}}{\gamma}\left\{2(2\Delta_A^2 - 2\Delta_A + 1)[\phi H + (1-\phi)L] - [\phi H + (1-\phi)L]\right\} \\
\doteqdot \quad & 2\frac{QP_0\sigma\sqrt{\tau}(\phi H + (1-\phi)L)}{\gamma}\left\{2(2\Delta_A^2 - 2\Delta_A + 1) - 1\right\} = 0
\end{aligned}
$$

C.36 Proof 4-10

Becuase $\frac{\partial a}{\partial \sigma} = \frac{-\frac{\partial f(\tau,L,H,a)}{\partial \sigma}}{\frac{\partial f(\tau,L,H,a)}{\partial a}}$, so we decompose it into $\frac{\partial f(\tau,L,H,a)}{\partial \sigma}$ and $\frac{\partial f(\tau,L,H,a)}{\partial a}$.

$$
\begin{aligned}
&\frac{\partial f(\tau,L,H,a)}{\partial \sigma}\\
= \quad & 2P_0\Big(\frac{a\sqrt{\tau}(e^{a\sigma\sqrt{\tau}}+e^{-a\sigma\sqrt{\tau}}-2)}{(e^{a\sigma\sqrt{\tau}}-e^{-a\sigma\sqrt{\tau}})^2}-\frac{2\phi L\sqrt{\tau}(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{(e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}})^2}-\frac{2P_0(1-\phi)H\sqrt{\tau}(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\Big)\\
& -\frac{P_0^2 Q}{\gamma}\Big\{\frac{H\sqrt{\tau}(1-\phi)\Delta_A^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}+\frac{H\sqrt{\tau}(1-\phi)(1-2\Delta_A)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\\
& +\frac{L\sqrt{\tau}\phi\Delta_A^2(e^{3\sigma H\sqrt{\tau}}-3e^{\sigma H\sqrt{\tau}}+3e^{-\sigma H\sqrt{\tau}}-e^{-3\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}+\frac{L\sqrt{\tau}\phi(1-2\Delta_A)(e^{3\sigma H\sqrt{\tau}}-6e^{\sigma H\sqrt{\tau}}+8-3e^{-\sigma H\sqrt{\tau}})}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\\
& -2\Big(\frac{\phi(e^{\sigma L\sqrt{\tau}}+e^{-\sigma L\sqrt{\tau}}-2)}{e^{\sigma L\sqrt{\tau}}-e^{-\sigma L\sqrt{\tau}}}+\frac{(1-\phi)(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}}}\Big)\\
& *\Big(\frac{\phi 2L\sqrt{\tau}(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}+\frac{(1-\phi)2H\sqrt{\tau}(e^{\sigma H\sqrt{\tau}}+e^{-\sigma H\sqrt{\tau}}-2)}{(e^{\sigma H\sqrt{\tau}}-e^{-\sigma H\sqrt{\tau}})^2}\Big)\Big\}\\
\doteqdot \quad & \Big\{\frac{P_0 a\sqrt{\tau}}{2}-\frac{P_0(1-\phi)H\sqrt{\tau}}{2}-\frac{P_0\phi L\sqrt{\tau}}{2}\Big\}\\
& -\frac{P_0^2 Q(1-\phi)}{\gamma}\left\{H\sqrt{\tau}\Delta_A^2 2\sigma H\sqrt{\tau}+H\sqrt{\tau}(1-2\Delta_A)\sigma H\sqrt{\tau}\right\}\\
& -\frac{P_0^2 Q\phi}{\gamma}\left\{L\sqrt{\tau}\Delta_A^2 2\sigma L\sqrt{\tau}+L\sqrt{\tau}(1-2\Delta_A)\sigma L\sqrt{\tau}\right\}\\
& -\frac{P_0^2 Q}{2\gamma}(\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau})(\phi L\sqrt{\tau}+(1-\phi)H\sqrt{\tau})
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial f(\tau,L,H,a)}{\partial \sigma}\\
\doteqdot \quad & \Big(\frac{P_0 a\sigma\sqrt{\tau}}{2\sigma}-\frac{P_0(1-\phi)\sigma H\sqrt{\tau}}{2\sigma}-\frac{P_0\phi\sigma L\sqrt{\tau}}{2\sigma}\Big)-\frac{Q}{\gamma}\Big\{\frac{(1-\phi)(\sigma H\sqrt{\tau})^2(\Delta_A^2-2\Delta_A+1)+P_0^2\phi(\sigma L\sqrt{\tau})^2(\Delta_A^2-2\Delta_A+1))}{\sigma}\Big\}\\
& +\frac{Q}{\gamma}\frac{P_0^2}{2\sigma}(\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau})^2\\
= \quad & \frac{2}{\sigma}\Big\{\frac{P_0\sqrt{\tau}}{4}[a\sigma-\sigma H-\sigma L]-\frac{Q}{\gamma}\Big[\frac{P_0^2(\Delta_A^2-2\Delta_A+1)}{2}((1-\phi)(\sigma H\sqrt{\tau})^2+\phi(\sigma L\sqrt{\tau})^2)\Big]\\
& +\frac{Q}{\gamma}\Big[\frac{P_0^2}{4}(\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau})^2\Big]\Big\}
\end{aligned}
$$

$\because E(W_A) - \frac{Q}{\gamma}Var(W_A) - c = 0$ (equilibrium condition)

$$
\begin{aligned}
\Rightarrow \quad & \frac{P_0\sqrt{\tau}}{2}[a\sigma-\phi\sigma L-(1-\phi)\sigma H]-\frac{Q}{\gamma}\Big\{\frac{P_0^2(2\Delta_A^2-2\Delta_A+1)}{2}\left[(1-\phi)(\sigma H\sqrt{\tau})^2+\phi(\sigma L\sqrt{\tau})^2\right]\Big\}\\
& +\frac{Q}{\gamma}\Big\{\frac{P_0^2}{4}\left[\phi\sigma L\sqrt{\tau}+(1-\phi)\sigma H\sqrt{\tau}\right]^2\Big\}-c=0
\end{aligned}
$$

Therefore, we know if $c > \frac{P_0\sqrt{\tau}}{4}[a\sigma-\sigma H-\sigma L] = \frac{E(W_A)}{2}$, then $\frac{\partial f(\tau,L,H,a)}{\partial \sigma} > 0$.

and we can conclude $\frac{\partial a}{\partial \sigma} = \frac{-\frac{\partial f(\tau,L,H,a)}{\partial \sigma}}{\frac{\partial f(\tau,L,H,a)}{\partial a}} < 0$

The condition is the same as what makes maturity effect happen.

$$
\begin{aligned}
&\frac{\partial a}{\partial \sigma}\\
= \quad & \frac{-\frac{\partial f(\tau,L,H,a)}{\partial \sigma}}{\frac{\partial f(\tau,L,H,a)}{\partial a}}\\
= \quad & \frac{-1}{\sigma^2}\Big\{[a\sigma-\sigma H-\sigma L]-\frac{Q}{\gamma}[2P_0\sqrt{\tau}(\Delta_A^2-2\Delta_A+1)((1-\phi)(\sigma H)^2+\phi(\sigma L)^2)]\\
& +\frac{Q}{\gamma}[P_0\sqrt{\tau}(\phi\sigma L+(1-\phi)\sigma H)^2] < 0
\end{aligned}
$$

$$\begin{aligned}
&\frac{\partial b}{\partial \sigma} \\
= \quad & \frac{-\frac{\partial f(\tau,L,H,b)}{\partial \sigma}}{\frac{\partial f(\tau,L,H,b)}{\partial b}} \\
= \quad & \frac{1}{\sigma^2}\{[-b\sigma+\sigma H+\sigma L]-\frac{Q}{\gamma}[2P_0\sqrt{\tau}(\Delta_A^2-2\Delta_A+1)((1-\phi)(\sigma H)^2+\phi(\sigma L)^2)] \\
& +\frac{Q}{\gamma}[P_0\sqrt{\tau}(\phi\sigma L+(1-\phi)\sigma H)^2] > 0
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial a}{\partial \sigma}-\frac{\partial b}{\partial \sigma} \\
= \quad & \frac{1}{\sigma^2}\{-[a\sigma-\sigma H-\sigma L]-[-b\sigma+\sigma H+\sigma L] \\
& +\frac{2Q}{\gamma}[2P_0\sqrt{\tau}(\Delta_A^2-2\Delta_A+1)((1-\phi)(\sigma H)^2+\phi(\sigma L)^2)]-\frac{2Q}{\gamma}[P_0\sqrt{\tau}(\phi\sigma L+(1-\phi)\sigma H)^2]\} \\
= \quad & \frac{1}{\sigma^2}\{-[a\sigma+b\sigma]+\frac{4P_0Q\sqrt{\tau}}{\gamma}[(\Delta_A^2-2\Delta_A+1)((1-\phi)(\sigma H)^2+\phi(\sigma L)^2)]-\frac{2P_0Q\sqrt{\tau}}{\gamma}[(\phi\sigma L+(1-\phi)\sigma H)^2]\} < 0
\end{aligned}$$

C.37 Model Free Implied Volatility Calculation

(a)CM IMV

The following is the CBOE formula for VIX index, denoted as CM IMV in this paper.

$$\sigma_{CM}^2 = \frac{2}{\tau}\sum_i \frac{\Delta K_i}{K_i^2} e^{r\tau} Q(K_i) - \frac{1}{\tau}\left[\frac{F}{K_0} - 1\right]^2$$

$CM\ IMV = \sigma_{CM} * 100$

τ	:Time to expiration
F	:Forward index level derived from index option prices
K_0	:First strike below the forward index level, F
K_i	:Strike price of i^{th} out-of-the-money options; a call if $K_i > K_0$, and a put if $K_i < K_0$; both put and call if $K_i = K_0$
ΔK_i	:Interval between strike prices. $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$
r	:Risk-free interest rate to expiration
$Q(K_i)$	:The asking price/bidding price for each options with strike K_i.[1]

For the lowest strike, ΔK is simply the difference between the lowest strike and next higher strike. Similarly, ΔK for the highest strike is the difference between the highest strike and next lower strike. Additionally, the forward index is calculated by put-call-parity equation.

$$F = strike\ price + e^{r\tau}(\ Call\ price - Put\ price\ )$$

For VIX calculation, the CBOE identifies the strike price at which the absolute difference between the call and put prices is smallest. The call and put prices in equation are prices of the contracts with identified strike. In our empirical work, we directly pick up the nearest term contracts with the strike that is closest to the spot price to calculate forward index. Sometimes this index is also called effective forward price.

(b)JT IMV

The formula of JT IMV follows.

$$\sigma_{JT}^2 = \frac{1}{\tau}\sum_{j=1}^{M}[g(\tau, K_i) + g(\tau, K_{j-1})]\Delta K$$

$JT\ IMV = \sigma_{JT} * 100$

m	: The number of toal toal strike
ΔK	: Interval between strike prices $\Delta K = \frac{(K_{\max} - K_{\min})}{m}$ $K_j = K_{\min} + j\Delta K$
$g(\tau, K_j)$	$= \frac{[C^F(\tau, K_j) - \max(0, F_0 - K_j)]}{K_j^2}$
F_0	:Forward asset price at time 0 $F_0 = S_0 * e^{r\tau}$
$C^F(\tau, K)$	$= C(\tau, K) * e^{r\tau}$

[1] In VIX index calculation, $Q(K_i)$ is midpoint of the bid-ask spread for each option with strike K_i.

The formula of JT IMV is similar to CM IMV. However, while CM used OTM put options, JT employed the value, the difference between call options price and its intrinsic value. The loading of ΔK is the same as CM IMV calculation, if all intervals between two consecutive strikes are the same. Because the interval between two consecutive strikes could change over different price level, using fixed loading ΔK specified in JT is not as ideal as using real interval between strike prices. Therefore we use $\frac{K_{i+1}-K_{i-1}}{2}$ to calculate JT IMV. Furthermore, Because the payoff of the derivatives contracts excludes the dividend of the underlying asset, the spot price, containing the dividend, is not a good reference for forward price. Again, we use the effective forward price rather than use $S_0 * e^{r\tau}$ to be the forward asset price in JT IMV calculation.

(c)CM (1998) IMV

Here we also include original formula proposed by Carr and Madan. The variance for time $[0, \tau]$ is

$$\sigma^2_{(0,\tau)} = e^{r\tau}\left\{\int_0^K \tfrac{2}{K^2}P(K,\tau)dK + \int_k^\infty \tfrac{2}{K^2}C(K,\tau)dK\right\}$$

$$\sigma^2_{(0,\tau)} = 2\left\{\sum_{K_{\min}}^{K_0} \tfrac{\Delta K_i}{K_i^2}e^{r\tau}P(K,\tau) + \sum_{K_0}^{\infty} \tfrac{\Delta K_i}{K_i^2}e^{r\tau}C(K,\tau)\right\}$$

$$\sigma^2_{CMO} = \frac{\sigma^2_{(0,\tau)}}{\tau}$$

$$CMO\ IMV = \sigma_{cmo} * 100$$

K_0:First strike below the forward index level, F
K_i:Strike price of i^{th} out-of-the-money options; a call if $K_i > K_0$, and a put if $K_i < K_0$; both put and call if $K_i = K_0$

In comparison with the formula of CBOE IMV, the original formula doesn't include expected return, $[\frac{F}{K_0} - 1]$. We denote this IMV as CM (1998) and show that the spread based on CM (1998) also demonstrates the same patterns in Appendix C-1.

(d)CMITM IMV

We can also utilize information of ITM options. After deriving effective forward price, F, we apply put-call-parity equation again to get implicit prices of OTM from prices of ITM options. Then we plug implicit OTM prices into the CBOE VIX formula to calculate new IMV measure, denoted as $CMITM\ IMV$. The similar patterns are found and plotted in Appendix C-1.

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