

ESSAYS ON FINANCIAL FRICTIONS AND BUSINESS CYCLES

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ESSAYS ON FINANCIAL FRICTIONS AND BUSINESS CYCLES

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In this dissertation I explore the relationship between the frictions in a country's financial market and its business cycle movements. It is well known that the financial market is far from perfect, and shocks originating in such market could have sizable impact on the real economy. On the other hand, evolvement in the financial market could also be a reflection of the real economy. For example, economic downturn often leads to high borrowing cost for a country in the international financial market. The essays in this dissertation present an analysis of this two-way relationship, both qualitatively and quantitatively.

The first essay studies the link between country credit spreads – defined as the difference between a home country's cost of borrowing from the international credit market and the world riskless interest rate – and the domestic business cycle fluctuations. By combining both empirical and theoretical analysis, this essay shows that deteriorating credit markets are both reflections of a declining economy and a major factor that depresses economic activity. This study uses a quarterly dataset over the period 1972Q1 to 2010Q1 for South Korea.

The second essay probes the importance of financial shocks in creating business cycles in the United States. It starts from a theoretical dynamic stochastic generating equilibrium model, which identifies positive financial shocks as those that drag down the corporate net worth while raising domestic output. An empirical analysis later uses this property to identify financial shocks and study their importance in creating

business cycle movement for the U.S. in the past fifty years. This property is in stark contrast to technological shocks, which raise both corporate net worth and total output.

BIOGRAPHICAL SKETCH

Yankun Wang is a Ph.D. student in the Department of Economics at Cornell University. Yankun's research interests are in the areas of Macroeconomics and Financial Economics, especially the study of Macro-Finance linkage. Prior to coming to Cornell, she had studied at Sun Yat-sen University in China and University of Cambridge in UK. She was a visiting student at Harvard University during the academic year of 2008 to 2009 under Sage Fellowship.

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Chapter 1

Introduction

This dissertation studies the two-way relation between a country's corporate sector borrow costs and its business cycle movements. Macroeconomics has a long-standing tradition, starting from Irving Fisher and John Maynard Keynes if not earlier authors, that emphasizes the role of credit market conditions in the propagation of cyclical fluctuations. Deteriorating credit markets are both reflections of a declining economy and a major factor that depresses economic activity. My goal in this dissertation is to study this interaction both qualitatively and quantitatively. Both essays in this dissertation achieve this goal, but from different angles.

In the first essay, I disentangle the complicated interrelations among country interest spreads, world interest rates, terms of trade, and business cycles in a small open economy. The key mechanism is the link between country spreads – defined as the difference between a home country's cost of borrowing abroad and the world riskless interest rate – and domestic business cycle fluctuations. On the one hand, high country spreads are reflections of a declining economy; on the other hand, poor credit conditions could severely damage economic activities. In studying this mechanism I develop both an empirical and a theoretical analyses. I find that U.S. interest rate shocks explain about 10 percent of the movements in aggregate activity in South Korea during the sample period; terms of trade shocks account for about 20 percent of aggregate fluctuations; and country spread shocks explain about 27 percent.

The second essay is an attempt to quantify the importance of financial shocks in creating business cycle movements in the United States. This is achieved in two steps. In the first part I embed the financial accelerator of Bernanke, Gertler and Gilchrist (1999) into a stylized dynamic stochastic general equilibrium model and study the transmission of financial and technology shocks. It turns out that both gross domestic output and corporate net worth increase after a positive technology shock, while a positive financial shock could drag down the corporate net worth but increase total output. Such properties are then used as identification conditions in the structural vector auto-regression analysis in the second part

the paper. It finds that technology shocks account for up to 40 percent of movements in total output for the U.S. during the past fifty years, while financial shocks account to about 10 percent of business cycle movement.

Chapter 2

Country Spreads and Balance Sheets

1 Introduction

In this paper, I disentangle the complicated interrelations among country interest spreads, world interest rates, terms of trade, and business cycles in a small open economy.

Times of high interest rates are typically associated with economic depressions while low interest rates are associated with economic expansions. Specifically, what is the relationship between country interest spreads – defined as the difference between a country’s cost of borrowing abroad and the world riskless interest rate – and macroeconomic conditions? This question has been studied previously. Recent examples include Eichengreen and Mody (2000), Neumeyer and Perri (2001), Uribe and Yue (2005) and Weigel and Gemmill (2006).

Eichengreen and Mody (2000) finds that higher credit quality leads to lower spread, thus confirms the view that market fundamentals contribute to the determination of country spread. Weigel and Gemmill (2006) uses bond prices to investigate how the creditworthiness of four Latin American countries is influenced by global, regional and country specific factors. They conclude that the credit worthiness of these four emerging markets is driven mainly by a set of factors which are related to stock market in the region as well as in the United States. Not much is said, however, about the fact that movements in domestic economy may be caused partly by variation in country interest rate itself. Neumeyer and Perri (2001) represents another extreme of the spectrum by assuming that the country spread follows an exogenous process. They find that interest rate is an important factor for explaining business cycles in emerging economies.

In reality, both country spreads and the level of economic activities are endogenously determined, and they closely interplay with each other. As a matter of fact, macroeconomics has a long-standing tradition, beginning with Irving Fisher and John Maynard Keynes if not earlier authors, that emphasizes the role of credit market conditions in the propagation of cyclical fluctuations. As Gertler (1988) discusses, deteriorating credit market conditions are

both reflections of a declining real economy and a major factor depressing economic activity. In particular, Bernanke, Gertler and Gilchrist (1998) develops a “financial accelerator” property in that endogenous developments in credit markets work to propagate and amplify shocks to the macroeconomy.

In this paper I first follow Bernanke, Gertler and Gilchrist (1998) by introducing endogenously determined financial market frictions through the balance sheet effects in a small open economy DSGE model. I then validate the theoretical results from this model by comparing them with an empirical time series econometric analysis.

The key mechanism in my theoretical analysis is the link between country spreads and domestic business cycle fluctuations. Deteriorating credit markets are both reflections of a declining economy and a major factor that depresses economic activity. Specifically, I assume that domestic entrepreneurs resort to international credit markets to cover the difference between investment needs and their net worths. In the presence of credit market frictions and with the total amount of capital investment held constant, this model of lending with costly state verification implies that the external finance premium, or country spread, depends inversely on domestic entrepreneurs’ net worths. This inverse relationship arises naturally because less net worth implies greater potential divergence of interests between borrowers and creditors, leading to increased agency costs. In equilibrium, foreign creditors must be compensated for higher agency costs by a larger premium. Since the borrowers’ net worths are usually procyclical, the external finance premium will be countercyclical, enhancing the fluctuations in business cycles.

Three kinds of shocks are considered in the theoretical analysis: shocks to credit spread, shocks to world interest rate and shocks to country terms of trade. A positive credit spread shock increases borrowing cost, which in turn depresses the economy, causing drop in both gross domestic investment and output. World interest rate shocks can affect the business fluctuations through two different channels. First an increase in US interest rate, which is taken to be the world interest rate in this paper, can increase country’s borrowing cost through the no-arbitrage condition. Second, rising world interest rate brings down country spread. As a result, the movement of country interest rate to world rate shocks is not one-to-one. In contrast, terms of trade shocks have direct impact on the fundamentals, thus shifting

the country spread. Similar issues have been examined in Uribe and Yue (2006), which also models endogenously determined country spread and aggregate output. However, their paper uses short-run restrictions to identify world interest rate shocks and country spread shocks, which is harder to justify compared with the identification scheme used in this paper. Furthermore, in their paper the theoretical analysis is limited to the case in which the law of motion of the country spread is exogenously given, and does not address why country spread depends upon variables such as output or the world interest rate. Therefore, it lacks microfoundation for the determination of country spread. The current paper addresses this issue and has a microfoundation for country spread. In a sense, this paper is an extension of Uribe and Yue (2006).

In my empirical analysis I introduce a VAR-based study of the transmission mechanism of the shocks described above. This is achieved through analyzing a six-variable VAR system, including total output, total investment, trade balance to output ratio, country spread, terms of trade and world interest rate. The dataset is from South Korea, covering the period 1972 to 2010 examined quarterly. Yielding reliable information about the transmission mechanism requires a credible identification scheme. This is achieved in two steps. First, the impulse responses of business cycles to terms of trade shock and world interest rate shock are calculated without imposing any constraints other than that both terms of trade and world interest rate are exogenous processes to the small open economy. Second, the identification of country spread shocks is obtained by imposing sign restrictions on the impulse response functions to a country spread shock, as in Uhlig (2005). Such sign restrictions avoid the conventional short-run or long-run restrictions, which are usually hard to justify and subject to debate. Applying sign restrictions to identify country spread shocks leads to more robust results. Two kinds of results can be derived through this exercise. First are the estimated impulse responses of all three shocks after imposing a Bayesian framework to achieve exact identification for country spread shocks. Second is the percentage of variations in aggregate output which could be explained by the three shocks. This measures the contribution and relative importance of various shocks to the movement of business cycles.

All the results from the VAR analysis are then contrasted with those derived from a DSGE model. It is shown that the impulse response functions to these shocks are broadly

consistent with the findings in the empirical VAR model.

The rest of the paper is organized as follows: Section 2 presents the theoretical DSGE model. Section 3 parameterizes the theoretical model and derives theoretical impulse response functions. Section 4 introduces the empirical model and identifies country spread shocks, terms of trade shocks, and world interest rate shocks in this context. Section 5 concludes. The appendices contain detailed discussions on the dataset, the Bayesian VAR estimation strategy, and the financial contracting problem.

2 A Small Open Economy with Financial Accelerator

The theoretical model starts from a standard dynamic stochastic general equilibrium (DSGE) model, but departs from it to include the balance sheet effects. In this model there are two types of agents in the domestic economy: risk-averse consumers, and risk-neutral consumers. The risk-averse consumer owns both the retail firms and the wholesale good production firms, and behaves like a typical representative household in a dynamic stochastic economy. The risk-averse consumer chooses consumption, holds bonds and provides labor to the wholesale firms. The risk-neutral consumer rents capital to wholesale firms, and for this reason, I call him “entrepreneur” for the rest of the paper. Entrepreneurs finance investment in excess of their own net worth by borrowing from foreigners, and they only consume when they depart from the scene. The product of the wholesale firm – the wholesale good – is transformed into final consumption goods by the retail firms. Adding retail firms to the model permits us to incorporate price inertia in a tractable way by separating price setting decisions from borrowing and lending decisions, as discussed later.

2.1 Risk-Averse Consumers

Suppose the preferences of risk-averse consumers are identical and are described by the following function

$$U(C_t, L_t), \tag{1}$$

where

$$C_t = \left[(1 - \psi)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \psi^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \quad (2)$$

In the above expression, $\psi \in [0, 1]$ measures home-bias, and η measures substitutability between domestic and foreign goods. The period utility function $U(\cdot, \cdot)$ is continuous and second order differentiable, and satisfies the following conditions: $U_C(\cdot, \cdot) > 0$, $U_{CC}(\cdot, \cdot) < 0$, $U_L(\cdot, \cdot) < 0$, and $U_{LL}(\cdot, \cdot) > 0$.

Each risk-averse consumer works, consumes, and invests his or her savings in riskless domestic bonds. Thus, the individual budget constraint is given by

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + B_{H,t+1} = W_tL_t + \Pi_t + (1 + i_{H,t})B_{H,t}, \quad (3)$$

for $t = 0, 1, 2, \dots$, where $P_{H,t}$ is the price of domestic good, $P_{F,t}$ is the price of an imported foreign good expressed in domestic currency, Π_t is the profit from the retail firms, $i_{H,t}$ is the interest paid on domestic bonds, and $B_{H,t}$ is the holding of domestic bond throughout period t .

The optimal allocation of consumption between the domestic and imported goods is determined by

$$C_{H,t} = (1 - \psi) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad (4)$$

and

$$C_{F,t} = \psi \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t,$$

where $P_t \equiv [(1 - \psi)(P_{H,t})^{1-\eta} + \psi(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI), and $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$.

The risk-averse consumers choose $C_{H,t}(j)$ for $j \in [0, 1]$, $C_{F,t}$, L_t and $B_{H,t}$ to maximize the expected discounted utility (3) subject to budget constraint (4). Note that since I assume all the risk-averse consumers are identical to each other, in equilibrium $B_{H,t} = 0$. Solving the risk-averse consumers' problem yields standard first order conditions for consumption and labor supply

$$U_{C,t} = (1 + i_{H,t})\beta E_t \left[U_{C,t+1} \frac{P_t}{P_{t+1}} \right], \quad (5)$$

and

$$\frac{W_t}{P_t} = \frac{-U_{L,t}}{U_{C,t}}. \quad (6)$$

2.2 Firms

Firms are owned by risk-averse consumers. At the beginning of each period, entrepreneurs purchase raw capital, convert it into usable capital, and combine it with hired labor to produce domestic goods in such firms. The production of the firms is competitive, and uses a constant return to scale technology

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad (7)$$

where Y_t is aggregate output of domestic goods, K_t is the aggregate usable capital input provided by entrepreneurs in the previous period, L_t denotes labor input, and A_t is an exogenous technology parameter.

The labor required in this production is supplied by the risk-averse consumers. In every period, the representative firm maximizes profits given by

$$P_t Y_t - W_t L_t - R_t K_t,$$

subject to the technology constraint. The solution to its problem is standard:

$$R_t K_t = \alpha P_t Y_t; \quad (8)$$

$$W_t L_t = (1 - \alpha) P_t Y_t. \quad (9)$$

2.3 Risk-Neutral Entrepreneurs

Entrepreneurs play an important role in this model, because they finance investment partly with loans from foreigners, and foreign borrowing is subject to financial frictions. In principle, many factors can contribute to imperfections in credit market, e.g., information asymmetry or enforcement problems. In this paper, however, I follow a number of previous papers in assuming a “costly state verification” (CSV) problem of the type first analyzed by Townsend (1979), in which lenders must pay a proportional “auditing cost” to observe an individual

borrower's realized return, while the borrower observes the return for free. This formulation allows for a simple, but natural way to introduce country spread into the model.

It is convenient to start to describe the entrepreneurs' behavior at the end of period t . At this time, the entrepreneurs have some net worth, $P_t N_t$, expressed in domestic currency, and they have access to the world capital market. The safe world interest rate for foreign currency, borrowed through period $t+1$, $i_{F,t+1}$, is known at the end of period t . Let S_t be the real exchange rate at period t . The effective foreign interest $i_{F,t+1}^e$, defined as $i_{F,t+1}(S_{t+1}/S_t)$, is the actual world interest rate in terms of domestic goods. The fluctuations in the effective world interest rate is captured by assuming that its deviation from the steady-state level is exogenously given and follows an AR(1) process

$$\widehat{i}_{F,t+1}^e = \chi_{i_F} \widehat{i}_{F,t}^e + \epsilon_{t+1}^{i_F}. \quad (10)$$

This formulation is desirable because it is consistent with the specification of world interest rate in the VAR analysis.

An entrepreneur can invest in capital for the next period, which he finances by both his own net worth and borrowings from foreign creditors. In assembling the domestic final goods and imports into capital, the entrepreneur follows the following rule

$$K_{t+1}^H = (1 - \psi^*) \left(\frac{P_{H,t}}{Q_t} \right)^{-\eta^*} K_{t+1}, \quad (11)$$

$$K_{t+1}^F = \psi^* \left(\frac{P_{F,t}}{Q_t} \right)^{-\eta^*} K_{t+1}, \quad (12)$$

and

$$Q_t = [(1 - \psi^*)(P_{H,t})^{1-\eta^*} + \psi^*(P_{F,t})^{1-\eta^*}]^{\frac{1}{1-\eta^*}}, \quad (13)$$

where K_{t+1} is the capital investment in period $t+1$, K_{t+1}^H is the domestic goods used in assembling capital K_{t+1} , K_{t+1}^F is the foreign goods used in assembling capital K_{t+1} , and Q_t is the aggregate price index for capital. As a result, the entrepreneur's budget constraint is:

$$P_t N_t + S_{t+1} D_{t+1} = Q_t K_{t+1}. \quad (14)$$

The entrepreneur is risk-neutral, and chooses D_{t+1} and K_{t+1} to maximize his profit from capital investment. The details of his borrowing behavior and the fundamental assumptions justifying it can be found in Appendix C. Here, only the main aspects of his behavior are described.

In the absence of financial frictions, the expected yield on capital would be equal to the effective world interest rate. However, with costly state verification, this is not the case anymore. There turns out to be a wedge between the expected return to investment and the effective world interest rate. This wedge is denoted as ρ_t , the external finance premium. More specifically,

$$1 + \rho_{t+1} = \frac{\frac{\mathbf{E}_t(R_{t+1})}{Q_t}}{(1 + i_{F,t+1}^e)}. \quad (15)$$

Appendix C shows that the risk premium can be expressed as an increasing function of the value of investment relative to net worth. More specifically,

$$1 + \rho_{t+1} = F\left(\frac{Q_t K_{t+1}}{P_t N_t}\right). \quad (16)$$

There is a disturbance term, ϵ_{t+1}^ρ , is called country spread shock in this model. It is the shocks to the monitoring cost in this model. The distribution of ϵ_{t+1}^ρ , which corresponds to country spread shock in the VAR analysis, is exogenously given.

Entrepreneurs are assumed to have finite horizons, and each entrepreneur has a constant probability δ of surviving to the next period, implying an expected lifetime of $1/(1 - \delta)$. This assumption captures the phenomenon of ongoing creation and demise of firms. I also assume the birth rate of entrepreneurs to be such that the fraction of agents who are entrepreneurs is constant. Entrepreneurs only consume when they die, and the composition of their consumption is the same as the risk-averse consumers. For convenience, I also assume that capital fully depreciates after each period. With such formulation, an entrepreneur's net worth evolves according to

$$P_t N_t = \delta \left[R_t K_t - \Phi(1 + \rho_t) R_t K_t - (1 + i_{F,t+1}^e)(Q_{t-1} K_t - P_{t-1} N_{t-1}) \right], \quad (17)$$

where $\Phi(1 + \rho_t)$ is the deadweight loss associated with monitoring cost in the financial contract, and I have used the entrepreneur's budget constraint. Appendix C shows that this monitoring cost is an increasing function of risk premium, thus $\Phi'(\cdot) > 0$.

2.4 Equilibrium

To describe the dynamic general equilibrium, I now combine various sectors of this economy.

The VAR analysis has explored the effects of terms of trade shocks on the domestic economy. Terms of trade in this model can be defined as

$$Z_t = \frac{P_{H,t}}{P_{F,t}},$$

the ratio of export prices and import prices, both measured in domestic currency. Similar to the empirical analysis, I stipulate that Z_t follows an AR (1) process. Specifically,

$$Z_{t+1} = \chi_Z Z_t + \epsilon_{t+1}^Z,$$

where χ_Z is the autocorrelation of the terms of trade process as specified in the VAR system, and ϵ_{t+1}^Z across time are i.i.d. innovations with mean zero and standard deviation σ_Z .

In describing the model, market clearing conditions for wholesale goods and labor have been implicitly used, while market clearing condition for the domestic final goods is still left unspecified. Domestic goods may be consumed by both risk-averse consumers and risk-neutral consumers(entrepreneurs), or used by entrepreneurs in assembling capital, or sold to foreigners. Because I do not model the foreign sector explicitly, I simply assume that the amount of exports, χ_t , is a decreasing function of terms of trade.¹ Thus the market for domestic final goods will clear if

$$C_{H,t}(j) + C_{H,t(j)}^E + K_{H,t+1}(j) + \chi(j) = Y_t(j),$$

for $j \in [0, 1]$, where $C_{H,t(j)}^E$ denotes the consumption of good j of entrepreneurs. Strictly speaking, monitoring cost should be included into this equation. Under reasonable parameterization, however, this factor has no perceptible impact on the model's dynamics. Thus I omit it here.

The dynamic stochastic equilibrium is defined in the usual way after specifying stochastic processes for ϵ^ρ , the country spread shock and ϵ^i , the monetary shock. Although I have not explicitly introduced an expression for money demand, it would be straightforward to amend the model to include this feature.

¹For the Korea data set we use, the correlation between export and terms of trade is -0.84.

3 Endogenous Country Spread

3.1 Functional Forms and Parameter Values

In calibrating the model, the time unit is set to be one quarter. First, I add the feature of external discount factor such that the model will be stationary for a broader set of parameter values. Suppose that the discount factor for risk-averse consumers depends on their consumption and labor supply for each period but they do not internalize this fact. Or, one can think that the discount factor depends not on the risk-averse consumer's own consumption and labor supply, but rather on the average per capita levels of those variables. More specifically,

$$\beta_{t+1} = \Delta(\tilde{C}_t, \tilde{L}_t)\beta_t, \quad \text{for } t \geq 0,$$

where \tilde{C}_t and \tilde{L}_t denote the average per capita consumption and labor supply, and in equilibrium, $\tilde{C}_t = C_t$ and $\tilde{L}_t = L_t$.

Following Mendoza (1991), I use the following functional form for risk-averse consumers' preference

$$U(C, L) = \frac{[C - w^{-1}L^w]^{1-\gamma}}{1-\gamma} - 1,$$

$$\Delta(\tilde{C}, \tilde{L}) = \left[1 + \tilde{C} - w^{-1}\tilde{L}^w\right]^{-\Psi_1},$$

and set the value of the parameters at $\gamma = 2$, $w = 2.955$ and $\Psi_1 = 0.10$. The implied subjective discount factor in the steady state is 0.962. $\tau = 3$ implies a labor supply elasticity of 1/3. The elasticity of substitution between differentiated home goods, ϵ , is set to be 6, implying a steady state markup of 20 percent. $A_R = 1$, $A = 1$, $\varphi = 0.3$ and $\alpha = 0.35$ are standard values in the business cycle literature. θ is set to be 0.75, consistent with an average period of one year between price adjustment. For the baseline calibration I assume that export is exogenously given and remains at a constant level. To determine the exact value of μ , I set the steady state bankruptcy level at 10 percent. The range of the uniform distribution of the idiosyncratic shock to capital returns is set to be $[0.2, 1.8]$. These values can be varied without affecting the results too much. In other words, the formulation is robust. The parameters ψ and ψ^* are both chosen to be 0.5, representing the case of no

home bias. η and η^* are both 3 which is again a standard value. For the monetary policy, $\alpha_\pi = 2$, $\alpha_y = 0.25$, and $\alpha_i = 0.85$, which are in line with actual data. The distribution of the terms of trade shocks and world interest rate shocks in the theoretical model follows the estimation in the VAR analysis.

Although there is no readily available value for δ , I estimate it by choosing the value which minimizes the distance between the estimated impulse response functions and the theoretically derived ones. Those pairs of impulse responses with larger estimated standard error are penalized more heavily in the minimization problem.

We will discuss the details of the results from the theoretical model after introducing the empirical analysis.

4 Empirical Analysis with a VAR Model

This section presents the VAR model and identifies three kinds of shocks: country spread shocks, terms of trade shocks and world interest rate shocks. The dynamic system in the model includes six variables: gross domestic output, gross domestic investment, trade balance to output ratio, country spread, terms of trade and world interest rate.

The dataset used in this analysis is quarterly data over the period 1972Q1 to 2010Q1 for South Korea. Foreign economic assistance was essential to South Korea's recovery from the Korean War in the 1950s and to Korea's economic growth in the 1960s. During the mid-1960s, however, South Korea's economy was growing so rapidly that the United States phased out its aid program to Korea, and assistance from the United States ended in the early 1970s. From then on South Korea had to meet its need for investment capital on the competitive international market and the country became increasingly integrated into the international capital market. It is from this moment our analysis begins. Foreign loans have played significant role in Korea's economic development, but in the Asian financial crisis in the late 1990s, dollarized foreign debts greatly increased the vulnerability of the economy. Therefore, it is of particular relevance to explore the relationship between country spreads and business cycles for South Korea.

4.1 Model Specifications

The starting point of the structural VAR analysis is the reduced form VAR of order p and dimension m

$$Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \dots + B_p Y_{t-p} + u_t, \quad t = 1, \dots, T, \quad (22)$$

where Y_t is $m \times 1$ vector of data at time $t = 1-p, \dots, T$; B_t is a coefficient matrix of size $m \times m$ and u_t is the one-step ahead prediction error with variance-covariance matrix Σ . Neither intercept nor a time trend is included since all the data to be used later is guaranteed to be stationary and has mean zero by HP filtering.

In this paper Y_t is a vector consisting of 6 variables:

$$Y_t = \left[\widehat{y}_t, \widehat{i}_t, tbr_t, \widetilde{cs}_t, \widehat{tot}_t, \widetilde{i}_{F,t} \right]',$$

where y_t denotes real gross domestic output, i_t denotes real gross investment, tbr_t denotes the trade balance to output ratio, cs_t denotes country spread, tot_t denotes terms of trade, $i_{F,t}$ denotes world interest rate, and u_t is the one period ahead prediction error with covariance matrix Ω . A hat $(\widehat{})$ on y_t , i_t and tot_t denotes the log series detrended by HP filter². Country spread is measured as the difference between the real corporate bond rate, after adjusting for the currency risk, and the world interest rate, which is defined as the real three-month U.S. treasury bill rate. Output, investment and trade balance are all seasonally adjusted. I have not included some measure of country debt into the system because adding external debt to the GDP ratio does not improve the fit of the model due to the high positive correlation of external debt to trade balance. More details on the data are provided in Appendix C.

In estimating the VAR system, it is assumed that both tot_t and $i_{F,t}$ follow a simple univariate AR(1) process. This assumption is reasonable because South Korea is a small open economy³, and unlikely to have a sizable impact on the world interest rate and its own

²Some economists have argued that HP filter, while working pretty well for developed countries, is not the most appropriate choice for developing countries. Although South Korea is an OECD country, it has rather volatile business cycle movements. So Baxter-King band-pass filter is also applied for robustness check. Since the results do not vary significantly from the case of HP filter, we omit the results here.

³In year 2007, the world output is \$54.62 trillion after adjustment for purchasing power parity while South Korea's GDP is \$1.206 trillion, which is about 2.21 percent of world output, according to CIA official statistics.

terms of trade.

The VAR system is estimated by standard OLS technique. To choose the appropriate lag length for the system, the Akaike Information Criterion (AIC) is calculated⁴. The result points to the choice of AR(1) specification. As a result, p is equal to 1.

4.2 Identification of External Shocks

World interest rate shocks and terms of trade shocks are external to the domestic economy, thus the corresponding identification schemes are different from the scheme for credit spread shocks. This subsection discusses the identification of these two external shocks.

The goal in the identification process is to decompose the prediction error u_t into economically meaningful or fundamental shocks. In the end, what interest us is examining the impulse responses to such fundamental shocks instead of the prediction errors. Suppose there are a total of 6 fundamental innovations which are mutually independent. Independence of the fundamental innovations is an appealing assumption since otherwise there would remain some unexplained causal relationship between them. To identify the fundamental shocks v_t , I assume there is a matrix A such that $u_t = Av_t$. The j th column of A then represents the immediate impact on all the variables in the system of the j th fundamental innovation, one standard error in size. Since both terms of trade and world interest rate have exogenous movements, the last two elements in v_t coincide with the prediction errors in the AR(1) processes for terms of trade and world interest rate, after normalization of the variance⁵. As for the credit spread shock, without loss of generality, let us assume that it resides in the fourth position in v_t . Note that there is no need to specify the first three elements in v_t , since they are not of interest here.

⁴The AIC is -22.37 for the AR(1) specification, -20.69 for the AR(2) specification, and -15.60 for the AR(3) specification.

⁵The correlation between the two prediction errors is 0.04. Thus it seems reasonable to assume they are independent.

By the specifications above, matrix A takes the following form:

$$A = \begin{bmatrix} \hat{A} \\ e_5 * s_{tot} \\ e_6 * s_\rho \end{bmatrix},$$

where \hat{A} is a 4 by 6 matrix, e_5 is the 1 by 6 unit vector with the fifth element being 1, e_6 is a 1 by 6 unit vector with the last element being 1, s_{tot} and s_ρ are the standard deviations of terms of trade shocks, and world interest rate shocks respectively. One restriction on A emerges from the covariance structure

$$\Omega = E[u_t u_t'] = A E[v_t v_t'] A'. \quad (23)$$

Simple accounting shows that there are nine degrees of freedom in specifying A^6 , hence further restrictions are needed to achieve the identification of A . Usually this is done by either (1) choosing A to be a Cholesky factor of Σ as in Sims (1986) or (2) separating transitory from permanent components as in Blanchard and Quah (1986) or (3) imposing structural relationships between the fundamental innovations and the one-step-ahead prediction errors as in Bernanke (1986) or Sims (1986).

Here, I propose a different strategy. Because we are only interested in the last three elements in v_t , it is only necessary to identify the last three columns of A to get the impulse responses. Due to the special structure of matrices A , the last two columns of A can be computed easily from the following relationships:

$$\Omega_5 = \begin{bmatrix} s_{tot} a_{15} \\ s_{tot} a_{25} \\ s_{tot} a_{35} \\ s_{tot} a_{45} \\ s_{tot}^2 \\ 0 \end{bmatrix} \quad \text{and} \quad \Omega_6 = \begin{bmatrix} s_\rho a_{16} \\ s_\rho a_{26} \\ s_\rho a_{36} \\ s_\rho a_{46} \\ s_\rho^2 \\ 0 \end{bmatrix},$$

⁶Since Ω is symmetric, the covariance structure imposes $(6 \times 5)/2 = 15$ conditions on the elements of A . There are $6 \times 4 = 24$ elements of A that need to be specified. Thus we have $24 - 15 = 9$ degrees of freedom left. Strictly speaking, this accounting condition, proposed by Rothenberg(1971), is a necessary but not sufficient condition for exact identification. Rubio-Ramirez, Waggoner and Zhua (2005) contains one such example.

where a_{ij} is the i th row, j th column element in A , Ω_5 and Ω_6 are the fifth and sixth columns of Ω respectively. The confidence intervals can be obtained by the bootstrap method.

4.3 Identification of Country Spread Shocks

Identifying the country spread shocks is equivalent to identifying the fourth column of A . This is done by imposing sign restrictions on the impulse responses of output, investment, and country spread itself in response to such a shock. As Uhlig (2005) points out, although researchers when carrying out the VAR analysis appeal to certain informational ordering about the arrival of shocks, they like the results to look “reasonable,” which amounts to putting some “sign restrictions” implicitly. Now, by explicitly introducing sign restrictions on impulse responses, it becomes possible to distinguish between assumptions and conditions, thus making the VAR analysis more convincing. It is therefore desirable to impose some sign restrictions that are broadly acceptable while leaving the question of interest open. Formally stated, the sign restrictions are:

Assumption 1 : A country spread shock will not cause a drop in country spread itself, and will not cause a rise in investment, i.e., the impulse response of country spread to a country spread shock is nonnegative, while the impulse responses of investment to a country spread shock is nonpositive at horizon $k = 0, 1, \dots, K$.

When imposing sign restrictions, one needs to decide for how long these sign restrictions will hold after an initial shock. I have tried a variety of time intervals, and the results are very robust across the different values for K . For the results presented here K is equal to 5.

Second, I adopt a Bayesian approach and supplement the identification assumption A1 by imposing a prior distribution on the coefficients in the VAR system in order to tackle the issue of nonexact identification. The Bayesian framework has the advantage of easy interpretation for results like confidence intervals. Details of this are contained in Appendix B.

4.4 Impulse Response Functions

With an estimated VAR system (1) and an identification strategy, we are now ready to address the following questions. First, how do terms of trade shocks affect domestic variables and country spread? Second, how do world interest rate shocks affect the macroeconomy and the country spread? Third, what are the potential responses of macrovariables to a country spread shock? Fourth, how important are all these shocks in explaining movements of aggregate activity in emerging countries? I answer these questions with the help of impulse response functions and variance decompositions.

Figure 1 depicts the impulse response functions to a unit innovation of terms of trade in a solid line with squares. The broken lines depict two-standard-deviation bands⁷, which measures the uncertainty of the calculation of impulse responses. In response to an unanticipated terms of trade shock, trade balance-to-GDP ratio falls. Note that although I do not maintain the assumption that shocks take some time to affect domestic variables, the responses of gross output and investment to a terms of trade innovation reach their peaks after three quarters. The effects on the trade balance-to-GDP ratio is relatively persistent with a half life of about 10 quarters. Note that output rises to a terms of trade innovation, while trade balance-to-GDP ratio falls. Comparing the relative magnitude of the responses tells us that trade balance increases with terms of trade shocks.

Figure 2 depicts the impulse response functions of the variable in the VAR system (1) to a one percentage innovation in world interest rate, together with the two-standard-deviation error bands. Output and investment drop right after such a shock due to the no-arbitrage condition and the increase in borrowing cost. The fact that country spread increases for a considerable period of time after an unanticipated world rate shock reflects the feedback channel from business cycle variables to country spreads. Thus, other than a direct channel that affects the country spread by decreasing it, there is an indirect channel for a world interest rate to increase the country spread through the financial accelerating mechanism. The net effect is a prolonged rise in country spread.

Figure 3 depicts the impulse response functions of domestic variables to an innovation in country spread, one standard deviation in size. In response to an unanticipated country spread shock, the country spread itself increases and reaches the peak after three quarters.

⁷All the bands in this section are computed by the Bootstrap Method.

Output and investment respond as one would expect. Output initially drops by 0.5 percent and then reverts gradually to the preshock level. Investment is more volatile in comparison. It decreases by more than three percent immediately upon the impact. The adverse spread shock produces an expansion in trade balance, and this is reflected in the fact that the increase in trade to output ratio far exceeds the absolute value of the decrease in output. The point estimates of the response functions of output and investment are qualitatively similar to those associated with an innovation in world interest rate.

A remarkable feature of the impulse responses functions across these figures is the fact that effects on country spread build up in tandem with the deterioration (or improvement, in the case of a positive terms of trade shock) of aggregate economic activities. In all three of the figures, the maximum effects on country spread occur a few quarters after the initial impact. This shows the existence of a feedback channel from macroeconomic conditions to borrowing cost. On the other hand, output drops by 0.5 percent after a one-standard-deviation innovation in country spread. Thus, the relationship between borrowing cost and business cycles is indeed two-way.

4.5 Variance Decomposition

Figure 4 depicts the variance decomposition for gross domestic output at different horizons. Black solid line with squares shows the fraction of the variance of one-period-in-advance forecasting errors on aggregate output explained by terms of trade shocks, world interest rate shocks, and country spread shocks, respectively. Broken lines depict the two-standard-deviation error bands. The solid blue lines are the same results from the theoretical analysis. Note that as the forecasting horizon approaches infinity, the decomposition of the variance of the forecasting error coincides with the decomposition of the unconditional variance for the gross domestic output series.

According to the estimate of the VAR system given in Equation (1), innovation in the U.S. interest rate explains about 10 percent of movements in aggregate activity at business cycle frequency⁸ in South Korea during the sample period. At the same time, country spread

⁸Business cycle frequency is typically defined in time series of frequencies ranging from 6 quarters to 32 quarters (Stock and Watson, 1999). For the purpose of the present discussion, I associate business-cycle

shocks account for about 27 percent of aggregate fluctuations. Thus, around 37 percent of business cycle movements in South Korea is explained by disturbances in financial variables. Terms of trade shocks can explain up to 20 percent of business cycle movements.

4.6 Theoretical and Estimated Impulse Response Functions

We are now ready to generate the impulse response functions implied by the theoretical model and compare them with those estimated for the VAR system Equation (1). Using these predictions from theory of business cycles as a metric allows us to assess the plausibility of the shocks identified in the empirical VAR analysis. If the estimated shocks imply similar business cycle fluctuations in the empirical model and in the theoretical ones, one can conclude that according to the proposed theory, the identified shocks are plausible.

Figure 5 compares the impulse response functions to an unanticipated shock in terms of trade. Figure 6 shows the effect of a world rate shock in both empirical and theoretical settings. Figure 7 shows the impulse responses to an innovation to country spread shocks. Overall, the model replicates the data relatively well, and most points belong to the estimated two-standard-error bands from the empirical model. Therefore, the plausibility of the empirical model is confirmed.

5 Conclusion

There is a complicated interrelation among country spreads, world interest rates and business cycle movements in a small open economy. This paper disentangles these interconnections by combining empirical and theoretical analyses. First it introduces a VAR-based analysis of the transmission mechanism for shocks to terms of trade, shocks to world interest rate, and shocks to country spread. This is achieved through analyzing a six-variable dynamic system, including total output, total investment, trade balance-to-output ratio, country spread, terms of trade and world interest rate for South Korea. Second, all the results of the VAR analysis are used as a benchmark to be compared with those derived from a DSGE model with fluctuations with the variance of the forecasting error at a horizon of about five years. This choice of 20 quarters falls in the middle of the typical window.

financial accelerating mechanism.

I find that country spreads affect aggregate economic activity and they respond to macroeconomic fundamentals at the same time. The world interest rate has an impact on the borrowing cost not only through no-arbitrage condition but also through feedback from macroeconomic variables to country spreads. Terms of trade shocks, which have direct impact on trade balance and output, also influence country spread and thus affect gross domestic output even further. U.S. interest rate shocks explain about 10 percent of movements in aggregate activity in South Korea during the sample period, terms of trade shocks accounts for about 20 percent of aggregate fluctuations, while country spread shocks explain about 27 percent of it.

Appendix A

This appendix describes the dataset used in the empirical analysis. The dataset contains quarterly data for South Korea over the period 1972Q1 to 2010Q1, for a total of 153 quarters. Quarterly series for GDP, total investment and trade balance are retrieved from the OECD database.

World interest rate series is constructed by using the three-month U.S. treasury bill rate adjusted for the U.S. inflation. Terms of trade series is obtained from IMF's International Financial Statistics as the ratio of export unit price and import unit price. Country spread is defined as the difference between the real corporate bond rates for South Korea and real world interest rate described above. The corporate bond rate series is available from Global Financial Data, a commercial database, and listed as series InKORD. However, since the bond is not denominated in dollars, rather in South Korean won, there exists intrinsic currency risk in the bond's returns which is not modeled here. To extract currency risk from the corporate bond rate, I first estimate the percentage change on the three month average exchange rate of South Korean won with respect to U.S. dollar as an AR(1) process. The one-period-ahead prediction of percentage changes in exchange rates in this process is then subtracted from the corporate bond series to obtain the currency risk adjusted bond returns. Another option to construct country spread for emerging economies is to use J.P.Morgan's Emerging Markets Bond Index Plus (EMBI⁺). However, such index only starts recording after 1994 and is only

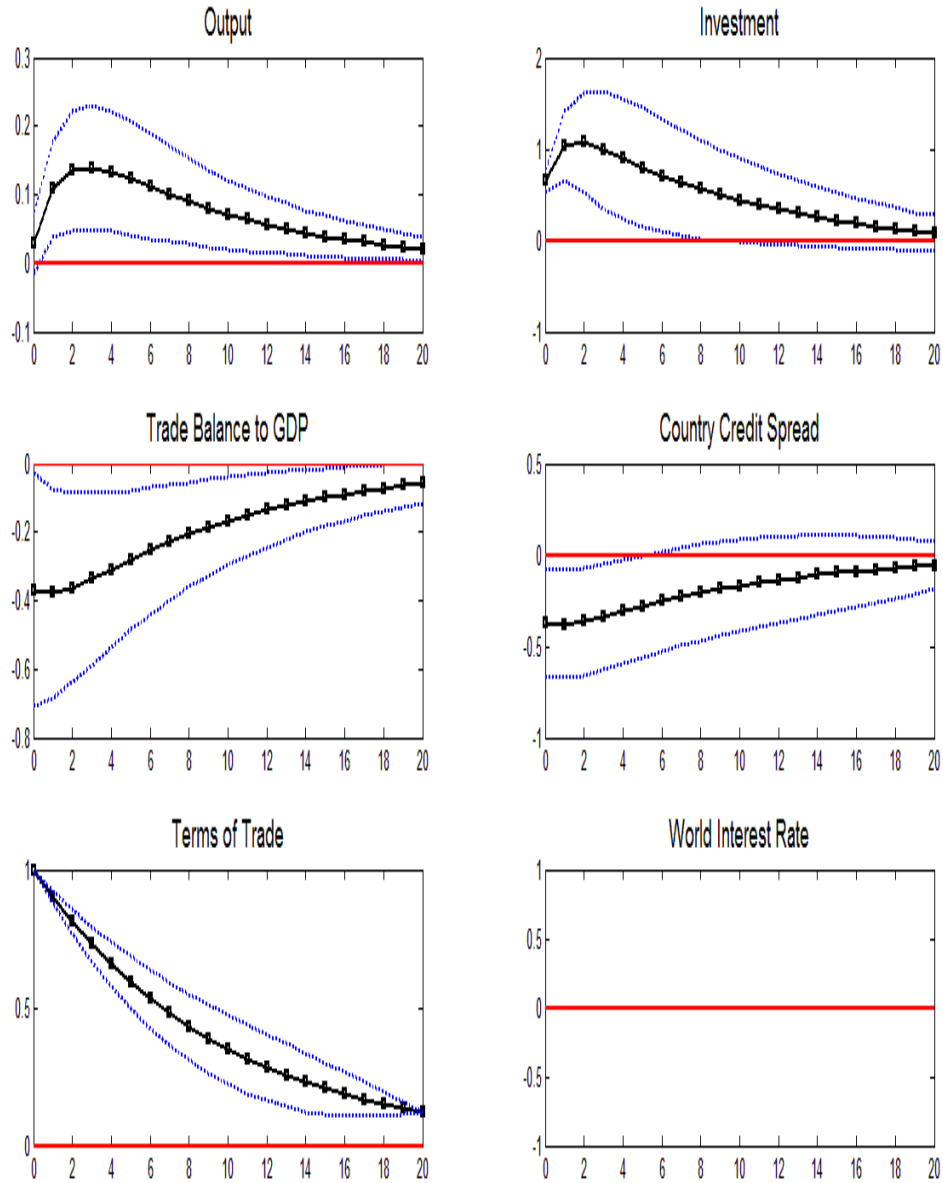


Figure 1: Impulse response functions to a terms of trade shock by structural VAR analysis. The solid line with squares is the estimated impulse response functions, and the dotted lines are the two-standard-deviation error bands.

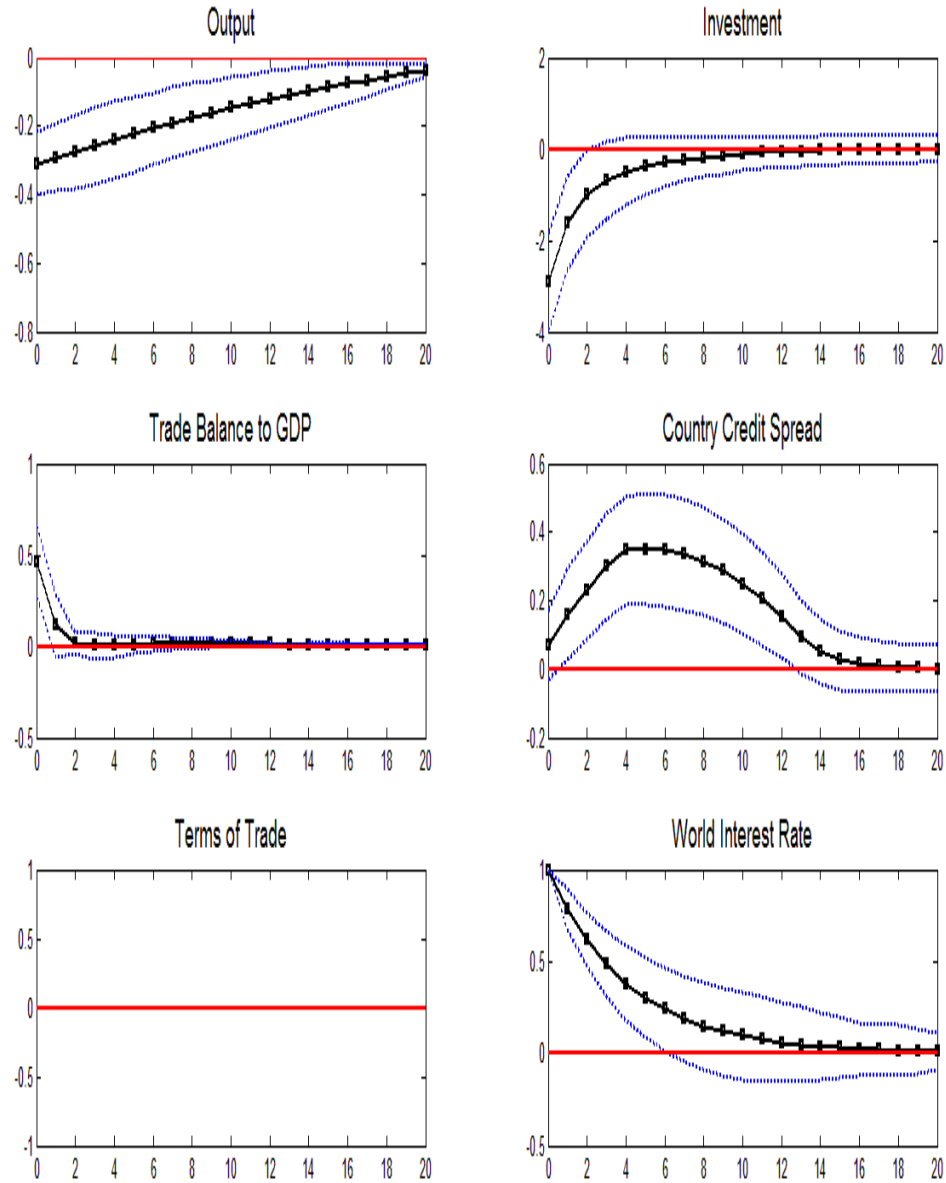


Figure 2: Impulse response functions to a world interest rate shock by structural VAR analysis. The solid line with squares is the estimated impulse response functions, and the dotted lines are the two-standard-deviation error bands.

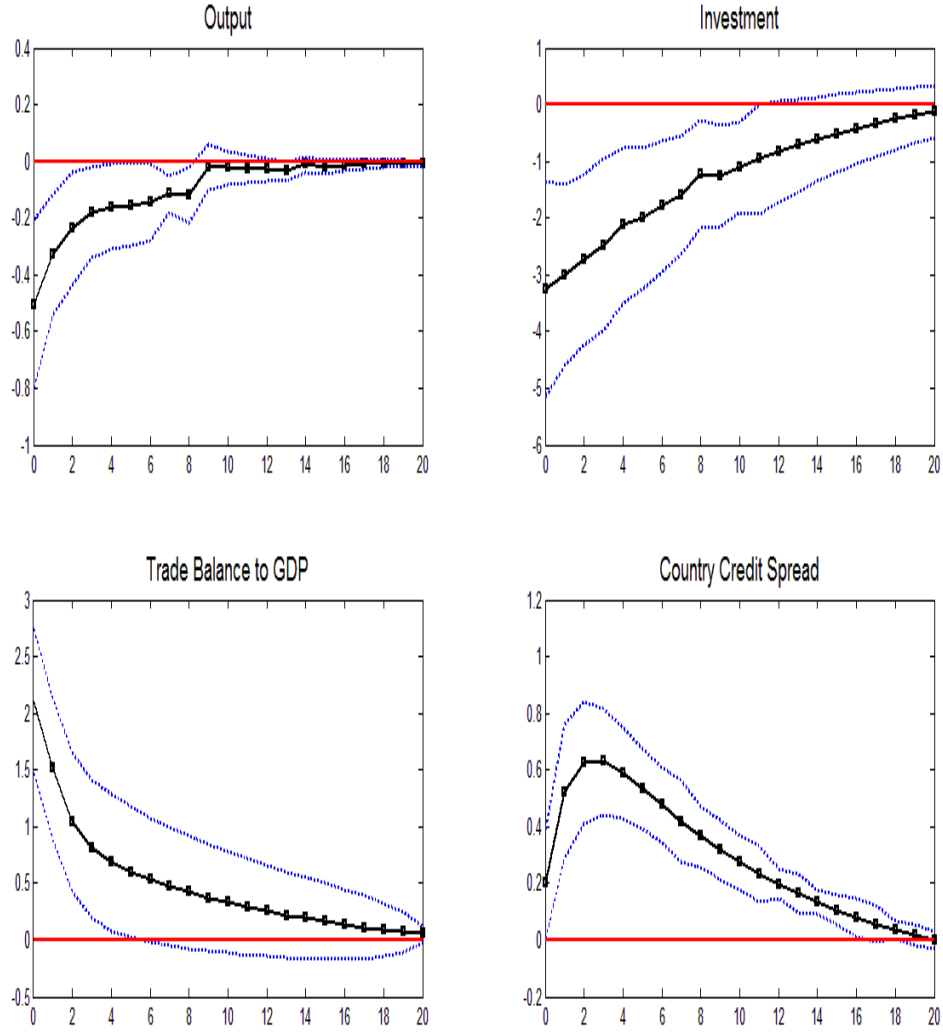


Figure 3: Impulse response functions to a country credit spread shock by structural VAR analysis. The solid line with squares is the estimated impulse response functions with sign restrictions, and the dotted lines are the two-standard-deviation error bands.

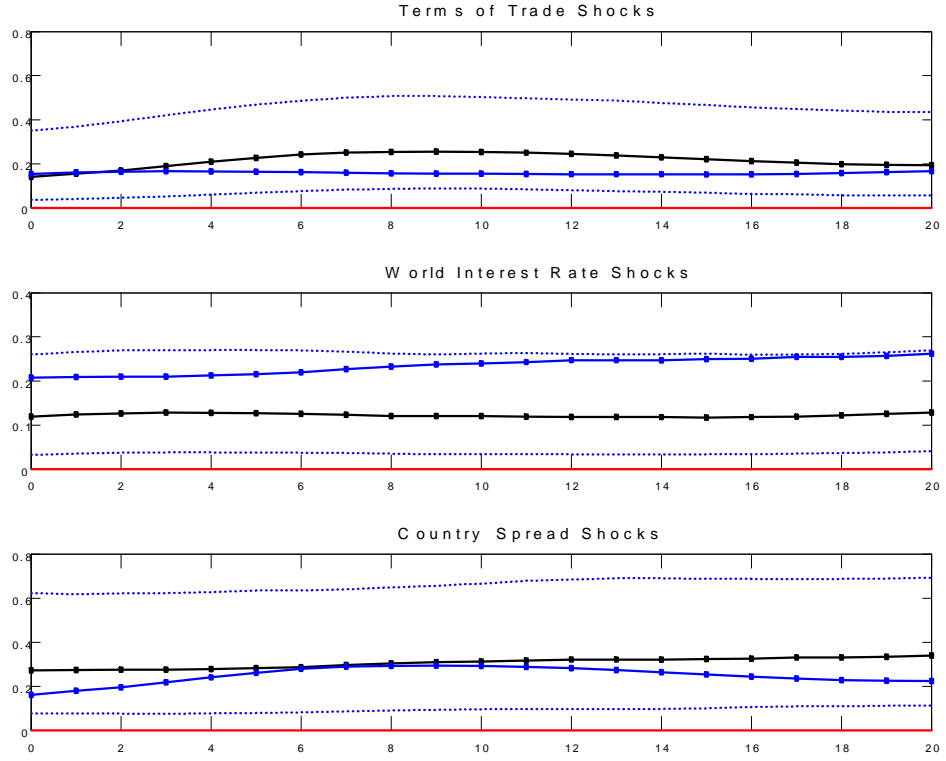


Figure 4: Percentage of variances on forecasting errors for gross domestic output explained by terms of trade shocks, world interest rate shocks and country spread shocks. The solid black lines are from the empirical analysis; the solid blue lines are from the theoretical analysis. The dotted lines are 16% and 84% empirical quantiles.

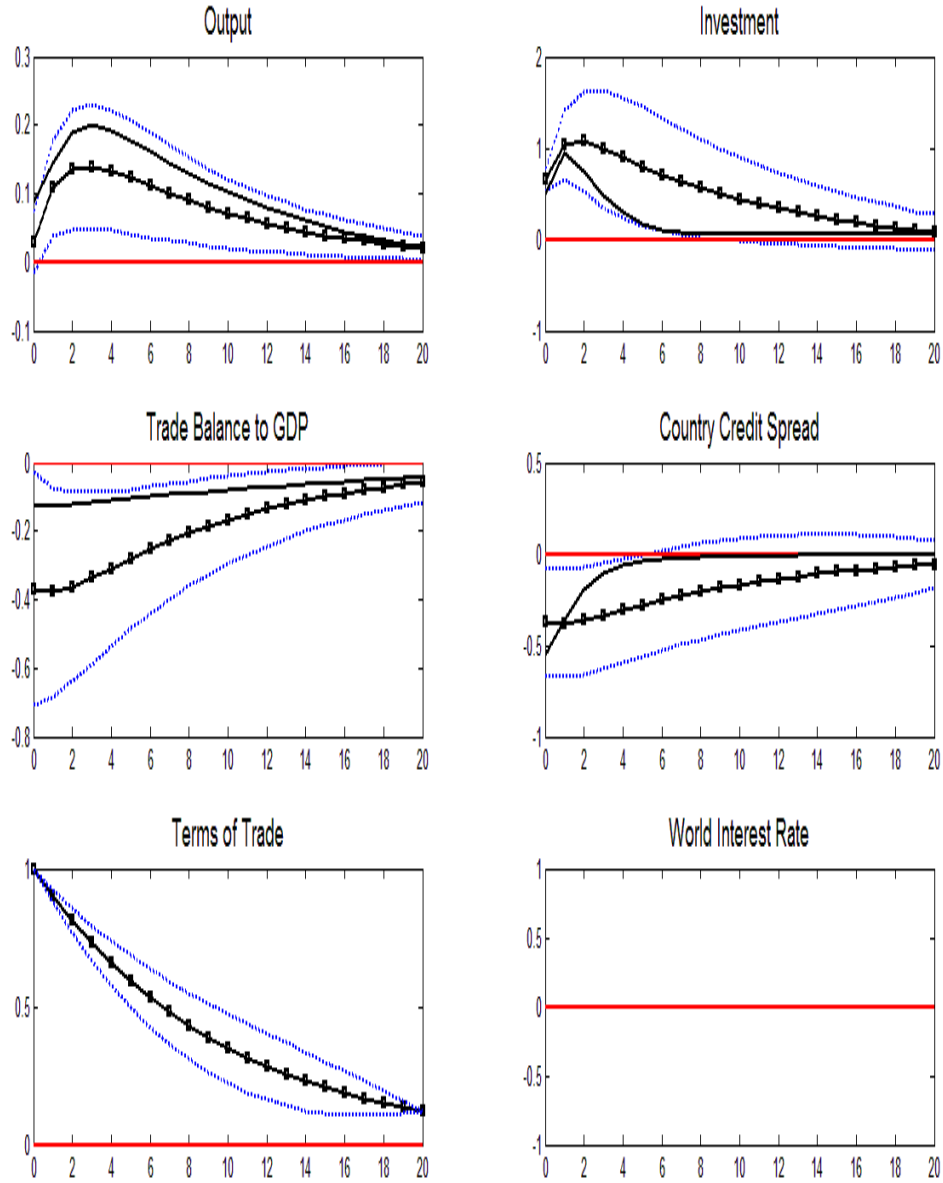


Figure 5: Theoretical and estimated impulse response functions to a terms of trade shock. The solid line without squares is the impulse response functions from the theoretical model. The solid line with squares is the estimated impulse response functions from structural VAR analysis, and the dotted lines are the two-standard-deviation error bands.

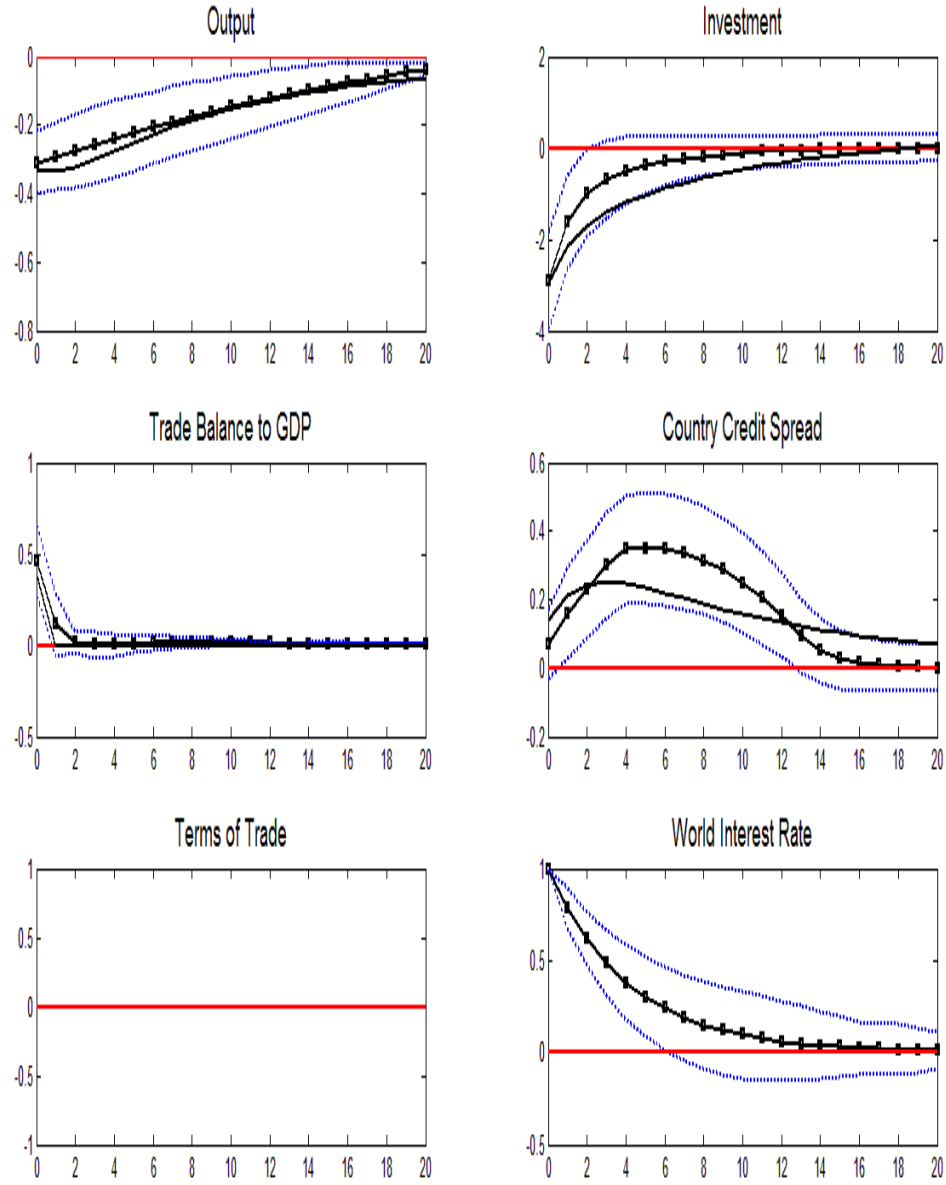


Figure 6: Theoretical and estimated impulse response functions to a world interest rate shock. The solid line without squares is the impulse response functions from the theoretical model. The solid line with squares is the estimated impulse response functions from structural VAR analysis, and the dotted lines are the two-standard-deviation error bands.

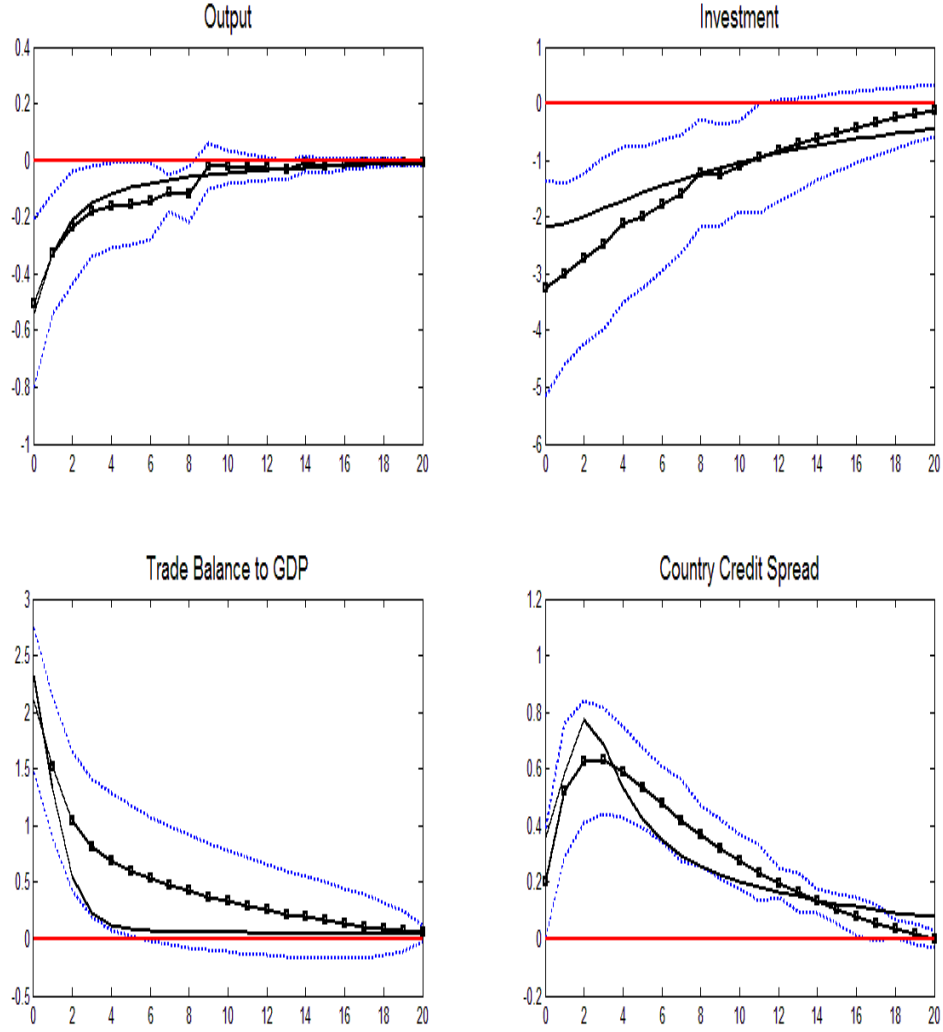


Figure 7: Theoretical and estimated impulse response functions to a country spread shock. The solid line without squares is the impulse response functions from the theoretical model. The solid line with squares is the estimated impulse response functions from structural VAR analysis, and the dotted lines are the two-standard-deviation error bands.

available for some emerging economies. Given the openness of South Korea, corporate bond rates are a reasonably good approximation of the borrowing rate on the international financial market.

Appendix B

This appendix contains details about the identification of country spread shocks using sign restrictions.

Let a be the column in A which corresponds to the country spread shocks. The following three lemmas proved in Uhlig (2005) are critical for identification.

Lemma 1 *If a vector $a \in R^m$ is a column of matrix A with $AA' = \Sigma$, and $\tilde{A}\tilde{A}' = \Sigma$ is the Cholesky decomposition of Σ , there is a m -dimensional vector b of unit length such that $a = \tilde{A}b$.*

Lemma 2 *Let $r_i(k) \in R^m$ be the vector response at horizon k to the i th shock in a Cholesky decomposition of Σ . The impulse response $r_a(k)$ for a is then simply given by*

$$r_a(k) = \sum_{i=1}^m b_i r_i(k).$$

Lemma 3 *Let $E_t[Y_{t+k}] - E_{t-1}[Y_{t+k}]$ be the k -period-ahead forecast revision due to the arrival of new data at date t . The fraction $\Theta_{a,j,k}$ of the variance of this forecast revision for variable j , explained by the shock corresponding to vector a in A , is given by*

$$\Theta_{a,j,k} = \frac{(r_{a,j}(k))^2}{\sum_{i=1}^m (r_{i,j}(k))^2},$$

where the index j corresponds to variable j .

The key to identifying country spread shocks lies in the identification of the vector a . Lemma 1 provides the criterion for eligible candidates of a ; Lemma 2 allows for appropriate impulse response analysis; Lemma 3 is used to obtain variance decomposition.

Given some VAR coefficient matrices $B = [B'_1, \dots, B'_p]$, some variance-covariance matrix Σ , and some parameter K , let $\Upsilon_T(B, \Sigma, K)$ be the set of vector a_T which satisfies Assumption 1. Because it is obtained from inequalities, this set will either be empty or contain

many elements, which makes exact identification impossible. This problem will be solved by adopting a Bayesian framework.

In this framework I supplement the identification assumption A1 by imposing a prior on $\Upsilon_T(B, \Sigma, K)$ in order to tackle the issue of nonexact identification. Let $\mathbf{B} = [B_1, \dots, B_p]'$ and assume that the parameters $(\mathbf{B}, \Sigma, b_T)$ are jointly drawn from a prior on $R^{p \times m \times m} \times P^{m \times m} \times U^m$, where $P^{m \times m}$ is the set of all $m \times m$ positive definite matrix and U^m is the unit sphere of m dimensional vectors. More specifically, I assume the prior is proportional to a Normal-Wishart distribution in (\mathbf{B}, Σ) whenever the resulting impulse responses from $(\mathbf{B}, \Sigma, b_T)$ satisfies the sign restrictions, and zero elsewhere. The flat prior on the unit sphere for b_T guarantees that reordering the variables and choosing a different Cholesky decomposition will not yield different results.

A proper Normal-Wishart distribution is parameterized by a mean coefficient matrix $\bar{\mathbf{B}}$ of size $mp \times m$, a positive definite mean covariance matrix \bar{S} with size $m \times m$, a positive definite matrix \bar{N} with size $mp \times mp$, and a real number $v \geq 0$ to describe the uncertainty about (\mathbf{B}, Σ) around $(\bar{\mathbf{B}}, \bar{S})$. The Normal-Wishart distribution specifies that Σ^{-1} follows a Wishart distribution $W_m(\bar{S}^{-1}/v, v)$ with $E[\Sigma^{-1}] = \bar{S}^{-1}$, and that the coefficient matrix in its columnwise vectorized form, $vec(\mathbf{B})$, follows a Normal distribution $N(vec(\bar{\mathbf{B}}), \Sigma \otimes \bar{N}^{-1})$. In this paper, I use a weak prior that $\bar{N}_0 = 0$, $v_0 = 0$, \bar{S}_0 and $\bar{\mathbf{B}}_0$ are arbitrary. If $\hat{\mathbf{B}}$ and $\hat{\Sigma}$ are the MLE estimates for (\mathbf{B}, Σ) , the posterior distribution will be such that $\bar{\mathbf{B}}_T = \hat{\mathbf{B}}$, $\bar{S}_T = \hat{\Sigma}$, $v_T = T$, $\bar{N}_T = \mathbf{X}'_T \mathbf{X}_T$, where $\mathbf{X}_T = [X_1, \dots, X_T]'$ and $X_t = [Y'_{t-1}, Y'_{t-2}, \dots, Y'_{t-p}]'$.

To draw inferences from the posterior, first take n_1 draws from the Normal-Wishart posterior distribution and for each of these draws, n_2 draws for b_T from the m -dimensional unit sphere. For each draw, I calculate the impulse responses and check whether the sign restrictions are satisfied. If they are, this is a valid draw and is thus kept. Statistics of interest such as error bands are calculated from all the draws kept. Here $n_1 = n_2 = 1000$.

Appendix C

This appendix provides a detailed analysis of the optimal financial contract, justifying our specification of the relation between risk premium and capital demand. The argument outlined here closely follows Bernanke, Gertler and Gilchrist (1998).

Suppose entrepreneurs and foreign creditors are both risk-neutral, and the supply of credit is competitive. The contracting problem happens between a single entrepreneur, indexed by j , and foreign lenders in period t . For tractability, I consider only one period contract. At the time of contracting, entrepreneur j 's net worth in nominal terms, $P_t N_t(j)$, risk-free world interest rate, $i_{F,t+1}$, and price of capital Q_t are all known. The wholesale firms are competitive with constant return to scale technology, and thus earn zero profit. The entrepreneurs' income stream comes from the returns on capital. Thus the financial contract is based on the returns from capital. Let profits per unit capital in terms of foreign currency be $\omega_{t+1}(j)(R_{t+1}/S_{t+1})$, where S_{t+1} is the nominal exchange rate in period $t + 1$, and ω_{t+1} is an idiosyncratic shock with $E(\omega_{t+1}) = 1$. I assume the c.d.f. of ω_{t+1} , $F(\omega_{t+1})$, and the p.d.f. of ω_{t+1} , $f(\omega_{t+1})$, are both public information, and $\omega_{t+1}(j)$ is i.i.d. across j and t . Without causing confusion I will simply denote them as ω ⁹. For now, assume also that period $t + 1$ aggregate return on capital in dollars, R_{t+1}/S_{t+1} , is known. Crucially, I assume ω is unknown to both parties prior to the investment decision; however, after the investment decision, ω is observed freely by the entrepreneur, while foreign creditors have to pay a proportional monitoring cost $\mu\omega(R_{t+1}/S_{t+1})$.

The credit contract specifies two functions $(I(\omega), E(\omega, K))$ such that if state ω is monitored, $I(\omega) = 1$, otherwise $I(\omega) = 0$, and the state contingent repayment schedule $E(\omega, K)$ to the foreign creditors is nonnegative if entrepreneur rent capital K to the wholesale firm.¹⁰ For a given K , if an entrepreneur is not monitored, he will always claim that he is in the state ω^* where the repayment $E(\omega^*, K)$ is the lowest among all the states which are not monitored. As a result, in the optimal contract, for those states which are not monitored, $E(\omega, K)$ is a constant for a given K . Let $\bar{\omega}$ be the cut off value such that if $\omega \geq \bar{\omega}$, the entrepreneur will not be monitored; if $\omega < \bar{\omega}$, monitoring will occur. The constant repayment $E(\omega, K)$ when $\omega \geq \bar{\omega}$ is equal to the total return of capital at state $\bar{\omega}$, $\bar{\omega}(R_{t+1}/S_{t+1})K_{t+1}$. When a monitoring cost is incurred, the entrepreneur receives nothing, while the foreign creditors receive $(1 - \mu)\omega(R_{t+1}/S_{t+1})K_{t+1}$ in residual claims net of monitoring costs.¹¹

⁹I do not specify the exact sources of the idiosyncratic shocks. They could come from various sources, but the combined effect is to vary the return to capital.

¹⁰The index j is suppressed.

¹¹See Williamson (1987) for a detailed analysis.

To provide enough incentive for the foreign creditors to participate in such a contract, it must be the case that

$$\begin{aligned}
& [\bar{\omega}(1 - F(\bar{\omega})) + (1 - \mu) \int_0^{\bar{\omega}} \omega f(\omega) d\omega] (R_{t+1}/S_{t+1}) K_{t+1} \\
&= (1 + i_{F,t+1}) D_{t+1} \\
&= (1 + i_{F,t+1}) (Q_t K_{t+1} - P_t N_t) / S_t,
\end{aligned} \tag{A1}$$

where the second equality follows from the fact that the loan must equal the difference between capital investment and the entrepreneur's net worth.

Following Bernanke, Gertler and Gilchrist (1998), I define $\Gamma(\bar{\omega})$ as the expected gross share of capital returns going to the lender:

$$\Gamma(\bar{\omega}) \equiv \int_{\omega_a}^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega} \int_{\bar{\omega}}^{\omega_b} f(\omega) d\omega,$$

where (ω_a, ω_b) is the support of ω . Similarly, let's define $\mu G(\bar{\omega})$ as the expected monitoring costs

$$\mu G(\bar{\omega}) \equiv \mu \int_{\omega_a}^{\bar{\omega}} \omega f(\omega) d\omega.$$

If the hazard rate $h(\omega)$, $f(\omega)/[1 - F(\omega)]$, satisfies the condition that $\omega h(\omega)$ is increasing in ω , one can easily prove the following lemma:

Lemma 4 *There exists a $\bar{\omega}^* \in (\omega_a, \omega_b)$ such that*

- (i) $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) < 0$ for $\bar{\omega} > \bar{\omega}^*$;
- (i) $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) = 0$ for $\bar{\omega} = \bar{\omega}^*$;
- (i) $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) > 0$ for $\bar{\omega} < \bar{\omega}^*$.

Note that $\lim_{\bar{\omega} \rightarrow \omega_a} \Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = 0$, $\lim_{\bar{\omega} \rightarrow \omega_b} \Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = 1 - \mu$. Since $\Gamma(\bar{\omega}) - \mu G(\bar{\omega})$ is increasing on $(\omega_a, \bar{\omega}^*)$ and decreasing on $(\bar{\omega}^*, \omega_b)$, the optimal contract will always choose $\bar{\omega} < \bar{\omega}^*$.

Formally the optimal contract problem is:

$$\begin{aligned}
& \max_{K_{t+1}, \bar{\omega}} (1 - \Gamma(\bar{\omega})) (R_{t+1}/S_{t+1}) K_{t+1} \\
& s.t. \quad [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] (R_{t+1}/S_{t+1}) K_{t+1} = (1 + i_{F,t+1}) (Q_t K_{t+1} - P_t N_t) / S_t.
\end{aligned}$$

To simplify, let's first define the premium on external funds as

$$1 + \rho_{t+1} = \frac{R_{t+1}S_t}{Q_tS_{t+1}(1 + i_{F,t+1})}$$

and define the capital/net worth ratio as

$$\Lambda_t = \frac{Q_tK_{t+1}}{P_tN_t}.$$

The contracting problem can be rewritten as

$$\max_{\Lambda_t, \bar{\omega}} (1 - \Gamma(\bar{\omega}))\Lambda_t$$

$$s.t. \quad [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1})\Lambda_t = \Lambda_t - 1.$$

Define λ as the Lagrangian multiplier on the lenders' participation constraint, the first order conditions for an interior solution to this problem can be written as:

$$\bar{\omega}: \quad \Gamma'(\bar{\omega}) - \lambda[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})] = 0; \tag{A2}$$

$$\Lambda_t: \quad [1 - \Gamma(\bar{\omega})] + \lambda[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1}) - \lambda = 0; \tag{A3}$$

$$\lambda: \quad [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1})\Lambda_t = \Lambda_t - 1. \tag{A4}$$

Note that (A2) and (A3) imply that the optimal cutoff depends only on the external finance premium $(1 + \rho_{t+1})$, and thus is independent of the entrepreneur's net worth.

Combining (A2) and (A3) one can derive $\bar{\omega}$ as a function of the external finance premium from the following equation

$$[1 - \Gamma(\bar{\omega})] + \left[\frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \right] [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1}) - \left[\frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \right] = 0.$$

Under the maintained assumption that $\omega h(\omega)$ is increasing in ω , Implicit Function Theorem can be applied to prove the following results.

Lemma 5 (i) *The optimal cutoff $\bar{\omega}$ is an increasing function of $(1 + \rho_{t+1})$, the risk premium, or, expressed in inverse form,*

$$1 + \rho_{t+1} = \Delta(\bar{\omega}) \text{ with } \Delta'(\cdot) > 0.$$

(ii) The ratio of investment relative to net worth, Λ_t , is an increasing function of risk premium $1 + \rho_{t+1}$.

$$1 + \rho_{t+1} = F(\Lambda_t) \text{ with } F'(\cdot) > 0.$$

For completeness, let's include an expression for Λ_t after solving $\bar{\omega}$:

$$\Lambda_t = \frac{1}{1 - [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1})}.$$

So far I take the aggregate return per capital (R_{t+1}/S_{t+1}) as given in the partial equilibrium analysis. If (R_{t+1}/S_{t+1}) is uncertain at the time of contracting, the preceding analysis survives intact with (R_{t+1}/S_{t+1}) replaced with its expectation at time t .¹²

Appendix D

This section provides an analytic expression for $\Gamma(\bar{\omega})$ and $\Gamma(\bar{\omega}) - \mu G(\bar{\omega})$ where ω has a uniform distribution over $[a, b]$ with $E(\omega) = 1$, which is the specification I use in calibrating the theoretical model. Note that $\omega h(\omega)$ is increasing in ω for uniform distribution. Since

$$F(\omega) = \frac{\omega - \omega_a}{\omega_b - \omega_a}, \quad f(\omega) = \frac{1}{\omega_b - \omega_a},$$

one can derive that

$$\Gamma(\bar{\omega}) = \frac{1}{\omega_b - \omega_a} \left(-\frac{\bar{\omega}^2}{2} + \bar{\omega}\omega_b - \frac{\omega_a^2}{2} \right), \quad \text{and}$$

$$G(\bar{\omega}) = \frac{1}{\omega_b - \omega_a} \left(\frac{\bar{\omega}^2}{2} \right).$$

¹²Appendix A in Bernanke, Gertler and Gilchrist (1998) has detailed derivation.

Chapter 3

Financial Shocks and Business Cycles

6 Introduction

Technology shocks have been well studied in the dynamic stochastic general equilibrium (DSGE) literature in creating business cycles. Other shocks, such as monetary shocks and preference shocks, are later introduced into the literature. However, the recent financial crisis, also dubbed “the Great Recession”, urges us to think the following question: could shocks originating in the financial markets play a significant role in creating the business cycle movements? Economists are used to studying shocks originating from the real side of the economy, but it is hard to convince ourselves that the recent economic downturn, which officially started from Fall, 2008, was actually caused by shocks from the real side; rather, it seems that it has its roots in the financial markets. This motivates me to explore the importance of financial shocks, together with technology shocks, in creating macroeconomy fluctuations. I start from a standard DSGE model, then use the defining characteristics found in the model to identify both financial and technology shocks in a structural vector auto-regression (VAR) analysis for the U.S.

In the early literature of real business cycles, there usually exists just one representative household. As a result, actual financial flows do not occur in the equilibrium and financial structures do not matter. On the other hand, Macroeconomics has a long-standing tradition of emphasizing the role of credit market conditions in the propagation of cyclical fluctuations. There exists a two-way interaction between the development in the credit market and that in the real economy: deteriorating credit market conditions are both reflections of a declining real economy and a major factor depressing economic activity. Bernanke, Gertler and Gilchrist (1999)¹³ is one of the early attempts to develop a “financial accelerator” property in DSGE models such that endogenous developments in credit markets work to propagate and amplify shocks to the macroeconomy. This is the framework I adopt in the

¹³Henceforth BGG (1999).

theoretical model. However, BGG (1999) focuses on studying the transmission of technology and monetary policy shocks, while here I study financial shocks.

In this paper I follow BGG (1999) by studying endogenously determined credit market frictions through the balance sheet effects, while introducing an external financial market which allows financial shocks to be defined in a simple and tractable way. The key mechanism involves the link between external finance premium, defined as the difference between marginal productivity of capital and riskless rate, and the corporate net worth in a closed economy. Specifically, I assume entrepreneurs, the capital providers, have to resort to banks to cover the difference between investment and their net worth. With credit market frictions present, and with the total amount of capital investment held constant, standard model of lending with costly state verification implies that the external finance premium depends inversely on entrepreneurs' net worth. This inverse relationship arises naturally because less net worth implies greater potential divergence of interests between borrowers and creditors, thus increased agency cost. In equilibrium, banks as creditors must be compensated for higher agency cost by a larger premium. As a result, a positive financial shock which increases banks' profits in the external financial market, increases credit flows and requires less net worth as collateral, thus a decrease in borrowers' net worth. This is in sharp contrast with the effects of technology shocks. Although a positive technology shock also increases credit flows, the agency cost is mitigated by the increase in productivity and as a result, external finance premium falls and entrepreneurs' net worth increases.

To determine the relative contribution of financial and technology shocks in creating business cycle movements in the U.S., I adopt the methodology used in Uhlig (2005) to identify these shocks using sign restrictions in a structural VAR analysis. Specifically, a positive financial shock is identified through the requirement that for the first 10 periods, such shocks must increase output and decrease corporate net worth. In other words, if a shock has such effects for the first 10 periods after its occurrence, we call it a positive financial shock. While a positive technology shock is a shock such that both total output and corporate net worth increase for the first 10 periods. The data set I use includes output, investment, loans made to nonfarm corporate from commercial banks, net worth of nonfarm nonfinancial

corporate, and credit spread¹⁴ for U.S. from 1957Q1 to 2009Q2. The results find that although technology shocks contribute about 40 percent of the business cycle fluctuations, financial shocks can still explain about 10 percent of variation in such movements.

This chapter is organized as follows. Section 7 presents the model economy. Section 8 discusses calibration and the transmission mechanism of the model. Section 9 contains the structural VAR analysis. Section 10 concludes.

7 The Model

The model features a financial accelerator in the general framework of a DSGE model. It is a real model, and is kept deliberately simple without monopolistic competition and nominal rigidities since the focus here is on the corporate net worth and external finance premium. The way I model the financial accelerator largely follows BGG. However, to their exposition a few new features are added that allow the financial shocks to be introduced into the model in a meaningful way.

The model distinguishes households, entrepreneurs, firms and banks in a closed economy. Households are infinitely lived and make intertemporal as well as intratemporal decisions over consumption and labor supply. Entrepreneurs borrow from the banking sector, produce usable capital and rent it to firms. Firms operate on competitive markets and choose the optimal levels of capital and labor for production. Banks supply credit in a competitive market to the entrepreneurs, pay back interest on households' deposits, and invest in an outside financial market whose details are not explicitly modeled in this paper. A negative shock to the return from the investment in the outside market will force the banks to increase the rates charged on loans, and I call such a shock a financial shock, or more specifically, a banking capital shock.¹⁵

The key for the working of the financial accelerator lies in the entrepreneur sector. First of all, since entrepreneurs are different from households, the model does not collapse into a representative agent framework, and there is nontrivial borrowing and lending in equi-

¹⁴Credit spread is defined as the difference between prime loan rate and Fed Funds rate in the data.

¹⁵We could also think of this as a positive productivity shock to the banking sector.

librium. Financial frictions arise from asymmetric information in the relationship between borrowers, the entrepreneurs, and lenders, the banks. It is assumed that the lenders must pay a positive monitoring cost to observe the true performance in the borrowers' production of usable capital. The optimal financial contracts thus dictates that lenders will only incur this monitoring cost when the borrowers declare bankruptcy, in which case the banks seize everything net from monitoring cost from the entrepreneurs' profit for the period. Note that monitoring cost in the model should be interpreted in more general terms as proxying for all kinds of expenses associated with debtor bankruptcy, such as accounting and legal expenses or liquidation losses. Such friction of costly state verification causes loans to be charged at a premium over the risk-free interest rate and give an important role to borrowers' balance sheets. Existing literature emphasizes the countercyclical behavior of the financial premium originated from the balance sheet effects, usually triggered by a productivity shock or a monetary policy shock. In this paper, however, increasing risk premium is not necessarily the passive reflection of declining real economy: even if the real economy is in perfectly healthy condition, a contractionary financial shock, or more specifically, a negative shock to banking capital, or a sudden increase in bankruptcy cost, could cause change in net worth and external financing premium.

One of the goals of this paper is to rigorously assess the quantitative importance of this mechanism. To do so, we now describe the objectives and constraints of agents in this model in more detail.

7.1 Households

There is a continuum of households of unit mass. Households are infinitely-lived agents with an identical utility function which is additively separable in consumption, C_t , and labor, H_t , i.e.

$$\sum_{\tau=0}^{\infty} \beta^{\tau} E_t \left[\frac{1}{1-\sigma} (C_{t+\tau})^{1-\sigma} - \frac{1}{1+\gamma} (H_{t+\tau})^{1+\gamma} \right],$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, σ^{-1} is the elasticity of intertemporal substitution, and γ^{-1} is the elasticity of labor supply. Households choose the appropriate level of consumption and labor supply to optimize this utility function, while

the flow budget constraint holds with equality and households' wealth accumulation satisfies the transversality condition.

Households' income comes from supplying labor to firms at competitive real wages, W_t , from the ownership of the competitive firms which rebate their profits Π_t to households every period, and from interests earned on their one-period nominal deposits in the banking system, B_t . With this disposable income, households finance their consumption, C_t , and open new deposits, B_{t+1} . Accordingly, the households' sequence of budget constraints is described by:

$$C_t + B_{t+1} = W_t H_t + (1 + r_t^B) B_t,$$

where r_t^B is the short-term interest rate offered to the depositors.

Households' optimization yields the standard first-order conditions for consumption-savings and labor supply.

$$\frac{1}{1 + r_{t+1}^B} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right],$$

$$W_t = H_t^\gamma C_t^\sigma,$$

plus the appropriate no-Ponzi scheme condition.

7.2 Production

There is a continuum of mass one of perfectly competitive firms. Firms combine capital rented from the entrepreneurs with the labor provided by the households to produce the only good in this economy, according to the following Cobb-Douglas technology, i.e.,

$$Y_t = F(K_t, L_t) \equiv e^{a_t} K_t^\alpha L_t^{1-\alpha},$$

where Y_t is the output of this good, K_t is the capital rented by firms, and L_t is the labor demanded. The productivity shock, a_t , follows an AR(1) process of the following form

$$a_t = \rho_a a_{t-1} + \epsilon_t^a,$$

where ϵ_t^a is a zero mean, and serially uncorrelated innovation. The parameter $0 < \rho_a < 1$ determines the persistence of the productivity shock, and $\sigma_a^2 > 0$ is the volatility of its innovation.

Firms maximize their static profit, i.e.,

$$\Pi_t = Y_t - (1 + r_t^F)K_t - W_t H_t,$$

subject to the technological constraint above, where $1 + r_t^F$ is the nominal price per unit of capital rented from the entrepreneurs. The optimization of firms results in the following well-known rules,

$$1 + r_t^F = F_K(K_t, L_t),$$

$$W_t = F_L(K_t, L_t).$$

Firms make zero profits in every period (i.e., $\Pi_t = 0$), and the households who own them do not receive any dividends.

7.3 Entrepreneurs

The entrepreneur sector, in which the financial accelerator originates and external finance premium is determined, is the most important sector in this model. Entrepreneurs finance the capital they rent to firms partly with loans from banks, and such loans are subject to financial frictions. In principle, many factors can contribute to imperfections in credit market, e.g., information asymmetry or enforcement problems. In this paper, however, I follow the trend after Bernanke, Gertler and Gilchrist (1999) in assuming a “costly state verification” (CSV) problem of the type first analyzed by Townsend (1979), in which lenders must pay a proportional “auditing cost” to observe an individual borrower’s realized return, while the borrower observe the return for free. This formulation allows for a simple, but natural way to introduce external finance premium and net worth into the model.

Suppose there is a continuum of entrepreneurs over a total mass of 1. It is convenient to start to describe a representative entrepreneur’s behavior at the end of period t . At that time, the entrepreneur has some net worth, N_t , and has access to external findings from banks. Let D_t denote the amount of loan the entrepreneur borrows. The total amount of capital the entrepreneur could potentially invest in period $t + 1$ is defined by

$$K_{t+1} = N_t + D_{t+1}.$$

However, the entrepreneur can not rent this amount directly to the firms. He has to apply a linear technology, which is subject to an idiosyncratic shock, ω , to convert the capital defined above as usable capital. This usable capital can then be rented directly to firms. Specifically, this linear technology is ωK_{t+1} , where the p.d.f. of ω follows $f(\omega) \sim \text{Uniform}(\omega_a, \omega_b)$ and $E(\omega) = 1$. Note that ω is unknown to both the entrepreneur and the lender prior to the investment decisions. After the realization of the idiosyncratic shock, the lender only observe ω by paying a proportionate monitoring cost, $\mu\omega(1 + r_t^F)K_{t+1}$. For now, assume that period $t + 1$ aggregate return on capital, $1 + r_t^F$, is known. Lenders are assumed to be competitive financial intermediaries who earn zero profits in equilibrium and are able to diversify away idiosyncratic credit risk.

The credit contract specifies two functions $(I(\omega), E(\omega, K))$ such that if state ω is monitored, $I(\omega) = 1$, otherwise $I(\omega) = 0$, and the state contingent repayment schedule $E(\omega, K)$ to the creditors is nonnegative if entrepreneur rent capital K to the wholesale firm.¹⁶ For a given K , if an entrepreneur is not monitored, he will always claim that he is in the state ω^* where the repayment $E(\omega^*, K)$ is the lowest among all the states which are not monitored. As a result, in the optimal contract, for those states which are not monitored, $E(\omega, K)$ is a constant for a given K . Let $\bar{\omega}$ be the cut off value such that if $\omega \geq \bar{\omega}$, the borrower will not be monitored; if $\omega < \bar{\omega}$, the borrower will declare bankruptcy and monitoring will occur.¹⁷ The constant repayment $E(\omega, K)$ when $\omega \geq \bar{\omega}$ is equal to the total return of capital at state $\bar{\omega}$, $\bar{\omega}(1 + r_{t+1}^F)K_{t+1}$. When monitoring cost is incurred, the entrepreneur receives nothing, while the creditors receive $(1 - \mu)\omega(1 + r_{t+1}^F)K_{t+1}$ in residual claims net of monitoring costs.¹⁸

To provide enough incentives for the banks to participate in such a contract, it must be the case that banks profit could cover its cost within the same period. I assume banks' profits come from two sources: providing loans to entrepreneurs and investing in an outside financial market. Specifically, each period banks allocate a certain amount of funds, equal to proportion s of the loans made, and invest it in a financial market whose details are not explicitly modeled. The only assumption about this market is that its gross rate of return

¹⁶The index j is suppressed.

¹⁷Strictly speaking, ω takes different values for each time period t . For notational convenience, here I write ω instead of ω_t , $\bar{\omega}$ instead of $\bar{\omega}_t$, when ambiguity does not arise.

¹⁸See Williamson (1987) for a detailed analysis.

during period $t + 1$, denoted as Φ_{t+1} , is exogenously given and follows an AR(1) process

$$\Phi_{t+1} - \bar{\Phi} = \rho_{\Phi}(\Phi_t - \bar{\Phi}) + \epsilon_{t+1}^{\Phi},$$

where $\bar{\Phi}$ is the mean of the process Φ_t , ρ_{Φ} measures the persistence of Φ_t , and ϵ_t^{Φ} is a white noise process with variance σ_{Φ}^2 . This assumption allows me to capture the phenomenon of deleveraging in a tractable way and ϵ_t^{Φ} , shocks to the exogenous financial market returns, become shocks to banking capital. Such shocks originate from the financial market, yet has real impact on the business cycle variables through the banks' lending channel, and in this sense, I also call them "financial shocks".¹⁹

The participation constraint for the banking sector becomes:

$$\begin{aligned} & [\bar{\omega}(1 - F_{\omega}(\bar{\omega})) + (1 - \mu) \int_0^{\bar{\omega}} \omega f(\omega) d\omega] (1 + r_{t+1}^F) K_{t+1} + \Phi_{t+1} (s D_{t+1}) \\ = & (1 + r_{t+1}^B) B_{t+1}, \end{aligned} \quad (A1)$$

where F_{ω} is the c.d.f. of the idiosyncratic shock ω , and B_{t+1} is the total deposit from households over period $t + 1$.

Following BGG (1999), we define $\Gamma(\bar{\omega})$ as the expected gross share of capital returns going to the lender plus the monitoring cost:

$$\Gamma(\bar{\omega}) \equiv \int_{\omega_a}^{\bar{\omega}} \omega f(\omega) d\omega + \bar{\omega} \int_{\bar{\omega}}^{\omega_b} f(\omega) d\omega,$$

where (ω_a, ω_b) is the support of ω . Similarly, let's define $\mu G(\bar{\omega})$ as the expected monitoring costs

$$\mu G(\bar{\omega}) \equiv \mu \int_{\omega_a}^{\bar{\omega}} \omega f(\omega) d\omega.$$

If the hazard rate $h(\omega)$, $f(\omega)/[1 - F(\omega)]$, satisfies the condition that $\omega h(\omega)$ is increasing in ω , we can easily prove the following lemma:

¹⁹The assumption that banks' investment in the external financial market is proportional to the credit provided to entrepreneurs is made to avoid modeling banks' portfolio choice problem. In real data the correlation of other investment and corporate loans at U.S. commercial banks is slightly positive after HP filtering at quarterly frequency.

Lemma 6 *There exists a $\bar{\omega}^* \in (\omega_a, \omega_b)$ such that*

- (i) $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) < 0$ for $\bar{\omega} > \bar{\omega}^*$;
- (ii) $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) = 0$ for $\bar{\omega} = \bar{\omega}^*$;
- (iii) $\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega}) > 0$ for $\bar{\omega} < \bar{\omega}^*$.

Note that $\lim_{\bar{\omega} \rightarrow \omega_a} \Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = 0$, $\lim_{\bar{\omega} \rightarrow \omega_b} \Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = 1 - \mu$. Since $\Gamma(\bar{\omega}) - \mu G(\bar{\omega})$ is increasing on $(\omega_a, \bar{\omega}^*)$ and decreasing on $(\bar{\omega}^*, \omega_b)$, the optimal contract will always choose $\bar{\omega} < \bar{\omega}^*$. This guarantees that the optimization problem is well-defined.

Formally we can state the optimal contract problem as:

$$\begin{aligned} \max_{K_{t+1}, \bar{\omega}} \quad & (1 - \Gamma(\bar{\omega}))(1 + r_{t+1}^F)K_{t+1} \\ \text{s.t.} \quad & [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + r_{t+1}^F)K_{t+1} + \Phi_{t+1}(sD_{t+1}) = (1 + r_{t+1}^B)(1 + s)D_{t+1}, \end{aligned}$$

where the credit market clearing condition $B_t = (1 + s)D_t$ is used.

To simplify, let's first define the premium on external funds as

$$1 + \rho_{t+1} = \frac{1 + r_{t+1}^F}{(1 + r_{t+1}^B)(1 + s) - \Phi_{t+1}s}.$$

Note that when defining the same concept in BGG (1999), the denominator is $1 + r_{t+1}^B$, instead of $(1 + r_{t+1}^B)(1 + s) - \Phi_{t+1}s$. This adjust by Φ_{t+1} reflects the introduction of the external financial market. If the gross return from this market is larger than the deposit return, the denominator as the reserved return for banks, will be lower than the deposit return, since banks only want to break even, and this is made easier by high profits obtained from the financial market. Similarly, if Φ_t drops significantly or even become negative, banks reserve return will become very high, possibly much higher than the deposit return.

If we define the capital/net worth ratio as

$$\Lambda_t = \frac{K_{t+1}}{N_t},$$

the contracting problem can be rewritten as

$$\max_{\Lambda_t, \bar{\omega}} (1 - \Gamma(\bar{\omega}))\Lambda_t$$

$$s.t. \quad [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1})\Lambda_t = \Lambda_t - 1.$$

Define λ_t as the Lagrangian multiplier on the lenders' participation constraint, and L_t the Lagrangian for this problem, the following optimality conditions are obtained:

$$\frac{\partial L_t}{\partial \bar{\omega}}: \quad \Gamma'(\bar{\omega}) - \lambda[\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})] = 0; \quad (A2)$$

$$\frac{\partial L_t}{\partial \Lambda_t}: \quad [1 - \Gamma(\bar{\omega})] + \lambda[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1}) - \lambda = 0; \quad (A3)$$

$$\frac{\partial L_t}{\partial \lambda_t}: \quad [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1})\Lambda_t = \Lambda_t - 1. \quad (A4)$$

Note that (A.2) and (A.3) imply that the optimal cutoff depends only on the external finance premium $(1 + \rho_{t+1})$, thus is independent of entrepreneur's net worth.

Combining (A.2) and (A.3) we can derive $\bar{\omega}$ as a function of $1 + \rho_{t+1}$ from the following equation

$$[1 - \Gamma(\bar{\omega})] + \left[\frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \right] [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1}) - \left[\frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega}) - \mu G'(\bar{\omega})} \right] = 0.$$

Under the maintained assumption that $\omega h(\omega)$ is increasing in ω , Implicit Function Theorem can be applied to prove the following results.

Lemma 7 (i) *The optimal cutoff $\bar{\omega}$ is an increasing function of $(1 + \rho_{t+1})$, the risk premium, or, expressed in inverse form,*

$$1 + \rho_{t+1} = \Delta(\bar{\omega}) \text{ with } \Delta'(\cdot) > 0.$$

(ii) *The ratio of investment relative to net worth, Λ_t , is an increasing function of risk premium $1 + \rho_{t+1}$.*

$$1 + \rho_{t+1} = \Xi(\Lambda_t) \text{ with } \Xi'(\cdot) > 0.$$

For completeness, let's include an expression for Λ_t after solving $\bar{\omega}$:

$$\Lambda_t = \frac{1}{1 - [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 + \rho_{t+1})}.$$

Although defining the variable $1 + \rho_{t+1}$ greatly simplifies this problem and leads to a better understanding of the underlying mechanism of the model, it is not the "credit spread" defined in the usual sense. Furthermore, the marginal productivity of capital in the definition of $1 + \rho_{t+1}$ is unobservable. This motivates us to define an other premium, which I call "credit spread", as the difference between entrepreneurs' effective borrowing rate and the riskless deposit rate, as in Woodford and Curdia (2009). For a representative entrepreneur, with probability $F_e(\bar{\omega})$, he declares bankruptcy and does not pay back to the banks; with probability $1 - F_e(\bar{\omega})$, he pays a certain proportion of his total profit, $\bar{\omega}(1 + r_{t+1}^F)K_{t+1}$ back to the banks. As a result, the effective borrowing rate for the entrepreneur is

$$\begin{aligned} & \frac{(1 - F_e(\bar{\omega}))\bar{\omega}(1 + r_{t+1}^F)K_{t+1}}{D_t} \\ &= (1 - F_e(\bar{\omega}))\bar{\omega}(1 + r_{t+1}^F)\Lambda_t. \end{aligned}$$

Thus the credit spread $1 + \rho_{cs}$ is

$$1 + \rho_{cs} = \frac{(1 - F_e(\bar{\omega}))\bar{\omega}(1 + r_{t+1}^F)\Lambda_t}{1 + r_{t+1}^B}.$$

As we have stated before, $1 + \rho_{t+1}$ is always an increasing function of $\bar{\omega}$, thus an increasing function of Λ_t . However, this is not the case for $1 + \rho_{cs}$. For fixed levels of r_{t+1}^F and r_{t+1}^B , it can be proved that $1 + \rho_{cs}$ is an increasing function of both $\bar{\omega}$ and Λ_t if ω follows a Uniform distribution; although this not true for a general distribution of ω . With the movement in r_{t+1}^F and r_{t+1}^B , the relationship between $1 + \rho_{cs}$ and $\bar{\omega}$ becomes ambiguous.

So far we take the aggregate return per capital $(1 + r_{t+1}^F)$ as given in the partial equilibrium analysis. If $(1 + r_{t+1}^F)$ is uncertain at the time of contracting, the preceding analysis survives intact with $(1 + r_{t+1}^F)$ replaced with its expectation at time t .²⁰

Entrepreneurs have a finite horizon with a constant probability δ of surviving to the next period, and they only consume when they die. This assumption is made to capture the phenomenon of ongoing births and deaths of firms, and to guarantee that they will always remain dependent on external funds. I also assume that the birth rate of entrepreneurs to be such that the total mass of entrepreneurs is always 1. Also, capital fully depreciates after each period. With such formulation, an entrepreneur's net worth N_t evolves according to

$$N_t = \delta [(1 - \Gamma(\bar{\omega}))(1 + r_{t+1}^F)K_{t+1}].$$

²⁰Appendix A in BGG (1999) has detailed derivation.

7.4 Market Clearing and Equilibrium

The labor market clears if $H_t = L_t$. Likewise, credit market clears if the total amount of deposits, B_{t+1} , meets the demands for loans supplies and external financial market investment,

$$B_{t+1} = D_{t+1} + sD_{t+1}.$$

The good market clearing condition is

$$C_t + \mu G(\bar{\omega}_{t-1})(1 + r_t^F)K_t + (1 - \delta) [(1 - \Gamma(\bar{\omega}_{t-1}))(1 + r_t^F)K_t] = Y_t,$$

where the second and third component on the left hand side are the monitoring cost and entrepreneurs' consumption respectively. The dynamic stochastic equilibrium is defined in the usual way for the specified distributions of the exogenous processes $\{a_t\}$ and $\{\Phi_t\}$.

8 Transmission of Shocks in the Model

8.1 Functional Forms and Parameter Values

In calibrating the model, the time unit is meant to be one quarter. The subject discount factor is chosen to be 0.988; σ is set to be 1, implying the elasticity of intertemporal substitution is also 1. Similarly, both γ and the elasticity of labor supply are 1. α , measuring the share of capital income on firms' total profit, is set to be 0.42. Technology process has AR(1) coefficient of 0.9, as standard in the literature, and the standard deviation of technology shocks is $\frac{0.007}{1-\alpha}$.

It is less straightforward to choose the values for parameters in the debt contract problem. Parameter δ , captures the ongoing birth and death of firms, and coefficient μ , measuring bankruptcy cost, can not be directly estimated from available data. However, their values could be pinned down from the steady state level of capital/net worth ratio, which is around 2 for U.S. during the sample period, and the bankruptcy rate in U.S., which is around 3 percent per quarter, or 12 percent annually. The implied value of δ is around 0.87, while the implied level μ is around 7.5%. The idiosyncratic shock ω follows a uniform distribution on $[\omega_a, \omega_b]$, where $\omega_a = 0.45$ while $\omega_b = 1.55$. The results turn out to be pretty robust to the choice in these parameters.

The introduction of the external financial markets requires us to determine the value of s , measure the ratio of total investment in the external financial market and total amount of loans. The average level of total investment for U.S. commercial banks during the sample period is about 532 billions, while the total amount loans made from commercial banks for the same period is around 473 billions. Thus s is set to be the ratio of the two, around 1.12. The mean level of return from the external market is chosen to be 1.08, with AR(1) coefficient of 0.8. The standard deviation of financial shocks is 0.02.

8.2 Theoretical Impulse Response Functions

The transmission mechanism of technology and financial shocks are illustrated in the following figures.

Figure 8 and Figure 9 are the impulse responses of variables in the model with respect with a positive technology shock, one standard deviation in size. Since capital fully depreciates in this model, the variable k measuring capital is the same as investment here. In response to a positive tech shock, output, consumption and investment all increase by about the same magnitude. The same could be said for credit flows. As I have discussed before in the model specification, the default probability, determined by $\bar{\omega}$, the external finance premium, ρ , and the capital/net worth ratio Λ , always move in the same direction. After the technology shock, the default risk first decreases since entrepreneurs are able to collect more rents from capital if productivity of firms increases, holding everything else constant. After a few periods, however, the other channel of increased agency cost begins to dominate due to the increased credit flows, which brings external finance premium even higher than the level before the shock. Entrepreneurs' net worth, due to the dominant effect of increased productivity, is always increasing.

Figure 10 and Figure 11 are the impulse responses to a positive financial shock. A sudden increase in the return from the outside financial market for the banks implies better credit conditions for borrowers. As in the case of technology shocks, output, consumption and investment all go up, although investment does so by a larger magnitude than the other two. Unlike the case of technology shocks, however, default probability, external finance premium

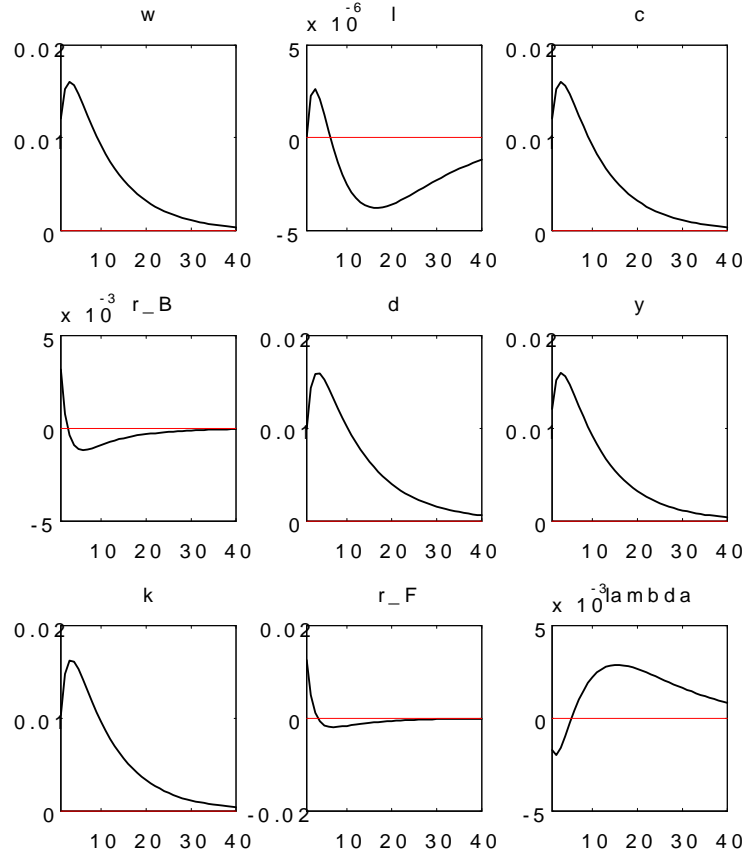


Figure 8: Impulse response functions after a positive technology shock. Each panel depicts the movement of one variable in the model after such a shock. w represents wages, d measures capital flows, c measures consumption, y measures output, k measures investment, while λ measures capital/net worth ratio.

and capital/net worth ratio all shoot up in this case, since the increase in debt flows without any improvement in productivity raises the concern on default. Note that entrepreneurs' net worth decreases in this case, which could be explained from two different perspectives. On the one hand, net worth is used by entrepreneurs as collateral in this model. When the credit conditions is more relaxed, less collateral, thus less net worth is needed. On the other hand, net worth is solely determined by the rent income collected by entrepreneurs. If capital supplied is increase, holding everything else constant, the marginal productivity of capital must fall. This property of decreasing net worth while increasing output becomes the defining characteristics of financial shocks.

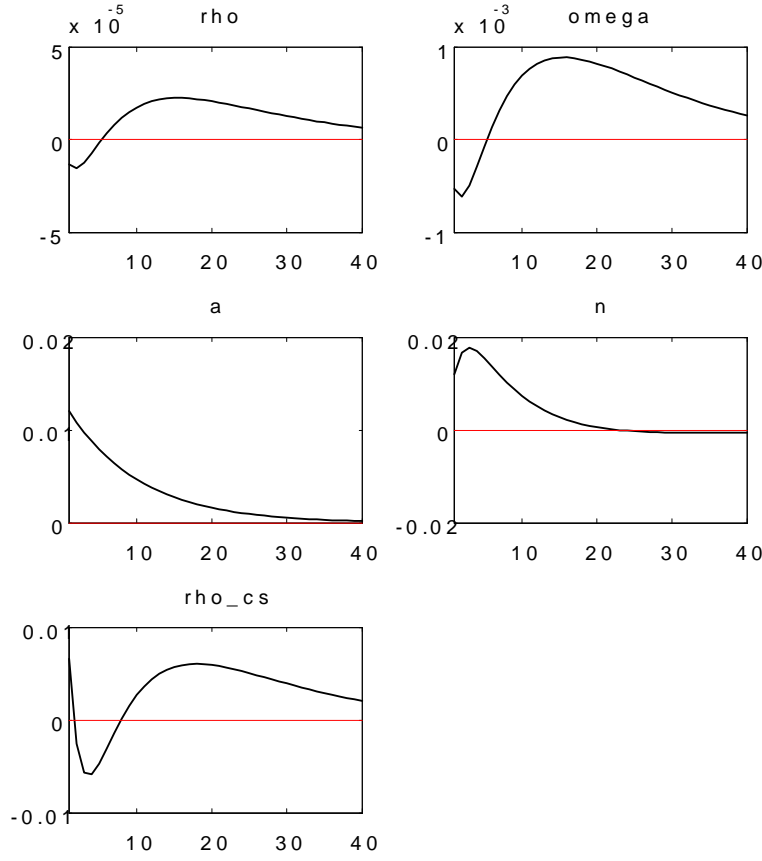


Figure 9: Impulse response functions after a positive technology shock. Here ρ represents external finance premium, ω represents the cutoff level of the idiosyncratic technology, n measures corporate net worth, while a measures technology.

9 Structural VAR Analysis

The key question in this chapter is the relative contribution of productivity and financial shocks in business cycle movement. Figure 12, 13 and 14 juxtapose movements in the net worth of nonfarm nonfinancial corporate business in U.S. from 1957Q1 to 2009 Q2 with the business cycle movements of GDP, gross private investment and nonfinancial corporate loans, respectively. It is well-known that both investment and credit flows are procyclical. However, it is surprising to see how strongly procyclical net worth is after keeping only the movement within business cycle frequencies. Figure 15 depicts the credit spread, defined as

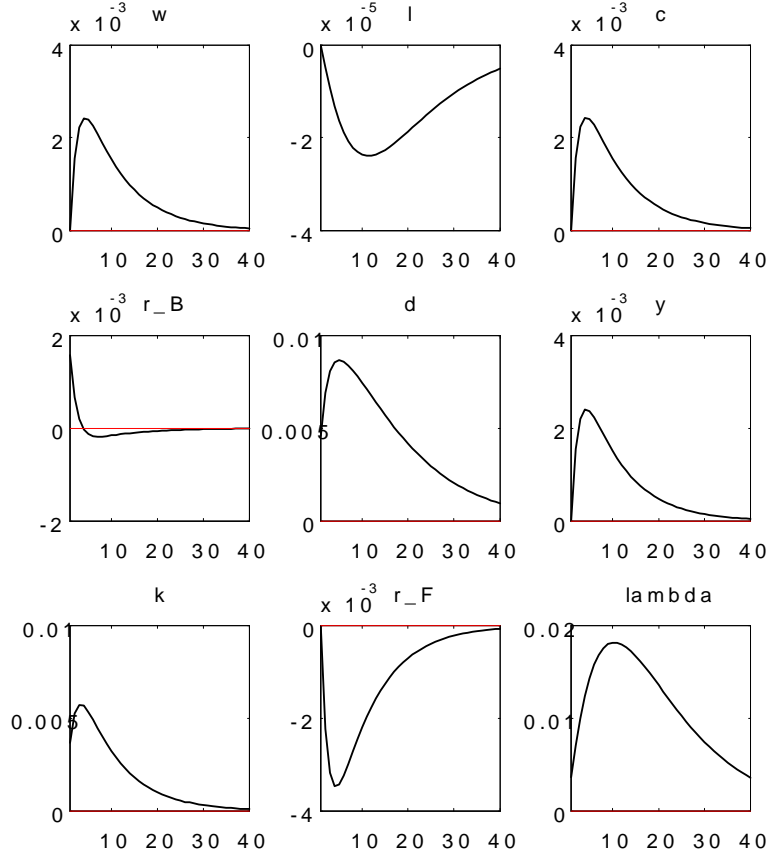


Figure 10: Impulse response functions after a positive financial shock. Each panel depicts the movement of one variable in the model after such a shock. w represents wages, d measures capital flows, c measures consumption, y measures output, k measures investment, while λ measures capital/net worth ratio.

the difference between prime loan rate and Fed Funds rate, together with the filtered GDP for U.S. Although the credit spread becomes very flat in recent years, over the whole sample period it is counter-cyclical. Simply looking at these graphs would suggest that a shock which changes output and net worth in the same direction, such as a technology shock, will play a dominant role in creating business cycle movements.

However, such eyeball econometrics could be potentially deceptive: many things are going on simultaneously in the economy, and one must be careful when making claims based on simple correlation. Thus, in the rest of this section I will follow the lead of Sims (1972, 1980) and proceed to analyze the key question with the aid of vector autoregression. In

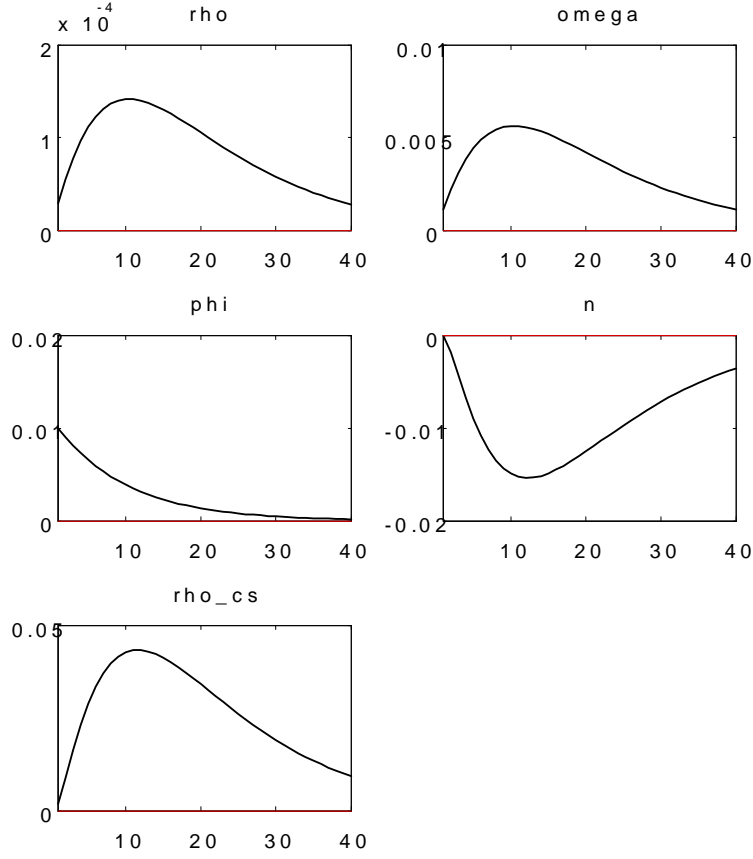


Figure 11: Impulse response functions after a positive financial shock. Here ρ represents external finance premium, ω represents the cutoff level of the idiosyncratic technology, n measures corporate net worth, while a measures technology.

particular, I adopt the methodology in Uhlig (2005) and others and use sign restrictions in the identification of structural shocks.

9.1 Data

The data set includes five time series, output, investment, amount of loans, corporate net worth and credit spread for U.S. over the sample period of 1957Q1 to 2009Q2. The output and investment data are downloaded from BEA and adjust by GDP deflator. The loan data are quarter commercial and industrial loans from all commercial banks for U.S., downloaded

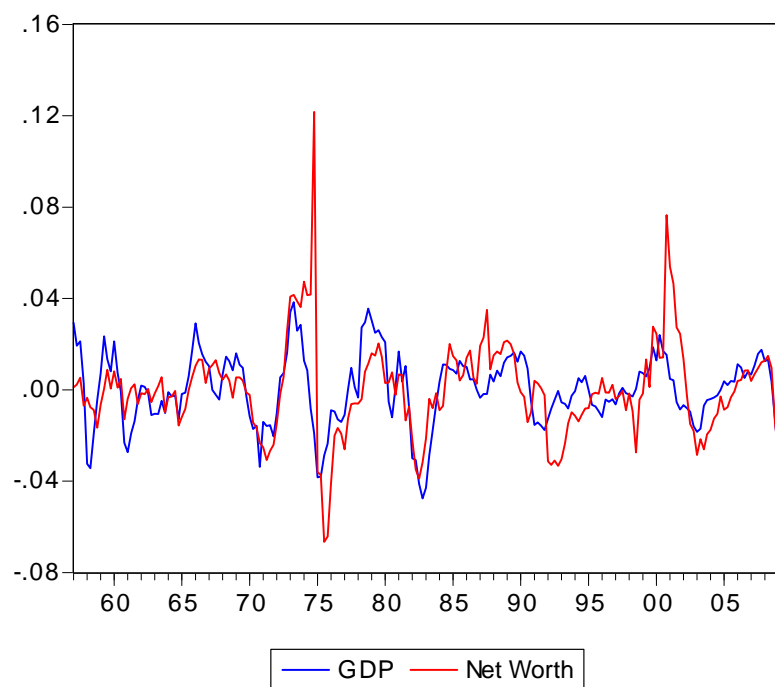


Figure 12: This figure contrasts movements in GDP and corporate net worth after HP filtering.

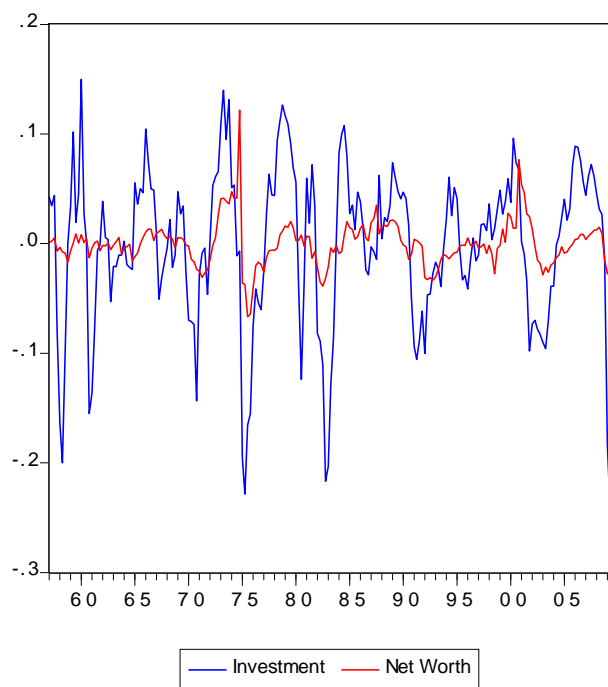


Figure 13: This figure contrasts movements in investment and corporate net worth after HP filtering.

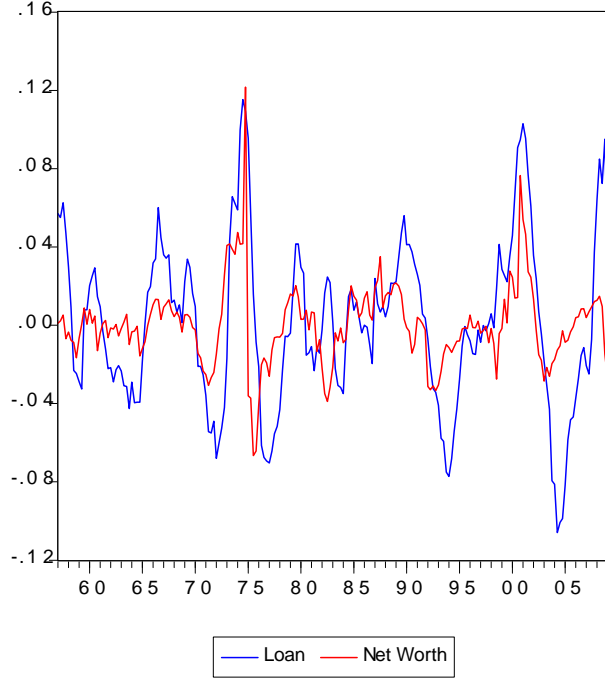


Figure 14: This figure contrasts movements in corporate loans and net worth after HP filtering.

from FED. Net worth series is series FL102090115 from Fed Flow of Funds which measures net worth at historical cost of U.S. nonfarm nonfinancial corporate balance sheet. All the series are seasonally adjusted, and HP filtered after taking logs. Credit spread is defined as the difference between quarterly bank prime lending rate from Fed and the Fed Funds Rate.

9.2 The Method

The method presented here closely follows Uhlig (2005). The starting point is the reduced form VAR of order(p) and dimension m

$$Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \dots + B_p Y_{t-p} + u_t, \quad t = 1, \dots, T,$$

where Y_t is $m \times 1$ vector of data at date $t = 1-p, \dots, T$; B_t is a coefficient matrix of size $m \times m$ and u_t is the one-step ahead prediction error with variance-covariance matrix Σ . Neither intercept nor a time trend is included since all the data to be used later is guaranteed to be

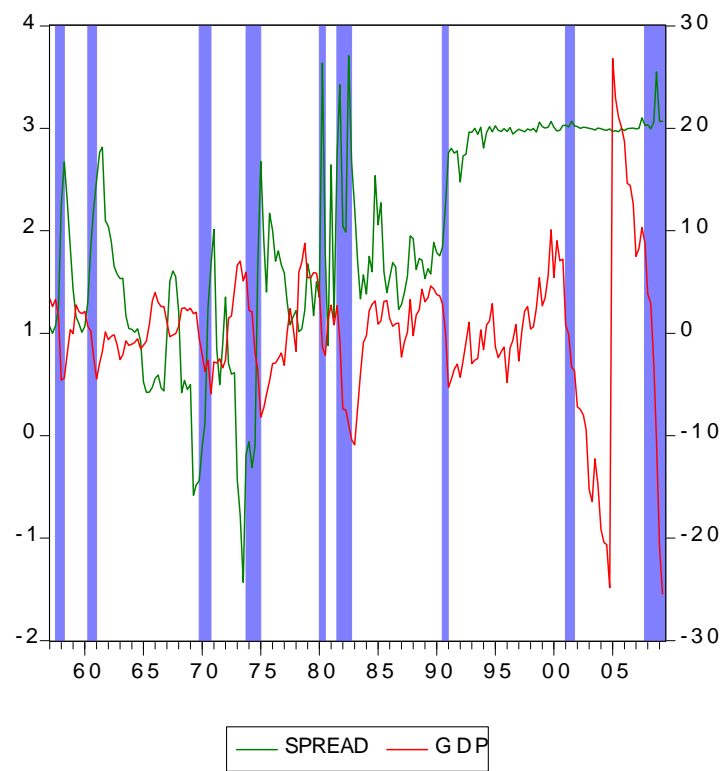


Figure 15: This figure contrasts movements in GDP and credit spread after HP filtering.

stationary and has mean zero by HP filtering.

Although it is standard to estimate such a system, what usually interest people are not the prediction errors u_t , but rather some more fundamental shocks with economic meaning in them. Suppose that there are a total of m fundamental innovations, which are mutually independent and normalized to be of variance 1: they can be written as a vector v_t of size $m \times 1$ with $E[v_t v_t'] = I_m$. It is reasonable to assume that the fundamental shocks are independent since otherwise there would remain some unexplained causal relationship among them. In particular, assume that u_t and v_t are related through a matrix A such that $u_t = Av_t$. Thus the j th column of A is the immediate impact of the j th fundamental innovation on all variables, one standard error in size and in the positive direction. So far the only restriction of A comes from the covariance condition

$$\Sigma = E[u_t u_t'] = AE[v_t v_t']A' = AA'.$$

Simple accounting shows that there are $m(m-1)/2$ degrees of freedom in specifying A , hence further restrictions have to be imposed to achieve identification.²¹ Usually this is done through either (1): choosing A to be a Cholesky factor of Σ as in Sims (1986); (2). separating transitory from permanent components as in Blanchard and Quah (1986), or (3). imposing structural relationships between the fundamental innovations and the one-step ahead prediction errors as in Bernanke (1986) or Sims (1986). Here I proceed differently. Note that I am only interested in identifying technology shocks and financial shocks, which move output in the same direction while moving net worth in the opposite direction. Therefore, there is no reason to identify all the m fundamental innovations. Also, given that the characteristics of the shocks to be identified are given in the form of sign restrictions on impulse responses, it is natural to start from sign restrictions for identification.

The following three results, stated and proved in Uhlig (2005), turns out to be very useful

²¹Strictly speaking, this accounting condition, proposed by Rothenberg(1971), is a necessary but not a sufficient condition for exact identification. Unless the model is recursive system, it may not be identified even if there are $n(n-1)/2$ linear restrictions. Rubio-Ramirez, Waggoner and Zhua (2005) contains one such example. Their paper also presents an algorithm which is based on Uhlig (2005) but more efficient, especially for large VAR systems. However, for ease of presentation, I still use Uhlig's algorithm here. Also, since the model is relatively small, computational cost is not a big issue here.

for the identification.

Lemma 8 *If a vector $a \in R^m$ is a column of matrix A with $AA' = \Sigma$, and $\tilde{A}\tilde{A}' = \Sigma$ is the Cholesky decomposition of Σ , there is a m -dimensional vector b of unit length such that $a = \tilde{A}b$.*

Lemma 9 *Let $r_i(k) \in R^m$ be the vector response at horizon k to the i th shock in a Cholesky decomposition of Σ . The impulse response $r_a(k)$ for a is then simply given by*

$$r_a(k) = \sum_{i=1}^m b_i r_i(k).$$

Lemma 10 *Let $E_t[Y_{t+k}] - E_{t-1}[Y_{t+k}]$ be the k -step ahead forecast revision due to the arrival of new data at date t . The fraction $\Theta_{a,j,k}$ of the variance of the this forecast revision for variable j , explained by the shock corresponding to vector a in A , is given by*

$$\Theta_{a,j,k} = \frac{(r_{a,j}(k))^2}{\sum_{i=1}^m (r_{i,j}(k))^2},$$

where the index j corresponds to variable j .

The key of identifying the shocks of interest lies in the identification of the vector a . Lemma 3 provides the criterion for eligible candidates of a ; Lemma 4 and Lemma 5 allow us to perform appropriate impulse response analysis and variance decomposition.

The sign restrictions I impose on the shocks are: a positive technology shock leads to a nonnegative response in both output and corporate net worth; a positive financial shock, while leading to a nonnegative response in output, leads to a nonpositive response in corporate net worth. One may argue that there are many other shocks which could potentially have such effects on output and net worth, or combination of some fundamental shocks. One way to avoid this problem would be to identify the other shocks explicitly, but at the cost of many additional assumptions. Furthermore, this approach is not the only one prone to such problems; other identification schemes suffer similar problems. Now let me state the sign conditions explicitly.

Assumption 1 : Technology shocks correspond to a column vector a_T in A such that the impulse responses of technology shocks of output and net worth are nonnegative at horizons $k = 0, \dots, K$ for some $K \geq 0$.

Assumption 2 : Financial shocks correspond to a column vector a_F in A such that the impulse responses of financial shocks of output are nonnegative while those of net worth are nonpositive at horizons $k = 0, \dots, K$ for some $K \geq 0$.

Given some VAR coefficient matrices $B = [B'_1, \dots, B'_p]$, some variance-covariance matrix Σ , and some parameter K , let $\Upsilon_T(B, \Sigma, K)$ be the set of vector a_T which satisfies Assumption 1.²² Because it is obtained from inequalities, this set will either be empty or contain many elements, which makes exact identification impossible. This problem will be solved in two steps.

As a first step, it is informative to derive the set $\Upsilon_T(B, \Sigma, K)$ and present the entire possible range of impulse responses implied by $\Upsilon_T(B, \Sigma, K)$, provided that $\Upsilon_T(B, \Sigma, K)$ is not empty. Let \hat{B} and $\hat{\Sigma}$ be the OLS estimates of the reduced form VAR, and choose a value for K , there are at least two different approaches to derive the set $\Upsilon_T(B, \Sigma, K)$ and the implied range of impulse responses for technology shocks.

The first approach involves solving a constrained optimization problem. According to Lemma 4, finding the range of impulse responses is equivalent to finding some vector b_T from the unit sphere such that the implied impulse responses obtain the upper or lower bound while resulting vector a_T satisfying Assumption 1. Note that there is usually no a single b_T such that the response will be at the bound for all variables j or at all horizon k .

Or, this range could be obtained in a brutal force approach by generating many candidates for b_T , calculating their implied impulse response functions, checking the sign restrictions and deriving the upper and lower bounds. This would give us a consistent, although slightly biased estimates of the bounds. Specifically, draw b_T from a standard Normal distribution in R^m , normalize its length to be 1. Use Lemma 4 to compute the impulse response functions at all the relevant horizons $k = 0, 1, \dots, K$ and check the sign restrictions as specified in Assumption 1. Generate a large number of candidate draws for b_T and plot the maximum and minimum of the impulse responses for those $a_T \in \Upsilon_T(B, \Sigma, K)$. Since it is more straightforward to implement, this is the approach I adopt in this paper. Results could be seen in Figure 9. I will defer the discussion of this and all other results in the next subsection.

²²All the discussion about $\Upsilon_T(B, \Sigma, K)$ could be applied to $\Upsilon_F(B, \Sigma, K)$ after appropriate adjustment of the sign conditions. To avoid repetition, I will not discuss the construction of $\Upsilon_F(B, \Sigma, K)$ here.

As a second step, I adopt a Bayesian approach and supplement the identification assumption A1 by imposing a prior on $\Upsilon_T(B, \Sigma, K)$ in order to tackle the issue of nonexact identification. Let $\mathbf{B} = [B_1, \dots, B_p]'$ and assume that the parameters $(\mathbf{B}, \Sigma, b_T)$ are jointly drawn from a prior on $R^{p \times m \times m} \times P^{m \times m} \times U^m$, where $P^{m \times m}$ is the set of all $m \times m$ positive definite matrix and U^m is the unit sphere of m dimensional vectors. More specifically, the prior is proportional to a Normal-Wishart distribution in (\mathbf{B}, Σ) whenever the resulting impulse responses from $(\mathbf{B}, \Sigma, b_T)$ satisfies the sign restrictions, and zero elsewhere. The flat prior on the unit sphere for b_T guarantees that reordering the variables and choosing a different Cholesky decomposition will not yield different results.

A proper Normal-Wishart distribution is parameterized by a mean coefficient matrix $\bar{\mathbf{B}}$ of size $mp \times m$, a positive definite mean covariance matrix \bar{S} is size $m \times m$, a positive definite matrix \bar{N} is size $mp \times mp$ and a real number $v \geq 0$ to describe the uncertainty about (\mathbf{B}, Σ) around $(\bar{\mathbf{B}}, \bar{S})$. The Normal-Wishart distribution specifies that Σ^{-1} follows a Wishart distribution $W_m(\bar{S}^{-1}/v, v)$ with $E[\Sigma^{-1}] = \bar{S}^{-1}$, and that the coefficient matrix in its columnwise vectorized form, $vec(\mathbf{B})$, follows a Normal distribution $N(vec(\bar{\mathbf{B}}), \Sigma \otimes \bar{N}^{-1})$. Following Uhlig (2005), I use a weak prior such that $\bar{N}_0 = 0$, $v_0 = 0$, \bar{S}_0 and $\bar{\mathbf{B}}_0$ arbitrary. If $\hat{\mathbf{B}}$ and $\hat{\Sigma}$ are the MLE estimates for (\mathbf{B}, Σ) , the posterior distribution will be such that $\bar{\mathbf{B}}_T = \hat{\mathbf{B}}$, $\bar{S}_T = \hat{\Sigma}$, $v_T = T$, $\bar{N}_T = \mathbf{X}'_T \mathbf{X}_T$, where $\mathbf{X}_T = [X_1, \dots, X_T]'$ and $X_t = [Y'_{t-1}, Y'_{t-2}, \dots, Y'_{t-p}]'$.

To draw inferences from the posterior, I take n_1 draws from the Normal-Wishart posterior distribution and for each of these draws, n_2 draws of b_T from the m -dimensional unit sphere. For each draw, I calculate the impulse responses and check whether the sign restrictions are satisfied. If they are, this is a valid draw and is thus kept. Statistics of interest such as error bands are calculated from all the draws kept. For the computation in this paper, I have chosen $n_1 = n_2 = 1000$.

9.3 Results

In this section, I present some results of identifying financial and technology shocks using sign restrictions. Figure 16 shows the range of impulse response functions which satisfies the sign restrictions on technology shocks for periods $k = 0, 1, \dots, K$, where $K = 10$. The top row contains the results for real GDP and real investment, the middle row contains the

results for credit flows and net worth, while the bottom row contains the result for credit spread. In addition to the bounds, three randomly selected impulse responses which satisfy the sign restrictions are also depicted. Although none of the panels is able to determine the size of the instantaneous responses for the variables with no sign restrictions imposed, the potential range of the impulse is indeed greatly shrunk by this exercise. For example, from the figure it is unlikely that a technology shock will have any significant impact on credit spread at 2 years. Figure 17 contains a similar picture for a positive financial shock. After about 6 quarters, the effects of financial shocks are not longer visible.

The benchmark result from Bayesian analysis is contained in Figure 18 and Figure 19, showing the impulse responses from an expansionary technology shock and an expansionary financial shock for $K = 10$. That is, for the first 10 period after a positive technology shock, the responses of output and net worth are required to be positive. According the results, output, investment and net worth react largely and positively immediately after the shock, typically rising by 20 to 50 basis points, then reverse course very slowly. The counter intuitive result is that the credit flows drops initially, although it climbs to a positive level after about 6 quarters. The results to a positive financial shock are more standard. Note that from the top panel, the initial response of GDP to a 100 basis point increase in the exogenous financial market is around 10 to 20 basis points, and then starts to decline soon.

What fraction of the variance of the k -step ahead forecast revision in, say, real GDP, or credit flows, could be explained by technology shocks, or financial shocks as we defined? The answer lies in Figure 20 and Figure 21 using Assumption 1 and Assumption 2 respectively and $K = 10$. The variables are ordered as in Figure 16. According to the median estimates, shown as the middle lines in this figure, technology shocks account for about 40 percent of GDP at its peak horizons, around 5 quarters and 18 quarters. Similar results holds for investment. Financial shocks accounts to 5 percent of variation in real GDP and real investment right after its occurrence, however, its effect starts to deteriorate very quickly. Note that financial shocks account for more than 20 percent of the forecast variance in net worth at the very short horizons.

10 Conclusion

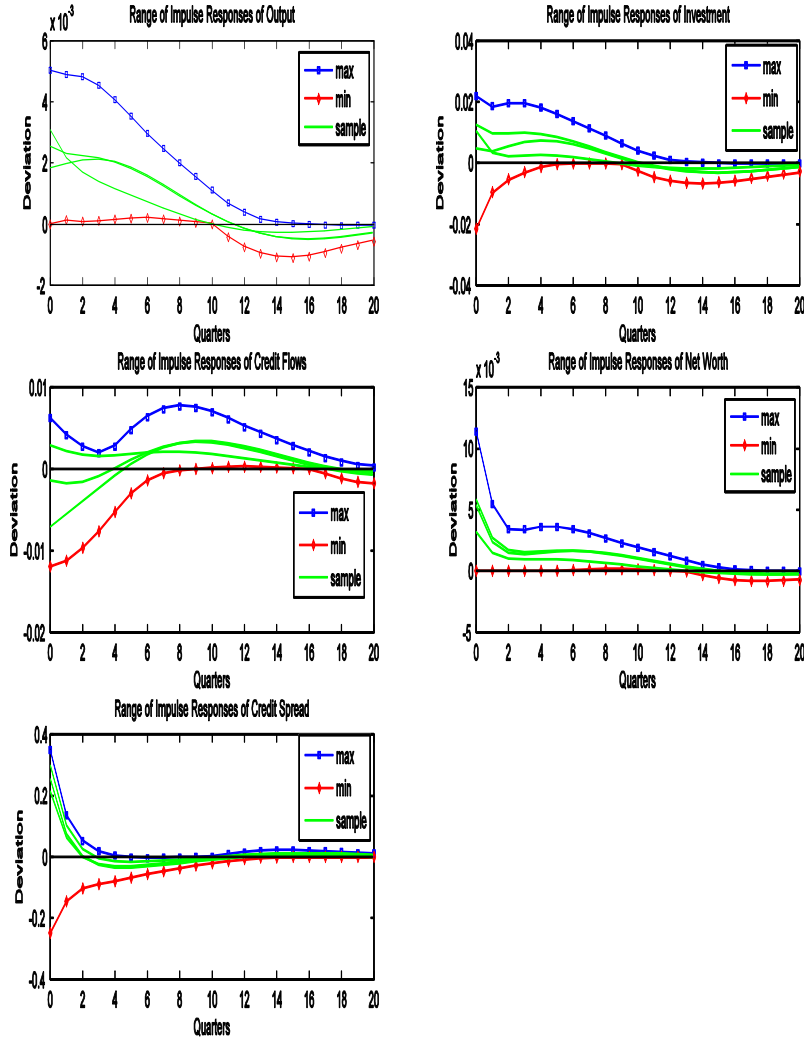


Figure 16: This figure shows the possible range of impulse response functions when imposing the sign restrictions Assumption1 for $K=10$ at the OLS point estimate for the VAR.

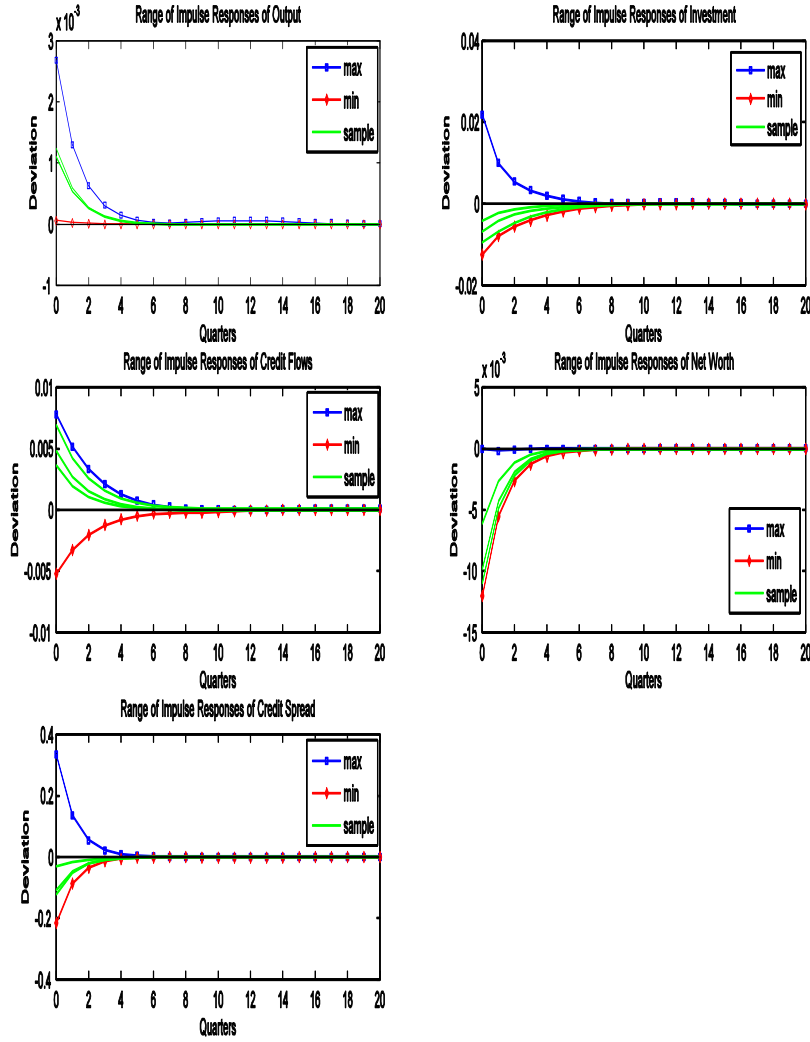


Figure 17: This figure shows the possible range of impulse response functions when imposing the sign restrictions Assumption2 for $K=10$ at the OLS point estimate for the VAR.

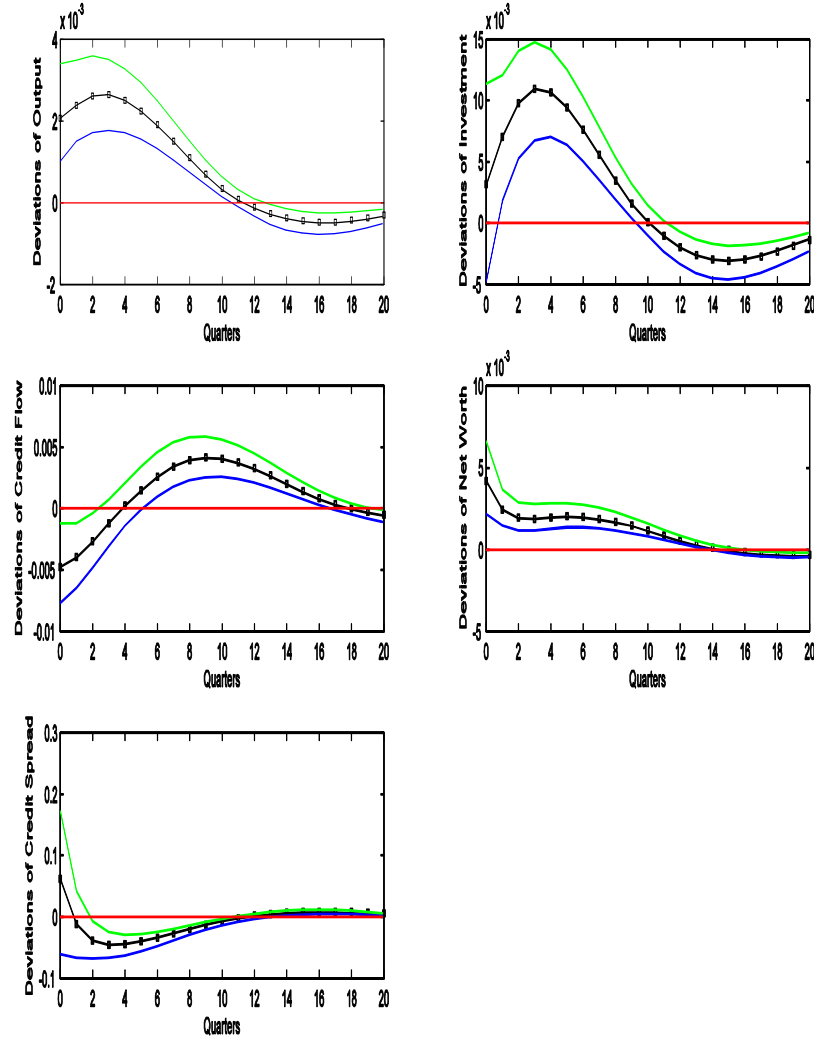


Figure 18: Impulse responses to an expansionary technology shock one standard deviation in size, using Assumption1 with $K=10$. The black line is the medium in the posterior distribution, and the error bands correspond to 25 and 75 percentile respectively.

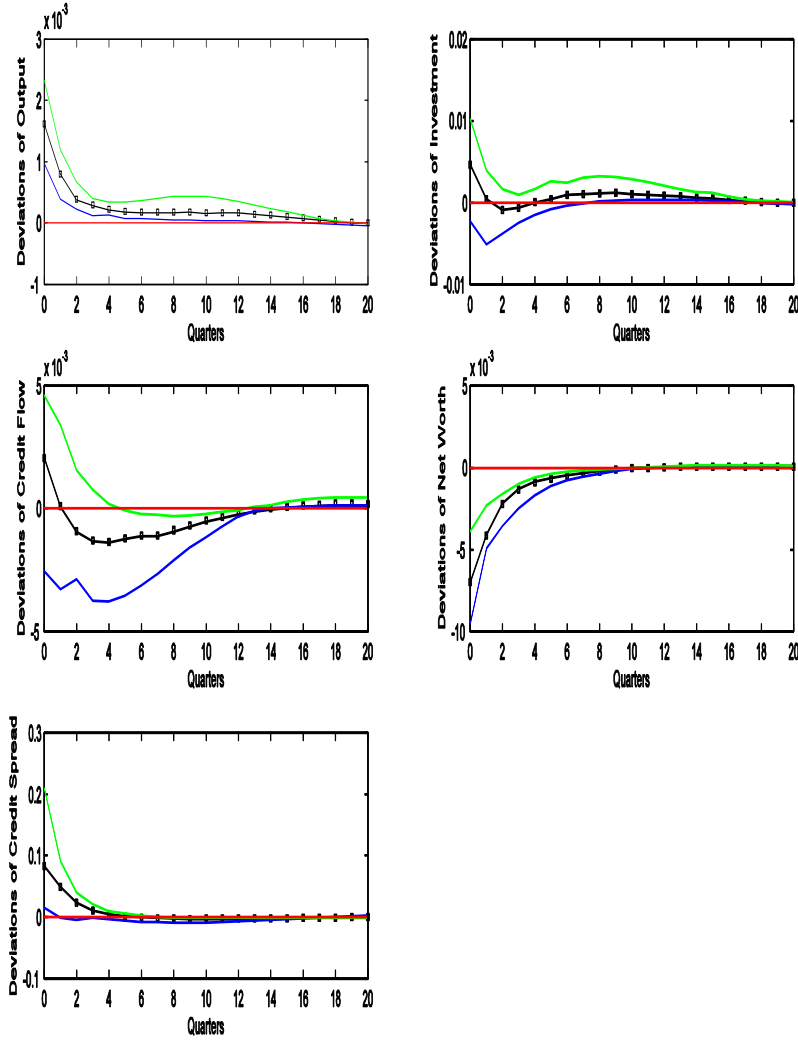


Figure 19: Impulse responses to an expansionary financial shock one standard deviation in *sizy*, using sign restrictions Assumption2 with $K=10$. The black line is the medium in the posterior distribution, and the error bands correspond to 25 and 75 percentile respectively.

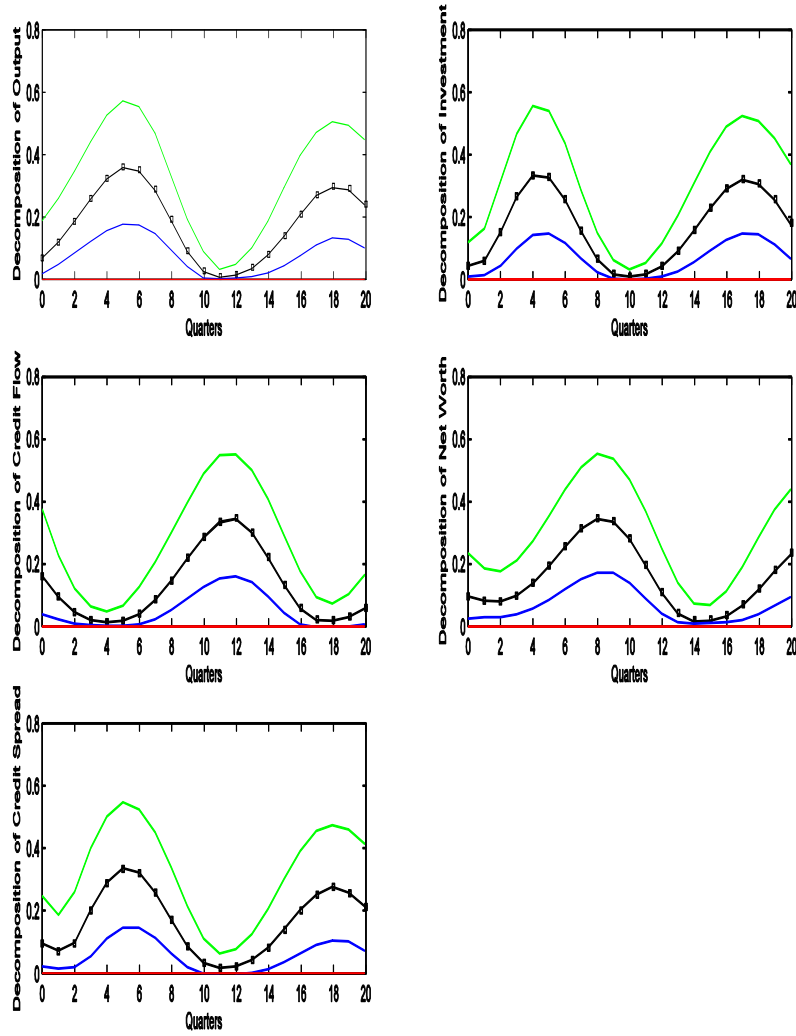


Figure 20: These plots show the fraction of the variance of the k-step ahead forecast revision explained by a technology identified by imposing Assumption 1. The error bands are 50% error bands around the median.

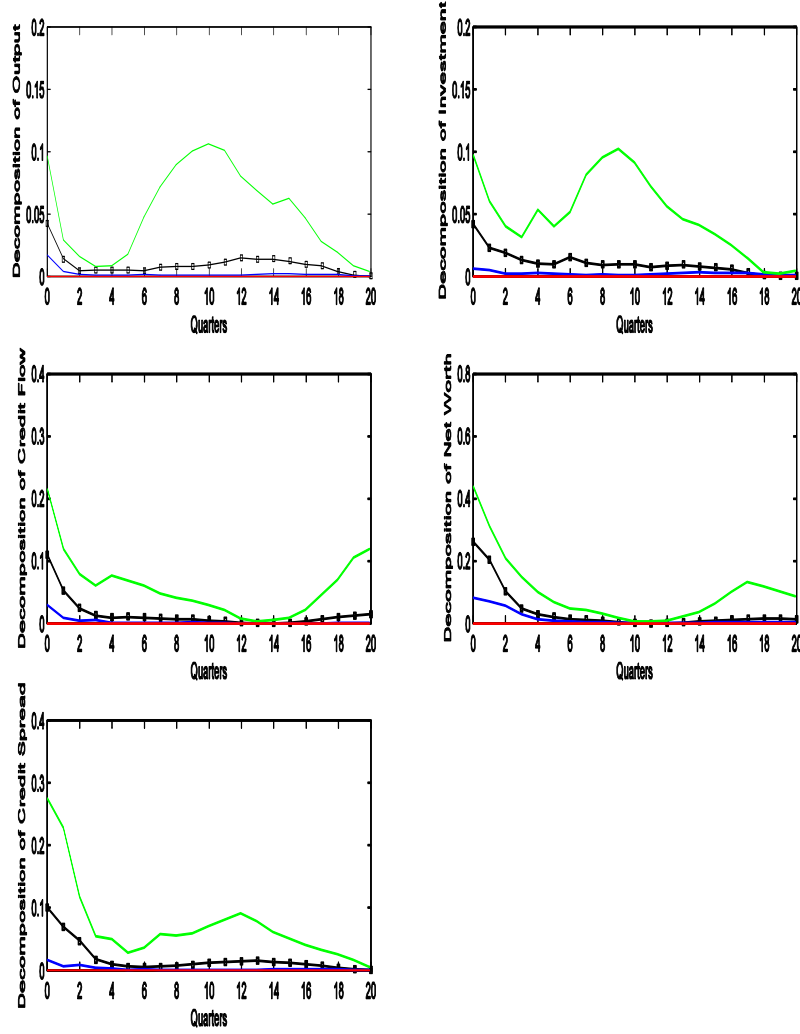


Figure 21: These plots show the fraction of the variance of the k-step ahead forecast revision explained by a financial shock identified by imposing Assumption 2. The error bands are 50% error bands around the median.

This paper has offered a theory-based approach to identify financial shocks. By introducing an exogenous financial markets into an otherwise standard dynamic general equilibrium model with financial accelerator, this paper first defines a financial shock as a shock to banking capital, and then explores its effects on the key variables such as output, investment and credit flows. Most importantly, although both a positive financial shock and a positive technology increase all the variables mentioned above, an expansionary technology shock tends to increase the entrepreneurs' net worth in the model, while an expansionary financial shock tends to decrease it. This difference provides the sign conditions needed for the identification.

With this framework in place, there are potentially more open questions that lie beyond the scope of this paper. For example, will the sign conditions identified in this paper be robust across different model specifications? Or, other than using the series of corporate net worth at historic cost, could we find a measure of "net worth" which captures the definition of entrepreneurs' net worth in the model as the capital rental income in a better way? These are all worth pursuing in future research.

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