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BEE 4530
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ENDOVENOUS LASER TREATMENT OF VARICOSE VEINS

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EXECUTIVE SUMMARY

Many men and women suffer from unsightly varicose veins typically found on the legs. This condition is caused by a failure of the leaflet valves in these veins, which prevent backflow and allow blood to return to the heart against gravity. The failure of the valves results in the veins becoming swollen and painful with an unpleasant appearance. To treat this condition, patients can undergo Endovenous Laser Treatment (ELT). During ELT, an optical fiber is inserted into the vein where it delivers infrared light, causing the vessels to shrink, close up, and direct blood flow to internal veins. The process is relatively safe as it is minimally invasive. However, there is still the possibility of thermal damage to the perivenous tissue from the laser light. Veins, also, can be damaged more than is necessary or even perforated during the process.

Using COMSOL, we modeled the heat transfer during ELT to determine which treatment time resulted in sufficient damage of the tunica intima and externa without causing irreversible damage to the perivenous tissue. A laser wavelength of 980nm is commonly used in this procedure. We replicated the effect of this wavelength on a small area during a single 1, 2, and 5 second pulse. This was simplified from the actual procedure, in which the laser is pulled through the vein at a slow, constant speed while pulsing. In COMSOL, we established a 2D axisymmetric geometry including the face of the laser, the vein lumen, vein wall, and the perivenous tissue beyond (Figure 2). In addition to the temperature equation, we implemented equations modeling laser decay, tissue damage, and blood flow in the vein lumen. Using the parameters and material properties found in Mordon et al[1], we were able to see the effects of each pulse time.

Our results indicated that a pulse time of 1 second resulted in permanent damage of the vein wall with minimal damage to the perivenous tissue. Both the 2 and 5 second pulses proved to be too strong. Those exposure times caused sufficient damage in the vein wall, but also caused a great deal of irreversible damage to the nearby perivenous tissue.

These results are relevant to sufferers of varicose veins and those researching methods to help treat this condition. Our model shows how the effectiveness of the ELT procedure is dependent on the combination of exposure duration and temperature. Optimization of time and temperature will increase the success of the treatment while decreasing the risk associated with possible perforations of the vein and damage to the perivenous tissue. Our results can be used as a foundation for further studies which can incorporate more complicated fluid flow, pullback speed, and perhaps the mechanical movement of the shrinking vein. Our study not only provided insight into how changes in pulse time affect treatment methods, but also a preliminary analysis of how blood flow affects this procedure.

INTRODUCTION

It is estimated that half of the population 50 years and older is affected with unsightly varicose veins, also known as spider veins[2]. Varicose veins are swollen and tortuous veins that appear to bulge from the skin with a red, blue, or flesh-colored hue. They are considered a symptom of venous insufficiency, a disease characterized by retrograde blood flow that can occur in low-pressure veins[3]. Varicose veins, as a result of venous insufficiency, are caused by faulty leaflet valves. If these valves weaken, blood flows backwards in the vein due to gravity and pressure. The blood can then collect and cause vein swelling[2]. The swelling may dilate the leaflet valves because they can no longer make contact in the lumen, at which point the veins continue to swell and become classified as varicose veins. They typically appear on the legs due to gravity and the relatively large pressure of body weight.

The most common symptom of varicose veins, other than their appearance, is aching pain and heaviness in the legs, especially in the evening or after exercise. In severe cases, the skin around the vein may darken due to blood leakage. Because affected veins no longer function as effectively, rashes on the outside of the skin, such as eczema, may develop due to waste product buildup. Restless leg syndrome commonly accompanies this condition[3]. Varicose veins are not a life-threatening disorder, but they can have some complications. Skin ulcers, or venous ulcers, commonly develop under the affected skin due to the buildup of blood. These are often painful and only heal once the vein is repaired. Blood clots can also form under the skin around varicose veins, known as thrombophlebitis. The blood clots are often firm and tender and uncomfortable when they come in contact with anything. Another serious concern is increased bleeding when a varicose vein is injured due to the built-up blood[2].

Varicose veins most commonly occur in the great saphenous vein (GSV), which is a part of the superficial venous system in the legs. These veins can be seen under the skin and are used primarily for cooling the body. When the body gets too hot, blood will be redirected from deep veins to superficial veins to facilitate heat transfer to the surroundings. Unlike the more important deep veins, superficial veins are not paired with an artery and carry significantly less blood[4]. Thus, if these veins become occluded, blood flow will be redirected towards internal veins.

Although varicose veins cannot always be prevented, steps can be taken to reduce the risk. The most important steps are healthy diet—low in salt and high in fiber—and exercise involving the legs. These will prevent body weight from putting pressure on the veins and also help improve circulation and increase vein strength. One should also avoid sitting or standing in the same position for prolonged periods of time, and especially not in high heels[2]. If the veins are not severe, a lifestyle change could provide an easy fix. For some, however, treatment is necessary. The five most common treatment methods are compression stockings, sclerotherapy, surface laser treatments, surgery, and endovenous laser treatment (ELT)[2].

Compression stockings are simple and used to treat minor cases of varicose veins. They are essentially socks that apply pressure to the legs in an attempt to assist with blood flow. Sclerotherapy is the most common method of treating of treating varicose veins. It is a simple outpatient procedure during which a doctor inserts a needle into the affected vein, often guided with ultrasound imaging, and injects a chemical that seals the vein closed. The vein then turns into scar tissue and fades in a few weeks. Surface laser treatment is the application of a laser on the skin outside of varicose veins. The laser damages the vein and causes it to fade. This treatment can be painful and takes two to five 20-minute sessions. Surgery is used for very large veins and involves the tying off of veins and physical removal. This technique, however, leaves scars behind and is not very common. The focus of this study is ELT, which has largely replaced surgery for large varicose veins because it is just as effective and minimally invasive[2].

During ELT, an optical fiber is inserted into the vein and emits laser light, typically at the infrared portion of the spectrum[6]. Under the guidance of ultrasound imaging, the catheter is inserted into the GSV through a small puncture and guided up to the groin or knee. The light is activated as the catheter is being withdrawn. The heat caused by laser light absorption damages the vein wall and causes the vein to become occluded, essentially destroying the entirety of the GSV. The vein will close and be no longer functional, and the blood that would have flowed through the vein will be rerouted to the healthier deep veins in the leg. The treatment is an outpatient procedure, requiring no sedation and taking about 2 hours to complete[6]. Complications due to the ELT procedure are very rare. The most common minor complications are bruising and induration. In regard to major complications, 0.5% of patients receive skin burns during treatment and 0.4% report having deep venous thrombosis. Despite these rare complications, 98.1% of patients have been shown to have closed veins after 60 months, so ELT is clearly a very effective and reasonable method for removing spider veins[6].

Despite the proven success that ELT has for the removal of varicose veins, research has yet to find the optimal exposure time to damage the vein without damaging the perivenous tissue. Treatment times are constantly being varied. Currently, treatment involves applying consistent flux as the laser is drawn down through the vein at a specified pullback speed. Numerous studies have examined different laser flux values to find the optimal laser power, but few have varied treatment time.

DESIGN OBJECTIVES

During endovenous laser treatment (ELT), the tunica intima and externa experience permanent thermal damage, which causes vein wall thickening and the eventual closing of the vein. Laser exposure times to the vein wall have a direct effect on how effective ELT can be, and so modeling this process can provide insight to the optimal laser exposure time while reducing the need for experimentation. We used COMSOL to model the effects of different laser pulse times with a 980nm laser on a 3mm diameter vein in the leg. The goals of our model are to:

1. Determine which pulse time (1, 2, or 5 s) for a 980 nm wavelength would result in the optimal vein damage for treatment with the least amount of perivenous tissue damage.

2. Define the desired tissue damage for vein wall thickening to be $\Omega = 0.53$ in the vein wall, where 1 is irreversible tissue damage to all cells and greater than 10 is perforation of the vein wall[7]. In the lumen and perivenous tissue, we are seeking to minimize the damage function to below $\Omega = 0.53$.

ELT PROCEDURE

Actual Procedure

In Figure 1, the ELT procedure is depicted on the left with an illustration and with an actual image on the right.

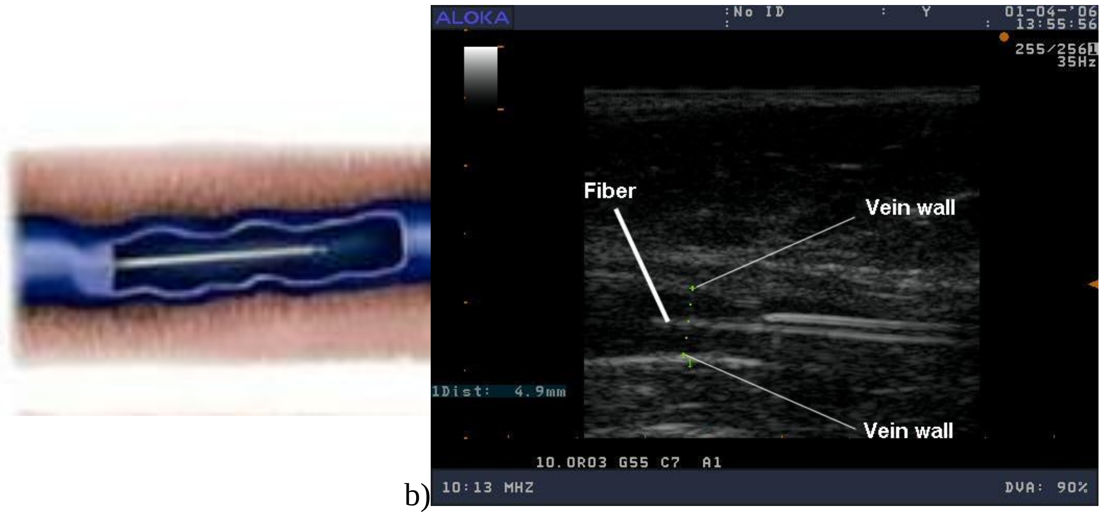

Figure 1. a) Illustration[8] and b) real image[1] of ELT procedure. A fiber-optic laser is inserted into the vein where it delivers light at a specified wavelength. This permanently damages the tunica intima and externa, increasing vein wall thickness and causing the vein to close.

The illustration shows that the laser is typically positioned in the center of the lumen with a tip that emits laser light radially, not axially. The blue material is the vein wall and the material outside of that is the perivenous tissue. Our schematic was created using these images.

Design Schematic

The axisymmetric geometry of the vein and laser has been simplified to a 2D axisymmetric geometry, so only half of the vein and tissue needs to be modeled. We modeled a 3mm diameter vein. The vein wall is 0.4mm thick, with the tunica intima at the inner boundary and the tunica externa at the outer boundary. The perivenous tissue extends 7.1mm from the vein wall to make our model semi-infinite. As in the actual procedure, the laser radiates outward towards the vein wall. The length of the laser that emits light is 0.6mm and is located directly in the middle of the lumen, which is one of the boundaries in our model. We assumed that the distance between the vein wall and laser is constant throughout the treatment. In COMSOL, each region is its own subdomain, so there are three subdomains: the lumen, the vein wall, and the perivenous tissue. To simplify the geometry so COMSOL can incorporate fluid flow, we extended the laser across the entire lumen so that the light doesn't come out of the tip but instead at a location along the laser's length. This prevented discontinuities caused by the sharp edge of the laser fiber.

The heat, laser light, and damage function governing equations were solved in all three subdomains. The laser light equation governs the distribution of light from the optical fiber to the tissues. In COMSOL, it was modeled as a species that diffuses with only a reaction rate that is equal to the rate of light dispersion. The damage function is a fraction that represents the number of damaged cells divided by the initial number of healthy cells. This equation is the basis of our study, as it describes the extent of damage that the ELT procedure causes. A value of 0.53 is considered to be irreversible damage of the tissue and a value of 1 represents complete necrosis. Blood flow exists only in the lumen subdomain and is modeled as plug flow. All governing equations, boundary conditions, and properties are listed in Appendix A.

Our schematic is depicted in Figure 2. We used this simplified depiction in order to design a model.

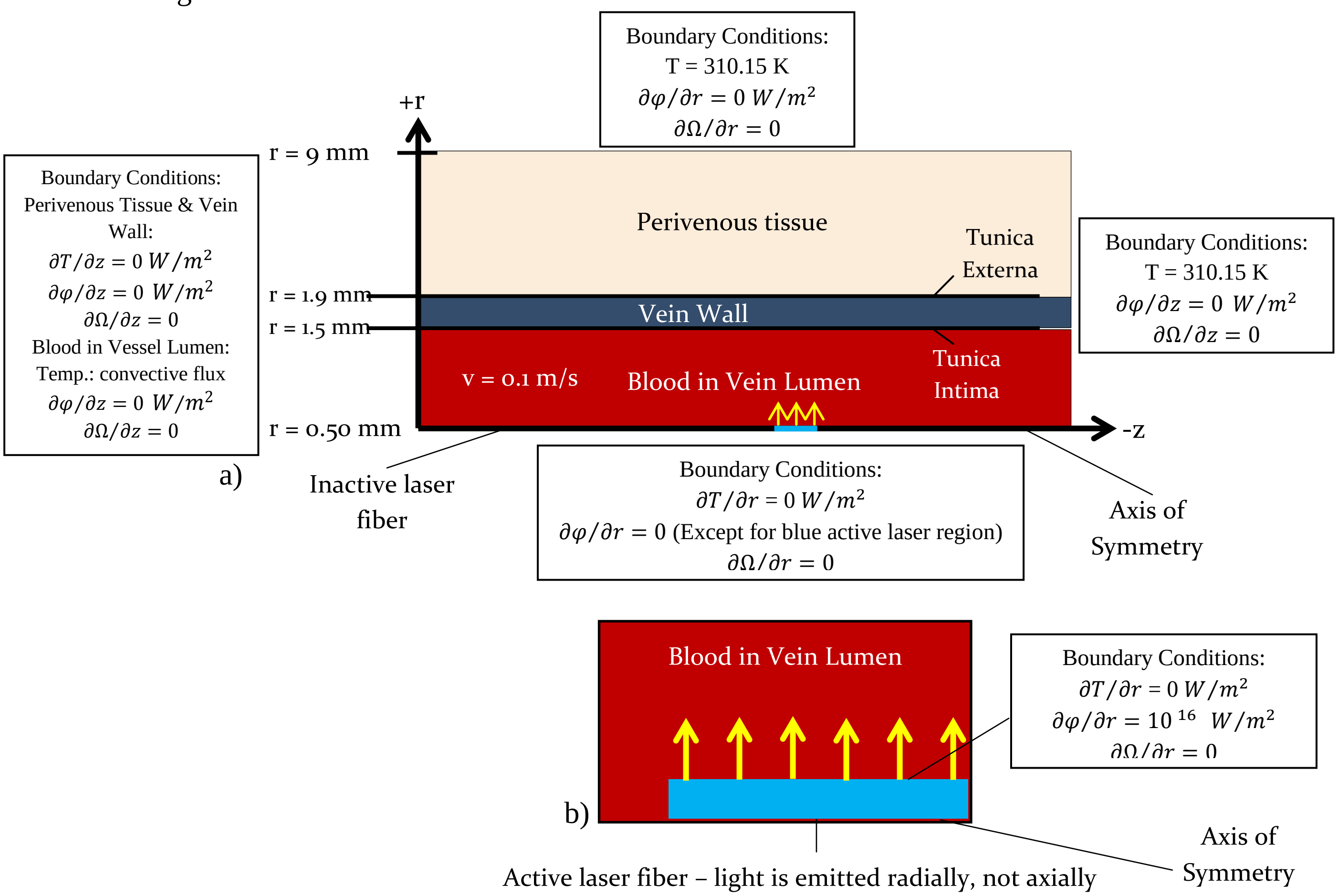

Figure 2. a) Schematic of model used to represent ELT. b) Schematic of optical laser fiber.

RESULTS & DISCUSSION

For a pulse time of 1 second, a plot of the temperature, laser light, and tissue damage and their respective scales can be found in Figure 3. Identical plots were generated for 2 and 5 second pulses.

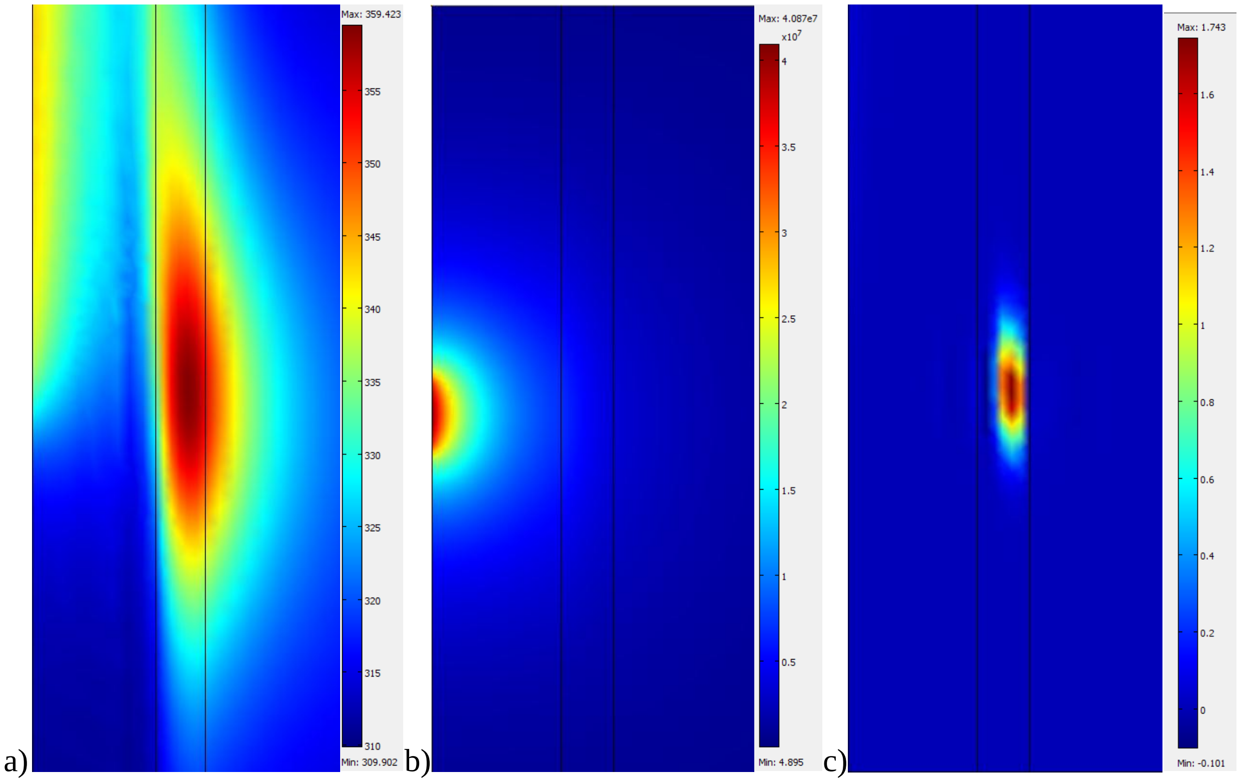

Figure 3. (a) Temperature profile of blood in lumen, vein, and perivenous tissue after a 1 second pulse. Range of temperature values is shown to the right. Min: 309.902 K Max: 359.423 K. (b) Light profile of blood in lumen, vein, and perivenous tissue after a 1 second pulse. Range of light values is shown to the right. Min: 4.895 Max: 4.087e7. c) Tissue Damage profile of blood in lumen, vein, and perivenous tissue after a 1 second pulse. Range of damage values is shown to the right. Min: -0.101 Max: 1.743.
Note: Images have been scaled to better show affected area.

In Figure 3c, we can see that the damage is concentrated in the vein wall, minimizing the damage to the perivenous tissue. For a 1 second pulse, $\Omega = 0$ in the perivenous tissue, which is well below the $\Omega = 0.53$ that would cause irreversible tissue damage. The damage throughout the majority of the treated section of the vein wall is above $\Omega = 0.53$, with the edges of the treated region just barely missing the irreversible damage threshold. There is a small region where the vein wall would experience cell necrosis due to thermal damage, but overall, the treatment is successful. The temperatures indicate that at t = 1s, the highest temperature reached is 359K (Fig 3a).

We quantified the volume of tissue that was permanently damaged ($0.53 < \Omega < 1$) and completely necrotic ($\Omega > 1$) in the vein wall and perivenous tissue. This analysis showed the effectiveness of each pulse time.

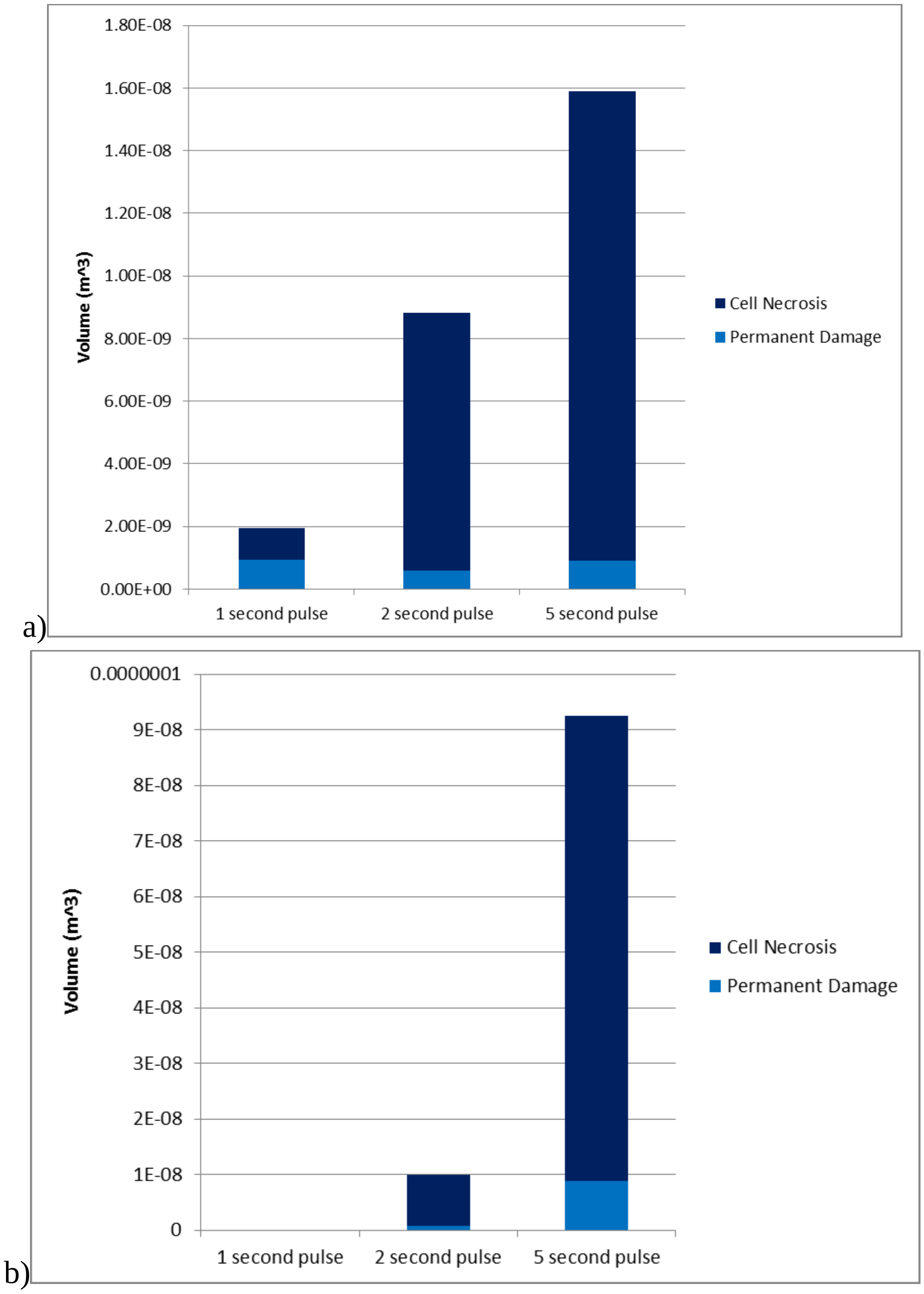

Figure 4. (a) Volume of permanently damaged tissue and volume of necrotic tissue in the vein wall for a 1, 2, and 5 second pulse. The amount of necrotic tissue in the vein wall for 2 and 5 second pulses indicate possible vein wall perforation. (b) Volume of permanently damaged tissue and volume of necrotic tissue in the perivenous tissue for a 1, 2, and 5 second pulse. There is no permanent damage or cell necrosis in the perivenous tissue for a 1 second pulse. The amount of necrotic tissue in the perivenous tissue for 2 and 5 second pulses indicate severe tissue damage and possible perforation in the perivenous tissue.

Figure 4 shows the volume of tissues that experienced permanent damage (Ω > 0.53) and cell necrosis (Ω > 1) for a 1, 2, and 5 second pulse. Figure 4a shows that the laser did sufficient damage to the vein wall without causing perforation in with the 1 second pulse. With the 2 and 5 second pulses, there is significant cell necrosis, and therefore danger of vein perforation which could cause a host of problems including bleeding and pain. In Figure 4b, it can be seen that there is no permanent damage or cell necrosis in the perivenous tissue for the 1 second pulse. The 2 and 5 second pulses show enough damaged volume to suggest perivenous tissue perforation and most likely extreme tissue damage which could be painful and harmful for the patient.

Through our model, we found that a 1 second pulse produces the most sufficient damage to the vein wall without damaging the perivenous tissue. A pulse of 2 or 5 seconds might create perforations in the vein wall and perivenous tissue, which could be very dangerous for the patient.

Sensitivity Analysis

We performed a sensitivity analysis in order to better understand our model. Sensitivity analysis gives us information about how our model works when parameters are varied by showing the influence of a certain parameter on the vein temperature. We changed the flux and blood, vein and perivenous tissue parameters by ±5% of its original value and calculated the percent change on vein temperature. The parameters varied were conductivity, specific heat, density, scattering coefficient and absorption coefficient. The sensitivity analysis using a 1 second pulse can be found in Figure 5.

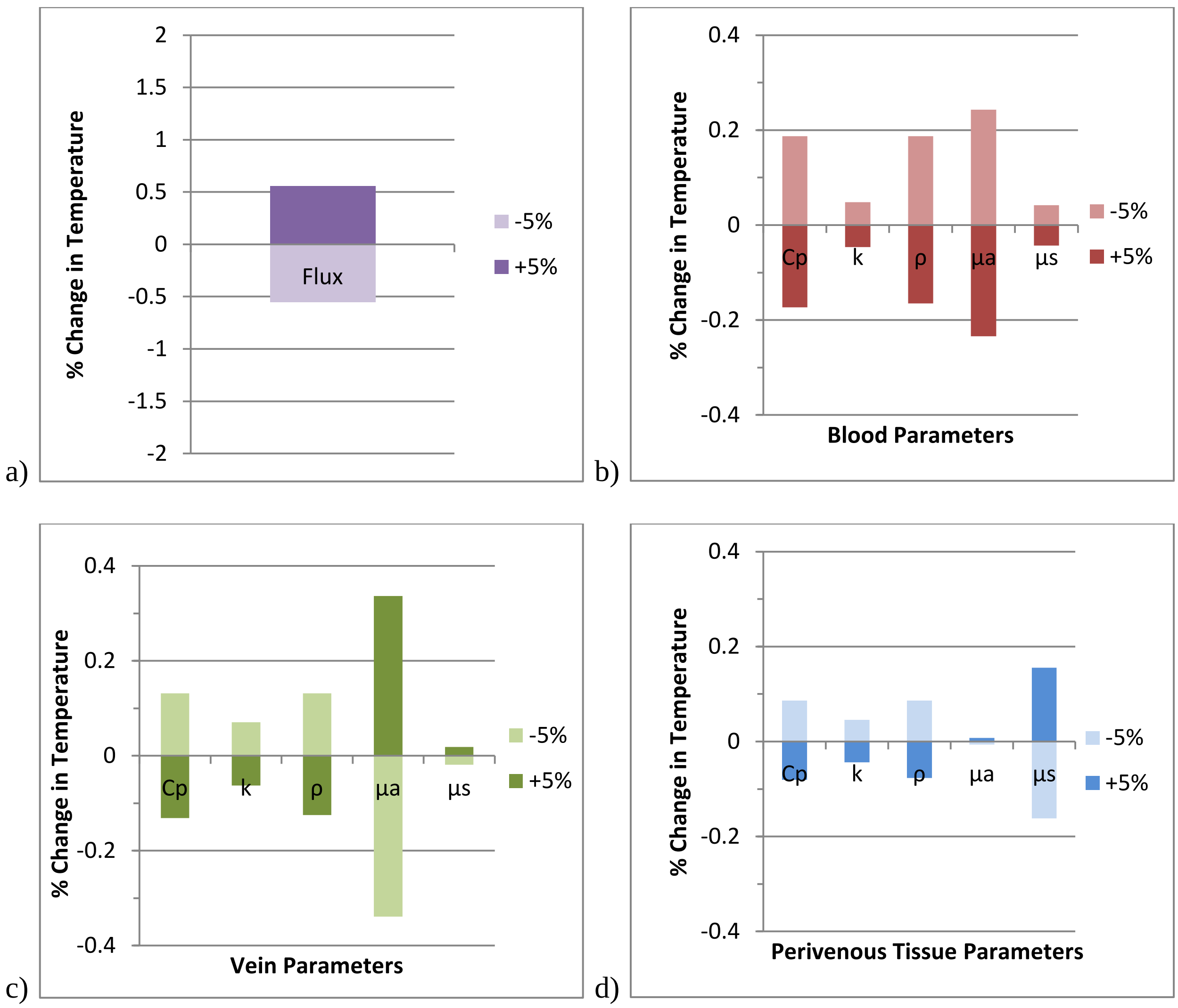

Figure 5. a) Sensitivity analysis for temperature with respect to flux change for a 1 second pulse. b) Sensitivity analysis for temperature with respect to changes in the blood parameters for a 1 second pulse. c) Sensitivity analysis for temperature with respect to changes in the vein parameters for a 1 second pulse. d) Sensitivity analysis for temperature with respect to changes in the perivenous tissue parameters for a 1 second pulse. Note: coordinates: r = 1.674 mm, z = 16.524 mm.

Figure 5a shows the percent changes in temperature when the flux was varied by ±5%. It can be seen that our model was most sensitive to changes in flux which suggests that we need to be accurate with our flux value in order for our model to work properly.

Figures 5b, 5c and 5d show the effect of varying blood, vein, and perivenous tissue parameters respectively. In Figure 5b, we can see that altering the blood absorption coefficient has the most effect on temperature. Similarly, Figure 5c shows that the vein absorption coefficient is the parameter that governs the temperature. Changes in the absorption coefficient of blood and vein produce significant temperature variations since this parameter influences the amount of laser energy that is being absorbed and therefore the temperature. Varying the absorptivity in the vein had a greater effect on the vein temperature compared to varying the blood absorptivity. Considering the effect of variation in parameters, the vein temperature was most sensitive to changes in the blood vessel parameters. This result is likely because the absorptivity in the blood is greater than in the vein and perivenous tissue. Figure 5d shows that variation of the perivenous tissue parameters caused the least amount of temperature change.

Accuracy Check

The paper published by Morton et al was used to check the accuracy of the solution that our model provides. Their model was similar to ours in that they used a 1.5mm vein with a laser that emits light at a wavelength of 980nm in 2 second pulses. From their results, they achieved damage in the vein walls at an order of magnitude between 10^1 and 10^3 [9]. Our results from the 2 second pulse, as shown in Figure 7, gave a damage value in the vein wall directly in front of the laser of approximately 1200, so our solution is within the bounds that have been determined by other studies. Morton et al did not, however, include blood flow in their design and so their damage function shows damage in the blood. We believe this discrepancy is acceptable because the blood flow that we modeled functioned to transport heat away, resulting in limited heating of the blood.

For the pulse times of 1 second and 5 seconds, we were unable to find any studies that matched our model as closely as Morton et al. Our model was validated with a pulse time of 2 seconds, however, and so we can compare that solution to the two other solutions. As expected, the pulse time of 1 second resulted in a very similar solution, but with a reduced temperature and damage values compared to the 2 second pulse. The 5 second pulse showed an increase in temperature and damage, but overall similar gradients compared to the other pulses. These results are expected because the exposure time of the laser directly affects the amount of heating in the tissue. With this reasoning we have determined that our model is accurate for all pulse times.

CONCLUSION

Our goal in developing this model was to determine the most effective laser exposure time during endovenous laser treatment of varicose veins. We defined the effectiveness of the exposure times in terms of our damage function: the desirable damage ($\Omega = 0.53$) should be at the vein wall, while the damage ($\Omega < 0.53$) should be minimal in the perivenous tissue. Three different exposure times (1, 2, and 5 seconds) were tested using a simplified two-dimensional axisymmetric geometry of a vein and previously published data regarding the thermal properties of human blood, veins, and tissue inputted into COMSOL. After 1 second, the desired amount of damage is contained to the vein wall; however, after 2 and 5 seconds, the damage spreads to the perivenous tissue. Thus, our results suggest that the optimal laser exposure time is 1 second. This is consistent with some existing publications, but may digress from others due to the various assumptions implemented during the development of this model[1]. The most notable of these assumptions was the incorporation of a constant blood flow velocity instead of the Navier-Stokes equation, an extremely high flux value, as well as the exclusion of laser pullback speed.

Although our model included simplifications, it is a step towards optimizing the ELT procedure. Our model is, in particular, a valuable tool in incorporating fluid flow into models of the ELT procedure, as many other models do not include this important factor. The addition of fluid eliminates damage to the blood. Blood damage occurs in models without fluid flow, but is not realistic to in vivo procedures. Blood flow also lessens the heating effects of the laser light by distributing the heat throughout the vein. Our model has also revealed that a one second pulse time yields what we defined to be an optimal result (Ω=0.53). Future models of ELT can build on our results to create a more accurate model to take a step closer to the ideal treatment.

Various improvements to our model, such as Navier-Stokes, pullback speed, material properties, an accurate flux value, and vein geometry, can be incorporated to further avail our findings to doctors performing these procedures. As more accurate models are created, different flux values can be paired with different pulse times to eventually lead to an optimized ELT procedure.

Design Recommendations

Our model, as with any model created with computer software, contains simplifications that must be made to digitally represent a physical reality. These simplifications include complexity of blood flow, laser movement, constant material properties, and vein geometry.

Complexity of Blood Flow

We initially attempted to model blood flow with the Navier-Stokes flow equation. This would have given an ideal parabolic flow through the vein, in which the blood flow is zero at the vein wall and at the laser catheter. Unfortunately, this created several complications with COMSOL and flow had to be simplified to plug flow with a constant velocity. The constant velocity assumption has several implications that make our model less realistic. First, it fails to model the variability of blood flow due to beating of the heart. Also, it makes the flow completely laminar, whereas in reality varicose veins have turbulent flow due the retrograde blood flow caused by faulty valves. We recommend that future studies incorporate Navier-Stokes into their blood flow model via an additional governing equation. Furthermore, heartbeats could be added by creating a function to represent the varying blood velocity occurring with each heartbeat. A further consideration might be the addition of turbulent flow into the model, as this does occur in the GSV.

Laser Movement

We did not include pullback speed of the laser. In reality, during ELT, the laser is slowly moved through the vein as it emits pulses of laser light. In our model, the laser is static and emits laser light at one location. We felt it unnecessary to implement a moving laser because our model is only for a single pulse, during which the movement of the laser is insignificant. To make this model more applicable to the real-world procedure, the position of active laser could be implemented in COMSOL as a function of pullback speed. This would ensure that multiple regions of the vein wall received treatment.

Constant Material Properties

Our model makes the assumption that material properties of tissue, such as the thermal conductivity and specific heat, are constant throughout the procedure. In reality, these values change with temperature. Future models can implement changes in these properties as functions of temperature in COMSOL.

Vein Geometry

Our geometry is a great simplification of true vein anatomy. Our model ignores the leaflet valves, combines the three layers of the vein wall into one, and excludes mechanical movement of the vein due to damage. Computer-Aided Design (CAD) imaging of an actual varicose vein can be used to add complexity to our simplified geometry. Mechanical movement of the vein can be incorporated via a geometry which changes as a function of the damage value in COMSOL.

Realistic Constraints

Our model uses a laser that emits light at a standard wavelength of 980nm, but the laser flux that it emits has not been found in literature. It was calculated with a separate, theoretical formula. As such, our model does not describe a laser that has been used with ELT. This makes our model difficult to implement in reality because the laser we specified will have to be manufactured, and the related cost may be too large for realistic implementation.

In addition, our model is strictly theoretical. Using the results of this study as a guide to perform ELT on a person is a serious safety concern and should not be considered. The results, however, are a step in the right direction. We have quantified how blood flow, pulse time, flux, and several parameters affect the heat transfer that occurs during ELT. Future studies can expand on this work to create a more realistic model that, for example, analyzes the entire length of the GSV or uses a laser flux that is already being manufactured. The potential is there for this study to be used as a foundation to improve the procedure of ELT.

APPENDIX A: MATHEMATICAL STATEMENT OF ELT

Governing Equations

- Temperature:

$$\frac{k}{\rho_t c_{pt}}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right)+\frac{Q}{\rho_t c_{pt}} = \frac{\partial T}{\partial t}+u\frac{\partial T}{\partial z}$$

Where $Q = \mu_a * \varphi$

Convection occurs only in the lumen. We assume plug flow with a constant blood velocity in the z direction.

- Laser Heat Generation (φ):

$$\frac{\partial \varphi}{\partial t} - \mathrm{D}\frac{\partial^2 \varphi}{\partial r^2} + c_* \mu_a \varphi = 0$$

Where $D = c_*[3(\mu_a + \mu_s')]^{-1}$

- Tissue Damage Concentration (Ω):

$$\frac{d\Omega}{dt} = Ae^{-E/RT}$$

Boundary and Initial Conditions

Boundary Conditions

- Temperature:
  - As r→∞, T = 37°C
  - There is axisymmetry, so at r = 0 mm, $-k\frac{\partial T}{\partial r} = 0$
  - The boundaries at z = 0 mm and z = 46 mm are assumed to be thermally insulated, as they are far away from the laser.
- Laser Light Generation:
  - Flux = 0 along all boundaries except for the optical fiber surface.
  - At r = 0.5 mm along the length of the optical fiber surface (600 microns), Flux = 1e16 W/m^2
- Tissue Damage Concentration:
  - At all boundaries, $\frac{\partial \Omega}{\partial r} = 0$

Initial Condition

- At t =0, T = 37°C
- At t =0, $\frac{\partial \varphi}{\partial r} = 0$
- At t =0, $\frac{\partial \Omega}{\partial r} = 0$

Material Properties

Table 1. All variables used for COMSOL analysis of Endovenous Laser Treatment. A description and value is included for each variable. All values obtained from Mordon, 2006[1] unless otherwise noted.

Description	Variable	Value
Initial temperature[10]	T	$310.15\ K$
Thermal conductivity, blood[10]	k	$0.492\ \frac{W}{m \cdot K}$
Thermal conductivity, vein / perivenous tissue[1]	k	$0.560\ \frac{W}{m \cdot K}$
Density, blood[1]	ρ_b	$1.05 \times 10^3\ \frac{kg}{m^3}$
Density, vein / perivenous tissue[1]	ρ_t	$1.05 \times 10^3\ \frac{g}{m^3}$
Specific heat of the blood[1]	c_{pb}	$3820\ \frac{J}{kg \cdot K}$
Specific heat, vein / perivenous tissue[1]	c_{pt}	$3780\ \frac{J}{kg \cdot K}$
Average blood velocity[11]	u_{blood}	$0.1\frac{m}{s}$
Power of the light source[1]	P_{laser}	15 W (980nm)
Scattering coefficient, blood[1]	$\mu_{s,b}{}'$	$600\,\frac{1}{m}$ (980 nm)
Scattering coefficient, vein tissue[1]	$\mu_{s,vt}{}'$	$2000\,\frac{1}{m}$
Scattering coefficient, perivenous tissue[1]	$\mu_{s,pt}{}'$	$1000\ \frac{1}{m}$
Absorption coefficient, blood[1]	μ_{ab}	$2800\,\frac{1}{m}$ (980 nm)
Absorption coefficient, vein tissue[1]	μ_{avt}	$100\ \frac{1}{m}$
Absorption coefficient, perivenous tissue[1]	μ_{apt}	$30\ \frac{1}{m}$
Speed of light[10]	c	$2.9 \times 10^8\ \frac{m}{s}$
Refractive index, vein[4]	n	1.4
Speed of light, vein[3]	c_*	$\frac{c}{n} = 2.1 \times 10^8\ \frac{m}{s}$
Gas Constant[10]	R	$8.314\ \frac{J}{K \cdot mol}$
Frequency Factor, blood[1]	A_b	$7.6 \times 10^{66}\ \frac{1}{s}$
Frequency Factor, vein / perivenous tissue[1]	A_t	$5.6 \times 10^{63}\ \frac{1}{s}$

Appendix B: Solution Strategy

Solver

- Within COMSOL, the UMFPACK solver was used.
- Automatic nonlinear solver, linear shape functions.
    - Quadratic shape functions caused graininess in final plot.

Time Step

- An initial time step of 1e-6s and a max time step of 0.01s were used.

Mesh Type

- An extremely fine free mesh was used produced a mesh with 5756 triangular elements.
- The mesh is finer near the optical fiber of the laser to accommodate for the high rate of change in light flux occurring in this region.

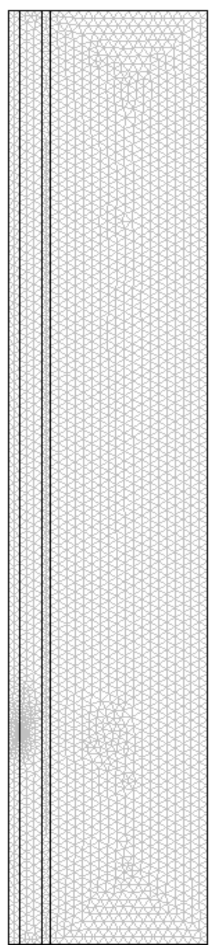

Figure 4. Mesh being used to solve model in COMSOL. Free mesh parameters and “finer” were used for this mesh. The mesh at the laser border was constrained to 5 elements due to a large gradient at that boundary.

Mesh Convergence

Using different mesh settings, we discovered that mesh convergence occurs approximately at 5756 elements (Fig. 5).

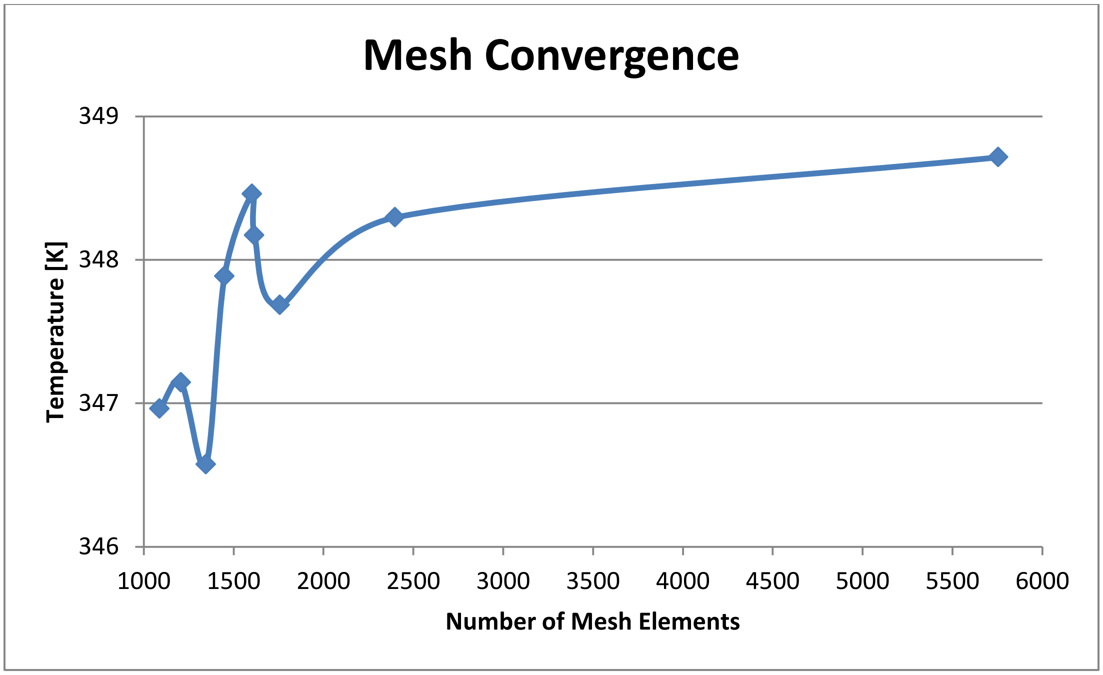

Figure 5. Mesh convergence based on temperature at the Tunica intima from t = 0 to 2s (coordinates: r = 1.674 mm, z = 16.524 mm). Mesh convergence has been achieved at 5756 elements.

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