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NON-LINEAR UTILITY PRICING AND TARGETING THE POOR

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NON-LINEAR UTILITY PRICING AND TARGETING THE POOR

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Abstract: This paper extends the standard analysis of optimal non-linear utility pricing with a single pricing schedule, to the operational and practical case where different schedules are implemented for different groups, with cross-subsidization across these groups. Apart from its theoretical interest, this joint treatment of non-linear pricing and cross-subsidization to achieve distributional objectives reveals features which are not present in the standard analysis. For example, it is found that under certain conditions the group which is being crosssubsidized should have an increasing block rate (or "lifeline") pricing schedule, while the group which is being cross-taxed should have a decreasing block rate (or "quantity discount") schedule. This puts into sharp relief the earlier "all or nothing" debate in the literature between these two structures. Our calculations also show that the gains from using two schedules instead of one can be substantial.

1. Introduction

With the accelerated move towards privatization of public utilities in developing countries, some old issues on public utility pricing and the poor have reemerged. Among the arguments for privatization are (a) increasing the efficiency management by insulting it from political pressures on day operations (eg hiring and firing decisions) and (b) greater economic efficiency by linking pricing to costs through the profit motive. And yet there are sufficient concerns about the possible impact of untrammeled market forces in these sensitive sectors, particularly about the distributional impact of pricing decisions, that privatization under the framework of regulation and oversight is an attractive alternative to completely free markets. The sorts of regulations that might be considered are to do with broad guidelines on pricing structure, and on crosssubsidization across groups.

These concerns are not of course new. Pricing structures designed to reflect quantity used and hence underlying income or wealth of the consumer are prevalent in both developed and developing countries. In developed countries, there has been much discussion of "lifeline rates" and other devices to give the poor lower prices for electricity (Diamopoulos, 1981). In developing countries, increasing block tariffs-i.e. "a price structure in which a commodity is priced up to a specified volume of use (block), then at a higher or several increasingly higher rates for additional blocks used" are common for water tariffs (Whittington,1992) as well as for other utilities. At the same time, differential price structures for rural versus urban areas are also found, justified on grounds of targeting predominantly poor populations.

Of course, there is a large literature on the shape of pricing schedules of regulated utilities. A recent review and exposition of non-linear pricing structures is available in Wilson (1993), which updates and extends the earlier synthesis by Brown and Sibley (1986). Earlier well known papers include those by Meyer(1975), Berg and Roth(1976), Roberts(1979),

Diamopoulos(1981) and Maskin and Riley(1984). This entire literature follows a common framework where there is a distribution of consumers differentiated by incomes or tastes who make choices on consuming the utility's output, and the utility then chooses the <u>single</u> pricing schedule to maximize some objective function-with differing weights given to distributional concerns in different studies. The literature shows that this single pricing schedule can have a range of characteristics, depending on the distribution of income, consumer tastes, and distributional concerns.

So much for single schedule analysis. And yet, in practice, it appears that there are different pricing schedules for different categories of consumers. And cross-subsidization across groups residential versus commercial, rural versus urban, government versus private, etc. - has been a staple descussion item in the policy arena. The object of this paper is to consider the twin issues of pricing structure and cross-subsidization jointly, in a framework where the distributional concerns are made explicit. It will be seen that this joint analysis highlights a number of features and raises a number of concerns not present in the conventional analysis. The optimal pricing structure within a group will be seen to be intimately connected to the structure of cross subsidization across groups, and to display surprising features.

The plan of this paper is as follows. Section 2 lays out the basic theoretical model of the optimal non-linear pricing with two distinguishable groups, and highlights the main qualitative features of the optimal pricing structures. It turns out that qualitative analysis cannot get us very far in gaining insights into the features of the optimal schedules in each of the two groups. Section 3 moves to a discussion of numerical solutions, and sets out the main results of the paper. Section 4 concludes the paper.

2. The model

The model developed here is an adaptation of the standard

model of non-linear pricing (see Wilson, 1993). We assume that the consumers can be divided by the utility into two mutually exclusive and exhaustive groups and that consumers are unable to switch between these groups. The groups are indexed 1 and 2. thought of rural/urban, can be as resident/commercial, etc.. Within groups consumers differ with respect to their income, denoted by real number y. This is distributed with continuous density function f_i with $f_i(y) \ge 0$, i =1,2, on closed interval $[y_0,y_t]$ where y_0 and y_t denote the lower and upper limits to the income distribution. Without loss of generality we assume that the mean of group 1's income exceeds the mean of group 2's income - we refer to group 1 as the richer group and group 2 as the poorer or "needy" group, even though there are income overlaps as between the two groups. population share of group 1 is Θ , and that of group 2 1- Θ .

We assume that there are two goods in the economy; a composite good, x, and the good, q (subject to nonlinear pricing) supplied by the utility. We assume that preferences are identical within groups but differ between them. Thus consumers who belong to group i, have identical concave utility function

$$(1) u_i = u_i(x_i, q_i),$$

where $u_i \in C^2$, $\partial u_i/\partial q_i > 0$, $\partial u_i/\partial x_i > 0 \ \forall \ q_i, x_i \geq 0$. We will further assume that q is a normal good. It is typical in nonlinear pricing literature to exclude income effects a priori. The usual motivation for ignoring income effects when constructing tariffs for services offered to household consumers is that their income elasticies are small and/or their residual income, x, is large in relation to their expenditures on the nonlinearly priced good or services, q. These assumptions cannot always justified and it will become clear that the properties of an optimal pricing schedule may crucially depend on income effects.

The pricing schedule is given by functions $R_i\left(q_i\right)$. If a consumer wishes to purchase an amount q_i then he or she must pay

an amount R_i to the utility. We assume that R_i is monotone and differentiable. The rate of change of total payment with respect to a change in quantity purhased $R_i{}'(q) = dR_i(q)/dq$ is called marginal price.

A consumer with income y chooses q so as to maximize (1) subject to

$$(2) xi + Ri(qi) = y.$$

In mathematical form this becomes the problem (P)

(3)
$$\max_{q_i, x_i} u_i(x_i, q_i)$$

subject to

$$q_i \ge 0$$

$$x_i + R_i(q_i) = y.$$

$$u_i(x_i, q_i) \ge u_i(y, 0)$$

The last constraint says that consumers have the option of leaving the market altogether and pay nothing to the public utility. Thus a participation constraint is required.

We assume that the utility (or the utility regulator) applies separate nonlinear pricing schedules to groups 1 and 2. The objective can be described by the utilitarian social welfare function

(4)
$$W = \int_{Y_0}^{Y_t} [\Theta u_1 f_1(y) + (1 - \Theta) u_2 f_2(y)] dy$$

The utility determines optimal schedules $R_i(q_i)$, i =1,2, by maximizing (4) subject to profit constraint

(5)
$$\int_{Y_0}^{Y_c} [\Theta \pi_1(q_1(y)) f_1(y) + (1-\Theta) \pi_2(q_2(y)) f_2(y)] dy \ge 0$$

where $\pi_1(.) = R_1(q_1(y)) - cq_1(y) - G$, $\pi_2(.) = R_2(q_2(y)) - cq_2(y) - G$, c is marginal cost (constant) and G is fixed cost). There is an additional constraint, that given the pricing schedule R_i , i=1,2, each consumer determines his or her consumption by solving

problem (P).

Next we formulate the nonlinear pricing problem of the public utility or government programme as an optimal control problem subject to state and control constraints. Let us define the utility of an y-consumer, when $q_i(y)$ and $x_i(y)$ are optimally chosen, as the maximum value function

(6)
$$v_{i}(y) = \max_{q_{i}, x_{i}} \{u_{i}(x_{i}, q_{i}) : x_{i} + R_{i}(q_{i}) - y = 0\}$$
$$q_{i}, x_{i}$$
$$= U_{i}(q_{i}(y), R_{i}(y), y).$$

By differentiating (6) with respect to y we obtain

(7)
$$dv_i/dy = [\partial U_i/\partial q_i][dq_i/dy]$$

$$+ [\partial U_i/\partial R_i][dR_i/dy] + \partial U_i/\partial y$$

Making use of the condition, implied by consumer's utility maximization

(8)
$$[\partial U_i/\partial q_i][dq_i/dy] + [\partial U_i/\partial R_i][dR_i/dy] = 0$$
 we have

(9)
$$dv_i/dy = \partial u_i/\partial x_i = \partial U_i/\partial y$$
.

This is also called a self-selection condition or incentive compatability condition.

As q is a normal good it is obvious that all consumers cannot be at a global maximum unless the following constraint holds

(10)
$$dq_i/dy \ge 0$$
 for all $y \in [y_0, y_t]$.

Defining the consumer's marginal rate of substitution between product and income

(11)
$$W_i(q(y), R(y), y) = -[\partial U_i/\partial q_i]/g_i(q, R, y)$$

where $g=-U_R$ is type y's marginal utility of income, and preferences are taken to satisfy the so called Mirrlees-Spence restriction that

(12)
$$\partial w_i/\partial y > 0$$
.

This means that increasing y increases the marginal willingness to pay for the utility output. In fact (12) is a sufficient assumption to ensure that (6) is equivalent to (9) and q increase with income y (See Mirrlees, 1976). We assume that condition (12) holds so that we can substitute the individuals utility maximization problem by weaker condition (9).

 $v_i(y)$ is continuous and strictly increasing in x_i . Thus (7) can be inverted so that $x_i(y) = h_i(v_i(y), q_i(y), y)$. Furthermore, we may eliminate $R_i(q)$ by the condition (2). Now $q_i(y)$ and $v_i(y)$ can be treated as state functions and $r_i(y) = dq_i/dy$ for all $y \in [y_0, y_t]$ as a control variable. Thus we first calculate an optimal allocation and then subsequently derive by condition (2) the marginal price schedule that implements this allocation. Now we can formulate the nonlinear pricing problem as an optimal control problem as follows

The nonlinear pricing problem (Q)

(13)
$$\max_{\mathbf{q}_{i}, \mathbf{v}_{i}} \mathbf{W} = \int_{0}^{\mathbf{y}_{t}} \left[\Theta \mathbf{v}_{1}(\mathbf{y}) f_{1}(\mathbf{y}) + (1 - \Theta) \mathbf{v}_{2}(\mathbf{y}) f_{2} d\mathbf{y} \right]$$

subject to state equations

$$dv_1/dy = (\partial U_1/\partial y) \quad \forall y \in [y_0, y_t]$$

$$dv_2/dy = (\partial U_2/\partial y) \quad \forall y \in [y_0, y_t]$$

$$dq_i/dy = r_i(y), \quad i = 1, 2, \quad \forall y \in [y_0, y_t]$$

and the constraints

(15)
$$\begin{aligned}
Y_{t} \\
\Theta[y-h_{1}(v_{1},q_{1})-cq_{1}(y)-G]f_{1}(y) \\
Y_{0} \\
+(1-\Theta)[y-h_{2}(v_{2},q_{2})-cq_{2}(y)-G]f_{2}(y)\}dy &\geq 0.
\end{aligned}$$

$$v_{i} \geq u(y,0), r_{i}(y) \geq 0$$

Furthermore in numerical solution we use the constraints that $q_1(y) \ge 0$ and $q_2(y) \ge 0 \ \forall y \in [y_0, y_f]$.

It is self-evident that if $v_1 = v_2$ and $f_1 = f_2$, the optimal policy yields $R(q) = R_1(q) = R_2(q)$. Thus we only have an

interesting situation when either preferences or distributions or both are different in different groups. It is also obvious that in general additional instruments can only improve the level of social welfare, since it is always open to the utility to set a common schedule.

Differentiating the Lagrangean of the problem (Q) with respect to $q_i(.)$ and $v_i(.)$ gives the first order conditions. When consumers with different incomes are not bunched together these conditions imply a pattern of marginal prices satisfying

(16)
$$R'_{1}[q_{1}] = c - \mu_{1}(y) [g_{1}(\partial w_{1}/\partial y)/\Theta \lambda f_{1}(y)]$$

$$R'_{2}[q_{2}] = c - \mu_{2}(y) [g_{2}(\partial w_{2}/\partial y)/(1-\Theta)\lambda f_{2}(y)]$$

where λ is the multiplier on the profit constraint and

(17)
$$\mu_{1}(y) = \int_{Q}^{Q} [(\lambda/g_{1}) - W'] [\exp - \int_{M}^{Y} (\partial U_{y}/\partial R_{1})/g_{1}] f_{1} dm$$

$$\mu_{2}(y) = \int_{Q}^{Y} (1 - \Theta) [(\lambda/g_{2}) - W'] [\exp - \int_{M}^{Y} (\partial U_{y}/\partial R_{2})/g_{2}] f_{2} dm$$

$$V_{0}$$

are the multipliers on the incentive compatibility condition from the two groups. In (16) we have used the fact that $\partial [U_y]/\partial q = gw_v$. (17) satisfies the transversality conditions

(18)
$$\mu_i(y_0) = \mu_i(y_t) = 0.$$

Using (16) and (17) it can be proved that

(19)
$$\mu_{i}(n) < 0 \quad \forall y \in [y_{0}, y_{t}].$$

We can see from (16) that the optimum distortion between marginal price and marginal cost in different groups depends upon several factors, μ , g, w_y , λ and f. A structure of marginal prices is based on three components. The first is the marginal cost. The second term arises purely from the incentive nature of the problem. We can interpret $\mu(y)$ as measure of the social value of providing a transfer to consumers with an income above y. λ is the social value of funds to the utility.

On the basis of equations (16) to (19) we can find some

general qualitative properties of schedules. It turns out that the qualitative properties of $R_i(q)$ are the same as in the single schedule model. We have the following list of properties:

- (P1) $R_i' \ge c$ for all q
- (P2) $R_{i}'=c$ at the upper end-point of the schedule
- (P3) $R_{i}'=c$ at the bottom end if there is no bunching at the lowest $income^{1}$
- (P4) R_{i} ' is strictly greater than c for all q_{i} such that $q_{i}(y_{0})$ $< q_{i} < q_{i}(y_{f})$.

The above well known results provide us with benchmark for group specific pricing schedules but do not tell us anything in detail about the shapes of these schedules and how they depend upon parameters such as group mean incomes of group inequalities. In order to address the issues the literature and policymakers are grappling with, we have to move to numerical simulations.

3. Numerical simulations²

We can provide better understanding of the form of optimal schedules through numerical simulations. The calculations were carried out for the following utility functions

(20)
$$u_i(x_i, q_i) = (1-\alpha_i) \ln(x_i+\epsilon) + \alpha_i \ln(q_i+\epsilon) \quad i=1, 2$$

(20')
$$u_i(x_i, q_i) = -[(1-\alpha_i)/(x_i+\epsilon)]-[\alpha_i/(q_i+\epsilon)]$$
 $i=1,2$

where α is the weight on q and $(1-\alpha)$ the weight on x. ϵ is a small positive constant to assure that (20) is well defined for

¹If there is bunching, then the consumers on the lower end do not pay marginal prices which are equal to marginal costs.

² The nonlinear problem was solved by the FORTRAN program MISER3. This program has been developed to solve a general class of optimal control problem with constraints. The constraints are allowed to be of equality as well inequality type. The program is based on the concept of the control parametrization technique: to transform an optimal control problem to a mathematical programming problem. The detailed usage of MISER3 is described in the User's guide (see Jennings, Fisher, Teo and Goh, 1990).

all $q_i, x_i \geq 0$. To focus on income distribution, we assume that preferences are identical across the two groups. Incomes in different groups are taken to be distributed according to a lognormal distribution (μ_i, σ_i) . The objective function of utility is the concave transformation of each consumer's utility in different groups, reflecting society's distributional preferences:

(23)
$$W = -(1/\beta)e^{-\beta v}$$

where ß expresses the degree of inequality aversion. The results are given for two different forms of the objective function: & = 0 (in this case we define & = $\verb"v"$) corresponding to the classical utilitarian case and & = $\verb"5"$. The average (marginal) cost of production $\verb"c"$ = 1 and fixed cost is zero.

Figure 1 presents our base run (we set ϵ at 0.005) throughout the paper. The degree of inequality aversion ϵ is set at zero and the expenditure share of the utility commodity, α , is set at 10%. We consider two specifications of population shares, Θ = 0.3 and Θ = 0.7, which capture "relatively large" and "relatively small" numbers of the poorer group, respectively.

With Θ = 0.3, Figure 1 demonstrates a pattern of marginal prices which decline with income over most of the range (see north-east panel). Thus, optimally, we have a quantity discount rather than an increasing block rate structure. Moreover, notice that with Θ = 0.7 the structure in group 2, the poorer group, is very different. Now for most of the income range the marginal price increases with income (and hence quantity premium), although of course at the very top it falls to marginal cost as by the qualitative result in (P2). Thus we see both quantity discount (for the rich group) increasing block rates (for the poor group).

Why does such a pattern occur? One clue is to be found in the extent of cross-subsidization. With Θ = 0.7 this runs to around 60% of the expenditure by the poorer groups on the utility's output. Viewing this as a single group, this is a large "negative profit requirement". This is analogue to the revenue

requirement in models of optimal income taxation, and it is argued in Immonen et al. (1998) that when revenue requirement is negative and large i.e. optimality calls for large subsidy on distributional grounds, the pattern of taxation will involve a large subsidy to the poorest in the poorer group, clawed back by increasing marginal tax rates in the same group. The analog in the case of utility pricing is an increasing marginal price structure over a large portion of the income range. When the proportion of the poorer group in the population is large, the subsidy per capita is smaller and hence this effect is less likely to arise.

Some sense of the analytical basis of the numerical results can be found from (16). From it we know that the variation of the optimal marginal prices with level of purchases is a complex matter. One consideration, however, is the variation of μ_i with y. It is straightforward to show that μ_i starts and finishes with a value of zero (the transversality condition) and has an U-shape in-between. Intuitively, μ_i measures the social value of giving a direct poll subsidy to consumer with an income above y. At low level of income μ tends to decrease with y and it reaches the minimum point at which λ = W'g. When the profit requirement is low, so is λ , the social value of funds to the utility. Then a reduction in the profit requirement can be expected to shift the point at which $\mu(y)$ has a minimum to the right. Thus $\mu(y)$ will continue to decrease further into the distribution than would otherwise be the case.

Figure 2 shows the effect of going to the CES utility function (20') - this is essentially the case with elasticity of substitution between the utility produced commodity and the other good set at half rather than 1. The population share of group 1 is set at 30% once again. It is seen that there is once again an increasing block rate structure in marginal prices over much of income range - it is not till almost 80% the population is crossed that the marginal price starts declining back to marginal cost, as it must. There is also an increasing marginal price phase in the pricing structure for the richer group, but this is much smaller. Figure 3 and 4 conduct sensitivity

exercises with respect to inequality aversion and the mean differences between the groups. The results are as expected. Increased inequality aversion raises the marginal price schedule for the better off group and lowers it for the poorer group, as does greater inequality between the two groups.

Our calculations also allow us to gauge the relative gains from using two schedules versus being restricted to only one. The gain in the base case (Fig 5) with Θ = 0.3, is 1.5% - the gain of the poorer group is 5%. In the CES-case (Fig 6) with Θ = 0.3 the gain of the poorer group is 8%. Although not shown in the figures as Θ increases to 0.7, the average gain reaches 4%. With Θ = 0.7 and α = 0.3, the gain rises to 15%.

4. Conclusions

We have developed a model in which the utility implements different price schedules on different groups of the population. These schemes reflect both consumers special circumstances and incomes. The available analytical results are limited, and we have to employ numerical simulations. The simulations suggest that the gains from the appropriate use of group tariffs can be substantial. They also suggest unexpected pattern of marginal prices, including combinations where the poorer group gets "lifeline rates" while the richer group gets "quantity premium". This indicates that the "all or nothing" debate between these two structures may need to become much more nuanced in the future.

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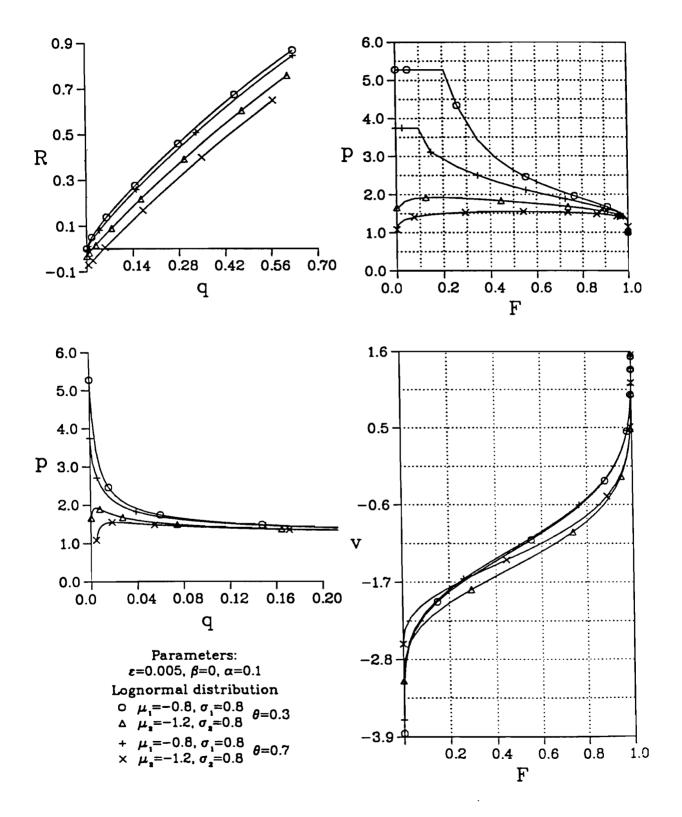


Figure 1

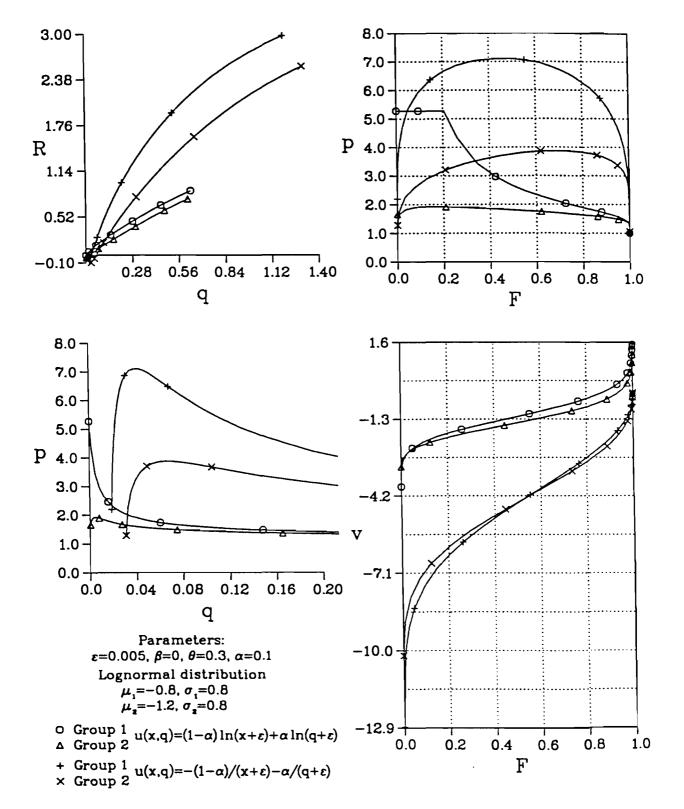


Figure 2

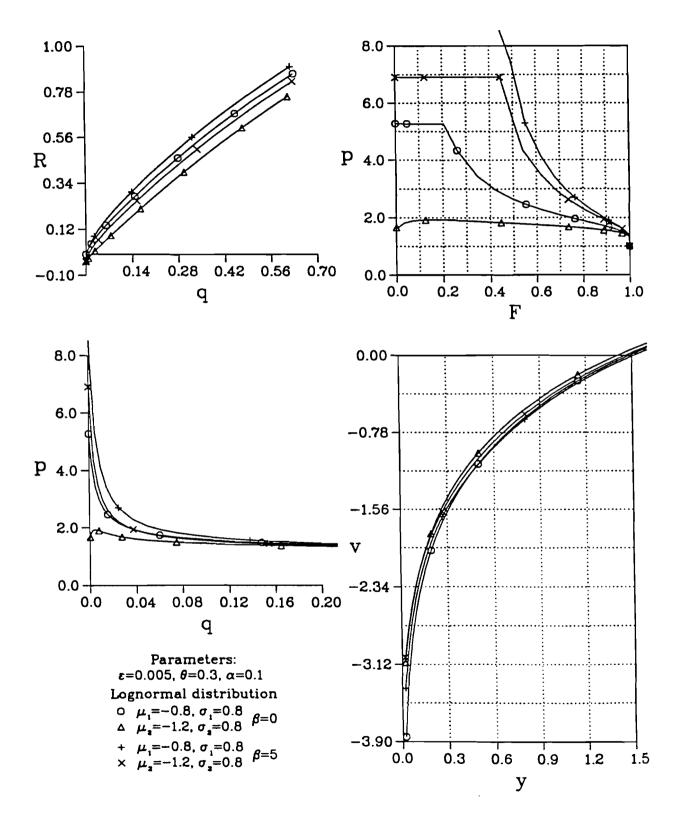


Figure 3

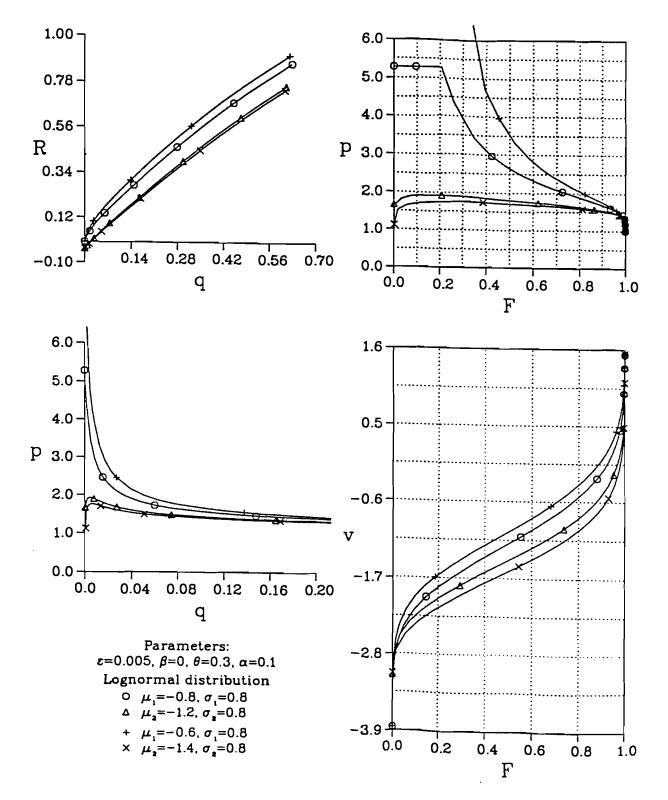


Figure 4

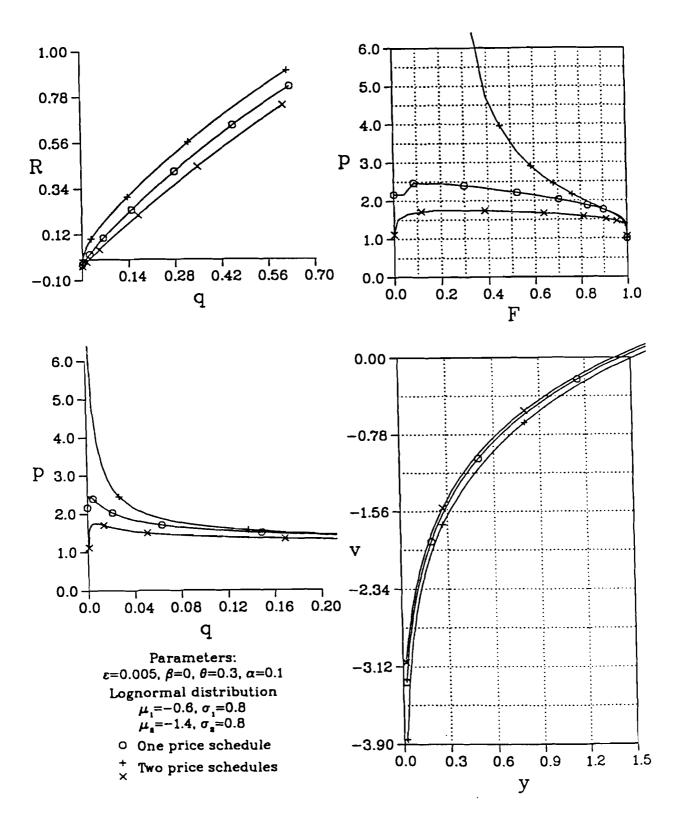


Figure 5

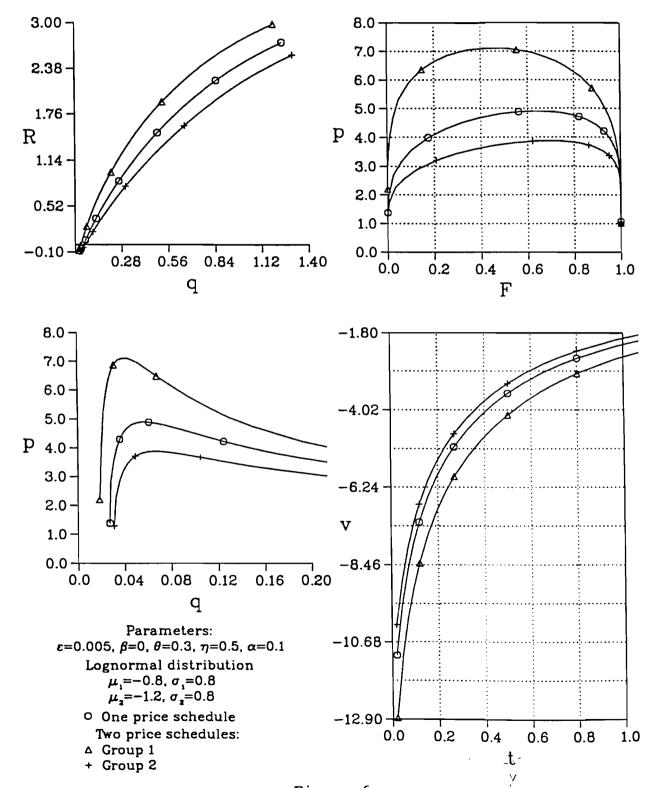


Figure 6

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