Water vapor transport across an arid sand surface - non-linear thermal coupling, wind-driven pore advection, subsurface waves, and exchange with the atmospheric boundary layer

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This document provides a nomenclature of variables, their typical units, subscripts and superscripts, and abbreviations. The symbol (-) indicates a dimensionless quantity.

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Nomenclature - Roman

$\bar{Y}$	mean water vapor mass fraction in Eq. (47) (-)
$\bar{\ }$	denotes a time-averaged value
$\breve{Y}$	local water vapor mass fraction around a spherical grain (Appendix G) (-)
$\dot{q}''_{\mathrm{rad}}$	net radiation energy flux (W/m^2)
$\dot{q}''_{\mathrm{wind}}$	wind-driven convective thermal energy flux (W/m^2)
$\dot{W}''$	vapor mass flux at radius r (kg/m^2.s)
$\dot{W}''_s$	water vapor flux at the grain surface (kg/m^2.s)
$\dot{W}''_{\mathrm{DIFF}}$	vapor mass flux in the diffusion limit around a grain (kg/m^2.s)
$\dot{W}''_{\mathrm{HKS}}$	vapor flux at the grain surface from HKS theory (kg/m^2.s)
ℓ^*	dimensionless characteristic depth of evanescent Y-wave decay (-)
ℓ_0	characteristic water film thickness (nm)
ℓ_c	sensor characteristic length (mm)
ℓ_w	water film thickness (nm)
$\hat{\mathbf{X}}$	unit vector along the wind (-)
$\hat{\mathbf{Y}}$	unit vector across the wind (-)
$\hat{R}$	fundamental gas constant $\simeq 8.314$ J/mole.K
$\Im$	denotes the imaginary part
$\imath$	$\imath^2 = -1$ (-)
$\mathbb{L}$	Lewis number $\mathbb{L} \equiv D_{\mathrm{st}}/(\varpi\alpha)$ (-)
$\mathbb{R}$	density ratio $\mathbb{R} \equiv \rho_p\nu/[\rho_{\mathrm{st}}(1-\nu)]$ (-)
$\mathbb{U}_M$	measured unsteady term associated with vapor mass fraction, Eq. (29) (-)
$\mathbb{U}_T$	measured unsteady term associated with temperature, Eq. (29) (-)
$\mathcal{A}$	shear stress coefficient, analogous to $\mathcal{C}$ (-)
$\mathcal{B}$	shear stress coefficient, analogous to $\mathcal{D}$ (-)
$\mathcal{C}$	pressure coefficient in Eq. (35) (-)
$\mathcal{D}$	pressure coefficient in Eq. (35) (-)
$\mathcal{X}$	distance along the wind direction $\hat{\mathbf{X}}$ (m)
$\mathcal{Y}$	distance perpendicular to the wind direction $\hat{\mathbf{Y}}$ (m)
$\Re$	denotes the real part
ABL	atmospheric boundary layer
BC	boundary condition
cRIO	compact Reconfigurable Input Output device
$\mathrm{Da_{II}}$	Damköhler's "second ratio" $\mathrm{Da_{II}} \equiv \tau_{\mathrm{DIFF}}/\tau_{\mathrm{HKS}} = k_m d/(2\rho D)$ (-)
DFT	discrete Fourier transform
FPGA	Field Programmable Gate Array
$\mathrm{Gr}_L^{1/2}$	square root of a Grashof number based on L and buoyancy-driven speed $\mathrm{Gr}_L^{1/2} \equiv \rho[gL(T_s - T_T)/T_T]^{1/2}L/\mu$ (-)
HKS	Hertz-Knudsen-Schrage
ML	microlysimeter
MW	molar mass, a.k.a. molecular weight (kg/mole)
NRW	non-rainfall water
ODE	ordinary differential equation of a single independent variable
PCB	printed circuit board
PDE	partial differential equation of several independent variables
Pr_t	turbulent Prandtl number (-)
Ra	Rayleigh number for natural convection within sand (-)
Re_X	Reynolds number based on wind speed and distance $\mathcal{X}$ from the dune leading edge along $\hat{\mathbf{X}}$ (-)
$\mathrm{Re}_{X_{\mathrm{crit}}}$	critical value of Re_X for transition to turbulence (-)
RH	relative humidity (-)

RH_c	relative humidity in the calibration chamber (-)
RH_e	relative humidity at equilibrium (-)
Sc	Schmidt number $\mathrm{Sc} \equiv (\mu/\rho)/D$ (-)
Sc_t	turbulent Schmidt number (-)
St_X	mass transfer Stanton number, Eq. (H5) (-)
TBL	turbulent boundary layer
TDR	time-domain reflectometry
$\tilde{}$	tilde denotes a Fourier-transformed variable
*	denotes a dimensionless quantity
$_s$	surface subscript
$_\ell$	subscript denotes the upper edge of the turbulent mass transfer boundary layer
a	half guard width of the capacitance probe (mm)
a_0	mean δp^* in Eq. (38) (-)
A_c	coefficient in the characteristic Eq. (48) (-)
a_i	Fourier series coefficient in Eq. (38) (-), $i = 1, .., 2^{N+1}$
a_m	coefficient in Eq. (57) (-)
A_s	coefficient in Appendix E (-)
a_T	Soret coefficient (-)
a_{h_D}	empirical constant for unstable thermal transport during the day, Eq. (D2) (-)
a_{h_N}	empirical constant for stable thermal transport at night in Eq. (D2) (-)
a_{m_D}	empirical cst for unstable momentum transport during the day, Eq. (56) (-)
a_{m_N}	empirical constant for stable momentum transport at night in Eq. (56) (-)
b	distance of outermost sensor side to probe vertical centerline (mm)
B_c	coefficient in the characteristic Eq. (48) (-)
b_i	Fourier series coefficient in Eq. (38) (-), $i = 1, .., 2^{N+1}$
B_s	coefficient in Appendix E (-)
c	distance of innermost sensor side to probe vertical centerline (mm)
C'_c	coefficient in the characteristic Eq. (52) (-)
C_0	sensor capacitance in air (fF)
C_c	coefficient in the characteristic Eq. (48) (-)
c_p	specific heat of air (by mass) at constant pressure (J/kg.K)
C_s	coefficient in Appendix E (-)
D	water vapor diffusion coefficient (m^2/s)
d	grain diameter (μm)
$d_{2,3}$	Sauter mean diameter (Appendix B) (μm)
D_{eff}	effective water vapor diffusion coefficient (m^2/s)
D_{st}	water vapor diffusion coefficient at T_{st} and p_{st} (m^2/s)
$d_{p,q}$	particle diameter moment (Eq. B4) (μm)
E_i	coefficient of species i, $i = 1, 2$ (-), see Appendix E
f	oscillator (a.k.a. clock) frequency (kHz), or evanescent wave frequency (Hz)
f^*	dimensionless evanescent wave frequency $= fJ$, Eq. (47) (-)
f''	function in Eq. (14) (-)
f'	function in Eq. (14) (-)
f_1^*	dimensionless reference evanescent wave frequency (-)
f_M	normalized particle-size-distribution by mass (Appendix B, 1/μm)
f_N	normalized particle-size-distribution by number (Appendix B, 1/μm)
f_p	pressure derivative function in Eq. (24) and Appendix C (-)
f_T	temperature derivative function in Eq. (24) and Appendix C (-)
f_Y	mass fraction derivative function in Eq. (24) and Appendix C (-)
g	gravitational acceleration ($\mathrm{m/s}^2$)
H	effective Hamaker coefficient (J)

h	ripple ondulation (cm)
h_0	amplitude of surface ondulation (cm)
J	one day = 24 hr
K	sand permeability (m^2)
k^*	dimensionless evanescent wavenumber (-), Eq. (47)
$k^{*\pm}$	dimensionless evanescent wavenumber in Eq. (50) (-)
K_e''	imaginary part of the effective dielectric constant (-)
K_e'	real part of the effective dielectric constant (-)
k_i	series coefficient of K_p in Eq. (13), $i = 0, .., 3$ (-)
$k_i^{*\pm}$	imaginary part of $k^{*\pm}$ (-)
k_m	rate constant of first-order evaporation reaction ($kg/m^2.s$)
K_p	equivalent material dielectric constant of moist sand (-)
$k_r^{*\pm}$	real part of $k^{*\pm}$ (-)
k_s	effective sand conductivity (W/m.K)
k_T	Soret factor in Eq. (E1) (-)
L	Monin-Obukhov length (m)
L^*	dimensionless Monin-Obukhov length $= L/\sqrt{\alpha J}$ (-)
M^*	ratio of molar masses of air and water $M^* \equiv \mathrm{MW_{air}}/\mathrm{MW_{H_2O}} \simeq 1.61$ (-)
M_i	coefficient of species i, $i = 1, 2$ (-), see Appendix E
m_i	molecular mass of species i, $i = 1, 2$ (kg)
N	there are 2^N cells in the $\hat{\mathbf{X}}$ and $\hat{\mathbf{Y}}$ directions of the central domain (-)
n_d	temperature-dependence-diffusion exponent in Eq. (22) (-)
n_i	number density of species i, $i = 1, 2$ ($1/m^3$)
p	air pressure (Pa)
p_a	ambient pressure (Pa)
$p_{\mathrm{H_2O}}$	partial pressure of water vapor (Pa)
p_{sat}	saturation pressure (Pa)
p_{st}	standard pressure $= 1.013 \times 10^5$ Pa
p_{A_0}	characteristic pressure in Antoine's law (Pa)
q^*	dimensionless heat flux $\equiv \dot{q}''_{\mathrm{wind}}/[\rho_{\mathrm{st}} c_p T_{\mathrm{st}} (\alpha/J)^{1/2}]$ (-)
Q_i	coefficient of species i, $i = 1, 2$ (-), see Appendix E
Q_{12}	coefficient in Eq. (E2) (-)
R	circular electric field line radius (mm)
r	radial distance from the center of an isolated spherical grain (m)
S_i	coefficient of species i, $i = 1, 2$ (-), see Appendix E
T	temperature (°C or °K)
t	time (s or day)
t^*	dimensionless time $\equiv t/J$ (-)
t_0^*	dimensionless time around the onset of aeolian transport (-)
t_1^*	dimensionless time around the end of aeolian transport (-)
T_B	temperature at the bottom thermometer on weather station (°C or °K)
T_c	calibration temperature (°K)
T_T	temperature at the top thermometer on weather station (°C or °K)
$T_{1/n}$	temperature excursion in Eq. (10) (°C)
$t_{1/n}$	phase lead time in Eq. (11) (s)
T_ℓ	temperature of the liquid film on a grain (°K)
T_∞	mean temperature in Eq. (10) (°K)
T_{st}	standard temperature = 298.15 K
T_{A_0}	reference temperature in Antoine's law (°K)
T_{A_a}	characteristic temperature in Antoine's law (°K)
T_i	temperature excursion in Eq. (10), $i = 1, .., 4$ (°C)

t_i	phase lead time in Eq. (11), $i = 1, .., 4$ (s)
T_v	vapor phase temperature around grains (°K)
U	wind speed (m/s)
u	seepage interstitial velocity (μm/s)
u^*	dimensionless seepage interstitial velocity $\equiv u\sqrt{J/\alpha}$ (-)
u^*_s	dimensionless seepage interstitial velocity on the surface (-)
U_∞	wind speed far above the sand surface (a.k.a. bulk wind speed) (m/s)
V	guard voltage amplitude in sand (V)
v^*_s	dimensionless seepage superficial velocity on the surface, Eq. (31) (-)
V_0	guard voltage amplitude in air (V)
v_ℓ	vertical velocity on top of the turbulent mass transfer boundary layer (m/s)
v_τ	shear velocity (m/s)
v^*_τ	dimensionless shear velocity $= v_\tau\sqrt{J/\alpha}$ (-)
W	vertical height (or spatial resolution) of any capacitance probe sensor (mm)
x	downward distance from the sand surface (mm)
x^*	dimensionless depth $\equiv x/\sqrt{\alpha J}$ (-)
x_c	distance of circular electric field line center to probe vertical centerline (mm)
x_i	downward distance of sensor i from the sand surface (mm), $i = 1, .., 15$
Y	water vapor mass fraction (-)
Y_a	ambient water vapor mass fraction (-)
Y_e	water vapor mass fraction at equilibrium (-)
Y_i	initial water vapor mass fraction in the bulk (-)
Y_i	water vapor mass fraction (-) at sensor i, $i = 1, .., 15$
Y_s	water vapor mass fraction at the surface (-)
Y_u	upper envelope of the Y signal during aeolian transport (-)
Y_∞	water vapor mass fraction far above the sand surface (-)
$Y_{\min}$	smallest detected water vapor mass fraction (-)
Y_{sat}	saturated water vapor mass fraction (-)
Y_l	lower envelope of the Y signal during aeolian transport (-)
Z	sensor impedance in sand (MΩ)
z	altitude above the sand or desert surface; dune topographical elevation (m)
Z_0	sensor impedance in air (MΩ)
z_0	turbulence hydrodynamic roughness (μm)
z_B	lowest instrument altitude on the weather station (m)
z_T	highest instrument altitude on the weather station (m)
z_ℓ	altitude on top of the turbulent mass transfer boundary layer (m)

Nomenclature - Greek

α	effective sand thermal diffusivity (m^2/s)
α_m	hybrid thermal diffusivity $\equiv k_s/(\rho c_p)$ (m^2/s)
$\chi_{\rm air}$	mole fraction of dry air
$\chi_{\rm H_2O}$	mole fraction of water vapor
$\delta\tau^*$	dimensionless evanescent wave time scale (-)
$\delta\tilde{p}^*$	Fourier transform of δp^* (in 2D, m^2; in 1D, m)
$\delta\tilde{p}^*_0$	Fourier transform of δp^* at cell $i = 0$ (1D Fourier transform to find coefficients in Eq. (38), m)
$\delta\tilde{p}^*_i$	Fourier transform of δp^* at cell, $i = 1, .., 2^{N+1}$ (Eq. 38, m)
$\delta\tilde{z}$	Fourier transform of dune topographical elevation z (in 2D, m^3; in 1D, m^2)
δp^*	dimensionless pressure excursion (-)
ΔT	temperature change from bottom to top thermometers on weather station (°K)
δt	time scale (s)
Δx	apparent stratum wavelength from capacitance probe (cm)
δY	amplitude of evanescent wave of water vapor mass fraction in Eq. (47) (-)
ΔY_a	ambient water vapor mass fraction excursion from bottom to top hygrometers on weather station (-)
δ_m	mass transfer thickness of the TBL (m)
$\dot{\Psi}''$	diffusive vapor flux through the ABL ($kg/m^2.s$)
$\dot{\Psi}''_\ell$	diffusive vapor flux on top of the TBL ($kg/m^2.s$)
$\dot{\Psi}''_\infty$	diffusive vapor flux far above the sand surface ($kg/m^2.s$)
$\dot{\Psi}''_s$	diffusive vapor flux at the surface ($kg/m^2.s$)
$\dot{\Sigma}''$	net vapor flux into the TBL at the surface ($kg/m^2.s$)
$\dot{\Sigma}''_\infty$	net upward vapor flux through top of the TBL ($kg/m^2.s$)
ϵ_0	dielectric permittivity of free space $\simeq 8.854$ fF/mm
γ_0	typical scale of u_s/τ_K used to render colors in Fig. 8 (m/s^2)
κ	von Kàrmàn constant (-)
κ_c	accommodation coefficient of condensation (-)
κ_e	accommodation coefficient of evaporation (-)
λ_0	single-mode wavelength (m)
λ_i	wavelength of harmonic i in Eq. (38), $i = 1, .., 2^{N+1}$ (m)
μ	air dynamic viscosity (kg/m.s)
ν	solid volume fraction of sand (-)
Ω	water mass fraction adsorbed on grains (-)
Ω_d	adsorbed water mass fraction on grains of class size d (-)
Ω_e	adsorbed water mass fraction at equilibrium (-)
Ω_{1_d}	characteristic water mass fraction adsorbed on grains of size class d (-)
Ω_1	characteristic water mass fraction adsorbed (-)
$\Omega_{d,e}$	adsorbed water mass fraction at equilibrium on grains of class size d (-)
Ω_{p_0}	equilibrium absorbed water mass fraction on a typical pharmaceutical powder as RH $\to 0$ (-)
Ω_{p_1}	isotherm slope for a typical pharmaceutical powder (-)
ω_{X_0}	single-mode wavenumber along $\hat{\mathbf{X}}$ (1/m)
ω_{X_i}	wavenumber of mode i along $\hat{\mathbf{X}}$, $i = 1, .., 2^{N+1}$ (1/m)
ω_X	wavenumber along $\hat{\mathbf{X}}$ (1/m)
ω_Y	wavenumber along $\hat{\mathbf{Y}}$ (1/m)
ϕ_0	phase lag of pressure excursions from topographical variations (rad)
ϕ_h	Monin-Obukhov correction function for heat transfer (-)
ϕ_m	Monin-Obukhov correction function for momentum transport (-)
ϕ_Y	Monin-Obukhov correction function for mass transfer (-)

Φ_{2_h}	integral function in Eq. (D5) (-)
Ψ_s^*	dimensionless vapor diffusion flux $\dot{\Psi}''$ through the surface $\equiv \dot{\Psi}''/[\rho_{\rm st}\sqrt{\alpha/J}]$ (-)
ρ	moist air density (kg/m^3)
ρ_b	bulk density of the sand bed (kg/m^3)
ρ_p	material density of sand grains (kg/m^3)
ρ_s	moist air density at the surface(kg/m^3)
ρ_w	liquid water density (kg/m^3)
$\rho_{\rm H_2O}$	partial density of water vapor (kg/m^3)
$\rho_{\rm st}$	air density at $T_{\rm st}$ and $p_{\rm st}$ (kg/m^3)
Σ^*	dimensionless vapor flux into the ABL (Eq. 54) (-)
Σ^*_∞	dimensionless vapor flux aloft (Eq. 57) (-)
σ_i	molecular diameter of species i, $i = 1, 2$ (Å)
σ_{12}	mean molecular diameter of species 1 and 2 (Å)
$\tan\varphi$	"loss tangent" $\tan\varphi \equiv K_e''/K_e'$ (-)
$\tan\varphi_0$	characteristic "loss tangent" (-)
τ	characteristic time of return to equilibrium (s)
τ^*	dimensionless characteristic time of return to equilibrium $= \tau/J$ (-)
$\tau^*_{\rm HKS}$	τ^* in the HKS theory (-)
τ^*_1	dimensionless evanescent wave reference time (-)
τ_d	characteristic time of return to equilibrium of grains of class size d (s)
τ_K	characteristic time $\tau_K \equiv K\rho(v_\tau/U)^2/[\mu(1-\nu)]$ defined in Eq. (36) (μs)
$\tau_{\rm DIFF}$	characteristic time in the diffusion limit around a grain (μs)
θ	bearing of wind direction $\hat{\mathbf{X}}$, clockwise from North as viewed from above, see Fig. 8 (rad)
θ_w	water volume fraction in the modeling framework of Shao, et al. (2021) (-)
$\theta_{w_{\rm min}}$	smallest water volume fraction detected by Shao, et al. (2021) (-)
ϖ	tortuosity (-)
ξ	$\xi \equiv \ln z$ ($\ln$ m)