SCRAMBLE VERSUS CONTEST COMPETITION IN A TWO PATCH SYSTEM

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Abstract: In this paper, we study a Susceptible-Infected-Susceptible (*SIS*) discrete-time model with two competing strains and distinct demographic dynamics. We use two different recruitment functions to model the population demography. Discrete models are capable of generating complex dynamics. Analytic and numerical methods are used in this study to separate the impact of demography on the epidemic process. The key issues addressed in this work are those of coexistence and/or competitive exclusion of competing strains in a population with complex dynamics. The principle of competitive exclusion is the most prevalent under the assumptions of this model.

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Scramble Versus Contest Competition In a Two Patch System

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Abstract

The population dynamics in two patch discrete-time systems are different if they are modeled by the Logistic and Verhulst equations. First, we analyze one - patch systems modeled with each of the equations. Then we study two patch systems, where the local dynamics between the two patches are coupled with Logistic equations regulated by the scramble competition. We also compare this with the Verhulst equation where the behavior is regulated by contest competition. It is known that fractal basin boundaries of attractors could occur in two-patch systems regulated by the scramble competition. We show that such complex dynamics do not occur in contest competition with dispersion between patches.[5]

1 Introduction

A patch or habitat is a continuous area of space with all the necessary resources for the persistence of a local population and separated by unsuitable habitat from other patches (at any given time, a patch may be occupied or empty)[3]. In this paper we consider a single species in two patch system that are coupled with dispersion.

We study the dynamics of populations in the two patches. Our main interest are two-patch models with contest and scramble competition. We studied these two cases and found that their dynamics are very different; the first one is modeled by a Verhulst equation and we observed that the

population is maintained; the second one is modeled by a Logistic equations and the population is not maintained: large population densities lead to very low densities in the next generation.

This work is organized as follows. In section [2] we consider single patch models whose dynamics are regulated by time-discrete Logistic and Verhulst equations, respectively. In section [3] we study the dynamics of a population en two patches where local dynamics and dispersion are considered; specifically, we use a Verhulst equation (which resembles contest competion) and follow closely A.Hastings' work ([6]), which uses a Logistic equation (scramble competition).
The results and discussion are presented in section [5]. Section [4] is an overview of A. Hastings'work [4]. The conclusions anf future work are presented in section [5].

2 Models and Analysis for a Single Patch

First, we analyze the dynamics of a single patch discrete - time system with the logistic equation. Then we use Verhulst equation to do the same analysis.

2.1 CASE 1: Logistic Model

There are many mathematical models that predict the behavior of populations. We now discuss the discrete - time *logistic model of population growth.* Let x_n =population size of the species at generation n. The the discrete - time logistc model is:

$$x_{n+1} = f(x_n) \tag{1}$$

where

$$f(x_n) = rx_n(1 - x_n) \tag{2}$$

is named the logistic function.[3]

Here r is a positive constant that depends on ecological conditions and is a measure of the of population growth.

If $x_n = 0$ (i.e., no individual is present), then $x_{n+1} = 0$. If $x_n = 1$, then $x_{n+1} = 0$. Thus, to understand the growth and decline of the population under this model, we must iterate the logistic function $f(x) = rx(1-x)$. We will show that this simple change gives rise to a very rich mathematical theory [3,10].

In practical applications, the logistic equation requires x_n never to exceed 1. Otherwise subsequent iterations of some points diverge towards $(-\infty)$ (wich means the population becomes extinct).[3]

The logistic function has a fixed point at x when

$$f_r(x) = x. \tag{3}$$

The solutions for equation (3) are: $x_\infty = 0$ and $x_\infty = 1 - \frac{1}{r}$.

If $r \leq 1$, then f has only one non negative fixed point. If $r > 1$, then f has two fixed points $0, (1 - \frac{1}{r})$

The stability of the fixed points is analyzed through the following theorem from [4,7,8].

Theorem 2.1. *Let $f : I \to I$ be a differentiable function at a fixed point x_∞ in the interior of I.*

1) If $|f'(x_\infty)| < 1$, then x_∞ is an attracting fixed point (asymptotically stable).

2) If $|f'(x_\infty)| > 1$, then x_∞ is a repelling fixed point (unstable).

3) If $|f'(x_\infty)| = 1$, then x_∞ can be stable, unstable or neither.

In reference to the theorem above, we have that, $x_\infty = 0$ is stable if $r < 1$, in other case is unstable.

$x_\infty = 1 - \frac{1}{r}$ is asymtotically stable if $1 < r < 3$ and unstable for $r > 3$.

The stability for fixed point $x_\infty = 1 - \frac{1}{r}$ changes in $r = 3$. In this case we can say that the fixed point undergoes a flip-bifurcation at $r = 3$ does indeed spawn a 2-cycle. We solve the equation

$$f^2(x) = x. \tag{4}$$

we obtain that

$$x_1 = \frac{(r+1) + \sqrt{(r-3)(r+1)}}{2r^2}$$

$$x_2 = \frac{(r+1) - \sqrt{(r-3)(r+1)}}{2r^2}$$

are the points period-2. They are real numbers and positive whenever $r > 3$

We examine the stability of a 2-cycle through the following theorem, from [4,7,8].

Theorem 2.2. *Let $f : I \to I$ be a differentiable function at a period-n point x_∞ in the interior of I.*

1) If $|(f^n)'(x_\infty)| < 1$, then x_∞ is an attracting period-n point (asymptotically stable).

2) If $|(f^n)'(x_\infty)| > 1$, then x_∞ is a repelling period-n point (unstable).

3) If $|(f^n)'(x_\infty)| = 1$, then x_∞ can be stable, unstable or neither.

We have $|(f^2)'(p)| < 1$ hence $|4 + 2r - r^2| < 1$ which implies that $3 < r < 1 + \sqrt{6}$.

In reference to the theorem above, we have that x_1 and x_2 are period -2 stables points, if $3 < r < 1 + \sqrt{6}$. If $r > 1 + \sqrt{6}$, then the period-2 cycle becomes unstable.

As r is increased even further, x_1 and x_2 in turn lose their property of stability to other states (with periods 4,8,etc). This period - doubling phenomenon continues until $r = 3.83$ when periodic solutions whose periods are not powers of 2 begin to appear, but these solutions are unstable. For $r > 3.83$ there is a periodic solution of period K for every integer K, but different initial values give different solutions. There also solutions whose behavior is apparently random; such solutions are called chaotic [2].

2.2 Case 2. Verhulst equation

Now we study a single patch that is modeled in accordance with the Verhulst equation.

Let x_n = population size of the species at generation n, the Verhulst equation is given by

$$x_{n+1} = \frac{rx_n}{x_n + A} \tag{5}$$

where r is the carrying capity of population, and $A = \frac{r}{g(0)}$ is the growth speed ratio Here the reproduction curve is:

$$f(x) = \frac{rx}{x + A} \tag{6}$$

The fixed points are $x_\infty = 0$ and $x_\infty = r - A$.
By the theorem (2.1), we are able examine to stability of these fixed points. If $r < A$, then f has only one fixed point $x = 0$ and is asymtotically stable. Hence, every solution tends to the zero under n iterations [2].

If $r > A$, then f has two fixed points $x_\infty = 0$ and $x_\infty = r - A$.
By the theorem (2.1) , $x_\infty = 0$ is a repelling fixed point (unstable) and $x_\infty = (r - A)$ is an attracting a fixed point (asymtotically stable).

We can conclude that for $r > A$, every positive solution, regardless of initial value x_0 tends to the limit $x_\infty = r - A$. In the Verhulst equation unlike the Logistic equation there is no possibility of period - doubling or chaotic behavior, or even of stable oscillations.[3]

3 Models and Analysis for a Two Patch

In this section, we consider a two - patch single species population model with dispersal, where the local dynamics is given by the Logistic and the Verhulst equation.

3.1 Logistic Equation System

To understand the dynamics between two patches governed by the logistic equation we review the article of [Alan Hastings [6]]. In this article he analyzed the dynamics with dispersion given by the logistic model. (In other words he study the dynamics where the two patchs (patch 1 and patch 2) are coupled by dispersion).

The dynamics can be represent by the following diagram.

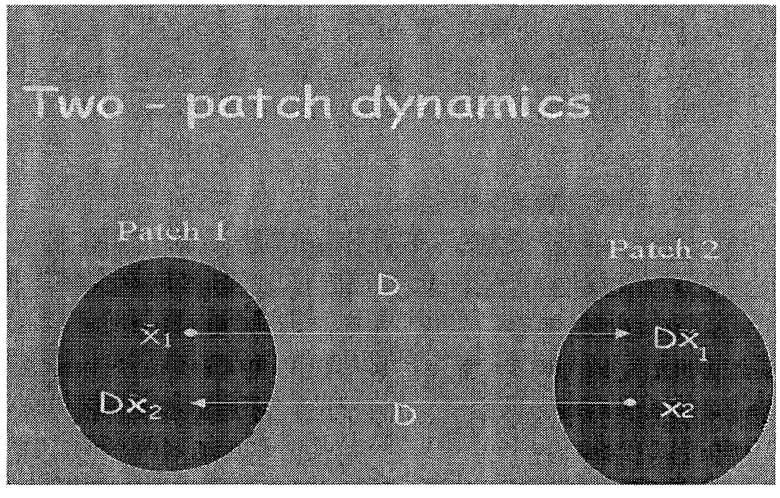

Figure 1: Dynamics between two- patches. The parameter D represented the dispersion.

Let $x_i(t)$, be the population size in patch i at time t, before the local dynamics have taken place, and $\tilde{x}_i(t)$, be the population size after the local dynamics have occured, but before the dispersal phase.

The equation which illustrates these dynamics has the general form

$$\tilde{x}_i(t) = f[r_i, x_i(t)] \tag{7}$$

where the parameter r is the measure of the how fast the population grows, and where f is the map version of the discrete time logistic model.

$$f(r, x) = rx(1 - x). \tag{8}$$

The two patches will be coupled by passive movement, or simple exchange of a fixed fraction of the population for each year . The dispersal phase is describe by the following equations:

$$x_1(t+1) = \tilde{x}_1(t) - D\tilde{x}_1(t) + D\tilde{x}_2(t) \tag{9}$$

and

$$x_2(t+1) = \tilde{x}_2(t) - D\tilde{x}_2(t) + D\tilde{x}_1(t). \tag{10}$$

Where D is the fraction of the population that is exchanged.
Using the Logistic equation $f(r,x) = rx(1-x)$, we obtain

$$x_1(t+1) = r_1 x_1(t)(1-x_1(t))(1-D) + Dr_2 x_2(t)(1-x_2(t)) \tag{11}$$

and

$$x_2(t+1) = r_2 x_2(t)(1-x_2(t))(1-D) + Dr_1 x_1(t)(1-x_1(t)). \tag{12}$$

The author in his article states used numerical and analytical approaches for to understand the dynamics. He analyzed where the local population dynamics when the two patches haved equal srowth rates $r_1 = r_2 = r$. He founded two general classes of asymtoptic solutions for different values of the growth parameter r. In the first type of solution, an *in phase solution*, the populations sizes in the two patches are always the same $x_1 = x_2$. The second type of solution, an *out of phase solution* the populations sizes in the two patches are always different $x_1 \neq x_2$.

The bifurcation diagram allow us to see the dependence of the dynamic on the growth parameter r, and determine the regions of stability and existence of the in phase and the out phase solutions. Figures (2) and (3) show for a particular value of the coupling $D = 0.10$, and the identical growth rates r in two patches $r = r_1 = r_2$.

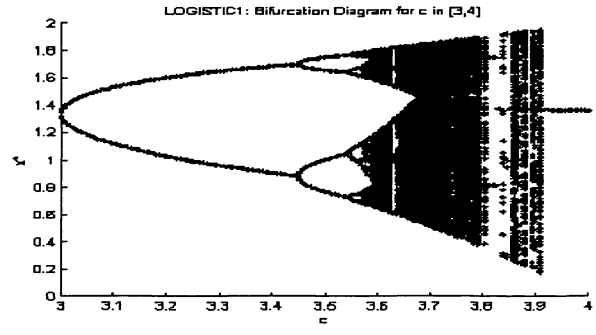

Figure 2: Bifurcation diagram for the coupled logistic model, with identical growth rates r, and $D = 0.10$ in two patches. It shows the in phase solution, with equal population size for both patches $x_1 = x_2$ here the total population size is plotted against r.

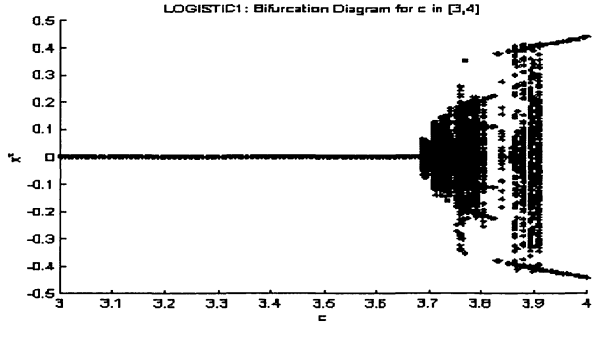

Figure 3: Bifurcation diagram for the coupled logistic model, with identical growth rates r and $D = 0.10$ in two patches. It shows the out of phase solutions, where the sizes differ, $x_1 \neq x_2$; here the difference in population sizes $x_1 - x_2$ is plotted against r

Finally, in the case of identical dynamics in two patches, the author studied the existence of sensitive dependence on initial conditions due to the existence of a fractal basin boundary. For $r = 3.8$ and $D = 0.15$, there are three different asymtotically solutions; any initial condition is attracted to a period - 2 solution or to period-4 solution or to chaotic attractor.

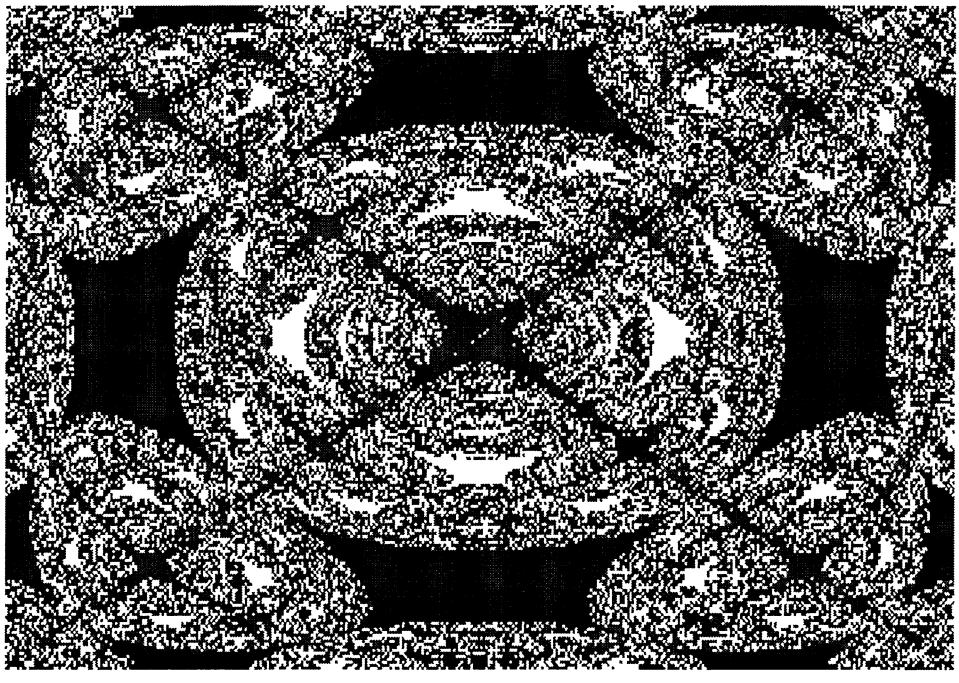

Figure 4: Illustration of the fractal basin boundary, initials conditions that are gray areas are attracted to the period two solution while the ones that are white areas are attracted to the period four solution. Initial conditions which form diagonals on the figure, end up at other solutions are typically chaotic.

We analyze only the cases with the same growth rate in both patches, for $0 \leq D \leq \frac{1}{2}$. The analytical expression for a period two cycle and its stability are given in the Appendix.

3.2 Verhults Equation System

In this section, we analyze the dynamics of a two-patch model governed by the Verhulst equation.The following equations show the dynamics

$$x_1(t+1) = (1-D)\frac{r_1 x_1(t)}{x_1(t)+A} + \frac{Dr_2 x_2(t)}{x_2(t)+A} \tag{13}$$

and

$$x_2(t+1) = (1-D)\frac{r_2 x_2(t)}{x_2(t)+A} + \frac{Dr_1 x_1(t)}{x_1(t)+A}. \tag{14}$$

If $r_1 = r_2 = r$, the system reduces to

$$x_1(t+1) = (1-D)\frac{r x_1(t)}{x_1(t)+A} + \frac{Dr x_2(t)}{x_2(t)+A} \tag{15}$$

and

$$x_2(t+1) = (1-D)\frac{r x_2(t)}{x_2(t)+A} + \frac{Dr x_1(t)}{x_1(t)+A}. \tag{16}$$

To find the fixed points, we make the following change the variables.

Let

$$y_1 = x_1 + x_2 \tag{17}$$

and

$$y_2 = x_1 - x_2. \tag{18}$$

So x_1 and x_2 are equivalent to

$$x_1 = \frac{y_1 + y_2}{2} \tag{19}$$

and

$$x_2 = \frac{y_1 - y_2}{2}. \tag{20}$$

Hence we have for $y_1(t+1)$ and $y_2(t+1)$ the following expressions:

$$y_1(t+1) = \frac{rx_1(t)}{x_1(t)+A} + \frac{rx_2(t)}{x_2(t)+A} \tag{21}$$

and

$$y_2(t+1) = \frac{rx_1}{x_1+A}(1-2D) - \frac{rx_2}{x_2+A}(1-2D). \tag{22}$$

Replacing (19) and (20) in (21) and (22), we have

$$y_1(t+1) = \frac{r(y_1+y_2)}{y_1+y_2+2A} + \frac{r(y_1-y_2)}{y_1+y_2+2A} \tag{23}$$

and

$$y_2(t+1) = \frac{r(y_1+y_2)}{y_1+y_2+2A}(1-2D) + \frac{r(y_1-y_2)}{y_1+y_2+2A}(1-2D). \tag{24}$$

3.2.1 Fixed Points

The condition for fixed points establish that:

$$y_1(t+1) = y_1 \tag{25}$$

and

$$y_2(t+1) = y_2. \tag{26}$$

So, replacing equation (23) and (24)

$$\frac{r(y_1+y_2)}{y_1+y_2+2A} + \frac{r(y_1-y_2)}{y_1+y_2+2A} = y_1 \tag{27}$$

and

$$\frac{r(y_1+y_2)}{y_1+y_2+2A}(1-2D) + \frac{r(y_1-y_2)}{y_1+y_2+2A}(1-2D) = y_2. \tag{28}$$

Solving for y_1 and y_2, we obtain the following six fixed points:

$$p_1 = (0,0),$$

$$p_2 = (2(r-A), 0),$$

$$p_3 = ((2r-A), 2[\sqrt{r^2 - Ar(1-2D)}],$$

$$p_4 = ((2r-A), -2[\sqrt{r^2 - Ar(1-2D)}],$$

$$p_5 = (0, 2[\sqrt{A^2 - Ar(1-2D)}],$$

$$p_6 = (0, -2[\sqrt{A^2 - Ar(1-2D)}].$$

We check the condition for fixed points and we find that only $p_1 = (0,0)$ and $p_2 = (2(r-A), 0)$ are true fixed points. These fixed points in terms of $x_1 and x_2$ become $p_1 = (0,0)$ and $p_2 = (r-A, r-A)$. Then if $r < A$ only exists one fixed point $(0,0)$. If $r > A$, we have two fixed points, they are $(0,0)$ and $(r-A, r-A)$

3.2.2 Stability for Fixed Points

The stability of the solutions is determined as follows. Let

$$F(x_1, x_2) = 2r[\frac{y_1^2 + 2Ay_1 - y_1^2}{(y_1 + 2A)^2 - y_2^2}]$$

and

$$G(x_1, x_2) = \frac{4Ary_2(1-2D)}{(y_1 + 2A)^2 - y_2^2}.$$

The Jacobian is given by

$$J = \begin{pmatrix} \frac{\partial F(x_1,x_2)}{\partial y_1} & \frac{\partial F(x_1,x_2)}{\partial y_2} \\ \\ \frac{\partial G(x_1,x_2)}{\partial y_1} & \frac{\partial G(x_1,x_2)}{\partial y_2} \end{pmatrix}. \tag{29}$$

and after calculating the partial derivatives, we obtain

$$\begin{pmatrix} (2Ar(\frac{1}{(2A+y_1-y_2)^2)}+\frac{1}{(2A+y_1+y_2)^2}) & \frac{-8Ar(2A+y_1)y_2}{((2A+y_1)^2-y_2^2)^2)} \\ \frac{8A(-1+2D)r(2A+y_1)y_2}{((2A+y_1)^2-y_2^2)^2} & \frac{-4A(-1+2D)r((2A+y_1)^2-y_2)^2}{((2A+y_1)^2-y_2^2)^2} \end{pmatrix}. \tag{30}$$

By evaluating the Jacobian at the point $(0,0)$, we have

$$J = \begin{pmatrix} \frac{r}{A} & 0 \\ 0 & \frac{(1-2D)r}{A} \end{pmatrix}. \tag{31}$$

We find the eigenvalues

$$\lambda_1 = \frac{r}{A}$$

$$\lambda_2 = (1-2D)\frac{r}{A}$$

If $0 \leq D \leq 1/2$ with $r < A$, the fixed point (0,0) is stable.

We evaluate the fixed point $(r-A, r-A)$ in the Jacobian and we obtain

$$J = \begin{pmatrix} \frac{A}{r} & 0 \\ 0 & \frac{(1-2D)A}{r} \end{pmatrix} \tag{32}$$

The eigenvalues are: $\lambda_1 = \frac{A}{r}$ and $\lambda_2 = \frac{(1-2D)A}{r}$
If $r > A$, and $0 \leq D \leq 0.5$ then $(0,0)$ is a fixed point unstable (saddle) and $(r-A, r-A)$ is stable.

3.2.3 Globally Stability

Theorem 3.1. *i) In system, given by equations* (15) *and* (16), *if* $r < A$, *then* $(0,0)$ *is globally stable.*
ii) In system, given by equations (15) *and* (16)*if* $r > A$, *then* $(0,0)$ *is unstable and* $(r-A, r-A)$ *is globally stable in* $(0,\infty)x(0,\infty)$.

Proof:

Now, we show that every point has a bounded orbit, under F iterations.

First, we define, the reproduction function F for system (2) F: $[0,\infty]$x$[0,\infty]$, is defined by

$F(x_1,x_2) = [(1-D)x_1g_1(x_1) + Dx_2g_2(x_2),\ (1-D)x_2g_2(x_2) + Dx_1g_1(x_1)]$

where

$g_1(x_1) = \frac{r_1x_1}{(x_1+A)}$ and $g_2(x_2) = \frac{r_2x_2}{(x_2+A)}$

If $x_1g_1(x_1) \leq x_2g_2(x_2)$

Then,

$$F_1(x_1,x_2) = (1-D)x_2g_2(x_2) + Dx_2g_2(x_2)$$

$$F_1(x_1,x_2) \leq x_2g_2(x_2)$$

and,

$$F_2(x_1,x_2) = Dx_1g_1(x_1) + (1-D)x_2g_2(x_2)$$

$$F_2(x_1,x_2) \leq x_2g_2(x_2)$$

If $x_1g_1(x_1) > x_2g_2(x_2)$, then

$$F_1(x_1,x_2) \leq x_1g_1(x_1)$$

and

$$F_2(x_1,x_2) \leq x_1g_1(x_1)$$

So,

$$F_1(x_1,x_2) \leq max[x_1g_1(x_1), x_2g_2(x_2)]$$

$$F_2(x_1, x_2) \leq max[x_1 g_1(x_1), x_2 g_2(x_2)]$$

with

$g_1(x_1) = \frac{rx_1}{x_1+A}$ and $g_2(x_2) = \frac{rx_2}{x_2+A}$

If

$$x_1 \leq r - A$$

we have,

$g_1(x_1) \leq r - A$ and $g_2(x_2) \leq r - A$

Since

$$F_1(x_1, x_2) \leq x_1 g_1(x_1) \leq r - A$$
$$F_2(x_1, x_2) \leq x_2 g_2(x_2) \leq r - A$$

If: $T = [0, r - A]$x$[0, r - A]$

Then T is F-invariant.Hence every point has a bounded orbit the region $T = [0, r - A]$x$[0, r - A]$.

Now we demostrate that if $r < A$, then $(0, 0)$ is a globally stable. Let $V(x_1, x_2)$ a Lyapunov function such that

$$V(x_1, x_2) = x_1 + x_2$$

We have,

$$x_1 + x_2 = x_1 g_1(x_1) + x_2 g_2(x_2)$$

Thus,

$$V(F(x_1, x_2)) = x_1 g_1(x_1) + x_2 g_2(x_2)$$

$$V(F(x_1, x_2)) = \frac{rx_1}{x_1 + A} + \frac{rx_2}{x_2 + A}$$

$$V(F(x_1, x_2) \leq \frac{rx_1}{A} + \frac{rx_2}{A}$$

$$V(F(x_1, x_2) \leq \frac{r}{A}(x_1 + x_2)$$

But,

$$\frac{r}{A}(x_1 + x_2) < (x_1 + x_2)$$

So,

$$V(F(x_1, x_2)) < v(x_1, x_2)$$

For all non zero point (x_1, x_2) in $[0, \infty)$x$[0, \infty)$ The part ii) is for to demostrate.

$$\frac{r}{A} < 1$$

4 Competition among members of the same specie

In this section we will define the concepts of scramble competition and contest competition. We will begin by defining competition amoung different species and individuals of same species. The term *competition* is defined as a "manifestation of the struggle for existence in which two or more organisms of the same or of different species exert a disadvantageous influence upon each other because their more or less active demands exceed the immediate supply of their common resources ([11])".

Among a specie, individuals compete for food, habitat, and other limited resources; this increases the mortality rate. This effect is more noticeable if the initial population density is high. This type of behavior is

called Intraspecific competition.The effect of intraspecific competition is always measurable as a density dependent process.

4.1 Scramble and Contest Competition

There exist two extreme forms of competition given by Nicholson [9], called "contest" and "scramble" competition.

In *contest competition* each successful animal gets all it requires the unsuccessful animals get unsufficient for survival reproduction.Thus, a certain number of individuals can be maintained, at the expense of the others.[2]
In *scramble competition* the available resources are partioned equally among all individuals. Hence as the population increases the rate of growth decreases.[1,9,11]

For the difference equations $x_{n+1} = rf(x)$, where f(x) is called the growth function per-capita. If f(x) is

$$f(x) = x_n - x_n^2$$

$$x_{n+1} = rx_n(1 - x_n)$$

is the Logistic equation.
And
If

$$x_{n+1} = r\frac{x_n}{x_n + A}$$

is the Verhulst equation.

The Verhulst equation is monotone increasing, and the Logistic equation rises to a maximum and then fall involves the nature of the intra-species for resources. Functions with a maximum corresponds to *"scramble competition"*, and functions monotone corresponden a *"contest competition"* [2].

If we have a single patch, biologically for the scramble competition, by the graph of the Logistic equation map which is a parabola. We note that initially, the population is small and the resource is abundant. After the function reaches a maximum, the resource begins to decrease. Since the resource is equally divided among the individuals, each individual receives a

very small amount.Thus, the individuals begin to die. This effect is transfered to the next generation. In scramble competition, the population is not maintained.[1,9,11]

For the *scramble competition* regulated by the Logistic equation equation

$$N_{t+1} = rN_t(1 - \frac{N_t}{k}$$

As $N_t \to \infty$' then $N_{t+1} \to 0$

In contest competition regulated by Verhulst's equation, we have an increasing section that grows assymptotically until it reaches the value $x_{n+1} = r$.

For *contest competition* regulated under the Verhulst equation, we have

$$N_{t+1} = \frac{rN_t}{A + N_t}$$

As $N_t \to \infty$ then $N_{t+1} \to r$

Thus, in contest competition the population is maintained. The resources are shared unequally.

If we have a population in two patches and they are regulated by the scramble competition, the situation is more complex than the case in a single patch. In two patches depending of the initial conditions the population will be attracted a stable two-period point or a stable four period point or a chaotic solution. The population in two patches regulated by the Verhulst equation es equal than a single patch.

5 Conclusions and future works

From a two patch system we can conclude:

- In contest competition, the dispersion does not impact the global dynamics.
- In scramble competition, the dispersion does impact the global dynamics.

- However, the scramble competition models do not generate qualitatively equivalent basins of attractors. This case is showed by Eratstenes Flórez in his project.

The future work can be:

- Analyze a model with local dynamics regulated by the equation $x_{n+1} = \frac{rx_n^2}{A+x_n^2}$, which has an Allee effect, multiple equilibria and not complex dynamics (closer to contest competition).
- Look at a multipach system with Verhults and the above model.
- Study the non-symetric case. Non- symetric growths rates and different dispersion rates.

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6 Appendix

In this section we will calculate, the results give for the author in his article.We get the conditions for the determination of the stability of systems of difference equations. We are e changing co-ordinates to the total population size.

$$y_1 = x_1 + x_2 \tag{33}$$

$$y_2 = x_1 - x_2 \tag{34}$$

Where y_1 is the total population andy_2 is the difference between two populations.

we have for x_1 and x_2:

$$x_1 = \frac{y_1 + y_2}{2} \tag{35}$$

$$x_2 = \frac{y_1 - y_2}{2} \tag{36}$$

We obtain for $y_1(t+1)$ and $y_2(t+1)$ the following equations

$$y_1(t+1) = r_2\tilde{x}_2(1-\tilde{x}_2) + r_1\tilde{x}_1(1-\tilde{x}_1) \tag{37}$$

$$y_2(t+1) = (2D-1)r_2\tilde{x}_2(1-\tilde{x}_2) + (1-2D)r_1\tilde{x}_1(1-\tilde{x}_1) \tag{38}$$

With $r_1 = r_2$ replacing Eq.(19) y Eq.(20) in Eq.(21) y Eq.(22) we have:

$$y_1(t+1) = ry_1 - \frac{y_1^2 + y_2^2}{2} \tag{39}$$

$$y_2(t+1) = (1-2D)ry_2(1-y_1) \tag{40}$$

The numerical solutions show that the out of phase solution is constant. Thus we will check for the existence ans stability of solution of the form

$$\hat{y}_1(t+1) = \hat{y}_1(t) \tag{41}$$

$$\hat{y}_2(t+2) = \hat{y}_2(t) \tag{42}$$

We will find $\hat{y}_2(t+2)$, of Eq.(24) we have

$$\hat{y}_2(t+2) = (1-2D)r\hat{y}_2(t+1)(1-\hat{y}_1(t+1) \tag{43}$$

So,

$$\hat{y}_2(t+2) = (1-2D)^2r^2\hat{y}_2(1-\hat{y}_1)[1-r(\hat{y}_1-(\frac{\hat{y}_1^2+\hat{y}_2^2)}{2})] \tag{44}$$

By the Eq.(26)

$$\hat{y}_2 = (1-2D)^2r^2\hat{y}_2(1-\hat{y}_1)[1-r(\hat{y}_1-(\frac{\hat{y}_1^2+\hat{y}_2^2)}{2})] \tag{45}$$

Here we have two solutions

$$\hat{y}_2(t) = 0 \tag{46}$$

and

$$(1-2D)^2r^2(1-\hat{y}_1)[1-r(\hat{y}_1-(\frac{\hat{y}_1^2+\hat{y}_2^2)}{2})] = 0 \tag{47}$$

The Eq.(29) is for a fixed point or in phase solution.
Furthermore, by the Eq.(25), we have the equation

$$\hat{y}_1(t+2) = r[\hat{y}_1(t+1) - [\frac{(y_1(t+1))^2+(y_2(t+1)^2)}{2}] \tag{48}$$

Replacing Eq.(30) in Eq.(31)

$$r^2(1-2D)^2(1-\hat{y}_1)^2 = 1 \tag{49}$$

The Eq.(32) has two solutions for $\hat{y}_1(t)$. They are

$$\hat{y}_1 = 1 - \frac{1}{r(1-2D)} \tag{50}$$

and

$$\hat{y}_1 = 1 + \frac{1}{r(1-2D)} \tag{51}$$

With Eq.(33) and Eq.(34) we can find the solutions for $\hat{y}_2(t)$. So

$$\hat{y}_2 = \sqrt{2\hat{y}_1(1-\frac{1}{r}) - \hat{y}_1^2} \tag{52}$$

Then the period -2 points out of phase are:

$$P : [1 - \frac{1}{r(1-2D)}, \sqrt{2\hat{y}_1(1-\frac{1}{r}) - \hat{y}_1^2}] \tag{53}$$

$$Q : [1 + \frac{1}{r(1-2D)}, \sqrt{2\hat{y}_1(1-\frac{1}{r}) - \hat{y}_1^2}] \tag{54}$$

The before equations must write in function of thex_1 and x_2 give en Eq.(1) and Eq.(2). So for the point Q

$$\hat{x}_{11} = \frac{1}{2}\left[1 + \frac{1}{r(1-2D)} + \sqrt{2(1+\frac{1}{r(1-2D)})(1-\frac{1}{r}) - \frac{1}{r(1-2D)}^2}\right] \tag{55}$$

and

$$\hat{x}_{12} = \frac{1}{2}[1 + \frac{1}{r(1-2D)} - \sqrt{2(1+\frac{1}{r(1-2D)})(1-\frac{1}{r}) - \frac{1}{r(1-2D)}^2} \tag{56}$$

We must check if efectively the P and Q points found are period-2 points. We examine that:

$$f(\hat{x}_1, \hat{x}_2) = \hat{x}_{11}$$

and

$$g(\hat{x}_1, \hat{x}_2) = \hat{x}_{12}$$

Where $f(\hat{x}_1, \hat{x}_2)$ and $g(\hat{x}_1, \hat{x}_2)$ are given by Eq.10 and Eq. (11) evaluate in $(\hat{x}_1, \hat{x}_2)$. The following expressions were find with the aid program Mathematica.

$$\hat{x}_1 = \frac{1 - r(-1 + 2D) - \sqrt{(r^2 - 2r)(1 - 2D)^2 + (1 - 4D)}}{2(-1 + 2D)} \tag{57}$$

$$\hat{x}_2 = \frac{1 - r(-1 + 2D) + \sqrt{(r^2 - 2r)(1 - 2D)^2 + (1 - 4D)}}{2(-1 + 2D)} \tag{58}$$

$$\hat{x}_{11} = \frac{-1 - r(-1 + 2D) - \sqrt{(r^2 - 2r)(1 - 2D)^2 - 3 - 4D}}{2(-1 + 2D)} \tag{59}$$

$$\hat{x}_{22} = \frac{-1 - r(-1 + 2D) + \sqrt{(r^2 - 2r)(1 - 2D)^2 - 3 - 4D}}{2(-1 + 2D)} \tag{60}$$

$$f(\hat{x}_1, \hat{x}_2) = \frac{1 - r(-1 + 2D) - \sqrt{(r^2 - 2r)(1 - 2D)^2 + (1 - 4D)}}{2(-1 + 2D)} \tag{61}$$

$$g(\hat{x}_1, \hat{x}_2) = \frac{1 - r(-1 + 2D) + \sqrt{(r^2 - 2r)(1 - 2D)^2 + (1 - 4D)}}{2(-1 + 2D)} \tag{62}$$

$$f(\hat{x}_{11}, \hat{x}_{22}) = \frac{-1 - r(-1 + 2D) + \sqrt{(r^2 - 2r)(1 - 2D)^2 - 3 - 4D}}{2(-1 + 2D)} \tag{63}$$

$$g(\hat{x}_{11}, \hat{x}_{22}) = \frac{-1 - r(-1 + 2D) - \sqrt{(r^2 - 2r)(1 - 2D)^2 - 3 - 4D}}{2(-1 + 2D)} \tag{64}$$

We must prove that $f[x_1, x_2]q$ and $g[x_1, x_2]$ have constant solutions.
Then

$$f(\hat{x}_1, \hat{x}_2) + g(\hat{x}_1, \hat{x}_2) = \hat{x}_1 + \hat{x}_2 \tag{65}$$

and

$$f(\hat{x}_{11}, \hat{x}_{22}) + g(\hat{x}_{11}, \hat{x}_{22}) = \hat{x}_{11} + \hat{x}_{22} \tag{66}$$

We calculate

$$f(\hat{x}_1, \hat{x}_2) + g(\hat{x}_1, \hat{x}_2) = \frac{1 - r(1-2D)}{(-1+2Dr)}$$

and

$$\hat{x}_1 + \hat{x}_2 = \frac{1 - r(1-2D)}{(-1+2Dr)}$$

The identify (49) is satisfied
for the other point, also we check the identity

$$f(\hat{x}_{11}, \hat{x}_{22}) + g(\hat{x}_{11}, \hat{x}_{22}) = \frac{-1 - r(1-2D)}{r(-1+2D)}$$

and

$$\hat{x}_{11} + \hat{x}_{22} = \frac{-1 - r(1-2D)}{r(-1+2D)}$$

as we wait the identify (50) is satisfied.
Also, we must demostrate that

$$f^2(\hat{x}_1, \hat{x}_2) - g^2(\hat{x}_1, \hat{x}_2) = \hat{x}_1 + \hat{x}_2 \tag{67}$$

and

$$f^2(\hat{x}_{11}, \hat{x}_{22}) - g^2(\hat{x}_{11}, \hat{x}_{22}) = \hat{x}_{11} + \hat{x}_{22} \tag{68}$$

we calculate

$$f^2(\hat{x}_{11},\hat{x}_{22}) - g^2(\hat{x}_{11},\hat{x}_{22}) = -\frac{\sqrt{(r^2-2r)(1-2D)^2+(1-4D)}}{2(-1+2D)r}$$

and

$$\hat{x}_1 + \hat{x}_2 = -\frac{\sqrt{(r^2-2r)(1-2D)^2+(1-4D)}}{2(-1+2D)r}$$

Tha identify (51) is satisfied
we calculate

$$f^2(\hat{x}_{11},\hat{x}_{22}) - g^2(\hat{x}_{11},\hat{x}_{22}) = -\frac{\sqrt{(r^2-2r)(1-2D)^2-3+4D)}}{2(-1+2D)r}$$

and

$$\hat{x}_{11} + \hat{x}_{22} = -\frac{\sqrt{(r^2-2r)(1-2D)^2-3+4D)}}{2(-1+2D)r}$$

Then identify (52) is satisfied.

With the before demostration, we show with the points $(\hat{x}_1,\hat{x}_2)$ and $(\hat{x}_{11},\hat{x}_{22})$ are period-2.

7 References

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