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ANNOTATED COMPUTER OUTPUT

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ANNOTATED COMPUTER OUTPUT FOR SPLIT PLOT DESIGN: BMDP 2V

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ANNOTATED COMPUTER OUPUT FOR SPLIT PLOT DESIGN: BMDP 2V

by

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ABSTRACT

In order to provide an understanding of covariance analyses for split plot designs, analyses are first conducted on a data set without the covariate. Then an analysis with the covariate is performed. The third example has a different experiment design for the whole plots and the covariate is constant for all split plots within a whole plot. Computer packages may only give portions of the analysis correctly. Some packages require two procedural calls to obtain a portion of the correct results. BMDP 2V falls in the above category.

INTRODUCTION

This is part of a continuing project that produces annotated computer output for the analysis of balanced split plot experiments with covariates. The complete project will involve processing three examples on SAS/GLM, BMDP/2V, SPSS-X/MANOVA, GENSTAT/ANOVA, and SYSTAT/MGLH. Only univariate results are considered. We show here the results from BMDP 2V.

For Example 1, the data are artificial and were constructed for ease of computation; the experiment design for the whole plots is a randomized complete block and the split plot treatments are randomly allocated to the split plot experimental units within <u>each</u> whole plot. Example 2 is the same as Example 1 except that a covariate varies from split plot to split plot. The data for Example 3 come from an experiment wherein the whole plot treatments were laid out in a completely randomized design and the split plot treatments were randomly allotted to the split plot experimental units within each whole plot. The value of the covariate varies from whole plot to whole plot but is constant for all split plots within a whole plot treatment.

We present the elementary computational steps. Simple hypothetical data are used for the first two examples so that it is easy to provide all detailed computations to illustrate how each number is obtained. Some readers may wish to skip the detailed computations. The third example comes from Winer (1971). The detailed computations are given in his book (p. 803).

Data SP-1

Split plot data with whole plots arranged in randomized complete block design (hypothetical data)

	Whole plot treatment									
	W1					W2				
	split plot treatment					split plot treatment				
Block	S_1	S_2	S_3	S_4	Total	S_1	S_2	S_3	S_4	Total
1	3	4	7	6	20	3	2	1	14	20
2	6	10	1	11	28	8	8	2	18	36
3	6	10	4	4	24	10	8	9	13	40
Total	15	24	12	21	72	21	18	12	45	96

Total and Means

	Blocks (8 observations) Total	Mean		W(whole plots) (12 observations) Total	Mean		S(split plot) (6 observations) Total	Mean
1	40	5	W1	72	6	S_1	36	6
2	64	8	W2	96	8	S_2	42	7
3	64	8				S_3	24	4
Grand Total		168				S_4	66	11
Grand Mean		7						

Model: $Y_{ijk} = \mu + \rho_j + \tau_i + \delta_{ij} + \alpha_k + (\alpha\tau)_{ik} + \epsilon_{ijk}$

μ = mean

ρ_j = effect of block j

δ_{ij} = error (a) effect

ϵ_{ijk} = error (b) effect

τ_i = effect of whole plot i

α_k = effect of split plot k

$(\alpha\tau)_{ik}$ = effect of interaction of whole plot i and split plot k

Analysis of Variance

Source	(*)	df	SS
B (Blocks)	$= R(\rho \mid \mu,\tau,\alpha,\alpha\tau)$	2	48
W (whole plot treatments)	$= R(\tau \mid \mu,\rho,\alpha,\alpha\tau)$	1	24
B×W (error (a))	$= R(\delta \mid \mu,\rho,\tau,\alpha,\alpha\tau)$	2	16
S (split plot treatments)	$= R(\alpha \mid \mu,\rho,\tau,\alpha\tau)$	3	156
S×W (interaction of S and W)	$= R(\alpha\tau \mid \mu,\alpha,\tau,\rho)$	3	84
(**) S×B:W (error (b))	$= R(\epsilon \mid \mu,\alpha,\tau,\alpha\tau,\rho)$	12	112
Total (Corrected for mean)	$= R(\rho,\tau,\delta,\alpha,\alpha\tau,\epsilon \mid \mu)$	23	440
Mean	$= R(\mu)$	1	1176
Total (Uncorrected for mean)	$= R(\mu,\rho,\tau,\delta,\alpha,\alpha\tau,\epsilon)$	24	1616

(*)Notation follows that of Searle(1971). Since the design is balanced, $R(\rho \mid \mu,\tau,\alpha,\alpha\tau) = R(\rho \mid \mu)$, etc. The simpler notation is used later.
(**) S×B:W means S×B within W.

Calculations of sums of squares:

$N = 2\cdot 3\cdot 4 = 24$, $\bar{Y} = 7$

$$R(\mu,\rho,\tau,\delta,\alpha,\alpha\tau,\epsilon) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{4} Y_{ijk}^2 = (3^2 + 6^2 + 6^2 + \cdots + 18^2 + 13^2) = 1616$$

$$R(\mu) = N\bar{Y}^2 = 24\cdot(7)^2 = 1176$$

$$R(\rho,\tau,\delta,\alpha,\alpha\tau,\epsilon \mid \mu) = 1616 - 1176 = 440$$

$$R(\rho|\mu) = R(\mu,\rho) - R(\mu) = \frac{(40^2+64^2+64^2)}{8} - 1176 = 1224 - 1176 = 48$$

$$R(\tau|\mu) = R(\mu,\tau) - R(\mu) = \frac{(72^2+96^2)}{12} - 1176 = 1200 - 1176 = 24$$

$$R(\delta|\mu,\rho,\tau) = R(\delta,\mu,\rho,\tau) - R(\mu,\rho) - R(\tau,\mu) + R(\mu)$$

$$= \frac{(20^2+28^2+24^2+20^2+36^2+40^2)}{4} - 1224 - 1200 + 1176$$

$$= 1264 - 1224 - 1200 + 1176 = 16$$

$$R(\alpha|\mu) = R(\alpha,\mu) - R(\mu) = \frac{(36^2+42^2+24^2+66^2)}{6} - 1176 = 1332 - 1176 = 156$$

$$R(\alpha\tau|\mu,\alpha,\tau) = R(\alpha\tau,\mu,\alpha,\tau) - R(\mu,\alpha) - R(\mu,\tau) + R(\mu)$$

$$= \frac{(15^2+24^2+12^2+21^2+21^2+18^2+12^2+45^2)}{3} - 1332 - 1200 + 1176$$

$$= 1440 - 1332 - 1200 + 1176 = 84$$

$$R(\epsilon|\mu,\rho,\delta,\alpha,\tau,\alpha\tau) = R(\epsilon,\mu,\alpha,\rho,\delta,\tau,\alpha\tau) - R(\mu,\rho,\tau,\delta) - R(\mu,\alpha,\tau,\alpha\tau) + R(\tau,\mu)$$

$$= 1616 - 1264 - 1440 + 1200 = 112$$

Data SP-2

Data SP-2: Data SP-1 with the following covariate Z which varies with split plot

Covariate (Z)

	whole plot									
	W1					W2				
	S_1	S_2	S_3	S_4	Total	S_1	S_2	S_3	S_4	Total
B_1	1	2	1	2	6	2	0	2	4	8
B_2	2	2	0	4	8	4	1	3	4	12
B_3	3	5	2	0	10	3	2	4	7	16
Total	6	9	3	6	24	9	3	9	15	36

Totals and Means

	blocks (8 observations) Total	Mean		W (whole plot) (12 observations) Total	Mean		S (split plot) (6 observations) Total	Mean
1	14	14/8	1	24	2.0	1	15	2.5
2	20	20/8	2	36	3.0	2	12	2.0
3	26	26/8				3	12	2.0
Grand						4	21	3.5
Total	60	2.5						

Model: $Y_{ijk} = \mu+\rho_j+\tau_i+\delta_{ij}+\alpha_k+(\alpha\tau)_{ik}+\beta_1(\bar{Z}_{ij.}-\bar{Z}_{...})+\beta_2(Z_{ijk}-\bar{Z}_{ij.})+\epsilon_{ijk}$

ρ_j = effect of jth block $\qquad$ τ_i = effect of ith whole plot

α_k = effect of kth split plot $\qquad$ β_1 = whole plot regression slope

β_2 = split plot regression slope $\qquad$ δ_{ij} = error a $\qquad$ ϵ_{ijk} = error b

Table of sum of squares and products

Source	df	YY	YZ	ZZ
B	2	48	18	9
W	1	24	12	6
B×W (error a)	2	16	4	1
S	3	156	33	9
S×W	3	84	33	21
S×B:W (error b)	12	112	17	20
Mean	1	1176	420	150
Total	24	1616	537	216

YY column is the same as in SP-1, ZZ column is computed in the same fashion. Thus, only computations for YZ column are illustrated.

$$\text{Total}_{YZ} = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{4} Y_{ijk}\cdot Z_{ijk}$$
$$= 3(1)+ 6(2)+\cdots+ 14(4)+ 18(4)+ 13(7) = 537$$

$$\text{Mean}_{YZ} = N\bar{Y}_{...}\bar{Z}_{...} = \frac{168\cdot 60}{24} = 420$$

$$B_{YZ} = \frac{\sum_{j=1}^{3}\left(\sum_{i=1}^{2}\sum_{k=1}^{4} Y_{ijk}\right)\left(\sum_{i=1}^{2}\sum_{k=1}^{4} Z_{ijk}\right)}{2\cdot 4} -420 = \frac{40(14)+64(20)+64(26)}{8} - 420$$
$$= 438 - 420 = 18$$

$$W_{YZ} = \frac{\sum_{i=1}^{2}\left(\sum_{j=1}^{3}\sum_{k=1}^{4} Y_{ijk}\right)\left(\sum_{j=1}^{3}\sum_{k=1}^{4} Z_{ijk}\right)}{3(4)} - 420 = 432 - 420 = 12$$

$$B\times W_{YZ} = \frac{\sum_{i=1}^{2}\sum_{j=1}^{3}(\sum_{k=1}^{4} Y_{ijk})(\sum_{k=1}^{4} Z_{ijk})}{4} - 438 - 432 + 420$$

$$= 454 - 438 - 432 + 420 = 4$$

$$S_{YZ}: \sum_{k=1}^{4} \frac{(\sum_{i=1}^{2}\sum_{j=1}^{3} Y_{ijk})(\sum_{i=1}^{2}\sum_{j=1}^{3} Z_{ijk})}{2(3)} - 420 = 453 - 420 = 33$$

$$S\times W_{YZ}: \frac{\sum_{i=1}^{2}\sum_{k=1}^{4}(\sum_{j=1}^{3} Y_{ijk})(\sum_{j=1}^{3} Z_{ijk})}{3} - 453 - 432 + 420$$

$$= 498 - 453 - 432 + 420 = 33$$

$S\times B{:}W_{YZ}$:

$$\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{4} Y_{ijk}Z_{ijk} - 454 - 498 + 432$$

$$= 537 - 454 - 498 + 432 = 17$$

Analysis of Variance and Covariance

Source		df	SS
B (block)	$= R(\rho \mid \mu,\tau)$	2	48
W (whole plot treatment)	$= R(\tau \mid \mu,\rho,\beta_1)$	1	3.4286
Regression (a)	$= R(\beta_1 \mid \mu,\rho,\tau)$	1	16.0
B×W (error (a))	$= R(\delta \mid \mu,\rho,\tau,\beta_1)$	1	0.0
S (split plot treatment)	$= R(\alpha \mid \mu,\rho,\tau,\alpha\tau,\beta_2)$	3	84.243
S×W (interaction of S and W)	$= R(\alpha\tau \mid \mu,\rho,\tau,\alpha,\beta_2)$	3	37.474
Regression (b)	$= R(\beta_2 \mid \mu,\rho,\tau,\alpha,\alpha\tau)$	1	14.450
S×B: W (error (b))	$= R(\epsilon \mid \mu,\rho,\alpha,\tau,\alpha\tau,\beta_2)$	11	97.550

$$\hat{\beta}_1 = B\times W_{YZ} \,/\, B\times W_{ZZ} = 4/1 = 4$$

$$\hat{\beta}_2 = S\times B{:}W_{YZ} \,/\, S\times B{:}W_{ZZ} = 17/20 = 0.85$$

The SS's adjusted by regression on Z are illustrated below:

$R(\rho|\mu) = 48$, remains same since it is not of interest to adjust for Z on the blocks.

$$R(\tau,\delta|\mu,\rho,\beta_1) = (W_{YY} + B\times W_{YY}) - \frac{(W_{YZ} + B\times W_{YZ})^2}{W_{ZZ} + B\times W_{ZZ}}$$

$$= (24 + 16) - \frac{(12 + 4)^2}{6 + 1} = 40 - \frac{256}{7} = 3.4286$$

$$R(\delta|\mu,\rho,\tau,\beta_1) = B\times W_{YY} - \frac{(B\times W_{YZ})^2}{B\times W_{ZZ}} = 16 - \frac{4^2}{1} = 0$$

$$R(\tau|\mu,\rho,\beta_1) = R(\tau,\delta|\mu,\rho,\beta_1) - R(\delta|\mu,\rho,\tau,\beta_1)$$

$$= 40 - \frac{256}{7} - 0 = 3.4286$$

$$R(\beta_1|\mu,\tau,\rho) = \frac{(B\times W_{YZ})^2}{B\times W_{ZZ}} = \frac{4^2}{1} = 16$$

$$R(\alpha,\epsilon|\mu,\rho,\tau,\alpha\tau,\beta_2) = (S_{YY} + S\times B{:}W_{YY}) - \frac{(S_{YZ} + S\times B{:}W_{YZ})^2}{S_{ZZ} + S\times B{:}W_{ZZ}}$$

$$= (156 + 112) - \frac{(33+17)^2}{9+20}$$

$$= 268 - 86.207 = 181.793$$

$$R(\alpha\tau,\epsilon|\mu,\rho,\alpha,\tau,\beta_2) = (S\times W_{YY} + S\times B{:}W_{YY}) - \frac{(S\times W_{YZ} + S\times B{:}W_{YZ})^2}{S\times W_{ZZ} + S\times B{:}W_{ZZ}}$$

$$= 84 + 112 - \frac{(33+17)^2}{21+20} = 196 - 60.976 = 135.024$$

Note: $R(\alpha,\epsilon|\mu,\beta_2)$ and $R(\alpha\tau,\epsilon|\mu,\alpha,\tau,\beta_2)$ are intermediate steps for later use.

$$R(\beta_2|\mu,\rho,\alpha,\tau,\alpha\tau) = \frac{(S\times B:W_{YZ})^2}{S\times B:W_{ZZ}} = \frac{17^2}{20} = 14.450$$

$$R(\epsilon|\mu,\rho,\alpha,\tau,\alpha\tau,\beta_2) = S\times B:W_{YY} - \frac{(S\times B:W_{YZ})^2}{S\times B:W_{ZZ}} = 112 - \frac{17^2}{20} = 112 - 14.45 = 97.55$$

$$R(\alpha|\mu,\rho,\tau,\alpha\tau,\beta_2) = R(\alpha,\epsilon|\mu,\rho,\tau,\alpha\tau,\beta_2) - \text{SS error b} = 181.793 - 97.55$$
$$= 84.243$$

$$R(\alpha\tau|\mu,\rho,\alpha,\tau,\beta_2) = R(\alpha\tau,\epsilon|\mu,\rho,\alpha,\tau,\beta_2) - R(\epsilon|\mu,\rho,\alpha,\tau,\alpha\tau,\beta_2)$$

$$= 135.024 - 97.55 = 37.474$$

Data SP-3

Split plot data with plots arranged in a completely randomized design and a covariate Z that is constant within the whole plot. (Winer, 1971, p. 803)

whole plot	Subject	Split plots B_1 Y	B_2 Y	Z	Total Y
A_1					
	1	10	8	3	18
	2	15	12	5	27
	3	20	14	8	34
	4	12	6	2	18
A_2					
	5	15	10	1	25
	6	25	20	8	45
	7	20	15	10	35
	8	15	10	2	25
	Total	132	95	39	227
	Mean	16.5	11.9	4.88	

Model: $y_{ijk} = \mu + \tau_i + \delta_{ij} + \alpha_k + (\tau\alpha)_{ik} + \beta_1(Z_{ij} - \bar{Z}_{..}) + \epsilon_{ijk}$

τ_i = A effect (whole plot) $\quad\delta_{ij}$ = error (a) $\quad\epsilon_{ijk}$ = error (b)

α_k = B effect (split plot) $\quad\beta_1$ = whole plot regression slope

Analysis of variance and covariance

Source		df	SS
A (whole plot)	$= R(\tau \mid \mu, \beta_1)$	1	44.492
Regression	$= R(\beta_1 \mid \mu, \tau)$	1	166.577
Error (a)	$= R(\delta \mid \mu, \tau, \beta_1)$	5	61.298
B (split plot)	$= R(\alpha \mid \mu, \tau, \alpha\tau)$	1	85.563
A×B (interaction)	$= R(\tau\alpha \mid \mu, \tau, \alpha)$	1	0.563
Error (b)	$= R(\epsilon \mid \mu, \tau, \alpha, \tau\alpha)$	6	6.375

Table of SS and products

Symbol	Y^2	ZY	Z^2
W	68.06	12.38	2.25
E(a)	227.88	163.00	159.50
S	85.563	0	0
WS	0.563	0	0
E(b)	6.375	0	0

$$\hat{\beta}_1 = \frac{163.00}{159.50} = 1.02$$

Since the computations are illustrated in Winer (1971, p. 803-5) we have omitted them here.

SP-1: Control Language

Control Language is typed upper case and comments are in bold and lower case.

```
/PROBLEM  TITLE IS 'SP-1: WHOLE PLOT ARRANGED IN RCB'.
/INPUT    VARIABLES ARE 6.
          FORMAT IS FREE.
/VARIABLE NAMES ARE BLOCK, WHOLE, Y1, Y2, Y3, Y4. ⟹ input variables
/DESIGN   GROUPING ARE BLOCK, WHOLE.    ⟹ whole plot classification
                                          variables
          DEPENDENT ARE Y1, Y2, Y3, Y4.  ⟹ specifies Y measured for
                                            four levels of split plot
          LEVEL IS 4.  ⟹ specifies four levels of split plot
          NAME IS SUBPLOT.  ⟹ split plot classification variable
          EXCLUDE IS 12.  ⟹ model specification for whole plot: assumes
                             full factorial. This statement requests the
                             BLOCK (1) by WHOLE (2) interaction to be
                             pooled with SSE(a)
          PRINT.  ⟹ requests residuals and predicted values
/END
1 1 3 4 7 6        ⟹  data organized with all split values for a partic-
2 1 6 10 1 11          ular BLOCK by WHOLE combination on the same line
3 1 6 10 4 4
1 2 3 2 1 14
2 2 8 8 2 18
3 2 10 8 9 13
```

SP-2: Control Language

Note: The whole plot and split plot analysis must be requested in separate runs. The first run gives the correct whole plot analysis but an incorrect split analysis (details follow in the annotations). The second run gives the correct split plot analysis.

Whole Plot Run

```
/PROBLEM   TITLE IS 'SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT'.
/INPUT     VARIABLES ARE 10.
           FORMAT IS FREE.
/VARIABLE  NAMES ARE BLOCK, WHOLE, Z1, Z2, Z3, Z4, Y1, Y2, Y3, Y4.
/DESIGN    GROUPING ARE BLOCK, WHOLE.
           DEPENDENT ARE Y1, Y2, Y3, Y4.
           COVARIATE ARE Z1, Z2, Z3, Z4. ==> yields an analysis of
           LEVEL IS 4.                       covariance
           NAME IS SUBPLOT.
           EXCLUDE IS 12.
           RESIDUAL IS MEAN.
           PRINT.
/END
1 1 1 2 1 2 3 4 7 6
2 1 2 2 0 4 6 10 1 11
3 1 3 5 2 0 6 10 4 4
1 2 2 0 2 4 3 2 1 14
2 2 4 1 3 4 8 8 2 18
3 2 3 2 4 7 10 8 9 13
```

Split Plot Run

```
/PROBLEM    TITLE IS 'SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT'.
/INPUT      VARIABLES ARE 5.
            FORMAT IS FREE.
/VARIABLE   NAMES ARE BLOCK, WHOLE, SUBPLOT, Z, Y.
/DESIGN     GROUPING ARE BLOCK, WHOLE, SUBPLOT.
            DEPENDENT IS Y.
            COVARIATE IS Z.
            EXCLUDE IS 13,123.  ==> model specification for split plot:
            PRINT                   program assumes full factorial.  This
/END                                statement specifies certain factors
1 1 1 1 3                           to be pooled in SSE(b).
1 1 2 2 4
1 1 3 1 7
1 1 4 2 6  ==> for split plot analysis data must be organized in a
:              similar arrangement with only one datum per line
:
:
3 2 4 7 13
```

SP-3: Control Language

Note: This example unlike the other two covariate examples requires only one run. This is because we are using a CRD and no special design specification is required.

```
/PROBLEM    TITLE IS 'SP-4: WHOLE PLOTS ARRANGED IN CRD WITH CONSTRAINT
                              COVARIATE
/INPUT      VARIABLES ARE 4.
            FORMAT IS FREE.
/VARIABLE   NAMES ARE A, Z, Y1, Y2.
/DESIGN     GROUPING IS A.
            DEPENDENT ARE Y1, Y2.
            COVARIATE IS Z, Z.   ==> BMDP requires that two covariables be
            LEVEL IS 2.              specified here to match the number of
            NAME IS B.               levels of the split plot. Since Z is
            PRINT.                   constant for both levels, its name is
/END                                 entered for both.
1 3 10 8
1 5 15 12
1 8 20 14
1 2 12 6
2 1 15 10
2 8 25 20
2 10 20 15
2 2 15 10
```

References

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Federer, W.T., and Henderson, H.V. (1979), Covariance Analysis of Designed Experiments X Statistical Packages: An Update, Proc., Comp. Sci. and Stat.: 12th Ann. Sym. on the Interface.

Searle, S.R., Linear Models, Wiley, N.Y., (1971), 532pp.

Searle, S.R., Hudson, G.F.S., and Federer, W.T. (1985), Annotated Computer Output for Covariance-Text, BU-780-M, Biometrics Unit Mimeo Ser., Cornell University, Ithaca, NY.

Winer, B.J., (1971), Statistical Principles in Experimental Design, McGraw-Hill Book Company, New York: 907pp.

1PAGE 1 BMDP2V

BMDP2V - ANALYSIS OF VARIANCE AND COVARIANCE WITH REPEATED MEASURES.

BMDP Statistical Software, Inc.
1964 Westwood Blvd. Suite 202   Phone (213) 475-5700
Los Angeles, California 90025   Telex 4992203

Program Version: April 1985
(VM/CMS)
Manual Edition: 1983, 1985 reprint. State NEWS in the PRINT
paragraph for a summary of new features.

APRIL 20, 1987 AT 16:43:45

PROGRAM CONTROL INFORMATION

```
/PROBLEM   TITLE IS 'SP-1: WHOLE PLOT ARRANGED IN RCB'.
/INPUT     VARIABLES ARE 6.
           FORMAT IS FREE.
/VARIABLE  NAMES ARE BLOCK, WHOLE, Y1, Y2, Y3, Y4.
/DESIGN    GROUPING ARE BLOCK, WHOLE.
           DEPENDENT ARE Y1, Y2, Y3, Y4.
           LEVEL IS 4.
           NAME IS SUBPLOT.
           EXCLUDE IS 12.
           PRINT.
/END
```

Control Language (see page 9 for details)

Data Organization - all split plot values for a particular BLOCK by WHOLE combination must be in the same observation. See control language on page 9 for the exact data layout.

```
PROBLEM TITLE IS
 SP-1: WHOLE PLOT ARRANGED IN RCB

 NUMBER OF VARIABLES TO READ IN. . . . . . . . .        6
 NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. .        0
 TOTAL NUMBER OF VARIABLES . . . . . . . . . . .        6
 NUMBER OF CASES TO READ IN. . . . . . . . . . . TO END
 CASE LABELING VARIABLES . . . . . . . . . . . .
 MISSING VALUES CHECKED BEFORE OR AFTER TRANS. . NEITHER
 BLANKS ARE. . . . . . . . . . . . . . . . . . . MISSING
 NUMBER OF WORDS OF DYNAMIC STORAGE. . . . . . .   32362

 VARIABLES TO BE USED
     1 BLOCK       2 WHOLE       3 Y1          4 Y2          5 Y3  → Each variable is assigned a number and
     6 Y4                                                            will be referred to by its number below

 INPUT FORMAT IS
 FREE

 MAXIMUM LENGTH DATA RECORD IS   80 CHARACTERS.

 DESIGN SPECIFICATIONS  → Interpretation of DESIGN paragraph

         GROUP =    1   2  → Variables 1 and 2 (BLOCK & WHOLE) make up the "grouping" or whole plot variables
         DEPEND =   3   4   5   6  → Variables 3,4,5 and 6 make up the dependent variable, Y
         LEVEL =    4  → 4 levels of the split plot variable, SUBPLOT

                                                                        INTERVAL RANGE
  VARIABLE      MINIMUM  MAXIMUM  MISSING CATEGORY CATEGORY  GREATER LESS THAN
  NO. NAME       LIMIT    LIMIT    CODE     CODE     NAME     THAN    OR = TO

    1 BLOCK
                                          1.00000 *1.00000
                                          2.00000 *2.00000     → BMDP assigned names for the levels of the two whole plot variables
                                          3.00000 *3.00000
    2 WHOLE
                                          1.00000 *1.00000
                                          2.00000 *2.00000
```

NOTE--CATEGORY NAMES BEGINNING WITH * WERE GENERATED BY THE PROGRAM.

NUMBER OF CASES READ.   6
BMDP2V SP-1: WHOLE PLOT ARRANGED IN RCB

GROUP STRUCTURE

BLOCK	WHOLE	COUNT	
*1.00000	*1.00000	1	→ **Frequency of observations in each BLOCK by WHOLE combination for Y1-Y4**
*1.00000	*2.00000	1	
*2.00000	*1.00000	1	
*2.00000	*2.00000	1	
*3.00000	*1.00000	1	
*3.00000	*2.00000	1	

SUMS OF SQUARES AND CORRELATION MATRIX OF THE ORTHOGONAL COMPONENTS POOLED FOR ERROR 2 IN ANOVA TABLE BELOW

0.20000	1.000		
3.00000	0.000	1.000	
16.80000	-0.327	-0.945	1.000

SPHERICITY TEST APPLIED TO ORTHOGONAL COMPONENTS - TAIL PROBABILITY   0.0000

Tests assumption that the orthogonal components for the split plot, SUBPLOT are independent and have equal variances (sometimes referred to as symmetry assumption) The above information on the orthogonal components of the split plots are useful when the levels of the split plot are ordered and the investigator is concerned about the assumption of independence and equal variances on the levels of the split plot. However, it will be shown on SP-1 page 5 that the "error 2" or SSE(b) that BMDP computs for this design is incorrect making the above values also incorrect.

CELL MEANS FOR 1-ST DEPENDENT VARIABLE

									MARGINAL
j = BLOCK	=	*1.00000	*1.00000	*2.00000	*2.00000	*3.00000	*3.00000		
i = WHOLE	=	*1.00000	*2.00000	*1.00000	*2.00000	*1.00000	*2.00000		
	SUBP LOT = k								
Y1	1	3.00000	3.00000	6.00000	8.00000	6.00000	10.00000		6.00000
Y2	2	4.00000	2.00000	10.00000	8.00000	10.00000	8.00000	$\} Y_{ijk}$	7.00000 $\} \bar{Y}_{\cdot\cdot k}$
Y3	3	7.00000	1.00000	1.00000	2.00000	4.00000	9.00000		4.00000
Y4	4	6.00000	14.00000	11.00000	18.00000	4.00000	13.00000		11.00000
MARGINAL	$\bar{Y}_{ij}$ =	5.00000	5.00000	7.00000	9.00000	6.00000	10.00000		7.00000 = $\bar{Y}_{\cdots}$
COUNT		1	1	1	1	1	1		6

BMDP2V SP-1: WHOLE PLOT ARRANGED IN RCB

ρ_j = block effect

τ_i = variety effect (whole plot)

α_k = fertilizer effect (split plot)

ANALYSIS OF VARIANCE FOR 1-ST

DEPENDENT VARIABLE - Y1 Y2 Y3 Y4

	SOURCE	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.	GREENHOUSE GEISSER PROB.	HUYNH FELDT PROB.
	MEAN $SSM = N\bar{Y}^2 = 24(7)^2$ =	1176.00000	1	1176.00000	147.00	0.0067		
	BLOCK $R(\rho\|\mu)$	48.00000	2	24.00000	3.00	0.2500		
	WHOLE $R(\tau\|\mu)$	24.00000	1	24.00000	3.00	0.2254	3.0 = 24.0/8.0	
1	ERROR $R(\delta\|\mu,\rho,\tau)$	16.00000	2 MSE(a)	8.00000				
	SUBPLOT $R(\alpha\|\mu)$	156.00000	3	52.00000	15.60	0.0031	0.0544	0.0031
	SB	92.00000	6	15.33333	4.60	0.0428	0.1719	0.0428
	SW $R(\alpha\tau\|\mu,\alpha,\tau)$	84.00000	3	28.00000	8.40	0.0144	0.0964	0.0144
2	ERROR incorrect → see note below	20.00000	6	3.33333				

ERROR TERM

EPSILON FACTORS FOR DEGREES OF FREEDOM ADJUSTMENT

ERROR TERM	GREENHOUSE-GEISSER	HUYNH-FELDT
2	0.3490	1.0000

The "2 error" or SSE(b) reported here is incorrect. This means that Greenhouse-Geisser & Huynh-Feldt are adjusted p-values to be used when the symmetry assumptions on the split plot are not met (see sphericity test on SP-1 page 3). However, these values are incorrect for this design because BMDP has computed the SSE(b) incorrectly. See the note below.

The "2 error" or SSE(b) reported here is incorrect. This means the F statistics and associated tail probabilities for the split plot portion of the analysis are also incorrect. To obtain the correct SSE(b) it is necessary to pool the SS's labelled SB(SUBPLOT*BLOCK) with the SS's labelled "Z error". I.E. 92+20 = 112. The F statistics for SUBPLOT(F) and SW(SUBPLOT*WHOLE) need to be recalculated using this corrected SSE(b).

$$F_{SUBPLOT} = \frac{156/3}{112/12} = 5.57$$

$$F_{SUBPLOT*WHOLE} = \frac{84/3}{112/12} = 3.0$$

OF SQUARES	CORRESPONDING RESIDUALS
1	R0
2	R1, R2, R3, R4

BMDP2V SP-1: WHOLE PLOT ARRANGED IN RCB

CASE	BLOCK	WHOLE	P0 R3	P1 R4	P2	P3	P4	R0	R1	R2
1	*1.00000	*1.00000	$\hat{Y}_{11\cdot}$ = 4.00000	-2.00000	0.00000	0.00000	2.00000	RO_1 = 1.00000	0.00000	-1.00000
			2.00000	-1.00000						
2	*2.00000	*1.00000	7.00000	-1.00000	3.00000	-5.50000	3.50000	0.00000	0.00000	0.00000
			-0.50000	0.50000						
3	*3.00000	*1.00000	7.00000	0.00000	3.00000	-0.50000	-2.50000	-1.00000	0.00000	1.00000
			-1.50000	0.50000						
4	*1.00000	*2.00000	6.00000	-2.00000	-4.00000	-2.00000	8.00000	-1.00000	0.00000	1.00000
			-2.00000	1.00000						
5	*2.00000	*2.00000	9.00000	-1.00000	-1.00000	-7.50000	9.50000	0.00000	0.00000	0.00000
			0.50000	-0.50000						
6	*3.00000	*2.00000	9.00000	0.00000	-1.00000	-2.50000	3.50000	1.00000	0.00000	-1.00000
			1.50000	-0.50000						

ERROR TERM	SUM OF SQUARES	RECOMPUTED FROM RESIDUALS	RELATIVE ERROR
1	16.00000	16.00000	0.00000
2	20.00000	20.00000	0.00000

$$\bar{Y}_{11\cdot} - \hat{Y}_{11\cdot} = 5.0 - 4.0 = 1.0 = RO_1$$

CPU TIME USED 0.183 SECONDS

PO & RO are whole plot predicted values and residuals, respectively. P1-P4 and R1-R4 are supposed to be split plot predicted values and residuals respectively but are computed from an incorrect value for SSE(b) (see previous page) and should therefore be ignored.

SP-2. SP-1 data with a covariate added and varying from split plot to split plot, first run.

```
BMDP2V - ANALYSIS OF VARIANCE AND COVARIANCE WITH REPEATED MEASURES.

Copyright (C) Regents of University of California.

  BMDP Statistical Software, Inc.
  1964 Westwood Blvd.  Suite 202        Phone (213) 475-5700
  Los Angeles, California  90025          Telex  4992203

Program Version: April 1985
                 (VM/CMS)
Manual  Edition: 1983, 1985 reprint.  State NEWS in the PRINT
                 paragraph for a summary of new features.

    APRIL 20, 1987  AT 16:44:22

PROGRAM CONTROL INFORMATION

/PROBLEM   TITLE IS 'SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT'.
/INPUT     VARIABLES ARE 10.
           FORMAT IS FREE.
/VARIABLE  NAMES ARE BLOCK, WHOLE, Z1, Z2, Z3, Z4, Y1, Y2, Y3, Y4.
/DESIGN    GROUPING ARE BLOCK, WHOLE.
           DEPENDENT ARE Y1, Y2, Y3, Y4.
           COVARIATE ARE Z1, Z2, Z3, Z4.
           LEVEL IS 4.
           NAME IS SUBPLOT.
           EXCLUDE IS 12.
           RESIDUAL IS MEAN.
           PRINT.
/END
```

Data Organization - all split plot values for Z and Y or a particular BLOCK by WHOLE combination must be in the same observation. See control language on page 10 for exact data layout.

} **Control Language (see page 10 for details)**

```
PROBLEM TITLE IS
SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

NUMBER OF VARIABLES TO READ IN. . . . . . . . .      10
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. .       0
TOTAL NUMBER OF VARIABLES . . . . . . . . . . .      10
NUMBER OF CASES TO READ IN. . . . . . . . . . . TO END
CASE LABELING VARIABLES . . . . . . . . . . . .
MISSING VALUES CHECKED BEFORE OR AFTER TRANS. . NEITHER
BLANKS ARE. . . . . . . . . . . . . . . . . . . MISSING
NUMBER OF WORDS OF DYNAMIC STORAGE. . . . . . .   32362

VARIABLES TO BE USED
     1 BLOCK         2 WHOLE         3 Z1            4 Z2             5 Z3
     6 Z4            7 Y1            8 Y2            9 Y3            10 Y4

INPUT FORMAT IS
FREE

MAXIMUM LENGTH DATA RECORD IS   80 CHARACTERS.

DESIGN SPECIFICATIONS

         GROUP  =    1   2
         DEPEND =    7   8   9  10
         COVAR  =    3   4   5   6
         LEVEL  =    4

                                                                      INTERVAL RANGE
  VARIABLE       MINIMUM  MAXIMUM  MISSING CATEGORY CATEGORY  GREATER LESS THAN
  NO. NAME        LIMIT    LIMIT    CODE     CODE     NAME      THAN    OR = TO

    1 BLOCK
                                              1.00000 *1.00000
                                              2.00000 *2.00000
                                              3.00000 *3.00000
    2 WHOLE
                                              1.00000 *1.00000
                                              2.00000 *2.00000
```

NOTE--CATEGORY NAMES BEGINNING WITH * WERE GENERATED BY THE PROGRAM.

NUMBER OF CASES READ. 6

BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

GROUP STRUCTURE

BLOCK	WHOLE	COUNT
*1.00000	*1.00000	1
*1.00000	*2.00000	1
*2.00000	*1.00000	1
*2.00000	*2.00000	1
*3.00000	*1.00000	1
*3.00000	*2.00000	1

SUMS OF SQUARES AND CORRELATION MATRIX OF THE ORTHOGONAL COMPONENTS POOLED FOR ERROR 2 IN ANOVA TABLE BELOW

0.01156	1.000		
3.07111	-0.907	1.000	
16.65067	0.802	-0.979	1.000

INSUFFICIENT DEGREES OF FREEDOM FOR SYMMETRY TEST

Like in SP-1, these values on the orthogonal components are incorrect because they are based on "Error 2" or SSE(b) which is computed incorrectly. See SP-1 page 3 and 5 for details.

CELL MEANS FOR 1-ST COVARIATE

	SUBP LOT = k								MARGINAL	
j = BLOCK =		*1.00000	*1.00000	*2.00000	*2.00000	*3.00000	*3.00000			
i = WHOLE =		*1.00000	*2.00000	*1.00000	*2.00000	*1.00000	*2.00000			
Z1	1	1.00000	2.00000	2.00000	4.00000	3.00000	3.00000		2.50000	
Z2	2	2.00000	0.00000	2.00000	1.00000	5.00000	2.00000	Z_{ijk}	2.00000	$\bar{Z}_{\cdot\cdot k}$
Z3	3	1.00000	2.00000	0.00000	3.00000	2.00000	4.00000		2.00000	
Z4	4	2.00000	4.00000	4.00000	4.00000	0.00000	7.00000		3.50000	

MARGINAL $\bar{Z}_{ij\cdot}$ =	1.50000	2.00000	2.00000	3.00000	2.50000	4.00000	2.50000 = $\bar{Z}_{\cdots}$
COUNT	1	1	1	1	1	1	6

CELL MEANS FOR 1-ST DEPENDENT VARIABLE

j = BLOCK	=	*1.00000	*1.00000	*2.00000	*2.00000	*3.00000	*3.00000		MARGINAL
i = WHOLE	=	*1.00000	*2.00000	*1.00000	*2.00000	*1.00000	*2.00000		
	SUBP LOT = k								
Y1	1	3.00000	3.00000	6.00000	8.00000	6.00000	10.00000		6.00000
Y2	2	4.00000	2.00000	10.00000	8.00000	10.00000	8.00000	Y_{ijk}	7.00000 $\bar{Y}_{\cdot\cdot k}$
Y3	3	7.00000	1.00000	1.00000	2.00000	4.00000	9.00000		4.00000
Y4	4	6.00000	14.00000	11.00000	18.00000	4.00000	13.00000		11.00000

MARGINAL $\bar{Y}_{ij\cdot}$ =	5.00000	5.00000	7.00000	9.00000	6.00000	10.00000	7.00000 = $\bar{Y}_{\cdots}$
COUNT	1	1	1	1	1	1	6

BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

ANALYSIS OF VARIANCE FOR 1-ST
DEPENDENT VARIABLE - Y1 Y2 Y3 Y4

ERROR TERM	SOURCE		SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.	GREENHOUSE GEISSER PROB.	HUYNH FELDT PROB.	REGRESSION COEFFICIENTS
	BLOCK $R(\rho\|\mu,\tau,\beta_1,\alpha,\alpha\tau)$		15.60000	2	7.80000************		0.0000			
	WHOLE $R(\tau\|\mu,\rho,\beta_1,\alpha,\alpha\tau)$		3.42857	1	3.42857************		0.0000			
	1-ST COVAR $R(\beta_1\|\mu,\rho,\tau,\alpha,\alpha\tau)$		16.00000	1	16.00000************		0.0000			4.00000 = $\hat{\beta}_1$
1	ERROR SSE(a)		0.00000	1	0.00000					
	SUBPLOT		105.22500	3	35.07500	8.89	0.0190	0.1182	0.0190	
	SB	SEE NOTE BELOW	77.81667	6	12.96944	3.29	0.1063	0.2624	0.1063	
	SW		50.23889	3	16.74630	4.24	0.0768	0.1992	0.0768	
	1-ST COVAR		0.26667	1	0.26667	0.07	0.8053			0.13333
2	ERROR		19.73333	5	3.94667					

EPSILON FACTORS FOR DEGREES OF FREEDOM ADJUSTMENT

ERROR TERM	GREENHOUSE-GEISSER	HUYNH-FELDT
2	0.3372	1.0000

POOLED REGRESSION COEFFICIENTS → The BMDP manual states that this estimate is "obtained from the weighted pooled cross-product matrix" ... and is used to compute adjusted cells means". This is probably of no interest to the experimenter.

1-ST COVARIATE 4.00000

The "2 error" or SSE(b) reported here is incorrect. This causes the SS's for the covariate in the above analysis to be incorrect also, thereby causing the SS's for SUBPLOT and SUBPLOT*WHOLE labelled (SW) to be adjusted incorrectly. The reason "2 error" or SSE(b) is wrong is because the SS's for SUBPLOT*BLOCK (labelled SB) should be pooled with "2 error". Another call to BMDP is required to get the correct split plot portion of the analysis and this begins on SP-2 page 8.

ERROR SUM
OF SQUARES   CORRESPONDING RESIDUALS

1 = SSE(a)   R0
2 ≠ SSE(b)   R1, R2, R3, R4

1PAGE   4 BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

$\bar{Y}_{11\cdot} - \hat{Y}_{11\cdot} = 5.0 - 5.0 = 0.0 = RO_1$

CASE	BLOCK	WHOLE	P0 / R3	P1 / R4	P2	P3	P4	R0	R1	R2
1	*1.00000	*1.00000	$\hat{Y}_{11\cdot}$ = 5.00000	-2.03333	-0.03333	0.03333	2.03333	RO_1 = 0.00000	0.03333	-0.96667
			1.96667	-1.03333						
2	*2.00000	*1.00000	7.00000	-1.06667	2.93333	-5.56667	3.70000	0.00000	0.06667	0.06667
			-0.43333	0.30000						
3	*3.00000	*1.00000	6.00000	0.10000	3.10000	-0.46667	-2.73333	0.00000	-0.10000	0.90000
			-1.53333	0.73333						
4	*1.00000	*2.00000	5.00000	-1.96667	-3.96667	-2.03333	7.96667	0.00000	-0.03333	0.96667
			-1.96667	1.03333						
5	*2.00000	*2.00000	9.00000	-0.93333	-0.93333	-7.43333	9.30000	0.00000	-0.06667	-0.06667
			0.43333	-0.30000						
6	*3.00000	*2.00000	10.00000	-0.10000	-1.10000	-2.53333	3.73333	0.00000	0.10000	-0.90000
			1.53333	-0.73333						

ERROR TERM	SUM OF SQUARES	RECOMPUTED FROM RESIDUALS	RELATIVE ERROR
1	0.00000	0.00000	******************
2	19.73333	19.73333	0.00000

ADJUSTED CELL MEANS ARE NOT COMPUTED WHEN THE INCLUDE OPTION IS USED. → **BMDP cannot compute adjusted means for anything other than straight factorials.**

CPU TIME USED   0.213 SECONDS

P0 and R0 are whole plot predicted values and residuals, respectively. P1-P4 and R1-R4 are supposed to be split plot predicted values and residuals but are computed from an incorrect value for SSE(b) (see previous page) and should therefore be ignored (correct values are computed in the run that follows).

SP-2 Split Plot, second run

```
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  BMDP Statistical Software, Inc.
  1964 Westwood Blvd.  Suite 202         Phone (213) 475-5700
  Los Angeles, California  90025           Telex  4992203

Program Version: April 1985
                 (VM/CMS)
Manual  Edition: 1983, 1985 reprint.  State NEWS in the PRINT
                 paragraph for a summary of new features.

    APRIL 10, 1987  AT 15:36:42

PROGRAM CONTROL INFORMATION

/PROBLEM    TITLE IS 'SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT'.
/INPUT      VARIABLES ARE 5.
            FORMAT IS FREE.
/VARIABLE   NAMES ARE BLOCK, WHOLE, SUBPLOT, Z, Y.
/DESIGN     GROUPING ARE BLOCK, WHOLE, SUBPLOT.
            DEPENDENT IS Y.
            COVARIATE IS Z.
            EXCLUDE IS 13,123.
            PRINT.
/END
```

Data Organization: Data arranged with only one Z and Y measurement per observation. See control language on page 10 for exact data layout.

Control Language (see page 10 for details)

```
PROBLEM TITLE IS
SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

NUMBER OF VARIABLES TO READ IN. . . . . . . . .       5
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. .       0
TOTAL NUMBER OF VARIABLES . . . . . . . . . . .       5
NUMBER OF CASES TO READ IN. . . . . . . . . . . TO END
CASE LABELING VARIABLES . . . . . . . . . . . .
MISSING VALUES CHECKED BEFORE OR AFTER TRANS. . NEITHER
BLANKS ARE. . . . . . . . . . . . . . . . . . . MISSING
NUMBER OF WORDS OF DYNAMIC STORAGE. . . . . . .   32362

VARIABLES TO BE USED
     1 BLOCK       2 WHOLE       3 SUBPLOT     4 Z           5 Y

INPUT FORMAT IS
FREE

MAXIMUM LENGTH DATA RECORD IS   80 CHARACTERS.

DESIGN SPECIFICATIONS

         GROUP =    1   2   3
         DEPEND =   5
         COVAR =    4
```

VARIABLE NO. NAME	MINIMUM LIMIT	MAXIMUM LIMIT	MISSING CODE	CATEGORY CODE	CATEGORY NAME	INTERVAL RANGE GREATER THAN	INTERVAL RANGE LESS THAN OR = TO
1 BLOCK							
				1.00000	*1.00000		
				2.00000	*2.00000		
				3.00000	*3.00000		
2 WHOLE							
				1.00000	*1.00000		
				2.00000	*2.00000		
3 SUBPLOT							
				1.00000	*1.00000		
				2.00000	*2.00000		
				3.00000	*3.00000		
				4.00000	*4.00000		

NOTE--CATEGORY NAMES BEGINNING WITH * WERE GENERATED BY THE PROGRAM.

NUMBER OF CASES READ.   24

BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

ρ_j = block effect

τ_i = whole plot effect

α_k = subplot effect (split plot)

β_2 = slope for split plot

ANALYSIS OF VARIANCE FOR 1-ST
DEPENDENT VARIABLE - Y

	SOURCE		SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.	REGRESSION COEFFICIENTS
	BLOCK	IGNORE*	20.20862	2	10.10431	1.14	0.3551	
	WHOLE	IGNORE*	6.10385	1	6.10385	0.69	0.4244	
	SUBPLOT	$R(\alpha \mid \mu,\rho,\tau,\alpha\tau,\beta_2)$	84.24310	3	28.08103	3.17	0.0678	
	BW	IGNORE*	9.45000	2	4.72500	0.53	0.6014	
	WS	$R(\alpha\tau \mid \mu,\rho,\tau,\alpha,\beta_2)$	37.47439	3	12.49146	1.41	0.2923	
	Z	$R(\beta_2 \mid \mu,\rho,\tau,\alpha,\alpha\tau)$	14.45000	1	14.45000	1.63	0.2281	$0.85000 = \hat{\beta}_2$
1	ERROR	SSE(b)	97.55000	11	8.86818			

*These whole plot effects were estimated correctly in the 1st call to BMDP on SP-2 page 6 and should be ignored here.

BMDP2V SP-2: SP-1 WITH COVARIATE VARYING WITH SPLIT PLOT

CASE	BLOCK	WHOLE	SUBPLOT		PREDICTD	RESIDUAL = split plot residuals
1	*1.00000	*1.00000	*1.00000	$\hat{Y}_{111}$ =	3.57500	-0.57500 = 3.0 - 3.575 = -0.575
2	*1.00000	*1.00000	*2.00000		6.57500	-2.57500
3	*1.00000	*1.00000	*3.00000		3.42500	3.57500
4	*1.00000	*1.00000	*4.00000		6.42500	-0.42500
5	*2.00000	*1.00000	*1.00000		6.00000	0.00000
6	*2.00000	*1.00000	*2.00000		8.15000	1.85000
7	*2.00000	*1.00000	*3.00000		4.15000	-3.15000
8	*2.00000	*1.00000	*4.00000		9.70000	1.30000
9	*3.00000	*1.00000	*1.00000		5.42500	0.57500
10	*3.00000	*1.00000	*2.00000		9.27500	0.72500
11	*3.00000	*1.00000	*3.00000		4.42500	-0.42500
12	*3.00000	*1.00000	*4.00000		4.87500	-0.87500
13	*1.00000	*2.00000	*1.00000		4.00000	-1.00000
14	*1.00000	*2.00000	*2.00000		3.00000	-1.00000
15	*1.00000	*2.00000	*3.00000		1.00000	0.00000
16	*1.00000	*2.00000	*4.00000		12.00000	2.00000
17	*2.00000	*2.00000	*1.00000		8.85000	-0.85000
18	*2.00000	*2.00000	*2.00000		7.00000	1.00000
19	*2.00000	*2.00000	*3.00000		5.00000	-3.00000
20	*2.00000	*2.00000	*4.00000		15.15000	2.85000
21	*3.00000	*2.00000	*1.00000		8.15000	1.85000
22	*3.00000	*2.00000	*2.00000		8.00000	0.00000
23	*3.00000	*2.00000	*3.00000		6.00000	3.00000
24	*3.00000	*2.00000	*4.00000		17.84999	-4.85000

NOTE: If the correct error SSE(b) sums of squares are to be obtained, we need to use the line "EXCLUDE IS 13,123." However, this does not allow the means to be adjusted for covariance. If the above statement is deleted from the program, adjusted means will be obtained but they are incorrect because SSE(b) is computed incorrectly.

ERROR TERM	SUM OF SQUARES	RECOMPUTED FROM RESIDUALS	RELATIVE ERROR
1	97.55000	97.55000	0.00000

ADJUSTED CELL MEANS ARE NOT COMPUTED WHEN THE INCLUDE OPTION IS USED.

CPU TIME USED 0.216 SECONDS

SP-3. Whole Plots Arranged in CRD With Constant Covariate Within Whole Plot

```
BMDP2V - ANALYSIS OF VARIANCE AND COVARIANCE WITH REPEATED MEASURES.

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  BMDP Statistical Software, Inc.
  1964 Westwood Blvd.  Suite 202          Phone (213) 475-5700
  Los Angeles, California  90025            Telex  4992203

Program Version: April 1985
                 (VM/CMS)
Manual  Edition: 1983, 1985 reprint.  State NEWS in the PRINT
                 paragraph for a summary of new features.

      MAY  1, 1987  AT 14:21:52

PROGRAM CONTROL INFORMATION

/PROBLEM  TITLE IS 'SP-3: WHOLE PLOTS ARRANGED IN CRD WITH
CONSTANT COVARIATE'.
/INPUT    VARIABLES ARE 4.
          FORMAT IS FREE.
/VARIABLE NAMES ARE A, Z, Y1, Y2.
/DESIGN   GROUPING IS A.
          DEPENDENT ARE Y1, Y2.
          COVARIATE IS Z, Z.
          LEVEL IS 2.
          NAME IS B.
          PRINT.
/END
```

Data Organization - all split plot values for a particular SUBJECT by A combination must be in the same observation. See Control Language on Page 11 for the exact data layout

} Control Language (see page 11 for details)

```
PROBLEM TITLE IS
SP-3: WHOLE PLOTS ARRANGED IN CRD WITH CONSTANT COVARIATE

NUMBER OF VARIABLES TO READ IN. . . . . . . . .        4
NUMBER OF VARIABLES ADDED BY TRANSFORMATIONS. .        0
TOTAL NUMBER OF VARIABLES . . . . . . . . . . .        4
NUMBER OF CASES TO READ IN. . . . . . . . . . . TO END
CASE LABELING VARIABLES . . . . . . . . . . . .
MISSING VALUES CHECKED BEFORE OR AFTER TRANS. . NEITHER
BLANKS ARE. . . . . . . . . . . . . . . . . . . MISSING
NUMBER OF WORDS OF DYNAMIC STORAGE. . . . . . .   32362

VARIABLES TO BE USED
     1 A             2 Z             3 Y1            4 Y2

INPUT FORMAT IS
FREE

MAXIMUM LENGTH DATA RECORD IS   80 CHARACTERS.

DESIGN SPECIFICATIONS

         GROUP =    1
         DEPEND =   3   4
         COVAR =    2   2
         LEVEL =    2

                                                                    INTERVAL RANGE
  VARIABLE      MINIMUM  MAXIMUM  MISSING CATEGORY CATEGORY  GREATER LESS THAN
  NO. NAME       LIMIT    LIMIT    CODE     CODE     NAME     THAN    OR = TO

    1 A
                                            1.00000 *1.00000
                                            2.00000 *2.00000

NOTE--CATEGORY NAMES BEGINNING WITH * WERE GENERATED BY THE PROGRAM.

NUMBER OF CASES READ. . . . . . . . . . . . . .        8
```

BMDP2V SP-3: WHOLE PLOTS ARRANGED IN CRD WITH CONSTANT COVARIATE

GROUP STRUCTURE

A	COUNT
*1.00000	4
*2.00000	4

CELL MEANS FOR 1-ST COVARIATE

	B	A = *1.00000	*2.00000	MARGINAL
Z	1	4.50000	5.25000	4.87500
Z	2	4.50000	5.25000	4.87500
MARGINAL		4.50000	5.25000	4.87500
COUNT		4	4	8

STANDARD DEVIATIONS FOR 1-ST COVARIATE

	B	A = *1.00000	*2.00000
Z	1	2.64575	4.42531
Z	2	2.64575	4.42531

CELL MEANS FOR 1-ST DEPENDENT VARIABLE

	A	=	*1.00000	*2.00000	MARGINAL
		B			
Y1		1	14.25000	18.75000	16.50000
Y2		2	10.00000	13.75000	11.87500
	MARGINAL		12.12500	16.25000	14.18750
	COUNT		4	4	8

STANDARD DEVIATIONS FOR 1-ST DEPENDENT VARIABLE

	A	=	*1.00000	*2.00000
		B		
Y1		1	4.34933	4.78714
Y2		2	3.65148	4.78714

BMDP2V SP-3: WHOLE PLOTS ARRANGED IN CRD WITH CONSTANT COVARIATE

τ_i = A effect (whole plot) α_k = B effect (split plot)
β_1 = Z slope for whole plot

ANALYSIS OF VARIANCE FOR 1-ST
DEPENDENT VARIABLE - Y1 Y2

	SOURCE	SUM OF SQUARES	DEGREES OF FREEDOM	MEAN SQUARE	F	TAIL PROB.	REGRESSION COEFFICIENTS
	A(whole plot)$R(\tau\|\mu,\beta_1,\alpha,\alpha\tau)$	44.49160	1	44.49160	3.63	0.1151	
	1-ST COVAR $R(\beta_1\|\mu,\tau,\alpha,\alpha\tau)$	166.57680	1	166.57680	13.59	0.0142	$1.02194 = \hat{\beta}_1$
1	ERROR SSE(a)	61.29820	5	12.25964			
	B(split plot)$R(\alpha\|\mu,\tau,\beta_1,\alpha\tau)$	85.56250	1	85.56250	80.53	0.0001	
	BA $R(\alpha\tau\|\mu,\tau,\beta_1,\alpha)$	0.56250	1	0.56250	0.53	0.4943	
2	ERROR SSE(b)	6.37500	6	1.06250			

ERROR SUM OF SQUARES	CORRESPONDING RESIDUALS
1	R0
2	R1, R2

BMDP2V SP-3: WHOLE PLOTS ARRANGED IN CRD WITH CONSTANT COVARIATE

$= \bar{Y}_{11\cdot} - \hat{Y}_{11\cdot} = 9.0 - 10.6 = -1.6$

CASE	A		P0	P1	P2	R0	R1	R2
1	*1.00000	$\hat{Y}_{11\cdot} =$	10.59208	2.12500	-2.12500	-1.59208	-1.12500	1.12500
2	*1.00000		12.63597	2.12500	-2.12500	0.86403	-0.62500	0.62500
3	*1.00000		15.70180	2.12500	-2.12500	1.29820	0.87500	-0.87500
4	*1.00000		9.57014	2.12500	-2.12500	-0.57014	0.87500	-0.87500
5	*2.00000		11.90674	2.50000	-2.50000	0.59326	0.00000	0.00000
6	*2.00000		19.06033	2.50000	-2.50000	3.43965	0.00000	0.00000
7	*2.00000		21.10422	2.50000	-2.50000	-3.60423	0.00000	0.00000
8	*2.00000		12.92868	2.50000	-2.50000	-0.42868	0.00000	0.00000

ERROR TERM	SUM OF SQUARES	RECOMPUTED FROM RESIDUALS	RELATIVE ERROR
1	61.29820	61.29820	0.00000
2	6.37500	6.37500	0.00000

ADJUSTED CELL MEANS FOR 1-ST DEPENDENT VARIABLE

	B	A = *1.00000	*2.00000
Y1	1	14.63323	18.36677
Y2	2	10.38323	13.36677

The means $\bar{Y}_{i\cdot k}$ are adjusted only for the whole plot covariate because the covariate is constant for all split plots within one whole plot. The formula for adjusting $\bar{Y}_{i\cdot k}$ means is

$$\bar{Y}_{i\cdot k}\ \text{adj} = \bar{Y}_{i\cdot k} - \hat{\beta}_1(\bar{Z}_{i\cdot\cdot} - \bar{Z}_{\cdot\cdot\cdot})$$

An example for $\bar{Y}_{1\cdot 1}$

$$\bar{Y}_{1\cdot 1}\ \text{adj} = 14.25 - 1.02(4.5 - 4.9) = 14.63$$

P0 and R0 are whole plot predicted values and residuals, respectively. P1-P2 and R1-R2 are split plot predicted values and residuals.

$$Y_{ijk} = P0 + R0 + Pj + Rj$$

e.g.

$$Y_{111} = 10.59208 - 1.59208 + 2.125 - 1.125 = 10.00$$