Shelter Island II

Gell-Mann

~~Reta~~ Renormalizability ~~of~~ of present theory.

W = X boson discovered.

Parameters, eg. ~~of~~ coupling of Higgs boson

Why 3 families? How many Higgs bosons? Unified Yang-Mills theory. Is renormalizability a good criterion?

Peccei-Quinn theory Higglet, also called ~~axia~~ axion. Could have very small coupling, large $\langle\phi\rangle$, then unobserved.

"Dial" parameters to near zero. Limit of high energy.

10^{15} GeV. Coupling constant $\sim 1/40$

m_τ / m_b is OK

m_μ / m_s not OK, if our m_s is OK.

SO(10) somewhat prettier. It takes guts to extrapolate GUTS to 10^{15} GeV. Is 10^{15} = Planck mass of 10^{19}?

~~Proton decay~~, ~~family transitions~~ by neutral ~~curr~~ currents.

Matter > antimatter to be explained. SU_5

SO_{10} gives 3 neutrino masses.

~~Boson QCD~~

Small masses compared to unification mass.

$$\frac{\mu}{M_{unif}} \sim \exp\left(-\frac{\text{const}}{\alpha_{unif}}\right)$$

Since $\alpha_{unif} \simeq \frac{1}{40}$, this explained OK

If you dial ~~masses~~ constant small, they stay small i.e. renormal.

Fundamental fields to be called haplons, others composite. Higgs might be comp.

$n=1$ supersymmetry, works to some extent. Goldstino breaks ~~sy~~ supersymmetry. ~~Quantities can be~~ only be estimate.

Supergravity $n=8$, many particles $1+8+28+70$ etc. particles, completely renormalizable.

Curiosity: Triplet of color & triplet of families

Ellis, Gaillard, Zumino (1980). All known particles are composite except graviton.

String theories No. of scales, Regge ~~tk~~ trajectories

$N=4$ supergravity & $N=4$ Super-Yang-Mills

Super-strings II, no open strings. $N=8$ supergrav., no supermass. Appears finite.

Renormalization of 0-point energy. Takes revenge in gravity. Cosmological constant in astronomy 10^{-120}.

Quantum (bubbles) (fluctuations) (Hawking)

Inflationary Universe Linde-Guth: Cosmological constant jumps from finite to zero. Hawking: no problem.

Weinberg

Calc. of Fine Structure Constants

Charges from Extra Dimensions

Kaluza 21, Klein 26, Mandel

5 dimensions Minkowski$^4 \times S^1$

S^1 in tight circle, radius ρ

Mass ℓ/ρ ρ undetermined

Charge $e = \sqrt{16\pi G}/\rho$ for $\ell = 1$

$1/\rho \simeq 10^{17}$ GeV, $\rho \sim 10^{-31}$ cm

Higher Dimensions 4 + N dimensions

De Witt 1964, many other papers

Symmetry bringing you back to same pt.

rms of circumference replaces $2\pi\rho$

Some progress in calc. ratios of coupling constants.

f massless matter fields in 4+N dimensions

Energy density $f\,\rho^{-4-n}$

$\rho^2 \sim Gf$ $\qquad e \simeq \frac{\sqrt{G}}{\rho} \simeq \frac{1}{\sqrt{f}}$

Classical gravitat. fields. Large f.

Gauge coupling constants < 1.

Size of fine structure constant has not changed since quasars.

If N is odd, no possible additions to $\mathcal{L}$ can eliminate dimen

$$g^{vac} = \begin{matrix} \eta_{\mu\nu} & 0 \\ 0 & \tilde{g}_{nn}(y) \end{matrix}$$

$$\Gamma_{eff} = \int d^4x \; V_{eff}(\tilde{g})$$

$$V_{eff} = \int d^N y \sqrt{\tilde{g}} \; \frac{\tilde{R}(y) + \bar{\Lambda}}{16\pi \bar{G}} \qquad + \underset{\text{matter loops}}{V(\tilde{g})}$$

$\bar{\Lambda}$ cosmological constant in 4+N dimens.

$$\frac{N-1}{8\pi G\rho^2} = V(\rho) - \frac{\rho}{N}\frac{dV(\rho)}{d\rho}$$

$$\rho^2 = \frac{8\pi G (N+4)\ C_N}{N\ (N-1)}$$

C_N's of order 1.

Quantize in space ρ. Integrals obtained

Appelquist & Chodos calc. results, for bosons $C_N > 0$ for $N = 3$ to 19, negative for $N = 1$ or $\gtrsim 21$.

For fermions <u>and</u> bosons, all OK if $N = 7, 11$ etc.

Embarassing that C_N is small, bec.

$$g \simeq \frac{N}{\sqrt{C_N}}$$

This is couplg. const. at 10^{17} GeV $=$ ~~10^{19}~~ $\frac{\text{Planck}}{\sqrt{16\pi}}\ g$

Curvature term $\propto -\rho^{N-2}$

$\bar{\Lambda}\Omega \propto \rho^N$

Matter term $V \sim r^{-4}$ prevents collapse

<u>Increase temp.</u> effect. : V gets term $\rho^N T^{N+4}$, is < 0

Crit. temp. order 10^{17} GeV (Kaluza-Klein.

2-loop divergences, 1^{st} approx. is good.

He thinks there is no finite th. incl. gravity.

$p\bar{p}$ machine $1.7 \cdot 10^9$

$W \to e\nu$

~~Calorimeter~~

Hi energy, large angle isolated electron

Transverse en. of electron 20 – 50 GeV

" " of ν (missing mass)

about 20 events of W

Max. likelihood fit $M_W \simeq 81 \pm 2$ GeV

Rapidity fits V–A

Transverse momentum of W abo.

$\sigma \cdot B = 2$ nb

Candidates for Z^0.

1. One event $\mu^+\mu^-$ $M_Z = 95 \pm 10$ GeV
2. e^+e^-, one $E = 49$ GeV

Possibly $M_Z = 102 \pm 7$

3. e^+e^- $M = 93$
4. " 98?

Vicki

Kramers 1948 (Ann Arbor) suggested calc. diff. betw. E (2s, H). Shelter Isl. gave actual Lamb shift.

Lamb Old expts. Lamb's history

1938-51 Columbia

H, D $n = 2$

He^+ $n = 2$

QED K, L

1951-56 Stanford

H $n = 3, 4$

He $2\,^3P_J$, $3\,^3P_J$

1956-62 Oxford

Laser theory

1962-74 Yale

Very high Rydberg states of He

Experiments, Shifts 1957. + MHz

Triebwasser, D.L. .77(6)

R.S. (70) .90(6)

L.P. (75) .892 (20)

N.A.U. (79) .862 (20)

L Pipkin (81) .845 (9)

Calculation

Erickson (71) .930 (10)

Mohr (75) .884 (13), with r_p = .862 (12)

Kinoshita: Mohr is correct

Kinoshita

Heard abt. g-2 in Japan

1947 Nafe, Nelson, Rabi hfs ≠ Dirac

Breit suggested g ≠ 2

Kusch, Foley after

1948 Schwinger $\alpha/2\pi$

1957 Sommerfied 4th order

Suura $g_\mu - g_e$

1967 $\alpha_\mu - \alpha_e$ 6th order, by renormaliz. group & mass singularity theorem

Parker, Taylor, Langenberg

1969 – 82 Kinoshita etc.

$a_e^6 = 1.176\ 5 \quad (\alpha/\pi)^3$

$= 24.45 \quad (\alpha/\pi)^3$

72 Electron-weak

77 Van Dyck, Schwinberg, Dehmelt, free electron

$a = 1,159,652,40\ (20) \cdot 10^{-11}$

~~ab~~ a_μ at Cern

81 Dehmelt 652 200 (40) $\cdot 10^{-12}$

222 (50

Theory to 8th order − 0.8 (1.5)

$\alpha^{-1} = 137.035\ 963\ (15)$ Williams, Olsen (79)

a_e theory = 1.159 652 460 (47 theory,

127 error in α)

a_μ Expt. 1,165,937 (12) $\cdot 10^{-9}$

Theor. 1,165,920 (3,5) $\cdot 10^{-9}$

of which 65.5 (3.4) is hadron contrib., weak inter. $2 \cdot 10^{-9}$

Improve error in hadron contrib.

Determine α from g-2

$\alpha(g-2)$ = 137.035 992 (5 exp.) (5 theor.)

muonium hfs 988 (21 exp.) (2.7 theor.)

Josephson 963(15) Williams, Olsen

Quantized Hall 968 (23) [illegible]

μ hfs

$$R_{Hall} = \frac{vB}{i} = \frac{h}{e^2 n}$$

Self-energy

$$W = -\frac{2e^2}{3\pi\hbar c^3}\int_0^K k\,dk \sum \frac{|v_{mn}|^2}{E_n - E_m + k}$$ My paper

$$W_0 = \quad " \quad \int k\,dk \frac{(v^2)_{mm}}{k}$$ free electron

v has only diagonal elements

$$(v^2)_{mm} = \sum_n |v_{mn}|^2$$

$$\therefore W - W_0 = + \quad " \quad \int dk \sum (E_n - E_m) \frac{|v_{mn}|^2}{E_n - E_m + k}$$

$$= \frac{"}{m^2}\sum (E_n - E_m)|p_{mn}|^2 \ln \frac{K}{|E_n - E_m|}$$

(principal value)

$K = mc^2$

$$\sum (E_n - E_m)|p_{mn}|^2 = -\hbar^2 \int \psi_m^* \nabla V \cdot \nabla \psi_m \, d\tau$$

$$= \frac{1}{2}\hbar^2 \int \nabla^2 V \psi_m^2 \, d\tau = 2\pi\hbar^2 e^2 Z \psi_m^2(0)$$

Must then calc.

$$\frac{\sum (E_n - E_m)\, p_{mn}^2 \ln (E_n - E_m)}{\sum (E_n - E_m)\, p_{mn}^2} = \ln (E_n - E_m)_{Av}$$

$$(E_n - E_m)_{Av} = 17.8 \text{ Ry}$$

PR 72, 339 (47)

~~T. D. Lee~~ Nambu

Tomonaga started covariant theory in '43, was ready when Lamb shift was discovered

T. D. Lee

Time as dynamical variable

Relativ. Q Mech $\phi(r, t)$ operators, $\vec{r}, t$ parameters.

Discrete mechanics, classical

Within T, have N space-time positions

$$A = \sum_n \tfrac{1}{2} v_n^2 - \tfrac{1}{2}\left(V(r_n) + V(r_{n-1})\right)\} \cdot (t_n - t_{n-1}$$

$$\frac{\partial A}{\partial t_n} = \tfrac{1}{2} v_n^2 = \tfrac{1}{2}\left[V(\vec{r}_n) + V(\vec{r}_{n-1})\right] = E_{n+1}$$

$$v_n = \frac{\vec{r}_n - \vec{r}_{n-1}}{t_n - t_{n-1}}$$

Reproduce uniform motion, harmonic oscillator.

Non-Rel quantum

$$\mathcal{G}(\tau) = \int e^{iA} J \prod_n d^3 r_n \, dt_n$$

$$A = \sum \frac{(r_n - r_{n-1})^2}{2(t_n - t_{n-1})} = \tfrac{1}{2}(t_n - t_{n-1})\left[V(r_n) + V(r_{n-1})\right]\}$$

$(\vec{r}_n)_{op}$ $(t_n)_{op}$

Unitarity is questioned. There may be problems only in order $e^{-T/l}$, where l is elementary time unit.

Lamb shift expt. requires that $l < 1.6 \cdot 10^{-14}$ cm

Relativistic Q M

Massless Scalar

$$G = e^{-A} \int \prod d^4x_i \, d\phi_i$$

$\rho = \ell^{-4}$ fundamental constant

$$A = \frac{1}{2} \sum \lambda_{ij} (\phi_i - \phi_j)^2 - j_i \phi_i$$

like Kirchhoff law

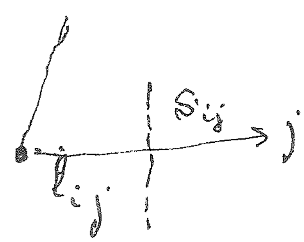
Draw a hypersphere thru any group of D+1 sites, then D+1 sites form a cluster.

Theorem: Form a D-simplex for every cluster, using its sites as vertices. Non-overlapping simplices.

$\sum \lambda_{ij} (\phi_i - \phi_j) = j_i$ $\sum$ all sites linked to i.

~~When $j_i = 0$, then $\phi_i = \phi_0 + \vec{E}$~~

Choose $\lambda_{ij} = \dfrac{S_{ij}}{l_{ij}}$



ϕ_i is contin. fn. of $\vec{r}_1, \ldots \vec{r}_N$

Gives no. of links, plaquettes

Gauge theory. Wilson uses cubic lattice, gives confinement easily. Lee's random lattice should give transition to weak coupling. not easily.

Calculated transition from strong to weak coupling, is smooth.

SU(3), 4^4 lattice. Av. sweeps for each β is 40,000

Gets very sharp max. in specific heat as fn. of β.

<u>Jackiw</u>

Non-perturbative calc. Skyrme suggestions.

Infinities tamed only recently.

Local field dynamics, synthesis of QCD and flavor.

<u>States</u> need spontaneous symmetry breaking

Solitons: 1-D: kink, 2-D vortex, 3-D monopole.

Solve classical field,

Weak coupling g:

Mass = g^{-2} · charact. mass

Stability: or energy barrier topologic qu. no.

Interactions: Soliton-soliton g^{-1}

Excitation.

bosons of integral spins ⟷ fermions

Fractional fermion no.

2. Processes

Yang-Mills th. tunneling betw. states of same energy.

Band formation - Tunneling

Learned topology from mathematicians

1. Yang-Mills gauge transform.

Non-Abelian gauge theory $U = e^{iT^a\theta_a}$

Not all deformable to unitary transf.

Large gauge transf. $|state\rangle = e^{i\theta}|state\rangle$

Analog of Bloch theory $\psi(x+a) = e^{i\theta}\psi(x)$

Axial vector anomaly : conseq. of gauge th.

Physical Results

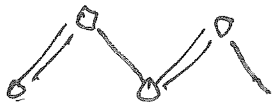
1. $\pi^0 \to 2\gamma$
2. quark - lepton balance
3. No chiral $U(1)$

Uncertain results!

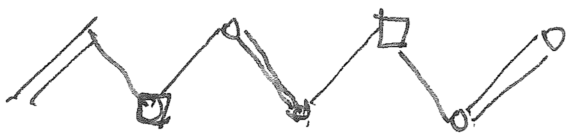
4. CP violation by vacuum angle
5. Monopole
6. Mon.-induced baryon decay
7. Color breaking by monopoles

Domain boundaries are solitons in solid state physics

Poly-acetylene 2 ground states



2 solitons



Count links betw. solitons

Quantization

1. Dynam. variables : E, ang. momentum $\sim \hbar$
2. Soliton number : No $\hbar$, topology
3. Qu. of parameters in Lagrangian

Dirac monopole

$$\mathcal{L} = \frac{1}{2} m \dot{r}^2 + \frac{e}{c} \dot{r} \cdot A$$

Action, $\mathcal{L}$ not invariant. Change of $\mathcal{A} = 2\pi\hbar\, n$

$$\therefore\ eg = \hbar c \frac{n}{2}$$

(2) <u>3-D gauge theory</u>

Eq. of motion gauge invariant

$$\text{Action} = \int d^3x\, \mathcal{L} \rightarrow \text{Act.} + n\, \frac{8\pi^2}{g^2}\, w$$

$$\frac{4\pi m}{g^2} = \hbar n$$

(3) N Weyl fermion doublets

Integrate out fermions

SU(2) chiral currents.

Fermion no. conserved, but not gauge invariant.

(4) Effective current algebra

Wess-Zumino action, constant ±

Physical results

2) Topological mass of gauge field.

4) Anomaly term in σ models

Skyrme: Non-linear σ-model

Witten, Balachandran find low levels of baryo[ns] t' Hooft

Elementary particles & black hole

Entropy of black hole $S = 4\pi M^2$ (perhaps divide by M)

Particles $\lambda_{compt} \sim 1/M$ Black hole $R \sim M$

Planck mass $\lambda \sim R$ (Schwarzschild)

Emission of γ's by black hole

Marshak

~~1)~~ Cosmic ray meson at sea level (37). J.G. Wilson found $\sigma_{scatt} \lesssim 1\%$ of theory, σ_{prod} in upper atmos. $\cong \sigma_{nuc}$ (Schein). Discrepancy (Nordheim & Hebb, 1939). Conversi, Pancini & Piccioni (1945-7)

2. Nuclear force: $1/r^3$ divergence, Møller - Rosenfeld, Strong couplg. theory (Pauli etc.)

3. Japanese (Yukawa, Tomonaga, Sakata, Inoue). Paper reached U.S. at end 47.

4. Fermi, Teller & Weisskopf found slowing down time 10^{-13} sec. Real discrepancy, Oppie instruction to Shelter Island ±

5. Marshak's 2-meson theory. Powell π meson. Underground penetration. Kink at 1000 m due to π lifetime.

6. Clarification (1948)

Planck Mass

$$\frac{GM}{r^2} = \frac{v^2}{r} \qquad v^2 = GMr \qquad v^2 r^3 = G^2 M^2 r^3$$

$$Mvr = \hbar$$

$$M^2 v^2 r^2 = \hbar^2 = G M^3 r^3$$

Hawking 3/6/83

~~Super~~ N = 8 Supergravity could be the ultimate theory

1) It may be finite to all orders

2) It unifies gravity w. ~~matter~~ & other forces

But

1) Gd. state is anti-de Sitter space w. large negative cosmo, $\Lambda = -e^2 M_p^2$

2) Observed universe not supersymmetric

3) Not correct spectrum of particles

Break symmetry:

One-loop divergences

$$\Delta I = A\chi + B\int \Lambda^2 dV$$

χ = Euler number

$B = 0$ for $N \geq 5$

$A = 3 - N$ for $N \geq 3$

Add counterterm $-K\chi$ to effective action

Topological coupling constant K obeys renormalization group eq.; soln.:

$$K = K_0 - 5 \log l_x / l_p$$

l_p Planck length, l_x typical length scale.

Fluctuations of .

$$\Lambda = \Lambda_0 + \sqrt{\rho} + 8\pi \frac{\mu_0}{M_p^2}$$

fluctuations? bubbles

1) Either form or scalar field w/o kinetic term leads to a range of different values of $\Lambda_{apparent}$ in path integral
2) Negative values of $\Lambda_{apparent}$ are suppressed
3) Positive values lead to a small Euclidean 4-volume
4) Most favored is $\Lambda = 0$.

~~Minkes~~ Guth — Inflationary Universe

1. One or more phase transitions
   $T = 10^{14}$ GeV
2. Magnetic monopoles exist
   $M = 10^{16}$ GeV
3. Baryon finite life $10^{30} - 10^{33}$ yrs.

Standard theory

$$\left(\frac{\dot R}{R}\right)^2 = \frac{8\pi}{3} G\rho \;, \quad \frac{\dot R}{R} = H$$

Metric increasingly flat as $t \to 0$
Mass density dominated by radiation of

$$T^2 = \frac{1}{40}\frac{M_p}{t}$$

$\rho = 10^{75}\ g\ cm^{-3}$

$R = 10\ cm$

Problems 1. Magn. monopoles, topological knots

$n_M = \xi^{-3}$ ξ = correlation length

$\xi <$

$n_M / s \geq 10^{-13}$, s = entropy density

remains constant since then, gives

$\Omega > 10^{12}$

2. Horizon problem. Large-scale homogeneity.

Rad. decoupled at $T \simeq 4000\,°K$. when $t \simeq 10^5$ yr.

Horizon length = distance of light travel since $t = 0$

= 90 times t at decoupling

3. Flatness Problem

Today, $0.01 < \Omega < 10$

$$\rho_c = \frac{3H^2}{8\pi G}$$

Then at $T = 10^{14}$ GeV must have

$\Omega = 1 \pm \mathcal{O}(10^{-49})$

bec. Ω gets increasingly far from 1

4. Smoothness Problem

Early universe very unstable vs. perturbations.

$\frac{\delta \rho}{\rho} \simeq \frac{1}{\sqrt{N}} \sim 10^{-29}$ originally

$\frac{\delta \rho}{\rho}$ prop. to t, grows to 10^{+15}

If you start at 10^{-25} s at $T = 10^{9}$ GeV

New Inflationary Universe

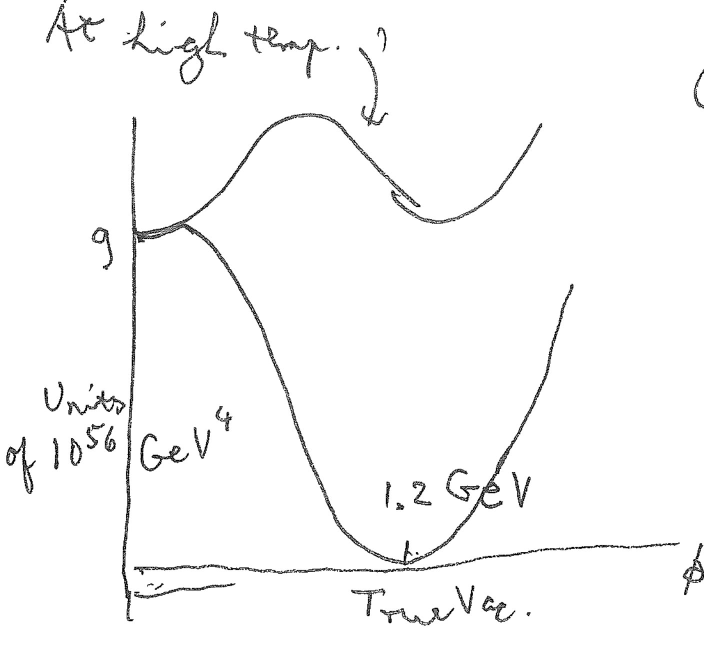
A single bubble can (probably) expand enough to encompass

SU_5 Coleman-Weinberg potential

$$\Phi = \sqrt{\frac{2}{15}}\,\phi \begin{pmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ 0 & & & & -3/2 \end{pmatrix}$$

At origin, $\frac{\partial^2 V}{\partial \phi^2} = \frac{\partial^3 V}{\partial \phi^2} = 0$. Here have false vacuum

$V(0) \simeq 10^{56}$ GeV^4



Crit. temp. when 2 minima equal. $\langle \Phi \rangle 0$

Supercools w. $\langle \Phi \rangle = 0$ to $T < T_c$

$\rho \to \rho_0$ (energy density of false vac.)

Flat Space (de Sitter).

All perturbations die out.

$\dot{R}/R$ = const.

<u>Observed universe</u>

Baryon no. 10^{78-79}

~~Matter energy~~ energy = 10^{78-80} GeV

Electric chge. = color chge. = ang. momentum $\simeq 0$

False vacuum is the source of all matter energy, and ~~entropy~~

Universe devoid of all conserved quantities

Ref. Tryon; Brout, Englert & Spindel; Gott; 3 others

<u>Phase Transition</u>

ϕ fluctuates, $\xi \sim \chi^{-1} \sim 10^{-11}$ proton diameters

ϕ rolls classically down potential

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} = -\frac{\partial V}{\partial \phi}$$

Exponential expansion much faster than rolling

$R(t)$ grows by factor $> 10^{25}$, ξ grows to > 10 cm

<u>Solution of problems</u>

1. Monopole

2. Horizon: Univ. comes from a region of correlated ϕ.

3. Flatness

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3} G\rho - \frac{k}{R^2}, \quad k \simeq 1$$

While ϕ starts to roll, $\rho \simeq \rho_0$, $k/R^2 \to 0$ by inflation

4. Smoothness

Qu. fluctuations in $\phi(\vec{x})$

$\frac{\delta\rho}{\rho}$ are scale-invariant

Mining SU_5 gives 10^2 (too much)

Hawking & Moss ~ 1

Geometric hierarchy $\sim 10^{-4}$ (perfect)

Linde

Problems

1. SU5 Coleman-Weinberg
2. Supersymmetric version

   No baryons.
3. Supergravity coupled to matter (primordial inflation)

   Inflation large, $\delta\rho/\rho \sim 10^{-4}$ — Ellis, Nanopoulos

   Monopoles? Reheating & baryons??

   Monopoles produced after inflation

   No phase transitions.

4. Improve model (Linde)

$$V = 3\mu^6(1 - \alpha^2\varphi^2 + \frac{\alpha}{4}\varphi^4)$$

Duration of inflation $(3 \cdot 10^{11}\ \text{GeV})^{-1}$

Extent of inflation e^{500}

~~First~~ Pathology

$$\langle\varphi^2\rangle = \frac{H^3}{4\pi^2} t \qquad H = \text{Hubble}$$

After inflation,

$$\varphi = \langle\varphi^2\rangle \sim H$$

Universe splits into many regions, dragons, etc.

No monopoles

Chaotic Inflation

$$V(\varphi) = \frac{\lambda}{4}\varphi^4$$

$$\varphi = \varphi_0 \exp\left(-\sqrt{\frac{\lambda}{6\pi}} M_p t\right) \qquad \text{if } \varphi \geq M_p$$

$$H^2 \geq 20\, M^2(V)$$

$$V(\varphi) \leq M_{Planck}$$

5. Inflation inevitable in chaotic

Chaos ⇒ Universe w. small fluctuation

~~Linde~~ Feynman QCD

1-Flav quarks

2+1 dimensions (renormalization)

SU(2)

No dynamic quarks

Outline

Why gap to lowest glueball

Confinement of static quarks

Nature of string or bag

Color correlation hard to maintain

QED fluctuations

2-D space	3-D space
$E \sim 1/L$	
$B^2 L \sim 1/L$	$B^2 L^3 \sim 1/L$
$B \sim L^{-3/2}$	$B \sim 1/L$
$A \sim L^{-1/2}$	$A \sim 1/L$
$\int A\, dl = \text{Flux} \sim L^{1/2}$	$\text{Flux} \sim \ln L$

~~S(A)~~ $\vec{A}$ is color vector.

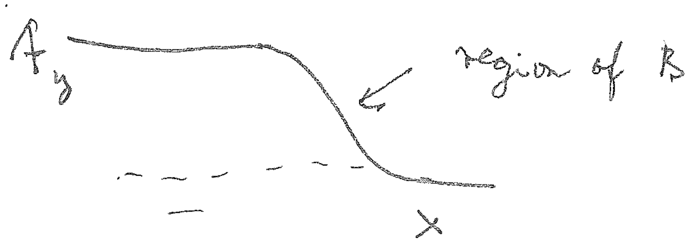
Euclidean space-time

Action $\dot{A}_x^2 + \dot{A}_y^2 + (\partial_x A_y - \partial_y A_x - A_x A_y)^2 = \mathcal{A}$

~~[illegible]~~ exp $\int \exp - \frac{1}{2}\mathcal{A}$

If there is no state which ~~can~~ can decay r·

Why can A_y not diffuse? Bec. then $\partial_x \dot{A}_y$ becomes large



$$A = \lambda \nabla \lambda$$