Ion control in closed growing systems with inert media: controller settings and modes of operation

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ABSTRACT

The recent market introduction of ion-selective sensors in horticulture removes one of the barriers towards accurate control of the supply of individual ions to greenhouse crops cultivated in soil-less closed water systems. In previous work, controllers have been designed that are able to compensate for transpiration and nutrient uptake by tracing a set point for pulse-averaged drain flow and individual ion concentrations in the drain, based on ion-selective sensor information.

The objective of the current work is to investigate the desired operation mode of the nutrient controller under the assumption of successful constant drain flow control. Two cases are distinguished: demand satisfying control and supply regulating control. Using fundamental mass balances and transport equations, conditions are derived to which set-points of the constant drain concentration controllers should obey in order to ensure non-inhibiting nutrient supply. It is concluded that uptake regulation below the demand is most likely difficult to achieve with a drain concentration controller, whereas it is very suitable for demand satisfying control.

KEYWORDS. Greenhouse, Irrigation systems, Nutrients, Control

INTRODUCTION

Closed water systems have been introduced in horticultural practice both for economic and environmental reasons. They are currently operated by adjusting the recipe of the incoming water on the basis of irregular chemical analysis of the return water, collected in reservoirs. Moreover, there is widely used EC and pH control to influence the nutrient solution on-line.

The advent of ion-specific electrodes in principle allows a much more accurate control of supply to the plants than is possible by the current EC control. The ratio of uptakes of various ions, and also the rate itself, may change over time, due to differences in the plant's needs (e.g. Voogt, 2002). Using ion-specific electrodes would allow for faster adjustment of incoming water, and also opens the possibility for feedback compensation of the actual uptake. Moreover, by on-line monitoring and control, the high drain percentages currently kept for safety reasons can be reduced, thus saving on disinfection and environmental costs.

There are two different views on the goal of the nutrient supply system. Traditionally, the supply system is intended to provide the plants with everything it needs. The composition of the plant together with growth determines the demand. The actual mechanisms for ion uptake are complex, and different for different ions and different plants. Some ions, e.g. Ca^{2+} in tomato, are, at least partly, taken up passively (Ho *et al.*, 1995). Others, like K^+ and NO_3^- are taken up actively, although the terms 'active' and 'passive' should not be taken too literally, as the plant's chemistry almost always plays a part (Marschner, 1995). This means that in those cases the plant itself regulates the actual uptake. As long as the transport of ions through the substrate is sufficient to meet the demand, the uptake will be dictated by the plant. The purpose of the supply system is then to create conditions in the substrate mat such that transport limitation is prevented. We call this 'demand satisfying supply'.

In some cases, it has been observed that the quality of crops can be influenced by manipulating the nutrient supply (Sonneveld and Welles, 1988; Sonneveld and Van den Burg, 1991; Drews, Schondorf and Krumbein, 1995). In order to make this happen, the nutrient uptake itself must be controlled. The purpose of the supply system is then to control the nutrient supply rate to create uptakes different from the demand. We call this 'uptake regulating supply'.

This paper gives a theoretical analysis on how uptake is related to nutrient supply with the irrigation water, with the main goal to develop a method to derive suitable controller settings – i.e. set-points, tuning parameters, and modes of operation - for a feed-back controller that tries to provide feedback compensation of the actual uptake, with emphasis on demand satisfying supply in water rich soil-less cultivation systems.

FEED BACK COMPENSATION OF UPTAKE

Gieling (2001) has developed and tested feed-forward feedback controllers to compensate for the water uptake. Solar radiation was used as the forward signal. The controller manipulates the inflow in order to keep the pulse-averaged drain flow constant. It appears that the resulting control signal closely follows the transpiration of the plant. He also developed controllers to compensate for the nutrient uptake of individual ions, using essentially the same principle by controlling the ion concentration in the drain.

The question is, how the settings of these controllers should be chosen in order to have either demand satisfying supply or uptake regulating supply. The controller options could be called ‘demand satisfying control’ and ‘uptake regulating control’, respectively.

FRAMEWORK FOR MODELLING TRANSPORT TO THE ROOTS

Figure 1 shows an arrangement of perpendicular vertical roots with root radius r_o in a slice of substrate with thickness H. To each root, a hexagonal cylinder belongs, which is approximated by a radial cylinder with radius r_1 (De Willigen and Van Noordwijk, 1994). If the root density, expressed in root length per unit substrate volume, is given by L_{rv}, then

$$r_1 \approx \frac{1}{\sqrt{\pi L_{rv}}} \qquad (1)$$

Supply of water and nutrients take place via the horizontal faces of the block in Figure 1.

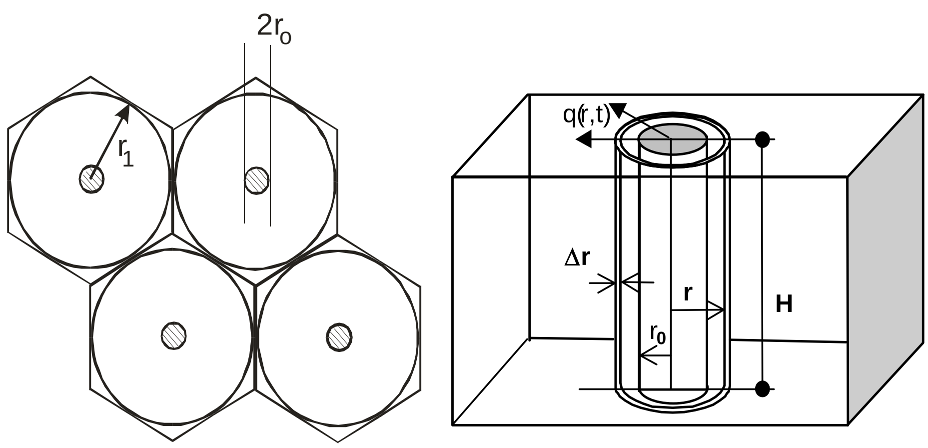

Figure 1. Schematic representation of root hair in a horizontal slice of the substrate (left: top view of several roots)

Barber (1962) was the first to indicate that nutrients are transported towards the roots both by mass flow, as well as by diffusion. The mass flow due to water uptake entrains the ions and transports them to the root surface. Let C_x be the so called influx concentration, specific for each ion, defined by $C_x=u/w$, where u and w are the ion and water uptake rates per unit roots surface, respectively. If the flux transported by the mass flow is larger then the uptake flux, which occurs if the concentration in the feed is larger than C_x, then back diffusion must occur to compensate for this, since the uptake itself is dictated by the plant. This is possible only if a concentration gradient is built up towards the roots. If the mass flux is lower than the uptake, the concentration at the root surface will go down which will create a gradient towards the roots, thus provoking additional transport due to diffusion. As long as the combined transport by diffusion and mass flow is sufficient to support the demand, no inhibition occurs. Since diffusion is concentration driven, it is clear that the total supported flow depends upon the concentration, as well as on the hydraulic conductivity of the substrate – for water transport – as well as the diffusion coefficient – for nutrient transport.

In practice nutrients are supplied in pulses. In this context one can think of a schematization where the vertical flow due to the pulse sets the initial conditions within the root cylinder for the period after the pulse. The transport is from then on governed by non-stationary flow and diffusion towards the roots. It is therefore not sufficient to consider steady state conditions; rather, dynamics must be taken into account, as will be done in the next sections.

MODELLING TRANSPORT OF WATER TOWARDS THE ROOTS

The transport of water from the bulk of the liquid towards the roots can be modelled using Richards' equation, which is a combination of a mass balance and Darcy's law. The boundary condition at the root surface is given by the water uptake, which, in turn, is determined by the transpiration of the plant. Heinen (1997) gives a more detailed description of the process. Here, it suffices to state that in wet saturated substrates the pressure head gradient towards the roots is very small. Under such conditions the evaporative needs of the plant are restricted by the root hydraulic conductance only, which cannot be altered by the control. The volumetric water density flux q from the bulk of the liquid to the roots is then given by

$$q\{r_o, t\} = w\{t\} \qquad (2)$$

where the water uptake rate per unit root surface w is dictated by the plant. In the sequel, the reasonable assumption will be made that in a controlled system with non-zero drain, using common horticultural substrates, the substrate mat will always stay sufficiently wet to ensure that the transpiration will not be restricted by the availability of water in the root zone.

MODELLING THE TRANSPORT OF NUTRIENTS TOWARDS THE ROOTS

The concentration profiles and transport of nutrients from the bulk of the liquid towards the roots can be derived from a nutrient mass balance:

$$\frac{\partial\left(\theta\{r,t\}C\{r,t\}\right)}{\partial t} = -\frac{1}{r}\frac{\partial(q\{r,t\}rC\{r,t\})}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(D\{\theta\{r,t\}\}r\frac{\partial C\{r,t\}}{\partial r}\right) \qquad (3)$$

where θ is the volumetric water content, and D the diffusion coefficient, which is a function of θ. This equation applies to a nutrient that is not subject to adsorption or chemical or biochemical reactions, although first order adsorption would be easy to accommodate (by writing $K_a+\theta$ in stead of θ in the left hand term). Eqn. (3) shows that the transport of nutrients towards the roots is partly due to the mass flow of water, and partly due to diffusion. Note that under the hydraulic conditions described above it is not necessary to compute $q\{r,t\}$ from a hydraulic model using Richards equation in the horizontal plane. The water flux is simply determined by the water balance. The distribution of the radial component of the water flux depends on the assumption on how the water in the root cylinder is replenished. If the water is considered to be replenished from outside the cylinder, the distribution is given by

$$q\{r,t\} = \frac{r_o}{r} q\{r_o,t\} = \frac{r_o}{r} w\{t\} \tag{4}$$

A slightly different formula applies under the assumption of uniform replenishment within the cylinder.

In situations where the nutrient uptake flux u is driven by the plant's demand, the following Neumann boundary condition at the root surface is appropriate

$$u\{t\} = w\{t\}C\{r_o,t\} - D\{\theta\{r_o,t\}\}\left.\frac{\partial C\{r,t\}}{\partial r}\right|_{r_o} \tag{5}$$

which is valid for either replenishment assumption. In general, Eqn. (3) can only be solved numerically. By dropping the dependency of the diffusion coefficient on θ, De Willigen and Van Noordwijk (1994a,b) succeeded in deriving analytical nutrient concentration profiles, both for boundary condition (5), as well as for a zero sink condition. The latter applies when the transport is not able to deliver the demanded nutrients, thus leading to a boundary condition of zero at the root surface. Starting from a sufficiently high initial concentration, first condition (5) applies, followed by a zero sink condition when the nutrient concentration gradually drops. As to the assumption of water replenishment they found that there was little difference between uniform replenishment and replenishment at the cylinder perimeter in both situations.

The solutions have a complicated form, not very suitable for direct use. However, Heinen (1997) uses their results to derive equations that specify the maximum rate of nutrient transport that can be supported by the substrate. The next section describes these, and also provides an alternate, more simplified form.

TRANSPORT SUPPORTED NUTRIENT SUPPLY

Let us now define U and W to be the nutrient and water uptake per unit horizontal area of substrate. The maximum possible nutrient transport towards the roots per unit surface area of substrate U^s is given by

$$\frac{U^s}{\overline{C}} = \pi H L_{rv} D \frac{\rho^2 - 1}{G\{\rho,\sigma\}} \tag{6}$$

where

$$\rho = \frac{r_1}{r_o} = \frac{1}{r_o\sqrt{\pi L_{rv}}} \quad \text{and} \quad \sigma = -\frac{q}{4H\pi L_{rv} D} \tag{7ab}$$

and

$$G\{\rho,\sigma\} = \frac{1}{2(\sigma+1)}\left(\frac{1-\rho^2}{2} + \frac{\rho^2(\rho^{2\sigma}-1)}{2\sigma} + \frac{\rho^2(\rho^{2\sigma}-1)(\sigma+1)}{2\sigma(\rho^{2\sigma+2}-1)} + \frac{(1-\rho^{2\sigma+4})(\sigma+1)}{(2\sigma+4)(\rho^{2\sigma+2}-1)}\right) \tag{8}$$

The concentration $\overline{C}$ is the average concentration in the root cylinder.

A simpler approximation to compute the limiting nutrient transport can be derived from Passioura (1963). Solving the non-stationary diffusion equation (3) without water transport, i.e. using the boundary condition

$$j\{r_o,t\} = -D\left.\frac{\partial C\{r,t\}}{dr}\right|_{r=r_o} \tag{9}$$

he states that the diffusive term can be replaced by

$$D\left.\frac{\partial C\{r,t\}}{dr}\right|_{r=r_o} = \frac{Df}{r_o}\left(C_{init} - C\{r_o,t\}\right) \tag{10}$$

In this expression f is a monotonically decreasing function of DT/r_o^2 , where T is the time elapsed since the constant initial bulk concentration C_{init} was applied. The dimensionless factor f ranges from 2.7 for very short times to 0.6 for long times. Using a typical D = 0.1 cm^2 d^{-1} and r_o = 0.02 cm, 'long' means a couple of hours.

The term Df/r_o can be viewed as an effective transport coefficient D/δ where δ is the penetration depth. As long as the penetration depth is less then r_1, i.e. as long as $r_o/f<r_1$ the initial concentration can be replaced without much error by the bulk concentration C_b in the region between δ and r_1. Since only a small part of the total nutrient mass will be in the root zone, C_b is roughly equal to $\overline{C}$. The approximation of Eqn. (10) is used to obtain a macroscopic steady state nutrient balance over a root cylinder with radius δ in the presence of additional mass flow, by stating that the total nutrient uptake per unit root length $2\pi r_o H u$ must equal the diffusive transport at the root surface described by Eqn. (10) plus a mass flow at the surface of the cylinder, which is simply equal to $2\pi r_o H w\overline{C}$ per unit root length with the assumptions above. Multiplying by the total length of roots per unit substrate surface area, which is L_{rv} , finally results in the relation

$$U = W\overline{C} + 2\pi HL_{rv}Df\left(\overline{C} - C_o\right) \qquad (11)$$

Note that for a plant controlled U this relation provides an estimate of the concentration of the nutrients at the root surface. In view of the approximations involved, f can be seen as a calibration parameter. The maximum possible supported nutrient transport occurs when C_o=0. Casting the final result in the same form as Eqn (6) yields

$$\frac{U^s}{\overline{C}} = W + 2\pi HL_{rv}Df \qquad (12)$$

NUMERICAL COMPARISON

Expressions (6) and (12) are compared for conditions that are typical for rockwool that is often used as horticultural substrate. The results are shown in Figure 2. The approximate equation is linear, whereas the more elaborate formula is not. However, it is quite possible to tune the linear approximation by a proper choice of f to fairly represent the situation. The incentive to introduce the linear approximation is first, to make a link between previous literature on the subject, like Passioura (1963), and the more detailed developments underlying Equation (6). Second, as will become clear in the next section, the linear equation is useful to present a graphical interpretation of the interplay between the demand of the plant, and the transport that can be supported by the conditions in the substrate. In figure 2, a value of f was chosen that ensures the approximate curve to be conservative. The effect of the mass flow to the maximum supported uptake rate is also shown in Figure 2 (dashed lines); it appears that the effect is quite small.

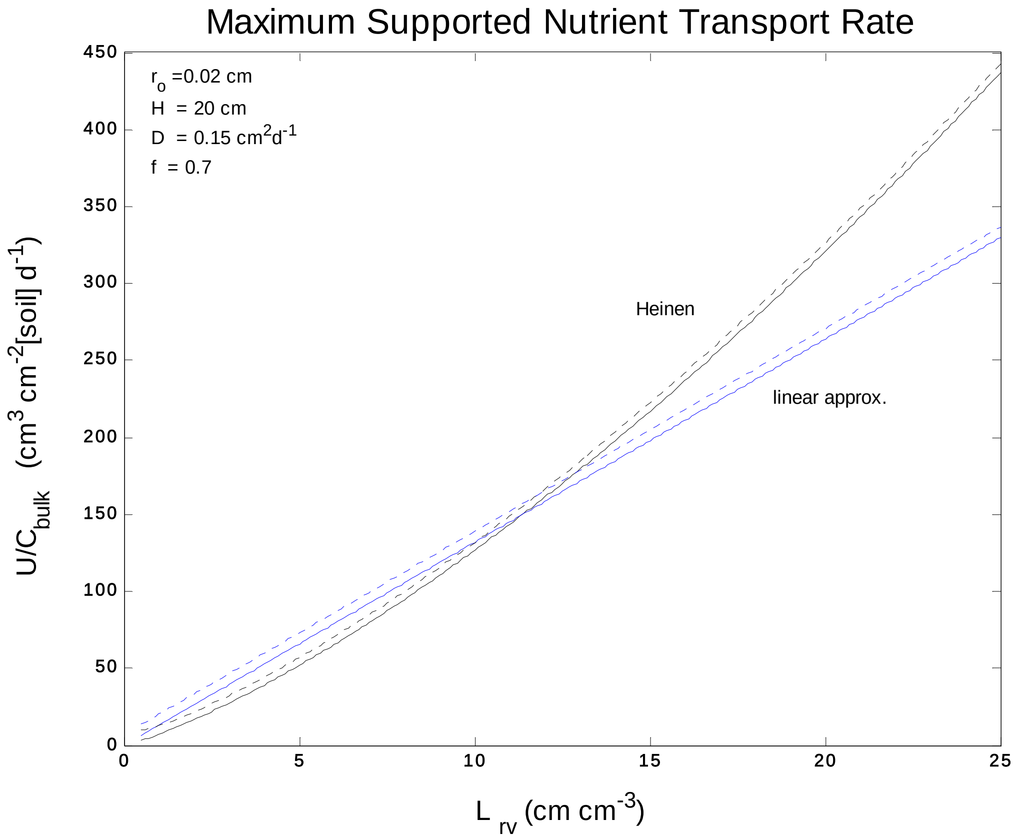

Figure 2. Maximum uptake rate per unit average bulk concentration as function of the specific root density, according to Eqn. (6), labeled 'Heinen' and Eqn. (12), labeled 'linear approx.' with f=0.7. The solid line is without water uptake, the dashed line for a water uptake of 7 cm d^{-1}. Other data: r_o = 0.02 cm, H = 20 cm, D = 0.15 cm^2 d^{-1}.

CONTROL OF THE NUTRIENT UPTAKE BY THE BULK CONCENTRATION

The solutions to the partial differential equations of the type (3) such as the solutions underlying Eqn. (6) as well as the approximate expression (11) represent a relationship between U, W, $\overline{C}$ and C_o . Such a relationship can be seen as a constraint condition of the general form

$$g\{C_o, U, W, \overline{C}, \Pi\} = 0 \tag{13}$$

where Π represent parameters which depend upon the root density and the diffusion coefficient in the substrate. We will call relation (13) a 'transport support line'. This condition defines the concentration at the root surface when the transport is sufficient to support the nutrient uptake.

Up till this point, we have assumed that the uptake would only be restricted when the root concentration is virtually zero. Suppose, however, that the uptake does show a dependency on the concentration, which in general can be formulated as

$$h\{C_o, U, \Pi\} = 0 \qquad (14)$$

For instance, the uptake from aqueous solutions in hydroponics is sometimes modelled by the Michaelis-Menten kinetics

$$U = U_{\max} \frac{C_o - C_{r\min}}{K + C_o - C_{r\min}} \qquad C_o \geq C_{r\min} \qquad (15)$$

which is in the form of Eqn. (14). Other forms are quoted in the review by Le Bot *et al.* (1998). It should be noted that plants rooted in soils and substrates can show apparent Michaelis-Menten behaviour at much higher half saturation constants than the value of K above. However, from the analysis above it is clear that such uptake relations are an artefact that can be attributed to transport resistance in the soil or substrate. As a consequence, the parameters will show wide variability depending upon the actual situation. It should also be noted that the kinetic expression most likely is not time invariant, and may depend upon the previous history. For instance, the maximum uptake rate will depend upon the root density, and if the plant experiences nutrient shortages for a longer time, it will respond by expanding the root system, thereby increasing U_{max}.

Despite the limited value of kinetic nutrient uptake expressions, it is interesting to see what might happen when the demand is larger than the supply supported by the substrate. The actual root concentration and the actual uptake should satisfy both Eqns. (13) or (11) and (14) or (15). The uptake rate can be plotted against C_o. The result is shown in Figure 3. The 'transport support line' Eqn. (11) can be written as

$$U = (W + B)\overline{C} - BC_o \qquad (16)$$

where $B = 2\pi HL_{rv}Df$. The slope of this line is $-B$, whereas the intercept is given by $(W + B)\overline{C}$. An increase in the mean bulk concentration shifts the line up. The same is true for increase in the water uptake. The intersection with the curved line representing the kinetics yields the root concentration C_o where uptake is balanced by transport. Obviously, if the mean bulk concentration is reduced (and/or the water uptake rate), the equilibrium point shifts towards lower concentrations at the root surface, and consequently to lower uptake rates. When the purpose is to prevent nutrient limitation, the root zone concentration should be kept in the range where the kinetics is virtually zero order, i.e. in the right side section of the graph. The bulk concentration that supports this can be read as the parameter that marks the transport support lines in Figure 3. In figure 3, the transport support lines are straight lines, because equation (16) was derived with the linear approximation.

Several ions have kinetic uptake curves that are rather steep at low concentrations. This means that when the purpose is to control the uptake, the overall system should regulate the root concentration in a very narrow range. This, together with the dynamic nature of the kinetic curve makes it practically unfeasible to control the nutrient uptake by manipulating the bulk concentration. It seems better to try to build an observer for the uptake, and then control the time-averaged uptake directly with the incoming mass. Alternatively, the desired uptake may be computed from a crop model, which is then used to calculate the required time averaged supply.

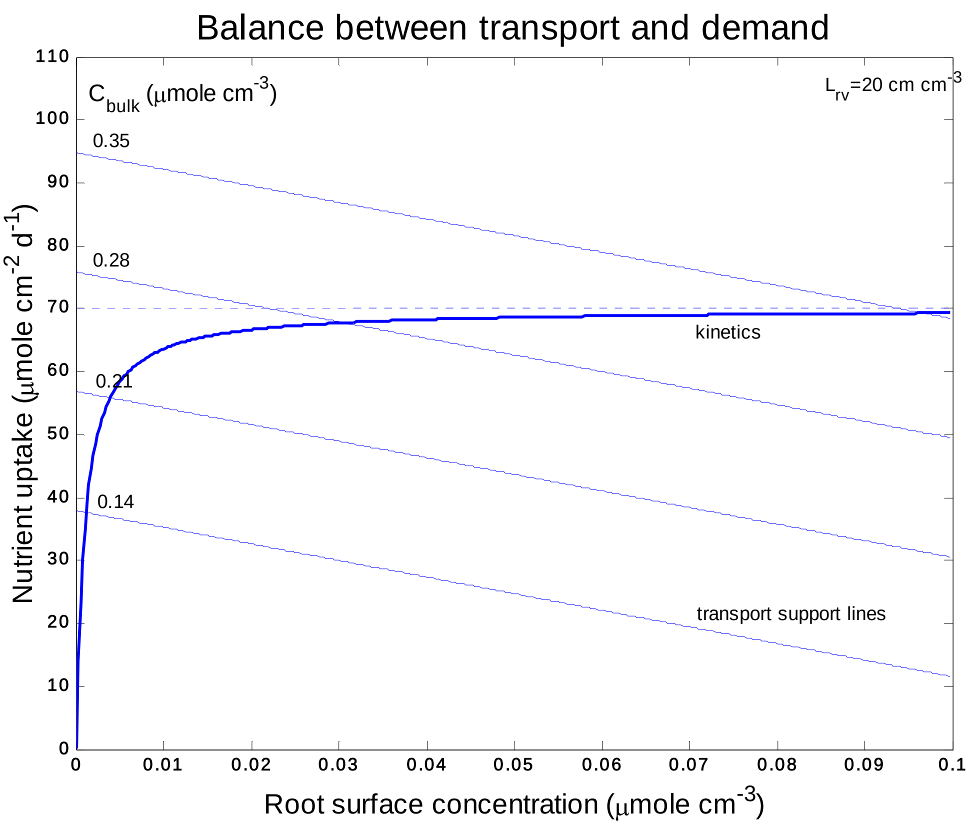

Figure 3. Nutrient uptake rate, U, versus concentration at the root surface C_o. 'Transport support lines' are given for various bulk concentrations, $\overline{C}$, according to Eqn. (11) with W = 7 cm d^{-1}. Other data: H = 20 cm, D = 0.15 cm^2 d^{-1}, L_{rv} = 20 cm cm^{-3}, f=0.7. Kinetic uptake and transport towards roots are in equilibrium at the intersection of lines. See text for more explanation.

Figure 3 also indicates that when the demand becomes lower, which is equivalent to lowering the kinetics asymptote, the concentration at the root surface increases. In fact, the concentration at the roots can become higher than the mean bulk concentration. This is more easily the case if the intercept of the transport line is dominated by the water uptake, because the same intercept can be achieved with a higher water uptake and a lower bulk concentration. In those cases the mass flow transports more ions to the roots than are actually taken up.

INFLUENCING THE MEAN BULK CONCENTRATION

The analysis above applies for a thin layer of thickness H. The concentration $\overline{C}$ cannot be controlled independently in every layer. In stead, only the incoming concentration can be manipulated. The information needed to control the $\overline{C}$ distribution must be derived from the concentration at the drain. This requires a simulation model, describing the vertical transport of water and nutrients, and using the minimum of the plant demand and the supply supported by transport based on Eqn. (6) or (12) as the actual uptake.

A view remarks can be made without simulation. A time averaged balance over the full substrate column in steady state yields

$$\overline{F_d C_d} = \overline{F_d C_{in}} + \overline{W(C_{in} - C_x)} \qquad (17)$$

where C_x is the so called influx concentration, defined as U/W. If on average $C_{in}=C_x$, it follows that the drain concentration equals the incoming concentration: $C_d = C_{in} = C_x = \overline{C}$ and there is no vertical gradient in the column. When $C_{in}>C_x$ then there is a concentration effect, and $C_d>C_{in}$. The bulk concentration will be between C_{in} and C_d. If $C_{in}<C_x$ we will have $C_d<C_{in}$. Whatever the situation, the averaged steady state bulk concentration will always be within the incoming concentration and the drain concentration (assuming a continuous water flow). The calculation of possible nutrient limitation can use the most conservative of these two.

CONTROLLER STRATEGY

The equations can be used to recommend controller setting for feedback control:

<u>Case 1: Demand Satisfying Control</u>

The minimum setting follows directly from Eqn. (12) or (6). If the maximum allowable concentration is called C_o^{UL}, then an upper limit can be computed from Eqn. (11) or (16). The final result is

$$\frac{U}{W+B} < \overline{C} < \frac{U}{W+B} + \frac{B}{W+B} C_o^{UL} \qquad (18)$$

In a system where the drain flow is kept constant, a rise in the drain nutrient concentration indicates a decrease in uptake. The incoming concentration can then be reduced, provided that the transport limitation conditions are not violated. The converse is true when the drain concentration decreases. This is exactly what a drain concentration controller would do. If a crop model is available, or if an estimate of the crop nutrient uptake from crop growth can be made, the set-point can be adjusted within the bounds given by Eqn. (18).

Case 2: Uptake regulating control

A feed-back system using the drain return concentration directly is probably not feasible, because of the very low concentrations needed. In cases where these low concentrations are still measurable, constant drain concentration control will regulate the uptake, but it is clear from Figure 3 that the actual value of the uptake is difficult to assess due to the sensitivity of the uptake to the concentration, and due to the uncertainties in the uptake kinetics.

CONCLUSION

When the demand is lower than the supply that can be supported, feedback compensation of the uptake is feasible and provides a good solution to individual ion control. The set point should be chosen larger than the lower limit in Eqn. (18).

Damage by too high concentrations at the root surface can be avoided by choosing the set-point lower than the upper bound in Eqn. (18).

It will be almost impossible to perform uptake control by manipulating the bulk concentration for ions with low limit concentration, such as K, N. The uptake is proportional to the concentration in this range, but the concentrations are very low. A system based upon an observer plus a mass balance computation to compute the required mass flux is probably more successful.

Dynamical simulations with a 2-D model are underway to confirm these findings under dynamic conditions. Experiments have been envisaged to test a constant drain concentration controller for demand satisfying control, as well as to provide information for the design of an observer based predictive controller for uptake regulating control.

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NOMENCLATURE

B	parameter $2\pi HL_{rv}Df$	
C	molar concentration	μmole cm^{-3}
D	diffusion coefficient	cm^{2} d^{-1}
f	dimensionless parameter; Eqn. (10)	
F	flow rate	cm^{3} d^{-1}
G	defined by Eqn. (8), dimensionless	
H	thickness of the substrate slice	cm
j	molar flux towards the root surface	μmole cm^{-2}[root surface] d^{-1}
K	half saturation concentration	μmole cm^{-3}
L_{rv}	root density	cm cm^{-3}[substrate]
r	radial distance	m
u	specific nutrient uptake rate	μmole cm^{-2}[root surface]d^{-1}
U	areal nutrient uptake rate	μmole cm^{-2}[substrate area]d^{-1}
q	specific water flux density	cm^{3} cm^{-2}[root surface] d^{-1}
w	specific water uptake rate	cm^{3} cm^{-2}[root surface]d^{-1}
W	areal water uptake rate	cm^{3} cm^{-2}[substrate area]d^{-1}
δ	penetration depth	cm
ρ	dimensionless radius; Eqn.(7ab)	
σ	dimensionless water uptake; Eqn.(7ab)	
θ	volumetric water content	cm^{3}[water] cm^{-3}[substrate]

subscripts

max	maximum
o	at the root surface
1	at the edge of the root cylinder (Figure 1)
in	incoming
init	initial
x	at the influx through the root boundary

superscript

s	saturated
UL	upper limit