BU-966-M

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LENGTH OF PRODUCTIVE LIFE OF DAIRY COWS:
I. JUSTIFICATION OF A WEIBULL MODEL

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Received November 3, 1987

ABSTRACT

The length of productive life of 39,683 grade Holstein cows milked in 150 large herds in New-York State between 1981 and 1986 was analyzed by modeling their hazard, which is a measure of their probability of being culled. Animals still alive when the analysis was performed were assigned a "censored" record equal to the current value of their length of productive life. The concept of hazard allows an adequate statistical treatment of these censored records. The proportional hazards models considered involve a baseline hazard function and log-linear time-dependent explanatory variables affecting culling rate. These include a herd x year effect, a stage of lactation x lactation number effect and a within herd and lactation level of milk production effect (normalized rank based on 305 ME milk yield).

A semi-parametric analysis - for which the baseline hazard function is completely unspecified (Cox's regression) - showed that the assumption of proportional hazards is appropriate, that all the effects in the model are highly significant and that the baseline hazard function can be closely approximated by a Weibull hazard function of the form $\lambda(t) = \lambda\rho\,(\lambda t)^{\rho-1}$. Such an approximation greatly simplifies computations and facilitates further genetic and nongenetic studies on longevity of dairy cows.

Key words: Holstein-Friesian, stayability, nonlinear model, Cox model, Weibull model, length of productive life

INTRODUCTION

Longevity is a highly desirable quality of a dairy cow : total profit and profit per day of life have been shown to be related to longevity (1, 2, 17, 21) : when herdlife increases, fewer heifers need to be raised and replacement costs are decreased. But culling decision usually occurs long before senescence. Consequently, geneticists have developed the concept of **stayability** (or **survivability**) to characterize the capability for a cow to remain productive in her herd over time (13, 14, 22).

When reason for leaving the herd is not considered, this ability can be referred to as **true stayability**. It also measures the dairyman's perception of the value of the cow. However, it may be of interest to distinguish between disposal mostly *beyond* the control of dairy managers such as the sale of a profitable but sterile cow (*involuntary* culling) and *voluntary* disposal of a healthy but not profitable cow. Van Arendonk (26) showed that if involuntary culling is decreased, a higher voluntary culling rate can be applied, resulting in a larger profit for the farmer. The aptitude to delay involuntary disposal will be called **functional stayability**.

Many different measures of stayability have been proposed: age, number of lactation, length of productive life or lifetime production at time of disposal. Computation of these measures requires the knowledge of the culling date. But it is usually impossible or useless to wait until all the animals of interest have disappeared from the herd before starting any analysis. To overcome this difficulty, early indicators of true stayability such as the proportion of cows still alive at a given time T_0 (e.g. 48 months) or at the beginning of a given lactation number have been used. But such measures suffer severe drawbacks: many different T_0 can be chosen and a substantial loss of information exists: cows culled one day or one year before T_0 are treated alike. Also, linear models are not adequate to analyze such binomial data : at T_0, a cow is alive or not (8, 16).

A continuous measure such as the length of productive life (LPL) seems more desirable. LPL is defined to include animals still alive at the time of the analysis. The corresponding records, which represent a lower bound of the eventual LPL's are called **censored** records and the existence of censored records is referred to as **censoring**. Records from cows sold for dairy purposes are also considered as censored (13).

Specific statistical methods dealing with censoring have been developed (7, 18, 20) but because they are quite complex, they have not been used by animal breeders until recently (15, 22, 23, 24, 26). The objective of this paper is to show in which direction the models proposed by these authors may be improved to more properly describe the culling process as it occurs on the farm. A particular approach on how functional stayability can be approximately estimated is also suggested.

MATERIALS and METHODS

General approach

The analysis of censored survival data is based on the use of special modeling distributions such as the **hazard function**. If T is the nonnegative random variable representing the failure time of a cow, the hazard function $\lambda(t)$ is defined as:

$$\lambda(t) = \lim_{\delta \to 0} \frac{\mathrm{Prob}\,[\,t \le T < t+\delta \mid T \ge t\,]}{\delta} \qquad (18) \qquad [1]$$

i.e. $\lambda(t)$ specifies the instantaneous rate of failure at time t, conditional upon survival up to t. Here, hazard is intuitively synonymous with **relative culling rate**. In many cases, the exact nature of the density function f(t) or the survivor function $S(t) = \mathrm{Prob}\,(T \ge t)$ is not known in the population under study but some information is available on how the failure rate $\lambda(t)$ changes over time. Note also that :

$$S(t) = \exp\left[\, -\int_0^t \lambda(u)\, du \right] \qquad [2]$$

The most popular regression model based on the concept of hazard function is the **Proportional Hazards** (PH) model, for which the hazard $\lambda(t) = \lambda(t\,;\mathbf{z_i})$ for animal i is the product of a time-dependent term $\lambda_0(t)$ related to the aging process (the **baseline hazard function**) and a "*stress-dependent* " term $e^{\mathbf{z_i}'\boldsymbol{\beta}}$ representing how the vector of covariates $\mathbf{z_i}$ influences failure rate, independently of time (5). Hence:

$$\lambda(t\,;\mathbf{z_i}) = \lambda_0(t)\, e^{\mathbf{z_i}'\boldsymbol{\beta}} \qquad [3]$$

Therefore, the hazards of two animals i and i' are assumed to be always proportional with hazards ratio $e^{(\mathbf{z_i} - \mathbf{z_{i'}})'\boldsymbol{\beta}}$. The baseline hazard function can have a known parametric form; e.g. if $\lambda_0(t) = \lambda$ = constant, the corresponding **baseline survivor curve** is exponential: $S_0(t) = \exp(-\lambda t)$. If $\lambda_0(t) = \lambda\rho\ (\lambda t)^{\rho-1}$ for some λ and ρ, T follows a **Weibull** distribution : $S_0(t) = \exp(-(\lambda t)^\rho)$).

Using the concept of "partial likelihood", Cox (5, 6) proposed a method for the estimation of the effects $\boldsymbol{\beta}$ in the PH model, which does not require any assumption about the form of $\lambda_0(t)$. In the Cox's regression, estimates of $\boldsymbol{\beta}$ are obtained by maximizing the logarithm L_1 of a "partial" likelihood of the form:

$$L_1 = \sum_{i \in \{unc.\}} \left[z_i'\boldsymbol{\beta} - \log\left\{ \sum_{m \in Risk(T_{[i]})} e^{z_m'\boldsymbol{\beta}} \right\} \right] \quad [4]$$

where : $T_{[1]} < ... < T_{[n]}$ are the ordered n observed (**uncensored**) failure times;

{unc.} is the set of uncensored cows;

Risk $(T_{[i]}) = \{ m ; T_m \geq T_{[i]}\}$ is the set of animals at risk at $T_{[i]}$, i.e. alive just prior to $T_{[i]}$.

If, as it often happens in practice, failure times are recorded in a way allowing for ties between some individuals - e.g. same number of days of productive life - an approximation of the partial likelihood is given by:

$$L_2 = \sum_{i \in \{unc.\}} \left[\left[\sum_{k_i \in D(T_{[i]})} z_{k_i}'\boldsymbol{\beta} \right] - d_i \log\left\{ \sum_{m \in Risk(T_{[i]})} e^{z_m'\boldsymbol{\beta}} \right\} \right] \quad [5]$$

(Peto, in (5))

where d_i , the k_i's and $D(T_{[i]})$ are the number, the indices and the set of cows actually failing at $T_{[i]}$.

$$\hat{S}_0(t) = \exp\left\{ - \sum_{i \mid T[i] < t} \left[\frac{d_i}{\sum_{m \in \mathrm{Risk}(T_{[i]})} e^{z_m'\beta}} \right] \right\} \quad (3) \qquad [6]$$

In some cases, the PH assumption is not tenable for all the factors of interest. A possible alternative which retains the simplicity of the PH model and known as **stratification** is the definition of a different baseline hazard function $\lambda_{0j}(t)$ for each level j of one particular factor. This was the approach chosen by Smith (22, 23) in his analysis of age at disposal of dairy cows: records were "stratified" by year of birth of the cows. Problems related with Smith's model are discussed in (12).

A much more powerful generalization of the PH model is the use of **time-dependent covariates**. In that case, the exponential part in [3] is allowed to vary with time:

$$\lambda(t\,; z_i(t)) = \lambda_0(t)\; e^{z_i(t)'\beta} \qquad [7]$$

Estimation of β in a Cox's PH model with time-dependent covariates can lead to extremely tedious computations: at each failure time $T_{[i]}$, the values of $e^{z_m'\beta} = e^{z_m(t)'\beta}$ in [4] or [5] vary. Howe ver, if zm(t) is a very simple function of time, such as a piecewise constant function, $\sum_{m \in \mathrm{Risk}(T_{[i]})} e^{z_m(t)'\beta}$ in [4] or [5] can be computed in a more efficient way than in the general case (12). In that situation, it is assumed that within each interval for which $z_m(t)$ is constant the PH assumption holds but that the hazards ratio changes from one such interval to the next. But even then, computations are still very tedious and such a model cannot be applied to very large data sets necessary for routine sire evaluation.

On the other hand, when the baseline hazard function $\lambda_0(t)$ in [1] has a parametric form, estimation of β and $\lambda_0(.)$ is generally easier (7, 18). Consequently, the following approach is chosen here: on a data set of moderate size, the Cox's version of the PH model is fit and the baseline survivor function $S_0(t)$ is estimated. Then the PH assumption is checked and the estimate of $S_0(t)$ is compared - using goodness-of-fit and cross-validation tests - to a known parametric form: the Weibull distribution. This choice results from the simplicity of the Weibull survivor function ($S_0(t) = \exp(- (\lambda t)^\rho)$) allied with its flexibility: a Weibull regression can model constant ($\rho = 1$), increasing ($\rho > 1$) and decreasing ($\rho < 1$) hazard rates. It is the simplest generalization of the exponential survivor distribution. If an approximation of $\lambda_0(t)$ and $S_0(t)$ with a parametric model is possible, further analyses would be greatly facilitated.

Data set

Only grade cows are considered here: culling policies in grade and registered herds are known to be markedly different and should not be treated alike: registered cows are kept longer, are culled less on milk production and more on type or for dairy purposes (10, 11).

In the Northeast Dairy Record Processing Laboratory (DRPL) AI sire file, the exact failure date of cows culled before 1981 was not recorded when failure occurred after more than 305 days of lactation. To avoid the problems associated with such truncated records (12), the period of study was restricted to January 1981 - February 1986, i.e., to the years for which complete information is available.

The data set includes the length of productive life (culling date - first parturition date, in days) of 39,683 grade Holstein cows milked in 150 large herds in New-York state. Admittedly, this data set is not representative of the whole grade population but this restriction to large herds (190 to 849 cows per herd over the whole period) is a compromise between the need to constrain the estimation problem to a reasonable size by limiting the number of herds and the desire to base conclusions on more precise estimates of the herd x year effect. LPL records of cows sold for dairy purposes or assumed alive on March 1, 1986 were considered as censored: 47% of the total number of records were censored.

Models

Our principal objective was to describe as precisely as possible the main factors affecting the culling process. Two models were envisioned here.

Management practices and culling policies are controlled by the dairy manager and influenced by the herd environment: they are likely to affect the LPL of all the cows in a same herd in a similar fashion. Therefore, a herd effect h_j is included in the model and its change over time is simulated by a step function, for which jumps are arbitrarily assumed to occur on January 1, each year.

Stage of lactation is regarded as another essential factor determining the probability for a cow of being culled, i.e., her hazard. For example, during the first months of lactation, milk production is maximum, reproductive status does not affect profitability and salvage value is generally low : culling at that point seems less likely than for cows of the same age but reaching a later stage of lactation. A piecewise constant stage of lactation effect $p_k(t)$ is defined in order to isolate three "biological periods" ("early", "middle", "end of lactation and dry period").

Finally, two cows may freshen the same day at the same age, one for the xth time and the other for the (x+1)st time. A lactation number effect $q_l(t)$ is added to treat differently cows managed more or less intensively than others.

The first model (**model A**) is written:

$$\lambda(t) = \lambda_{jkl}(t) = \lambda_0(t) \ \exp \{ h_j(t) + p_k(t) + q_l(t) \} \qquad [8]$$

where : $\lambda_0(t)$ is a completely arbitrary baseline hazard function,

$h_j(t)$ is the jth herd x year effect,

$p_k(t)$ is the kth stage of lactation effect (from day 0 to day 29 after parturition, from day 30 to 249, and from day 250 to the beginning of the next lactation),

$q_l(t)$ is the lth lactation number effect (lactation 1, 2 , 3 to 5, 6 and more).

Note that $h_j(t)$ is a function of the *calendar* time whereas $p_k(t)$ and $q_l(t)$ are step functions of *biological* time, dependent on date of parturition.

Although the estimation of sire genetic merit is our ultimate goal, sire effects are completely ignored in this part : sires are expected to have a rather small effect on the LPL of their daughters. Heritability of stayability is known to be quite low. The other effects described above are intuitively believed to have a more drastic effect on culling rate that the genetic make-up of the cow. Moreover, if sires are to be included in the Cox's PH model, fewer herds have to be selected in order to constrain the estimation problem to a reasonable size. In such a reduced date set, inevitably, each sire would have very few daughters and the precision of their estimate would be very poor. The adequacy of the model would be difficult to assess.

Low milk yield has been described as the major reason for voluntary disposal of a cow. Hence, a correction of LPL for milk production should reveal differences between animals for reasons for disposal other than production: differences in voluntary culling due to type, old age or general health and above all, differences in involuntary culling caused for instance by infertility, illness, chronic mastitis, etc.... Therefore, such a correction would represent a first step *toward* the study of what was previously defined as functional stayability.

A time-dependent "within herd and lactation level of milk production effect" $r_m(t)$ is added to the previous model to form **model B:**

$$\lambda(t) = \lambda_{jklm}(t) = \lambda_0(t) \, \exp\{h_j(t) + p_k(t) + q_l(t) + r_m(t)\} \qquad [9]$$

$\lambda_0(t)$, $h_j(t)$, $p_k(t)$ and $q_l(t)$ are defined as in [8]. $r_m(t)$ is the effect associated with the mth class of milk production. These classes are defined in a specific way trying to simulate the actual voluntary culling process as it is performed on the farm. In particular, it is believed that *relative* milk production (compared to the other cows present in the same herd at the same time) plays a larger role in the culling decision than actual yield. In practice, each record (lactation) of a cow is assigned a milk production class in the following way, illustrated in figure 1:

[figure 1 here]

i) 305 days Mature Equivalent (305ME) records are sorted within herd and year separately for first and later parities.

ii) ranks within herd-year are standardized by computing their expected normal scores.

iii) these expected normal scores are divided into 9 classes of equal importance (each of probability 11.1%).

Records for which the 305ME production is not known (mainly lactations not terminated at the end of the study period) are assigned to a tenth group.

Goodness-of-fit and model validation

The adequacy of the two models proposed was checked in several ways:

1) A test for the proportional hazards assumption is based on the concept of

generalized residuals developed by Cox and Snell (4). Generalized residuals for observations T_i are functions $e_i = g_i(T_i; \beta, \mathbf{z_i})$ such that the e_i's are independent and identically distributed, with known distribution. For example, in the case of failure times, it can be shown that the random variable:

$$e_i = \int_0^{T_i} \lambda(u ; \mathbf{z_i})\, du \qquad [10]$$

follows an exponential distribution with parameter 1 (4). The generalized residual e_i represents the sum of the hazards that animal i encountered during its life.

A test of the proportional hazards assumption is obtained by checking whether the *estimated* generalized residuals $\hat{e}_i$ constitute a random sample from a unit exponential distribution, where :

$$\hat{e}_i = \int_0^{y_i} \hat{\lambda}(u ; \mathbf{z_i})\, du \qquad [11]$$

with $y_i = T_i$ if animal i is uncensored or $y_i = C_i$ (censoring time) if the animal i is censored.

In practice, the ordered (uncensored) $\hat{e}_i$ are plotted against the expected order statistics of a unit exponential with the same censoring pattern. If the resulting line strongly deviates from a straight line with slope 1 and going through the origin, the proportional hazards assumption is rejected.

2) The need for the inclusion of a particular group of covariates in the model is checked by a forward stepwise procedure based on the large sample likelihood ratio test (19). If $\hat{\boldsymbol{\beta}}_{(1)}$ represents the maximum likelihood (ML) estimate of $\boldsymbol{\beta}_{(1)}$ in a reduced model including only covariates $\mathbf{z}_{(1)}$ and if $\hat{\boldsymbol{\beta}}$ denotes the ML estimate of $\boldsymbol{\beta} = (\boldsymbol{\beta}_{(1)}, \boldsymbol{\beta}_{(2)})$ in the extended model including covariates $\mathbf{z} = (\mathbf{z}_{(1)}, \mathbf{z}_{(2)})$, the procedure to test $H_0 : \boldsymbol{\beta}_{(2)} = \mathbf{0}$ is to compare the value of $2\,[\,L_2(\hat{\boldsymbol{\beta}}) - L_2(\hat{\boldsymbol{\beta}}_{(1)})\,]$ to a χ^2 distribution with ν degrees of freedom, where ν is the dimension of $\boldsymbol{\beta}_{(2)}$.

3) In the case of the Weibull proportional hazards model, we have:

$$S_0(t) = \exp\,[\,-(\lambda t)^{\rho-1}\,] \qquad [12]$$

and therefore:

$$\log\,[\,\log S_0(t)\,] = \rho \log t + \rho \log \lambda \qquad [13]$$

Hence the adequacy of the Weibull model in a study of LPL records can be assessed by looking at the quality of the regression of $\log\,[\,-\log \hat{S}_0(t)\,]$ on $\log t$, where $\hat{S}_0(t)$ is the estimated baseline survivor curve, as computed in [6]. The slope and the intercept of the regression line also provide crude estimates of the Weibull parameters λ and ρ (20).

4) To definitely confirm the validity of the Weibull model as an approximation of the Cox's semi-parametric model, a **cross-validation** test was performed: two subsets S1 and S2 of the initial data set were randomly created and Weibull versions of models A and B - i.e., for which the baseline hazard function is a Weibull hazard - were fit on both subsets. The following likelihood function of the observed failures given the model (7,18) was maximized:

$$L = \left[\prod_{m \in \{unc.\}} \lambda(y_m ; \mathbf{z_m}(y_m)) \right] \left[\prod_{m' \in \{unc., cens.\}} S(y_{m'} ; \mathbf{z_{m'}}(t)) \right] \quad [14]$$

where {unc.} and {cens.} are the sets of uncensored and censored cows.

The two sets of estimates for $\boldsymbol{\beta}$, ρ and λ are then compared and for each subset, generalized residuals are computed using the ML estimates of $\boldsymbol{\beta}$, ρ and λ obtained from the same subset or from the other. The distribution of both sets of generalized residuals is then compared to that of a censored unit exponential. If the model is correct, the same fit should be observed whatever the origin of the estimates.

At the same time, a check for the existence of interactions between Stage of Lactation (SL) and Lactation Number (LN) effects in models A and B was performed through a slight modification of these models: a SL x LN effect g_{kl} is defined to replace p_k and q_l in [8] and [9]. Models A and B are modified as:

$$\lambda(t) = \lambda_0(t) \ \exp\{h_j(t) + g_{kl}(t)\} \qquad \text{(Model A*)} \qquad [15]$$

$$\lambda(t) = \lambda_0(t) \ \exp\{h_j(t) + g_{kl}(t) + r_m(t)\} \qquad \text{(Model B*)} \qquad [16]$$

In absence of interaction, $g_{kl}(t) = p_k + q_l$ for all k and l.

Nine SL x LN classes are defined (3 SL classes defined as previously and only 3 LN classes - first, second, third and more). In contrast with the Cox's model, only records from cows calving *for the first time* after January 1, 1981 can be used in the Weibull model: none of these cows had started a sixth lactation before the end of the study period (February 1986). S1 and S2 include respectively 13,797 and 13,842 LPL records and two-thirds of these records are censored. This proportion of censored records is quite large: it illustrates the need for an different statistical treatment of the two types of records. Indeed, some herd-year "subclasses" include only censored records. It should not be considered that these subclasses do not contain any information. The absence of uncensored records simply indicates that the average hazard was particularly low in those herd-years.

RESULTS and DISCUSSION

The estimation of 757 effects (750 herd x year + 3 SL + 4 LN effects) and 777 effects (757 + 20 within herd x lactation level of production effects) for the Cox's models A and B was performed by maximum likelihood using a very efficient method for numerical optimization, known as the BFGS algorithm ((9), chapter 8). This algorithm mimics the well known Newton's algorithm but replaces the exact evaluation of the matrix of second derivatives of the log-likelihood by an approximation of this matrix in some optimal way.

For the estimation of these same effects when the Weibull models A* and B* are fit, the matrix of second derivatives of the likelihood L in [14] is very sparse and its inversion is simple: therefore, the Newton's algorithm can be used.

The likelihood ratio tests used to check the importance of the factors in models A and B reveal that all the factors included have a very highly significant effect ($p<0.001$) on a cow's hazard. Estimates of herd x year effects range from -3.96 to 1.32. Note that an estimate of 1.0 means that in the herd considered, the **relative culling rate** is $e^{1.0}=2.7$, i.e. a cow in this herd is 2.7 times more likely to be culled at any time t than a cow in an "average" herd. Figure 2 presents the distribution of herd x year estimates for model A.

(figure 2 and table 1 here)

The estimates of the stage of lactation and lactation number effects are presented in table 1. As expected, relative culling rates increase considerably with stage of lactation. A cow *finishing* her lactation has a probability of being culled exp [0.77 - (-0.63)] = 4.06 times larger than a cow of the same productive age *starting* her lactation. Relative culling rate is also larger in first lactation than later on, especially when differences in milk production between young and old cows are taken into account (model B). A cow finishing her first lactation will be at a much higher risk of being culled than another *of the same age* in the middle of her second lactation.

(figure 3 here)

Within herd x lactation level of production (WHLP) effects for model B are presented in figure 3. The two curves for first and later lactations are smooth, monotone and almost parallel: there is no interaction between this factor and lactation number. WHLP effects increase continuously at an approximately quadratic rate. But as far as the relative culling rate ($\exp(\hat{r}_l(t))$) is concerned, the increase is slow and almost linear from production classes 1 to 7 and then very sharp for the last two classes.

Cows in the last milk production class in first lactation are about 10 times more likely to be culled at any time t than cows in the first class and almost 4 times more than cows in the seventh class.

This trend was expected but these results suggest that dairymen actually base their voluntary culling decision - maybe only intuitively - on a criterion closely related to the standardized - and therefore artificial - 305ME milk production.

For the tenth class of milk production - which corresponds to cows with unknown 305ME records - the estimates of WHLP effects are extremely low (-1.98 in first lactation, -1.20 in later lactations) because most of the records assigned to this class are from the *last* lactation of *censored* cows and therefore, very few failures are actually observed in this category.

A regression of the estimated generalized residuals $\hat{e}_i$ on the expected order statistics o_i of a censored unit exponential distribution leads to the following equations:

$\hat{e}_i = -0.0003 + 1.005\ o_i$ $\qquad R^2 = 0.9997$ $\qquad$ for model A

$\hat{e}_i = -0.013 + 1.033\ o_i$ $\qquad R^2 = 0.991$ $\qquad$ for model B

The agreement with theoretical prediction when a proportional hazards model is adequate is excellent, especially for model A. The power of such a graphical test based on generalized residuals is unknown. Cox and Oakes ((7), p109) warn against an ill-considered positive interpretation of this kind of test for large data sets. However, in a preliminary analysis with some truncated - and therefore incorrect - records, this same test clearly detected a large discordance with the proportional hazards assumption (12). It can be concluded *at least* that there is no evidence here of a departure from the proportional hazards situation.

The slightly less satisfying behavior of the observed residuals in model B is probably due to an incorrect treatment of the animals with no 305ME record (grouped in the tenth level of production class): the hazard of these animals is compared with the hazard of other cows whose LPL record is adjusted for differences in milk production. However, this discrepancy is rather small: only 0.6% of the residuals deviate significantly from their expected value (see (12) p133).

A weighted regression of $\log[-\log \hat{S}_0(t)]$ on $\log t$ gives the following equations:

$\log[-\log \hat{S}_0(t)] = -11.20 + 1.48 \log t$ $\qquad R^2 = 0.991$ $\qquad$ for model A

$\log[-\log \hat{S}_0(t)] = -12.88 + 1.69 \log t$ $\qquad R^2 = 0.989$ $\qquad$ for model B

Therefore, the baseline hazard function can be well approximated by a Weibull hazard function. The values of the crude estimates of ρ (1.48 and 1.69) show that the baseline hazard of a cow increases with productive age. Estimates for λ are $\lambda = 5.3\ 10^{-4}$ and $\lambda = 4.9\ 10^{-4}$ respectively.

Figure 4 presents the values of the estimates of $\rho^{-1}\beta$ for the SL x LN effects when Weibull model A* is fit on the two subsets S1 and S2. The estimates obtained for both models are consistent, except for the first period of the first lactation. Indeed, the gap between the two estimates is easily explained by the difference for the number of cows actually failing in S1 and S2 (48 vs 68). This difference is entirely due to sampling. More interestingly, figure 4 shows that an interaction exists between SL and LN: in first lactation, cows are comparatively at a higher risk at the beginning and the middle of their lactation.

(figure 4 and table 2 here)

Finally, table 2 presents the regression equations of generalized residuals $\hat{e}_i$ for animals in S1 and S2 on the expected order statistics of a censored unit exponential distribution, when models A* and B* are fit and when estimates for $\boldsymbol{\beta}$, ρ and λ are obtained either from S1 or S2.

Clearly, the agreement between predicted and observed values is excellent for model A* : very similar results are obtained whatever the origin (S1 or S2) of the estimates used to compute the residuals. For model B*, the agreement is not as good. In particular, the slope corresponding to residuals in S1 computed with estimates from S2 is larger than when these estimates are from S1 itself (1.17 vs 1.03). However, regression equations tends to exaggerate this discordance. This is shown in figure 5: only a small fraction of the residuals strongly deviate from the theoretical straight line with slope 1 and more than 90% of the residuals behave as expected. Again, the observed discrepancies probably originate from the grouping of LPL records for which the 305ME milk yield is unknown.

(figure 5 here)

CONCLUSION

The results presented here suggest that the Weibull regression is well suited for an efficient analysis of LPL data, especially when its flexibility is enhanced by the use of time-dependent regression variables. The choice of a Weibull model largely alleviates the computation burden which limits the use of the Cox's model with time-dependent variables. Comparisons between populations are facilitated since the baseline hazard function can be described through only 2 parameters instead of a step function with many jumps. Also, the Weibull model is a particular type of proportional hazards model: an intuitive interpretation of the effects in the models remains very simple, through the concept of relative culling rate.

The inadequate treatment in models B and B* of records for which the 305ME milk yield is not known should be easy to correct: approximate 305ME records can be predicted from early lactation tests. When this is not possible (extremely short lactations), it can be assumed that the corresponding LPL records are censored at the end of the previous lactation. In any case, these models give encouraging results about the possibility of correcting LPL records for voluntary disposal.

Finally, models A* and B* can be extended to include transmitting abilities (i.e. sire effects) in order to detect genetic differences in culling rate of sires' daughters. Note that although the proportional hazards assumption is found satisfactory here, nothing guarantees that this is still the case for sire effects when they are added to models A* and B* . This will have to be considered as an approximation of the true situation. The validity of this assumption requires further investigation.

REFERENCES

(1) Bakker, J. J, R. W. Everett, and L. D. Van Vleck, 1980. Profitability index for sires. J. Dairy Sci. 63: 1334.

(2) Balaine, D.S., R. E. Pearson, and R.H. Miller, 1981. Profit functions in dairy cattle and effect of measure of efficiency and prices. J. Dairy Sci. 64: 87.

(3) Breslow, N. E., 1974. Covariance analysis of censored survival data. Biometrics. 30: 89.

(4) Cox, D. R., and E. J. Snell, 1968. A general definition of residuals (with discussion). J. R. Statist. Soc., B. 30: 248.

(5) Cox, D. R., 1972. Regression models and life tables (with discussion). J. R. Statist. Soc., B. 34:187.

(6) Cox, D. R., 1975. Partial likelihood. Biometrika. 62: 269.

(7) Cox, D.R., and D. Oakes, 1984. Analysis of survival data. Chapman and Hall, London.

(8) De Lorenzo, M. A., 1983. Non-linear estimation of dairy cow survival to fixed ages. Ph.D. thesis, Cornell Univ., Ithaca, NY; Univ. Microfilms Intl., Ann Arbor, M.I.

(9) Dennis, J.E., and R.B. Schnabel, 1983. Numerical methods for unconstrained optimization and nonlinear equations. Prentice-Hall, Englewood Cliffs.

(10) Dentine, M.R., B. T. Mc Daniel, and H. D. Norman, 1987. Evaluation of sires for traits associated with herdlife of grade and registered Holstein cattle. J. Dairy Sci. 70: 2623.

(11) Dentine, M. R., B. T. Mc Daniel, and H. D. Norman, 1987. Comparison of culling rates, reasons for disposal and yields for registered and grade Holstein cows. J. Dairy Sci. 70: 2616.

(12) Ducrocq, V., 1987. An analysis of length of productive life in dairy cattle. Ph.D. thesis, Cornell Univ., Ithaca, NY; Univ. Microfilms Intl., Ann Arbor, MI.

(13) Everett, R. W., J. R. Keown, and E. E. Clapp, 1976. Relationships among type, production and stayability in Holstein sire evaluation. J. Dairy Sci. 59: 1277.

(14) Everett, R. W., J. R. Keown, and E. E. Clapp, 1976. Production and stayability trends in dairy cattle. J. Dairy Sci. 59: 1532.

(15) Famula, T.R., 1981. Exponential stayability model with censoring and covariates. J.Dairy Science. J. Dairy Sci. 64: 538.

(16) Gianola, D., 1980. A method of sire evaluation for dichotomies. J. Anim. Sci. 51: 1266.

(17) Gill, G. W., and F. R. Allaire, 1976. Relationship of age at first calving, days open, days dry and herd life to a profit function of dairy cattle. J. Dairy Sci. 59: 1131.

(18) Kalbfleisch, J.D., and R. L. Prentice, 1980. The statistical analysis of failure time data. Wiley, New-York.

(19) Kay, R., 1977. Proportional hazard regression models and the analysis of censored survival data. Appl. Statist. 26: 227.

(20) Miller, R.G., 1981. Survival Analysis, Wiley, New-York.

(21) Norman, H. D., B. G. Cassell, R. E. Pearson, and G. R. Wiggans, 1981. Relation of first lactation production and conformation to lifetime performance and profitability in Jerseys. J. Dairy Sci. 64: 104.

(22) Smith, S. P., 1983. The extension of failure time analysis to problems of animal breeding. Ph.D. thesis, Cornell Univ., Ithaca, NY; Univ. Microfilms Intl., Ann Arbor, MI.

(23) Smith, S. P., and R. L. Quaas, 1984. Productive life span of bull progeny groups : failure time analysis. J. Dairy Sci. 67: 2999.

(24) Smith, S. P., and F. R. Allaire, 1986. Analysis of failure times measured on dairy cows: theoretical considerations in animal breeding. J. Dairy Sci. 69: 217 .

(25) Van Arendonk, J.A.M., 1986. Economic importance and possibilities for improvement of dairy cow herd life. Third world congress on genetics applied to livestock production, Lincoln, Nebraska, July 16-22,1986.IX: 95.

(26) Wolynetz, M. S., and M. R. Binns, 1983. Stayability of dairy cattle: Models with censoring and covariates. J. Dairy Sci. 66: 935.

Figure 1: Illustration of the definition of milk production classes.

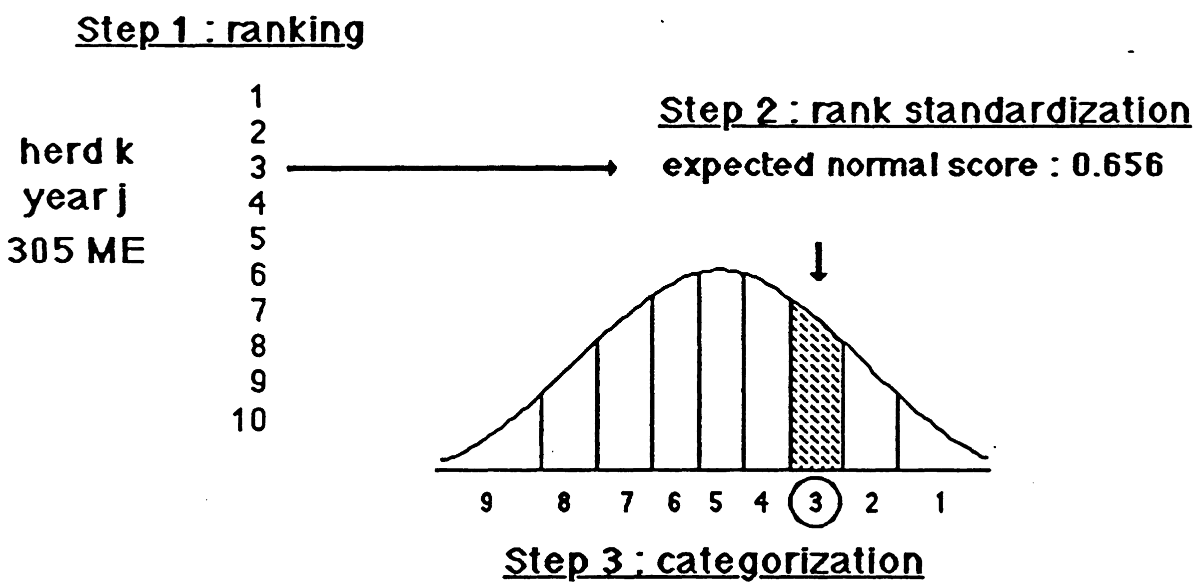
Figure 2 : Distribution of herd x year estimates for model A.

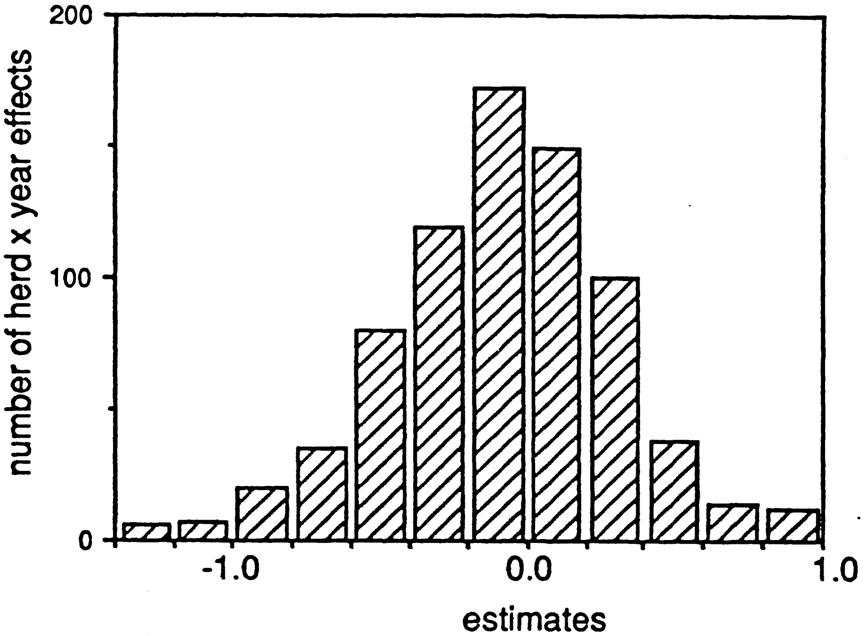
Figure 3 : Within herd x lactation level of production estimates for model B.

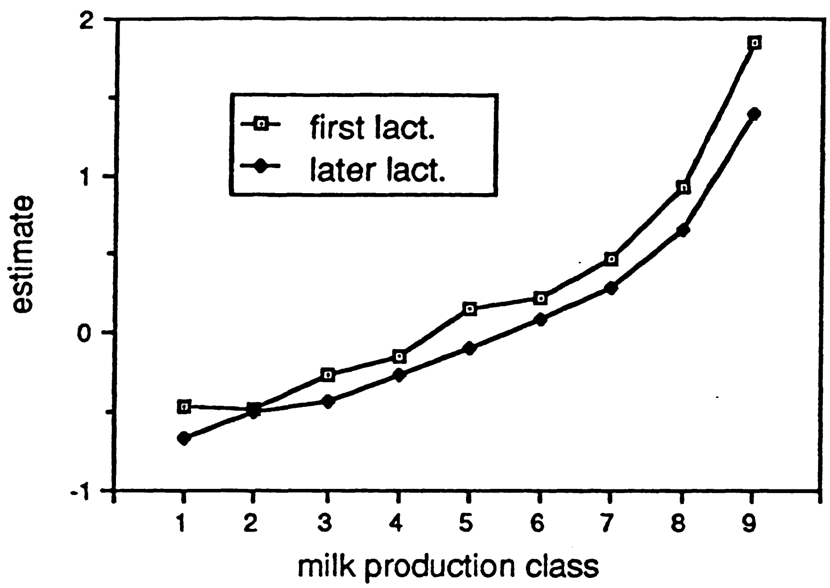
Figure 4 : Estimates of $[\rho^{-1}\ \beta_V]$ computed from data subsets S1 and S2 for model A*. (β_V is the Stage of lactation x Lactation number (SL x LN) effect and ρ is the slope parameter of the Weibull baseline hazard function).

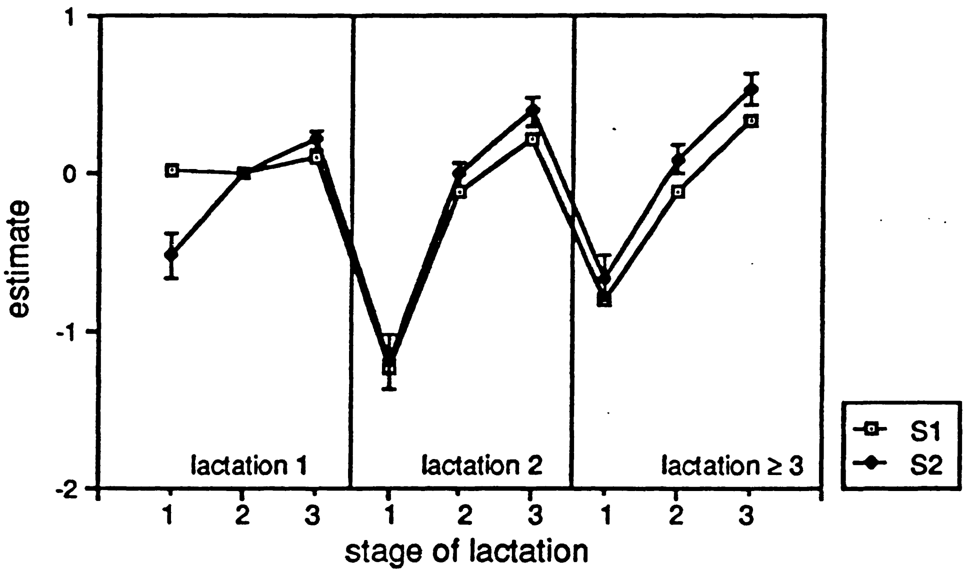
Figure 5 : Generalized residuals for model B.

Figure 1


Step 1 : ranking
herd k
year j
305 ME
1
2
3
4
5
6
7
8
9
10
Step 2 : rank standardization
expected normal score : 0.656
9 8 7 6 5 4 3 2 1
Step 3 : categorization


200
100
0
number of herd x year effects
-1.0
0.0
1.0
estimates


first lact.
later lact.
estimate
2
1
0
-1
1
2
3
4
5
6
7
8
9
milk production class


1
0
-1
-2
estimate
lactation 1
lactation 2
lactation ≥ 3
S1
S2
1
2
3
1
2
3
1
2
3
stage of lactation

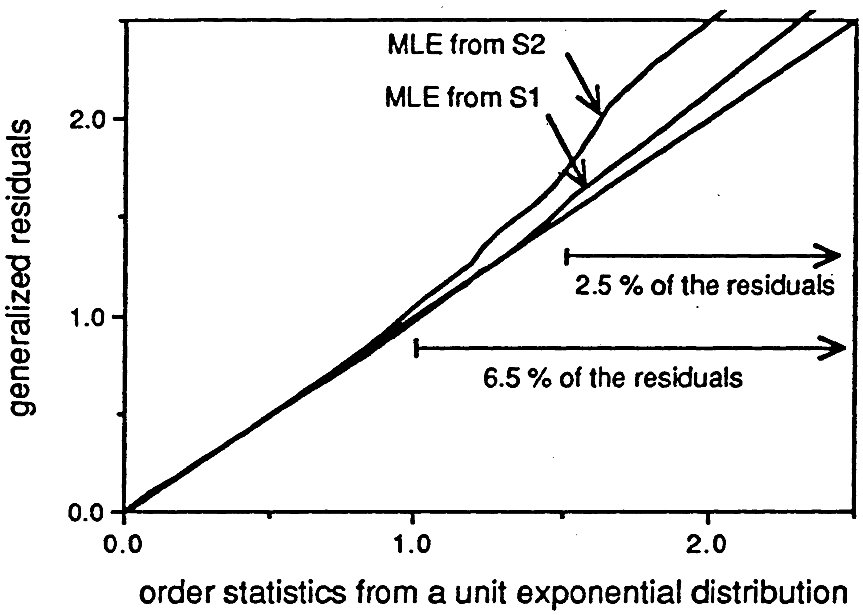

MLE from S2
MLE from S1
2.5 % of the residuals
6.5 % of the residuals
generalized residuals
2.0
1.0
0.0
0.0
1.0
2.0
order statistics from a unit exponential distribution

Table 1 : Maximum likelihood estimates of Stage of lactation and Lactation number effects for model A

Stage of lactation			Lactation number		
days after parturition	Model A	Model B	lactation	Model A	Model B
0 - 29	-0.77	-0.81	1	0.31	0.62
30 - 249	0.18	0.12	2	-0.03	0.15
250 - 0	0.63	0.66	3 to 5	-0.13	-0.26
			≥6	-0.14	-0.52

Table 2 : Regression equation of generalized residuals $\hat{e}_i$ for models A^* and B^* on the order statistics o_i of a censored unit exponential distribution

generalized residuals from:	estimate from data subset : S1	(R^2)	S2	(R^2)
	Model A^* :			
S1	$\hat{e}_i = 0.007 + 0.981\ o_i$	(0.999)	$\hat{e}_i = 0.003 + 1.002\ o_i$	(0.999)
S2	$\hat{e}_i = -0.003 + 1.019\ o_i$	(0.999)	$\hat{e}_i = 0.005 + 0.986\ o_i$	(0.998)
	Model B^* :			
S1	$\hat{e}_i = -0.013 + 1.031\ o_i$	(0.996)	$\hat{e}_i = -0.049 + 1.167\ o_i$	(0.984)
S2	$\hat{e}_i = -0.026 + 1.078\ o_i$	(0.982)	$\hat{e}_i = -0.021 + 1.051\ o_i$	(0.990)

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February 12, 1988

Dr. V. Ducrocq
Animal Science Department
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Ithaca, NY 14853

Dear Dr. Ducrocq:

Enclosed is a copy of the manuscript you submitted for publication in the Journal of Dairy Science. The reviewers recommend that the paper be published after you have considered their suggestions. If you can not accept all of the reviewers suggestions, please indicate those that were not accepted and the reasons why in your covering letter. Please consider these, make the necessary changes, and return two copies of the manuscript to me at my Columbus address as soon as possible.

Sincerely,

G.H. Schmidt, Editor
Journal of Dairy Science

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enc. ED#7474

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