[WALLSTREETBETS AND THE MADNESS OF CROWDS]

A Thesis

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ABSTRACT

Whether the current stock market follows the efficient market theory is a topic that has been studied all the time. We use Hurst exponent to test whether the price series and short interest series of GME and AMC affected in the WallStreetBet event have long-term memory. We hope to test whether there are price bubbles in the market and whether abnormal price fluctuations are related to short positions in this way. In the end, we found that even though the prices of GME and AMC rose sharply, their price sequences did not have long-term memory, but short interest sequences had long-term memory. This proves that market sentiment does not affect the price, on the contrary, the short squeeze caused by the excessive short position causes the excessively high price.

BIOGRAPHICAL SKETCH

Guanghan Xie comes from Beijing, China. She received her bachelor's degree of agricultural business from Oklahoma State University and China Agricultural University in 2019. Soon after, she started her master's degree at the Dyson School of Applied Economic and Management at Cornell University. During her graduate study in Cornell University, she focused on the research of agricultural finance and behavioral finance.

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CHAPTER 1: INTRODUCTION

Wisdom of financial fundamentals is the main orthodoxy that financial economists rely on to maintain their theories about the efficiencies in markets. Yet from time to time this orthodoxy goes astray and a sense of madness drives people to focus on an object and with some abandon, buy into it. The attractor to the object festers slowly with the few who buy in at profit and then more and then more. Like the R0 factor of an infecting virus the extraordinary popular delusions about the object give rise to economic (and social) instability. The R0 factor leads to an increase in speculative interest, attracting new buyers, assuring a further increase in the object's value, assuring in the next moment in time even more buyers, with the speculation building upon itself driving its own momentum in what ends up being a self-fulfilling prophecy. The dynamics of euphoria that drives the observed phenomenon discussed in this paper is as follows.

The common assumption in creating bubbles is that the bubble is driven by euphoric herd behavior. If we assume that the 'infection rate' is $p(t)$ then the number of investors in time follows

$$N_t = N_0 p(t) T^{p(t)}, t \rightarrow 1, T$$

This model assumes that herd behavior follows a power law. $p(t)$ is the basic reproduction (R0) value which indicates the infection rate and captures the exponential growth of herd behavior. For example, if $p(t) = 3$, then this means that three individuals active in the first instance leads to 9 in the 2nd instance and 27 in the 3rd

instance and so on. At any moment in time the new investors entering the fray is given by

$$\frac{\Delta N_t}{\Delta t} = t * N_0 \rho(t)^{t-1} * \frac{\partial \rho(t)}{\partial t}$$

This number is dependent on R0 which also changes in time. One can imagine $p(t)$ being of quadratic form, rising in numbers towards a maximum and then falling. The bubble model also involves price $P(t) = P(N_t)$ with the change in price at each moment in time being determined by

$$\frac{\Delta P(t)}{\Delta t} = \frac{\partial P(N_t)}{\partial N_t} \frac{\partial N_t}{\partial \rho(t)} \frac{\partial \rho(t)}{\partial t}$$

As the time derivative of R0 increases so too will prices; when it stabilizes the change in prices will fall to zero and as R0 begins to decline the prices fall.

Whether this structure holds in reality will be determined by further research, but it captures the main elements of euphoria as depicted by Galbraith (1994); There are those who are persuaded that some new price enhancing circumstances are in control, with an expectation that the market will steadily or increasingly rise, perhaps indefinitely. It is a herd adjustment to a new situation that resonates across social and financial boundaries. This group provides the fuel. In addition to these there are the more astute individuals who perceive the speculative mood of the moment and join the fray, but this group has little staying power and will sell with profits and exit, and so starts the eventual decline. When, and to what extent the decline will happen depends on euphoric intensity, but eventually the bubble will break.

The type of euphoria described above, and the consequence on the financial market is the subject matter of this paper. In January 2021, the sluggish US stock market under the influence of the COVID-19 pandemic was about to usher in a storm.

The leaders of the storm are a group of people who call themselves the *WallStreetBets* or WSB. *WallStreetBets* is a subreddit online forum. They use Reddit as a base camp and participate in discussing stock and options trading. This forum has become the world's largest gathering place of retail traders, with over 10.5 million registered members as of June 2021. Affected by the COVID-19 pandemic, many companies in the entertainment industry have suffered. Among them is GameStop Corporation (GME), an electronics retail company that sells game hardware and optical discs. In the past three years, due to the development of the network economy and the digitization of video games, revenue performance had continued to decline. Under the influence of the pandemic, GME's stock price proceeded to fall. Large investment institutions generally had a negative attitude towards GME's stock price based on fundamental analysis. On April 14, 2020, the news that 84% of GameStop's stock was short went out on Reddit.

From April to July 2020 GME's short float had exceeded 100% under the bearishness of Wall Street Capital companies. This paved the way for a potential short squeeze. At the same time, on Reddit, more small investors were participating in discussions and going long GME stocks. In contrast, short sellers led by Wall Street hedge fund Melvin Capital kept their pessimistic attitudes toward GME and the short float persisted.

On January 11, 2021, GME announced that Ryan Cohen had officially joined the board of directors. This professional manager who once founded Chewy, the nation's largest online pet supplies e-commerce company, added more confidence to single investors on Reddit. As the discussion of GME on Reddit increased, more people participated. This caused GME's stock price to rise by more than 60% in three

days, and the stock market bubble intensified. Since then, GME's stock price rose sharply on January 25, and has repeatedly fused and suspended trading due to the high increase rate. The madness of GME stock price has also attracted many other Wall Street financial institutions. Elon Musk also posted on Twitter to support small investors to take long positions. GME's stock price reached $347.51 at its peak. Such a huge market bubble ended when Robinhood, the online brokerage, restricted trading action on GME stock.

GME was not the only stock that was affected by the *WallStreetBets* event. On Reddit, the small investors gathered up and took long positions on some stocks which had high short of float and suffered from the COVID-19 pandemic. Among them was AMC Entertainment Holdings Inc, which ran movie theaters. The price of GME and AMC during the *WallStreetBets* incident is shown in Figure 1 and Figure 2 below.

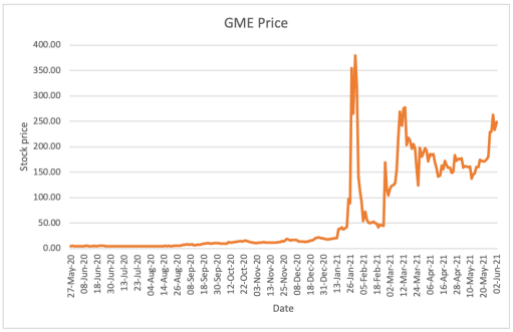

Figure 1: GME price

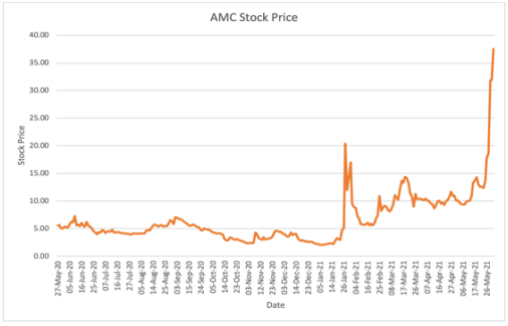

Figure 2: AMC stock price

The focus of this *WallStreetBets* event was that individual investors were away from the stock's value analysis, and the investment behavior for short-term speculation brought huge volatility to the market. Potential high short positions also provided soil for this event. In the *WallStreetBets* event, the impact of Reddit and other social media on the stock market became the focus of extensive discussion. The question raised by this *WallStreetBets* event is how much small retail investors matter to the market. Indeed, it is believed that this has been the first event in history that small-time investors actually confronted and defeated the traditional institutions, causing them serious damage. The individual traders formed an army on the Reddit forum r/WallStreetBets with no specific team leader and organization. This phenomenon was spontaneously initiated by *netizens* without any prior planning and triggered the short squeeze.

Under the efficient market hypothesis, all new information should immediately and fully reflect the market price. Moreover, it is unpredictable for the impact of any new information. Consequently, the returns of the market are essentially not influenced by the past. It implies the market returns should follow a random walk, characterized as a 'Brownian motion' in scientific terms. There are findings indicating that in a long period, the efficient market hypothesis could hold, while the WSB event could be a piece of evidence that under certain situations, the past could have a huge impact on the future.

The Economic Problem

A Brief History of Bubbles

In the general understanding, whenever asset prices deviate progressively and more rapidly from the path dictated by its economic fundamentals, it can be considered a bubble, which is stated as a ‘rational bubble’ (Santos and Woodford 1997). In contrast to this, a speculative bubble is a spike in asset prices to unsubstantiated levels, fueled by irrational speculative motivation, market sentiment, and momentum.

There have been many incidents in financial history where speculative bubbles caused market turbulence. The South Sea Company case in the United Kingdom, the Mississippi bubble incident, and the tulip mania in the Netherlands are known as the three biggest early European economic bubbles. Their emergence is inseparable from the imperfect legal system, compromised audit system, and information asymmetry during that time. Take the South Sea Company incident as an example. In the early 18th century, the British government raised funds by issuing bonds to the society. Operators in the company spread a lot of false positive news to the market, causing the South Sea Company's stock price to skyrocket. Speculators saw the opportunity and began to invest heavily in South Sea company stocks. After the British government promulgated a bill to curb the growth of the bubble company, the South Sea Company's potential crisis broke out. Investors were affected by panic and concentrated on selling stocks, causing stock prices to plummet. The South Sea company bubble incident became the most serious financial crisis in European capital markets before the Industrial Revolution.

Many scholars have studied the mechanism of bubble formation. A general asset price bubble is defined as a rational expectation of equilibrium that is higher than the dividend with probability one using a finite period general equilibrium model.

Both the research of Bick (1987) and Kreps (1982) argued that geometric Brownian motion (gBm) can be shown to be a rational expected equilibrium in the market, and all available information is immediately discounted to the current price. Horst (2005) believes that while some investors determine the price of an asset based on underlying fundamental value, ther investors use past observations to predict future asset prices. In these models' endogenous conversion between different types of market participants will cause market price fluctuations, and exhibit market chaotic behavior. This raises the possibility that stock prices can behave as a fractional Brownian motion (fBm) with some type of determinism and correlation between changes in prices. Scheinkman and Xiong (2003) applied the standard Brownian motion as the criterion for whether there is a speculative bubble in the market and attributed the occurrence of speculative bubbles to recursive expectations of traders to take advantage of mistakes by others, which is quite different from the 'rational bubble'.

The discovery of many market anomalies raises questions about the efficient market hypothesis. The efficient market hypothesis is an important concept in financial market research, which refers to the fact that all known information in the market will be immediately reflected in the price of securities. If a market is efficient, it means that changes in market prices are caused by all market-related information. Changes in market prices are the result of the optimization of resource allocation and are the core mechanism of market operations. This price change is rational and can reflect the current market supply and demand situation in a timely and accurate manner. In other words, no one can continuously profit by predicting stock prices. Historical information is not helpful in predicting future prices, so the time series of market returns should not have autocorrelation, that is, market memory. The

autocorrelation of the market in the short term is called short-term memory. If the market has short-term memory, it will be quickly eliminated by arbitrageurs. According to efficient market theory, the market should not have long-term autocorrelation, that is, long-term memory. Efficient market hypothesis needs to meet the assumptions of a competitive market. If long-term memory in the market persists then this suggests the presence of some underlying deterministic structure in price movements that arbitrageurs can exploit for risk free gain. The efficient market theory argues that due to the competition of arbitrageurs for excess returns in the market, the time series of returns generally do not have a significant trend. Even if there is a trend, it will be quickly eliminated by arbitrageurs. In the market structure corresponding to the efficient market hypothesis, the sequence of returns should be independent, linear, of finite variance, and obey a normal distribution.

Both the random walk and efficient market hypothesis (EMH) assume that there is no correlation in the time series of financial derivative returns, and the market has no memory of the past. This memoryless time series is referred to as the Markov property. Since the efficient market hypothesis has always been regarded as an important cornerstone of capitalist market theory, it has been subjected to more extensive empirical tests than any economic theory. A large amount of empirical data questioned its correctness. These tests that are different from theories mainly focus on the test of the normality assumption of EMH; the study of the relationship between volatility and the power law of time length; and market memory theory. The traditional EMH theoretical framework is basically a linear market hypothesis, and in the process of empirical analysis, it mainly uses geometric Brownian motion to

measure whether the market is effective. However, the actual market is not as concise as assumed by efficient market theory, but often manifests as a fractal random walk.

What precisely defines a 'bubble' is ubiquitous. By appearance a stock price bubble will show a strong and rapid rise, but by this definition any short-term or long-term movement in prices can be visualized in scale to capture a bubble. Random walks by definition follow a pattern of excursions with stopping times (the time in minutes, days, weeks that a stock remains above a baseline) tending to follow a Poisson distribution with many short excursions but a few very long excursions. This is in the nature of Brownian motion. But if these excursion patterns satisfy the Markov property with independence across price changes, it is difficult to assign price movements to collective euphoria. If changes in prices are recursive in nature, that is the change in prices between t and $t + s$ is driven by changes in prices between t and $t - s$, then the Markov property will fail and changes in prices will be correlated across time. Whether a stationary market exists, while violating the Markov property, is established by the concept of fractional Brownian motion. The economic problem addressed in this thesis is whether stock involved in *WallStreetsBets* transitioned from a geometric Brownian motion to a fractional Brownian motion. If such a transition is observed, then that would be a strong indicator that the price movements caused by *WallStreetBets* and the Reddit forums created a bubble.

Corresponding to the efficient market theory is the fractal market theory. In the 1960s, Professor Mandelbrot proposed that the return distribution of the stock market does not follow a normal distribution, but a fat-tailed distribution (Leptokurtic distribution), and the time series of prices does not satisfy the classic random walk model. The market risk under the fat-tail distribution is very different from the market

risk under the normal distribution assumption. The fat-tail distribution means that the market has very little volatility most of the time. Under the fat-tail distribution, the probability of small fluctuations is less than the probability under the assumption of normal distribution; the probability of having large fluctuations in the market is much greater than the probability of normal distribution. Financial risk may be underestimated if it is assumed that the time series of returns obey a normal distribution. In the 1990s, Professor Peters proposed the fractal market hypothesis on this basis. He believes that the essence of the financial market is a complex nonlinear dynamic system. The capital market consists of many independent investors. Each investor has a different investment cycle. This means that the market is affected by different investment behaviors, which provides sufficient liquidity for the stability of the market and ensures the smooth operation of the market. The efficient market theory believes that the liquidity of the financial market is provided by white noise traders. Fractal market theory believes that rational investors' differences of opinion due to their investment horizons have different expectations for the same market. The difference of market expectations provides the main transaction liquidity for the financial market, while white noise traders only provide a small part of the market liquidity. To some extent, this theory can explain some extreme events in which the sudden loss of liquidity in the financial market leads to sharp price fluctuations. This conclusion is different from the efficient market theory that market trends will quickly disappear under the actions of arbitrageurs.

Secondly, different market information also has different effects on investors. The efficient market hypothesis divides investors into rational investors and irrational investors. Rational investors invest based on market information, while non-rational

investors are regarded as white noise traders. Fractal market theory considers investors to be bounded rational, which is obviously different from the efficient market hypothesis. The efficient market hypothesis believes that market price is a summary of the reactions of different investors to market information. If there is no economic cycle in the capital market, then the market itself has no long-term trend. In other words, market volatility is mainly determined by transaction volume, market liquidity, and short-term information. However, if the market has a correlation with the economic cycle, it means that the market has a long-term memory, the market can be predicted, and the risk of investment will be reduced accordingly. The fractal market hypothesis theory proposes that the self-similar structure in the market may be caused by the existence of independent investors with different investment time periods in the market. There are many differences between fractal market theory and the characteristics of EMH. Among them, the most obvious characteristics include long-term memory, that is, information from the past will have a long-term impact on the future; and scale invariance, that is, similar statistical laws occur under different time scales.

The Research Problem

The abnormal stock price reflects a bubble that existed in the market. The rapid rise of stock prices in a short period of time violates the existing economic assumptions, that is, the law of geometric Brownian motion of stock prices follows. It is suspicious whether the discussion on the *WallStreetBets* online forum is sufficient to cause such huge market fluctuations. Most of the participants in the *WallStreetBets* online forum are retail investors. They are amateur investors who only access information from public market information. They are not informed investors and do

not meet the conditions of market manipulation in theory. We believe that in addition to the discussions on the *WallStreetBets* forum, there are other factors driving this small-scale stock market bubble. When combing through the storyline of the entire *WallStreetBets* incident, we found that *WallStreetBets* first discovered the investment opportunity of GameStop Corporation through their excessive short interest. Therefore, in addition to intuitively using the long-term memory of price data to represent stock market bubbles, we also collected short interest from GME and AMC companies that were affected by the *WallStreetBets* incident. We try to distinguish whether there is a possible bubble in the stock market through whether the short interest data has long-term memory.

The research problem is to investigate whether there was any economic impact on the basic properties of the stock prices i.e., did the *WallStreetBets* event create economic opportunism and did the underlying price dynamics signify a bubble. In addition, did the *WallStreetBets* have any influence on this stochastic pattern of short interest given the underlying and overt objectives of activist investors to put pressure on short sellers?

To investigate this, I evaluate two of the most significant stocks that were affected by this *WallStreetBets* incident, AMC Entertainment Holdings Inc (AMC) and GameStop Corp. (GME) and comparing them with Apple Inc (APPL) in the same time period. I take advantage of a natural experiment in which Reddit. Pre-chatter (date to date) and post-chatter to detect whether the underlying stochastic changes in form. Therefore, the overall objective of this thesis is to prove whether possible price bubbles can be found from the short interest in stocks. To achieve the overall objective, I use the Hurst measure for identifying persistence and memory in the

underlined prices of this stock that I evaluated. If we find that prices show increased persistence that is long-term memory, that could indicate that what occurred was a bubble. If, however, we find that the markoff property of independent random shocks across time, was maintained, that could indicate people and their actions but not memory caused the observed price rise and therefore, despite what might be observed in examining the graph of the stock prices, it is distinguishable from a bubble.

Short Squeeze

Market manipulations existed in every period of American securities industry. For example, the gold corner on Black Friday, September 24, 1869, corners on the Northern Pacific Railroad (1901), Soybean market corners (1977 and 1989), and treasury bond market corner in 1986. Those events in history all form a bubble in the securities market by extraordinary price increases and collapses in a short period of time. According to Wall Street finance analysts, GME's stock price showed an abnormally sharp rise that violated the analysis of market value, which was caused by an excessively high short position. In history, common short-squeeze incidents have been attacks on short-sellers by financial institutions with strong capital who we called a “large trader”. For the large traders, the assumption has been made by economists that they have no information and trades move prices only because of size or other players in the market believe that they are informed. The characteristic of a large trader is that they generate profit at no risk (Jarrow, 1992). They generally have the initiative to supply the stock by buying a certain stock in large quantities or secretly controlling the source of that stock. Then intentionally, they reduce the volume of such stocks that can be circulated in the market. As a result, the number of short stocks exceeds the number of outstanding stocks. By making the short-sellers unable to

obtain the corresponding delivery quantity on the due date or raising the stock price by the short position, they may be forced to carry out the margin call.

Different from the *WallStreetBets* event, the traditional short squeeze event is dominated by a group of retail holders. According to public market information, the short position of GME is very high. This shows that this stock has a financial risk that is excessively shorted. During that time, GameStop had over 138 percent of its float shares sold short, making it the most shorted stock in the U.S stock market. Secondly, this stock price is very low, therefore it is also very limited in the profit space. Thirdly, Michael Burry's support and Ryan Cohen's joining the company brought the good news in value analysis, so that this company's share price has the potential to rise. These elements are combined and found by *WallStreetBets.* That made the retail holders become information traders. With the extensive communication of social media, GME is almost inevitable from short squeezing. We believe that the key to this extremely high stock price is not caused by the unity of retail rebellion’s purchasing behavior. The essence of the increase in GME stock price is short squeeze. If this stock itself does not have a financial risk, then though the *WallStreetBets* are united, they are not able to compete with the financial hedge fund of Wall Street.

In this incident, it is possible that a short squeeze caused a stock bubble. Usually, we think that stock price is a random event, and its price fluctuates within a certain variance around the value of the company. But factors other than investor sentiment can cause a bubble. Perhaps if there are multiple sources of bubbles they should be labeled accordingly. For example, in our empirical examinations below we test for changes in fractional properties in prices and short positions. If changes to the Hurst measure in prices is observed then this would indicate investor euphoria, and

'price bubble' would be an appropriate label. If we find that the Hurst measure on short positions changes then this too can cause a bubble (in either direction) and it would be appropriate to label this a 'short bubble'. If both are in play, then perhaps a 'mixed bubble' label would be appropriate. However, some unexpected events led to the formation of a price bubble. How to find bubbles has become one of the hotspots of financial research. We believe that the stock market is a time series that satisfies a random walk. By calculating the Hurst exponent of a stock's short interest, the market sentiment can be tested, and the possible price bubble caused by the short squeeze can be found.

Brownian Motion

Brownian motion was originally used to describe the irregular motion trajectory of suspended particles hit by molecules, which is a phenomenon of random fluctuations. Brownian motion, indicated as B(t), usually satisfies three properties: (1) B(t) is an independent incremental process: for any $t > s$, $B(t) - B(s)$ is independent of the previous process; (2) B(t) has a normal distribution increment: $B(t) - B(s)$ satisfies a normal distribution with a mean value of 0 and a variance of $t - s$; (3) B(t) is a continuous path: for any $t \geq 0$, B(t) is a continuous function of t. After Einstein published a paper on Brownian motion in 1908, Brownian motion became the basic model of the random walk hypothesis. Einstein believed that the radius of the Brownian motion of a molecule can be measured by the square root of time: $R = t^{0.5}$.

The Hurst exponent is a statistic discovered on this basis to test the randomness of a time series. The random walk hypothesis and the efficient market hypothesis are unified. Its meaning is to say that because the price of securities on the market has

already reflected all the information available in the market, the remaining price fluctuations are due to random events that cannot be predicted and captured. Therefore, the price fluctuation of securities is a form of random walk. If we indicate $P(t)$ is the price of a stock at time t with uncorrelated increments, then under the random walk model hypothesis, $P(t+s) - P(t)$ should be independent of $P(t) - P(t-s)$ and of all past prices. Under the efficient market hypothesis and the random walk model, knowing the past prices will not help to predict the future price. The martingale model suggests that since the probabilities of a security's price to go up or go down are the same, therefore, the expected return on investing in that security should be the average return of the market. This point has been demonstrated by Mandelbrot in 1971 that every random walk without drift is a martingale system, and the price of a security in an efficient market is a martingale. Random walk hypothesis indicates a nonstationary time series, where its mean is constant throughout the time while the variance of the series is increased linearly.

Since it was found that the actual financial market is too complicated to be described by an idea as simple as the Markov model, Mandelbrot (1967) pointed out that using fractional Brownian motion can more accurately describe the fluctuations of the financial market while proposing the concept of a fractal market. On this basis, Peters (1994) systematically proposed the fractal market hypothesis and used fractional Brownian motion to describe the trajectory of the financial market.

In the financial market, the cycle analysis of time series has always been a very important issue. In the traditional efficient market hypothesis, price fluctuations in financial markets are usually considered to follow a random walk model. There are also other studies that believe that most time series in financial markets do not follow

the principle of random walk. The empirical research of De Bondt (1985) and Summers (1990) believes that the stock market exhibits the characteristics of mean reversion. But his research results have received many doubts. After entering the 1990s, a large number of studies have found that there is actually a non-periodic cycle in price variance in the financial market.

Rescaled Range (R/S) Analysis and the Hurst Exponent

Rescaled Range (R/S) Analysis is introduced into empirical research to study the non-cyclical characteristics of financial markets. Traditional cyclical research on financial markets is based on the assumption that the system being studied is random and normally distributed. But when the system being studied is nonlinear, traditional statistical methods no longer work. R/S analysis is a widely used nonparametric statistical method. Its main advantage is that it does not have to assume that the distribution of the tested time series follows a normal distribution. The robustness of R/S analysis results is not affected.

Hurst exponent was first proposed by British hydrologist H.E. Hurst, who studied the cycle of flood formation. Based on Einstein's research, the radius of molecular motion R has a similar relationship with time t: $R \sim t^H$, where H is the Hurst exponent. When H=0.5, it indicates that the time series is a standard Brownian motion, that is, geometric Brownian motion. The time series at this time can be described by a random walk, and it exhibits Markov characteristics. When $H \neq 0.5$, the possibility of a fractional Brownian motion to describe the time series exists. It is necessary but not sufficient, since there is always the possibility that $H > 0.5$ or $H < 0.5$ can arise by chance alone (Mandelbrot 1982). It means that the time series has long-term memory, as well as the level of persistence in the given signal.

Mandelbrot (1975) and Peters (1994) used the Hurst exponent to measure the strength of long-term memory to study the time series of market returns. Their results show that long-term memory is common in the time series of returns in stock markets. Their research results refute the classic efficient market hypothesis. Other studies on the time series of financial market returns, such as Matteo (2005), have also proved that they have a long-term memory and conform to the fractal market hypothesis. Since the Hurst exponent represents whether there is a long-term correlation in the time series, we believe that the Hurst exponent can be used as an index to perceive market sentiment. The Hurst exponent could be used to predict the direction of future price changes, and this index can be used as an early warning of market bubbles. According to some researchers, such as Eom and Choi (2008), they empirically investigated the relationship between the degree of efficiency and the predictability in financial time series data. They indicate that the Hurst exponent can be used as the measurement of the degree of market efficiency.

Brownian Motion, Stock Prices and Short Positions

Under conventional assumptions about the lognormality of prices we can assume that stock prices follow a geometric Brownian $\mu_i(P,t)$ being a linear drift, $\sigma_i(P,t)$ the variance, and dz_i a Wiener process. For the singular Brownian motion, the stochastic differential equation is

$$\frac{dP_i}{P_i} = \mu_i(P,t)dt + \sigma_i(P,t)dz_i$$

In our research, we will need to examine whether short interest has the same property. We assume a simple structure of short position that increases as prices rise and follows the power relationship. Assume short position follows a power rule as follows

$$S = AP^{\lambda}$$

Assume prices follow a Brownian motion

$$dP = \mu P dt + \sigma P dz$$

The first and second derivatives for that equation are

$$\frac{\partial S}{\partial P} = \lambda A P^{\lambda-1}$$

$$\frac{\partial^2 S}{\partial P^2} = \lambda(\lambda-1) A P^{\lambda-2}$$

According to Ito's Lemma, the transformation is given by Ito's Lemma

$$dF = \frac{\partial S}{\partial P} dP + \frac{1}{2}\frac{\partial^2 S}{\partial P^2} dP^2$$

The rest of the math follows by substitution

$$dF = \frac{\partial S}{\partial P}\left(\mu P dt + \sigma P dz\right) + \frac{1}{2}\frac{\partial^2 S}{\partial P^2}\left(\mu P dt + \sigma P dz\right)^2$$

$$dF = \lambda A P^{\lambda-1}\left(\mu P dt + \sigma P dz\right) + \frac{1}{2}\lambda\left(\lambda-1\right) A P^{\lambda-2}\left(\mu P dt + \sigma P dz\right)^2$$

$$dF = \lambda A P^{\lambda}\left(\mu dt + \sigma dz\right) - \frac{1}{2}\lambda\left(\lambda-1\right) A P^{\lambda}\left(\mu^2 dt^2 + \sigma^2 dz^2 + 2\mu\sigma dz dt\right)$$

$$dt^2 = 0, dz dt = 0 \text{ and } dz^2 = dt \quad \left(by\ Ito's\ Lemma\right)$$

$$dF = \lambda A P^{\lambda}\left(\mu dt + \sigma dz\right) - \frac{1}{2}\lambda\left(\lambda-1\right) A P^{\lambda}\sigma^2 dt$$

$$dF = \lambda S\left(\left(\mu - \frac{1}{2}\left(\lambda-1\right)\sigma^2\right)dt + \sigma dz\right)$$

The stochastic differential equation governing the dynamics of changes in the short position is therefore

$$dF = \lambda S[\left(\mu - \frac{1}{2}(\lambda-1)\sigma^2\right)dt + \sigma dz]$$

Therefore, under the assumed power relationship, the short position follows a Brownian motion with drift and volatility

$$E(dF) = \lambda S\left(\mu - \frac{1}{2}(1-\lambda)\sigma^2\right)T$$

$$VAR(dF) = \lambda^2 S^2 \sigma^2 T$$

$$\lambda > 0$$

Assuming the short position elasticity is $\lambda > 0$, this Brownian motion is going to be largely dictated by this measure. As λ rises, the rate of change in short positions decreases. Whether the short position behaves depicted in Equation $S = AP^{\lambda}$ is not to be proved in this paper and is provided for illustration purposes. However, if the market short position is related to market prices along this line of logic and can be represented by a twice differentiable function of the sort presented here, and the underlying price follows a Brownian motion, then our assumption that the short position follows a Brownian motion is justified. Therefore, we can use the Hurst exponent to determine whether the prices and short interest of the affected stocks have memory during the *WallStreetBets* event, so as to verify the authenticity of the efficient market theory.

CHAPTER 2: METHODOLOGY

In order to measure the impact of the *WallStreetBets* event on GME and AMC, I estimated the Hurst exponent. H. E. Hurst discovered a way to characterize biased random processes or fractional Brownian motion (fBm) in the 1950s. It was first proposed in hydrodynamics but has since found multiple applications in empirical finance. In contrast to a pure random walk, this biased process indicates a long-term dependency between observations, which is characterized as a long-term memory of

the market. A single event can affect all subsequent periods. The dependence between cycles is called "persistence".

Generally speaking, the prices of stocks, currencies, and financial derivatives follow a random walk process, which is consistent with geometric Brownian motion (gBm). This is a memoryless series that conforms to Markov nature. If a sequence contains some long-term memory, then it is referred to as a fractional Brownian motion (fBm). We use the rescaled range (R/S) Hurst measure in order to test for Brownian motion. R/S analysis is an insightful test for market dependency and market efficiency. In this article, I apply Hurst's R/S analysis to stock short interest and stock price, as a way to indicate the potential market bubble during the *WallStreetBets* event. Fractional Brownian motion (fBm) is a memorized stochastic process with multiple roots, and one of them has to be the unit root. On the other hand, the geometric Brownian motion (gBm) has only one root which is its unit root. We use the autoregression model for the unit root test. If a series follows the geometric Brownian motion (gBm), therefore, it satisfies the Markov property, and the Hurst exponent of that series should be 0.5. The fractional Brownian motion (fBm) will have a Hurst coefficient greater or smaller than 0.5.

When the Hurst exponent is greater than 0.5, the future price increase or decrease is positively correlated with the current price, which means that there is a positive memory in the price series. The existence of this kind of memory in the sequence makes the price sequence self-reinforcing. They seem to be random and independent of each other, but in fact they are deterministic and predictable. From the perspective of efficient market theory, it can be considered that part of the market is inefficient, and the price of the transaction product does not fully reflect all available

information. Regarding the relationship between efficient market hypothesis and Brownian motion random walk, other scholars, such as Kirman, have proved the relationship between the price of transaction products and random shocks in the past. (Kirman, 2009)

Since Brownian motion is very important in asset pricing and practice, it is extremely important to test whether prices and short interest sequences follow the law of Brownian motion in the existing natural experiment. In our paper, we show that we check that all data does not satisfy the Markov property of geometric Brownian motion, but satisfies the property of fractional Brownian motion. The general set of fractional Brownian motion (fBm) is described by

$$dx = \alpha x dt + \delta x dZ^H$$

Where dZ^H indicates a fBm with Hurst exponent that $H \in (0,1)$. Setting a time step or duration $k = \Delta t$, then the variance over Δt expresses as

$$E[x_{t+\Delta t} - x_t]^2 = \delta^2 (\Delta t)^{2H}$$

When H=0.5, then it is the sample of a geometric Brownian motion (gBm). The variance is linear in time which is consistent with the gBm feature. It implies that there is no dependence between observations and not long-term memory within the market. For $H < 0.5$, the covariance term decreases when the time step increases. It suggests that the value of the future is affected in the opposite direction of the past. In contrast, for $H > 0.5$, the covariance term increases when time step increases, and the future moves the same direction as the past (Turvey, 2007).

Dividing the left-hand side and reorganized the function

$$\frac{E[x_{t+\Delta t} - x_t]^2}{\delta^2} = \Delta t^{2H}$$

It is referred to as Scaled Variance Ratio. Since they are positive on both right and left hand, apply log theorem

$$log\left(\frac{E[x_{t+\Delta t}-x_t]^2}{\delta^2}\right) = 2H * log(\Delta t)$$

Hurst exponent can be calculated as

$$H = \frac{1}{2} * \frac{log\left(\frac{E[x_{t+\Delta t}-x_t]^2}{\delta^2}\right)}{log(\Delta t)}$$

I used the short interest rate and stock price data of GME and AMC during the *WallStreetBets* event, respectively, to calculate the Hurst exponent using the method shown above. At the same time, I also calculated the Hurst exponent before the *WallStreetBets* event with these two companies. I used GME stock price and short interest data from May 28, 2019 to June 11, 2020, and AMC's data from January 2, 2018 to May 4, 2019. The reason why I choose this period of time as the data source of my control group is because they have not experienced concentrated short selling during this period, and it can represent the normal activities of these two stocks when they did not experience the madness of crowds similar to the *WallStreetBets* event. As a horizontal comparison group, I chose Apple Inc's data to calculate the Hurst exponent within the two periods as the baseline to verify that for the US stock market, the random walk or gBm has been followed in the long-term, whether it's short interest or stock price.

CHAPTER 3: AUTOREGRESSION MODEL

In our definition of bubble, we argue that bubbles can only exist if the Markov property fails. There are two important aspects of this definition. The first is that the persistent time series will follow an autoregressive process with AR(n>1) which implies some form of underlying deterministic structure that may not be visually apparent. This is the fractional equivalent to underlying determinism in Chaos theory. The second important aspect is that even though the Markov property fails, the time series still must contain a unit root; that is the time series must still be non-stationary in levels prices and stationery in differences. If this second property fails, then the price series will either be tending towards infinite or zero. This is described in detail in Turvey and Wongsasutthikul (2016), who developed the relationship between the autoregressive model and fractional Brownian motion. The autoregressive approach is to identify whether unit roots exist in the series. The autoregression model is defined as this

$$Y_t = a_1 Y_{t-1} + a_2 Y_{t-2} + \ldots + a_q Y_{t-q}$$

The notation of the autoregression model is AR(p) where p is the order of the regression, which implies the price difference. a_1, a_2, …, a_q are the autoregression coefficients of the model. The lag of the autoregression model is indicated as q. For any AR(p) model with the order of the regression greater than 1, they could have multiple roots, however, if $\sum_{i=1}^{q} a_i = 1$, then it implies that it has one unit root among the complex roots. Therefore, our null hypothesis for a fractional Brownian motion is that $\sum_{i=1}^{q} a_i = 1$.

In our case, we have 256 observations for each system, therefore, the lag of our autoregression is 16. Since we used day-to-day opening prices and short interest rate for each stock, the order of our AR model is 1. We ran the AR(1) model on every

system. We made a logarithmic transformation on the short interest rate of APPL during the *WallStreetBets* event, due to the lack of variance of this system. It is reasonable since Apple Inc is the largest company in the world, there are relatively low short interest in this company. After we made the adjustment, our results showed that all of our tested systems had the AR estimation sum up to one, which indicates that all of them follow the fractional Brownian motion. With this fact in mind, we can use the Hurst exponent in order to test the long-term memory within the system.

CHAPTER 4: RESULTS

The analysis primarily used the real-time data of GME, AMC and APPL from publicly available sources[1] We used daily data on opening prices and short interest rate (Day to cover rate). The scaling property for Brownian motion should hold for opening, closing or settle price since being on the same time scale the percentage changes should be independent regardless. Therefore, we chose the opening price out of convenience. The results are shown below.

Estimated Hurst exponent summary table

	Hurst Exponent	**R-squared**	**Standard error**	**P-value**	
GME SI during WSB	0.81	99.60%	0.036	0.013	(1)
AMC SI during WSB	0.75	99.90%	0.017	0.005	(2)
GME P during WSB	0.56	98.90%	0.042	0.305	(3)
AMC P during WSB	0.56	98.70%	0.045	0.342	(4)
GME SI without WSB	0.62	99.80%	0.02	0.029	(5)
GME P without WSB	0.56	97.70%	0.06	0.427	(6)
AMC SI without WSB	0.6	99.60%	0.026	0.057	(7)
AMC P without WSB	0.6	99.90%	0.009	0.008	(8)
APPL SI during WSB	0.52	99.60%	0.022	0.476	(9)
APPL P during WSB	0.57	99.30%	0.034	0.163	(10)

[1] I collected the price and short interest data from https://www.ortex.com

APPL SI without WSB	0.48	99.20%	0.031	0.598	(11)
APPL P without WSB	0.55	99.50%	0.028	0.225	(12)

Table 1: Estimated Hurst exponent summary table

Table I (1) shows the estimated Hurst exponent for the GME short interest. The Hurst exponent was estimated to be 0.81 during the *WallStreetBets* event. The R-squared is 99.6 percent and the standard error is 0.036. That illustrated the goodness of the fit. (2) shows the estimated Hurst exponent for the AMC short interest during the *WallStreetBets* event, with an H of 0.75. It fits well as well with an R-squared of 99.9 percent and standard error of 0.017.

Table I (3) and (4) indicate the estimated Hurst exponent for the GME price (0.56) and AMC price (0.56) during the *WallStreetBets* event. They are all good fit with R-square of 98.8 and 98.7 per cent, standard error of 0.042 and 0.045. Table I (5) and (6) indicate that before the *WallStreetBets* event, the estimated Hurst exponent for the GME short interest is 0.62 and the Hurst exponent for the GME price is 0.56. The fitness is good with R-squares are 99.8% and 97.7%, and the standard errors are 0.02 and 0.06. For the AMC short interest and stock price, they show the same pattern. The estimated Hurst exponents are both 0.6 with high R-square and low standard errors.

We use Apple Co. (APPL), whose stock price and short interest are relatively stable, as the control group. The experimental results show that during the *WallStreetBets* event, APPL's short interest estimated Hurst exponent is 0.52, and the stock price H is 0.57. Before the *WallStreetBets* event, its results were 0.48 and 0.55. Likewise, they all have a high degree of fitness.

We did a unit root test on the data using the autoregression model. Due to the small changes in Apple's short interest and lack of necessary variance, we performed logarithmic transformation on Apple's short interest data in 2019. The results show

that all data has a unit root, so we believe that it confirms that all of our data follow fractional Brownian Motion, instead of geometric Brownian motion. Our previous Hurst exponent is true, valid and credible.

CHAPTER 5: DISCUSSION AND CONCLUSION

The orthodoxy, or conventional wisdom, on how capital markets operate is that everything knowable is captured in the price of stock. The next piece of information that puts a price in forward or reverse motion is unknowable and thus independent of the last point of information. This orthodoxy has led to the general presumption that randomness in the change in prices is composed of independent increments, and these take on the properties of Brownian motion. However, from time-to-time markets depart from this orthodoxy. For some, usually inexplicable, reason investors get euphoric about a particular thing. This results in rapid increases (or decreases) in asset prices that cannot be explained by chance. In fact, the momentum is driven by measurable correlation in price changes which violate the Markov property. As a result, stochastic price dynamics evolve from a geometric Brownian motion to a fractional Brownian motion. Identifying events that are fractional in nature is important to the general understanding of financial economics and asset pricing. It raises the possibilities that some astute investors can predict price changes and arbitrage the market for risk-free profits.

Of broader interest is the idea of speculative bubbles. The term ‘bubble’ has been used to describe many economic phenomena but in the financial economics literature it generally relates to moments in time where changes in stock prices become increasingly dependent. In this paper I argue that systematic changes in the Hurst

exponent can be used to establish the presence of a bubble and distinguish bubble-like observations from chance events. In other words, my thesis is that while a rapid rise and perhaps extended excursion in process is a sufficient condition to define a bubble, inter-temporal covariance is a necessary condition.

We believe that recent activity originating on forums in *WallStreetBets* has the ingredients of a bubble. The triggering event was a euphoric move by thousands of forum participants to collectively organize to press a short squeeze on hedge funds and other institutional investors that were shorting a number of retail and technology stocks. We have proved that all data conform to the fBm model, therefore, we hope to reflect possible bubbles through the long-term memory of data, so as to prevent the stock market from experiencing such violent abnormal fluctuations. Using the event as a natural experiment we measure Hurst coefficients on targeted stocks and short positions before and after the triggering event. With some surprise we show that while GME and AMC were generally (weakly) fractal before the triggering event, this did not change to any large or impressive degree after the event. In other words, the two stocks did not exhibit the characteristics of long-term memory in price. Instead, we find that the focus on the short squeeze was in fact the main event. During the *WallStreetBets* period, the Hurst exponent of the short interest of these two stocks increased significantly to 0.81 and 0.79. This is obviously different from before the *WallStreetBets* event. We believe that even if there is no long-term memory through stock prices and there is no price risk from the Hurst indicator, the bubble still exists. The long-term memory of short interest reflects the potential risk of a bubble. The substantial increase in the Hurst coefficient for short positions indicates that changes to short positions in real time were not independent but became increasingly

dependent resulting in large swings and variance. As short positions unwound, the demand for retail-available shares increased and forum users who were long on the stock would not release them immediately. This is what caused the rapid price increases observed.

Same as our assumption, despite what appears in the bubble, there is no evidence that the bubble is created by herd behavior or other behavioral characteristics that might be found in a bubble. The unique contribution of our research is that we prove that from the price sequence, the price always basically follows the fBm law. Although the price fluctuates sharply, the price itself does not have long-term memory. It is not a simple supply-demand relationship that leads to bubbles. Because the Hurst coefficient of the price series of GME and AMC during the *WallStreetBets* event was not significantly different from before the *WallStreetBets* event. They are all close to 0.5, so we can think of them as within the bounds of what might be consistent with a gBm (Turvey, 2007). This is the same as the result of our control appl. This proves that stock prices follow random walk patterns in the long run. Price fluctuations do not have long-term memory. There are many factors that cause market volatility. Since the market has no memory, we can treat it as random as possible. The best evidence to prove that this is a market bubble generated by short squeeze is that the hurst exponent of short interest of GME and AMC during the *WallStreetBets* event is far greater than 0.5. This shows that their short interest series have long-term memory and positive self-reinforcing functions. It is this kind of memory that led to short squeeze and thus the market bubble.

A related implication of our findings is the notion that even if a price series appears to be long-run geometric Brownian motion, this does not imply that within

that term there are not periodic moments of euphoria that bring about localized fractional excursions that can be construed as a bubble. This is a different notion from a long-term fractional process which is self-affine and will naturally result in multiple bubble-like excursions. This is unlikely to arise in any economic time series because it suggests that the bubbles are predictable, and if a bubble is predictable, it wouldn't be phenomenal. It would also then be arbitrable under rational expectations which would dampen the effect before it happens. Like what was observed with GME and AMC a bubble is most likely going to be a non-affine event which is not replicated to the same (or any) scale in the future. A bubble should be defined by its uniqueness at a moment in time and not part of a recurring pattern across time.

This natural experiment also rejected the idea that retail investors can defeat Wall Street capitalists. When the *WallStreetBets* incident broke out, many analysts believed that the rise in stock prices of stocks such as GME was a resistance and attack by retail investors to capital institutions. They emphasized that the power of social media brought together non-professional investors and produced the power to fight financial capital. Different from the pessimistic attitude of Wall Street investment bank analysts, it is their optimistic estimation of GME that makes his stock price soar. Our research shows that such analysis exaggerates the impact of *WallStreetBets* online forums on GME's stock price. It is the excessive short-selling of short-selling institutions that led to the occurrence of short squeeze.

The long-term memory of the price return sequence in the financial market is one of the focuses of many empirical studies. The results of our research are different from those of previous existing studies. In many previous studies, they believe that the statistical characteristics of financial market returns are significantly different from the

characteristics of the random walk model. However, in our research, during the *WallStreetBets* event, whether it is the affected stocks, GME and AMC, or the unaffected stock APPL, their price maintains similar features to random walks. This result shows that there is no long-term memory in their price series. We think this is similar to the assumption supported by the efficient market theory. From the perspective of behavioral finance, we believe that arbitrageurs can effectively reflect changes in market prices and eliminate short-term market anomalies. Although asset prices fluctuate sharply and deviate from equilibrium prices, future market prices are not affected by past prices, which means there is no persistence in the prices. However, the short interest series of GME and AMC exhibited significantly different characteristics from the random walk model showing that they express long-term memory. The fractal market hypothesis gives the possibility of another explanation.

We believe that the reason for the market anomaly may be due to the lack of liquidity caused by excessive short positions. One of the differences between the fractal market hypothesis and the efficient market theory is the explanation of the source of market liquidity. The traditional efficient market theory believes that white noise traders provide liquidity to the market, but the fractal market hypothesis believes that they only provide a small part of the liquidity. Traders' different reactions to market information and different expectations to the market provide liquidity. This view was confirmed in the *WallStreetBets* incident. When the GME stock price just about started to rise, the well-known hedge fund agency issued a report determining that GME was seriously overvalued and called on more investors to short GME. However, retail investors in the *r/WallStreetBets* forum continue to hold a positive attitude towards GME and keep their long position. Although investment institutions

and independent investors are rational and both have the same information, they have completely different interpretations of this information. They maintained opposite positions with each other, which should have provided sufficient liquidity for the market. However, the size of hedge funds is too large compared to independent investors, and the total trading volume of GME is relatively small, resulting in insufficient liquidity for a short period of time, then causing a short squeeze situation. This fractal market hypothesis provides a powerful explanation for the sudden collapse or skyrocketing in financial markets. The existing empirical research also shows that when the market collapses, it is always accompanied by the disappearance of investors and the lack of market liquidity. These results of empirical studies all support this hypothesis.

Finally, given the recency of events I elected to examine in detail only two of the stocks promoted in WallStreetBets and the Reddit forums. To this present day (July 27, 2021) the prices of GME and AMC have remained steadfastly high over their January 2021 prices. Eventually, like all unusual market phenomena and events it will come to an end, and the fractional behavior in short positions discovered in this paper will return to near-gBm levels. At that time a fuller investigation can be undertaken to evaluate the most affected stocks of the *Reddit-Rebellion.* To this end the techniques used in this paper can be more broadly applied with greater strength in data analysis of a longer time series, and with greater force of emerging research on different aspects of the WallStreetBets such as the sister study to this by fellow student Jiachen Sung (2021) who is examine how forum chatter affects GME, AMC and 13 other WallStreetBets stocks.

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