## LINEAR MODELS FOR UNBALANCED DATA

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## **CORRIGENDA**

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## Page/Line

145, 147, 149, 151 and 153/running head:

THE NO-INTERACTION MODEL should be TESTING FOR INTERACTIONS .

145/title to 5.4: THE NO-INTERACTION MODEL should be TESTING FOR INTERACTIONS .

XV/7: The no-interaction model should be Testing for interactions.

148/1: To b. add for the no-interaction model.

xv/9: After squares add for the no-interaction model.

151/12: To e. add corresponding to  $\Re(\mu_{ij} \mid \mu_i, \tau_j)$  .

xv/12: After hypothesis add corresponding to  $\Re(\mu_{ij}\mid \mu_i, \tau_j)$  .

Note: In the two preceding symbols  $\Re$  gets typeset as script cap R . Either none, or all, of the preceding 11 corrections should be made .

8/15: possibly should be possible

9/lower right: Section listings should be as follows:

4.6d	4.10	Chpts. 2
5.5a	5.4	4
9.1g	9.2	5
12.1a	10.3	10

<sup>\*26/6:</sup> The third  $\sum$  should be  $\sum_{i}$ .

45/4 up: SSSM should be SSM.

-1-

4

<sup>\*44/</sup>line after (91): The (98) should be (87).

<sup>\*</sup> Corrections identified 1989 - 1991.

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Every \sum should be \sum_{i}.
  *47/2 and 8:
    46/eq. (99):
                            (1/n) should be (1/n_i).
  \sqrt{57/9}:
                             Change Section 4.5a to Sections 4.6d and 4.7a.
    79/eq. (1):
                            \overline{y}_{ij}. / n_{ij} should be \overline{y}_{ij}. = y_{ij}. / n_{ij}.
  *80/7:
                            The last symbol, n_i, should be n_i.
  *83/9:
                            (13) and (14) should be (14) and (15).
  *83/11 up:
                            (13) should be (14).
                            \frac{14}{3} should be \frac{41}{3}.
  *85/(31):
  \sqrt{87/\text{Eq. (31)}}:
                            14/3 should be 41/3.
                                                            (41 \text{ not } 14)
  *90/12:
                            N should be \mathcal{N} . (script en)
  \sqrt{92/9}:
                            \mathbf{v}(\rho_i) should be \mathbf{v}(\hat{\rho}_i) . (a hat is needed)
   *93/4:
                            \tilde{y}... should be \overline{y}... (bar not tilde)
    93/16 up:
                            (48) should be (47).
                            In \frac{1}{8}(4\mu_{21}+\mu_{22}+\mu_{23}) the \mu_{23} should be 3\mu_{23} .
    93/12 up:
  *94/4:
                            + \mu_{13} should be 2\mu_{13} . (add 2)
  *94/19:
                            /n_i. should be /n_{ij}. (sub ij not i.)
   *94/2 below (50):
                            1/a should be 1/b.
                            /n_{ij} should be /n_{ij}. (sub ij not \cdotj)
   *95/3:
    97/4 up:
                            Proof. should be Proof of (i). [insert of (i)]
                            Put Q.E.D. on right margin .
    98/8:
    98/3 below eq.(56): Delete Q.E.D. .
   *99/9 up and 7 up:
                            y_{ij} should be y_{ijk}. (add k to sub)
\sqrt{102/10} up:
                            9.3j should be 9.2j
                            -\hat{y}...) should be -\hat{y}...) . (delete one dot)
*104/7 up:
*104/up:
                            n_i. should be n_i. (delete sub dot)
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<sup>√</sup> Corrections identified 1991-1997.

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*107/5:
                            n_i should be n_i. (add sub dot)
 *107/12 up:
                            SSA should be SSA_w.
                                                              (add sub w)
\sqrt{107/3} up:
                            (52) should be (40).
  107/last:
                            \mathbf{r}_k should be \mathbf{r}_j. (sub j not k)
  111/last line Table 48: The first term after the first = sign is \Re(\tau_j | \mu), but should be \Re(\mu_i | \mu).
                  \ensuremath{\mathfrak{R}} is to be typeset as script cap R .
\sqrt{112}/last line of Table 4.9: \gamma_i' should be \gamma_j' . (subscript j not i)
                             One should be ones . (add s)
  115/14:
 *117/last line before 4.10: sum should be sums.
                                                                 (add s)
\sqrt{121/2} below Table 11.11: 9.3g should be 9.2g.
                             \rho'_i should be \rho'_i. (sub i not j)
 *121/(112):
  122/Partitioning II:
                             The tens digits under the "Sum of squares" heading should be interchanged;
                             i.e., 83 should be 93 and 93 should be 83. Fraction remain unchanged.
                            \tau_i should be \tau_1 . (sub 1 not j)
 *122/7 up:
                            \tau_i should be \tau_1 . (sub 1 not j)
 *122/(113):
\sqrt{127/9}, Eqs (15),(16): The \sum_{k} should be \sum_{j}; and n_{i} should be n_{ij}.
\sqrt{129/4}:
                             y_{.j}. should be \overline{y}_{.j}. (add a bar)
                             In Y: the X:3,7 should be 3,7. (delete X:)
  131/4:
                            \mu_{12} and \mu_{13} should be \hat{\mu}_{12} and \hat{\mu}_{13} .
 *134/4 up:
                                                                             (add hats)
\sqrt{134/5} up:
                             add 'for Table 5.1' before are .
 *135/2 after (12): \rho_2^{\prime\prime} should be \rho_2^{\prime} . (single prime only)
  136/Table 5.2, line labeled Residual: Put a bar over y_{ij} so that \sum_{i} \sum_{j} \sum_{k} (y_{ijk} - y_{ij})^2
                                 should be \sum_{i} \sum_{j} \sum_{k} (y_{ijk} - \overline{y}_{ij})^{2}. (add a bar)
\sqrt{136/7} up:
                             Add a prime to \omega and \hat{\omega}.
\sqrt{143/\text{Eq.}(21)}:
                             The denominator 121 should be 21.
                             a. Fitting the model. should be Fitting the no-interaction model. .
   145/10 \text{ up}:
                             The second from right + should be = .
   147/last:
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\*156/Table 5.8: In the first (d.f.) column the last 1 (one) should be 2.

 $\sqrt{159/\text{Eq}}$  (55): Change what comes after the second equal sign to be

 $\frac{1}{2}(\mu_{11}+\mu_{12})-\frac{1}{3}(\mu_{21}+\mu_{22}+\mu_{23}) .$ 

167/last:  $> should be \geq .$ 

175/2: The  $-\frac{1}{2}\sigma^2$  should be  $-\frac{1}{3}\sigma^2$ .

\*182/Table 6.4E, line 2 of (a): Delete  $\mu$ +.

189/last: Q of (69) should be Q of (70) . [(69) should be (70)]

\*201/2 of Table 6.7: R(M) should be  $R(\mu)$ .

 $\sqrt{214/4}$ : After symmetric delete the comma and add and real (we deal only with real

matrices), .

 $\sqrt{216/12}$  up:  $\mathbf{A}^*\mathbf{A}\mathbf{A}^* = \mathbf{A}$  should be  $\mathbf{A}^*\mathbf{A}\mathbf{A}^* = \mathbf{A}^*$ . (add a star)

 $\sqrt{217/4}$  above Eq. (8): Replace positive semi with non-negative.

\*218/3 below (17): PX = QX should be PX' = QX'. (add primes)

 $\sqrt{223/1}$ st line of **Example**: Delete of after **G** and put after **X'X**.

\*224/3 below (35):  $\begin{bmatrix} \mathbf{X}_1'\mathbf{X}_1 & \mathbf{X}_1'\mathbf{X}_2 \\ \mathbf{X}_2'\mathbf{X}_1 & \mathbf{X}_2'\mathbf{X}_2 \end{bmatrix} = should be \begin{bmatrix} \mathbf{X}_1'\mathbf{X}_1 & \mathbf{X}_1'\mathbf{X}_2 \\ \mathbf{X}_2'\mathbf{X}_1 & \mathbf{X}_2'\mathbf{X}_2 \end{bmatrix} = . \text{ (add superscript minus)}$ 

 $\sqrt{224/2}$  up:  $(\mathbf{X}_2'\mathbf{M}_1\mathbf{X}_2)$  should be  $(\mathbf{X}_2'\mathbf{M}_1\mathbf{X}_2)^-$ . (add superscript minus)

\*224/last: 2 spaces are needed before  ${f I}$  .

226/7: f should be F. (cap F)

 $\sqrt{232/\text{last}}$ :  $\mathbf{D}_r$  should be  $\mathbf{\Delta}_r$ .

234/last line before -ii: used in should be used at (79) in .

 $\sqrt{237/6}$ : Between  $\sum_{i}$  and  $(\overline{y}_{i}, -\overline{y}_{..})^{2}$  add  $n_{i}$ .

240/E7.18: 7.2b should be 7.3b . (3 not 2)

 $\sqrt{246/9}$  up: Section 2.10 should be 8.11.

247/bottom: The 76 should be 78.

\*256/8:  $\mathbf{y} = \mathbf{X}\boldsymbol{\beta}^0$  should be  $\hat{\mathbf{y}} = \mathbf{X}\boldsymbol{\beta}^0$ . (add hat

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\sqrt{256/7} up:
                                 The boldface N should be italic feint.
   263/(74):
                                  The off-diagonal X_1'X_1 should be X_2'X_1.
\sqrt{263}/\text{Eq.} (74):
                                 The lower left submatrix should be X_2'X_1 not X_1'X_1.
\sqrt{264/3}:
                                 (60) should be (65).
                                 \mathbf{X}_1 \sigma^2 should be \mathbf{X}_1' \sigma^2. (add prime)
\sqrt{267/12}:
                                 The curly R should be plain cap R.
   268/6:
                                 The \tilde{\boldsymbol{\beta}}_2 should be \hat{\boldsymbol{\beta}}_2 . (hat, not tilde)
   269/12
   272/Eq. (91):
                                 The last term \mathbf{X}_2\boldsymbol{\beta}_3 should be \mathbf{X}_3\boldsymbol{\beta}_3 . (sub 2 should be 3)
   273/\text{line 5 of the body of Table 8.4:} + should be - .
\sqrt{276/7}:
                                 Replace Section 7.1d with (43), Sec. 7.3.
                                 (106) should be (107).
\sqrt{278/3}:
\sqrt{282/5} up:
                                 b\overline{z}_{i} should be b\overline{z}_{i}. (sub i)
                                 Each \beta should be \beta^0. (add superscript 0) (K'GK) should be (K'GK)<sup>-1</sup>. (add superscript -1)
   291/Eq. (146):
                                  The last K should be K'. (prime)
   291/Eq. (147):
                                 The first (\mathbf{K}'\boldsymbol{\beta} - \mathbf{m}) should be (\mathbf{K}'\boldsymbol{\beta} - \mathbf{m})'.
\sqrt{291/5} up:
                                 \mathbf{K}\boldsymbol{\beta} should be \mathbf{K}'\boldsymbol{\beta}. (prime on \mathbf{K})
   292/(v):
  *292/2 below (v):
                                  \geq should be \leq .
   295/16:
                                 (36) should be (66).
\sqrt{297/16} up:
                                 After element add (and every linear combination of elements)
                                 \mathbf{K}'\boldsymbol{\beta}^0 = should be \mathbf{K}'\boldsymbol{\beta}^0 - .
\sqrt{298/17}:
                                                                        (minus, not equals)
                                 After can be add (or can be rewritten so as to be)
\sqrt{299/8}:
\sqrt{301/8} up:
                                 \mathbf{K}_{i} should be \mathbf{k}_{i}. (lower case)
   305/(169):
                                  Delete the minus sign from the -2 in the matrix on the left .
                                 -16 + 11 + 0 should be -16 - 11 + 0.
   307/4:
                                 I - G_r X'X should be G_r X'X. (delete I -)
   307/11 up:
                                  This should be = 89 + \frac{1}{3}(-16 - 11 + 0) = 80.
   308/4:
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After BLUE change 20\frac{2}{3} to 80.
   308/6 and 7:
   308/2 lines below (179): XB should be X\beta.
                                                                           (beta not B)
                                     1/n should be 1/n; . (add subscript i)
\sqrt{309/5} up:
                                     The last X needs a prime: XV^{-1}y should be X'V^{-1}y.
   316/12:
                                     X_1 = should be X'_1 = .
   318/eq. (76):
                                                                               (add prime)
                                     This line ends with -\sum_i \mathbf{w}_i \tilde{\mathbf{y}}_i ... / \sum_i \mathbf{w}_i)^2. (Missing subscript i needed on two \sum symbols and one w)
   325/9:
                                     The -\overline{y}_{22}. should be +\overline{y}_{22}. (+ not -)
   325/E8.33:
   327/3:
                                     all should be cell .
  *333/3 up:
                                     u_{ij} should be \mu_{ij}. (new not you)
                                     Between the last ) and the period add \forall j.
   334/10:
                                     Delete + e.
   336/2 \text{ up}:
                                     Delete + e.
   336/last:
                                     Between SSE and \sum_{i} insert = .
   339/last:
                                     \overline{y}_{..}^2 should be \overline{y}_{i..}^2. (add sub i)
   352/2 up:
                                     \bar{y}_{ij}^2 should be \bar{y}_{ij}^2. (add sub dot)
    356/5 up:
    357/line 1 of Table 9.1E: H: \mu + should be H: 10\mu + ...
                                                                                                (insert 10)
    358/line 1 of Table 9.2E: H: \mu + should be H: 10\mu + ...
                                                                                                (insert 10)
  *379/3:
                                     9.5 should be 9.4.
                                                                      (4 \text{ not } 5)
                                     The matrix should be
    382/3-7:
                                    \mathbf{T} = \begin{vmatrix} -1/\mathbf{a} & -1/\mathbf{b} & -\mathbf{1}'_a/\mathbf{b} & \cdot \\ \mathbf{1}_a/\mathbf{a} & \cdot & -\mathbf{C}_a/\mathbf{b} & \cdot \\ \cdot & \mathbf{1}_b/\mathbf{b} & \mathbf{J}_{b\times a}/\mathbf{a}\mathbf{b} & -\mathbf{I}_b/\mathbf{a} \\ \cdot & \cdot & \cdot & \cdot \end{vmatrix}.
                                    \sum_{i=1}^{c} \frac{1}{n_{ijk}} \text{ should be } \sum_{k=1}^{c} \frac{1}{n_{ijk}} \text{ (under the right-most } \sum, \text{ the j should be k)}
    388/2 up:
                                     \mu_{ij}, should be \overline{\mu}_{ij}.
  *390/(29):
                                                                        (add bar)
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 $(\mu_{ijk} \;\; should \; be \;\; (\mu_{ijk'}) \;\; . \;\;\; [ the \; \mu_{ijk} \;\; after \; the \; ( \; needs \; a \; prime \; on \; the \; sub \; k]$ 

391/1 above (33):

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394/8 up:
                               -\mu_{22} = 0 should be +\mu_{22} = 0.
 394/7 up:
                               -\mu_{.32} = 0 should be +\mu_{.32} = 0.
 402/last:
                               -\mu_{22} = 0 should be +\mu_{22} = 0 .
                               [The preceding 3 corrections each consist of changing the last -to + .]
*415/2 up:
                               I - X(X'X)^{-}X should be I - X(X'X)^{-}X'.
                                                                                            (add prime)
                               In equations (20), (25) and (26) (\mathbf{X}'\mathbf{X})^{-1} should be (\mathbf{X}'\mathbf{X})^{-}.
 422, 3 and 4:
 423/5 up:
                               (18) should be (17).
                               (25) should be (24).
 423/3 up:
                               - should be + . (change minus to plus)
 423/last:
 424/13:
                               Zb should be \mu \mathbf{1} + \mathbf{Zb} . (add \mu \mathbf{1} + \mathbf{1})
                               R(\mathbf{b}/\mu) should be R(\mathbf{b}|\mu).
 424/14:
                                                                        (vertical line)
 425/2nd to last line of part (a) of Table 11.4: \mathbf{y}'\bar{\mathbf{y}} should be \mathbf{y}'\mathbf{y}.
                                                                                               (delete bar)
 425/3 up of part (b) of Table 11.4: R(\beta | \mu) should be R(\mathbf{b} | \mu).
                                                                                             (\mathbf{b} \quad not \quad \boldsymbol{\beta})
                               Add = \left\{ \mathbf{C}_{n_i} \right\} .
*427/(29):
                               b_2 = b should be b_2 = b_3.
 430/4 up:
                               z_1^* should be z_i^*. (sub 1 should be i)
 434/2nd to last:
                               b should be b . (bold not feint)
 435/3:
                               b should be b . (feint not bold)
 435/4:
                               c should be c . (bold not feint)
 435/2 up:
                               y_1^2 should be \overline{y}_i^2. (add bar and sub dot)
 443/last:
                               y_{ij}. should be \overline{y}_{ij}. (add bar)
 447/eq. (103):
                               -\overline{z}_{ij}. should be -\hat{\lambda}\overline{z}_{ij}. (insert \hat{\lambda})
 448/eq. (109):
                               \mathbf{P}_{1,\,yz} should be \mathbf{u}_{1,\,yz} . (P should be \mathbf{u})
 453/5 up:
                               \mathbf{U}_{2,\,zz} \ \ \mathit{should} \ \ \mathit{be} \ \ \mathbf{U}_{2,\,yz} \ \ . \qquad (\mathrm{yz} \ \mathrm{not} \ \mathrm{zz})
*453/9 up:
 454/6:
                               \mu should be \mu_i. (add sub i)
 454/last:
                               7 and 8 should be 8 and 9.
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2-4 should be 2-6 .

455/2:

455/14:  $b_2 = should be \tilde{b}_2 = .$  (add tilde)

459/12: Under the heading Type III

 $R(\dot{\beta} | \dot{\mu}, \dot{\alpha}, \dot{\beta})_{\Sigma}$  should be  $R(\dot{\beta} | \dot{\mu}, \dot{\alpha}, \dot{\phi})_{\Sigma}$ . (The second  $\dot{\beta}$  should be  $\dot{\phi}$ )

466/5: Delete (see Grid 12.1) .

466/10: In heading of (b) classes should be columns .

467/4: Using  $\ell_1 = \ell_2 = -1$  and then  $\ell_1 = 1 = \ell_2$  should be

Using  $\ell_2=1,\,\ell_3=-1$  and then  $\ell_2=1,\,\ell_3=0\,$  .

Note: The symbol  $\ell$  used here is just plain el .

467/6:  $\ell_1$  should be  $\ell_2$  . (sub 2 not 1)

\*468/(9):  $\beta_3$  should be  $\beta_2$  . (sub 2 not 3)

470/12:  $n_3 = 1$  should be  $n_3 = 2$ . (1 should be 2)

470/16:  $\overline{\mathbf{J}}_1$  should be  $\overline{\mathbf{J}}_2$  . (sub 2 not 1)

470/2-3 up: In f the  $\begin{bmatrix} 6 & 0 \\ 0 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 6 & 0 & -6 \\ 0 & 6 & 0 & -4 \end{bmatrix}$  should be  $\begin{bmatrix} 6 & -6 \\ 6 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 6 & -6 & 0 \\ 0 & 6 & 4 & -10 \end{bmatrix}$ .

489/2 below eq. (39): (103) should be (94).

506/eq. (80):  $\hat{\sigma}_i$  should be  $\hat{\sigma}_i^2$ . (add superscript 2)

506/12 up: add period after negative . Delete and they are only .

506/13: Delete whole line.