NOTES.

NOTES.

[1] (P. 5.) This letter is dated June 30, 1784, and others, written before the specification was drawn up, extend as far as the 22nd July of the same year. The specification is, however, dated April 28th, 1784, an inconsistency in Muirhead which seems to require explanation.

[2] (P. 9.) Through 113 chapters he is compelled to employ long descriptions for what we express by the word pump. For example: Chap. 1:—"Cette cy est une sorte de machine, par laquelle facilement et sans point de bruit l'on peut faire monter l'eau d'une fontaine ou d'un fleuve à une proportionnée haulteur . . . . Chap. xvii.: Ceste autre façon de machine, par laquelle l'on faict pareillement monter l'eau d'un lieu des en hault . . . . Chap. lvii. : L'effect de ceste autre façon de machine est de faire monter l'eau d'un canal à une juste haulteur," etc.

[3] (P. 11.) *Calcul de l'Effet des Machines.*

[4] (P. 11.) *Introduction à la Mécanique Industrielle.*—Compare also § 129, Chap xii. of this work.

[5] (P. 12.) From a philological point of view, "Kinematics" is incorrect,—Ampère should rather have said "Kinetics," (*Cinétique*). [It is of course impossible to make any alteration in this direction now in this country, where the word Kinetics has already obtained extended use in another sense, and there are good reasons for not making any change on the Continent, although the word Kinetics has not yet come into general use there.] It is in every way better to use the word with a K, as in the language from which it has been derived, than with the C which it owes to its transmission through the Latin and French.

[6] (P. 21.) Parerga ii., Chap. iii., § 41,—also Wille und Vorst. ii., Chap. xiv.

[7] (P. 35.) It is very remarkable, and has long ago given occasion for reflection, that we so seldom find definitions of the machine which agree with each other. The following examples show how uncertain, and often how altogether indefinite, have been the attempts made to define the machine even by those who must have known the thing itself.

Weisbach. "Machines are all those artificial arrangements, by means of which forces are made to act in a way differing from that in which they would otherwise have acted." Any tool whatever ; a needle, a pencil, &c., is therefore by itself a machine.

Poncelet. "The industrial . . . . machines have for their purpose the performing of certain work by the help of motors or moving forces provided for us by nature." An explanation full of restrictions, which gives us only one of the purposes of the machine.

Bresson. "A machine is a tool of which the general purpose is the transference of a force from its point of application to a position where it can act so as to overcome a resistance, and execute work which it would be difficult, and sometimes impossible for the same force to execute if applied directly." What then is a "tool?" And how does "sometimes" find its way into a scientific definition?

Rühlmahn. *Geostatik*, 3rd Edit., 1860. "By the name machine, we indicate a combination of rigid bodies, moveable and immoveable, into a rigid unalterable 'free' (*lose*) system, by means of which forces, through changes in their direction and magnitude, may be made to balance each other." He has explained in an earlier part of the work what a "free" system is. According to this definition, a suspended iron chain would be a machine; an hydraulic press, however, could not receive that name, for the water is not a rigid body.

Rühlmann. *Geostatik*, 2nd Edit., 1845. Almost exactly as Weisbach.

Rühlmann. *Allgemeine Maschinenlehre*, i. (1862). "The machine is a combination of moveable and immoveable rigid bodies which serves to receive physical forces and to transmit them, changing if required their direction and magnitude, in a manner suitable for the performance of definite mechanical work." Here are three definitions coming to us from the same pen: to which shall we trust?

Kayser. "Machines are arrangements which transmit the action of forces in order to balance or to overcome other forces, and to produce motions for definite purposes." This covers, *e.g.*, the tow-line of a ship.

Schrader. "A machine is an arrangement for the alteration of a given force." Very concise, but somewhat difficult to understand. What is it to "alter a given force"?

Wernicke. "A machine is a combination of bodies, of which the purpose is the accomplishment of any work by some disposable force." The first few words sound like a definition, the conclusion, however, becomes altogether indefinite.

Poppe. "By machines, we mean those artificial arrangements by which motions may with advantage be produced, prevented, or transmitted in definite directions." What has "advantage" to do with science? Motions also cannot be produced by "arrangements;" and so on.

Delaunay. (*Analytische Mechanik*, 1868.) "A machine is an apparatus which serves to transmit mechanical energy, or also, to make a force to aot at a point which does not lie in its own direction." Again, only characteristics, no explanation, no rigid definition; and that fatal "or also!"

Willis. "An instrument, by means of which we may produce any relations of motion between two pieces." We might call this definition an equation with two unknowns.

[I think Prof. Reuleaux is mistaken in giving this as Willis's definition of a machine. Willis merely says (Preface, 1st Ed., p. xiii.)—" . t . . instead of considering a machine to be an instrument by means of which we may change the direction and velocity of a given motion, I have treated it as an instrument by means of which . . .t. ," etc. He nowhere gives a formal definition of a machine, but in one place (Preface, p. iv.) describes it in a sentence which more nearly agrees with Reuleaux's definition than any other I have seen, although it certainly is still incomplete :—" Every machine will be found to consist of a train of pieces connected together in various ways, so that if one be made to move, they all receive a motion, the relation of which to that of the first is governed by the nature of the connection."]

Giulio. " An arrangement which is designed to receive motion from the action of a motor, to alter this motion, and to transmit it to an instrument formed so as to execute any kind of work." This gives certain characteristics of the machine, not, however, what it actually is.

Laboulaye. " We give the name machine to a system of bodies which is intended to transmit the work of forces, and consequently both to alter the intensity of the forces, and to make the velocity and direction of the motion produced such as is suitable for the purpose in view." What is a "system of bodies ? " is it sufficient that it should be "intended" for all this ? and so on.

Belanger. "A machine is a body (one body ?) or a number of bodies intended to receive at one point certain forces, and at other points to exert certain forces, the latter being in general different from the former, both in intensity and direction, and in the velocity of the point at which they are exerted." Again the "intention." The whole also is a description, not a definition.

Haton. " A machine is an apparatus which is intended to connect a motor with the material to be worked upon." Apparatus, motor, material to be worked on, connection ? From a logical point of view, how many riddles! The reader may fairly exclaim *Davus sum, non Oedipus !*

Lastly, Pierer's *Universal Lexikon* (? Hulsse). "Machine—an arrangement by which a motion, *i.e.*, a change of place or of form, may be given to a body, by which, that is, in general, some work may be done or mechanical effect obtained." This is only descriptive, and suits a multitude of things which are not machines.

[The definition I have quoted from Willis is certainly more accurate than any of those which have followed it, it is indeed so obviously better than most of them that it is matter for wonder that it has not been generally adopted. In order simply to shew to what extent indefinite definitions have passed current here also, and not from any desire to criticise, I may add the following to Prof. Reuleaux's catalogue :—

Hart (1844). " A machine is defined to be an instrument, by means of which a given force is caused to make equilibrium with a resistance to which it is either unequal, or not directly opposed."

Goodwin (1851). " Any contrivance by which force is transmitted from one point to another, or by means of which force is modified with respect to direction or intensity, is called a machine." Goodwin expressly includes as machines an oar, a poker, etc.

Galbraith and Haughton. " A machine is an instrument, by means of which pressure or motion may be transmitted from one point to another, and altered both in magnitude and direction."

Goodeve (1860). "A machine may be defined to be an assemblage of moving parts, constructed for the purpose of transmitting motion or force, and of modifying, in various ways, the motion or force so transmitted."

Rankine. "Machines are bodies, or assemblages of bodies, which transmit and modify motion and force. The word 'machine,' in its widest sense, may be applied to every material substance and system, and to the material universe itself; but it is usually restricted to works of human art, and in that restricted sense it is used in this treatise."

Todhunter. "Machines are instruments used for communicating motion to bodies, for changing the motion of bodies, or for preventing the motion of bodies."

Magnus (1875). "A machine is an instrument by means of which a force applied at one point is able to exert, at some other point, a force differing in direction and intensity."

I conclude that the authors, some of them very distinguished men, whom I have cited, can hardly have been satisfied with their own definitions, which, indeed, ought not strictly speaking to receive that name at all, so far as the machine is concerned. Almost without exception the machine is an "instrument," a "contrivance," an "assemblage of bodies," by means of which something is done or can be done. This something is the thing defined in each case, the machine itself is quite left out. It is instructive to note also how completely the idea of the "fixed link," which will be found presently to be of absolutely vital importance, is absent from most of them. Prof. Rankine recognised it,* (although it is excluded by his definition, if "a body" can be a machine), but it is expressly or tacitly excluded by most of the other writers.]

The reader must not be surprised that I have placed here beside each other names of such very various degrees of importance, nor that I have omitted others which are so well known, *e.g.*, Moseley, Redtenbacher, Jolly, Karmarsch, Holzmann, and among the older authors Langsdorf, Eytelwein and others. These authors give no definition of the machine. They consistently avoid it, going at once into classification and description. I have given so many examples in order to show the more clearly that no authoritative definition has ever yet been arrived at.

The older definitions, much more naïve than the modern attempts to grasp a multitude of phenomena, are by no means uninteresting. Leupold, for example, says (*Theatr. Macht*, 1724):—"A machine or engine † is an artificial work, by means of which some advantageous motion can be obtained, and something moved with a saving, either in time or in force, which would not be otherwise possible." [The word Rüstzeug, which occurs in this old definition, and which I have represented (I cannot say translated) by engine, continued in use in Germany until well on in the present century. Prof. Reuleaux traces it back to an adaptation by Zeising, 1607, of an old definition of Vitruvius.]

* See p. 24.

† "*Rüstzeug*," a word covering machinal arrangements of all kinds, and used very much as the word *engine* was used among us in Leupold's time;—*e.g.* "Let all the dreadful engines of war," &c.

All the definitions which I have given have a common property; they are entirely or chiefly descriptive, they do not go down to the essentials of the matter. In criticising them in this way I do not wish to be misunderstood; the question is not one of mere criticism, but of the relative importance of the fundamental propositions upon which a scientific study is built up, for the whole matter begins with the definition. It may be objected to this that the definition can in this case only be determined after the object defined has been thoroughly investigated,—its position at the beginning is therefore artificial. The statement is perfectly true: it is, and must be, true however for every definition. The beginning must presuppose the end. A scientific study is not a chronological account of its own investigations. It is not necessary, however, that the learner should know at the very commencement the full meaning of the definitions. The explanation and development of the subject, as they are carried on, refer him back always to these first propositions, which he finds, as he proceeds, to be a mirror in which he can see a reflection of each later one—until at the end of his work he discovers the full and complete justification of the definitions with which he started. It has seemed to grow more and more full of meaning as he advanced, until finally it has shown itself to be a true representation of the whole matter in a form of the greatest possible conciseness.

An incomplete or merely descriptive definition forming the key-stone of any branch of science reflects the condition in which that study must be. We shall have an opportunity of showing in the course of this work how the machine has gradually shaken itself free from the problems of Pure Mechanics. The student only of this science, in whose work the machine is merely an accident, does not feel the drawbacks of its incorrect definition. Nor do these present themselves even to the engineering student, so long as his science has not yet been put in the form of rigidly logical propositions. I cannot do better than quote here an excellent sentence from Mill (*Logic* I., bk. i., chap. viii., § 4) bearing upon the subject. He says:—"What is true of the definition of any term of Science, is of course true of the definition of a science itself; and accordingly the definition of a science must necessarily be progressive and provisional. Any extension of knowledge or alteration in the current opinions respecting the subject matter, may lead to a change more or less extensive in the particulars included in the science; and its composition being thus altered, it may easily happen that a different set of characteristics will be found better adapted as differentiæ for defining its name."

[8] (P. 47.) The immense importance of this proposition will appear greater to the reader presently than it can do at present. Its principle seems to have been hitherto entirely unrecognised. I have found but a single trace of it. This is in Chasles (*Aperçu historique sur l'Origine . . . des Méthodes en Géométrie*, 1837, note xxxiv., p. 408, *et seq.*, and later on), where he speaks of the elliptic chuck of Leonardo da Vinci. The consequence in that case of fixing another link of the kinematic chain Chasles has taken for an indication of the great law of duality, upon which he enlarges at great length. His reasoning, however, is not well grounded, and goes seriously wrong; we have here not duality, but a most characteristic plurality, which contains naturally

all the consequences which Chasles obtained artificially from the duality, and an immense number of others.

[I have used the word train as synonymous with mechanism through this work. I regret that it was omitted, by an oversight, in line 8 of p. 47. I would venture to suggest, in connection with this subject, that it would be really more accurate, and in every way better, if the three-bar, five-bar, &c., linkworks and cells (about which so much that is interesting has recently been written and done by Peaucellier, Sylvester, Hart, Kempe, and others) should rather be called four-bar, six-bar, &c., linkworks respectively. The numbers hitherto attached to them are those of the moving links only of the kinematic chain. I need hardly point out that the plane in which the two fixed points are supposed to be, and relatively to which some other point describes some particular curve, is in every sense as much a "bar" as any of the other links. In view especially of this most interesting subject finding its way from the hands of the mathematicians to those of the students, it seems a pity that this most important fact should not be recognised in the names given to them. Prof. Sylvester distinguishes between the chain and the mechanism or train in the special cases mentioned by calling them linkage and linkwork respectively.]

[9] (P. 72.) It seems suitable to call here particular attention to the fact that the two centroids are absolutely reciprocal, that is, that neither of them possesses, as a centroid, any property which the other has not. This may appear to be the case when (as in Fig. 21) one of the two curves is fixed. We see, however, from the above problem, that this difference of appearance may always be removed, and both the centroids made to move, if another link of the chain be fixed. The conditions of both pairs of centroids are identical, the fixing of one curve is merely accidental. The difference made by Poinsot between Poloid and Serpoloid is precisely the difference due to one of the curves being fixed—it cannot, however, at least in the study of machines, be justified. It must be given up in other investigations also, I think, for there is no real difference between the two curves; the particular distinction made is indeed more apt to confuse than to explain, for it seems to point to the existence of only two centroids in a moveable system, while there is no such limit to their number; we have already noticed one case in which there were six pairs in one mechanism. Mechanisms such as those of Fig. 135 also—the spur-wheel train—give further illustrations of the undesirability of making this distinction. There the centroids of *a : c* become a point, as do also those of *b : c*; they are therefore no longer visible as curves. The centroids of *d : b*, on the other hand, the only ones remaining as curves in the chain, both move if the latter be placed on *c*, and are therefore absolutely indistinguishable. The difference in name cannot therefore be logically justified, and in a Science, especially a young one, whatever cannot be logically justified should be carefully kept at a distance, and by no means taken up on mere grounds of convenience. I should not have spoken of the matter if it had not been for Prof. Aronhold's proposal to call the stationary and moving centroids by different names (*Polbahn* and *Polkurve* respectively), which has been too hastily accepted by the younger forces. [I believe Aronhold has now given up the use of the word Polkurve.] My experience has shown me a

hundred times that the students,—in spite of the reiterations of their teachers that no logical difference between the things exists—do make such a difference. I hope it may still not be too late to return to a correct nomenclature. If at any time it be necessary to distinguish between the two curves, it will be both correct and sufficient to do so by calling one the stationary and the other the moving centroid.

[10] (P. 75.) The base circles used in drawing the profiles of involute teeth are secondary centroids of this kind. The third centroid is the straight line, rolling on these centroids, of which a point generates the profiles.

[11] (P. 80.) [The theorem is contained in Poinsot's celebrated memoir *Théorie nouvelle de la Rotation des Corps*, presented to the French Institute in 1834, and afterwards published in an extended form in *Liouville's Journal*, vol. xvi. pp. 9-129 and 289-336 (March, 1851). A translation (by Whitley) of the original paper was published at Cambridge in 1834. The theorem referred to is stated as follows: "The most general motion of which a body is capable is . . . . that of a certain external screw which turns in the corresponding internal screw." The same paper contains also the first enunciation and proof of the theorem: "the rotatory motion of a body about an axis which incessantly varies its position round a fixed point is identical with the motion of a certain cone whose vertex coincides with this point, and which rolls, without sliding, on the surface of a fixed cone having the same vertex." The simple treatment of such a problem which we now adopt was not possible forty years ago; Poinsot's reasoning included the ideas of force and velocity, instead of merely the notion of change of position.]

[12] (P. 80.) [*Traité de Cinématique*, 1864. Belanger here re-states Poinsot's theorems of motions in a plane and about a point (pp. 55 and 58), and gives the following theorems respecting general motion in space (p. 59) :—"Tout mouvement continu d'un système invariable équivaut au roulement d'un cône lié au système, sur un autre cône qui aurait un mouvement de translation dans l'espace, le premier cône étant le lieu géométrique dans le corps en mouvement, des axes instantanés de sa rotation autour du point choisi; le second cône étant le lieu des mêmes axes instantanés dans le système de comparaison en translation" . . n . (p. 79) "Il faut ajouter ici que ce même mouvement équivaut à celui d'une surface réglée qui, étant liée au système, toucherait continuellement, suivant une génératrice, une autre surface réglée fixe dans l'espace, sur laquelle elle roulerait en glissant à chaque instant le long de la génératrice de contact des deux surfaces. En effet, les positions successives dans l'espace, de l'axe instantané de rotation et de glissement, forment une surface réglée, c'est le surface immobile ; et les positions de ce même axe instantané, relativement an système ou corps mobile, forment une autre surface réglée mobile, emportant le corps avec elle."

Prof. Ball points out (*Theory of Screws*, p. xix., etc.) that the discovery of the theorem given in § 12, the "canonical form" of the displacement of a rigid body, is due to Chasles. In view of Ball's most interesting investigations, as well as of the more elementary treatment of the subject now adopted in other books, I regret that Prof. Reuleaux adheres to the view taken on page 83, where twisting is expressed as a rolling about two axes, one at an infinite distance. This conception seems rather to add difficulty to, than to simplify, that of the twist pure and simple, which for every reason it appears to me better to treat as the ultimate and general case.]

[13] (pp. 64, 119 and 121). [It must, I think, be admitted that it is very desirable to have a somewhat clearer understanding as to the use of these names than has hitherto existed. I shall be very glad if the following table assist in any way in promoting this understanding. It shows the nomenclature I have myself used throughout the book, which differs somewhat from Professor Reuleaux's, but which agrees with that adopted by our best writers on the subject, so far as I have been able to make out a system from their references to these curves. The last column requires a word of explanation, the rest of the table, I think, explains itself. The case referred to there will be understood by a reference to Fig. 97. Two circles are there in internal contact, of which the larger rolls and the smaller is stationary. It may seem at first sight unnecessary to separate this case, for what I have called the pericycloid can always be described as an epicycloid. The necessity for having a separate name arises, however, from the fact that the curves described by points without and within the large rolling circle, that is the curtate and prolate peritrochoids, cannot be described as curtate or prolate epi-trochoids. The cardioid can be generated as a roulette as a special case both of the epi- and the pericycloid. I have retained "curtate" and "prolate" for want of better words, but they are sometimes singularly inappropriate to the external form of the curve, in Fig. 96 for example, where the larger ellipse is the curtate curve. Professor Cayley's kru-nodal and ac-nodal hardly seem adapted for popular use, and an excellent suggestion of Professor Clifford's, "looped" and "wavy," fails also in (external) suitability in the case of Fig. 96.

I use trochoid as the general name for the whole class of curves.]

DESCRIBING POINT.	TROCHOID.			
	EXTERNAL CONTACT.		INTERNAL CONTACT.	
	CIRCLE ROLLING UPON STRAIGHT LINE (LINEAR TROCHOIDS.)	CIRCLE ROLLING UPON CIRCLE. (EPI-TROCHOIDS.)	SMALLER CIRCLE, ROLLING. (HYPO-TROCHOIDS.)	GREATER CIRCLE, ROLLING (PERI-TROCHOIDS.)
On circle	Cycloid.	Epicycloid.	Hypocycloid.	Pericycloid.
Beyond circle	Curtate trochoid	Curtate epitrochoid.	Curtate hypotrochoid.	Curtate peritrochoid.
Within circle	Prolate trochoid.	Prolate epitrochoid.	Prolate hypotrochoid.	Prolate peritrochoid.

[14] (P. 119.) Chasles (*Aperçu historique sur l'Origine . . . . . . des Méthodes en Géométrie*, 1837), cites Cardano's *Opus Novum de Proportionibus Numerorum Motuum*, etc.

[15] (P. 121.*) Without the use of a model it is very difficult for anyone unaccustomed to this class of problems to realize fully the nature of the motion which occurs here. In my model the curve-triangle, UTQ, is etched upon a glass disc, the duangle, PVQW, is engraved upon the plate RPSQ. [Professor

* The reference is to the top line of p. 121, the 5 of the reference number has unfortunately been omitted in printing.

Reuleaux's beautifully made models are now (May 1876) at the Exhibition of Scientific Apparatus at South Kensington, where they will remain during the summer. By drawing the triangle ABC with its centroid upon paper, and the duangle with its centroid upon tracing paper, the rolling of the centroids and the relative motion of the elements can both be tolerably well followed. By this method I have found it very easy not only to examine the motion, but also to draw series of point-paths for these higher pairs of elements.]

[16] (P. 125.) For drawing these and similar roulettes I use a special three-legged compass, made for me by Herr J. Kern of Aarau (Switzerland). The third leg is jointed and its length also can be altered, so that obtuse as well as acute-angled triangles can be taken up, which could not be done with the old form of three-legged compass.

[17] (P. 125.) What a strong hold this idea has obtained is shown, for example, in the following passage from Weissenborn's *Cyclischen Kurven*, (Eisenach 1856) p. 3 :—" If the circle described about m_0 roll upon that described about M, and if the describing point B^0 describe the curve B_0 P_1 P_2 as the inner circle rolls upon the arc B_0b, then evidently, if the smaller circle be fixed and the larger one rolled upon it in a direction opposite to that of the former rotation, the point of the great circle which at the beginning of the operation coincided with B_0 describes the same line B_0 P_1 P_2. This "evidently" expresses the usual notion, and the one which is suggested by a hasty prejudgment of the case. In point of fact B_0 describes the pericycloid B_0 B′ B″, which certainly differs sufficiently from the hypocycloid B_0 P_1 P_2.

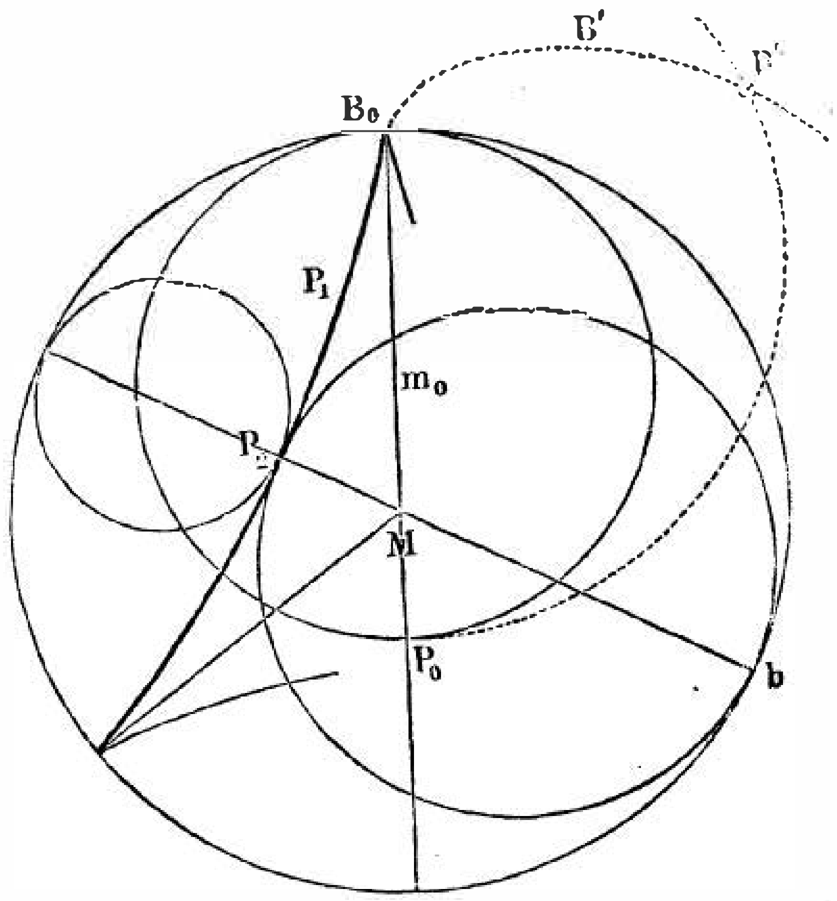


FIG. 445.

Centroids are also of very great assistance in understanding planetary movements, and are well suited to remove the difficulties which commonly occur in connection with their "real" and "apparent" motions. (I have a very instructive model for this purpose in the kinematic collection at Berlin.) If we test the ideas held by the majority of people upon this matter, we shall perceive how true a remark of Poinsot's (in the Memoir already referred to) still is,—" Mais s'il s'agit du mouvement d'un corps de grandeur sensible et de figure quelconque, il faut convenir qu'on ne s'en fait qu'une idée très-obscure."

[18] (P. 129.) These curves have often engaged the attention of mathematicians. Cf. Schlömilch's *Zeitschrift*, vol. ix., p. 209, Durège, *Ueber einige besondere Arten cyklischer Kurven*.

[19] (P. 145.) Open square figures, like ABCD, give square point paths if the centroid of the curved disc be a duangle (compare Pl. XII. Fig. 1). This occurs here, however, if QT be normal to PS. For we have, by similar triangles, $\angle 1PQ = \angle m_1 SP$, and hence $m_1 P = \frac{PR}{2e} \cos 1PQ = PS \sin 1PQ$, or (as $PR = PS$), $\tan 1PQ = \frac{1}{2}$. The form of the disc must therefore be so chosen that the tangent of the angle $m_1 SP = 0{\cdot}5$, or that $m_1 P = 0{\cdot}5\, m_1 S$. [I regret that the Figures on Pl. XII. have been transposed by mistake. Fig. 1 is referred to in the text as Fig. 2, and *vice versa.*]

[20] (P. 154). Willis (*Elements of Mechanism*, 2nd Edition, p. 90) gives Camus' theorem as follows :—

"If the pinion is to turn the wheel with a uniform force, the curve of its leaf, and that of the tooth of the wheel must be generated in the manner of epicycloids by one and the same describing curve, which must be rolled within the circle of the pinion to describe the inner form of the leaf, and on the outside of the circle of the wheel to describe the outer form of the tooth," etc. [Willis gives the quotation from Camus also in the original.]

[It is very interesting to compare the "solutions" given in Willis's third chapter with this portion of Reuleaux's work. Willis gives the method of § 32 as the "general solution," his other solutions are all for the special cases of circular centroids. With this limitation the first solution corresponds with § 34, the second and third are special cases of § 32, and the fourth corresponds with § 33. In a subsequent section he uses the method of § 35 for pin teeth.]

[21] (P. 154.) *Ann. Ph.* 1706, p. 379. De la Hire enunciates the proposition as follows :—"It is always possible to find a curve which, by revolving upon a given base-curve, shall generate by some describing point, in the manner of a trochoid, a second given curve ; provided that the normals from all points of the second curve meet the first." He gives as an illustration the production of a straigh line by rolling a curve upon a second straight line cutting the first. The describing curve, as can be easily seen, is a logarithmic spiral. If the lines are parallel the spiral becomes a circle.

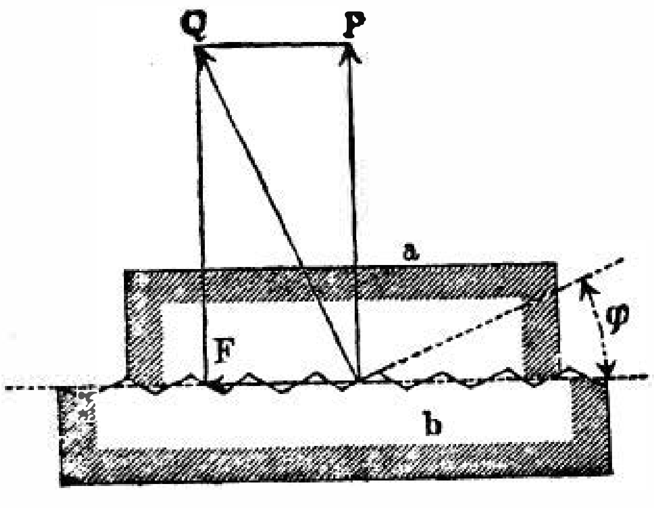


FIG. 446.

[22] (P. 171.) Suppose the two pieces *a* and *b*, Fig. 446, to be pressed together by some force, P, normal to their surfaces, and that these are covered with regularly formed teeth whose sides are inclined at a uniform angle, ϕ, in the way shown in the figure ; the resultant, Q, of the pressures upon the sides of the teeth opposed to any intended motion resolves itself into the resistance, P to the load and the resistance, F, to the force tending to produce sliding. F therefore is $= P \tan \phi$. We could in this way determine, by experiments on the "friction of rest," the mean angle of the roughnesses upon the two surfaces in contact.

I may be permitted to take this opportunity of pointing out the inexactness of the treatment of friction which is customary in text-books of elementary Mechanics, and which, in my opinion, is very unfavourable to its proper comprehension.

There is first the notion that friction prevents motion, but that it does not produce it. This is the view commonly taken in theoretical text-books. Weisbach, for example (*Theor. Mech.* i, 4th Edition, § 167) uses the heading, "the resistances of friction and rigidity," and says in the text, "In relation to the motion of bodies friction is a passive force or resistance, for it only prevents or retards motion, and never produces or aids it." Kayser (*Statik* § 161) says, "Friction may be regarded as a passive force, merely hindering motion, and never causing or aiding it." All writers do not express themselves so frankly, but the prevailing idea is certainly that friction is a "resistance," and substantially this is said by all. Rühlmann is an instance, and Wernicke, Mosele, too, and Poncelet, and even Duhamel. Among modern writers even the clear-headed Ritter has adopted the common mode of expression, indeed, I have found in text-books no exception whatever to the rule. And yet the idea is not founded upon any investigation as to its accuracy, and it stands in manifest contradiction to the known laws of motion, or more generally to the law of the conservation of energy. For friction is a force and must itself be treated as one, no matter whether or how it be derived from other forces. A multitude of other forces are derived in the same sense as it is. There is no real reason why this force (and the stiffness of cords, which is generally treated in the same way as friction) should be suddenly thrown out from all systematic connection with others, why it should be absolutely asserted that this force does not possess the essential characteristic of other forces, the capacity, namely, of producing or aiding motion,—why it should always appear with the negative sign. We have here a survival from the ancient Mechanics, from which the modern scientific treatment of the subject has in some places shaken itself entirely free, and against which, in others, it is still struggling.

It is necessary, of course, to prove my position, and it is at the same time exceedingly easy. Both in nature and in machinery there are numerous illustrations of the falsity of the statement that friction does not produce motion. The wind sets the surface of water in motion by friction; the wind itself is retarded by its friction upon the water, the latter being at the same time accelerated. The violin-bow sets the string in vibration by friction, in a way which Helmholtz describes as follows:—(*Sensations of Tone*, trans. by Ellis, p. 133)—"During the greater part of each vibration the string clings to the bow, and is carried on by it; then it suddenly detaches itself and rebounds, whereupon it is seized by other points in the bow and again carried forward." If a fast-running belt or cord be slid on to a stationary pulley, it slides upon it at first and then, by a more or less gradual operation, sets it in motion. The force which acts as a driving force from a velocity $= 0$ to a velocity equal to that of the band is friction. It resists the motion of the band, but accelerates that of the pulley. If we look into the matter more closely we shall find that in every single case friction both causes and hinders motion,—although the former may occur only as small alterations of form in the body acted upon. It is not even necessary to fall back upon pure mathematics, as might easily be done, and show that the retardation is itself a production of motion, motion, that is, with a changed sign. Neither upon practical nor upon scientific grounds, therefore, is there any justification for the popular view, the correction of which is much to be wished, although a number of connected errors may make such correction difficult.

"Every friction causes the disappearance of actual energy," says Helmholtz, in one of his excellent *Vorträge* (Hft. ii. p. 129). From this undeniable proposition it is only too easy to infer the other, and false one, "in every case friction causes only the disappearance of actual energy." That friction always causes energy to disappear, in no way contradicts the fact that it also produces energy. It is therefore not unnecessary to warn students that the proposition quoted, although in itself true, may lead to some very erroneous conclusions if it be blindly followed. Let us take the piston of the steam-engine for an example. The piston fits the cylinder closely, and loses not inconsiderably in actual energy by friction against its sides. Even the keen experimenter Hirn, however, did not succeed in detecting the slightest loss of energy from this cause. He himself gives the reason for this quite rightly in saying that the energy lost by friction appears again in the correspondingly raised temperature of the steam. Here friction causes the simultaneous disappearance and re-appearance of energy in such a way that at the end we can perceive nothing of the process. The proposition, "Every friction causes actual energy to disappear," by itself, would not be apparently substantiated by mere measurement; if it be given in an explanation of the nature of friction (for which purpose Helmholtz did *not* use it), it must be completed by the addition of the clause, "and also to reappear in another form."

I wish that writers upon Mechanics could be persuaded to give this matter a proper logical treatment, or at least to say something about it in elementary text-books. The further down the defect in logic be mended, the fewer corrections have to be made in the higher part of the work.

Another point to be mentioned is the statement of the *laws of friction*. Taking up almost any text-book of Mechanics whatever, we find the three following important propositions given:—(1) Friction is *proportional* to the normal pressure between the rubbing surfaces; (2) it is *independent* of their extent; (3) it is *independent* of the velocity with which sliding takes place between them. These are, as a whole, the propositions of Coulomb and Morin. Later, although now by no means recent, experiments have shown that they express the real phenomena of the case only within very narrow limits; that for those surfaces and velocities which are commonly to be found in machinery they do not apply; that in the latter, in fact, we must really read: *not proportional* and *not independent*. Indeed we know that machine-design has fallen into a hundred errors through its adhesion to the propositions of Coulomb and Morin, and in fact that in recent practice they have been entirely disregarded, and dimensions adopted which are altogether at variance with them. Is it not time that the experiments of Rennie, Hirn, Sella, Bochet, and others, were transferred from the notes to the text? It is to be desired equally for the sake of mechanical science itself and for its many practical applications.

[I am glad to be able to point to at least one excellent elementary text-book, in which the results of experiment are "transferred to the text,"—Prof. Ball's *Experimental Mechanics*. The extent to which inaccurate statements on this subject are made by writers of ability, who are perfectly well acquainted with the physical facts in question, is really extraordinary. The following sentences, for instance, occur on the first page of a well-known treatise on Friction :—(the italics are mine) " . . . The second class of forces, comprising the resistance of fixed obstacles, the resistance which fluid media oppose to the passage of bodies through them, and friction, are *not capable* of producing sensible motion, and *show their effects only* by the apparent destruction of the motion which has been caused by forces of the former class. These may be termed 'resisting' forces. . . . Resisting forces are *incapable of producing sensible motion*, and if they do not actually destroy it they do convert it into a species of motion which is, to sight, insensible."]

[23] (P. 203.) The series of phenomena adduced by Lubbock (*Origin of Civilization*, etc. Lond. 1870) in support of the theory of the unity of the human race (which is not to be confused with their growth from a single pair) is really extraordinary.

[24] (P. 205.) Chamisso (iv. 244) gives the following description of this and other methods :—

"In the Caroline Islands a piece of wood is fixed to the ground, and over it is held perpendicularly a second piece, about a foot and a half long, tolerably round, and of the thickness of one's thumb. This is caused to twirl by the palms of the hands, its lower and roughly pointed end being pressed against the fixed piece. The first slow regular motion is quickened and the pressure increased as the wood-dust, which is formed by the friction and collects round the borer, begins to carbonise. This dust is the tinder, and soon catches fire. The women of Eap possess wonderful facility in executing this process.

"In Radack and the Sandwich Islands they hold over the fixed piece of wood another which is about a span long and roughly pointed, and slope its upper end away from them at an angle of about 30°. It is held with both hands—the thumbs being placed below and the fingers above to improve the grip, and moved backwards and forwards in the plane of its slope through a distance of two or three inches. When the dust which collects in the groove formed by the friction begins to carbonise, the pressure and velocity are doubled.

"It is remarkable that in both methods the two pieces of wood used are of the same kind. They are best when of equally fine grain, not too hard and not too pliable. Both methods require practice, skill, and patience.

"The method of the Aleutians is the first of those above given, mechanically improved. They use the twirling-stick as they do the drill which they employ for other purposes. They hold and pull one end of the cord, which is twisted twice round it, with both hands; its upper end is passed through a piece of wood prepared for the purpose, which they hold in the mouth. We have seen two pieces of fir-wood thus used give fire in a few seconds,—a result which otherwise would have taken a much longer time.

"The same people also produce fire by striking together two stones rubbed with sulphur over dry moss strewn with the same material."

These accounts are clear and intelligible—peculiarities with which, unfortunately, we cannot always credit the corresponding descriptions of other travellers. It is much to be desired that expeditions to remote parts of the world should attach greater importance to objective observation of the technical industries of the natives, and should make their descriptions of these as full and accurate as possible, and unmixed with subjective additions. The fire-producing apparatus of these people is one of their most interesting possessions; it is in most cases of extremely great antiquity, and has formed the first step towards their other industrial operations. Many methods of fire producing have indeed been observed which have escaped the notice of writers on technical subjects. I may mention briefly three which have been orally described to me by esteemed friends.

Herr Jagor found the Malays employing the following method. A piece of dry bamboo, a foot long, is split up lengthways, and the tender inside bark which forms its inner coating is scraped together into a little ball in the middle of one of the halves. This half is then placed on the ground with the hollow side (and the little ball) downwards. The worker then splits so much away from the other half as to make it into a sharp-edged straight piece like a knife-blade. This he draws like a saw or file across the middle of the first piece, in which he has perhaps previously cut a little notch. This notch is widened and deepened by the sawing, and its edges get so hot that when at last a hole is made through the cane, the little ball of pith within catches fire.

Prof. Neumeyer saw a similar process used in New Holland. Instead of bamboo, wood was there used, and wherever it was possible a split log was used for the fixed piece. Some easily-kindled pith or other material was placed in the crack, and the process went on as above described.

Consul Lindau witnessed the following method of making fire in the Sandwich Islands. Some little stones of a kind which give sparks when struck together were placed, along with easily ignitable leaves, in a box formed from a large dry leaf, and then fastened to the end of a switch. This was twirled round in the air in a particular manner with great skill, so that the stones rattled against each other and the leaves caught fire.

The question of the invention or discovery of fire is not yet cleared up. Peschel in his excellent *Völkerkunde* (1874) deprecates premature conclusions on account of the scarcity of available material. Caspari (*Urgeschichte der Menschheit*, 1873) develops fully and carefully the hypothesis that the use of the borer may have led to its discovery, and his hypothesis is repeated and treated at great length in Baer-Hellwald's *Vorgeschichtliche Mensch* (p. 554 *et seq.*). Here some very remarkable survivals of primitive customs are pointed out as having been observed in Germany and England in the kindling of beacon-fires, and in Appenzell (Switzerland) as a child's play. (*See* Kuhn, *Herabkunft des Feuers*, Berlin, 1859; Caspari, as above, vol. i. p. 37; Schwartz, *Ursprung der Mythologie*, 1860, p. 142.) Herr Kuhn says that the case at Essede, among the Hanoverians, described by him, is not the only one which he has seen. The fire was lit by means of a horizontal pole, the ends of which were pivoted in hollows in two upright posts. In one of these hollows tow was placed, and this was kindled by twirling the pole rapidly in both directions. This was done by a cord twisted round it, pulled at both ends by men.

[25] (P. 206.) See Rau, *Drilling in Stone without Metal*, Smithsonian Report, 1868. In the cause of archæological science Rau has made the tremendous sacrifice of completing such a boring with his own hands. With a wooden borer such as we have described he pierced a hand plate of Diorite, 45 mm. thick, by making two hollows on opposite sides and meeting in the centre. He succeeded only after two years' (more or less intermittent) work. The form of the hole made is exactly that of the holes in numerous rude axes found in various places in Europe.

In the Ethnographic department of the Berlin Museum there are several excellent specimens of American work in rock crystal, among others a specially characteristic carved horse's head of something like 70 mm. long.

Mr. A. R. Wallace, in his *Narrative of Travels on the Amazon and the Rio Negro* (p. 278), says:—"I now saw several of the Indians with their most peculiar and valued ornament—a cylindrical, opaque, white stone, looking like marble, but which is really quartz imperfectly crystallized. These stones are from four to eight inches long, and about an inch in diameter. They are ground round, and flat at the ends, a work of great labour, and are each pierced with a hole at one end, through which a string is inserted, to suspend it round the neck. It appears almost incredible that they should make this hole in so hard a substance without any iron instrument for the purpose. What they are said to use is the pointed flexible leaf shoot of the large wild plantain, triturating with fine sand and a little water; and I have no doubt it is, as it is said to be, a labour of years. Yet it must take a much longer time to pierce that which the Tushuaúa wears as the symbol of his authority, for it is generally of the largest size, and is worn transversely across the breast, for which purpose a hole is bored lengthways, from one end to the other, an operation which I was informed sometimes occupies two lives. The stones themselves are procured from a great distance up the river, probably from near its sources at the base of the Andes; they are, therefore, highly valued, and it is seldom the owner can be induced to part with them, the chiefs scarcely ever."

[26] (P. 208.) This subject is most fully treated by Ginzroth, *Wagen und Fahrwerke der Griechen, Römer und anderer alter Völker*, Munich, 1817; also Weiss in several places. The four-wheeled vehicle was also in use, especially for carrying heavy loads. It bad fixed axles, and was tberefore much less easily guided than the two-wheeled one. In India there still exist, in the use of the natives, four-wheeled vehicles with a kind of moveable fore-carriage, an arrangement which must therefore be considered somewhat ancient. We know that the war-chariots of Porus were drawn into the immediate neighbourhood of the battle-field by draught oxen and not by horses. This may have occurred by no means unfrequently. It may well have happened that two of the empty chariots were then sometimes fastened together, the pole of the one to the frame of the other, and in this way a vehicle would be formed which had separate fore and hind carriages. The great ease with which such a compound vehicle could be guided must have struck its possessors, and may have led to the deliberate use of the revolving fore-carriage in regular vehicles.

[27] (P. 208.) Wooden war-chariots would be burnt by the conqueror for lack of horses to carry them away, iron vehicles he would render useless by

breaking some essential part of them, very much as we spike cannon. In 2 Sam. viii. 4, we have: "And David took from him a thousand and seven hundred horsemen, and twenty thousand footmen: and David houghed [rendered useless] all the chariots, and reserved of them an hundred chariots;" also Joshua xi. 6 . . . "Thou shalt hough their horses and burn their chariots with fire . . . (v. 9) Joshua . . . houghed their horses and burnt their chariots with fire." That the Jews had long known wheeled vehicles is evident from a passage in Numbers (vi.; 3—8), where six (wooden) wagons, each drawn by two oxen, are spoken of. The wheels of Solomon's laver carriages (about 1000 B.C.) were cast of bronze (1 Kings vi. 33), "their axle-trees and their naves and their felloes and their spokes were all molten."

[28] (p. 209.) The museum in Toulouse contains two remarkably well preserved antique bronze chariot-wheels of 54 cm. diameter, with naves 40 cm. long and 7 cm. diameter; casts of them are in the Roman-Germanic museum at Mainz. These wheels have five round spokes and deeply-recessed felloes; in the latter the rivets for fastening on the wooden rims still remain. The Esterhazy collection at Vienna, and the national museum at Pesth contain similar excellent specimens; an existing Egyptian carriage-wheel of wood is described and illustrated in Wilkinson, *The Ancient Egyptian*, vol. i. p. 383.

[29] (P. 209.) In the gradual development of the chariot-wheel in constructive form, the tire plays a very important part. It was evidently the metal rim which first made the wheel durable when used for quick motion along heavy roads. It was long, however, before the iron tire made in one piece was reached. Homer mentions in his celebrated description of the chariot of Juno, tires of copper (*Iliad*, v. 722, *et seq.*)—

"Quickly Hebe fixed on the chariot the rounded wheels
Of copper, eight-spoked, around an iron axle;
Their felloes, indeed, were of gold, imperishable, but around
Tires of copper were firmly fitted, a wonder to behold."

The last words show that there must have been great difficulty in completing the "firmly-fitted" tires (in segments?). That these were made of copper does not exclude the possibility that iron rims were also in use. Assyrian and ancient Persian relievos show chariots of many forms, most of them with smooth tires, scarcely distinguishable from the rest of the wheel. A few wheels are specially remarkable as showing rings of little projections (Fig. 447), like strings of beads all round the tire. Prof. Lindenschmidt, of Mainz, who called my attention to this peculiarity, solved the riddle at once. The projections are intended for the heads of nails. The whole wheel is covered with nails, driven into the wooden rim in close rows, their heads overlapping each other like scales. Among the discoveries in ancient burial-places in South Germany there are not a few iron tires about a metre diameter. They are always found in pairs, and are evidently the remains (after the rotting away of the wood), of the chariot wheels of the dead warrior, which had been buried along with him. These tires are covered with radial spikes on the inside, and their outer surfaces show the scale-like overlappings just mentioned. Close inspection

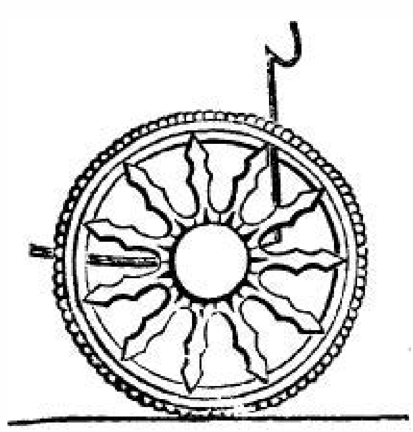


FIG. 447.

shows them to be simply the nails which had been driven into the wooden rim rusted together! The collection at Sigmaringen contains some beautiful specimens of them. They form evidently a very early step in the direction of making tires out of one piece.

I have found what appears to be a confirmation of Lindenschmidt's view in a model of a two-wheeled Chinese cart which was sent to the Vienna Exhibition of 1873. The tires are here made of iron, drawn out under the hammer; they are, however, very narrow, and on their outer surfaces are deeply stamped into forms resembling strings of beads. This appears to be simply a transference of the traditional outward form to the solid tire. The stamping is a fashion only, use and wont give value to this external form, although the new construction has made it worthless—a process which fashions of every kind experience. It may be noted here, also, that in the great Pompeian mosaic, the "Alexander-schlacht," the Persian chariot in the centre is represented as having a tire of nails made in the way we have described.

[The ancient wheels in the British Museum form a very interesting study. The Egyptian paintings show but few chariots; their wheels have always six spokes, and only one drawing (so far as I have noticed) shows any constructive details. This belongs to the 18th or 19th dynasty. The nave is made in one piece, and has sockets for the (round) spokes; at the end of each spoke is a tee-piece, which forms a socket both for the spoke and for the segments of the tire. Spokes and tire segments are coloured red, the nave and tee-pieces are left white.

The ancient Greek vases (*circa* 800—500 B.C.) show numerous racing chariots, sometimes in very great detail. A great number of their wheels have four spokes only, but the way in which the wheel is put together is not shown. The tires are very narrow and apparently (from some of the end views) made in segments. A number of the drawings seem to indicate that the spokes are flat, and very much wider (in the plane of the axle) at the nave than at the rim. Strengthening pieces are always used at the junction of the spokes with the rim. Upon one vase, a prize at an Athenian chariot-race about 700 B.C., chariot-wheels are very distinctly shown as having one pair of radial spokes only, these being crossed at right angles by two bars passing at a considerable distance on each side of the nave. In the bronze room there is a four-wheeled bronze brazier from Vulci, and two others from Eschara, none of them probably later than 600 B.C. The wheels are four or five inches diameter.

The most interesting wheels are, however, those of the Assyrian sculptures. Here the uses of three different forms of wheel can be distinctly noticed. The vehicles used for heavy carriages and drawn by oxen have four spokes only. The sculptor has not thought it worth while to show their constructive details, but the spokes are very broad and clumsy, and probably square in section, the tires or rims are also very heavy. The ordinary war chariot-wheels have commonly eight spokes. These are apparently round and fit in sockets formed upon the nave. The rim of the wheel is very deep, and is always shown as consisting of three concentric rings, of which the outer one, the tire, is much the deeper, and is made in segments. The rims are generally strengthened by two pairs of clips slipped on from the inside and reaching partly across the tire,—Prof. Reuleaux tells me that in some instances at least these have proved to be pieces of leather. The royal chariots, at least those of Sennacherib and Sardanapalus, not only have ornamented spokes and naves, but have also the nail tires mentioned by Prof. Reuleaux. These occur nowhere but on the royal chariots, so that they must have been the "latest improvement" in Assyria in the eighth century B.C. I have noticed wheels with more than eight spokes only upon one slab. One has sixteen, one thirteen and several twelve spokes. Some of these, however, are certainly intended for wheels belonging to the chariots of the people with whom the Assyrian are fighting.]

[30] (P. 209.) According to Herr Detring's own observation, so that the disc-wheel of the plaustrum forms a step in the growth of wheeled vehicles all the world over.

[31] (P. 210.) In Sanscrit the chariot is called *ratha.*

[32] (P. 210.) We may remember, for instance, the method of transporting the pillars of the Temple of Artemis in Ephesus, described by Vitruvius (x. chap 2). The master-builder Chersiphron fastened iron pins to the ends of the enormous cylindrical blocks of hewn stone, and laid upon these a wooden frame fitted with proper bearings for them. To this frame the draught oxen were yoked, and with their aid the pillars were dragged, after the manner of our street rollers, from the quarry to the site of the building—the same site upon which excavations have recently enabled us to appreciate the magnitude of the work, and the advantages of the method used in carrying it out.

[33] (P. 211.) It has been successfully shown by experiment that apparently blunt fragments, if they have crystalline edges, are specially suited for boring harder stones.

[34] (P. 212.) The rare form *tornator* is to be found in Jul. Firmicus (Mathesis, iv. 7) :—*facit quoque tornatores, aut simulacrorum sculptores.*

[35] (P. 212.) Among the specimens at the Berlin Museum which undoubtedly belong to the old kingdom, there are several which have certainly been turned in the lathe, and the latter must therefore have been used by the Egyptians between 2,000 and 3,000 years before our era. These are again vessels, partly of alabaster and serpentine (as Nos. 93 and 88), partly of marble and even granite (Nos. 62 and 100). The hypothesis of the connection between the lathe and the potter's wheel (which turned out most excellent work, as we see from the museum collection, even at that early date) seems to be supported by this.

[36] (P. 215.) Cf. Böckler, *Theatrum Mechanicum Novum,* Nuremberg, 1762, Plates 35, 36, 80. Neither in this whole work of 154 plates, nor in Rosberg's *Kunstlichem Abriss* &c. Nuremberg, 1610, do we find any apparent trace of our present belt-train. Arrangements for driving by a cord or rope twisted two, three, or four times round a pulley, are given by Ramelli, *Arteficiose Machine,* Paris, 1588, Plates 171, 175, 183.

[37] (P. 217.) There are specimens of Ancient Egyptian spindles in the Berlin Museum. Wilkinson, who mentions the Berlin specimens in his *Ancient Egyptians,* places beside them (Fig. 385, 1 to 5, vol. ii.) three illustrations of distaffs or portions of them, which he erroneously takes to be spindles also. He had apparently been misled by a note in an older catalogue. [Nos. 1 and 2, Fig. 168 have unfortunately been printed upside down.]

[38] (P. 220.) Dr. Wetzstein writes to me: "The word schaduff or shadoof

comes from the root *schadf*, which means to hang down to one side. This is very appropriate to the irrigating machine in question, because its lever, when not in action, always slopes downwards towards that side which is weighted with stone. The machine is not found in Syria, I have seen it only in Egypt." In the *Descr. de l'Egypte* (xviii., 2, p. 539, *et seq.*) the shadoof is also called *delú* (*delou*); at the junctions of water channels from thirty to fifty shadoofs are not unfrequently to be seen together.

39 (P. 225.) Endeavours in this direction are even now to be met with among a few cultivated nations. Baron Von Korff saw, as he told me, in Egypt, a gunsmith who, while both hands were busy with his iron work, used his feet in working a saw to cut the wood for his gun-stock. The Tartars, both men and women, although engaged in their domestic duties, seldom lay aside their great curved embroidering frames. We need only look at the European stocking-knitter, too, to see the connection between these customs and our own.

40 (P. 229.) A Spanish word, from the Arabic *nā-'ūrah*, so called from the snorting noise made by the emptying of the buckets:—*na'ara*, to snort (Heyse). Vitruvius also knew these wheels, which even in his time must have been of great antiquity (x. chap. v. [Vulgo x.]): . . . "Circa eorum frontes affiguntur pinnæ, quæ cum percutiuntur ab impetu fluminis, cogunt progredientes versari rotam, et ita modiolis aquam haurientes et in summum referentes sine operarum calcatura, ipsius fluminis impulsu versatæ, præstant quod opus est ad usum."

41 (P. 230.) A splendid example of this kind of machine stands in Zürich in the immediate neighbourhood of the Polytechnic School, a contrast which is humorous enough. It would be worth while to preserve at least drawings of this mammoth among machines, this doomed representative of a past epoch for the benefit of the coming race.

42 (P. 235.) [In the Patent Museum at South Kensington there are to be seen several wooden models of Watt's, of his proposed arrangements for obtaining rotary motion from the beam, as well as the sun-and-planet engine which for so long drove the machinery at his Soho factory. See also Note, p. 433.]

43 (P. 237.) It appears not to be well known, and may therefore be mentioned here, that the Greeks were perfectly well acquainted with the pulley. The Romans received both the thing itself and its name from the Greeks (cf. Vitruvius x. chap. iii, *De machinis tractoriis*). The three-sheaved tackle they called τρίσπαστος, the five-sheaved πεντάσπαστος, the multi-sheaved generally πολύσπαστος. These names were certainly better than ours, for we have seen (§ 43) that the characteristic part of the tackle is the stretched rope or cord, and not the revolving pulley or sheave. A mere fixed guide pulley the Greeks called ἀρτέμων.

44 (P. 237.) If we arrange the different forms of toothed wheels according to the increasing complexity of their theoretical treatment, we should have to adopt the order: spur-wheels, bevel-wheels, screw-wheels, hyperboloidal-wheels. It would, however, be a mistake to assume, without further inquiry, that this was the order of their natural development. As a matter of fact toothed wheels with crossed axes, and therefore with hyperboloidal axoids, appear to be the oldest, and to have led up to the conception of the simpler toothed wheels.

For we find wheels of the simplest possible form, consisting, namely, of nothing but a nave and radial spokes, in primitive water-lifting wheels, where the horizontal wheel-shaft was driven from a vertical one (cf. Fig. 50 of Ewbank's *Hydraulic and Other Machines*, 16th Edition, New York, 1870). The screw-wheels for parallel axes, the invention of which has been ascribed to an Englishman, White, are to be found in primitive Indian cotton ginning rollers (see a drawing in Leigh's *Modern Cotton Spinning*, London, 1873, as well as several complete machines in the Indian Museum, London). We may note also that toothed-wheels having intersecting axes, in the form of crown-wheel and pinion, have received more extensive application and attention in mill work, almost down to our own time, than spur-wheels. The latter, indeed, appear to come last in order, so that the real sequence of historical development is exactly the reverse of what we might expect, a hint that we must never confound what actually and practically lies nearest to us with what is geometrically simplest.

[45] (P. 238.) From the Arabic *sakai*, to water or supply water; *sakkū*, a water-carrier in eastern countries.

[46] (P. 275.) It is not uninteresting to compare the different statements on this subject. We may give a few specimens of them :—

Poppe, *Maschinenkunde* (1821), p. 81 :—"The lever, the wheel and axle, the pulley, the inclined plane, the wedge, and the screw are included under the name simple machines, simple engines (*Rüstzeuge*) or mechanical powers. From these all machines, even the most complicated, are constructed. Since, however, the theory of the wheel and axle and of the pulley is based upon the law of the lever, and the theory of the wedge and the screw upon the law of the inclined plane, we may reduce the number of simple machines to two, the lever and the inclined plane."

Here it is clearly and emphatically stated that all machines, "even the most complicated," are formed from the simple machines, and that the latter may be reduced to two. Read, however, the following :—

Langsdorf, *Maschinenkunde* (1826), i. p. 277 :—"Even in the older text-books we find machines divided into simple and compound, the latter being those which are formed by a combination of several of the former. The simple machines are limited to the lever, the pulley, the inclined plane, the wedge, the screw, and the wheel and axle. The immoveable inclined plane should not, however, be included, it is no more a machine than is the slope of a mountain. We do indeed find a moveable inclined plane in the wedge; inclined plane and wedge are not then machines of two different kinds. I put in their place the roller. . . . ." Here, then, it is said to be wrong to treat the inclined plane as a simple machine, while before it was treated as the foundation of several others.

Gerstner, *Handbuch der Mechanik* (1831), i. p. 73 :—"Machines are commonly divided into simple and compound. The simplest machine, which we shall first consider, is the lever. We shall then proceed to the wheel and axle, the pulley and the pulley-tackle (!), the inclined plane, the screw and the wedge. All these machines are simple machines ; compound machines consist always of a combination of several simple ones, after which, therefore, their treatment will come."

Kayser, *Handbuch der Statik* (1836), p. 460 :—"Machines are divided also into simple and compound. Strictly speaking only the cord (!), the lever and the inclined plane are simple machines. It is customary, however, to treat along with these also those into which compound machines can be resolved. These simple machines are seven in number, viz., the cord, the lever, the pulley, the wheel and axle, the inclined plane, the wedge, and the screw. They are also called machine organs or mechanical powers. Many writers do not reckon the cord among them."

Rühlman, *Mechanik* (1860), p. 231 :—"A machine of which no part is itself a machine is called simple, in the opposite case compound. The simple machines are the funicular machine, the lever, the pulley, the wheel and axle, the inclined plane and the wedge. Note : strictly speaking we need distinguish only three simple machines, the funicular machine, the lever, and the inclined plane, all the others may be resolved into these." This definition leaves something still wanting, and in itself it is a complete *petitio principii*. We have again the impossible derivation of the pulley from the lever.

Schrader, *Elemente der Mechanik und Maschinenlehre* (1860), p. 26 :—"The different kinds of simple machines. The originals of all simple machines are the lever and the inclined plane. From the lever are derived the pulley and the wheel and axle, and from the inclined plane the wedge and the screw. Note: in the lever the moving piece rotates, in the inclined plane it moves in a straight path." The pulley is, as usual, quite wrongly placed.

[I am sorry to say that the definitions of English authors have been no more satisfactory, as a rule, than those given above. Todhunter, for example, says :—"The most simple machines are called mechanical powers ; by combining these, all machines, however complicated, are constructed. These simple machines . . . . are usually considered to be seven in number; namely, the lever, the wheel and axle, the toothed-wheel, the pulley, the inclined plane, the wedge and the screw." We continually find, too, the loose expressions that the lever is a solid body "moveable about a fixed point," that it is "supported at one point" (as distinct from the wheel, which is supported at an axis), and so on.]

It is very remarkable that in all the examples we have given, with the exception of Langsdorf, the peculiarity of the screw as a simple machine is denied, although it is kinematically the general case of the three lower pairs, and ought therefore in every case to remain in the classification. The extraordinary confusion (for so we must call it) of ideas upon the subject arises from a peculiar misunderstanding which, so far as my experience goes, is very strongly rooted and may be only very slowly dislodged. It is that the similarity of the relations existing between the forces coming into action is mistaken for a similarity between the objects themselves. Because certain force-relations in the screw are conditioned similarly to those in the inclined plane, it does not follow that the two things are identical. Instead of examining the things themselves, people have concerned themselves with certain of their properties. The importance of the latter indeed cannot be disputed, but they ought logically to be kept apart from the actual nature of the combination of bodies to which they belong. When, on the other hand, the more recent writers apparently do away with the simple machines altogether, but in reality introduce them as "exercises," "examples," "applications" and so on, they have furthered a good result less than they believe. For, as we have seen in the text, there is really some truth at the bottom of these problems—no precautions have been able to expel the sense of this fact. We read between the lines the sentence of Horace: *Naturam expellas furea, tamen usque recurret!*

It is not easy to say up to what limits it may be advisable for general Mechanics to follow the methods which we have worked out. I believe, however, that it is certainly advisable that the simple machines should be treated in the way which our kinematic investigations have pointed out to us in elementary Mechanics. This cannot but be of use in increasing the tangibility and definiteness of the ideas which the scholars receive upon the subject.

To the question how far generally Mechanics should concern itself with machines, we may say that this should be the case in decreasing degree from the lower to the higher Mechanics. For those who want merely elementary notions of the subject, general Mechanics and the Mechanics of machinery are one and the same thing. The higher the studies be pursued the more distinctly their differences make themselves felt. Which of the many positions between the highest and the lowest should be adopted in the construction of a text-book must in every case be carefully considered. But before everything in my opinion elementary text-books of Mechanics deserve much more careful treatment, especially as to the logical arrangement of their contents, than they have hitherto received. They are too often deficient in that transparent clearness which we are entitled now to demand from Mechanics. We have already noticed this in reference to friction. How unconnected with everything else, also, the treatment of the strength of materials commonly is! Apart from certain internal peculiarities in the way of new data, to which I have called attention in the preface to the last edition of my *Constructeur*, the general treatment of the matter appears to me defective, and it is never made sufficiently distinct that the "strength of materials" stands simply in the same relation to rigid bodies as hydrostatics and hydraulics occupy to liquids and aerostatics and aerodynamics to gaseous bodies. (If we wished to include all under a general title we might use the words stereostatics and stereodynamics for that purpose.) All three branches treat of the inner mechanical forces—our latent forces of § 1—which give the material its existence; all three besides overlap each other in the limiting cases. On the other hand just the same separation can be made with fluids as is done in the separate treatment of the "strength of materials" with solids; the problems connected with their molecular condition can be separated from those concerning their relations as a whole to other bodies. Very valuable analogies show themselves between the three departments if they only be looked for. I believe that new life might be thrown into the whole study if its treatment were taken up afresh in the direction which I have pointed out. [I may take this opportunity of pointing out that we have already in English an excellent word, introduced by Prof. Rankine, for for what Prof. Reuleaux calls a "latent force," namely stress. I am not aware that it has any German equivalent. It is very much to be wished that engineers—and physicists too, for that matter—would agree to use this word where now strain is often employed, and to keep the latter for its more obvious meaning of deformation, for which it is far better adapted.]

[47] (P. 290.) [The lines A B and D C (Fig. 208) cut 1, 4 and 2, 3 in such a way as to make the alternate angles 1 and 3 (and also of course 4 and 2) equal. Hence the name a n t i - p a r a l l e l. See *e. g.* Reuleaux's *Constructeur*, 3rd ed., p. 71.]

[48] (P. 327.) It was Willis who first pointed out the nature of the conic crank-trains, and their analogy to the cylindric crank trains. (*Principles of Mechanism*, 2nd ed., 1870, p. 249, ff.) He called the trains "solid angular link work,' and indicated several of their more important forms and characteristics. He had not, however, the idea of the kinematic chain, and missed therefore, some of their most essential properties; as a kinematist of the old school, too, the fourth (fixed) link altogether escaped him, as also the possibility of inversion, and with these some very remarkable practical applications of the chain, of which we shall have more to say further on.

[49] (P. 341.) The treatment of compound chains belongs to the more difficult problems of Kinematics. I refer to them again in Chapter XIII., particularly in § 160. The full advantage of the idea of chain reduction only makes itself felt in the study of these applications of Kinematics. I recommend the teacher to give his pupils exercises in the re-completion of reduced chains.

[50] (P. 384.) I have certainly not exhausted the list of chamber-trains which have been constructed from $(C_3^{z} P^{\perp})^{a}$, although so many of them have been investigated. It is interesting to note that lately the crossed slider-crank chain (see § 73) has also been used in chamber-trains. Gibson's rotary steam-engine (*American Artisan*, Feb. 1874, p. 30) is an example of this; it is a combination of two trains of the form $(C_3'' P^{+})\frac{a}{c} - b$.

[51] (P. 399.) For the determination of the axoids of the links b and *d* in $(C_3^{\perp} C^{\perp})$ we have the following (see Fig. 448) :—

$$\frac{w_1}{w} = \frac{r}{r_1} = \frac{\sin \gamma}{\sin \gamma_1} = \frac{\cos a}{1 - \sin^2 \omega \sin^3 a}$$

We require to find the values of γ and γ_1 corresponding to different values of ω. We have

$$\gamma_1 = 180° - (\gamma + a),$$

hence

$$\sin \gamma_1 = \sin (\gamma + a) = \sin \gamma \cos a + \cos \gamma \sin a,$$

and from this, putting $\frac{\cos a}{1 - \sin^2 \omega \sin^3 a} = A$, we obtain

$$\sin \gamma = A \; (\sin \gamma \cos a + \cos \gamma \sin a).$$

$$\frac{1}{A} = \cos a + \cot \gamma \sin a,$$

whence

$$\cot \gamma = \frac{\frac{1}{A} - \cos a}{\sin a},$$

or, again inserting the quantity represented by A:

$$\cot \gamma = \frac{1 - \cos^2 a - \sin^2 \omega \sin^2 a}{\cos a \sin a} = \frac{\sin^2 a \cos^2 \omega}{\cos a \sin a},$$

that is, $$\cot \gamma = \tan a \cos^2 \omega.$$

a relation which also may easily be determined graphically, as is shown, for example, in Fig. 452. It gives the ratio $\frac{\cos^2 \omega}{1} = \frac{x}{\tan a}$, that is, $x \cot \gamma$.

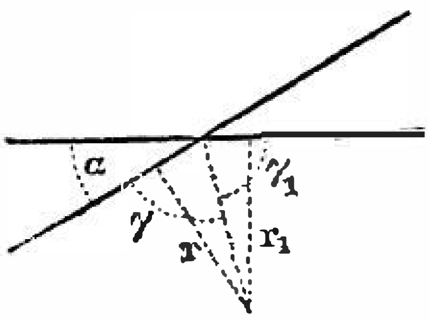


FIG 448.

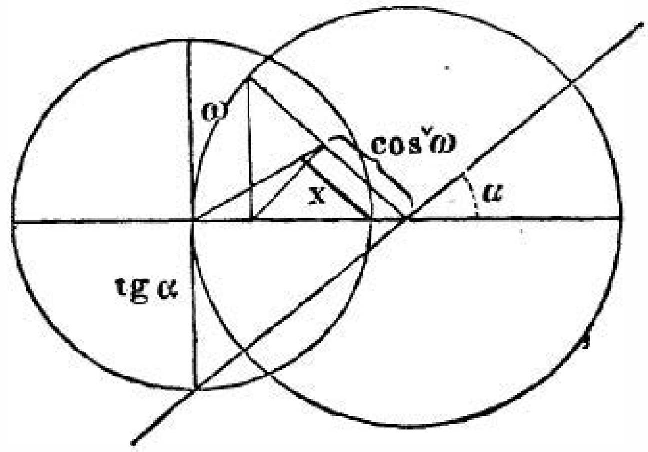
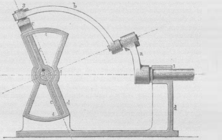


FIG. 449.

[52] (P. 400.) It is not difficult to add new forms synthetically to the many old ones which we have mentioned. Indeed, the foregoing investigations permit such problems to be given directly as exercises in a course of machine instruction. I do not intend to urge here the use of such exercises, which would be suitable only for very advanced students; they do not, however, essentially differ from those of modern chemistry, where the more advanced students in the laboratory are exercised in the synthetic development of new series of bodies. [I venture to go further in this matter than Prof. Reuleaux, and to hope that synthetic exercises may become both possible and popular among our students to a considerably greater extent than he suggests. The formation of chamber-trains alone gives immense scope for such exercises, and there are also other directions in which they could be worked without touching on problems of any serious difficulty. I know no kind of exercises which are likely to excite so much interest among the class of young men who in this country devote themselves to the profession of engineering.] I may give an example of this. It will be noticed that in the twelve conic chamber-trains formed from $(C_{\frac{1}{3}}^{\perp} C^{\angle})^a$ and $(C_{\frac{1}{3}}^{\perp} C^{\angle})^b$ the chambering $(V\mp)=c$, d is never used. This, might, however, be done as follows. Instead of making the element 4 of the link d into a diaphragm and guide for c (as in Fig. I., Pl. XXVIII.), it might be formed into a sector of a hollow cylinder with cylindrical openings at its ends, as in Fig. 450. In the chamber thus formed, the cross-section of which is similar to that of the former spheric-sector chambers, the element of c belonging to the pair 4 may be placed as a piston. This takes the form of a slice of the same cylinder, fitted with a shaft, the ends of which might project beyond the chamber. To these projecting ends an outer frame is fastened, which carries the cylinder, $C+$, of the pair 3 belonging to the link c. The link b has open cylinders both at 2 and at 3, and is simply a connecting-rod with its two open elements normal to each other. At 2 it is

paired with the crank $a = C+ \ldots \angle \ldots C+$, which (as before) finds its bearing 1 in the link d. In this way we obtain a chambertrain in which the piston c merely oscillatesabout its axis. Without discussing at all the usefulness of this form of engine, I maymention that something similartoit has already been made in Morton's disc-engine (*DeutscheGewerbezeitung*, 1857,



p. 31). Morton has, however, impressed by the form of the older disc-engines, madehispiston c and chamber d unnecessarily as portions of spheres, and prides himself on the fact thatthe piston of his machine does not make the wobbling motions of those of the older engines, and that it has not the slot,nor the chamberthe diaphragm and packing pieces formerly required. He has thought it necessary, however, to make the sides of his chamber with plane inner surfaces, parallel to the axis of 4, the piston lyingupon these in itstwo extremepositions.

[53] (P.413.) Inthe year before, 1858,apatentwas takenout (dated 14th April,and taken outthrough Newton's Agency),for a steam-engine having such shoe-sole-shaped wheelpistons. Special packing-pieces werefitted into theends of theteeth(*Propagation Industrielle*, iv.,1869,p. 179).

[54] (P. 437.) [This statement, I am sorryto say,does not apply to this country,where,so far as Iknow,the constructive elements assuchhave neveryet receivedany systematictreatment. I have had to make a fewsmall alterations in Chapter XI. on thisaccount, omitting a few sentences which applied solely to the existing continental treatment of certain details. Professor Reuleaux's discussion as to the subdivision and classification of the constructive elements has therefore nodirect bearing,as yet,npon English text-books, and hardly any upon English systems of instruction. I hope sincerely that such a state ofmatters may not long exist.]

[55] (P.461.) [I think we might call the whole class cam trains, under whichthe click-trainsand slider-cam trains would come as special cases. The latter bear to the cam-trains the same relation that $(C_3^{\prime\prime} P^{\perp})$ bears to $(C_4^{\prime\prime})$

The higher pairing which occurs here receives further consideratior in Chapter XIII. § 157.]

[56] (P. 498.) [The matter may also be looked at in a somewhat different way. The driver of the old engines formed an element of a sliding-pair, as t does still, for instance, in direct-acting pumping engines. The attempts at rotary engines seem to me attempts to replace this sliding-pair directly by a cylinder or turning-pair. Something equivalent to this is very frequently insisted on in descriptions and specifications, and has certainly, in a more or less indistinct form, been present in the minds of many inventors, who have persistently refused to see more than one moving part in their machines. It is one of the results most to be hoped from the acceptance of Professor Reuleaux's method of analysis, that thee energies of these and other ingenious minds may be turned into worthier channels.]

[57] (P. 522.) [§ 137, as it appears here, is a summary of a much longer treatment of the subject given by Reuleaux, which I hope may be published at length in another form. He discusses in some detail the present position of workmen on the Continent, and the way in which they have been affected by the machine and machino-facture. The circumstances of the case, as he describes them, differ in some very important respects from the circumstances attending similar industries in this country.]

[58] (P. 533.) [It will be remembered that strictly the pair (C) is not closed, for it has not of itself the cross profiles necessary to prevent axial motion. This is a general difference between the pairs formed from S and those formed from $\bar{H}$; the former may be made completely constrained in themselves, while in the latter (as Figs. 363 to 367 for instance) the necessary constraint in one or more directions is obtained only by the use of pair-, chain-, or force-closure. This closure being provided, however, the form of these higher pairs determines motion as absolutely as the closed forms of the lower ones.]

[59] (P. 538.) [A comparison of this table with that given at p. 543 of the German edition will show some points of difference between the two. These appeared to me necessary to bring the table and the text into complete agreement,—especially as several corrections (see Preface) have been made in the latter,—but it is right to say that I have not been able to submit them to Prof. Reuleaux, who was on his way to Philadelphia when they were made. For my own purposes I have used a different classification, which I need not give here. There is one detail connected with the notation of the higher pairs which might be modified, I think, with advantage. Prof. Reuleaux uses, *e.g.* the symbols $(C,_z)$ and $(C_z;)$ for different pairings of C with Z. The first form appears to me much the better, so that I would suggest the use of $(C;_z)$, $(C:_z)$ and so on, instead of $(C_z;)$, $(C_z:)$, etc. It would then be understood that a small letter used simply as a suffix indicated some quality of the element denoted by the capital letter after which it was placed, and possessed in common by the two elements of the pair, while if a stop of any kind (comma, semicolon, etc.) were marked between the two letters the meaning would be that the pair consisted of two dissimilar elements, paired in the manner pointed out by the stop. Thus (C_z), or more strictly $(C_z,)$, would indicate a pair of elements each of the form C_z, while $(C{,}_z)$ would be a pairing of C with Z, etc.]

[69] (P. 552.) A model of the mechanism shown in Fig. 395, exhibited at Vienna, was placed beside a very nearly related piece, the so-called skew-disc (*schiefe Scheibe*). In the *Bairischen Industrie und Gewerbeblatt*, 1874, p. 100, Herr Schedlbauer gives a theory of the motion in this mechanism, and shows that the link *e* makes swinging motions which are given by the formula $(r \tan a) \sin \omega$,—in which r is the constant distance 1·6, a the angle between 1 and 2, and ω the angle of turning of the link *a* relatively to the fixed link *f*. According to this the link *e* would have a simple harmonic motion. The constructive conditions of Herr Schedlbauer's mechanism are, however, somewhat different from those of Fig 395, and of my model. He assumes the link *b*, carrying an element of each of the pairs 2 and 3, to be of the form $C \ldots \perp \ldots P$, and also that the axis of the last-mentioned prism always intersects the axis of the prism C at a constant distance from 1. This would be a mechanism having for its formula,—beginning with the pair 1.—$(C^{\angle}C^{\perp}P^{\perp}C^{\perp}P''C'')_s$. In the train represented in Fig. 395 the link *e* makes motions which only approximate to simple harmonic oscillations, and which are given by the expression

$$y = \frac{(r \sin a) \sin \omega}{\sqrt{\cos^2\omega \cos^2 a + \sin^2\omega}}$$

if y be the distance of a point of *e* from its middle position and r the length 2·4.

The difference between the two motions is very small with a small angle at a, and may be generally neglected in the cases which occur in machine practice. I mention the matter merely as another illustration of what I have already noticed in note 46, that the motion of a mechanism, or more correctly one of the motions occurring in it, has been often investigated without any examination having been made of the actual combination itself by which that motion was produced. The latter is, however, in the case before us the more important part of the problem, for we have already an immense number of trains in which an exact or approximate simple harmonic motion occurs, [I have called the latter *distorted* harmonic motion], while the various forms of the train (C_6^{+}) have never yet received investigation.

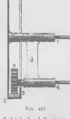


FIG 451

The steam-engine of Robertson, mentioned upon p. 551, contained one detail which deserves further mention. Robertson used, namely, according to the published description, the driving mechanism shown in Fig. 451, which has occasioned no little astonishment. Here *c* is a spur-wheel driven by the wheel *a* not by means of teeth but by the water *b* held in the hollow ring of *a*; or, as we should say more rightly, paired with *a*, for the water slides in *a* if the velocity of this wheel fluctuates. The water-ring *b* can therefore turn uniformly, although the wheel *a* moves with a variable speed received from the train (C_6^{+}). It is claimed that the transference of motion will be very "smooth"; at starting only a little water will be spattered about, afterwards the centrifugal force is sufficient to keep it against the rim of the wheel *a*. We have here an application of the pair C_z, Q_λ or $(C_{z,\lambda})$ belonging to order XVI. (§ 149.) The vessel $V-$, with which Q_λ is paired on the other side, has, on account of the sliding of the water-ring, the form C−. We might write the whole train therefore :—

$$C^+ \ldots \overset{a}{|} \ldots C^-, Q\gamma \ldots \overset{b}{\ldots} Q\gamma, C^+_z \ldots \overset{c}{|} \ldots \underline{C^+_= C^- \ldots \overset{d}{\|} \ldots C^-_=}$$

We note also again here how practical machine construction has taken the road already pointed out by our synthesis. The pairing of Q_λ with C_z is, as we know, nothing new in itself ; it exists, for instance, in the common water-wheel and other mechanisms ; the mechanism of Robertson is interesting only as an attempt to use the pairing in a free manner in a driving-train intended for common factory work.

INDEX.

INDEX.

D.

G.

H.

I.

J.

K.

L.

THE END.

LONDON:
R. CLAY, SONS, AND TAYLOR, PRINTERS,
BREAD STREET HILL,
QUEEN VICTORIA STREET