ON THE EFFICIENCY OF THE NEW GENETIC SELECTION INDEX

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Abstract

A new genetic selection index was proposed by the author in [1] for the case of unknown $E(\underline{Y})$. Here, the efficiency of this index is compared with the index for known $E(\underline{Y})$.

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1. Introduction

Consider the genetic model

$$\underline{Y} = X\underline{\beta} + Z\underline{u} + \underline{e}; \qquad \begin{pmatrix}\underline{e}\\ \underline{u}\end{pmatrix} \sim N\left[\underline{0}, \begin{pmatrix}E & 0\\ 0 & G\end{pmatrix}\right] . \qquad \text{--- (1)}$$

Suppose we want to predict a linear combination of the 'genetic' components of the model, say $\underline{\ell}'\underline{u}$. When $\underline{\beta}$ is known the best predictor for $\underline{\ell}'\underline{u}$ is given by $\underline{\ell}'\hat{\underline{u}}_{(1)}$ where

$$\hat{\underline{u}}_{(1)} = GZ'A^{-1}(\underline{Y} - X\underline{\beta}) \qquad \text{--- (2)}$$

and

$$A = ZGZ' + E \quad .$$

A new selection index $\hat{\underline{u}}_2$ was proposed in [1], viz.

$$\hat{\underline{u}}_{(2)} = GZ'P'(PAP')^{-}P\underline{Y} \qquad \text{--- (3)}$$

where

$$P = (I - X(X'A^{-1}X)^{-}X'A^{-1})$$

and hence a predictor of $\underline{\ell}'\underline{u}$ is $\underline{\ell}'\hat{\underline{u}}_{(2)}$.

We shall now show that even when $\underline{\beta}$ is known, (3) can be used and still achieve at least the same efficiency as (2) in terms of variances of the predicted values.

2. The efficiency

For any $\underset{\sim}{\ell}$, we have

$$V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(1)}\right) = \underset{\sim}{\ell}'GZ'A^{-1}ZG\underset{\sim}{\ell} \qquad \text{---- (4)}$$

$$V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right) = \underset{\sim}{\ell}'GZ'P'(PAP')^{-}PZG\underset{\sim}{\ell} \qquad \text{--- (5)}$$

and

$$\begin{aligned} Cov\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(1)}, \underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right) &= \underset{\sim}{\ell}'GZ'A^{-1}AP'(PAP')^{-}ZG\underset{\sim}{\ell} \\ &= \underset{\sim}{\ell}'GZ'P'(PAP')^{-}ZG\underset{\sim}{\ell} \\ &= V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right) \qquad \text{--- (6)} \end{aligned}$$

$$\begin{aligned} \therefore \left[V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right)\right]^2 &= \left[Cov\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(1)}, \underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right)\right]^2 \\ &\leq V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(1)}\right) \cdot V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right) \ . \qquad \text{--- (7)} \end{aligned}$$

Since $V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right) \geq 0$, (7) gives

$$V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right) \leq V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(1)}\right) \quad \text{for all } \underset{\sim}{\ell} \ .$$

Thus $\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}$ is a more efficient predictor of $\underset{\sim}{\ell}'\underset{\sim}{u}$ than $\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(1)}$ in general. It is easy to see that, when $\underset{\sim}{\beta}$ is known, $V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(1)}\right) = V\left(\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(2)}\right)$ since $\underset{\sim}{\ell}'\hat{\underset{\sim}{u}}_{(1)}$ is the minimum variance predictor in that case. The generalization to the case, where $V\begin{pmatrix}\underset{\sim}{e}\\ \underset{\sim}{u}\end{pmatrix} = \begin{pmatrix}E & C\\ C & G\end{pmatrix}$; $C \neq 0$ is straightforward.

Reference

[1] Nair, R. C. [1974] A note on the genetic selection index. BU-539-M in the Biometrics Unit Mimeo Series, Cornell University.