Multiplicity of Bidding Strategies in Reverse Auctions*

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Abstract

We study a reverse auction in which a buyer procures a single unit of good or service (such as a contract) from one of many competing sellers through auctions. Sellers have independent and identically distributed costs. Under certain conditions, we show the multiplicity of the symmetric equilibrium bidding strategies in the first-price reverse auction, contrasting with the well-known uniqueness result in the first-price forward auction. The multiplicity of bidding strategies is associated with the risk of very large costs.

Keywords: Auctions/bidding; mechanism design; reverse auction; procurement; supply chain management.

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1 Introduction

With the rise of the Internet and electronic commerce, auctions are increasingly used to determine prices and to allocate resources. Auctions can be administered fairly and efficiently online, eliminating costly negotiations due to clearly defined rules. The use of auctions is not merely limited to the transfer of goods among end-users (as in the case of art and antiques), but includes supply chain management. Auctions are used to distribute goods from upper-echelon producers to lower-echelon dealers and consumers. For nearly a century, the floral industry has been running auctions, some of which are now available through the Internet (e.g. BloomAuction.com in Canada). Electronics manufacturers and distributors have also increasingly rely upon auction sites such as eBay.com to liquidate their products.

Auctions are also increasingly used in the procurement side of a supply chain. For example, Covisint and Fast Buyer, business-to-business solution and product providers for the automobile industry founded by OEM's, provide online auction services. These auctions can be originated not only by a seller (in a forward auction), but also by a buyer (in a reverse auction). In a reverse auction, a buyer sets up an auction, specifies an auction type, and notifies qualified suppliers to submit their bids. Another application of auctions in procurement is the U. S. Government using reverse auctions to award contracts among competing bidders.

A growing number of papers study reverse auctions: Holt (1990); Dasgupta and Spulber (1990); Carey (1993); Gallien and Wein (2001); Teich et al. (2001); Jin and Wu (2002); Chen (2001); Compte and Jehiel (2002); Schummer and Vohra (2003); and Seshadri and Zemel (2001). Most of them are applications of auctions in the supply chain context. Few papers articulate the difference between the forward auction and the reverse auction. Single-unit auction theory is predominantly comprised of literature on the forward auction, and asserts that the reverse auction has equivalent properties. In a reverse auction, a bidder is a seller and prefers any price greater than his cost to losing his bid; whereas in a forward auction, a bidder is a buyer, and wants to pay less than his value of the object. The range of acceptable prices to a bidder is unbounded above in a reverse auction, yet bounded in a forward auction.

This difference is due to the existence of an implicit reserve price of zero in a forward auction, since both values and bids have to be nonnegative. In a reverse auction, there is no fixed natural reserve price, and bids can be arbitrarily high. The buyer is contractually bound to pay the price determined by the auction mechanism as well as the set of bids.

This dissimilarity results in significant supply chain implications, particularly in the design of sealed-bid reverse auctions. In reverse (forward) auctions, the seller (buyer) with the lowest (highest) bid wins the object. There are two common methods of determining the price of the object. In the first-price reverse (forward) auction, the price is the same as the winning bid; in the second-price reverse (forward) auction, the price is given by the lowest (highest) losing bid.

The buyer in a reverse auction often has the power to design the auction mechanism. Suppose that the U. S. government sets up an auction to procure a certain service. It is plausible to assume that each potential service provider has an independent and identically distributed cost. Then, is the expected cost to the risk-neutral government irrelevant to the type of sealed-bid reverse auction chosen? Suppose now that an automobile plant wants to buy a new press. Which type of auction should be used to attain the lowest expected cost? These are some of the questions addressed in this paper.

This paper makes the following strategic implications for the buyer who wants to procure using an auction. First, when there is no reserve price, the buyer should prefer the second-price auction over the first-price auction because the second-price auction admits only one symmetric equilibrium for bidding strategies. Thus, in the second-price auction, the behavior of bidders is more predictable and stable. Furthermore, the (unique) expected payment by the buyer in the second-price auction is less than or equal to any of the multiple equilibria for the first-price auction. Second, if a first-price reverse auction is used, the buyer should set a reserve price. The existence of the reserve price eliminates the multiplicity of bidding strategies, and makes forward and reverse auctions mathematically equivalent.

In this paper, we study single-unit single-period sealed-bid reverse auctions in which bidders are symmetric and have independently and identically distributed private costs. In particular, we examine first-price and second-price reverse auctions. For a first-price reverse auction, we derive symmetric bidding strategies by the sellers and the buyer's expected payment, and examine the impact of the seller's reserve price. The first-price reverse auction is compared with the second-price reverse auction.

The main contribution of this paper is to articulate how designing a first-price auction for procurement is different from designing it for distribution. In first-price forward auctions with private values, Milgrom and Weber (1982), Maskin and Riley (1984) and Lebrun (1999) show that there is a unique symmetric equilibrium. A number of papers address the uniqueness and multiplicity of bidding strategies with asymmetric strategies (Maskin and Riley (1996)), asymmetric bidders (Maskin and Riley (2000a); Lebrun (1998); Lebrun (1999)), affiliated signals (Maskin and Riley (2000b); Rodriguez (2000)), and common values (Bikchandani and Riley (1991); Milgrom and Weber (1982)).

The much-celebrated Revenue Equivalence Theorem due to Vickrey (1961) and its generalizations due to Myerson (1981) and Riley and Samuelson (1981) imply that the expected revenue to the seller is the same in the first-price and second-price forward auctions. Based on current literature, the buyer may be tempted to draw an analogous result in setting up a reverse auction. Klemperer (2002) warns that poorly understood economic theory may find inappropriate applications and yield unexpected results. We show that in the reverse first-price auction, in general, there are multiple symmetric Nash equilibria for bidding strategies. Each of these bidding strategies corresponds to a distinct expected payment by the buyer. The first-price reverse auction bidding strategy, corresponding to the lowest payment by the buyer, has the same expected payment as the (unique) bidding strategy of the second-price reverse auction. In the second-price reverse auction, by comparison, a simple extension of Vickrey (1961) shows an analogous result in the first-price auction that bidding one's own cost is a dominant strategy of every seller.

The second contribution is to study the impact of the buyer's reserve price in eliminating the multiplicity of bidding strategies and associated risk of very high costs, as well as maximizing the buyer's cost. Such benefits are consistent with a recent trend in the automobile industry. More buyers are setting reserve prices when they originate auctions, following Covisint's recommendation (personal communication).

In Section 2, we develop theoretical properties of the first-price reverse auction, including bidding strategies and expected buyer's payment. These findings are illustrated using exponential and uniform cost distributions in Section 3. Section 4 discusses why the multiplicity of bidding strategies in the first-price reverse auction occurs.

2 Reverse Auction

2.1 Model

In a single-unit reverse auction, the buyer wants to procure an indivisible object from one of $N \geq 2$ sellers. We use a symmetric independent private cost model. Each seller $i = 1, \cdots, N$ knows his private cost $c_i \geq 0$ of production. All bidders are symmetric. The unknown production costs are independent and identically distributed as a p.d.f. f and c.d.f. F, and have a continuous support J. Both the buyer and sellers are risk-neutral. This cost model is analogous to the standard private values model in the forward auction (e.g., Wolfstetter (1999) and Krishna (2002)). We assume that the unknown production cost has a finite mean. Each seller i submits a nonnegative bid b_i, and the buyer procures the object from the seller with the lowest bid. The price is determined by the lowest bid in the first-price reverse auction, and by the second-lowest bid in the second-price reverse auction.

2.2 Bidding Strategy

A symmetric bidding strategy β maps a bidder's cost c_i to a corresponding bid b_i, i.e., $b_i = \beta(c_i)$. It is well known that in the second-price reverse auction, bidding one's cost is a dominant strategy. We want to find a symmetric, increasing and differentiable Nash equilibrium bidding strategy for the first-price reverse auction.

Denote the infimum and the supremum of the support J of the cost distribution by $\underline{c}$ and $\overline{c}$, allowing the possibility of $\overline{c}$ being ∞. It follows from the nonnegativity of production costs, $0 \leq \underline{c} \leq \overline{c}$. We say J is a *right-open* interval if $J = [\underline{c}, \overline{c})$ or $J = (\underline{c}, \overline{c})$.

Proposition 2.1. *Suppose J is right-open. The symmetric, increasing and differentiable Nash equilibrium bidding strategies for the first-price reverse auction are characterized by*

$$\beta(c) = c + (1 - F(c))^{1-N}\{C - \int_{\underline{c}}^{c} (1 - F(u))^{N-1} du\} \tag{1}$$

where

$$\int_{\underline{c}}^{\overline{c}} (1 - F(u))^{N-1} du \leq C. \tag{2}$$

We remark that the left hand side of (2) is finite since the *ex ante* expected value $\int_{\underline{c}}^{\overline{c}}(1 - F(u))du$ of the production cost is finite and $\int_{\underline{c}}^{\overline{c}}(1 - F(u))^{N-1}du \leq \int_{\underline{c}}^{\overline{c}}(1 - F(u))du$.

Proof. Using a standard approach (e.g., Wolfstetter (1999)), Appendix A.1 shows that (1) subject to (2) is a necessary first-order condition. We show sufficiency.

From (1),

$$\beta'(z) = \frac{(N-1)f(z)}{(1 - F(z))^N}\{C - \int_{\underline{c}}^{c} (1 - F(u))^{N-1} du\} = \frac{(N-1)f(z)}{1 - F(z)}(\beta(z) - z).$$

From (1) and (2), we observe that β is a strictly increasing function satisfying $\beta(c) \geq c$. Let $\Pi(z, c)$ be the expected profit of a seller when his production cost is c and he bids $\beta(z)$, while all the other sellers follow β. It remains to verify that $\Pi(z, c)$ is maximized at $z = c$.

The expected profit $\Pi(z, c) = (1 - F(z))^{N-1}(\beta(z) - c)$ is the product of the probability of winning and the profit when the bidder wins. Thus,

$$\begin{aligned} \frac{\partial}{\partial z}\Pi(z, c) &= -(N-1)(1 - F(z))^{N-2} f(z)(\beta(z) - c) + (1 - F(z))^{N-1}\beta'(z) \\ &= (N-1)(1 - F(z))^{N-2} f(z)(c - z). \end{aligned}$$

It follows that $\Pi(z, c)$ is quasi-concave in z, and the maximum is attained at $z = c$. □

It is clear that higher bids lead to a higher payment by the buyer, and thus the bidding strategy with the minimal C is preferred by the buyer. The minimal C is obtained by replacing the inequality in (2) with equality. It can be shown that the symmetric bidding strategy corresponding to this minimal C in the first-price reverse auction yields the lowest possible expected payment by the buyer, which equals the expected payment in the second-price reverse auction. Also note that with this minimal C, $\beta(c) \to \overline{c}$ as $c \to \overline{c}$, whereas $\beta(c) \to \infty$ with all larger values of C.

2.3 Buyer's Reserve Price in the First-Price Reverse Auction

This section shows that in the first-price reverse auction, the buyer's reserve price eliminates the multiplicity of symmetric equilibrium bidding strategies. Suppose the buyer sets a reserve price R beyond which sellers are not allowed to bid, With $\overline{c} \leq R < \infty$, there is a unique equilibrium that corresponds to the equilibrium with the minimal C, and that with $R < \overline{c}$, there is a positive probability of no purchase being made. We assume that R is in the support of sellers' costs.

We denote by $\beta_C(\cdot)$ the bidding strategy given by (1) and scalar C. An analysis similar to the previous section shows that any symmetric bidding strategy in the first-price reverse auction must satisfy (1) where C satisfies the following two conditions. The first condition is $\beta_C(R) = R$. The second condition is (2) where the integral is taken in the support of costs no more than R; i.e. $\beta_C(c) \geq c$ for all $c \in [\underline{c}, R]$. Therefore, the choice of R determines the unique symmetric bidding strategy by specifying the constant $C = C(R)$ of integration in (1). It follows

$$C(R) = \int_{\underline{c}}^{R} (1 - F(u))^{N-1} du,$$

and

$$\beta_{C(R)}(c) = c + (1 - F(c))^{1-N} \int_{c}^{R} (1 - F(u))^{N-1} du. \tag{3}$$

It follows from the Revenue Equivalence Theorem that with the reserve price R, the bid-your-cost strategy of the second-price reverse auction and the bidding strategy $\beta_{C(R)}(\cdot)$ of the first-price reverse auction yield the same expected payment by the buyer.

3 Examples

3.1 Exponential Distribution of Costs

This section uses an exponential distribution of costs to illustrate a case where the supports for the seller's cost distribution and the buyer's payment are not bounded. We suppose the cost c_i of each seller i is distributed as an independent and identical exponential distribution with a rate parameter $\lambda > 0$, i.e. $f(x) = \lambda e^{-\lambda x} 1_{[x \geq 0]}$. Then, the support of the cost distribution is unbounded. We note that in the first-price forward auction, the exponential value distribution is considered by Krishna (2002).

Proposition 3.1. *In the first-price auction, if costs are distributed as an exponential distribution with rate parameter λ, then the increasing, symmetric and differentiable bidding strategy is given by*

$$\beta(c) = c + \frac{1}{\lambda(N-1)} + C_1 e^{\lambda(N-1)c}, \tag{4}$$

where $C_1 \geq 0$. Furthermore, the corresponding expected price is $\frac{1}{\lambda N} + \frac{1}{\lambda(N-1)} + \frac{C_1 N}{\lambda}$, which is at least the expected price of the unique bidding strategy of the second-price auction.

Proof. From $\int_0^c e^{-\lambda(N-1)u} du = (1 - e^{-\lambda(N-1)c})/(\lambda(N-1))$, we get

$$\beta(c) = c - e^{\lambda(N-1)c} \int_0^c e^{-\lambda(N-1)u} du + C_o e^{\lambda(N-1)c} = c + \frac{1}{\lambda(N-1)} + C_1 e^{\lambda(N-1)c}$$

for some scalar C_o and $C_1 = C_o - \frac{1}{\lambda(N-1)}$. Condition (2) implies $C_o \geq \int_0^\infty e^{-A(u)} du = \frac{1}{\lambda(N-1)}$, or equivalently, $C_1 \geq 0$.

For $C_1 \geq 0$, the bidding strategy $\beta(\cdot)$ is strictly increasing. The minimum of N exponential distributions with rate parameter λ is distributed as an exponential distribution with rate parameter λN. Denote the density of this distribution by $f^{(1)}(c) = \lambda N e^{-\lambda N c}$. The expected payment by the buyer is

$$\int_0^\infty \beta(c) f^{(1)}(c) dc = \frac{1}{\lambda N} + \frac{1}{\lambda(N-1)} + C_1 N \int_0^\infty \lambda e^{-\lambda c} dc = \frac{1}{\lambda N} + \frac{1}{\lambda(N-1)} + C_1 N.$$

We compare this to the expected payment in the second-price auction. The expected cost of the minimum of N exponential distributions with rate parameter is λN, and by the memoryless property, the expected cost of the gap between the first and the second order statistic is $\frac{1}{\lambda(N-1)}$. Thus the expected price in the second-price auction is the expectation of the second order statistics, which is $\frac{1}{\lambda N} + \frac{1}{\lambda(N-1)}$. □

Figure 1 illustrates multiple symmetric bidding strategies.

Now, suppose the buyer sets a reserve price $R > 0$. A seller does not participate in the auction if his cost is greater than R. Setting $\beta(R) = R$ specifies $C_1 = -\frac{\exp(-\lambda(N-1)R)}{\lambda(N-1)}$ in (4). If L is the shortfall penalty cost for failing procurement, the buyer minimizes expected cost if and only if R satisfies

$$\lambda R + e^{\lambda R} = \lambda L + 1. \tag{5}$$

See Appendix A.2 for derivation. For strictly positive shortfall penalty L, (5) implies $R < L$: the optimal reserve price for the buyer is strictly less than his shortfall penalty cost. This finding is consistent with the result that the optimal reserve price result in the forward auction should be at least the auctioneer's reserve price. (e.g. Myerson (1981); McAfee and McMillan (1987)).

3.2 Uniform Distribution of Costs

This section illustrates the case when the seller's cost has a uniform distribution. Suppose the cost c_i of each bidder i is distributed as a uniform $[0, 1)$ distribution, i.e. $f(c) = 1_{[0 \le c < 1]}$. The support of cost distributions is bounded, but not compact since the support is right-open. This model is very similar to the classical uniform $[0, 1]$ model.

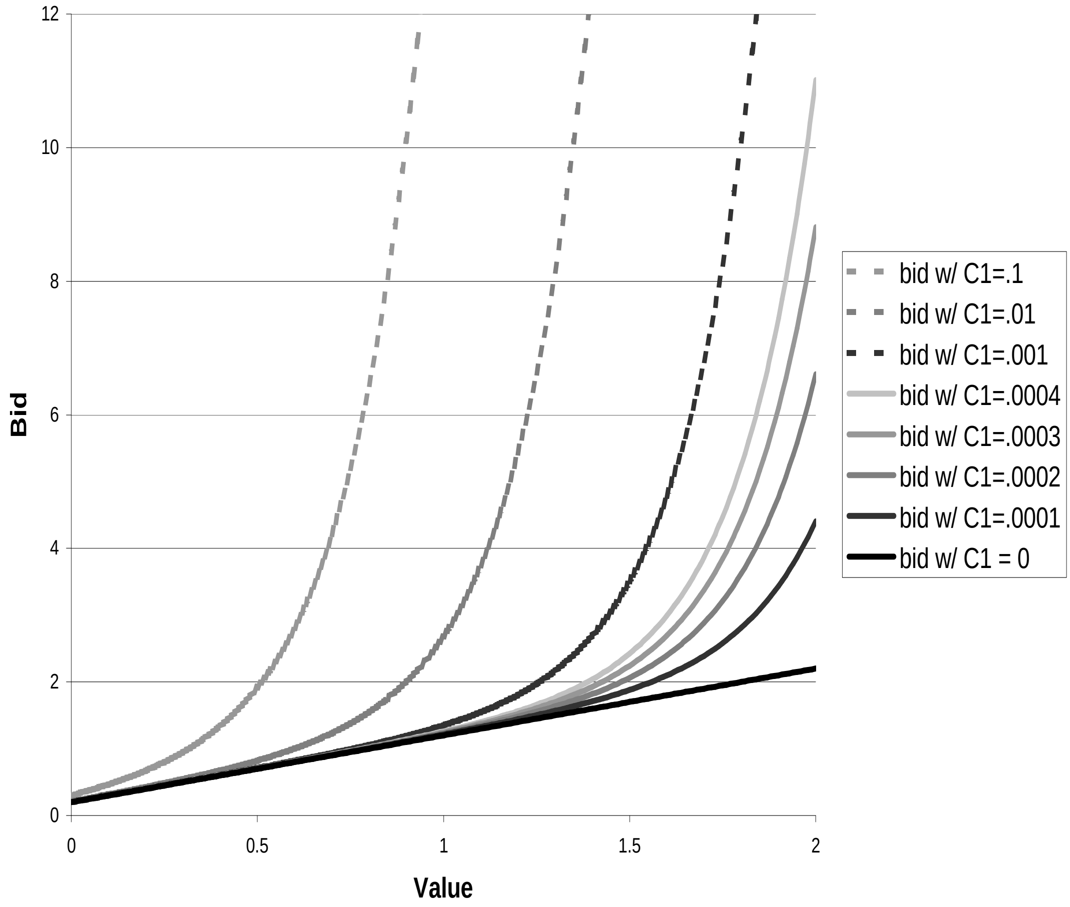

Figure 1: Multiple Symmetric Bidding Functions. There are 6 sellers bidding for sale. Costs are distributed as an i.i.d exponential distribution with rate parameter 1. See (4).

Proposition 3.2. *Suppose the uniform* $[0,1)$ *distribution of costs. Increasing, symmetric and differentiable bidding strategies are given by*

$$\beta(c) = \frac{1+(N-1)c}{N} + C_1(1-c)^{1-N},$$

for $C_1 \geq 0$. *The corresponding expected price is* $2/(N+1) + C_1 N$, *which is at least the expected price of the unique bidding strategy in the second price auction.*

Proof. Let $c \in ([0,1)$. Since $\int_0^c (1-u)^{(N-1)} du = -\frac{1}{N}(1-c)^N + \frac{1}{N}$, (1) implies

$$\beta(c) = c - \frac{\int_0^c (1-u)^{(N-1)} du}{(1-c)^{N-1}} + C_o(1-c)^{1-N} = \frac{(N-1)c}{N} + \frac{1}{N} + C_1(1-c)^{1-N}$$

for some scalar C_o and $C_1 = C_o - 1/N$. From $\int_0^1 (1-F(u))^{N-1} du = 1/N$, the condition (2) becomes $C_1 \geq 0$.

Letting $f^{(1)}(c) = N(1-c)^{N-1}$, the expected payment made by the buyer is

$$\int_0^1 \beta(c) f^{(1)}(c) dc \;=\; \frac{1}{N} + \frac{N-1}{N}(\frac{1}{N+1}) + C_1 N = \frac{2}{N+1} + C_1 N.$$

We recall that the expected payment of the second price auction, in which bidding at cost is the unique symmetric equilibrium, is the expected cost of the second order statistic. Assuming the uniform [0,1) distribution of costs, the expected payment is $2/(N+1)$, the expected cost of the second order statistics of N uniform [0,1) distributions. □

If there is a reserve price $R \in [1,\infty)$, then the symmetric Nash-equilibrium corresponding to $C_1 = 0$ is unique. The bidding strategy and the expected price do not depend on the exact value of the reserve price as long as it is at least 1. Therefore, the existence of an arbitrarily high reserve cost eliminates the multiplicity of bidding strategies. If $R \in (0,1)$, then it follows from $\beta(R) = R$ that $C_1 = -\frac{1}{N}(1-R)^N$ determines the unique symmetric bidding strategy.

4 Discussion

What causes the multiplicity of the symmetric equilibrium bidding strategy in the first-price reverse auction? The answers can be found in the non-compactness of the support of the cost distribution and the unboundedness of bids. Without either of these conditions, the multiplicity of the bidding strategy disappears.

Non-compactness of the support of the cost distribution: In economics literature, the distribution of incomplete information typically has a compact support. As a result, in auction theory, the distribution of bidders' values (costs) in a forward auction (reverse auction) usually has a support that is closed and bounded (e.g. Vickrey (1961); Riley and Samuelson (1981); Myerson (1981); Laffont and Tirole (1993)). Yet, there is no reason why the support of the cost distribution should be a closed interval. First, the support may be an open interval or a right-open interval. For instance, there is no convincing argument articulating why the classical uniform $[0,1]$ distribution is superior to the uniform $[0,1)$ model in Section 3.2. Furthermore, even in the case of a closed interval, say $[0,1]$, one can argue that it suffices to define the bidding strategy on a subset of probability of measure 1, such as $[0,1)$ or $(0,1)$. Second, the distribution of sellers' costs may have a support that is not bounded from above as in Section 3.1. Continuous distributions with unbounded support (such as exponential, gamma, normal, log-normal and log-logistic) have long been used to model behaviors in finance, marketing and operations management. Yet, auction literature has almost always assumed an upper bound for distributions of costs. (In forward auctions, this assumption translates to the existence of a lower bound on buyers' values, which is reasonable for the sale of an object. Krishna (2002) has an example with the exponential distribution of values in the first-price forward auction.) It is the first paper, to our knowledge, that introduces such generalizations to auction theory.

Unboundedness of bids: A bidding strategy is a mapping from the support of costs (or values) to a nonnegative real number. In the forward auction, the nonnegativity of bids provides an implicit but concrete reserve price of 0, and the uniqueness of bidding strategy follows. The derivation of the first-price forward auction bidding strategy in Wolfstetter (1999) and Lebrun (1999) can be modified to show the multiplicity of the bidding strategy if bidders were allowed to submit negative bids.

In the reverse auction, however, there is no natural corresponding upper bound on bids, and the sellers' bids may have an unbounded support. In the case that the support of costs is not bounded above (as in the exponential cost distribution in Section 3.1), neither the support of bids nor the support of the payment by the bidder is bounded above. One may argue that it is not a reasonable model since there is a finite amount of money in the world. This has merit if one adopts a purely mathematical point of view grounded in auction theory. However, we believe that considering an unbounded support might be useful for the following reasons: First, the value of the object is often very small compared to the amount of money the buyer possesses. In this situation the seller might perceive that the buyer has an essentially infinite amount of money and will take action according to this perception. Second, distributions with an unbounded support such as an exponential distribution are commonly used in finance and operations literature. Third, in real auctions, we observe very large bids, which cannot otherwise be adequately explained. The following report in *The Wall Street Journal* (August 4, 2000, p.A1) as quoted by Schummer and Vohra (2003) describes the speculation of bidders in the multi-unit auctions:

> What PECO and PPL did was offer much of their output at low prices so that the majority of their plants would be called into service. But knowing demand was so high, they offered power from their tiniest plants at vastly higher bids, in a way that often set the peak price for a number of hours. Consumers that day ended up paying millions of extra dollars for power.

It is consistent with the experimental results of Thomas et al. (2000), which finds that high price spikes are frequent and the supply curve looks like a hockey stick. A bidder's strategy of setting vastly high bids works if his competing bidders follow the same strategy. The tradeoff between the probability of winning and the price also exists in the first-price reverse auction.

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A Appendix

A.1 The Necessity of (1) in Proposition 2.1

We study the bidding behavior of the seller i as a function of his cost c_i given the distribution of others' bids. The seller i's problem is to choose his bid b_i as to maximize expected profit, which is the product of the probability that he wins the bid and the conditional expected price minus the cost of production given that he has won the unit. Alternatively, since the probability p_i that seller i submits the winning bid is a nondecreasing function of his bid b_i, it is convenient to consider bid $b_i(p_i)$ as a function of probability p_i. (We define $b_i(p_i)$ the smallest bid such that the probability that i wins the bid is at least p_i.) The expected profit associated with seller i's choice of p_i is $e_i(p_i) - c_i p_i$ where $e_i(p_i)$ is the expected payment to i. (In the first-price reverse auction, $e_i(p_i) = b_i(p_i)p_i$.) Let $p_i^*(c_i)$ maximize the expected profit function $e_i(p_i) - c_i p_i$.

It can be shown that e_i is differentiable in the interior of J, and consequently, the first-order condition of the expected profit yields

$$e_i'(p_i^*(c_i)) = c_i. \tag{6}$$

We obtain, by the Fundamental Theorem of Calculus,

$$e_i(p_i^*(c_i)) = \lim_{c \downarrow \underline{c}} e_i(p_i^*(c)) + \int_{\underline{c}}^{c_i} u \, dp_i^*(u). \tag{7}$$

which is a mathematical statement of the Revenue Equivalence Theorem. Now, (7) depends on the auction mechanism used only through p_i^* and $\lim_{c \downarrow \underline{c}} e_i(p_i^*(c))$. A reverse auction is *efficient* if the seller with the lowest cost wins the bid. All efficient reverse auctions have a common p_i^*. Thus, all efficient symmetric bidding strategies with the same expected payment $\lim_{c \downarrow \underline{c}} e_i(p_i^*(c))$ given that seller i's cost is $\underline{c}$, have the same expected payment by the buyer.

In the second-price reverse auction, bidding one's cost is the dominant strategy. Thus, $\lim_{c \downarrow \underline{c}} e_i(p_i^*(c))$ equals the second lowest cost of sellers given that seller i's cost is at the lowest possible value $\underline{c}$. This value of $\lim_{c \downarrow \underline{c}} e_i(p_i^*(c))$ specifies the unique solution to (7).

In the first-price reverse auction, we have $e_i(p_i) = b_i(p_i)p_i$, and $e_i'(p_i) = b_i(p_i) + b_i'(p_i)p_i$. It follows from (6),

$$\frac{c_i - b_i(p_i^*(c_i))}{p_i^*(c_i)} = \frac{d}{dp_i^*(c_i)} b_i(p_i^*(c_i)) = \frac{\frac{d}{dc_i} b_i(p_i^*(c_i))}{\frac{d}{dc_i} p_i^*(c_i)}.$$

Let $\beta(c_i) = b_i(p_i^*(c_i))$ where β is a symmetric, increasing and differentiable bidding strategy. The probability of winning becomes $p_i(\beta(c_i))$. Thus,

$$\beta(c_i) \;=\; c_i - p_i(\beta(c_i)) \cdot \frac{\frac{d}{dc_i}\beta(c_i)}{\frac{d}{dc_i} p_i(\beta(c_i))}. \tag{8}$$

As cost c_i increases, the probability $p_i(\beta(c_i))$ of winning weakly decreases. Thus, bid-inflation occurs; i.e., $\beta(c_i) \geq c_i$. It makes sense since no seller would want to incur a loss by submitting a bid below his production cost.

From the strict monotonicity and symmetry of β, the probability that a bidder wins the unit is the probability that his cost is lower than the cost of any other bidder. Thus, the probability $p(c_i) = p_i(b_i(c_i))$ of winning is a function of cost c_i only, and this function $p(\cdot)$ is common for all bidders. Equation (8) becomes

$$\beta(c_i) \quad = \quad c_i - p(c_i) \cdot \frac{\frac{d}{dc_i}\beta(c_i)}{\frac{d}{dc_i}p(c_i)}.$$

Since there are $N-1$ sellers other than i, we have $p(c_i) = (1-F(c))^{N-1}$ and $\frac{d}{dc_i}p(c_i) = -(N-1)f(c)(1-F(c))^{N-2}$, which imply

$$\beta(c) = c + \frac{\beta'(c)}{(N-1)} \cdot \frac{1-F(c)}{f(c)}. \tag{9}$$

We now solve this differential equation. Because $\underline{c}$ is the infimum of the support of costs, the solution to the above differential equation (9) is given by

$$\beta(c) = c - e^{A(c)} \int_{\underline{c}}^{c} e^{-A(u)} du + Ce^{A(c)}$$

where C is some scalar, and

$$A(c) = \int_{\underline{c}}^{c} (N-1) \frac{f(u)}{1-F(u)} du = (1-N)\log(1-F(c)).$$

Thus, we obtain (1). This is a necessary condition for a symmetric bidding strategy. Since $A(\cdot)$ is strictly increasing in cost c within the support of costs, the second term in (1) is strictly decreasing.

There is no incentive for bidders to win the auction at the price below their productions costs. It follows $\beta(c) \geq c$ for all possible c, which implies from (1) that C has to be big enough to ensure $\int_{\underline{c}}^{c} e^{-A(u)} du \leq C$ for all c in the support of costs. This condition is satisfied if and only if (2) holds. If the cost distribution has a bounded support, then the integral is proper, and taken over the entire support. It can be shown based on the finite mean of the cost distribution that the improper integral in the above expression is finite.

We remark the second factor $f(u)/(1-F(u))$ of the integrand of $A(c)$ is the hazard rate of the cost distribution.

A.2 Optimal Reserve Price: Exponential Distribution Case

A seller does not participate in the auction if his cost is greater than R. Setting $\beta(R) = R$ implies $C_1 = -\frac{\exp(-\lambda(N-1)R)}{\lambda(N-1)}$ in (4). The probability that the buyer will fail to purchase any unit is $(1-F(R))^N = e^{-\lambda NR}$. The expected total cost by the buyer is

$$\begin{aligned}
&\int_0^R \beta(c) f^{(N)}(c)dc + L(1-F(R))^N \\
= &\int_0^R c f^{(N)}(c)dc + \frac{1-e^{-\lambda NR}}{\lambda(N-1)} + C_1 \int_0^R e^{\lambda(N-1)c} f^{(N)}(c)dc + Le^{-\lambda NR} \\
= &\int_0^R \lambda N c e^{-\lambda Nc} dc + \frac{1-e^{-\lambda NR}}{\lambda(N-1)} + C_1 N \int_0^R \lambda e^{-\lambda c} dc + Le^{-\lambda NR}.
\end{aligned}$$

Since $\int_0^R \lambda N c e^{-\lambda Nc} dc = [-ce^{-\lambda Nc} - \frac{e^{-\lambda Nc}}{\lambda N}]_{c=0}^R = \frac{1}{\lambda N}(1-e^{-\lambda NR}) - Re^{-\lambda NR}$, the above expression becomes

$$\frac{1}{\lambda N}(1-e^{-\lambda NR}) - Re^{-\lambda NR} + \frac{1-e^{-\lambda NR}}{\lambda(N-1)} - \frac{e^{-\lambda(N-1)R}}{\lambda(N-1)} N(1-e^{-\lambda R}) + Le^{-\lambda NR}.$$

We want to find the optimal reserve price R for the buyer. Differentiation of the above expected total cost yields

$$Ne^{-\lambda(N-1)R} \cdot [\lambda R + e^{\lambda R} - \lambda L - 1].$$

Since $e^{-\lambda(N-1)R}$ is strictly positive, and $\lambda R + e^{\lambda R}$ is strictly increasing in R, the expected payment is quasi-convex in R. The optimal choice of R satisfies (5). With respect to R, the right-side is a constant no less than 1, and the left-side increases strictly from 1 to infinity as R goes from 0 to infinity. Thus, there must be a unique optimal R satisfying (5). Equation (5) is equivalent to $\lambda(L-R) = e^{\lambda R} - 1$, which is strictly positive for all $R > 0$.