Efficient Auction Mechanisms for Supply Chain Procurement

Rachel R. Chen*, Ganesh Janakiraman**
Robin Roundy**, and Rachel Q. Zhang*

* Johnson Graduate School of Management, Cornell University,
Ithaca, NY 14853
** Department of Operations Research and Industrial Engineering,
Cornell University, Ithaca, NY 14853

Abstract

We consider multi-unit Vickrey auctions for procurement in supply chain settings. This is the first paper that incorporates transportation costs into auctions in a complex supply network. We introduce three incentive compatible auction mechanisms. Two of them make simultaneous production and transportation decisions so that the supply chain is cost-efficient for the quantities awarded to the buyer, and the third determines the production quantities before the shipments. We show that considerable supply chain cost savings can be achieved if production and transportation costs are considered simultaneously. However, under the typical regular-overtime production cost structure, the buyer's payments in efficient auctions can be high so that the buyer may prefer inefficient auction protocols. We develop an efficient auction that can control the size of the buyer's payments at the expense of introducing uncertainty in the quantity acquired in the auction.

1 Introduction

Research and practice in operations management has emphasized integrated supply chains, where long-term relationships between suppliers and buyers of manufactured components facilitate optimization of total supply chain costs. There is a recent emphasis on using auctions in the supply chain as an effective and efficient means of achieving lower acquisition costs, lower barriers for new suppliers to enter a market, and consequently better market efficiency. Exploiting recent advances in information technology, such auctions can be carried out through the Internet, referred to as online auctions. Online auctions allow geographically diverse buyers and sellers to exchange goods, services, and information, and to dynamically determine prices that reflect the demand and supply at a certain point of time so that efficient matches of supply and demand can be realized. As pointed out by Lucking-Reiley (2000), online auctions often lead to lower information, transaction and participation costs, as well as increased convenience for both sellers and buyers, the ability for asynchronous bidding, and easier access to larger markets.

Most auctions are price-driven. However, there are other costs associated with integrating a supply chain, e.g., costs associated with transportation, capacity management, inventories, etc. Recently there have been escalating disputes in the electronics industry on fair pricing in supply chains. According to industry executives, OEMs and suppliers are ignoring the numerous additional costs that affect parts prices, such as shipping, logistics, and value-added services when they quote prices. They are passing such costs onto their manufacturing partners and distributors, squeezing profitability out of the channel (*Electronic Business News*, 11/19/2001). According to Mike Dennison, senior director of Global Procurement and Strategic Supply Chain Management at Flextronics International, "Suppliers must understand all the complexities of getting material from their dock to the OEM's and need to factor in the cost when they are quoting a price". This issue relates to more than a general understanding of terms and conditions, according to Andy Fischer, vice president and channel director for Avnet Electronics Marketing at Avnet Inc., "Continuity (of a supply chain) ... is not only channel related, it's also global. Service packages, demand curves, global locations all lead to different costs and requirements." As Chopra and Van Mieghem (2000) point out, accounting for these additional costs may be a key issue in B2B auctions. Yet, there is a lack of decision analysis that combines sophisticated OR/MS tools with economic insights to help supply chain practitioners in auctions and e-marketplaces (Keskinocak and Tayur 2001).

The goals of this paper are to design multi-unit auctions that achieve overall supply chain efficiency while taking into account production costs *and* transportation costs that have been ignored in most of the auction literature, and to illustrate some of the risks and tradeoffs associated with these auctions. For example, in manufacturing systems per-unit costs tend to be relatively flat if the regular-time capacity of the manufacturing system is not exceeded, and to grow rapidly when overtime or other measures are required to increase capacity in the short term. In an efficient auction, this property of manufacturing cost functions can lead to substantial payments to suppliers. We will see that the buyer may prefer an auction that is clearly inefficient for the supply chain, simply because it reduces the size of these payments. The buyer can control the size of these payments via a reservation price, at the expense of new risks associated with the quantity purchased in the auction.

The paper is organized as follows. We discuss some of the relevant literature in Section 2, and introduce our models in Section 3. In Sections 4 and 5 we discuss the analytical properties of our auction mechanisms. In Sections 5.3 and 6 we discuss payment reduction at the cost of uncertainty in the quantity purchased in the auction, and payment-induced distortions of buyer behavior, respectively. The paper concludes in Section 7.

2 Literature Review

Incorporating transportation costs into production and shipment decisions has been a rich research area in operations research. As an example, Sharp *et al.* (1970) consider a manufacturer who owns multiple production plants with non-linear production costs and provides products to multiple demand locations. The demands at different locations are known and the objective is to minimize the total production and transportation costs. When there is more than one production facility *and* production costs are non-linear, the common approach, which combines production and transportation costs on each arc of the supply network and makes production and transportation decisions based on a single cost function, does not apply. Recently, Holmberg and Tuy (1999) extend the classic transportation problem by allowing fixed costs of production at the supply nodes, and stochastic demand and convex non-linear penalty costs for being short at the demand centers. They reduce the problem to a simpler optimization problem and solve it using a branch and bound method. Both papers provide lists of references dealing with similar optimization problems. However, none of them considers pricing decisions using auctions.

Bidding behavior in auctions has been studied since 1950's using game theory and decision theory (Rothkopf 1969). We refer the reader to Vickrey (1961), Milgrom and Weber (1982), Klemperer (1999), McAfee and McMillan (1987), and Rothkopf and Harstad (1994) for classical auction theory. In a typical auction, there is a single buyer who wants to purchase a single unit or a number of units of a product. There are several potential suppliers (or sellers) in the market; each has her own cost structure. (Since the analysis for the case with a single seller and multiple buyers is the same, we will focus on the case with a single buyer and multiple sellers). To participate in the auction, each supplier is required to submit a bid to an auctioneer (also called a market intermediary or an agent). Based on these bids, the quantity to purchase from, and the payment to each supplier, are determined by the auctioneer through pre-specified rules known as a mechanism. A mechanism is incentive compatible (or induces truth-telling) if each supplier's dominant strategy is to submit her true production cost as her bid. Incentive compatibility is important because it is usually a requirement for an auction to be efficient. An efficient mechanism is one that guarantees an allocation of production quantities that minimizes the total system costs.

There are three dominant types of mechanisms for multi-unit auctions, known as Pay-as-You-Bid, uniform-price, and VCG auctions (e.g. second-price auctions in the single-unit setting, also known as Vickrey auctions) (Klemperer 1999). The Pay-as-you-Bid auction is self-explanatory. In a uniform auction a uniform price is paid for each unit purchased. The price can be either the first rejected or the last accepted bid. In economics literature it is well known that neither Pay-as-You-Bid nor uniform-price auctions is incentive compatible or efficient, whereas the family of mechanisms known as VCG auctions, attributed to Vickrey (1961), Clarke (1971), and Groves (1973), are both incentive compatible and efficient. In a VCG auction, the buyer's payment to a supplier is based not only on the bids submitted, but also on the contribution that the supplier makes to the system by participating in the auction. This payment structure motivates suppliers to improve their operational efficiency and lower their production costs, increasing the contribution they make to the system. The reader is referred to Nisan and Ronen (2001) for a general definition of the VCG family of auction mechanisms. In spite of the attractive properties of VCG auctions, they are not widely used in practice, largely due to the fact that human auctioneers may take advantage of the truth telling bidders and do not always act truthfully (Rothkopf *et al.* 1990).

Recently, we have seen research on auctions from OM's perspective. Beil and Wein (2001) consider a manufacturer who uses a reverse auction to award a contract to suppliers based on both prices and a set of non-price attributes (e.g., quality, lead time). Pinker *et al.* (2001) design sequential, multi-unit online B2C auctions to allocate a fixed amount of inventory over certain number of periods. Eso (2001) develops an iterative sealed bid auction (where bidders can not see others' bids) for selling excess capacity for an airline company. Vulcano *et al.* (2001) consider variants of the second-price auction (i.e., the single-unit Vickrey auction) in a multi-period revenue management problem where each individual bidder can be awarded at most one unit. Jin and Wu (2001) show that auctions can serve as a coordination mechanism for supply chains.

It is noted that most research on auctions limits its effort to either single-unit auctions or multiple-unit auctions under which each bidder wants at most one unit (Keskinocak and Tayur 2001). In this paper, we allow each supplier to be awarded multiple units. The main contribution of this paper to the OR/MS literature is the integration of production and transportation costs in multi-unit auctions that involves multiple supplier locations and buyer locations. Since the auction mechanisms that we propose fall into the VCG family, they are incentive compatible and produce efficient solutions for the supply chain when production and transportation decisions are made simultaneously.

As we mentioned above, VCG auctions are rarely used because of possible cheating by the auctioneer. In an online auction, however, the auctioneer can be a web site (virtual computer agent) which receives bids, and decides awards and payments based on some pre-specified algorithm so that this problem can be minimized or eliminated. Furthermore, the internet may make auctions more secure as bidders may come from all over the world and the algorithm can be designed not to reveal buyers' and suppliers' private information. Rapid advances in computational capability make it possible to conduct sophisticated allocation rules quickly and at low cost. With the recent advances in information technology, VCG auctions have the potential to become an important mechanism for online auctions. This is the first paper that uses the VCG auction mechanism in a complex supply chain setting that achieves global optimization of both production and transportation costs. This also opens the door to the solution of a host of supply chain optimization problems using these auctions.

3 Model Introduction

Consider a single buyer who has requirements, called consumption quantities, for a certain component at a set of geographically diverse locations. As in the standard auction literature, we assume the buyer has private valuation of the consumption quantities at the demand locations, which form the *consumption vector*, and she will act strategically to maximize her utility (i.e., her valuation minus her payments). The buyer's private valuation is given by a consumption utility function representing the dollar value of every possible consumption vector to the buyer. This could be, for example, the buyer's expected profit (excluding the acquisition costs which will be determined by the auction) that a consumption vector would bring in. Multiple suppliers, each of whom owns a set of production facilities, are available to satisfy these requirements. Every supplier has a production cost, which can be described as a convex function of the quantities that she produces at her production locations. Note that in economics, such a cost can already include a reasonable profit margin of the industry. We also assume that each supplier is a rational, self-interest player who is trying to maximize her own profit (i.e. the payment received minus her production cost). The convexity of a production cost function implies increasing marginal cost in the quantities produced, which includes the case where the overtime marginal production cost is higher than the marginal cost during regular production time. With general convex production costs, one can not simply combine the production and transportation costs and treat them as a single cost function. In addition, the buyer pays the transportation costs associated with every shipment from a supplier location to a buyer location. We assume that the buyer knows the per-unit transportation cost along each arc of the network, and there is no coalition among the suppliers, which is reasonable because it is relatively harder for suppliers to form a coalition in online auctions.

We consider three auction mechanisms, T, R, and S. All three of them require each supplier to submit a bid, a function of the quantity supplied by her, representing her desired prices. Although the three mechanisms have the same payment structure as that of the VCG family, they can sometime lead to very different payments.

Auction T: In this auction, the buyer submits a fixed consumption vector, $\mathbf{q}$, to the auctioneer, who will decide the quantities each supply location will provide to each demand center by minimizing the system production and transportation cost. Under the VCG payment structure, Auction T is incentive compatible and efficient. We investigate how the buyer determines her consumption vector, and show that the consumption vector picked by the buyer usually will not optimize the total supply chain (i.e., maximize the total net utility of the buyer and the suppliers). Although Auction T is incentive compatible and efficient, it may result in unreasonably high payments for the buyer.

Auction R: In this auction, the buyer submits a utility function (a proxy for her true consumption utility function) to the auctioneer, rather than a fixed consumption vector. The auctioneer treats this as if it were the profit (excluding acquisition costs) that the buyer makes by consuming $\mathbf{q}$, a consumption vector, and considers it when making production and transportation decisions. The quantities that the buyer will be awarded and her payments are the outputs of the auction.

Under the VCG payment structure, Auction R is incentive compatible for the suppliers, but NOT for the buyer. The auction mechanism meets the consumption vector $\mathbf{q}$ at minimum production and transportation cost and hence, is efficient for providing $\mathbf{q}$. It also typically results in a consumption vector $\mathbf{q}$ that fails to optimize the total supply chain.

Under Auction R, we show that the buyer's payments are always less than or equal to those in Auction T for providing the same level of consumption. We derive the buyer's optimal strategy assuming the buyer knows the suppliers' production costs (i.e., under complete information) and show that, in general, the buyer will not submit her true consumption utility function. We also consider Auction R under uncertainty in suppliers' production costs and investigate how the buyer's decision on her bidding function affects her awarded quantities and payments.

Auction S: To demonstrate the benefits of incorporating transportation costs into auctions, we examine Auction S. In Auction S, the buyer submits a fixed consumption vector to the auctioneer who will decide the production quantities at all supplier locations and the buyer's payments to the suppliers by minimizing the total production cost in the system. Transportation decisions are subsequently made to match the demand and supply at the lowest total transportation cost. It is obvious that Auction T achieves a lower total supply chain cost than Auction S. However, the buyer may prefer Auction S under certain circumstances. We found that the typical regular-overtime production cost structure can lead to higher payments in efficient auctions to distort buyer behavior.

Before introducing the detailed auction mechanisms, we define the notation and introduce our first assumption.

$$
\begin{aligned}
N &= \text{total number of supplier production facilities} \\
K &= \text{number of suppliers} \\
M &= \text{total number of buyer locations} \\
N^k &= \text{set of production facilities owned by supplier } k \\
k^n &= \text{index for the supplier that owns production facility } n \\
q_m &= \text{consumption at demand center } m,\ q_m > 0 \\
x_n &= \text{production quantity at production facility } n \\
y_{nm} &= \text{quantity shipped from production facility } n \text{ to demand center } m \\
z_{km} &= \sum_{n \in N^k} y_{nm}, \text{total quantity shipped to demand center } m \text{ by supplier } k \\
C_k(\mathbf{x}_k) &= \text{production cost function for supplier } k\ (\mathcal{R}^{|N^k|} \to \mathcal{R}) \\
F_k(\mathbf{x}_k) &= \text{bidding function from supplier } k\ (\mathcal{R}^{|N^k|} \to \mathcal{R}) \\
\tau_{nm} &= \text{cost for shipping one unit from production facility } n \text{ to demand center } m
\end{aligned}
$$

We use boldfaced letters to represent vectors or matrices, whose dimension will be clear from the context. In particular, $\mathbf{x}_k = (x_n : n \in N^k)$ and $\mathbf{z}_k = (z_{km} : 1 \leq m \leq M)$. For simplicity, we use $\partial f(\mathbf{x})$ to denote the set of all subgradients of $f(\cdot)$ at $\mathbf{x}$.

Assumption 1 *The production cost function $C_k(\cdot)$ and bidding function $F_k(\cdot)$ are non-decreasing convex and closed with $C_k(\mathbf{0}) = F_k(\mathbf{0}) = 0$. Furthermore, $C_k(\cdot)$ ($F_k(\cdot)$) is subdifferentiable at points where $C_k(\cdot) < \infty$ ($F_k(\cdot) < \infty$).*

4 Auction T

In this section, we study Auction T where the buyer submits a fixed consumption vector $\mathbf{q}$ to the auctioneer. Supplier k submits to the auctioneer a bid function $F_k(\mathbf{x}_k)$ for supplying $\mathbf{x}_k$ units, for which she incurs a production cost $C_k(\mathbf{x}_k)$, $\mathbf{x}_k \in \mathcal{R}^{|N^k|}$. The suppliers may or may not see the consumption vector.

As in any auction, the auctioneer will decide the quantities awarded to each of the suppliers, the amount transported from each supplier location to each of the buyer locations, and the payments made by the buyer to the sellers. Under Auction T, the auctioneer will minimize the sum of the accepted bids and the transportation costs, for a given consumption vector $\mathbf{q}$, as

$$\begin{aligned}
\text{Min.} \quad & \sum_{k=1}^{K} F_k(\mathbf{x}_k) + \sum_{n=1}^{N}\sum_{m=1}^{M} \tau_{nm} y_{nm} && (4.1)\\
\text{s.t.} \quad & \sum_{n=1}^{N} y_{nm} = q_m, \quad m = 1, \ldots, M; && (4.2)\\
& \sum_{m=1}^{M} y_{nm} = x_n, \quad n = 1, \ldots, N; && (4.3)\\
& y_{nm} \geq 0, \quad m = 1, \ldots, M, \quad n = 1, \ldots, N. && (4.4)
\end{aligned}$$

Let $\pi(\mathbf{q})$ be the optimal value of the objective function for a given $\mathbf{q}$. Define $\mathcal{Q} = \{\mathbf{q} : \mathbf{q} > \mathbf{0}, \pi(\mathbf{q}) < \infty\}$ and we restrict $\mathbf{q} \in \mathcal{Q}$ to ensure sufficient supply capacity. Since $F_k(\cdot)$ is closed, for any $\mathbf{q} \in \mathcal{Q}$, an optimal solution exists.

Let $(\mathbf{x}^T, \mathbf{y}^T)$ be an optimal solution, and $\pi^{-k}(\mathbf{q})$ be the optimal value of the objective function with the additional constraint $\mathbf{x}_k = \mathbf{0}$ (i.e., supplier k does not participate in the auction). The buyer will pay supplier k

$$\psi_k^T(\mathbf{q}) = \pi^{-k}(\mathbf{q}) - \pi(\mathbf{q}) + F_k(\mathbf{x}_k^T) \tag{4.5}$$

where $\pi^{-k}(\mathbf{q}) - \pi(\mathbf{q})$ is the bonus payment made to supplier k, representing the value she adds to the system by participating in the auction. The buyer pays supplier k her bid $F_k(\mathbf{x}_k^T)$ plus her contribution to the system. This payment scheme belongs to the general truth-inducing VCG family described in Nisan and Ronen (2001). Consequently, rational suppliers will bid their costs, $F_k(\mathbf{x}_k) = C_k(\mathbf{x}_k)$, irrespective of other suppliers' bids. Therefore, Auction T is incentive compatible for all suppliers (see Vickrey (1961), Clarke (1971) and Groves (1973)) and $\pi(\mathbf{q}) = \sum_{k=1}^{K} C_k(\mathbf{x}_k^T) + \sum_{n=1}^{N}\sum_{m=1}^{M} \tau_{nm} y_{nm}^T$, the minimum total supply chain cost for meeting the demand $\mathbf{q}$. That is, truth-telling implies that Auction T is efficient.

The total cost incurred by the buyer, $\kappa(\mathbf{q})$, is her total payments to the suppliers plus the transportation costs, i.e.,

$$\kappa(\mathbf{q}) = \sum_{k=1}^{K} \psi_k^T(\mathbf{q}) + \sum_{n=1}^{N}\sum_{m=1}^{M} \tau_{nm} y_{nm}^T = \sum_{k=1}^{K} \pi^{-k}(\mathbf{q}) - (K-1)\cdot \pi(\mathbf{q}). \tag{4.6}$$

4.1 Basic properties of Auction T

Here we list several properties of Auction T that will be used repeatedly in the subsequent sections.

Property 1 $\pi(\mathbf{q})$ *and* $\pi^{-k}(\mathbf{q})$ *are increasing convex functions of* $\mathbf{q}$*.*

Property 1 follows directly from the convexity of the production cost functions and linearity of the transportation costs in the convex program (4.1) – (4.4).

Property 2 *For any optimal solution* $(\mathbf{x}^T, \mathbf{y}^T)$ *for a given* $\mathbf{q} \in \mathcal{Q}$*, there exist nonnegative Lagrange multipliers,* $\mathbf{v} \in \mathcal{R}^M$ *and* $\mathbf{u} \in \mathcal{R}^N$ *associated with constraints* (4.2) *and* (4.3)*, such that the following results hold.*

1. $\mathbf{u}_k \in \partial C_k(\mathbf{x}_k^T)$ *for all* k*.*
2. $v_m = u_n + \tau_{nm}$ *if* $y_{nm} > 0$ *and* $v_m \le u_n + \tau_{nm}$ *if* $y_{nm} = 0$*. That is,* $v_m = \min_{1\le n\le N}\{u_n + \tau_{nm}\}$ *for all* m*.*
3. $\mathbf{v} \in \partial\pi(\mathbf{q})$ *and* $\mathbf{v} \in \partial\pi^{-k}(\mathbf{q} - \mathbf{z}_k^T)$ *where* $\mathbf{z}_{km}^T = \sum_{n\in N^k} y_{nm}^T$*,* $1 \le m \le M$*.*

The proof of Property 2 can be found in the Appendix. To understand Property 2, consider the case where $\pi(\cdot)$, $\pi^{-k}(\cdot)$ and $C_k(\cdot)$ are differentiable. Then the first two results in Property 2 become

$$\begin{aligned} u_n &= \frac{\partial C_{k^n}(\mathbf{x}_{k^n}^T)}{\partial x_n}, \\ \frac{\partial C_{k^n}(\mathbf{x}_{k^n}^T)}{\partial x_n} + \tau_{nm} &= \frac{\partial \pi(\mathbf{q})}{\partial q_m}, \text{ for } y_{nm}^T > 0, \\ \frac{\partial C_{k^n}(\mathbf{x}_{k^n}^T)}{\partial x_n} + \tau_{nm} &\ge \frac{\partial \pi(\mathbf{q})}{\partial q_m}, \text{ for } y_{nm}^T = 0. \end{aligned}$$

That is, suppliers who supply demand center m do so with the same marginal production-plus-transportation cost, and suppliers who do not supply demand center m have larger marginal costs at demand location m. The third result, $\mathbf{v} = \nabla\pi(\mathbf{q}) = \nabla\pi^{-k}(\mathbf{q} - \mathbf{z}_k^T)$, implies that the marginal supply chain cost remains the same if we take supplier k and her allocation, $\mathbf{z}_k^T$, out of the system. These results establish the relationships between the marginal production and transportation costs at any supply location and demand center, and the relationships between the marginal costs at any demand center with and without a supplier (and her bids).

Since $\mathbf{v}$ may not be unique in general, we define $V(\mathbf{q})$ as the set of all $\mathbf{v}$ that satisfy Property 2. That is, $V(\mathbf{q}) = \{\mathbf{v} : \exists$ some $\mathbf{u}$ such that $(\mathbf{u}, \mathbf{v})$ satisfies Property 2 for some $(\mathbf{x}^T, \mathbf{y}^T)$ given $\mathbf{q} \in \mathcal{Q}\}$. Note that $V(\mathbf{q})$ is nonempty and only depends on $\mathbf{q}$.

Property 3 *For any optimal solution $(\mathbf{x}^T, \mathbf{y}^T)$ for a given $\mathbf{q} \in \mathcal{Q}$, there exists $(\mathbf{x}^{T-k}, \mathbf{y}^{T-k})$ representing an optimal solution without supplier k such that $y_{nm}^{T-k}(\mathbf{q} - \mathbf{z}_k^T) = y_{nm}^T$ for $1 \leq m \leq M$ and $n \bar{\in} N^k$, and $\mathbf{x}_{k'}^{T-k}(\mathbf{q} - \mathbf{z}_k^T) = \mathbf{x}_{k'}^T$ for $k' \neq k$. That is, if we take supplier k and her awarded quantities $\mathbf{z}_k^T$ out of the system, then the production and transportation quantities at other suppliers in the auction with fixed demand $\mathbf{q} - \mathbf{z}_k^T$ remain the same. Furthermore, $\pi^{-k}(\mathbf{q} - \mathbf{z}_k^T)$ is the minimum cost for providing $\mathbf{q} - \mathbf{z}_k^T$ without supplier k, and*

$$\pi(\mathbf{q}) = \pi^{-k}(\mathbf{q} - \mathbf{z}_k^T) + C_k(\mathbf{x}_k^T) + \sum_{n \in N^k} \sum_{m=1}^{M} \tau_{nm} y_{nm}^T. \tag{4.7}$$

The proof for Property 2 is straightforward and hence, omitted here.

4.2 The Buyer's Choice of q

We now discuss how the buyer determines her consumption vector $\mathbf{q}$ in auction T and how her decision deviates from the one that optimizes the total supply chain. Assume that there exists a *Consumption Utility Function*, $U(\mathbf{q})$, that specifies the profit (excluding the acquisition costs that will be determined by the auction) the buyer will make by consuming $\mathbf{q}$. The buyer will choose the consumption vector $\mathbf{q}^B$ that maximizes her net utility $U(\mathbf{q}) - \kappa(\mathbf{q})$. This differs from $\mathbf{q}^A$, the consumption vector that optimizes supply chain by maximizing the total net utility of the buyer and the suppliers, $U(\mathbf{q}) - \pi(\mathbf{q})$. As $\kappa(\mathbf{q}) \geq \pi(\mathbf{q})$, one might expect $\mathbf{q}^B \leq \mathbf{q}^A$. This is true when $M = 1$, but may not always be true for $M > 1$ as the following example illustrates.

Consider a system with 2 suppliers ($K = 2$). Each owns a single production facility ($N = 2$) with production costs $C_1(x) = C_2(x) = x^2$. The buyer has 2 demand centers ($M = 2$) with consumption utility function $U(q_1, q_2) = 30,000 - 3[100 - (q_1 + q_2)]^2$ if $q_1 + q_2 \leq 100$ and $U(q_1, q_2) = 30,000$ otherwise. That is, there is zero utility associated with each additional unit above 100. Unit transportation costs are $\tau_{11} = \tau_{21} = \tau_{12} = 1$, and $\tau_{22} = \infty$. This implies that $\mathbf{y}_{22}^T = 0$ for all $\mathbf{q}$. Furthermore $q_2^B = 0$ because, if $q_2^B > 0$, then when supplier 1 is removed from the auction, the buyer's transportation cost would be infinite. Thus the buyer would have to make an infinite payment to supplier 1. Consequently, to maximize the buyer's net utility $U(q_1, q_2) - \kappa(q_1, q_2)$, the buyer will choose $q_2^B = 0$ and $y_{12}^B = 0$. This leads to

$$\begin{aligned} \pi(q_1, 0) &= q_1^2/2 + q_1, \\ \pi^{-1}(q_1, 0) &= \pi^{-2}(q_1, 0) = q_1^2 + q_1. \end{aligned}$$

Consequently the buyer's net utility is

$$U(q_1^B, 0) - \kappa(q_1^B, 0) = \max\left\{30,000 - 3(100 - q_1)^2 - (3q_1^2/2 + q_1)\right\}.$$

The solution is $(q_1^B, q_2^B) = (66.6, 0)$.

By contrast, total supply chain costs are minimized by solving

$$\begin{aligned} U(\mathbf{q}^A) - \pi(\mathbf{q}^A) = \max \quad & 30,000 - 3[100 - (q_1 + q_2)]^2 \\ & -[(y_{11} + y_{12})^2 + y_{21}^2 + y_{11} + y_{12} + y_{21}] \\ \text{s.t.} \quad & y_{11} + y_{21} = q_1, \\ & y_{12} = q_2. \end{aligned}$$

The result is $(q_1^A, q_2^A) = (64.17, 21.3)$, and $q_1^B > q_1^A$.

Although a VCG type auction is incentive compatible and efficient for any given $\mathbf{q}$, as pointed out by Ausubel and Cramton (1998), such an auction can be very expensive for the buyer. Consider 10 identical suppliers who meet a total demand of 90 units at a single demand center. Assume that transportation costs are negligible. At each supplier location, $C(x) = F(x) = x$ for $x \leq 9$ and $9 + 100(x - 9)$ for $x > 9$. Then it is optimal for all suppliers to produce 9 units at \$1.00 each and $\pi(90) =$ \$90. If we remove one supplier from the system, the rest of the suppliers will have to produce 9 more units at \$100 each, which leads to $\pi^{-k}(90) =$ \$981 for all k. The payments from the buyer are then given by $\psi_k = \pi^{-k}(90) - \pi(90) + C(9) =$ \$900 for all 10 suppliers. So the buyer pays a total of \$9,000 versus the real production cost of \$90. The reason for such a high payment is that, when capacity is tight and production costs are sharply convex for some suppliers, the total production costs may be significantly different when one of these suppliers is taken out of the system. The bonus payment can be very large, even for something produced at very low costs.

Another potential problem with Auction T is that when bidders are asymmetric, i.e., some suppliers have relatively high capacity, removing a supplier with large capacity may result in insufficient capacity and an infinite $\pi^{-k}(\mathbf{q})$ value.

5 Auction R

In this section, we propose a new auction mechanism, Auction R, that will result in much lower payments for the buyer and yet still induces the suppliers to bid their true costs. In this auction, the buyer submits a function $W(\mathbf{q})$ as a proxy for her true consumption utility, $U(\mathbf{q})$, rather than a fixed consumption vector as in Auction T. $W(\mathbf{q})$ is treated by the auctioneer as if it were the profit (excluding acquisition costs) that the buyer makes by consuming $\mathbf{q}$. We refer to $W(\cdot)$ as the buyer's bidding strategy and make the following assumption about $U(\cdot)$ and $W(\cdot)$.

Assumption 2 *$U(\mathbf{q})$ and $W(\mathbf{q})$ are increasing concave and closed with $U(\mathbf{0}) = 0$ and $W(\mathbf{0}) = 0$. Furthermore, $U(\cdot)$ $(W(\cdot))$ is subdifferentiable at points where $U(\cdot) < \infty$ $(W(\cdot) < \infty)$.*

As we will see, by submitting $W(\mathbf{q})$ the buyer is actually reporting a reservation price function for each unit she might acquire. It is well-known that one's true reservation price for any unit is the first derivative of her true consumption utility function and is usually NOT identical for all units. Instead, it may well be a decreasing function of the number of units one already has on hand. While in most auctions, the payment made to a supplier is determined by bids from all sellers, in Auction R, part or all of the payments will be determined by the buyer's reservation price. This prevents the buyer from making unreasonable payments.

The auctioneer solves the optimization problem $\min\{\pi(\mathbf{q}) - W(\mathbf{q})\}$ to determine an optimal consumption vector $\mathbf{q}^R$ and associated production quantities $\mathbf{x}^R$ and shipments $\mathbf{y}^R$. If there are multiple minimizers, we assume that the auctioneer will always choose an optimal consumption vector $\mathbf{q}^R$ with the maximum total purchase quantity.

Let $\mathbf{q}^{R^{-k}}$ be the solution to $\min\{\pi^{-k}(\mathbf{q})-W(\mathbf{q})\}$, $\Pi(W) = \pi(\mathbf{q}^R)-W(\mathbf{q}^R)$ and $\Pi^{-k}(W) = \pi^{-k}(\mathbf{q}^{R^{-k}})-W(\mathbf{q}^{R^{-k}})$. Following the VCG payment structure, the payment made to supplier k is

$$\psi_k^R(W) \quad = \quad \Pi^{-k}(W) - \Pi(W) + F_k(\mathbf{x}_k^R). \tag{5.8}$$

5.1 Properties of Auction R

In this section, we show that under (5.8), the suppliers will still bid their costs $C_k(\cdot)$, and the buyer pays less for purchasing $\mathbf{q}^R$ in Auction R than in auction T. We first introduce the following assumption.

Assumption 3 *There exists a finite consumption vector $\mathbf{q}^{max}$ such that $W(\mathbf{q}) = W(q_1, .., q_m^{max}, .., q_M)$ if $q_m > q_m^{max} - 1$ for all $\mathbf{q}$, for $W(\cdot)$ satisfying Assumption 2. That is, the buyer associates no utility to any consumption in excess of $q_m^{max} - 1$ at demand center m, $1 \le m \le M$.*

Assumption 3 guarantees that $\mathbf{q}^R < \mathbf{q}^{max}$.

Theorem 1 *Under assumption 3, truth-telling is the dominant strategy for all suppliers in auction R.*

We show that Theorem 1 holds by establishing that Auction R is equivalent to a T type auction. Consider a T type auction with fixed consumption vector $\mathbf{q}^{\max}$ and the buyer as an additional supplier. The buyer has a production facility with ample capacity co-located with each demand center. These production facilities can only supply their own demand centers and the transportation costs are naturally zero. The buyer is required to bid as a supplier and will bid the following convex function that satisfies Assumption 1,

$$F_B(\mathbf{x}) = \begin{cases} 0, & \text{if } x_m < 0 \text{ for some } m, \\ W(\mathbf{q}^{max}) - W(\mathbf{q}^{max} - \mathbf{x}), & \text{if } \mathbf{0} \le \mathbf{x} \le \mathbf{q}^{max}, \\ +\infty, & \text{otherwise,} \end{cases}$$

where $\mathbf{x}$ represents the vector of the buyer's production quantities. The auctioneer will then solve $\min_{\mathbf{0}\leq\mathbf{x}\leq\mathbf{q}^{\max}}\{\pi(\mathbf{q}^{\max}-\mathbf{x})+W(\mathbf{q}^{max})-W(\mathbf{q}^{\max}-\mathbf{x})\}$ or $\min_{\mathbf{0}\leq\mathbf{q}\leq\mathbf{q}^{\max}}\{\pi(\mathbf{q})+W(\mathbf{q}^{max})-W(\mathbf{q})\}$, which is equivalent to $\min_{\mathbf{0}\leq\mathbf{q}\leq\mathbf{q}^{\max}}\{\pi(\mathbf{q})-W(\mathbf{q})\}$. That is, this T type auction with fixed demand vector $\mathbf{q}^{max}$ and an additional dummy supplier is equivalent to the original Auction R.

In this equivalent T auction, the real suppliers will provide the actual consumption vector $\mathbf{q}^R$ and the dummy supplier, which has enough capacity at each demand center, will supply the remaining $\mathbf{x}_B^R = \mathbf{q}^{max} - \mathbf{q}^R$. The amounts the real suppliers will produce and ship to the demand locations, and the actual payments to the real suppliers are exactly the same as those resulting from Auction R. The existence of a dummy supplier does not affect suppliers' behavior in the equivalent T auction and, as a result, all suppliers will bid at their production costs.

As we can see, Auction R has the following nice properties. (i) It is still incentive compatible for the suppliers. (ii) By Theorem 1, $\pi(\mathbf{q}^R) = \sum_{k=1}^K C_k(\mathbf{x}_k^R) + \sum_{n=1}^N \sum_{m=1}^M \tau_{nm} y_{nm}^R$ is the minimum total system cost given $\mathbf{q}^R$. Hence, $(\mathbf{x}^R, \mathbf{y}^R)$ minimizes the total supply chain cost for producing and shipping $\mathbf{q}^R$.

If the buyer also submits her true consumption function $U(\mathbf{q})$, Auction R will maximize $U(\mathbf{q}) - \pi(\mathbf{q})$ and optimize the total supply chain. Interestingly, it is usually not optimal for the buyer to do so, as we will show in the next section when we discuss the buyer's optimal bidding strategy. The buyer will usually manipulate $W(\cdot)$ to gain some control over the total acquisition cost

$$\kappa_R(W) = \sum_{k=1}^K \psi_k^R(W) + \sum_{n=1}^N \sum_{m=1}^M \tau_{nm} \cdot y_{nm}^R = \sum_{k=1}^K \Pi^{-k}(W) - K \cdot \Pi(W) + \pi(\mathbf{q}^R).$$

Since $\mathbf{q}^{R^{-k}}$ minimizes $\{\pi^{-k}(\mathbf{q}) - W(\mathbf{q})\}$,

$$W(\mathbf{q}^R) - \pi^{-k}(\mathbf{q}^R) \quad \leq \quad W(\mathbf{q}^{R^{-k}}) - \pi^{-k}(\mathbf{q}^{R^{-k}})$$

for all k. Consequently

$$\kappa_T(\mathbf{q}^R) - \kappa_R(W) = \sum_{k=1}^K \left\{W(\mathbf{q}^{R^{-k}}) - \pi^{-k}(\mathbf{q}^{R^{-k}}) - \left[W(\mathbf{q}^R) - \pi^{-k}(\mathbf{q}^R)\right]\right\} \geq 0,$$

and we have the following theorem.

Theorem 2 *Let $\mathbf{q}^R$ be a consumption vector that results from $W(\mathbf{q})$ that the buyer submits in Auction R. The buyer will pay more to purchase the same quantity, $\mathbf{q}^R$, in Auction T. That is, $\kappa_R(W) \leq \kappa_T(\mathbf{q}^R)$.*

From Theorem 2, it appears that the buyer should prefer Auction R to Auction T. However as we will see subsequently, Auction T has an important advantage. If the buyer does not have complete information about the suppliers' production costs, she cannot predict $\mathbf{q}^R$ in advance. The degree of uncertainty in the quantity acquired favors Auction T.

5.2 The Buyer's Optimal Bidding Strategy

We now explore the optimal $W(\mathbf{q})$ that the buyer would submit with complete information under Auction R. We first derive lower bounds for the buyer's total payment $\kappa_R(W)$ for a given concave function $W(\mathbf{q})$. We then show that there exists a concave function $W^*(\mathbf{q})$ such that $\kappa_R(W^*)$ is at the best lower bound and the buyer's profit $U(\mathbf{q}) - \kappa_R(W)$ is maximized. To establish the lower bounds for the buyer's total payment, we first introduce the following lemma.

Lemma 3 *For any optimal solution $(\mathbf{x}^R, \mathbf{y}^R, \mathbf{q}^R)$ in auction R for a given $W(\cdot)$ satisfying Assumption 3, there exist nonnegative Lagrange multipliers $\mathbf{v} \in \mathcal{R}^M$, $\mathbf{u} \in \mathcal{R}^N$ and $\mathbf{u}_B \in \mathcal{R}^M$ that satisfy the following.*

1. *$\mathbf{u}_k \in \partial C_k(\mathbf{x}_k^R)$ for all k and $\mathbf{u}_B \in \partial F_B(\mathbf{x}_B^R)$.*
2. *By Assumption 3, $\mathbf{x}_B^R > \mathbf{0}$. Since the dummy production facilities only supply their own demand centers, $\mathbf{x}_B^R = \mathbf{y}_B^R$ and $\mathbf{v} = \mathbf{u}_B$. If we let $r_m = \min_{1 \le n \le N}\{u_n + \tau_{nm}\}$, then $r_m = v_m = u_{Bm}$ if $q_m^R > 0$ and $r_m \ge v_m = u_{Bm}$ if $q_m^R = 0$.*
3. *$\mathbf{r} \in \partial W(\mathbf{q}^R)$, $\mathbf{r} \in \partial \pi(\mathbf{q}^R)$, and $\mathbf{r} \in \partial \pi^{-k}(\mathbf{q}^R - \mathbf{z}_k^R)$.*

The proof of Lemma 3 is in the Appendix. Lemma 3 indicates that, if $W(\cdot)$, $\pi(\cdot)$, and $\pi^{-k}(\cdot)$ are differentiable, then the marginal utility equals to the marginal supply chain cost with and without supplier k at an optimal solution $\mathbf{q}^R$ and $\mathbf{q}^R - \mathbf{z}_k^R$. Since $\mathbf{r}$ may not be unique for any $\mathbf{q}^R$, let $\gamma(W, \mathbf{q}^R)$ be the set of all $\mathbf{r}$ defined in Lemma 3. That is, $\gamma(W, \mathbf{q}^R) = \{r : r \text{ is defined in Lemma 3 for some } (\mathbf{x}^R, \mathbf{y}^R, \mathbf{q}^R)\}$. We are now ready to establish the lower bounds of the buyer's total payment for given $W(\cdot)$.

Lemma 4 *For any function $W(\cdot)$ satisfying Assumption 3 and the resulting $\mathbf{q}^R$ in Auction R, $\kappa_R(W) \ge \mathbf{r} \cdot \mathbf{q}^R$, for any $\mathbf{r} \in \gamma(W, \mathbf{q}^R)$.*

The proof of Lemma 4 is in the Appendix. We now show that if $C_k(\cdot)$ is strictly increasing for all k, there exists $\bar{W}(\mathbf{q})$ such that the buyer's payment $\kappa_R(\bar{W})$ is at one of the lower bounds. Bidding at this function, the buyer will pay a uniform price for all the units purchased at the same demand location.

Let $\Gamma(\mathbf{q}) = \{\mathbf{r} : \mathbf{r} \in \gamma(W, \mathbf{q}^R) \text{ for some } W(\cdot) \text{ satisfying Assumption 3 and resulting in } \mathbf{q}^R = \mathbf{q}\}$. Then for any $\mathbf{r} \in \Gamma(\mathbf{q})$, $\mathbf{r} \in \partial \pi(\mathbf{q})$, $\mathbf{r} \in \partial \pi^{-k}(\mathbf{q} - \mathbf{z}_k^R)$, and $\mathbf{r} \in \partial W(\mathbf{q})$ for some $W(\cdot)$ by Lemma 3.

Lemma 5 *Assume that $C_k(\cdot)$ is strictly increasing for all k. Pick any $\bar{\mathbf{q}} \in \mathcal{R}^M$, and any $\mathbf{r}$ from $\Gamma(\bar{\mathbf{q}})$. If the buyer submits $\bar{W}(\mathbf{q}) = \sum_{m=1}^M \min\{q_m, \bar{q}_m\} \cdot r_m$ in Auction R, the resulting consumption vector is $\bar{\mathbf{q}}^R = \bar{\mathbf{q}}$ and the buyer's total payment is $\kappa_R(\bar{W}) = \mathbf{r} \cdot \bar{\mathbf{q}}$. That is, the buyer pays a uniform price r_m for all $\bar{q}_m$ units shipped to demand center m.*

The proof of Lemma 5 is in the Appendix. We now derive the buyer's strategy $W^*(\cdot)$ that maximizes her net utility $U(\mathbf{q}^R) - \kappa_R(W)$. As you will see, the structure of $W^*(\cdot)$ has the same form as that in Lemma 5 and the buyer needs to find the $(\mathbf{q}, \mathbf{r})$ pair, $\mathbf{r} \in \Gamma(\mathbf{q})$, that maximizes $\{U(\mathbf{q}) - \mathbf{r} \cdot \mathbf{q}\}$. For a given $\mathbf{q}$, let $\mathbf{r}^*(\mathbf{q}) \in \Gamma(\mathbf{q})$ maximize $\{U(\mathbf{q}) - \mathbf{r} \cdot \mathbf{q}\}$. Also, let $\mathbf{q}^*$ maximize $\{U(\mathbf{q}) - r^*(\mathbf{q}) \cdot \mathbf{q}\}$ and $\mathbf{r}^* = \mathbf{r}^*(\mathbf{q}^*)$.

Theorem 6 *The optimal function the buyer should submit is* $W^*(\mathbf{q}) = \sum_{m=1}^{M} \min\{q_m, q_m^*\} \cdot r_m^*$.

The proof of Theorem 6 is in the Appendix. By Theorem 6, the buyer's optimal strategy is determined by two parameters, $(\mathbf{r}^*, \mathbf{q}^*)$. To find $\mathbf{r}^*$, we need to identify the set $\Gamma(\mathbf{q})$. As you will see in the following lemma, $\Gamma(\mathbf{q})$ is actually equal to the set $V(\mathbf{q})$ defined in Section 3.1 and hence, is nonempty and only dependent on $\mathbf{q}$.

Lemma 7 $\Gamma(\mathbf{q}) = V(\mathbf{q})$, *for any* $\mathbf{q} \in \mathcal{Q}$.

The proof of Lemma 7 is in the Appendix. If $\pi(\mathbf{q})$ is differentiable for $\mathbf{q} \in \mathcal{Q}$, then $\Gamma(\mathbf{q}) = \nabla\pi(\mathbf{q})$ and we can solve for the optimal $\mathbf{q}^*$ by maximizing $\{U(\mathbf{q}) - \nabla\pi(\mathbf{q}) \cdot \mathbf{q}\}$. Otherwise we can use a differentiable function to approximate $\pi(\mathbf{q})$ and solve for $\mathbf{q}^*$ approximately.

With the buyer's optimal strategy specified, we can consider the example in Section 3 with 10 identical suppliers under Auction R. In that example, the total buyer's payment for providing 90 units under Auction T is \$9,000 while the total production cost is only \$90. Recall that there is only one demand center so we can ignore the subscript m. To make a meaningful comparison, we assume that $q^* = 90$. Then the optimal $r^* = \$1.00$, which is the marginal supply chain cost at $q^* = 90$, and $W^*(q) = \min(q, q^*)r^*$. Furthermore, $q^{*R} = q^* = 90$, $z_k^{*R} = 9$, $q^{*R} - z_k^{*R} = 81$, $\pi(q^{*R}) = \$90$, and $\pi^{-k}(q^{*R-k}) = \$81$ for all k. So the buyer's total payment is given by

$$\begin{aligned} \kappa_R(W^*) &= \sum_{k=1}^{10}[\pi^{-k}(q^{*R-k}) - W^*(q^{*R-k})] - \sum_{k=1}^{10}[\pi(q^{*R}) - W^*(q^{*R})] + \pi(q^{*R}) \\ &= \sum_{k=1}^{10}[W^*(90) - W^*(81)] + \sum_{k=1}^{10}[\pi^{-k}(81) - \pi(90)] + \pi(90) = 90 \end{aligned}$$

which happens to be the minimum supply chain cost.

Interestingly, when the buyer submits her optimal bidding function, $W^*(\mathbf{q})$, she actually pays a uniform price r_m^* for all the units shipped to demand center m. This uniform price is her submitted reservation price, as $W^*(\mathbf{q})$ is her submitted consumption utility. If $\pi(\cdot)$ is differentiable, $r_m^* = \frac{\partial\pi(\mathbf{q}^{*R})}{\partial q_m}$, the marginal supply chain cost at demand center m. Under such uniform payment structure, suppliers will try to bid low to get bids accepted, knowing the price is usually greater than their offered bids. Recall that uniform price auctions (e.g., using the first rejected or last accepted bid as the price for each purchased unit) are not in general incentive compatible (Ethier *et al.* 1999). Following the optimal strategy, Auction R yields a simple uniform price for each unit shipped to the same demand center and yet, the suppliers still have an incentive to tell the truth.

In summary, Auction R has the following nice properties. (1) It prevents unreasonably high payments in auctions, which may well be a reason for the buyer not to participate. Moreover, it results in lower payments for the buyer than Auction T. (2) The suppliers still have the incentive to submit their true cost functions. (3) When the buyer follows the optimal bidding strategy, her payments have a very simple uniform price structure. (4) For the consumption vector it provides, $\mathbf{q}^{*R}$, the total production and transportation cost is minimized and hence, Auction R is efficient. (5) Auction T will work only if any $K-1$ suppliers have sufficient capacity to provide all the demand and $\pi^{-k}(\mathbf{q})$ can be determined. In Auction R, the buyer plays a role as a dummy supplier with ample capacity and $\Pi^{-k}(W)$ always exists.

5.3 Auction R under Uncertainty

So far, we have assumed that the supplier's production cost functions $C_k(\mathbf{x}_k)$ are deterministic and known to the buyer. The buyer's optimal bidding function $W^*(\mathbf{q})$ uses this information. However, in reality it is more likely that the buyer has a probabilistic belief about her suppliers' production cost functions, rather than exact knowledge. In addition to facing uncertainty in her payments, she will face uncertainty in her consumption quantities, as we will see from numerical examples later on. In this section, we investigate how $W(\cdot)$ affects the mean and variance of the buyer's awarded quantities and her payments.

In general, finding an optimal concave function $W(\cdot)$ under incomplete information is very challenging. Therefore, we will concentrate on functions with the maximum-quantity, reservation-price structure we saw in the previous section. That is, $\bar{W}(\mathbf{q}) = \sum_{m=1}^{M} \bar{W}_m(q_m)$ where $\bar{W}_m(q_m) = \min(q_m, \bar{q}_m) \cdot \bar{r}_m$ with parameters $\bar{q}_m$ and $\bar{r}_m$, and we use a computational approach. Utility functions with such structure are easy for practitioners to understand and use. Although the outcome of the auction is uncertain, when the buyer submits $\bar{W}(\cdot)$, she actually sets $\bar{r}_m$ as her reservation price for the first $\bar{q}_m$ units at demand center m and her reservation price at zero for any unit beyond $\bar{q}_m$. So she at least knows that $\bar{\mathbf{q}}$ is an upper bound on the quantities she will be awarded, and that her maximum total payment is bounded from above by $\sum_{m=1}^{M} \bar{q}_m \bar{r}_m$. For the first $\bar{\mathbf{q}}$ units, sometimes their marginal costs can be too high, even though they are already the lowest from the supply chain, as each supplier is submitting her true cost, so the buyer is actually better off not buying all the $\bar{\mathbf{q}}$ units. If $\bar{r}_m$ is set high enough and there is enough capacity to supply $\bar{\mathbf{q}}$, the buyer will purchase exactly $\bar{\mathbf{q}}$ units. In Auction R, $\bar{r}_m$, instead of bids from suppliers, is sometimes used to determine the payments, so the buyer in general will pay less. If $\bar{r}_m$ is sufficiently large, Auction R and a T-type auction with fixed demand $\bar{\mathbf{q}}$ will result in the same awards and payments. In fact, Auction T can be viewed as a special case of Auction R.

Consider a system with nine suppliers ($K = 9$) and three demand centers ($M = 3$). Each supplier owns one production facility ($N = K = 9$) and has a piece-wise linear production cost function. For $n = 1, \cdots, 9$,

$$C_n(x_n) = \begin{cases} \alpha_n \cdot x_n, & \text{if } 0 \le x_n \le a_n, \\ \alpha_n \cdot a_n + \beta_n \cdot (x_n - a_n), & \text{if } a_n < x_n \le b_n, \\ \infty, & \text{otherwise.} \end{cases}$$

That is, supplier location n has a regular time capacity of a_n units with unit production cost α_n and an overtime capacity of $b_n - a_n$ units with unit production cost $\beta_n \ge \alpha_n$. We let $a_n \sim U(30, 45)$, $b_n \sim a_n + U(5, 10)$, $\alpha_n \sim U(5, 25)$ and $\beta_n \sim \alpha_n + U(5, 25)$. As transportation costs are common knowledge, we will create a supply chain network for our numerical study by randomly generating $\tau_{nm} \sim U(2, 12)$. The data are shown in Table 1. For simplicity, we assume that the maximum quantity $\bar{q}_m$ the buyer wants is the same at each demand center m, and we let $\bar{q} = 3\bar{q}_m$ denote the sum of the three maximum quantities. Furthermore, we only consider buyer bidding functions with a uniform reserve price $\bar{r}_m = \bar{r}$ for all m. Thus the only parameters in the buyer's bidding function are $(\bar{q}, \bar{r})$.

	Production Facilities n								
Demand Centers m	1	2	3	4	5	6	7	8	9
1	11.50	4.31	8.07	6.86	10.90	9.62	6.56	2.19	10.20
2	8.15	9.92	11.20	9.38	3.76	6.06	11.40	11.20	6.10
3	2.58	5.53	10.10	2.10	3.39	4.03	3.99	8.04	4.72

Table 1: Transportation cost τ_{nm}($) from Production Facility n to Demand Center m

For a fixed $(\bar{q}, \bar{r})$ pair, we generate the parameters and run Auction R to obtain a consumption vector and the buyer's total payment. Using this consumption vector as the input, we then run Auction T with the same parameters. We repeat this process 500 times, and calculate the expectation and variance of the purchase quantities and payments under Auctions R and T. We repeat the process for various values of $\bar{q}$ and $\bar{r}$. The results are shown in Tables 2 - 4 .

	Reserve price $\bar{r}$ ($)					
Maximum quantity $\bar{q}$	20	25	30	35	40	45
75	72.17	74.95	75	75	75	75
150	130.69	148.79	150	150	150	150
225	164.50	216.88	224.90	225	225	225
300	176.82	252.83	298.04	300	300	300
375	182.49	266.44	343.97	365.64	371.87	374.18
400	185.28	271.06	347.80	372.21	387.54	397.90

Table 2: Expected Total Purchase Quantity

	Reserve price $\bar{r}$ ($)					
Maximum quantity $\bar{q}$	20	25	30	35	40	45
75	0.117	0.015	0	0	0	0
150	0.198	0.037	0	0	0	0
225	0.268	0.101	0.007	0	0	0
300	0.289	0.159	0.023	0	0	0
375	0.291	0.177	0.069	0.031	0.019	0.008
400	0.293	0.171	0.070	0.047	0.046	0.040

Table 3: Coefficient of Variation of the Total Purchase Quantity

	Reserve price $\bar{r}$ ($)					
Maximum quantity $\bar{q}$	20	25	30	35	40	45
75	0.9759	0.9991	1	1	1	1
150	0.9544	0.9939	0.9999	1	1	1
225	0.9441	0.9841	0.9998	1	1	1
300	0.9411	0.9715	0.9861	0.9991	0.9999	1
375	0.9413	0.9682	0.8443	0.7284	0.7363	0.7678
400	0.9437	0.9691	0.8188	0.6653	0.6104	0.6201

Table 4: The Ratio of the Total Payments between Auction R and Auction T

In Auction R the buyer's total payments are always lower than those under Auction T for the same purchase quantities, so the entries in Table 4 are always less than or equal to 1. For any $\bar{q}$, as the reservation price $\bar{r}$ set by the buyer increases, uncertainty in the purchase quantity vanishes, and Auctions R and T become identical. If $\bar{q}$ is large and $\bar{r}$ is small, Auction R buys nearly all units that can be had at the reservation price, and $\bar{q}$ loses its relevance.

For moderate values of $\bar{q}$ and $\bar{r}$, if we carefully decrease $\bar{r}$ and increase $\bar{q}$, the expected purchase quantity can be maintained at a constant level (see Table 2). Costs will go down (see Table 4), but the degree of uncertainty in the total purchase quantity will go up (see Table 3). The buyer will seek an attractive tradeoff between the payments required, and variability in the quantities awarded. To quantify the nature of this tradeoff, we identified a series of pairs ($\bar{q}$,$\bar{r}$) that result in an expected total purchase quantity of 250 units. Figure 1 plots $\bar{q}$, the standard deviation of the total purchase quantity (values on the left axis), and the expectation and standard deviation of the buyer's total payment (values on the right axis). The standard deviation of the total payment is unimodal in our example. It is driven up by the uncertainty in the purchase quantity acquired as $\bar{r}$ decreases, and by the increase in total costs as $\bar{r}$ increases.

The primary tradeoff is between the total payment, and variability in the purchase quantity. The buyer can control the size of his payments via a reservation price, but only by accepting additional risks associated with the amount of material he acquires in the auction.

6 Auction S

Most literature in multi-unit auctions focuses on the cost of the product, ignoring other costs that will be determined by the outcome of an auction. To examine the benefits of incorporating transportation costs into auctions, we introduce Auction S, in which the auctioneer selects production quantities at supplier locations solely based on the suppliers' bids and on demand, and the transportation decisions are made subsequently. For any consumption vector $\mathbf{q}$ the buyer submits, the production quantities at different production facilities are determined by minimizing $\sum_{k=1}^{K} F_k(\mathbf{x}_k)$ subject to constraints (4.2)-(4.4). Let $\pi_S(\mathbf{q})$ and $\mathbf{x}^S$ be the optimal objective value and production vector, and $\pi_S^{-k}(\mathbf{q})$ be the counter part of $\pi^{-k}(\mathbf{q})$. The buyer's payment to supplier k in this auction, $\psi_k^S(\mathbf{q})$, is then given by

$$\psi_k^S(\mathbf{q}) = \pi_S^{-k}(\mathbf{q}) - \pi_S(\mathbf{q}) + F_k(\mathbf{x}_k^S).$$

Under this payment structure, the suppliers will still submit their true cost functions and $F_k(\mathbf{x}_k) = C_k(\mathbf{x}_k)$ for all k. The transportation quantities y_{nm}^S for all n and m are determined by solving the following optimization problem

$$\begin{aligned} \text{Min.} \quad & \sum_{n=1}^{N} \sum_{m=1}^{M} \tau_{nm} y_{nm} \\ \text{s.t.} \quad & \sum_{n=1}^{N} y_{nm} = q_m, \quad m = 1, \ldots, M, \\ & \sum_{m=1}^{M} y_{nm} = x_n^S, \quad n = 1, \ldots, N, \end{aligned}$$

and the buyer's total payment, $\kappa_S(\mathbf{q})$, will be given by

$$\kappa_S(\mathbf{q}) = \sum_{k=1}^{K} \psi_k^S(\mathbf{q}) + \sum_{n=1}^{N} \sum_{m=1}^{M} \tau_{nm} y_{nm}^S.$$

It is obvious that Auction S leads to lower total production costs but higher supply chain costs than Auction T. Our primary interests are the magnitude of the savings by running Auction T and how the buyer's payments differ in two auctions. We show that, under the typical regular-overtime production cost structure, Auction T can lead to payments that are large enough to distort buyer behavior. Intuitively, Auction T is most beneficial when the supplier locations and demand centers are geographically dispersed with distinct

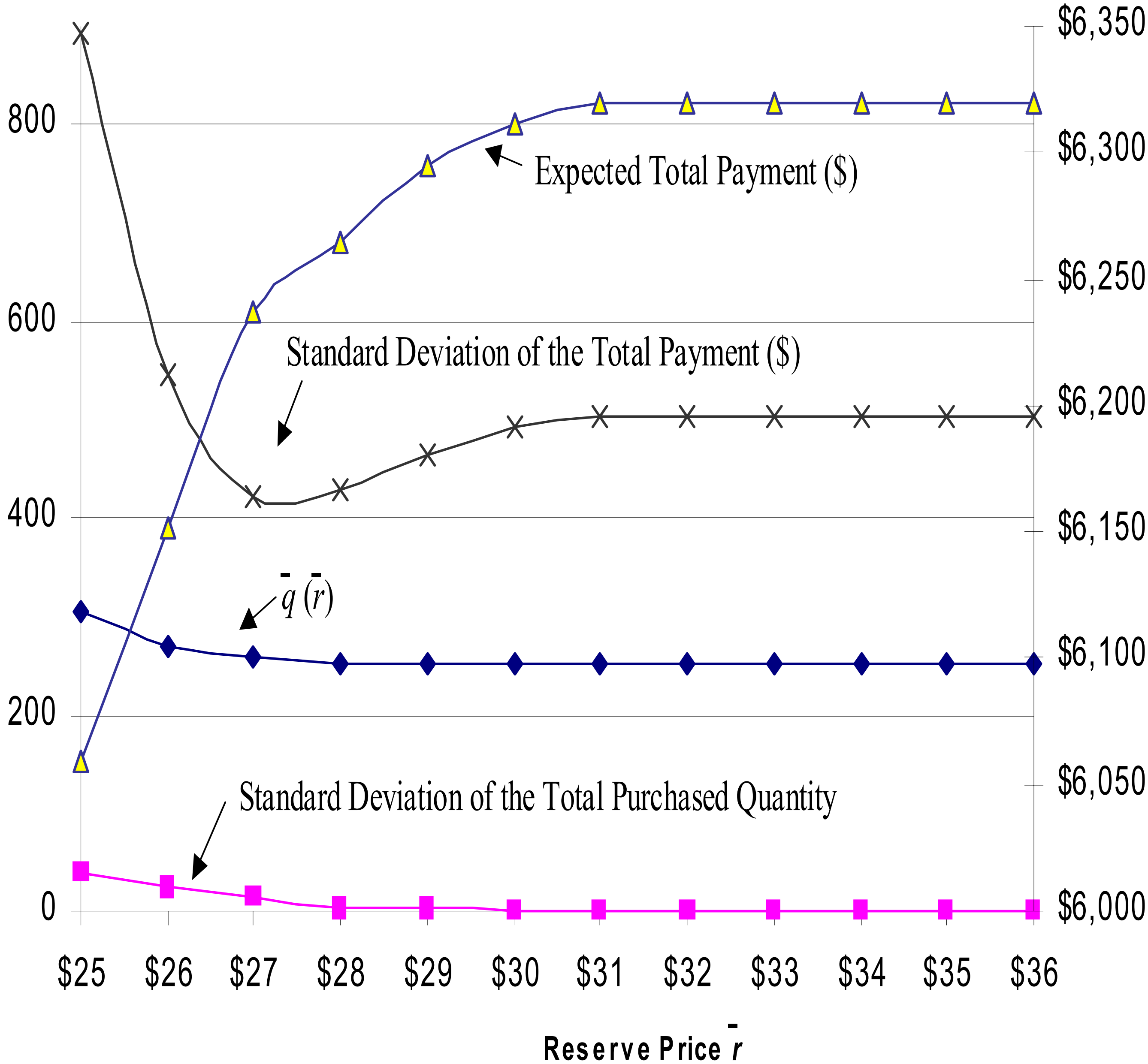

Figure 1: $\bar{q}(\bar{r})$, standard deviation of the awarded quantity, expectation and standard deviation of the buyer's payment

production and transportation costs. For example, in a global supply chain suppliers in North America usually have higher production costs while suppliers in Asia deliver products at higher transportation costs. So in our numerical study, we focus on a supply network where the production and transportation costs are negatively correlated.

As in Section 5.3, we consider a supply network with nine suppliers and three demand centers. Each demand center has the same fixed demand q_m and each supplier has only one production facility. The production cost function $C_n(\cdot)$ has a similar structure but with regular time capacity of $a_n = 40$, total capacity of $b_n = 60 > a_n$, unit regular time production cost of $\alpha_n \sim N(100, 10)$, unit overtime production cost of $\beta_n = 1.5\alpha_n$, and unit transportation costs $\tau_{nm} \sim N(50, 25)$ where (α_n, τ_{nm}) are negatively correlated with correlation $\rho \in \{-0.5, -0.25, 0\}$, $m = 1, ..., M$. For each (q, ρ) pair, we randomly generate a supply network and the costs, and run Auctions S and T to obtain the total production and transportation cost (π_T and π_S), the bonus payments (B_T and B_S) representing the suppliers' contributions, and the buyer's total payment (κ_T and κ_S). We repeat these activities 2,000 times and report the average total supply chain cost ($\bar{\pi}_T$ and $\bar{\pi}_S$), bonus payment ($\bar{B}_T$ and $\bar{B}_S$), and the buyer's total payment ($\bar{\kappa}_T$ and $\bar{\kappa}_S$) in Table 5. We also included in the table the percentage savings in total supply chain cost ($\frac{\bar{\pi}_S - \bar{\pi}_T}{\bar{\pi}_T}$), and buyer's total payment ($\frac{\bar{\kappa}_S - \bar{\kappa}_T}{\bar{\kappa}_T}$).

q_m	ρ	$\bar{\pi}_T$	$\bar{\pi}_S$	$\frac{\bar{\pi}_S - \bar{\pi}_T}{\bar{\pi}_T}$	$\bar{B}_T$	$\bar{B}_S$	$\bar{\kappa}_T$	$\bar{\kappa}_S$	$\frac{\bar{\kappa}_S - \bar{\kappa}_T}{\bar{\kappa}_T}$
50	-0.50	18259.4	20422.7	11.9%	1854.4	1203.0	20113.7	21625.7	7.5%
	-0.25	17823.8	19769.5	10.9%	2229.0	1200.2	20052.9	20969.7	4.6%
	0.00	17398.1	19040.4	9.4%	2553.8	1195.7	19951.9	20236.2	1.4%
75	-0.50	28129.6	30061.6	6.9%	3326.6	2284.0	31456.2	32345.6	2.8%
	-0.25	27612.7	29420.9	6.6%	4169.0	2336.8	31781.7	31757.7	-0.1%
	0.00	27151.8	28783.7	6.0%	4760.4	2292.7	31912.2	31076.4	-2.6%
100	-0.50	38561.6	39711.6	3.0%	5904.2	4393.8	44465.8	44105.5	-0.8%
	-0.25	38194.9	39528.3	3.5%	7268.1	4405.5	45463.0	43933.7	-3.4%
	0.00	37925.7	39399.7	3.9%	8064.5	4382.8	45990.2	43782.5	-4.8%
125	-0.50	49787.6	52163.0	4.8%	15549.2	13943.2	65336.8	66106.1	1.2%
	-0.25	49625.4	51832.1	4.5%	14608.2	13964.4	64233.6	65796.5	2.4%
	0.00	49309.5	51400.5	4.2%	14648.5	13947.1	63958.1	65347.6	2.2%
150	-0.50	62876.1	65804.1	4.7%	23105.7	21737.9	85981.8	87542.0	1.8%
	-0.25	62456.2	65523.7	4.9%	24355.2	21814.0	86811.4	87337.6	0.6%
	0.00	62094.0	65238.7	5.1%	25395.2	21768.0	87489.2	87006.6	-0.6%

Table 5: Auction T vs. Auction S

As we can see, Auction T results in savings in total supply chain costs ranging from 3.0% to 11.9%. As the level of correlation $|\rho|$ increases, the total supply chain costs in both auctions increase monotonically as expected. However, often times the bonus payments in Auction T are much higher than in Auction S so that the buyer actually pays more in Auction T. This seems especially true with low correlation between the production and transportation costs. As a result, the bonus portion of the total payments is usually higher in Auction T as shown in Table 6.

Note that a bonus payment to a supplier in a Vickrey type auction measures the supplier's contribution to the system by comparing the total supply chain costs with and without the supplier. The bonus payments can be higher in Auction T than Auction S for the following two reasons.

- While the contribution of a supplier in Auction S is solely based on the production costs, the contribution of a supplier in Auction T takes both the production and transportation costs into consideration. So a supplier has a higher impact on the total supply chain costs in Auction T than in Auction S and hence, receives a higher bonus payment. However, as correlation $|\rho|$ increases, the difference of the total unit production and transportation costs among all suppliers becomes smaller and hence the bonus payments in Auction T, in general, decrease. Often times, the difference between the supply chain costs outweighs the difference between the bonus payments, and Auction T leads to both lower total supply chain cost and total buyer's payment.

- Auction S allocates production to the lowest cost suppliers while Auction T allocates production to suppliers with the lowest production *and* transportation costs. Consequently, more overtime production is likely to be used in determining bonus payments in Auction T and the bonus payments can be significant, especially when the difference between regular time production costs and overtime production costs is big.

The behaviour of the buyer's total payments, which are the sum of the total supply chain cost and the bonus payments, is more complex. When demand is low ($q_m = 50$ in Table 5), there is ample capacity in the system and the difference between the total supply chain costs dominates the difference between the bonus payments. Hence, the buyer pays less in Auction T. As demand increases ($q_m = 75$, 100), more overtime capacity is involved in determining the bonus payments in Auction T and the buyer pays more in Auction T. As demand continues to increase ($q_m = 125$), a small amount of overtime production is needed to meet the demand in both auctions. The impact of the overtime costs is similar in both auctions and, in general, the buyer pays less in Auction T. However, at higher demand level ($q_m = 150$), the transportation costs cause the bonus payments to be much higher in Auction T than Auction S, especially when $|\rho|$ is small, and the buyer pays more in Auction T.

In most manufacturing systems, per-unit costs are relatively flat when the regular-time capacity of the manufacturing system is not exceeded, while the marginal per-unit production costs grow rapidly when overtime or other measures are required to increase capacity in the short term. As we can see, such cost structure can lead to higher total payments in Auction T, making Auction S more preferable to the buyer, although it is less efficient for the overall supply chain. One might conclude that this counter-intuitive situation is caused by our selection of the VCG payment mechanism. However under mild assumptions, the Revenue Equivalence Theorem (Engelbrecht-Wiggans 1988) indicates that any auction mechanism that results in an efficient solution will reward the suppliers with the same expected revenue.

q_m	ρ	$\bar{B}_T/\bar{\kappa}_T$	$\bar{B}_S/\bar{\kappa}_S$
50	-0.50	9.22%	5.56%
	-0.25	11.12%	5.72%
	0.00	12.80%	5.91%
75	-0.50	10.58%	7.06%
	-0.25	13.12%	7.36%
	0.00	14.92%	7.38%
100	-0.50	13.28%	9.96%
	-0.25	15.99%	10.03%
	0.00	17.54%	10.01%
125	-0.50	23.80%	21.09%
	-0.25	22.74%	21.22%
	0.00	22.90%	21.34%
150	-0.50	26.87%	24.83%
	-0.25	28.06%	24.98%
	0.00	29.03%	25.02%

Table 6: Bonus Portion of the Total Payment in Auction T and Auction S

This situation can be avoided when the total regular-time capacity is adequate, or the negative correlation between the production and transportation costs is high.

Table 6 indicates another interesting point. That is, the bonus portion of the total payments in both auctions is kept in a reasonable range when regular capacity is not very tight or when demand is low. This portion jumps to a much higher level when overtime production is used ($q_m = 125$, 150), as the bonus payments increase dramatically because of the big difference between regular-time and overtime production costs. This illustrates the importance of prudent capacity management in markets of manufactured components.

7 Conclusion

In this paper, we introduce multi-unit VCG auctions that incorporate both production costs and transportation costs in supply chains involving multiple supplier locations and demand centers. This is the first attempt to analyze auctions in complex supply chains. We propose two auction mechanisms with VCG payment structure, Auction T and Auction R. Both auctions are easy to implement and are incentive compatible for the suppliers. In Auction T, the purchase quantities are fixed, while the payments are uncertain and can be unreasonably high. So we introduce Auction R, under which the buyer also submits a bid, consisting of a dollar-denominated consumption utility function that the auctioneer uses when making production and transportation decisions. We show that under Auction R the buyer will always pay less than she would have paid to obtain the same quantity under Auction T.

Consequently, if the buyer can anticipate the bids of the suppliers (complete information), she will always prefer Auction R. We derive the buyer's optimal bidding function that maximizes her net utility under complete information. However, if the buyer lacks complete information, both the awarded quantities and the buyer's payments are random variables in Auction R. This adds a new element of risk that needs to be traded off against the cost savings that the buyer realizes with Auction R. In practice a computational approach can be used to make this tradeoff.

To illustrate the importance of incorporating transportation costs into auctions, we consider Auction S, under which the auctioneer awards quantities solely based on the suppliers' bids and the demand, and the transportation decisions are made subsequently. Although Auction S is incentive compatible, numerical examples show that considerable supply chain cost savings can be achieved by running Auction T. However, the buyer may favor Auction S due to lower total payments under certain circumstances. This is because, in Auction T, a supplier's contribution is measured not only by its production costs, but also the transportation costs. When the system-wide regular capacity is tight, overtime production and transportation costs together drive the payments up dramatically. The possible higher payments in Auction T can induce the buyer to favor the inefficient solution, Auction S. This illustrates the vital importance of prudent capacity management in markets of manufactured components.

This work can be easily extended to VCG auctions with one supplier and multiple buyers. Other costs that might arise when parts are purchased in supply chain settings can also be included, including fixed costs and other economies of scale. However with some of these extensions the required computational effort will increase. Future work also includes auctions in supply chains with multiple suppliers and buyers.

8 Appendix

Proof of Property 2: The first two results follow directly from Theorem 28.3 of Rockafellar (1970). We now prove that $\mathbf{v} \in \partial\pi(\mathbf{q})$. The proof for $\mathbf{v} \in \partial\pi^{-k}(\mathbf{q} - \mathbf{z}_k^T)$ is similar and is omitted here.

Let $(\mathbf{x}^T(\mathbf{q}'), \mathbf{y}^T(\mathbf{q}'))$ represent an optimal solution for $\mathbf{q}' \in \mathcal{Q}$. Then,

$$
\begin{aligned}
\pi(\mathbf{q}') - \pi(\mathbf{q}) &= \sum_{k=1}^{K}[C_k(\mathbf{x}_k^T(\mathbf{q}')) - C_k(\mathbf{x}_k^T)] + \sum_{n=1}^{N}\sum_{m=1}^{M}\tau_{nm}[y_{nm}^T(\mathbf{q}') - y_{nm}^T] \\
&\geq \sum_{k=1}^{K}\mathbf{u_k}\cdot[\mathbf{x}_k^T(\mathbf{q}') - \mathbf{x}_k^T] + \sum_{n=1}^{N}\sum_{m=1}^{M}\tau_{nm}[y_{nm}^T(\mathbf{q}') - y_{nm}^T] \\
&= \sum_{n=1}^{N}u_n\sum_{m=1}^{M}[y_{nm}^T(\mathbf{q}') - y_{nm}^T] + \sum_{n=1}^{N}\sum_{m=1}^{M}\tau_{nm}[y_{nm}^T(\mathbf{q}') - y_{nm}^T] \\
&= \sum_{n=1}^{N}\sum_{m=1}^{M}(u_n + \tau_{nm})[y_{nm}^T(\mathbf{q}') - y_{nm}^T]
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{n,m:y^T_{nm}(\mathbf{q})>0} v_m[y^T_{nm}(\mathbf{q}') - y^T_{nm}] + \sum_{n,m:y^T_{nm}=0} (u_n + \tau_{nm}) y^T_{nm}(\mathbf{q}') \\
&\geq \sum_{n,m:y^T_{nm}>0} v_m[y^T_{nm}(\mathbf{q}') - y^T_{nm}(\mathbf{q})] + \sum_{n,m:y^T_{nm}=0} v_m y^T_{nm}(\mathbf{q}') \\
&= \sum_{m=1}^{M} v_m \sum_{n=1}^{N} [y^T_{nm}(\mathbf{q}') - y^T_{nm}] = \sum_{m=1}^{M} v_m (q'_m - q_m) = \mathbf{v} \cdot (\mathbf{q}' - \mathbf{q}).
\end{aligned}
$$

The inequalities follow from the first two results and the nonnegativity of $y^T_{nm}(\mathbf{q}')$.

□

Proof of Lemma 3: Consider the equivalent T auction with fixed demand $\mathbf{q}^{max} = \mathbf{q}^R + \mathbf{x}^R_B$ resulting from a concave function $W(\cdot)$. Recall that the equivalent T auction has the minimum cost $\Pi(W) = \pi(\mathbf{q}^R) + F_B(\mathbf{x}^R_B)$. By Property 2, we have the first two results.

To show that $\mathbf{r} \in \partial W(\mathbf{q}^R)$, it suffices to prove that $\mathbf{r} \in \partial F_B(\mathbf{x}^R_B)$ by the definition of $F_B(\cdot)$. Consider any vector $\mathbf{x}'_B \in \mathcal{R}^M$ and, without loss of generality, $\mathbf{0} \leq \mathbf{x}'_B \leq \mathbf{q}^{max}$.

$$
F_B(\mathbf{x}'_B) - F_B(\mathbf{x}^R_B) \geq \mathbf{u}_B \cdot (\mathbf{x}'_B - \mathbf{x}^R_B) = \sum_{m:q^R_m>0} u_{Bm}(x'_{Bm} - x^R_{Bm}) + \sum_{m:q^R_m=0} u_{Bm}(x'_{Bm} - x^R_{Bm}).
$$

Note that $q^R_m = 0$ implies that $x^R_{Bm} = q^{max}_m$ and $x'_{Bm} - x^R_{Bm} \leq 0$. Since $0 \leq u_{Bm} \leq r_m$ if $q^R_m = 0$, and $u_{Bm} = r_m$ if $q^R_m > 0$, we have

$$
F_B(\mathbf{x}'_B) - F_B(\mathbf{x}^R_B) \geq \sum_{m:q^R_m>0} r_m(x'_{Bm} - x^R_{Bm}) + \sum_{m:q^R_m=0} r_m(x'_{Bm} - x^R_{Bm}) = \mathbf{r} \cdot (\mathbf{x}'_B - \mathbf{x}^R_B).
$$

The proof of Property 2 establishes the rest of the lemma.

□

Proof of Lemma 4: For any $r \in \gamma(W, \mathbf{q}^R)$, let $(\mathbf{x}^R, \mathbf{y}^R)$ be an solution associated with it. Rewrite $\kappa_R(W)$ as

$$
\kappa_R(W) = \sum_{k=1}^{K} [\pi^{-k}(\mathbf{q}^{R-k}) - W(\mathbf{q}^{R-k})] - K[\pi(\mathbf{q}^R) - W(\mathbf{q}^R)] + \pi(\mathbf{q}^R).
$$

Recall that the production and transportation quantities resulting from the T auction for given $\mathbf{q}^R$ are the same as those from the R auction with $\mathbf{q}^R$ as the output. Also, $z^R_{km} = \sum_{n \in N^k} y^R_{nm}$, the total quantity shipped to demand center m by supplier k. Hence, Property 3 leads to

$$
\begin{aligned}
\pi(\mathbf{q}^R) &= \pi^{-k}(\mathbf{q}^R - \mathbf{z}^R_k) + F_k(\mathbf{x}^R_k) + \sum_{n \in N^k} \sum_{m=1}^{M} \tau_{nm} y^R_{nm}, \\
(1 - K)\pi(\mathbf{q}^R) &= \pi(\mathbf{q}^R) - \sum_{k=1}^{K} \pi(\mathbf{q}^R) = -\sum_{k=1}^{K} \pi^{-k}(\mathbf{q}^R - \mathbf{z}^R_k),
\end{aligned}
$$

and

$$\kappa_R(W) = \sum_{k=1}^{K}[\pi^{-k}(\mathbf{q}^{R-k}) - \pi^{-k}(\mathbf{q}^R - \mathbf{z}_k^R) + W(\mathbf{q}^R) - W(\mathbf{q}^{R-k})].$$

For any $\mathbf{r} \in \gamma(W, \mathbf{q}^R)$, $\mathbf{r} \in \partial\pi^{-k}(\mathbf{q}^R - \mathbf{z}_k^R)$ and $\mathbf{r} \in \partial W(\mathbf{q}^R)$ by Lemma 3. As $\pi^{-k}(\cdot)$ is convex and $W(\cdot)$ is concave,

$$\begin{aligned} \pi^{-k}(\mathbf{q}^{R-k}) - \pi^{-k}(\mathbf{q}^R - \mathbf{z}_k^R) &\geq \mathbf{r} \cdot [\mathbf{q}^{R-k} - (\mathbf{q}^R - \mathbf{z}_k^R)] \\ W(\mathbf{q}^R) - W(\mathbf{q}^{R-k}) &\geq \mathbf{r} \cdot (\mathbf{q}^R - \mathbf{q}^{R-k}). \end{aligned}$$

Hence,

$$\kappa_R(W) \geq \sum_{k=1}^{K} \mathbf{r} \cdot [\mathbf{q}^{R-k} - (\mathbf{q}^R - \mathbf{z}_k^R) + (\mathbf{q}^R - \mathbf{q}^{R-k})] = \sum_{k=1}^{K} \mathbf{r} \cdot \mathbf{z}_k^R = \mathbf{r} \cdot \mathbf{q}^R.$$

□

Proof of Lemma 5: If the buyer submits $\bar{W}(\mathbf{q})$, the auctioneer will minimize $\{\pi(\mathbf{q}) - \bar{W}(\mathbf{q})\}$ to obtain the consumption vector.

We know that, for $\mathbf{r} \in \Gamma(\bar{\mathbf{q}})$, $\mathbf{r} \in \partial\pi(\bar{\mathbf{q}})$. By the way $\bar{W}(\cdot)$ is constructed, $\mathbf{r} \in \partial\bar{W}(\bar{\mathbf{q}})$. So $\mathbf{0} \in \partial\pi(\bar{\mathbf{q}}) - \partial\bar{W}(\bar{\mathbf{q}})$. By Theorem 23.8 of Rockafellar (1970), $\mathbf{0} \in \partial[\pi(\bar{\mathbf{q}}) - \bar{W}(\bar{\mathbf{q}})]$. As $\pi(\mathbf{q}) - \bar{W}(\mathbf{q})$ is a convex program, $\bar{\mathbf{q}}$ minimizes $\{\pi(\mathbf{q}) - \bar{W}(\mathbf{q})\}$. Let $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$ be an solution associated with $\mathbf{r}$ and $\bar{\mathbf{z}}_k = \sum_{n \in N^k} \bar{y}_{nm}$. Following the same arguments, $\bar{\mathbf{q}} - \bar{\mathbf{z}}_k$ minimizes $\{\pi^{-k}(\mathbf{q}) - \bar{W}(\mathbf{q})\}$ and, for any $\bar{\mathbf{q}}^{R-k}$ that minimizes $\{\pi^{-k}(\mathbf{q}) - \bar{W}(\mathbf{q})\}$,

$$\pi^{-k}(\bar{\mathbf{q}}^{R-k}) - \bar{W}(\bar{\mathbf{q}}^{R-k}) = \pi^{-k}(\bar{\mathbf{q}} - \bar{\mathbf{z}}_k) - \bar{W}(\bar{\mathbf{q}} - \bar{\mathbf{z}}_k). \tag{8.9}$$

Since $C_k(\cdot)$ is strictly increasing for all k, $\pi(\mathbf{q})$ is strictly increasing for any $\mathbf{q} \in \mathcal{Q}$. If there exist multiple solutions to $\min\{\pi(\mathbf{q}) - \bar{W}(\mathbf{q})\}$, the auctioneer will always choose an optimal consumption vector with the largest total purchase quantity as we assumed earlier. We claim that $\bar{\mathbf{q}}^R = \bar{\mathbf{q}}$. Suppose there exists an optimal solution with $q_m > \bar{q}_m$ for some m. Then $\bar{W}(\mathbf{q} \wedge \bar{\mathbf{q}}) = \bar{W}(\mathbf{q})$, but $\pi(\mathbf{q} \wedge \bar{\mathbf{q}}) < \pi(\mathbf{q})$. Thus $\mathbf{q}$ cannot be optimal.

The buyer's total payment is given by

$$\begin{aligned} \kappa_R(\bar{W}) &= \sum_{k=1}^{K}[\pi^{-k}(\bar{\mathbf{q}}^{R-k}) - \bar{W}(\bar{\mathbf{q}}^{R-k})] - \sum_{k=1}^{K}[\pi(\bar{\mathbf{q}}) - \bar{W}(\bar{\mathbf{q}})] + \pi(\bar{\mathbf{q}}) \\ &= \sum_{k=1}^{K}[\bar{W}(\bar{\mathbf{q}}) - \bar{W}(\bar{\mathbf{q}}^{R-k})] + \sum_{k=1}^{K}[\pi^{-k}(\bar{\mathbf{q}}^{R-k}) - \pi(\bar{\mathbf{q}})] + \pi(\bar{\mathbf{q}}) \\ &= \sum_{k=1}^{K}[\bar{W}(\bar{\mathbf{q}}) - \bar{W}(\bar{\mathbf{q}} - \bar{\mathbf{z}}_k)] + \sum_{k=1}^{K}[\pi^{-k}(\bar{\mathbf{q}} - \bar{\mathbf{z}}_k) - \pi(\bar{\mathbf{q}})] + \pi(\bar{\mathbf{q}}). \end{aligned}$$

The last equality follows from Equation (8.9). By Property 3,

$$\pi(\bar{\mathbf{q}}) = \pi^{-k}(\bar{\mathbf{q}} - \bar{\mathbf{z}}_k) + C_k(\bar{\mathbf{x}}_k) + \sum_{n \in N^k} \sum_{m=1}^{M} \tau_{nm} \bar{y}_{nm}.$$

Summing over k, we have

$$\pi(\bar{\mathbf{q}}) = -\sum_{k=1}^{K} [\pi^{-k}(\bar{\mathbf{q}} - \bar{\mathbf{z}}_k) - \pi(\bar{\mathbf{q}})],$$

and

$$\kappa_R(\bar{W}) = \sum_{k=1}^{K} [\bar{W}(\bar{\mathbf{q}}) - \bar{W}(\bar{\mathbf{q}} - \bar{\mathbf{z}}_k)] = \sum_{m=1}^{M} \left(\sum_{n \in N^k} r_m \cdot \bar{y}_{nm} \right) = \mathbf{r} \cdot \bar{\mathbf{q}}.$$

That is, the buyer pays a uniform price r_m, for all the units shipped to demand center m.

□

Proof of Theorem 6: If the buyer submits $W^*(\mathbf{q})$, by Lemma 5, the resulting consumption vector is $\mathbf{q}^{*R} = \mathbf{q}^*$, the buyer's total payment is $\kappa_R(W^*) = \mathbf{r}^* \cdot \mathbf{q}^*$, and the buyer pays a uniform price r^*_m for all units shipped to demand center m. For any $W(\mathbf{q})$ resulting in $\mathbf{q}^R$ as the output of Auction R, by Lemma 4,

$$U(\mathbf{q}^R) - \kappa_R(W) \leq U(\mathbf{q}^R) - \mathbf{r} \cdot \mathbf{q}^R$$

for all $\mathbf{r} \in \gamma(W, \mathbf{q}^R) \subseteq \Gamma(\mathbf{q}^R)$. For any $\mathbf{r} \in \Gamma(\mathbf{q}^R)$,

$$U(\mathbf{q}^R) - \mathbf{r} \cdot \mathbf{q}^R \leq U(\mathbf{q}^R) - r^*(\mathbf{q}^R) \cdot \mathbf{q}^R \leq U(\mathbf{q}^*) - \mathbf{r}^* \cdot \mathbf{q}^*.$$

So

$$U(\mathbf{q}^R) - \kappa_R(W) \leq U(\mathbf{q}^*) - \mathbf{r}^* \cdot \mathbf{q}^*.$$

That is, in Auction R, the buyer's utility is maximized by submitting $W^*(\mathbf{q})$.

□

Proof of Lemma 7: For any given $\mathbf{r} \in \Gamma(\mathbf{q})$, $\mathbf{r} \in \gamma(W, \mathbf{q}^R)$ for some W satisfying Assumption 3 with $\mathbf{q}^R = \mathbf{q}$. The associated $(\mathbf{x}^R, \mathbf{y}^R, \mathbf{u}, \mathbf{u}_B)$ satisfies Lemma 3 and hence, $(\mathbf{x}^R, \mathbf{y}^R, \mathbf{u}, \mathbf{r})$ satisfies Property 2. Therefore, $\mathbf{r} \in V(\mathbf{q})$.

For any given $\mathbf{v} \in V(\mathbf{q})$, there exists $(\mathbf{x}^T, \mathbf{y}^T, \mathbf{u})$ satisfying Property 2. Hence, $\mathbf{v} \in \partial\pi(\mathbf{q})$ and $\mathbf{v} \in \partial\pi^{-k}(\mathbf{q} - \mathbf{z}^T_k)$. We can construct $\bar{W}(\mathbf{q})$ as defined in Lemma 5 with parameter $\mathbf{v}$ instead of a $\mathbf{r}$ from set $\Gamma(\mathbf{q})$. By similar arguments in the proof of Lemma 5, $\bar{W}(\cdot)$ results in $\bar{\mathbf{q}}^R = \mathbf{q}$ and hence, $\mathbf{v} \in \gamma(\bar{W}, \mathbf{q})$. Therefore, $\mathbf{v} \in \Gamma(\mathbf{q})$ and $\Gamma(\mathbf{q}) = V(\mathbf{q})$ for any $\mathbf{q} \in \mathcal{Q}$.

□

9 References

Ausubel, L. and P. Cramton, 1998, "Demand Reduction and Inefficiency in Multi-Unit Auctions", *Department of Economics, University of Maryland, Working Paper No. 96-07.*

Beil, D. R. and L. Wein, 2001, "An Inverse-Optimization-Based Auction Mechanism to Support a Multi-Attribute RFQ Process, Working paper, Sloan School of Management, MIT.

Chopra, S. and J. Van Mieghem, 2000, "Which e-business is right for your supply chain?", *Supply Chain Management Review*, **4**(3), 32-40.

Clarke, E. H., 1971, "Multipart Pricing of Public Goods", *Public Choice*, **11**, 17-33.

Engelbrecht-Wiggans, R., 1988, "Revenue Equivalence in Multi-object Auctions", *Economic Letters*, **26**, 15-19.

Eso, M., 2001, "An iterative onling auction for airline seats.", Technical report, IBM T. J. Watson Research Center.

Ethier, R., R. Zimmerman, T. Mount, W. Schulze and R. Thomas, 1999, "A Uniform Price Auction with Locational Price Adjustments for Competitive Electricity Markets", *Electrical Power and Energy Systems*, **21**, 103-110.

Groves, T., 1973, "Incentive in Teams", *Econometrica*, **41**, 617-631.

Holmberg, K. and H. Tuy, 1999, "A production-transportation problem with stochastic demand and concave production costs", *Mathematical Programming*, **85**, 157-179.

Jin, M. and S. D. Wu, 2001, "Supplier Coalitions in eCommerce Auctions: Validity Requirements and Profit Distribution", Working paper, Department of Industrial and Systems Engineering, Lehigh University.

Klemperer, P., 1999, "Auction theory: A guide to the literature.", *Journal of Economic Surveys*, **13**(3), 227-186.

Keskinocak, P. and S. Tayur, 2001, "Quantitative Analysis for Internet-Enabled Supply Chains", *Interfaces*, **31**(2), 70-89.

Lucking-Riley, D., 2000, "Auctions on the Internet: What's Being Auctioned, and How?", *Journal of Industrial Economics*, **48**(3), 227-252.

McAfee, R. P. and J. McMillan, 1987, "Auction and bidding", *Journal of Economic Literature*, **25**(2), 699-738.

Milgrom, P. and R. Weber, 1982, “A theory of auctions and competitive bidding. ”, *Econometrica*, **50**(5), 1089-1122.

Nisan, N. and A. Ronen, 2001, “Algorithmic Mechanism Design”, *Games and Economic Behavior*, **35**(1/2), 166-196.

Pinker, E., A. Seidmann and Y. Vakrat, 2001, “Using Transaction Data for the Design of Sequential, Multi-unit, Online Auctions”, Working Paper, W. E. Simon Graduate School of Business Administration, University of Rochester.

Rockafellar, R. T., 1970, Convex Analysis, Princeton University Press, Princeton, New Jersey.

Rothkopf, M. H., 1969, “A Model of Rational Competitive Bidding”, *Management Science*, **15**(7), 362-373.

Rothkopf, M. H. and R. M. Harstad, 1994, “Modeling Competitive Bidding: A Critical Essay”, *Management Science*, **40**(3), 364-384.

Rothkopf, M. H., A. Pekec, and R. H. Harstad, 1998, “Computationally manageable combinatorial auctions”, *Management Science*, **44**(8), 1131-1147.

Rothkopf, M. H., T. J. Teisberg, and E. P. Kahn, 1990, “Why are Vickrey auctions rare?” *Journal of Political Economy*, **98**(1),94-109.

Sharp, J. F., J. C. Snyder and J. H. Greene, 1970, “A Decomposition Algorithm for Solving the Multifacility Production-Transportation Problem with Non-linear Production Costs”, *Econometrica*, **38**(3), 490-506.

Sullivan, L. and R. Lamb, “Rift Grows Over Pricing in Supply Chain”, *Electronics Business Network*, November 19, 2001, http://www.ebnews.com.

Vickrey, W., 1961, “Counterspeculation, Auctions, and Competitive Sealed Tenders”, *Journal of Finance*, **16**, 8-37.

Vulcano, G., G. van Ryzin and C. Maglaras, 2001, “Optimal Dynamic Auctions for Revenue Management”, Working Paper, Graduate School of Business, Columbia University.