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A THREE-ECHELON, MULTI-ITEM
MODEL FOR RECOVERABLE ITEMS

by

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ABSTRACT

The main objective of this paper is to develop a mathematical model for a particular type of three-echelon inventory system. The proposed model is being used by the Air Force to evaluate inventory investment requirements for alternative logistics structures. The system we will model consists of a group of locations, called bases, and a central depot. The items of concern in our analysis are called recoverable items, that is, items that can be repaired when they fail. Furthermore, each item has a modular or hierarchical design. Briefly, the model is used to determine the stock levels at each location for each item so as to achieve optimum inventory system performance for a given level of investment. An algorithm for computing stock levels for each item and location is developed and illustrated. Some of the ways the model can be used are illustrated with Air Force data.
I. INTRODUCTION

The main objective of this paper is to develop a mathematical model for a particular type of three-echelon inventory system. The model is being used by the Air Force to evaluate inventory investment requirements for alternative logistics structures. The system we will study consists of a group of locations, called bases, and a central depot. The items of concern in our analysis are called recoverable items, that is, items that can be repaired when they fail. Briefly, the model will be used to determine what the stock levels should be at each location for each item so as to achieve optimum inventory system performance for a given level of investment.

We assume that each item in the system has a hierarchical or modular design. By a hierarchically designed recoverable item we mean one that has components which are also recoverable items. In the Air Force context, when an aircraft fails, a recoverable assembly is often found to be faulty. It is removed from the aircraft and replaced by a serviceable assembly of the same type. If this failed assembly has a hierarchical design, it may be taken to a shop on the base where a faulty recoverable component may be identified. To return the failed assembly to a serviceable condition requires removing and replacing the defective recoverable component. For example, many avionics system components on newer aircraft have this type of hierarchical design, such as the radar target digital processor on the Air Force's new F-15 fighter. This assembly has many recoverable components which are, for the most part, integrated circuit boards. Recognizing and describing the hierarchical relationship among the recoverable
items is a major element of the model we will develop. As we will see, 
the relationship between the stocking of inventories of assemblies and 
recoverable components at one echelon and the performance at that and 
lower echelons is demonstrated through the equations for the average 
resupply time for each base. These equations are the backbone of the 
model.

The three-echelon system, as we have stated, consists of a group of 
bases and a depot. Each base is capable of performing only certain types 
of maintenance. Some bases, which form the second echelon, have extensive 
repair centers, called maintenance centers, collocated with them. The 
remaining bases, called operating bases, perform only a minimal amount 
of maintenance. These bases comprise the third and lowest echelon. By 
definition, an operating base does not have a collocated maintenance 
center. Lastly, the depot, the first echelon, has the capability to 
perform all types of repair.

Each customer demand occurs at a base and is always a requisition 
for a serviceable assembly. Furthermore, we assume each customer demand 
is triggered by the failure of an assembly. The failed assembly is 
then repaired at either a maintenance center or the depot. The location 
at which repair takes place depends only on the nature of the failure. 
Maintenance centers repair assemblies, using diagnostic equipment located 
in shops to isolate their defective components. In some instances, these 
components may also be repaired at the maintenance center, although in 
most real applications they are normally repaired at the depot. Since 
repair of assemblies is performed only at maintenance centers and the 
depot, components need to be stocked only at the depot and bases having a
collocated maintenance center. Assemblies, on the other hand, can be stocked at all locations.

Customer demands for serviceable assemblies are always satisfied by an organization at each base called base supply. If a serviceable spare assembly is immediately available from base supply, the customer demand is satisfied immediately. On the other hand, if no serviceable stock is on hand, the assembly is placed in a backorder status and the satisfaction of the customer demand is delayed. As we have described, the failed assembly is then either repaired at the base or sent to some higher echelon to be repaired.

Correspondingly, resupply of base supply can occur in one of two ways. If a failed assembly is repaired at a maintenance center, resupply occurs from the maintenance center; if the assembly is repaired at the depot, then resupply will occur from the depot. In either case, the organization that resupplies the base supply activity does so by exchanging a serviceable part for a failed part on a one-for-one basis. The resupply time—-that is, the time it takes to replace an assembly demanded from base supply with a serviceable one—depends on the source of resupply. For example, at a base having a collocated maintenance center, the average resupply time for base supply for an assembly equals the average repair time when the assembly is repaired at the maintenance center. This average repair time for the assembly clearly depends on the availability of the components needed to accomplish the repair. If adequate component stocks are on hand, repair will be completed with minimal delay. On the other hand, if repair of the assembly takes place at the depot, the average resupply time for base supply equals the average depot-to-base shipping time plus the expected waiting time before a serviceable assembly is available for
shipment to the base. This expected waiting time depends on the depot stock level for the assembly.

Additionally, in this three-echelon system we will assume the structure of the supply and maintenance system for an assembly family—an assembly and its subordinate components—can be represented by a tree as displayed in Fig. 1. Specifically, we assume that the set of bases can be partitioned into a collection of mutually exclusive and collectively exhaustive sets. Each set, which has exactly one maintenance center and a collection of operating bases logistically supported by the maintenance center, is called a Consolidated Support Family. Each operating base is assumed to receive all maintenance center-level resupply from the maintenance center in its Consolidated Support Family.

Bases at which a maintenance center is located may or may not have customers requesting serviceable assemblies. If the base has such customers, all requests for spare assemblies are made to the base maintenance center. The maintenance center performs all resupply for all customers at that base and is also a resupply point for all the operating bases in the same Consolidated Support Family. Some assemblies that fail at a base having a maintenance center are repaired at the base, but others may be sent to the depot for repair. We assume, for the sake of simplicity, that any failed assembly sent to a location for repair by a lower echelon base is not sent on and is repaired there.

In the next section we develop a mathematical model for the system we have described for a single hierarchically designed item. The model recognizes the hierarchical relationship among the recoverable
items, and it explicitly accounts for the relationship between the stocking of inventories of assemblies and components at one echelon and the performance at that echelon, as well as at other echelons. An analytic solution is obtained for the three-echelon stockage problem under the steady-state demand assumption. Under this assumption, the solution depends only on the mean resupply times rather than the resupply time distributions. Mathematical results are stated in terms of the Poisson assumption, but they can be readily extended to cover the compound Poisson case.

The third section contains a description of an algorithm for computing stock levels for the assembly and its components. A method for computing stock levels for systems consisting of a large number of assemblies is presented and illustrated in Section IV.

The model presented in this paper was developed originally to assist Air Force planners in their study of alternatives to the Air Force's
current two-echelon (depot-base) logistics system structure. In the current system, all bases have collocated maintenance centers. The model is being used to assess the differences in stockage requirements between the Air Force's current structure and the three-echelon structure described previously. Some of the ways the model can be used are illustrated in Section V with Air Force data. The illustrations provide some interesting insights into how inventory requirements change when a three-echelon system is operated rather than a two-echelon system.

The final section contains a brief summary and an example that indicates that using relatively sophisticated inventory models, such as the one described in this paper, will significantly improve system performance for the same level of investment over that obtained using simple models.
II. THE MODEL

The three-echelon system described in Section I is an extension of the two-echelon MOD-METRIC model [6]. For simplicity, we temporarily consider only one assembly family. Later we extend the results to the situation in which there are an arbitrary number of assembly families.

The model's objective is to determine the stock levels for the depot, bases with maintenance centers, and operating bases that minimize expected backorders for assemblies at all bases, subject to a constraint on total investment in assemblies and components. More precisely, a function measuring the expected backorder days is to be minimized. An assembly backorder exists whenever a demand for a serviceable assembly cannot be satisfied by base supply at the base at which the assembly failure occurred. Assembly resupply delays at the depot or a maintenance center are measured in the model only insofar as they influence backorders for assemblies at bases; component shortages are also measured indirectly. Observe that a backorder for an assembly at a base indicates that a customer demand is unsatisfied. Since components are only used to repair assemblies, a component backorder only delays repair of the assembly; it does not directly cause a customer's demand to be unsatisfied immediately. Consequently, the impact of assembly and component backorders on customer satisfaction is quite different. We describe the exact nature of the assembly/component interaction in detail later in this section.

Before presenting the mathematical model for this decision problem, we first state the underlying assumptions and then develop the average resupply time equations for assemblies for all bases. As will be shown, these equations are the backbone of the model. They represent
the manner in which assembly and component stock levels interact, and explicitly state how resupply capability for each echelon depends on the stock levels for all higher echelons. Having established these equations, we next determine the probability distributions describing the number of units in resupply for each location. Using these probabilities we can then calculate the expected number of assembly backorders outstanding at any time at each base; that is, we can state the model's objective function.

BASIC ASSUMPTIONS

The basis assumptions* underlying the model, in addition to those mentioned earlier, include:

1. Demand for assemblies at each base is a stationary Poisson process.
2. There is no lateral resupply among bases.**
3. All failed parts are repaired.
4. The probability of a failure of one assembly is independent of failures occurring for other assemblies.
5. Repair times are statistically independent.
6. There is no waiting or batching of items before starting the repair of any item.
7. The echelon at which repair is performed depends only on the complexity of the repair.
8. Each assembly failure repaired at a maintenance center is caused by a failure of at most a single component.

*A complete discussion of these assumptions and their implications is given in Ref. 7.

**This assumption is consistent with Air Force policy for computing recoverable item stock levels and was made for this reason.
AVERAGE RESUPPLY TIME EQUATION FOR ASSEMBLIES

After defining some necessary notation, we first derive the average resupply time equation for assemblies for each maintenance center and describe in detail the exact nature of the assembly/component interaction. Next, we develop the average resupply time equations for assemblies for both the bases having collocated maintenance centers and the operating bases subordinate to the maintenance center.

Let \( N = \{1, \ldots, n\} \) denote the set of locations having maintenance centers, and let \( N(k) \) denote the set of locations resupplied by \( k \in N \); let \( M = \{n+1, \ldots, n\} \) be the set of operating bases. An index \( j \) will refer to an operating base, an index \( k \) to a base having a collocated maintenance center, and \( 0 \) index will refer to the depot.

Let

\[
\begin{align*}
\lambda'_k & \triangleq \text{expected daily customer demand for assemblies at the base collocated with maintenance center } k, k \in N; \\
\lambda'_j & \triangleq \text{expected daily customer demand for assemblies at operating base } j, j \in M; \\
w_{vt} & \triangleq \text{probability that a failed assembly occurring at location } v \text{ is both repaired and resupplied by location } t; \\
\lambda_k & \triangleq \text{expected daily resupply requests for assemblies levied on maintenance center } k, k \in N.
\end{align*}
\]

The expected number of requests for resupply for assemblies levied on maintenance center \( k \) equals the expected number of daily assembly failures at the base collocated with maintenance center \( k \) plus the expected number of daily resupply requests for assemblies generated.
by lower echelon bases supported by maintenance center \( k \). Thus,

\[
\lambda_k = \lambda_k' + \sum_{j \in N(k)} w_{jk} \lambda_j' .
\]

Furthermore, let

\( r_k' \triangleq \) probability that an assembly failure at the base collocated with maintenance center \( k \) is repaired at maintenance center \( k \);

\( r_k \triangleq \) probability that an assembly arrival to maintenance center \( k \) is repaired there (see the next paragraph for a discussion of \( r_k \));

\( B_v \triangleq \) the expected assembly repair cycle time at location \( v \), measured in days, including repair time delay for unavailable components, \( v = 1, \ldots, n \);

\( A_{vt} \triangleq \) the expected order-and-ship-time between \( t \) and \( v \) for assemblies measured in days, where \( t = 0, \ldots, n \), and \( v = 1, \ldots, n \);

\( D \triangleq \) the expected depot repair cycle time for assemblies measured in days;

\( s_t \triangleq \) the stock level for the assembly at location \( t \).*

By assumption, all failed assemblies shipped from an operating base to maintenance center \( k \) are actually repaired at maintenance center \( k \); however, some assemblies that fail at the base collocated with maintenance center \( k \) are sent to the depot for repair. Then the expected number of failed assemblies arriving at maintenance center \( k \) each day that

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*The stock level is defined to be the on-hand plus on-order inventory minus backorders.
are repaired there equals the total number of maintenance center \( k \) expected daily resupply requests minus the expected number of assembly failures per day occurring at the base collocated with maintenance center \( k \) that require depot level repair. Thus the probability that an assembly arriving at maintenance center \( k \) will actually be repaired there is

\[
r_k = \frac{\lambda_k - \lambda_k' (1 - r_k')}{\lambda_k} = 1 - \frac{\lambda_k'}{\lambda_k} (1 - r_k').
\]

We are now ready to establish the equation for the average resupply time for assemblies for maintenance center \( k \), which we denote by \( T_k \). The expected assembly resupply time at maintenance center \( k \) equals the probability \( r_k \) that the assembly will be repaired at maintenance center \( k \) times the average maintenance center \( k \) assembly repair time, \( B_k \), plus the probability the assembly will be repaired at the depot, \( (1 - r_k) \), times the average depot-to-maintenance center \( k \) resupply time.

The average depot-to-maintenance center \( k \) resupply time equals the average assembly order-and-ship-time, \( A_{k0} \), plus the expected number of days before a serviceable assembly is available at the depot for shipment to the base. This depot delay time can be found using the following formula: expected delay days per demand equal the expected number of assemblies being delayed at any point in time—the expected number of depot backorders—divided by the expected daily depot demand rate for assemblies. Let

\[
\lambda = \sum_{t=1}^{n} w_t \lambda_t',
\]

the expected number of daily demands for assembly resupply placed on the depot.
It follows from assumptions 1 and 4 that \( p(x|\lambda D) \)--the probability that \( x \) assemblies are in resupply at the depot given that the expected demand over the depot resupply cycle is \( \lambda D \)--has a Poisson distribution with mean \( \lambda D \). Thus the expected number of delay days experienced by each assembly resupplied by the depot can be expressed as

\[
H(s_0) \equiv \frac{\text{expected depot backorders given the depot assembly stock level } s_0}{\text{expected daily depot demand for assemblies}},
\]

or

\[
H(s_0) = \frac{1}{\lambda} \sum_{x > s_0} (x - s_0)p(x|\lambda D).
\]

Combining these observations, we see that average assembly resupply time at maintenance center \( k \) can be expressed as

\[
T_k = r_k B_k + (1 - r_k)(A_k 0 + H(s_0)).
\]

This equation indicates how the depot stock affects the average resupply time for assemblies at maintenance center \( k \).

But \( T_k \) also depends on the component stock levels. The average assembly repair cycle time \( B_k \) at maintenance center \( k \) is the sum of two terms. The first reflects the portion of the repair cycle time related to the operation of the maintenance and transportation systems.

In particular, this term represents administrative delay time plus queuing time, plus fault isolation time, plus component remove-and-replace time. It also includes transportation time if the assembly is sent to maintenance center \( k \) for repair by an operating base. Denote this portion of the average assembly repair cycle time by \( R_k \).

The second term reflects expected delay in completing an assembly's repair due to the shortage of serviceable components. If a particular component is the cause of the assembly's failure and no serviceable
component of that type is on hand, then the assembly repair time is lengthened. Consequently \( B_k \) depends on component stock levels at both maintenance center \( k \) and the depot. Let \( G_k(s_{1k}, \ldots, s_{mk}; s_{10}, \ldots, s_{m0}) \equiv G_k \) represent the average delay days per demand in maintenance center \( k \) assembly repair given component \( i \) stock level \( s_{ik} \) at base \( k \) and component \( i \) depot stock level \( s_{10} \), where \( m \) represents the number of different components in the assembly. We will now develop an explicit expression for \( G_k \).

Recall we assume that if an assembly is repaired at a maintenance center, at most one component needs to be replaced. Then the expected delay in assembly repair time at maintenance center \( k \), given the failure of component \( i \), is the expected number of components of type \( i \) at maintenance center \( k \) on which delay is being incurred at any point in time—the expected backorders for component \( i \) at maintenance center \( k \)—divided by the expected component \( i \) daily removal or demand rate at maintenance center \( k \). Denote this conditional expected delay by \( g_{ik} \), that is,

\[
g_{ik} \triangleq \frac{\text{expected backorders for component } i \text{ at maintenance center } k \text{ at any point in time}}{\text{expected daily removal rate for component } i \text{ at maintenance center } k'},
\]

or

\[
g_{ik} \triangleq \frac{1}{\lambda_{ik}} \sum_{x > s_{ik}} (x - s_{ik}) p(x \mid \lambda_{ik} T_{ik}),
\]

where
\( \lambda_{ik} \) \( \Delta \) average number of daily removals of component \( i \) at maintenance center \( k \);

\( r_{ik} \) \( \Delta \) probability that if component \( i \) fails at location \( k \), the component will be repaired at maintenance center \( k \);

\( B_{ik} \) \( \Delta \) average component \( i \) repair time at maintenance center \( k \);

\( A^i_{k0} \) \( \Delta \) average component \( i \) order-and-ship-time from the depot to maintenance center \( k \);

\( D_i \) \( \Delta \) average depot repair cycle time for component \( i \);

\( T_{ik} \) \( \Delta \) average resupply time for component \( i \) at maintenance center \( k \).

The average resupply time equation \( T_{ik} \) for component \( i \) at maintenance center \( k \) equals the probability \( r_{ik} \) that the component is repaired at maintenance center \( k \), times the average maintenance center \( k \) repair time, \( B_{ik} \), for component \( i \), plus the probability the component will be repaired at the depot, \( 1 - r_{ik} \), times the sum of the depot-to-maintenance center order-and-ship-time, \( A^i_{k0} \), and the expected number of days before a serviceable component is available at the depot for shipment to the maintenance center. We denote this latter delay by \( H_{ik}(s_{i0}) \), where

\[
H_{ik}(s_{i0}) \equal{} \frac{\text{expected number of unsatisfied depot demands for component } i \text{ at any point in time given the depot stock level for component } i \text{ is } s_{i0}}{\text{expected daily depot demand rate for component } i},
\]

or

\[
H_{ik}(s_{i0}) \equal{} \frac{1}{\theta_i} \sum_{x > s_{i0}} (x - s_{i0}) p(x|\theta_i, D_i),
\]

and where
\[ \theta_i = \sum_{k=1}^{n_i} \lambda_{ik}(1 - r_{ik}), \]

and

\[
p(x|\theta_{Di}) = \text{the probability of } x \text{ units of component } i \text{ in depot resupply at any point in time}
\]

\[
e^{-\theta_{Di}} (\theta_{Di})^x / x!.
\]

Thus

\[
T_{ik} = r_{ik}B_{ik} + (1 - r_{ik})(A_{k0}^i + H_{ik}(s_{i0})).
\]

The probability that an assembly failure repaired at maintenance center \( k \) is caused by component \( i \) is \( \lambda_{ik}/r_k\lambda_k \). Then the expected delay time in repair of an assembly at maintenance center \( k \) due to the unavailability of component stock is found by multiplying the conditional delays \( g_{ik} \) by \( \lambda_{ik}/r_k\lambda_k \) and summing over component types. Thus

\[
G_k = \sum_{i=1}^{m} \frac{\lambda_{ik}}{r_k\lambda_k} g_{ik}.
\]

We have now seen that \( B_k \), the average repair time for an assembly at maintenance center \( k \), can be represented as the sum of two terms, \( R_k \) and \( G_k \). We therefore have shown that the average resupply time for an assembly at maintenance center \( k \) can be represented as

\[
T_k = r_kB_k + (1 - r_k)(A_{k0} + H(s_0))
\]

\[
= r_k(R_k + G_k) + (1 - r_k)(A_{k0} + H(s_0)).
\]
This equation indicates how the depot stock level for the assembly and the maintenance center \( k \) and depot component stock levels affect the assembly resupply time at maintenance center \( k \).

We now develop the average resupply time equation for base \( k \), \( k \in N \). Since the base \( k \) is physically collocated with the maintenance center, no assembly stock will be allocated exclusively to it. Immediate resupply is assumed to be always available (zero lead time) for customers at the base from the maintenance center, assuming serviceable stock is on hand. From a system's viewpoint there is no advantage to allocating exclusive stock to the base since all assemblies assigned there, by assumption, would be unavailable for redistribution. This would degrade expected system performance. Since all resupply for customer demands at base \( k \) comes from maintenance center \( k \), the average resupply time for an assembly for base \( k \), call it \( T'_k \), equals the expected number of delay days before a serviceable assembly becomes available at maintenance center \( k \). Therefore,

\[
T'_k = \sum_{x>s_k} \frac{(x-s_k)p(x|\lambda_k T_k)}{\lambda_k}, \quad k \in N.
\]

The average resupply time equation for operating base \( j \), call it \( T_j \), can be found using the same method we used to determine \( T'_k \). Let us temporarily assume that location \( j \) receives resupply from maintenance center \( k \). Then \( T_j \) equals \( r_j \), the probability that the failed assembly is repaired at base \( j \), times the average base repair time for the assembly, \( B_j \), plus the probability \( w_{jk} \) that the assembly is repaired at maintenance center \( k \), times the sum of maintenance center \( k \) to operating base \( j \) order-and-ship time, \( A_{jk} \), and the expected delay in
shipment due to the unavailability of a serviceable assembly at maintenance center $k$ (call this quantity $H_k(s_k)$), plus the probability that the assembly is shipped to the depot for repair, $w_{j0}$, times the sum of the depot-to-base $j$ order-and-ship time, $A_{j0}$, and the expected delay before a serviceable assembly is available at the depot for shipment to the base, $H(s_0)$. In general, let $g(j) \in N$ denote the maintenance center for which $w_{jk} > 0$. Then we may express the average resupply time for operating base $j$ as

$$T_j = r_jB_j + w_{j,g(j)}(A_{j,g(j)} + H_g(j)(s_{g(j)})) + w_{j0}(A_{j0} + H(s_0)).$$

The average number of days a resupply request for an assembly levied on maintenance center $k$ is delayed before a serviceable assembly becomes available for shipment given the stock level $s_k$ was denoted by $H_k(s_k)$. This function is

$$H_k(s_k) \triangleq \frac{\text{expected maintenance center } k \text{ assembly backorders at any point in time given the maintenance center } k \text{ stock level of } s_k}{\text{expected daily assembly demand at maintenance center } k}$$

or

$$H_k(s_k) \triangleq \frac{1}{\lambda_k} \sum_{x > s_k} (x - s_k)p(x|\lambda_k, T_k),$$

where $p(x|\lambda_k, T_k)$ is the probability of $x$ assemblies in the maintenance center $k$ resupply system. In the expression, $p(x|\lambda_k, T_k)$ is approximated by a Poisson distribution whose mean is $\lambda_k T_k$. 

MATHEMATICAL STATEMENT OF THE MODEL

The goal of the model is to find the assembly and component stock levels for each location that minimize the system's average number of backorders for customer demands for assemblies outstanding at any point in time subject to a restriction on inventory investment. For each operating base, that is, for each $j \in M$, we express the average number of outstanding backorders at any time for customer demands for assemblies as

$$\sum_{x > s_j} (x - s_j)p(x|\lambda_j T_j),$$

where $s_j$ represents the stock level for the assembly at base $j$. Recall that no stock is explicitly allocated to the base at a location having a maintenance center. All stock at base $k$ is under the administrative control of the maintenance center. Thus the average number of customer backorders for assemblies at base $k$ at any time are

$$\sum_{x > 0} xp(x|\lambda_k T_k') = \lambda_k T_k'.$$

Therefore the objective function for the model is

$$\sum_{j \in M} \left\{ \sum_{x > s_j} (x - s_j)p(x|\lambda_j T_j) \right\} + \sum_{k \in N} \lambda_k T_k'.$$

Note that the backorder expression for each base depends on its average resupply time.

The inventory investment constraint in the model states that the system investment in assemblies and components cannot exceed some maximum
value. If

\[ c = \text{the unit cost of an assembly}, \]
\[ c_i = \text{the unit cost of component } i, \]
\[ s_{it} = \text{stock level for component } i \text{ at location } t, \text{ and} \]
\[ C = \text{the available budget}, \]

the mathematical representation of the investment constraint is

\[ c \sum_{t=0}^{n} s_t + \sum_{i=1}^{m} c_i \sum_{t=0}^{n} s_{it} \leq C. \]

Combining the above, we write the mathematical statement of the model as follows:

\[ \min \sum_{j \in M} \left\{ \sum_{x \geq s_j} (x - s_j) p(x \mid \lambda_j' T_j) \right\} + \sum_{k \in N} \lambda_k' T_k' \]

subject to \[ c \sum_{t=0}^{n} s_t + \sum_{i=1}^{m} c_i \sum_{t=0}^{n} s_{it} \leq C, \] (P)

where \( s_t \) and \( s_{it} \) are non-negative integers, and \( t = 0, \ldots, n \) and \( i = 1, \ldots, m \).

We will call this problem P.
III. AN ALGORITHM FOR DETERMINING STOCK LEVELS

The model's objective function represents the total system backorders existing at any point in time for customer demand for the assembly. As stated in the previous section, the expected backorder expression for the assembly for each operating base, that is, for each $j \in M$, is

$$\sum_{x > s_j} (x - s_j)p(x|\lambda_j, T_j),$$

which depends on $T_j$. But $T_j$ is a function of both depot and maintenance center assembly and component stock levels. Similarly, the expected backorder expression for customer demands for assemblies for each $k \in N$ depends on depot assembly and component stock levels as well as maintenance center stock levels for components. Consequently, problem $P$ is not a separable programming problem. Furthermore, the objective function need not be convex.

The strategy we employ to solve problem $P$ circumvents these difficulties. Specifically, we will solve a finite sequence of subproblems, each corresponding to a fixed investment in assemblies. For a fixed total budget $C$, it is possible to purchase either 0, 1, ..., or $Q$ assemblies, where $Q$ is the greatest integer less than or equal to $C/c$. The proposed algorithm requires evaluating the solution—at least implicitly—to $Q + 1$ subproblems, one for each possible investment in assemblies. Each subproblem can be stated as follows:
\[
\min \left\{ \min_{s_{it}} \sum_{j \in M} \sum_{x > s_j} (x - s_j)p(x | \lambda'_j T_j) + \sum_{k \in N} \lambda'_k T'_k : s_{it} \text{ is fixed for all } i \text{ and } t \right\}
\]

where \( s_{it} \) is fixed for all \( i \) and \( t \) (thereby establishing the component delay in \( T_k \)), and \( s_t \) and \( s_{it} \) are non-negative integers;

\[
\sum_{t=0}^{n} s_t = \bar{N},
\]

where \( \bar{N} \) represents the number of assemblies available for distribution, and \( \bar{N} \) is the greatest integer less than or equal to

\[
\left( C - \sum_{i=1}^{m} c_i \sum_{t=0}^{n_i} s_{it} \right) / \bar{N} \right) \}
\]

Consequently, each subproblem can be partitioned into two parts, one corresponding to components and the other to assemblies. The first part establishes the manner in which a limited budget \( (C - \bar{cN}) \) is allocated among the \( m \) components. Once a specific allocation of components to the depot and maintenance centers has been determined, the expected delay in assembly repair time due to components is known. This in turn affects the resupply time and ultimately the expected backorders for assemblies at each base. The optimal allocation of the \( \bar{N} \) assemblies among the bases and depot—which corresponds to the second portion of the above problem—is obtained knowing the expected delay in assembly repair time at each maintenance center.
Suppose \( U = C - cN \) dollars are available for investment in components. How should it be allocated among the \( m \) components? Clearly, we should make the investment so that the total expected customer backorders for assemblies are reduced by the greatest amount. If all \( n_1 \) Consolidated Support Families are identical, it is not hard to show that this corresponds to an allocation in which the stock levels are selected so that total weighted expected delay in assembly repair due to components is minimized, where the weights reflect the expected number of daily assembly failures repaired at a maintenance center. Although only an approximation in cases where the Consolidated Support Families are not identical, we will use this objective to determine the allocation of the available \( U \) dollars among the components for each subproblem. A considerable amount of experimentation was accrued by the Air Force Logistics Command using this type of approximation in the MOD-METRIC model [4]. The approximation produced the optimal allocation in all cases. Thus, the component stock levels in each of the \( Q + 1 \) subproblems are obtained by solving the following problem, called Problem Pl:

\[
\begin{align*}
\min & \sum_{k \in N} r_k \lambda_k \mathbb{E}_{k} = \min \sum_{k \in N} \sum_{i \geq s_{ik}} (x - s_{ik}) p(x | \lambda_{ik} T_{ik}) \\
\text{subject to} & \sum_{i=1}^{m} \left( c_{i1} s_{i10} + \sum_{k=1}^{n_1} c_{i1k} s_{ik} \right) \leq U, \\
\end{align*}
\]  

(P1)

where \( s_{it} \) is a non-negative integer.

Observe that minimizing the total weighted expected delay due to components is equivalent to minimizing total component backorders. The solution to this
two-echelon component problem can be easily obtained using the method described in either Ref. 3 or Ref. 5.

When the component stock levels have been established, we must then determine the optimal method for allocating the $\bar{N}$ assemblies among the depot and bases. In particular, we must solve the following problem, called problem P2.

$$\min \sum_{j \in M} \sum_{x \in S_j} (x - s_j)p(x | \lambda_j T_j) + \sum_{k \in N} \lambda_k T_k$$

subject to $\sum_{t=0}^{n_1} s_t = \bar{N},$ (P2)

where $s_t$ is a non-negative.

Due to the interaction of stock levels among echelons, problem P2 is neither convex nor separable. We therefore employ a simple partitioning procedure to obtain its solution. The algorithm for solving this three-echelon problem is based on the system's nested tree structure as displayed earlier in Fig. 1. The algorithm works up the tree by solving a sequence of independent two-echelon subproblems, one set of problems for each Consolidated Support Family; the solution to these problems are then combined in an appropriate way to solve problem P2. We now discuss the algorithm for solving problem P2 in detail.

Suppose the depot stock level is fixed at $s_0$, and assume that a total of $\bar{N}_k$ assemblies are available for allocation to all bases in Consolidated Support Family $k$. Then the optimal allocation of the $\bar{N}_k$ assemblies among the bases can be found by solving the following problem, called problem P3:
\[ B_k(\overline{N}_k; s_0) \triangleq \min \lambda' \Gamma' + \sum_{j \in N(k)} \sum_{x \succ s_j} (x - s_j)p(x|\lambda'_j \Gamma_j) \]

subject to \( s_0 \) fixed,

\[ \sum_{j \in N(k)} s_j + s_k = \overline{N}_k, \text{ and} \]

\( s_j \) a non-negative integer, \( j \in N(k), s_k \in R_k, \)

where \( R_k \) represents a set whose elements are the candidate values for \( s_k \). This problem may not be convex and is not separable. To obtain its solution we solve the subproblems

\[ h(s_k, s_0) \triangleq \min \sum_{j \in N(k)} \sum_{x \succ s_j} (x - s_j)p(x|\lambda'_j \Gamma_j) \]

subject to \( s_0 \) and \( s_k \) fixed,

\[ \sum_{j \in N(k)} s_j = \overline{N}_k - s_k, \text{ and} \]

\( s_j \) a non-negative integer,

via marginal analysis (valid because of the convexity of the objective function). Then the solution to problem P3 is found by solving

\[ \min_{s_k \in R_k} h(s_k, s_0) + \lambda'_k \Gamma_k \]

Since the optimal value of \( \overline{N}_k \) is unknown, Problem P3 is solved for all values of \( \overline{N}_k \in R_k \), where \( R_k \) represents the set of possible total family \( k \) stock levels.
To solve problem P2 we use the solutions obtained for each Consolidated Support Family. More specifically, to solve P2 we solve problem P4:

$$B(\overline{N}) = \min_{k=1}^{n_1} B_k(\overline{N}_k; s_0)$$

subject to

$$\sum_{k=1}^{n_1} \overline{N}_k + s_0 = \overline{N},$$

$$\overline{N}_k \in \overline{R}_k, \quad s_0 \in R_0,$$

where $R_0$ represents the set of candidate depot assembly stock levels. A dynamic programming algorithm is used to compute the optimal solution.

The amount of effort required to solve problems P3 and P4 depends on the cardinality of the sets $R_0$, $R_k$, and $\overline{R}_k$. Fortunately, the number of stock levels that need to be explicitly considered for any location or Consolidated Support Family is generally not large. This is chiefly due to the nature of the functions $H(s_0)$ and $H_k(s_k)$, which rise very sharply for stock levels below the mean demand, and approach 0 rapidly for stock levels above the mean. Experiments [4] on similar problems indicate that the cardinality of the $R_0$ and $R_k$ sets should rarely exceed 10.

To find $\overline{R}_k$, we may first compute the total expected daily removals for family $k$, call it $\overline{A}_k$. An estimate of the average family $k$ resupply time, $\overline{r}_k$, is found by weighting the expected resupply times for each location in the family by the proportion of family $k$ daily demand occurring at that location, and then summing these quantities over locations.

---

*An illustration of this fact is given in Ref. 4, p. 479.*
An estimate of the depot and base \( k \) optimal stock levels obtained, for example, using the method described in Ref. 5 is employed to estimate the value of \( T_j \) and \( T_k \) used in the averaging. Using these values we solve problem P5:

\[
\begin{align*}
\min & \sum_{k=1}^{n_1} \sum_{x=S_k}^{s_k} (x - \bar{s}_k)p(x | \bar{\lambda}_k, \bar{T}_k) \\
\text{subject to} & \sum_{k=1}^{n_1} \bar{s}_k = \bar{N} - \bar{s}_0, \text{ and} \end{align*}
\]

where \( \bar{s}_0 \) is the estimate of the optimal depot stock level. Marginal analysis is used to obtain the optimal solution since the objective function is convex. \( \bar{R}_k \) is constructed based on the estimate \( \bar{s}_k \). The minimum element of \( \bar{R}_k \) can be set at \( \max\{a\bar{s}_k, \bar{s}_k - b\} \) and the largest value at \( \min\{c\bar{s}_k, \bar{s}_k + d\} \). The values of \( a, b, c, \) and \( d \) can be selected as a function of the size of \( \bar{s}_k \). For larger values of \( \bar{s}_k \), the range should be larger. Limited computational experience on a similar problem using this technique has shown that a maximum cardinality of 15 for \( \bar{R}_k \) is adequate [6]. However, the best method for determining \( R_0 \), \( R_k \), and \( \bar{R}_k \) remains an open question.

Combining the above observations, we can state a basic algorithm for determining item stock levels:

**Initialization Step:** Establish an upper and lower bound constraint on assembly investment. Let \( u \) and \( \ell \) represent these upper and lower
limits, and \( z' \) represent the best known objective function value. Set 
\( z' = \infty \) and \( U = C - \lambda \). Assume \( C \) is an integer multiple of \( c \).

**Step 1.** Solve problem P6:

\[
\min \sum_{k} r_{k} \lambda_{k} G_{k}
\]

subject to \( \sum_{i=1}^{m} \left( c_{i} s_{i0} + \sum_{k=1}^{n_{1}} c_{i} s_{ik} \right) \leq U, \) \( \) (P6)

where \( s_{ik} \) is a non-negative integer.

**Step 2.** Solve problem P7:

\[
\min z = \sum_{k=1}^{n_{1}} \lambda'_{k} T'_{k} + \sum_{j \in M} \left\{ \sum_{x > s_{j}} (x - s_{j}) p(x|\lambda'_{j} T_{j}) \right\}
\]

subject to \( \sum_{t=0}^{n} c s_{t} = C - U, \) \( \) (P7)

where \( T_{j} \) and \( T'_{k} \) are calculated using the stock levels computed in Step 1, and \( s_{t} \) is a non-negative integer.

**Step 3.** If \( z > z' \), go to Step 4; otherwise, set \( z' = z \) and retain the corresponding stock levels as the incumbent stock levels. Go to Step 4.

**Step 4.** Decrement \( U \) by \( c \). If \( C - U > u \), stop; otherwise, return to Step 1.
The algorithm outlined above suggests a rather tedious method for establishing the optimal investment level in assemblies and components. We will now see that the number of assembly budgets that need to be explicitly examined is generally quite small. First observe that the values of $u$ and $l$ should be selected considering the marginal impact of investment in components on expected backorders for customer demands for assemblies. The marginal impact is negligible when investment in components is large; on the other hand, assembly resupply times are increased substantially, thereby increasing total assembly system backorders, when the investment in components is relatively low. Roughly stated, we would like to allocate the available budget $C$ in such a way that the marginal reduction in backorders for customer demands for assemblies per dollar invested in components equals the marginal reduction per dollar invested in the assembly. The values of $u$ and $l$ should reflect this goal.

It is easy to obtain an estimate of the optimal total component investment. Suppose we estimate component and assembly depot stock levels, perhaps using the method described in Ref. 5. The total cost of this investment can be determined and subtracted from the available budget $C$. We next assume that each of the $n_1$ Consolidated Support Families have the same demand rates for assemblies and the same number of operating bases and that all demand for assemblies takes place at the corresponding base collocated with a maintenance center. Then a crude estimate of the optimal investment in components for each $k \in N$ corresponds to the investment level for which the partial derivative of maintenance center $k$'s average resupply time with respect to dollar investment in components at
the maintenance center $k$ equals $1/c$. We can then easily estimate
the optimal total system investment in components by multiplying the
Consolidated Support Family estimate by $n_1$ and adding to this value
the estimated required depot component investment.

Once $u$ and $l$ have been established, the search for the optimal
partitioning of the budget is simplified by exploiting the apparent
strict quasi-convexity of the total expected customer back-orders for
assemblies as a function of investment in assemblies. Using the
Fibonacci search algorithm, we see that it is necessary to examine only
a very small number of assembly investment levels explicitly. For
example, if $Q = 600$, only 13 problems need to be solved explicitly.
Each problem requires solving two subproblems. The first subproblem
has the form of problem P6 in Step 1 of the algorithm, and the second
subproblem has the form of problem P7 in Step 2. The value of $U$, of
course, corresponds to a specific total investment in assemblies.

Figure 2 displays the results of applying the proposed algorithm to
one assembly/component family for the Air Force's F-15 aircraft. The
graph relates total expected customer backorders for assemblies to the
proportion of the total system budget invested in assemblies. As indi-
cated on the graph, approximately two-thirds of the total budget should
be allocated to the assembly. Investing either a greater or lesser
proportion of the total budget in the assembly increases total back-
orders. A substantial misallocation of the available budget can
seriously degrade system performance. For example, investing one-half
rather than two-thirds of the budget in the assembly causes expected
customer backorders for assemblies to double.
Fig. 2 -- System backorders for assemblies as a function of the percentage of total budget allocated to assemblies
IV. MULTIPLE ASSEMBLY PROBLEMS

We have developed a model of a three-echelon inventory system for one assembly type and its subordinate components. If this model could not be easily extended to multiple assembly problems, it would be of little practical use. We now demonstrate how it can be extended.

In practice, problem P is solved for a finite number of budgets $C_1^i, C_2^i, \ldots, C_{q_i}^i$ for each assembly family $i$. The number of budgets explicitly examined, $q_i$, depends, in practice, on the expected assembly failure rate. Using the data obtained when solving these $q_i$ problems, it is possible to plot performance versus investment. Figure 3 illustrates this trade-off data for one P-15 assembly family. A piece-wise linear function can then be constructed to approximate the entire performance

![Graph](image)

Fig. 3 -- System backorders for assemblies as a function of various levels of total investment
versus investment trade-off curve for this assembly family as shown in Fig. 4. If this curve is not convex, then replace it by its greatest convex minorant.

Fig. 4 -- Piece-wise linear approximation of performance vs. investment relationship

After the convex performance/investment trade-off curves are developed for each assembly family, they are combined to produce a curve relating total customer backorders for all types of assemblies as a function of investment in all assembly and component types. This curve is constructed by applying a simple marginal analysis algorithm. The first point on the system performance curve corresponds to the total
expected customer backorders for an assembly type $i$ when investing the minimal amount, $C_i$, in assembly family $i$.

Let $B_i(C_j)$ represent the total expected customer backorders for assembly $i$ given the investment in assembly family $i$ is $C_j$. Then the first point on the system performance curve is $\sum_i B_i(C_i^i)$ corresponding to an investment of $\sum_i C_i^i$. Next compute

$$\Delta_i^1 = \frac{B_i(C_i^1) - B_i(C_i^2)}{C_i^2 - C_i^1}$$

for each assembly family. Then $\Delta_i^1$ measures the marginal reduction in system customer backorders for assemblies per dollar invested in assembly family $i$. Suppose $\Delta_i^k = \max \Delta_i^1$. Then the second point on the curve is $\sum_i B_i(C_i^i) - \Delta_i^1$ corresponding to an investment of $\sum_i C_i^i + C_2^k - C_1^k$. Now compute

$$\Delta_i^k = \frac{B_i(C_i^k) - B_i(C_i^{k+1})}{C_i^{k+1} - C_i^k}$$

and find the minimum of $\Delta_i^1,\Delta_i^2,\ldots,\Delta_i^k,\ldots,\Delta_i^{I}$, where $I$ represents the number of assembly families in the problem. The third data point is determined by computing the new total backorders and new total investment. Continue in this manner until all the available individual assembly family data have been used.

We illustrate the algorithm with a two-assembly family example.

Table 1 shows the results of solving problem $P$ several times for each of the two families. In particular, the data in the table represent the investment versus backorder data for each individual family. Table 2
Table 1
DATA FOR TWO F-15 ASSEMBLY FAMILIES

<table>
<thead>
<tr>
<th>Budget ($)</th>
<th>Assembly Family 1</th>
<th>Assembly Family 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Assembly Backorders</td>
<td>Budget ($)</td>
</tr>
<tr>
<td>231,804</td>
<td>.1747</td>
<td>1,036,100</td>
</tr>
<tr>
<td>251,204</td>
<td>.1108</td>
<td>1,168,100</td>
</tr>
<tr>
<td>270,604</td>
<td>.0736</td>
<td>1,300,100</td>
</tr>
<tr>
<td>290,004</td>
<td>.0448</td>
<td>1,432,100</td>
</tr>
<tr>
<td>309,404</td>
<td>.0303</td>
<td>1,564,100</td>
</tr>
<tr>
<td>328,804</td>
<td>.0178</td>
<td>1,682,400</td>
</tr>
<tr>
<td>350,530</td>
<td>.0114</td>
<td>1,814,400</td>
</tr>
<tr>
<td>367,604</td>
<td>.0069</td>
<td>--</td>
</tr>
</tbody>
</table>

Table 2
REDUCTION IN BACKORDERS PER DOLLAR INVESTED FOR EACH INCREMENT IN INVESTMENT FOR EACH ASSEMBLY

<table>
<thead>
<tr>
<th>Assembly Family 1</th>
<th>Assembly Family 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_1 = 3.2938 \times 10^{-6}$</td>
<td>$\Delta_1^2 = 1.9409 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\Delta_2 = 1.9175 \times 10^{-6}$</td>
<td>$\Delta_2^2 = 1.8000 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\Delta_3 = 1.4845 \times 10^{-6}$</td>
<td>$\Delta_3^2 = 9.2955 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Delta_4 = 7.4742 \times 10^{-7}$</td>
<td>$\Delta_4^2 = 7.1970 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Delta_5 = 6.4432 \times 10^{-7}$</td>
<td>$\Delta_5^2 = 4.9619 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Delta_6 = 2.9457 \times 10^{-7}$</td>
<td>$\Delta_6^2 = 2.6288 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Delta_7 = 2.6355 \times 10^{-7}$</td>
<td>--</td>
</tr>
</tbody>
</table>

contains the values of the marginal reduction in backorders per dollar invested for each incremental investment level for each assembly. These values are denoted $\Delta^i_j$, where

$$\Delta^i_j = \frac{B^i_j(C^i_j) - B^i_{j+1}(C^i_{j+1})}{C^i_{j+1} - C^i_j}.$$  

The results of applying the algorithm to this two-assembly example are given in Table 3. These data show how system performance—total customer backorders for assemblies—depends on system investment.

These data can then be used to determine what the individual assembly family investment levels should be so that a target system budget or performance goal is achieved. For example, suppose planners decide that approximately $1.9$ million is available for investment in these two assembly families. The closest tabulated value corresponds to a total investment of $1,892,904$. This point in turn corresponds to an investment of $328,804$ in the first assembly family and an investment of $1,564,100$ in the second assembly family.

The tabulated values can be used in a second way as well. Suppose the planners decide they want no more than .1 expected customer backorders attributed to these two assemblies at any point in time. Then a budget of $2,032,930$ must be made available for these assembly families with $350,530$ and $1,682,400$ budgeted for the first and second families, respectively.
Table 3

BACKORDER AND INVESTMENT DATA
FOR THE COMBINED SYSTEM

(Two assembly families)

<table>
<thead>
<tr>
<th>Investment ($)</th>
<th>Backorders</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,267,904</td>
<td>1.0327</td>
</tr>
<tr>
<td>1,287,304</td>
<td>.9688</td>
</tr>
<tr>
<td>1,419,304</td>
<td>.7126</td>
</tr>
<tr>
<td>1,438,704</td>
<td>.6754</td>
</tr>
<tr>
<td>1,570,704</td>
<td>.4378</td>
</tr>
<tr>
<td>1,590,104</td>
<td>.4090</td>
</tr>
<tr>
<td>1,722,104</td>
<td>.2863</td>
</tr>
<tr>
<td>1,741,504</td>
<td>.2718</td>
</tr>
<tr>
<td>1,873,504</td>
<td>.1768</td>
</tr>
<tr>
<td>1,892,904</td>
<td>.1643</td>
</tr>
<tr>
<td>2,011,204</td>
<td>.1056</td>
</tr>
<tr>
<td>2,032,930</td>
<td>.0992</td>
</tr>
<tr>
<td>2,050,004</td>
<td>.0947</td>
</tr>
<tr>
<td>2,182,004</td>
<td>.0600</td>
</tr>
</tbody>
</table>
V. AN EXAMPLE OF ANALYSIS USING THE MODEL

As we have mentioned, the main reason for developing the model described in this paper was to assess the impact of inventory investment in recoverable spares of changing the Air Force's current logistics system structure. To illustrate the use of the model in this regard, we postulated an operating environment involving three bases. In this example, aircraft correspond to customers. We assume there are two squadrons of F-15 aircraft stationed in the first base; one squadron of F-15 is stationed in each of the other two bases. Flying activities per aircraft are assumed to be the same at each of the three bases. A maintenance center is assumed to exist at each base. The average repair cycle time is assumed to be 4 days, order and shipping time from depot to each of the bases is 12 days, and the depot repair cycle time is 52 days for assemblies.

We selected 18 high demand assemblies related to the F-15 avionics system as our data base. The number of components associated with these 18 assemblies is 224 items. For these assemblies, 80 to 95 percent of the malfunctioning items can be repaired at the maintenance center at the base. These avionics type items were selected for analysis because it would make sense in the real world to consolidate their maintenance at some central location.

We made five sets of computer runs, and for each run we generated a tradeoff curve between inventory investment and performance expressed in terms of the expected number of backorders on assemblies at the three bases:
Case 1 is the base case in which the structure of the logistics system remains the same as the current one. In other words, maintenance is performed at maintenance centers at each location, and other system parameters are the same as those described above.

In Case 2 it is assumed that repairs will be performed at the largest base, namely base 1. Thus base 1 has a maintenance center and bases 2 and 3 do not. It is assumed to take an average of four days to ship defective assemblies to base 1 from bases 2 and 3, and to ship serviceables back from base 1 to bases 2 and 3. Shipping time from base 1 to the depot, and depot repair times remain the same as in the base case.

In Case 3 all parameters remain unchanged from Case 2 except that it is assumed that the proportion of repairs that cannot be accomplished at the maintenance center for every assembly has been reduced by 50 percent. The reduction of 50 percent is hypothetical and is not based on any engineering study. Under this structure, however, the Air Force has in some actual tests reduced the proportion of items that have to be returned to the depot for repair by roughly this amount.

In Case 4 we based our calculation on the same proportion of failures being repaired at the depot as in the base case and also using the same system parameters, except the shipping time from bases 2 and 3 to base 1, and the transportation time for shipping items to base 1 from bases 2 and 3, have been reduced from 4 to 2 days. This was done to check the effect of the responsiveness of the transportation system on this alternative type of structure.

Finally, in Case 5 we assume that the shipping time can be set at two days, and it takes only two days to ship to the maintenance center.
Furthermore, we assume that the proportion of repairs that cannot be accomplished at the maintenance center has been reduced as in Case 3.

The results are shown in Fig. 5. Each curve summarizes the analysis corresponding to each of the five cases described above. Each curve portrays the impact on the performance of the support system as a function of investment in inventory of spares and conditioned on system parameters as described above. The performance is stated in terms of the number of assembly backorders throughout the system. For example, if we take the base case trade-off curve, we see that for an investment of $45 million, there will be 9 backorders on the average.

A comparison of Case 2 with the base case shows that introducing a change in the logistics structure of the type described would imply that additional spares requirements of nearly $5 million would be needed to maintain the same level of performance. However, Case 3 results show that if the maintenance capability at the intermediate level could be enhanced to the extent that a greater proportion of defective assemblies could be fixed at the maintenance center instead of having to be shipped to the depot, additional spares requirements would be minimal. Even without the assumed improvement in the maintenance productivity, if it takes only two days to ship serviceable and broken assemblies from operating bases to the maintenance center, then the alternative structure does not require any additional spares investment as depicted in the comparison between Case 1 and Case 4. It was mentioned earlier that in the alternative structure, an additional requirement for assemblies may be offset by a reduction in component stockage. When Case 4 was compared to Case 1 at
Fig. 5 -- Analysis of structures: maintenance centralization vs. decentralization
a performance level of 25 assembly backorders, it was found that the composition of stockage had changed as follows: For Case 1, inventory investment for components was $13.8 million and for the assemblies, $27.5 million. For Case 4, they were $11.3 million and $28.4 million, respectively. Thus under the hypothesized operating conditions, a saving in component stockage investment more than offset a need for more assemblies.

Finally, Case 5 suggests that if the alternative three-echelon structure can improve maintenance productivity as well as rely on a highly responsive transportation system, economic gains in the area of spares requirements are possible.
VI. SUMMARY AND CONCLUDING COMMENTS

A three-echelon, two-indentured inventory model was developed that can be used to establish assembly and component stock levels for an arbitrary number of assembly families. The model's development was based on demonstrating how assembly and component stock levels influence the average resupply time equations and ultimately the expected customer backorders outstanding at any time at any location for assemblies. Furthermore, an algorithm was presented for computing item stock levels for each location.

The model has been compared with two other approaches: (1) the Air Force's initial provisioning technique [1] and (2) a METRIC-like optimization model.

The Air Force initial provisioning technique is not an optimization model. Requirements are established by determining the quantity of each item needed to fill the steady-state resupply system pipeline. Neither cost nor the assembly/component interactions are considered in this approach. Normally, too large a fraction of total investment is allocated to assemblies when using this approach. The second approach is an optimization model, whose objective is to minimize total assembly and component backorders subject to a constraint on total inventory investment. However, the assembly/component relationship is not considered. The model usually allocates too large a proportion of a given budget to components because they are generally less expensive than assemblies, and both assembly and component backorders are considered to be equally undesirable in the model. The assumptions upon which this model is developed are the same as assumptions 1) through 7) stated in Section II.
To illustrate these observations, the two alternate approaches were compared with the proposed model. For a set of F-15 fire control system data, aircraft related assembly backorders more than doubled when using the two alternate approaches. The same total target budget was, of course, used in all cases. As this test indicates, ignoring the hierarchical relationship between the assemblies and its components can degrade expected system performance substantially for a given level of investment. Stated in another way, ignoring this relationship causes an over-investment in spares to achieve a specific system performance goal.

Ostensibly, the model's main use would be to determine inventory levels for each location. The model, however, was developed primarily as a tool for investigating the impact on both supply performance and investment of changing the Air Force's two-echelon supply system to a three-echelon system. Specifically, Air Force planners are interested in examining how such a change affects the requirement for logistics resources, and inventory investment in particular. Thus, in addition to being simply a mechanism for computing stock levels, the model can be effectively employed to answer many questions related to the design of a logistics system. For example, issues that can be addressed concern the number and citing of maintenance centers, the impact of changing pipeline times on inventory investment, and the way that repair capability—measured in the model in terms of the probability that an item is repaired at a particular location—alters the investment in inventory.
REFERENCES


