CS&E—Using Computational Methods to Analyze Dynamical Systems

John M. Guckenheimer is a mathematician doing research on dynamical systems and their applications. Dynamical systems theory elucidates general phenomena occurring in the solution of ordinary differential equations. Sometimes called "chaos theory," it explains how deterministic systems can give rise to unpredictable, seemingly random dynamics in a robust manner. The broad applicability of these theories is astonishing. Guckenheimer has collaborated with physicists, chemists, engineers, and biologists, and has published work in all of these disciplines. His book, *Nonlinear Oscillations, Dynamical Systems, and Bifurcation of Vector Fields*, co-authored with Philip Holmes, has been a basic reference and text for 20 years.

To imagine dynamical systems, think of the example of several planets moving in space around a fixed sun according to the laws of gravitation. The positions and velocities of the planets determine their subsequent positions and velocities. Now imagine bundling all of these quantities into a single object (a list of 18 numbers, if there are three planets, and 24 if there are four). Then regard this object as a single point moving in a high dimensional "phase space." Dynamical systems theory seeks to answer questions about this motion in phase space.

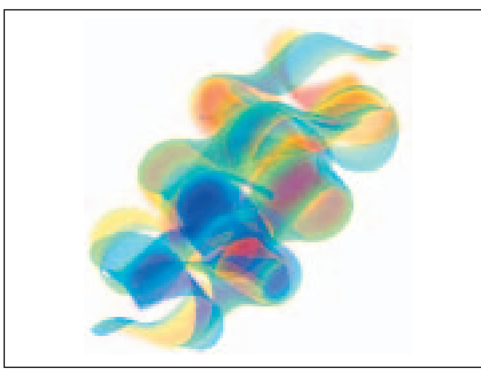
The initial value problem is: Where will the system go from a given starting point? Will the motions from two nearby starting points remain close to one another or will the system display "sensitive dependence on initial conditions?" Which motions are "periodic"— return to their initial starting points? Do all motions eventually lead to the same place, or are there multiple "attractors" that represent the fate of different sets of initial conditions? These same questions are relevant to dynamical systems that represent chemical reactions in a cell, electrical currents in a circuit or the brain, the motion of an animal or a robot, or the number of individuals catching a disease.

Algorithms that compute solutions to the initial value problem are one of the mainstays of scientific computing. However, it is more difficult to compute answers to the questions posed above. Guckenheimer develops better algorithms for providing answers to some of these questions. In particular, he has worked extensively on the problem of computing periodic solutions to dynamical systems.

Many biological processes are rhythmic—the heartbeat, a steady walking gait, circadian rhythms of sleep, and the menstrual cycle of women are a few in our daily lives. This motivates Guckenheimer's emphasis on determination of periodic solutions and their properties. The problem becomes more challenging when the dynamical systems have multiple timescales. The periodic solutions may be punctuated, with fast motion during parts of the cycle and slower motion during the rest. Action potentials in the heart and the nervous system are examples.

Dynamical models of the nervous system illustrate how these computational methods are applied. Guckenheimer has a long-standing collaboration with the laboratory of Ronald M. Harris-Warrick, Neurobiology and Behavior, studying the stomatogastric ganglion, a small neural network in lobsters that controls rhythmic motions of part of the digestive system. Action potentials of motor neurons in the ganglion are the signals that direct muscles of the lobster foregut to contract. Thus, the timing of these action potentials provides coordination of the foregut contractions. These same processes are involved in motor control throughout the animal kingdom.

The action potentials themselves arise from the dynamics of membrane channels, molecules with pores that open and close to allow the flow of ions into and out of a neuron. These ionic flows can be modeled as a dynamical system. The activity patterns of the neurons depend on synaptic inputs from other neurons and the influence of neuromodulators. Mathematicians use computational methods to analyze these dynamics. For example, they determine under what conditions a neuron will remain silent without firing action potentials and under what conditions it will fire action potentials in varied patterns.



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