

ESSAYS IN LABOR AND ORGANIZATION ECONOMICS

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## ESSAYS IN LABOR AND ORGANIZATION ECONOMICS

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The following dissertation is a collection of three independent essays. The first two essays contribute to the literature on Organization Economics. The third essay contributes to the literature on data confidentiality.

Essay 1, "Turnover as a Gateway to Symmetric Information," explores high-ability turnover in highly competitive labor markets. Why do workers who are successful at a given firm decide to leave? Essay 1 asserts that such movement is driven by the presence of asymmetric information. In particular, it is shown that when competing firms have less knowledge of a worker's ability than his current firm, there exists an incentive for high-ability workers to leave their current job in pursuit of a higher wage. Such an incentive generates a set of testable predictions. The predictions are tested using the personnel records from the management of a medium-size firm in the US financial services industry. The data is consistent with the theory.

Essay 2, "Piece-Rates, Salary, Performance and Job Level," explores the effect of monitoring and hierarchy on compensation structure. Previous work has shown that monitoring worker effort is more difficult at lower levels of the hierarchy, and, simultaneously, that compensation should rely more on salary payments than piece-rate payments when effort is more difficult to monitor. Essay 2 formalizes these ideas in a simple model of moral hazard. The model generates a set of predictions about

how salary, bonus and performance should vary across levels of the hierarchy. The predictions are tested using the same data as Essay 1 and strong support is found.

Essay 3, “Synthetic Data and Risk of Disclosure,” explores how well synthetic data protects confidential data. Using a unique Census dataset and 4 synthetic implicates, the risk of disclosure is found to be quite small. In a secondary analysis, the effectiveness of distance-based and probabilistic re-identification methods are also explored. Contrary to previous experiments it is found that probabilistic re-identification outperforms distance-based. Further, it appears that the difference in performance is driven by the number of matching variables: as more matching variables are added, the success rate of probabilistic matching increases more quickly.

## BIOGRAPHICAL SKETCH

Dr. Bryan Ricchetti received his B.A. in Economics from Hamilton College in May 1999. He graduated Phi Beta Kappa, Magna Cum Laude and with Honors. From July 1999 – July 2002 he worked as a Research Analyst at MDRC in Manhattan, NY (a non-profit, non-partisan, research organization) studying the economics of welfare and poverty under the supervision of Dr. Charles Michalopoulos. During his time at MDRC, Dr. Ricchetti's interest in the methods and applications of economic analysis were cemented. He matriculated to Cornell University in August 2002 to pursue his Ph.D. He graduated in August 2007. The following three essays are the end result of his training. They are tied together in spirit by the desire to reduce complex outcomes to simpler origins.

*I would like to dedicate my work to my parents, Paul and Janet Ricchetti, and my brothers, Matt and Eric, who each, through their actions and words, taught me the value of independent thought and hard work.*

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# **CHAPTER 1: TURNOVER AS A GATEWAY TO SYMMETRIC INFORMATION: TESTING PATTERNS OF ENTRY INTO PERSONNEL RECORDS**

## **1.1. Introduction**

In the past thirty years there has been a movement in labor economics to better understand phenomena associated with allocation of skill within the firm. The firm is no longer a “black box,” and the complexities of the decisions made within are now better understood. This movement has grown with the availability of detailed, personnel data that allows researchers to better understand the dynamic environment inside real-world firms; promotion chains, hierarchical structure, wage paths, career paths, etc.... In turn, a large theoretical literature aimed at explaining these dynamics has evolved. Following Doeringer and Piore’s (1971) hypothesis that internal labor markets are “protected” from the influence of the external labor market, the literature examining career paths has predominantly focused on career paths within a single firm. The literature has found that the typical career path starts at a low job level known as a “port of entry” and then proceeds upward as the firms learns about the workers skills. However, the empirical literature has also found evidence that career paths quite often involve movement between firms, even at high levels of the promotion hierarchy.<sup>1</sup> This type of movement between firms, however, has not been studied as thoroughly as the upward movement within firms. The goals of the current paper are to (1) develop a theoretical framework consistent with mid-career movement between firms at high job levels and (2) test the implications of the framework using personnel data.

Starting with Akerlof (1970), economists have analyzed the consequences of unobservable quality in product markets. A large literature has explored mechanisms

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<sup>1</sup> Baker, Gibbs and Holmstrom (1994a) is representative of the literature in this regard. They find significant external entry at all job levels.

through which sellers can reveal information to buyers to avoid adverse selection problems.<sup>2</sup> In the current paper, I apply this idea to the labor market to develop a turnover mechanism that ties turnover to level of the hierarchy. I consider a 3-period model with asymmetric learning. Unlike the standard literature, however, I grant the worker self-knowledge of his ability. With self-knowledge the worker is in a situation analogous to a firm in a market with unobservable quality; by revealing information about his ability to competing firms he can improve his sales price (i.e. wage). In order to allow workers to take advantage of their self-knowledge I grant them access to a networking technology through which they can reveal their ability to a competing firm at a fixed cost. Only high-ability workers will find such an activity profitable. When a match parameter is included in production high-ability networking will lead to high-ability turnover.

A secondary consequence of self-knowledge is that the economy endogenously transitions from asymmetric to symmetric information. Even if a worker does not find it profitable to network early in his career, self-knowledge allows him to credibly threaten turnover later in his career. This threat of turnover forces the inside firm to relinquish its private information as workers age. As such, each worker starts his career in a situation of asymmetric information and eventually moves to a situation of symmetric information. This feature of the model is novel.<sup>3</sup>

When I apply the model to a labor market with multi-level firms where job level is observed publicly it is shown that high-ability workers at *each level* of the firm have an incentive to turnover. This incentive arises because the public observability of job level attaches wages to jobs. High-ability workers within a level must then threaten turnover to receive a wage in line with their ability. Further, the transition

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<sup>2</sup> Nelson (1970, 1974, 1978) and Milgrom and Roberts (1986).

<sup>3</sup> Milgrom and Oster (1986) consider both asymmetric and symmetric environments in the same model, but the environments are exogenous. Pitchick (2006) is closer in spirit to my model, as she allows workers to escape asymmetric information through investment in “visible” human capital.

from asymmetric to symmetric information implies the firm's promotion decision changes for older workers. These two features of the model lead to the following testable implications in personnel data: (1) the wage increase due to promotion decreases with experience, (2) a higher proportion of new hires than incumbents are promoted in the period after entry, (3) new hires enter the firm at the top and bottom of the wage distribution of their given job level, and (4) new hires have more variable future promotion paths than incumbents.

Using the sample of white males from the Baker, Gibbs and Holmstrom (1994a, b) (BGH, hereafter) dataset, I test the above predictions. I first develop a methodology that ties the theory to the data. The paramount issue is translating the stylized timing of the theory into the more complex timing of the data. I use the data as a guide in this process to limit the sample to workers to whom the theory most likely applies. I then use parametric and semi-parametric regression to test the predictions of the model on the relevant sample. The tests demonstrate that the data is consistent with the model. Further, complementary evidence for entry behavior has been found by Treble et. al (2001) and Acosta (2006). Outside of the personnel economics literature, there is also strong evidence that within specific sectors (high technology, academia, etc...) high ability turnover is prevalent.<sup>4</sup> The model is also potentially consistent with such behavior.

Lastly I consider alternative explanations for the behavior in the data. The strength of the model relative to the alternative explanations is its consistency with a *set* of empirical findings and the fact that it explicitly ties turnover to job level. Several other models can explain a single finding, but none seem to explain the set examined.

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<sup>4</sup> Saxenian (1994) and Shih (2004) show evidence of high skill turnover in Silicon valley. Coupe et al. (2006) in academia.

The paper proceeds as follows. Section 1.2 reviews the related literature and puts the model in context. Section 1.3 introduces a simple model with a simple firm structure that highlights the intuition of the turnover mechanism. In Section 1.4 I enrich the production environment to a multi-level firm with dynamic promotion and develop testable predictions for personnel data. Section 1.5 discusses the methods for testing the model and presents empirical results. Section 1.6 discusses potential alternative explanations. Section 1.7 concludes.

## **1.2. Related Literature**

The paper's main contribution is to the literature on personnel and organization economics, with a particular contribution to the literature on careers in organizations. The seminal empirical work in this literature is BGH (1994a, 1994b). Following their work, other papers have explored personnel records (Lazear (1999), Treble et al. (2001), Gibbs and Hendricks (2004) are examples). These papers have revealed important patterns in promotion, wage, and career paths in real world firms. Along with the empirical literature there is a large theoretical literature considering careers in organizations. Important work in this literature includes Harris and Holmstrom (1982), Lazear and Rosen (1981), and more recently Gibbons and Waldman (1999, 2006). In general this literature has focused on career paths within a single firm. A smaller literature considers movement between firms. Chan (1996) is an example from this literature. He considers the possibility of external entry in the context of a tournament model. In his model, when the firm promotes a worker it can choose between external and internal candidates. Chan shows that the firm will show "favoritism" towards the internal candidate. As noted in the introduction, the main contribution of my paper to this literature is to offer a framework that is consistent with a set of facts about entry in personnel data.

My paper also contributes to a smaller literature on external entry and asymmetric information. Greenwald (1986) is the seminal work in this literature. Greenwald considers the importance of adverse selection in labor markets, arguing that workers who are fired and seeking work in the external, secondary market will be of average lower quality than internal workers. Bernhardt and Scoones (1993) consider the possibility of management turnover in a situation of asymmetric information. In their model, when the inside firm promotes a worker an outside firm can attempt to steal him by paying to observe his ability. In similar work Banerjee and Gaston (2004) consider the possibility of a “raiding firm” stealing a worker away from the inside firm after observing a signal of the worker’s ability. They show that if the signal of ability is “noisy enough” and counter-offers are costly, then the inside firm is forced to offer a “pooling wage” that puts their high ability workers at risk. Shankar (2005) and Jagdish (2006) also consider models of turnover in environments of asymmetric information. Like my paper and Banerjee and Gaston, they consider the incentive for high-ability workers to turnover. Both papers, however, consider this incentive in very specific labor market settings (Shankar considers the high-tech industry and Jagdish the market for CEO’s). Acosta (2006) gives careful empirical consideration to the difference in outcomes between workers hired externally and those promoted from within. Finally, Ghosh (2007) considers an integrated model of promotion dynamics similar to mine in which he considers the effect of entry and exit on wages. Although he does not rely on asymmetric information his work is relevant.

Finally, my paper contributes to the literature on learning in labor markets. The literature has two main branches: symmetric learning (Farber and Gibbons (1996) is a modern treatment) and asymmetric learning (Greenwald (1986), Waldman (1984), Bernhardt (1995)). A novel contribution of my model is that it endogenously links the two environments. In the model, workers start in a situation of asymmetric

information and move to a situation of symmetric information over time. Milgrom and Oster (1987) consider both asymmetric and symmetric information in a single model, but assume the information environment is exogenous to the worker. Pitchick (2006) endogenizes the choice via investment in “visible” capital. Pitchick considers a labor market in which self-aware workers strategically invest in “visible” human capital. The investment allows the outside firm to learn their ability with some probability. She shows that such a model can explain the negative seniority wage-premium in academia, as the investment in visible capital induces a wage jump early in a worker’s career, but she does not consider the transition from asymmetric to symmetric information more broadly. Pinkston (2006) is closest in spirit to my model. He considers a model in which the inside firm has an informational advantage, but the outside firms observe signals every period. In equilibrium the outside firms can bid away the workers, and asymmetric information goes away in the long run.

In terms of modeling techniques, the central idea of the paper has a long history in the IO literature. In response to Akerlof’s (1970) insight regarding adverse selection, the IO literature has considered strategies which high-quality sellers can implement to avoid the fate of adverse selection (Nelson (1970, 1974, 1978), Milgrom and Roberts (1986), Johnson and Waldman (2003)). Jin and Leslie (2003) offer a nice test of this idea in the restaurant industry. My model also contains a “search” component. Starting with the seminal work by Burdett (1978) and later Mortensen, there is a large literature on worker search (Mortensen (2003)). This literature tends to be more macro in nature, looking at flows in and out employment at an aggregate level. My paper builds a very specific micro foundation for how the search process might work in a competitive, ability-sensitive labor market. Workers can “search” at a specific outside firm and reveal their ability.

Lastly, the use of “self-knowledge” is not common in the asymmetric learning literature. Ricart i Costa (1988) and Pitchick (2006) are the main exceptions. In Ricart i Costa, a menu of self-selecting contracts is offered by the market to risk-averse workers as a way for them to signal their ability. Workers, knowing their own ability, use the contracts to separate themselves. Pitchick was discussed above.

### **1.3. Asymmetric Information and Self Knowledge**

In this section I introduce the baseline model. I consider a model of asymmetric information where workers have self-knowledge and the economy lasts for 3 periods. Further, I define a “networking technology” that allows the worker to reveal his ability to an outside firm. Throughout this section the organizational structure of the firm is kept simple in order to focus on the effects of self-knowledge and networking. In Section 1.4 the firm’s organizational structure is expanded to a multi-level hierarchy where job assignment is publicly observable. The model then allows for a set empirical predictions.

#### **A) The Model**

Workers are in the economy for 3 periods. They supply labor inelastically and seek to maximize their lifetime income. There are three firms. In any given period a worker’s current employer is referred to as the “inside firm”, I, while the other two firms are referred to as the outside firms, O1 and O2. There is a mass one of workers whose “innate ability,”  $\theta_i$ , is distributed uniformly over  $[\theta_L, \theta_H]$ . A worker’s “effective ability” is  $\eta_{it} = \theta_i f(t)$  where  $f(0)=1$ ,  $f'(t) > 0$  and  $f''(t) \leq 0$ . This formulation of effective ability captures the fact that a worker’s ability grows over time in the labor market at a speed determined by his innate ability.  $f(t)$  can be interpreted as general human capital. A simpler function of ability would suffice, but when I enrich the

model in Section 1.4 to consider a multi-level firm this formulation of effective ability is necessary to generate promotion dynamics.

Output for firm  $j$  in period  $t$  equals  $Y_{jt} = \sum_i \eta_{it}(1 + s) + \sigma_{ij}$  where  $\sigma_{ij}$  is a match parameter,  $s$  is specific capital that equals 0 if a worker is in his first period with the firm, and  $S > 0$  otherwise.  $\sigma_{ij}$  follows a two point distribution; with probability  $p$  it is  $\sigma_H$ , a good match, and with probability  $1-p$  it is  $\sigma_L$ , a bad match. I normalize  $\sigma_L$  to 0. The match parameter is included in the model to allow for the possibility of turnover. It is assumed that  $2\sigma_H > \theta_H f(2)S$  and that  $\theta_L f(1)S > p\sigma_H$ . The first assumption ensures that a worker with a good match at an outside firm and a bad match at the inside firm is more valuable to the outside firm. The second assumption ensures that a worker with a bad match at the inside firm is worth more than a worker with an unknown match ( $p\sigma_H$ ) at an outside firm. Further it is assumed  $\theta_i$  is independent of  $\sigma_{ij}$ ,  $\sigma_{ij}$  and  $\sigma_{ik}$  for  $j \neq k$  are independent, and that  $\sigma_{ij}$  is re-drawn by the firm if a worker leaves.<sup>5</sup>

For the cost  $C$  a worker can “network” with an outside firm  $k$ , which allows the outside firm to learn  $\theta_i$  and  $\sigma_{ik}$  and the worker to learn  $\sigma_{ik}$ .<sup>6</sup> This captures the idea that a worker can meet with an outside firm and demonstrate his ability through interviews, tests, consulting. Once contacted by the worker, the outside firm could also contact workers inside the inside firm to learn  $\theta_i$ . Formally workers who choose to network have  $n_i = 1$ , otherwise  $n_i = 0$ . It is assumed that  $n_i$  is only observed by the outside firm at which the worker networks and the inside firm. It is also assumed that workers randomize between the two outside firms when networking.

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<sup>5</sup> The fact that  $\sigma_{ij}$  is re-drawn ensures that a worker’s outside option does not depend on  $\sigma_{ij}$ , which allows the period 2 wage for workers with  $n_i = 0$  to be perfectly revealing. One interpretation of the assumption is that  $\sigma_{ij}$  represents an idiosyncratic match between worker  $i$  and his co-workers, or his particular job duties, at firm  $j$ . If a worker leaves firm  $j$  then this match will have to be re-established upon re-entry.

<sup>6</sup> Appendix A shows that workers can not profit from networking in period 1.  $n_i = 1$  in period 1 will raise the worker’s expected wage in period 2, but due to competition in period 1 the increase in expected wage in period 2 will be exactly offset by a lower period 1 wage. Thus, his lifetime income is identical whether  $n_i = 1$  or  $n_i = 0$ . This result is driven by the fact that the worker does not know  $\theta_i$  in period 1.

In period 1, firms do not observe  $\theta_i$  or  $\sigma_{ij}$  and compete via wage offers for the workers. After period 1, the worker's current firm *and* the worker learn  $\sigma_{ij}$ . Further they observe output and perfectly infer  $\theta_i$ . Note that the fact that the worker learns  $\theta_i$  after period 1 is the assumption of "self knowledge." All firms remember  $\theta_i$  if a worker leaves. It is assumed that  $f(t)$  is known by all market participants. Finally, the period 2 wage is used as signal of ability by outside firms in period 3. This arises because the nature of the period 2 auction is such that the outside firms can observe the winning offer.<sup>7</sup>

After production in period 1, the following wage setting game is played between the inside firm, the outside firms, and the worker, who all seek to maximize expected profit/wages:

### Period 2

- (1) The worker learns  $\sigma_{ij}$  and  $\theta_i$ , and chooses  $n_i$  in  $\{0,1\}$ .
- (2) If  $n_i = 0$  only the inside firm learns  $\theta_i$  and  $\sigma_{iI}$ . If  $n_i = 1$  the networked firm also learns  $\theta_i$  and  $\sigma_{iO}$ .
- (3) The inside firm and the outside firms engage in an ascending value auction for the worker.<sup>8</sup>
- (4) The worker chooses the offer that will generate the highest total expected wage for periods 2 and 3, conditional on  $\theta_i$  and  $\sigma_{ij}$ .

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<sup>7</sup> By similar reasoning, networking choices in period 2 are observed by all firms at the beginning of period 3.

<sup>8</sup> The literature on asymmetric information in labor markets has a standard "winner's curse" result in which the informed firm always wins the worker at a wage equal to the productivity of the lowest ability worker of a given signal (Milgrom and Oster (1987)). We model the bidding as an ascending auction because it not only generates the winner's curse, but also a wage equal to marginal product for workers in symmetric information. Because both situations are considered in the model, the ascending auction is a good fit. Further it is realistic.

### Period 3

- (1) The worker chooses  $n_i$  in  $\{0,1\}$ .
- (2) The inside firm learns  $\sigma_{ij}$  and  $\theta_i$  for all workers new to the firm in Period 2 who did not network. The outside firms observe the period 2 wage offers and period 2  $n_i$  for all workers.
- (3) The inside firm and the outside firms engage in an ascending value auction for the worker.
- (4) The worker chooses the highest offer.

The ascending value auction is conceptualized as follows: all participating firms make increasing counter offers until only one firm remains (Lazear (1986) uses a similar wage setting process).<sup>9</sup> Once a firm drops out it can not re-enter. The equilibrium concept of the game is a special case of the proper equilibrium (Myerson 1978). A proper equilibrium refines the trembling hand perfect equilibrium in which players make “smaller” mistakes on actions with higher payoffs. In the context of the auction game a “mistake” is when a firm makes a wage offer other than that which is optimal. I consider the special case in which the firms only make mistakes on  $\theta_L$  workers, as they are the least costly workers to lose. I use this refinement to eliminate equilibria in which the outside firm bids a wage higher than it would be willing to pay if there was a positive probability the worker would accept. See Appendix 1 for a formal definition of the equilibrium. Once this refinement is imposed there is a unique equilibrium. Finally, note that because the period 2 wage is observed with a lag, there is also a signaling game embedded in the larger bidding game.

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<sup>9</sup> Unlike standard ascending auctions, the last firm remaining may have an incentive to raise the wage offer because of the “threat” of turnover. If the period 2 wage offer of the inside firm generates less lifetime income than that from turnover, the worker will leave. Bernhardt and Scoones’ (1993) idea of pre-emptive wage offers is similar in spirit. Also, it is assumed that once a firm drops out it can observe the winning offer, but can not re-enter. Thus, the period 2 wage can be used in period 3 to refine beliefs about ability

B) Equilibrium with High-Ability Networking and No Turnover:  $p=0$

In order to focus on the basic logic of the model, I first consider the case when  $p=0$ , i.e. when there is no match parameter in the production function. In this case, workers have the incentive to reveal information to the market but there is no turnover in equilibrium. Proposition 3.1 formalizes the main features of the equilibrium in this case:

Proposition 3.1: When  $p=0$  the unique equilibrium is characterized by:

- (i) In period 2 all workers with  $\theta_i > \theta^N = \theta_L + C/f(1)$  choose  $n_i = 1$ , stay with the current firm, and receive a wage of  $\theta_i f(1) + S\theta_i f(2)$
- (ii) In period 2 all workers with  $\theta_i \leq \theta^N = \theta_L + C/f(1)$  choose  $n_i = 0$ , stay with the original firm, and receive a wage of  $\theta_L f(1) + S\theta_i f(2)$
- (iii) In period 3 all firms learn  $\theta_i$  from the period 2 wage and workers receive a wage of  $\theta_i f(2)$ .

Proof: In Appendix 1.

Consider first workers who do not network. Result (ii) states that they will receive a wage associated with a  $\theta_L$  worker plus a premium of  $S\theta_i f(2)$ . The wage is associated with a  $\theta_L$  worker because of the winner's curse. This is a standard result in the asymmetric information literature. The premium of  $S\theta_i f(2)$  captures the signaling aspect of the period 2 wage. The inside firm knows that a worker who does not network can leave the firm in period 2, reveal his ability to an outside firm, and ensure a high wage in period 3. Because the period 2 wage is observable, by offering the premium  $S\theta_i f(2)$  the inside firm reveals the worker's ability to the outside firms and pre-empts the worker's decision to turnover. That is, the inside firm *intentionally*

abandons its informational advantage in order to eliminate the worker's incentive to leave. An immediate consequence of this is that the economy endogenously moves to a situation of symmetric information in period 3. Result (iii) captures this transition.

A second implication of the premium  $S_{\theta;f}(2)$  is that workers who the market believes to be identical receive different wages. This result differs from standard models of asymmetric information which predict that a worker's wage will be determined entirely by the market's belief (Ricart i Costa (1988) is an exception). The fact that there is variation in wages across seemingly identical workers is consistent with empirical observation.<sup>10</sup>

Next consider workers who network. Result (i) states that only high ability workers will find it profitable to network in period 2. In a situation analogous to markets with unobservable quality, networking allows the worker (the seller) to reveal information to competing firms (potential buyers). If a worker does not network the outside firms will not be able to distinguish him from other workers in period 2 and he will receive the low wage from result (ii). Because networking is costly, only relatively high-ability workers will find it profitable.<sup>11</sup>

A final point regarding Proposition 3.1 is that the amount of networking will depend on the magnitude of  $C$ . As  $C$  gets large, no one will network (full asymmetric information in period 2), and as  $C$  approaches 0 all workers will network (full symmetric information in period 2). For the purposes of understanding high ability turnover, it is assumed in the rest of the paper that  $C$  satisfies the conditions for high-ability networking. In the baseline model this requires  $0 < C < \theta_{Hf}(1)$ .

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<sup>10</sup> For example, in BGH (1994a) it is clear that there is variation in wages within job levels and observable skill groups. Simple models of asymmetric information predict that this should not occur.

<sup>11</sup> Note that there are no efficiency differences between the period 2 and 3 equilibria. The turnover threat only has distributional effects; it redistributes rents across workers. Later, when we consider a multi-level firm with promotion, the 3<sup>rd</sup> period equilibrium will be more efficient than the 2<sup>nd</sup> period equilibrium.

C) Equilibrium With High Ability Networking and Turnover:  $0 < p < 1$

In order to generate turnover in the above model some amount of randomness, or firm heterogeneity, needs to be introduced. In this subsection I consider the case of  $0 < p < 1$ . Under this parameterization it is possible that the outside firm will outbid the inside firm on workers who network. A technical difference is that there will be two networking cut-offs rather than one, conditional on the realization of  $\sigma_{iI}$ . Workers with high match values at the inside firm will have a lower networking cut-off as the return to revealing their ability is slightly higher. Proposition 3.2 formalizes the equilibrium:

Proposition 3.2: When  $0 < p < 1$  the unique equilibrium is as follows:

- (i) In period 2 workers with  $\theta_i > \theta_N(\sigma_{ij})$  choose  $n_i = 1$ . If  $\sigma_{iI} = 0$  and  $\sigma_{iO} = \sigma_H$  they turnover and receive  $W_2 = \theta_i f(1)(1+S)$ . Otherwise they stay with the inside firm and receive  $W_2 = \theta_i f(1) + S\theta_i f(2) + 2\sigma_{iO} - p\sigma_H$ .
- (ii) In period 2 workers with  $\theta_i \leq \theta_N(\sigma_{ij})$  choose  $n_i = 0$ , stay with the inside firm, and receive  $W_2 = \theta_i f(1) + S\theta_i f(2) + p\sigma_H$ .
- (iii) In period 3, all firms learn  $\theta_i$ . Workers who turned over in period 2 receive  $W_3 = \theta_i f(2)(1+S) + p\sigma_H$ . All other workers receive  $W_3 = \theta_i f(2) + p\sigma_H$ .

Proof: In Appendix 1.

Proposition 3.2 is analogous to Proposition 3.1. Results (i) and (ii) state that relatively high ability workers choose to network, while relatively low-ability workers do not.<sup>12</sup> Result (iii) states that the economy transitions from asymmetric to

<sup>12</sup> The period 2 wages in Proposition 3.2 differ from the wages in Proposition 3.1 because of the presence of the match parameter in production and the possibility of turnover.

<sup>12</sup> The period 2 wages in Proposition 3.2 differ from the wages in Proposition 3.1 because of the presence of the match parameter in production and the possibility of turnover.

symmetric information by period 3. The major difference with Proposition 3.2 is that workers who network *and* have a bad match at the inside firm will now turnover if they find a good match at the outside firm. Thus, a proportion  $p(1-p)$  of workers who network will turn over in equilibrium.

Finally, Corollary 3.1.1 formalizes the result of high-ability turnover:

Corollary 3.3.1: The average ability of those who turnover is greater than those who do not.

Proof: Note that  $E(\theta_i) = E(\theta_i | t^*=1)P(t^*=1) + E(\theta_i | t^*=0)P(t^*=0)$ . Workers who turnover have ability uniformly distributed ( $\sigma_{ij}$  is independent of  $\theta_i$ ) on  $(\theta_N(0), \theta_H]$ .  $\theta_N(0) > \theta_L$  then implies  $E(\theta_i | t^*=1) > E(\theta_i)$ , which then implies, by the first expression,  $E(\theta_i | t^*=1) > E(\theta_i | t^*=0)$ .

The goal of this section was to introduce the logic of the model in a simple production environment. The model builds a micro foundation for why high ability workers may turnover; they do so as a strategic response to the informational advantage their current employer has over competing employers. In the next section I embed the baseline model into a production environment with dynamic promotion. I then show that the model generates a set of predictions for personnel records regarding entry and promotion.

#### **1.4. Asymmetric Information, Self Knowledge and Dynamic Promotion**

In this section I embed the baseline model in a richer production environment which will allow for testable predictions in personnel data. I look at a multi-level firm with a dynamic promotion process. I continue to assume that ability is not publicly

observable. However, it is now assumed that job level is observable. In such an environment the wage for a worker who does not network will be a function of the lowest ability worker within his job level. As a consequence the promotion decision of the firm is tied to the turnover decision of the worker. I show that the model produces a series of testable predictions about entry and promotion in personnel data.

#### A) The Model

The model is identical to the baseline model with two enrichments. First, firms now consist of two job levels rather than just one, where job  $j$  has a lower return to effective ability than job  $j + 1$ . Formally, production at job  $j$  in time  $t$  equals  $y_{ijt}(\eta_{it}) = (1+s)(a_j + b_j\eta_{it}) + \sigma_{ij}$  where  $s$  and  $\sigma_{ij}$  are defined the same as in the baseline model. It is assumed that  $a_j > a_{j+1}$  and that  $b_j < b_{j+1}$ , which implies a unique cutoff level,  $\eta'$ , above which workers are more productive at job 2. It is assumed that  $\theta_L(f(2)) < \eta' < \theta_H(f(1))$  which implies that some workers are more productive in job 2 in each period  $t$ . Second, job assignment,  $j$ , is publicly observable. This enriches the information environment as job assignment now serves as a signal of ability to the outside firm.

This production function is used in Gibbons and Waldman (1999, 2006). They show that in a world of symmetric learning it captures a wide set of empirical findings about intra-firm promotion and wage dynamics. Thus I believe it is a good starting point for linking internal promotion and external entry.

Finally, I restrict the parameters as above:  $(a_2 + b_2\theta_H f(2))S < 2\sigma_H$  and  $(a_2 + b_2\theta_L f(1))S > p\sigma_H$ . These assumptions are analogous to assumptions in the baseline model. The first ensures a worker with a good match at an outside firm and a bad match at the inside firm is more valuable to the outside firm. The second ensures that a worker with an unknown match at an outside firm and a bad match at the inside firm is more valuable to the inside firm. Further, I assume  $E(\theta_i) < \eta'$  to insure that all

workers start their careers in job 1. In words, this assumption implies that a worker who just entered the market is more productive at job 1 than job 2.

After production in period 1, a 2-period wage setting game analogous to the baseline model is played. The only difference is the addition of the promotion decision. Each period after the worker chooses  $n_i$  in stage (1), there is an extra stage before the wage bidding in which the inside firm publicly announces job assignment, conditional on  $n_i$ .

## B) Equilibrium

As in the baseline model the equilibrium specifies optimal networking decisions and equilibrium turnover results for the workers. Further it also specifies optimal promotion decisions of the firm. For notational purposes define  $\theta^*$  as the equilibrium promotion cut-off for a worker with  $n_i = 0$ :

Proposition 4.1: The unique equilibrium is as follows:

- (i) In period 2, all workers with  $n_i = 1$  and  $\theta_i f(1) \geq \eta'$  are promoted. For workers with  $n_i = 0$ , there exists a cut-off,  $\theta^*$ , such that  $\theta^* f(1) > \eta'$  and all workers with  $\theta_i \geq \theta^*$  are promoted.
- (ii) In period 2 all workers in job 1 with  $\theta_i > \theta^{N1}(\sigma_{ij})$  choose  $n_i = 1$ . If  $\sigma_{i1} = 0$  and  $\sigma_{ij} = \sigma_H$  they turnover. All other workers choose  $n_i = 0$  and remain with the inside firm.
- (iii) In period 2 all workers in job 2 with  $\theta_i < \theta^*$  or  $\theta_i > \theta^{N2}(\sigma_{ij})$  choose  $n_i = 1$ . If  $\sigma_{i1} = 0$  and  $\sigma_{ij} = \sigma_H$  they turnover. All other workers choose  $n_i = 0$  and remain with the inside firm.
- (iv) In period 3 all firms learn  $\theta_i$  and all workers with  $\theta_i f(2) \geq \eta'$  are promoted

Proof: See Appendix 1.

Result (i) addresses the promotion decision of the firm. It states that workers who choose  $n_i = 0$  are promoted inefficiently ( $\theta_i f(1) \geq \theta^* f(1) > \eta'$ ), while workers who choose  $n_i = 1$  are promoted efficiently ( $\theta_i f(1) \geq \eta'$ ). Both results are standard in their respective literatures. Workers in asymmetric information are under-promoted because the act of promotion sends a positive signal of ability to outside firms which drive up their wage offers, while workers in symmetric information are promoted efficiently because the act of promotion does not reveal any new information.<sup>13</sup> The fact that the worker endogenously chooses between the two outcomes in the model, however, is novel. Finally, note that in period 3 *all* workers are promoted efficiently. This result is also analogous to the baseline model; in the long run, asymmetric information disappears from the economy because the inside firm is forced to abandon its information advantage.

Results (ii) and (iii) address optimal networking decisions and subsequent turnover. Result (ii) states that there is high-ability turnover in job 1. This result is analogous to the prediction of high-ability turnover from the baseline model. Result (iii) states that there is “two-sided” turnover from job 2. This result is unique to the enriched model. The turnover at the “top” of job 2 ( $\theta_i > \theta^{N2}(\sigma_{ij})$ ) is analogous to the turnover in the baseline model; such workers network to avoid the low wage in job 2. The bottom end of the two-sided turnover [ $\eta' / f(1), \theta^*$ ] is made up of workers who face the prospect of being inefficiently assigned to job 1. Intuitively, these workers can be thought of as high-ability workers from job 1 who network to avoid the low wage of job 1. However, because  $\theta_i f(1) > \eta'$ , such workers are efficiently assigned to the low end of job 2 when they network. Figure 1.1 captures this dynamic graphically.

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<sup>13</sup> Waldman (1984a) shows that promotion is inefficient in the asymmetric case. The symmetric case is straight forward.



**Figure 1.1: Networking Cutoffs, Job 2**

As a technical note, the distance between  $\eta'/f(1)$  and  $\theta^*$  in Figure 1.1 will be a function of  $S$ , and the distance between  $\theta^*$  and  $\theta^{N2}(0)$  will be a function of  $C$ . Thus, as I vary the parameters  $C$  and  $S$  of the model, I can vary the cut-points  $\theta^*$  and  $\theta^{N2}(\sigma_H)$ . For the purpose of two-sided turnover, it is sufficient to assume that  $S$  and  $C$  are such that  $\theta_{M2}$ , the median ability in job 2, is in  $[\theta^*, \theta^{N2}(0)]$  (See Appendix 1).

### C) Testable Predictions

The equilibrium has three main features: (1) transition from asymmetric information to symmetric information between periods 2 and 3, (2) high-end turnover on  $\theta_i$  in job 1, and (3) two-sided turnover on  $\theta_i$  in job 2. In this section I derive testable predictions of the model regarding variables that are observable in personnel records. In the following discussion I will use the term “new hire” to refer to a worker who enters a given job from an outside firm in a given period, and “incumbent” as a worker internally assigned to a given job in a given period.

I start by deriving a testable prediction for the transition from asymmetric to symmetric information. As noted earlier, the wage premium upon promotion for workers in asymmetric information is larger than that for workers in symmetric information because promotion serves as a positive signal of ability to the outside

firms. To formalize the discussion, define the wage premium at promotion as the wage a worker receives if promoted minus the wage he would receive if not promoted.<sup>14</sup> Denote this premium for worker  $i$  in Period  $t$  as  $W_{it}(P)$ . Proposition 4.1 then implies the following corollary:

Corollary 4.1: The average wage premium upon promotion for workers in period 2 is larger than the average wage premium upon promotion for workers in period 3.

Formally,  $E(W_{i2}(P)) > E(W_{i3}(P))$ .

Proof: See Appendix 1.

The intuition for Corollary 4.1 is as follows. In period 2 there will be a subset of workers in asymmetric information who have a large wage premium for promotion due to signaling. In period 3, however, all workers are in symmetric information and have a relatively small wage premium for promotion. As a consequence, the average wage premium upon promotion will be larger in period 2. Corollary 4.1 thus offers a testable implication of the transition from asymmetric to symmetric information. In the data older workers should have smaller wage increases due to promotion than younger workers.

The second feature of the equilibrium is high end turnover on  $\theta_i$  into job 1. Because  $\theta_i$  is unobservable we can not directly test this feature. However, because  $\theta_i$  determines promotion, high end turnover on  $\theta_i$  implies a testable prediction regarding promotion rates in period 3 for new hires and incumbents. Corollary 4.2 formalizes this prediction:

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<sup>14</sup> To be precise,  $W_{it}(P)$  is defined using an off-equilibrium-path comparison. In equilibrium a worker is either promoted or not promoted.  $W_{it}(P)$  is the equilibrium wage increase for a given worker at promotion minus the off-equilibrium wage increase if the given worker were not being promoted.

Corollary 4.2: In period 3, a randomly chosen new hire from period 2 is more likely to be promoted into Level 2 than a randomly chosen incumbent from period 2.

Formally,  $P(\text{Promotion in Period 3} \mid t_2^*=1) > P(\text{Promotion in Period 3} \mid t_2^*=0)$ .

Proof: In Appendix 1.

The logic of Corollary 4.2 is as follows. As noted above, period 3 promotion is a function of  $\theta_i$ . Specifically, all workers above the efficient promotion cut-off  $\eta'/f(2)$  will be promoted in period 3. High-end turnover on  $\theta_i$  in job 1 implies that new hires will have a range of  $\theta_i$  to the right of incumbents, which then implies the proportion of new hires above the promotion cut-off,  $\eta'/f(2)$ , will be larger than that of incumbents. In the data in the period after entry we should see a higher proportion of new hires than incumbents promoted.

The third feature of the equilibrium is two-sided entry on  $\theta_i$  into job 2. Two-sided entry on  $\theta_i$  into job 2 translates into two testable predictions. The first of these predictions is two-sided entry into the wage distribution of a given job level:

Corollary 4.3: Define  $W = W(\theta_i, t^*)$  as the function that maps  $\theta_i$  and turnover outcomes,  $t^*$ , into wages for job 2. Let  $W^M$  be the median wage in job 2. For an arbitrary  $\epsilon$ , there exists a range of the wage distribution,  $(W^M - \epsilon, W^M + \epsilon)$ , where new hires will not enter into job 2

Proof: See Appendix 1.

In words, Corollary 4.3 states that new hires will enter job 3 only at the relatively high end or the relatively low end of the wage distribution. This prediction follows immediately from two-sided entry on  $\theta_i$  into job 2 and the fact that wages are an increasing function of  $\theta_i$ . Together these two facts imply that there will be a range in

the middle of the wage distribution where new hires do not enter. This range corresponds to the range in the middle of the ability distribution where workers do not turnover,  $\theta_i$  in  $(\theta^*, \theta^{N^2}(0))$ , which, as noted earlier in the paper, contains the median ability of job 2. Importantly, Corollary 4.3 is directly testable because we can observe both the worker's starting wage and the wage distribution of a given level in the dataset.

The second prediction related to two-sided entry on  $\theta_i$  into job 2 is that new hires should have more variance in  $\theta_i$  relative to incumbents. Because new hires are of either high or low ability for job 2, their variance in ability is higher than incumbents who occupy the whole range of ability in job 2. Corollary 4.4 formalizes this statement:

Corollary 4.4: Relative to the average worker in job 2, new hires into job 2 will have higher variance in  $\theta_i$  compared to incumbents promoted into job 2 from within. Formally,  $E((\theta_i - \theta_{2M})^2 | t^*=1, j=2) > E((\theta_i - \theta_{2M})^2 | t^*=0, j=2)$ , where  $\theta_{2M}$  represents the mean of innate ability on job 2.

Proof: See Appendix 1.

Corollary 4.4, however, is not directly testable because  $\theta_i$  is not observable in the data. Further, because the model contains only 2 job levels Corollary 4.4 can not be translated into a statement about future promotion for new hires in job 2.

To translate Corollary 4.4 into a testable implication I consider an extended version of the model in which  $T > 3$  periods and  $J > 2$  levels. In such a model the larger variance in  $\theta_i$  for new hires in job 2 translates into larger variance in future promotion outcomes at levels  $j > 2$ . In particular, at a given future period  $t + n$ , new hires will be more likely to have reached higher levels of the firm but also more likely

to be stuck in the level of entry.<sup>15</sup> Such a prediction is testable as promotion outcomes in the data are observable. Though it should be noted this is not a direct test of Corollary 4.4. Rather it is a suggestive implication consistent with two-sided entry that would hold in an extended model with more periods and more levels.

### **1.5. Empirical Analysis**

The goal of this section is to test the 4 corollaries above using personnel data. An important issue in testing the corollaries is translating the simple environment of the theoretical model into the more complex environment of a real world firm. A major focus of this section is developing an empirical strategy that properly links the theory to the data.

It should be noted that previous empirical work by BGH and Treble et., al. (1999) found suggestive evidence supporting 3 of the above findings (promotion of new hires in period after entry, two-sided entry, and more variable future promotion patterns of new hires). However, these papers are predominantly descriptive. They do not formally test the patterns in the data, nor do they offer a theoretical framework. Acosta (2006), on the other hand, does conduct formal statistical tests regarding different promotion outcomes for new hires and incumbents. He does not, however, use a formal theoretical framework to guide his tests. Thus, I am not the first to examine entry behavior empirically, but my analysis is novel in that it is guided by a theoretical framework *and* implements formal statistical tests of the framework's predictions.

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<sup>15</sup> I have shown that this result holds in the extended model.

## A) The Data

I test the above predictions on the sample of white, male employees from the same dataset analyzed in BGH (1994 a,b). The data include yearly personnel records from 1969 to 1988 for all such managerial employees of a US firm in the financial services industry. The original sample analyzed by BGH included females and nonwhite males, for a total of 68,437 employee-years of data. The sample of white males has 50,556 employee-years. The dataset was originally constructed using raw data from the firm's personnel records. The key feature of the data is that it allows researchers to identify the hierarchical structure of the firm and observe how workers move through it over time. The 8-level hierarchy was constructed by BGH by analyzing transitions between job titles.

The key variables in my analysis are promotion, salary, education, potential labor market experience, job level and entry. All variables in the data set are measured at the end of the year (December 31) for each employee in that year. As such, the exact date of a change in job title, salary, or entry is not observed. Most of the variables are observed for each employee for each of the sample years. I discuss the issue of missing values in detail in the appendix.

I define a promotion as a transition from a lower job level to a higher job level across two periods. I do not distinguish between one and two-step promotions as almost all promotions are one step (over 96 percent). Salary is measured in 1988 dollars, deflated by the CPI. The salary variable does not include bonuses. Information on bonuses is only available for 1981 to 1988. Following BGH, I do not use this information.<sup>16</sup> Education categories are constructed from an underlying "years of education" variable. Specifically, I construct dummy variables for high

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<sup>16</sup> Using the full sample, BGH found that only twenty-five percent of employees received bonuses in the 1981 through 1988 time period. Further these workers were heavily concentrated in the highest levels of the job ladder, which will be dropped from all of my analyses below.

school graduate, bachelors, masters, and Ph.D. degree.<sup>17</sup> Following BGH, I construct potential labor market experience as:  $\text{experience} = \text{age} - \text{education years} - 6$ . Lastly, entry refers to entry into the dataset. Such entry could in principle be entry from non-managerial levels of the same firm. Given the distinct change in job tasks this entry is considered equivalent to external entry. Thus, all entry is interpreted as external entry.

I restrict the sample in a few basic ways. First, I focus only on levels 1-4 of the hierarchy. In their original paper BGH note that promotion and entry dynamics at levels 5-8 may be different than at levels 1-4. Further 97% of the employees in my sample are in levels 1-4 and issues such as slot constraints (which I abstract away from) become more important at levels 5-8. See Appendix 1 for a further discussion of this issue.

I also restrict the time frame of the data. I drop the first year, 1969, from all analyses. I have no information on a worker's history before 1969 so it is not possible to distinguish new hires from other workers in 1969. I also drop the final two years, 1987 and 1988. As BGH note these two years have large data errors in the level classification. In particular half of new hires have missing level information. These are both obvious problems for my analysis of entry.

I use these data because I am looking at turnover decisions that are strategically tied to a worker's position in the firm hierarchy. The BGH data was built specifically to address phenomena such as these. Second, the theoretical model is best suited for labor markets in which production is sensitive to ability and in which many firms are competing for ability. The BGH data looks at workers in management positions in the service sector. I believe such work is highly sensitive to ability and highly competitive across similar firms.

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<sup>17</sup> A high school graduate is defined as a worker with 12-15 years of education, a bachelor's, 16-17 years of education., an MBA or other master's degree holder, 18-19 years of education, a Ph.D., 20 or more years of education. Because education is interpreted as a signal in my analysis, categorizing workers by degree status is more appropriate than total years of education.

## B) Methodology: Linking the Theory to the Data

A strength of the model is that the theoretical firm structure translates easily into the structure in personnel records. Workers are assigned to job levels and are easily identified as new hires or incumbents. As such, the predictions of the model apply nicely to the variables in the data. On the other hand, the information structure on worker ability and the stylized, 3-period timing in the theoretical model do not translate directly into the data. In this subsection I outline a strategy for properly translating these features.

In the theoretical model it is assumed that  $\theta_i$  is not known by outside firms at the beginning of period 2. In the data, however, there are variables such as experience and education that vary across workers, are correlated with ability, *and* are publicly observable. It is almost certain that the firm and workers in the data set use this information when making decisions. The theoretical model does not contain such variables. To address this issue I consider the logic of the model *under the assumption* that it does. The logic offers a natural way to interpret the empirical tests.

Suppose the theoretical model included publicly observable variables that vary across workers and are correlated with  $\theta_i$ . Outside firms would use such variables to refine their beliefs about  $\theta_i$ . In particular, each combination of the observable skills would act as a signal of ability. The market's belief about  $\theta_i$  would be the same for all workers with the same values of observable skills. Workers within a signal would then make their networking choice based on the wage of the lowest ability workers within their signal. The model's dynamics would then exist *within* each signal and the predictions of the model would hold even after controlling for observable skills. Below, I implement all of my tests with and without controls for observable skill as a way to address this issue.

The second main empirical issue is translating the simplified, stylized timing of the model into the more complex timing of the data. All of the testable predictions apply directly to period 2 of the theoretical model; all turnover occurs in period 2 and the transition from asymmetric to symmetric information occurs between periods 2 and 3. Thus, the tests of Corollaries 4.1 - 4.4 should focus on the real world analogue of period 2.

In the theoretical model it takes exactly 1 period for all workers to learn their ability and prepare to network. In the real world this process is likely more complex. There is likely variation across workers in their ability and/or preferences to communicate with outside firms in order to break asymmetric information. It may take some workers 3 years, while it may take other workers 10 years, or a whole career. An immediate consequence is that the timing of period 2 will vary across workers in the data. To formalize this idea, define  $\Sigma_i^2$  as the potential labor market experience of worker  $i$  when “period 2” begins. The fact that  $\Sigma_i^2$  varies across workers implies a range of  $\Sigma_i^2$ ,  $[\Sigma_L^2, \Sigma_H^2]$ . My empirical strategy is to use the data to estimate  $[\Sigma_L^2, \Sigma_H^2]$ , and run my tests *only* on observations that fall in this “period 2” window. To take advantage of the information I have in the data, I will let  $\Sigma_i^2$  depend on education categories. That is, for a given education category I will define the window  $[\Sigma_L^2, \Sigma_H^2]$  and include only workers in the window in my tests. To identify the window  $[\Sigma_L^2, \Sigma_H^2]$  for each education category I examine the distributions of entry and experience by level and education. The window should be characterized by active entry rates, and should occur in early/mid career. As such, I drop relatively young workers and relatively old workers who are “stable” in a turnover sense.

I first consider workers in levels 2-4. Table 1.1 presents the distributions of potential experience by education category for (a) the basic sample, (b) new hires into level 1, and (c) new hires into levels 2-4. Further, Figures A1.1 – A1.4 in Appendix 1

plot entry rates by labor market experience for each education category. A clear pattern emerges when examining panel (a) and the plots of entry rates. For each education category the point at which entry rates stabilize coincides well with the median of the experience distribution. That is, entry is most common when workers are relatively “young.” Such behavior is exactly what characterizes “period 2” in the model. As such, I conclude that good estimates for  $\Sigma_H^2$  for workers in levels 2-4 are

**Table 1.1: Distribution of Potential Experience by Education**

<b>Education</b>	<b>Mean</b>	<b>Median</b>	<b>Min</b>	<b>Max</b>	<b>Count</b>
<b><u>Basic Sample, Levels 1-4</u></b>					
High School	25.07	24	4	50	12,260
Bachelors	15.86	14	0	45	13,351
Masters	13.42	12	-1	43	7,323
PhD	11.19	9	-4	37	1,074
<b>Total</b>					34,008
<b><u>New Hires into Levels 1</u></b>					
High School	16.88	15	4	46	918
Bachelors	7.36	6	0	41	1,343
Masters	6.23	5	-1	39	595
PhD	3.56	2	-4	28	59
<b>Total</b>					2,915
<b><u>New Hires into Levels 2-4</u></b>					
High School	19.01	18	5	45	918
Bachelors	12.71	11	1	41	540
Masters	10.27	9	1	34	473
PhD	7.02	6	-3	27	77
<b>Total</b>					2,008

the points in Figures A1.1 – A1.4 at which entry rates stabilize. For workers with a high school diploma, bachelor’s, master’s, and PhD these points occur at, respectively, 25 years, 18 years, 15 years and 10 years of experience. All workers above these cut-offs are interpreted as “period 3” workers and dropped from the analysis. As with all

cut-offs discussed in this section, I vary these cut-offs in analyses not reported in the paper to ensure that the results are robust. They are.

I next identify  $\Sigma_L^2$  for workers in levels 2-4. Although most workers in levels 2-4 are likely in at least “period 2” of the model, it is possible that the firm may assign very talented workers directly into higher levels upon first entering the labor market. Such workers should be considered “period 1” workers and dropped from the analysis. The last panel of Table 1.1 addresses this issue. As is clear, there is a small set of workers in each education class that enter levels 2-4 with very little experience. I drop workers below the 1<sup>st</sup> percentile of the experience distribution for each education category. Again, I vary this cut-off in analyses not reported here.

Lastly I identify  $\Sigma_L^2$  and  $\Sigma_H^2$  for workers in level 1. Level 1 is quite different from levels 2-4 because, as BGH note in their analysis, it is a “port of entry” into the hierarchy. The entry rate is greater than .40 in level 1, while it is between .06 and .10 in levels 2-4. As such, it is likely that most workers in level 1 are “period 1” workers from the theoretical model, i.e. workers new to the labor market for management. However, it is possible that some older workers in level 1 are better interpreted as “period 2” workers. The middle panel of Table 1.1 presents the distribution of potential experience by education category for new hires into level 1. I look for workers who are relatively “young” for the overall sample, but relatively “old” for the sample of new hires into level 1. Noting that the 90<sup>th</sup> percentile of the experience distribution of new hires into level 1 coincides well with the median of the experience distribution for the whole sample, I believe workers in level 1 between the 75<sup>th</sup> and 90<sup>th</sup> percentile are old enough to be out of “period 1” but young enough to still be in “period 2.” As such I keep them in the analysis.

In all of the tests below I impose the restrictions described above. Again, in analyses not reported in the paper I vary all of the cut-offs. The findings presented are

not sensitive to the cut-offs. For information about how the restrictions affect the sample size see Appendix 1, Table A1.2.

### C) Testing the Predictions

In all of the regressions presented below, I define a vector of controls,  $X_{it}$ . For each test I fit the model with several different specifications of  $X_{it}$ . In the first specification I leave  $X_{it}$  empty. In the second I include year dummies (an indicator variable for each year in the time frame). The year dummies control for things that vary over time and influence outcomes of interest (i.e. conditions in the macro economy (Oyer (2006)).<sup>18</sup> In the last specification I include dummies for education categories and a quadratic in years of potential experience along with the year dummies. The quadratic in experience is consistent with the assumption that  $f''(t) < 0$ . The fullest specification is interpreted as discussed above; the dynamics of the model should hold even after controlling for observable skills that vary across workers. For Corollary 4.1 the fullest specification of  $X_{it}$  is slightly different because the outcome of interest (wage change) is first-differenced. I discuss this below. Lastly, I pool the observations across level and control for level rather than run regressions by level. If the regressions by level differ from the pooled regressions it will be noted in the text.

I begin with Corollary 4.1. The testable implication of Corollary 4.1 is that the average wage premium associated with promotion should decrease with worker experience. As noted in Section 1.4, the wage premium is defined as the wage increase upon promotion minus the wage increase the worker would have received if he were *not* promoted. In order to make the proper comparison I implement a 3-step method. I first estimate the wage increase a worker would have received had he *not* been promoted. I then take the actual wage increase a worker received at promotion

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<sup>18</sup> The data includes large macro cycles in exit, entry and average salary.

and subtract the estimated wage increase had he *not* been promoted to get the predicted wage premium due to promotion, as defined in the theoretical model. Finally, I use the predicted wage premium as the dependent variable to test how it varies with experience.<sup>19</sup>

To estimate the wage increase a worker would have received had he *not* been promoted, I run the following regression on the sample of workers who were not promoted. Let  $\Delta Z_{it+1} = Z_{it+1} - Z_{it}$ :

$$(1a) \Delta \ln w_{it+1} = \alpha_1 + \beta_1 \Delta \text{Exp}_{it}^2 + \beta_2 \text{Bachelor}_{it} + \beta_3 \text{Master}_{it} + \beta_4 \text{PhD}_{it} + \beta_5 \text{Level}_{it} + \beta_5 Y_{it} + \varepsilon_{it}$$

Equation (1a) includes a difference in experience squared to capture  $f''(t) < 0$ , controls for education, level of the firm and a vector of year dummies,  $Y_{it}$ . I use the parameter estimates from (1a) to predict the wage increase of each worker who was promoted had he *not* been promoted. That is, for each observation of a promoted worker I plug the values for Exp, Bachelor, Master, PhD, Level and,  $Y_{it}$  into (1a) with the estimates of  $\beta_1 - \beta_5$ . Define this predicted wage increase as  $\Delta \ln w_{it}^p$ . The theoretical concept of wage premium at promotion is then captured by  $\Delta \ln w_{it} - \Delta \ln w_{it}^p$ .<sup>20</sup>

Given this representation of the wage premium at promotion, I run the following regression on the sample of promoted workers:

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<sup>19</sup> This method is equivalent to the method in Devaro and Waldman (2006).

<sup>20</sup> In the context of the theoretical model, workers who are not promoted are of systematically lower ability than those promoted. Because the wage increase depends on ability, using such workers to predict the wage increase of promoted workers if *not* promoted is biased. I control for this problem by transforming wages to logs. Given that the log wage is linear in the log of ability, and that I am analyzing differences in wages, the ability bias is “differenced” out at the log level. See Proof of Corollary 4.1 for more discussion

$$(1b) \Delta \ln w_{it} - \Delta \ln w_{it}^p = \alpha_1 + \beta_1 \text{Exp}_{it} + \beta_2 T_{it} + \beta_3 X_{it} + \varepsilon_{it}$$

Corollary 4.1 then implies that  $\beta_1 < 0$ . That is, the wage premium associated with promotion should decrease with experience.  $T_{it}$  is a vector of job transition dummies. These dummies control for the exact transition between levels associated with the promotion.  $X_{it}$  represents a set of controls that I vary across specifications. In the first specification  $X_{it}$  is empty, in the second it includes year dummies, and in the third it includes year dummies as well as controls for education. The results are reported in Table 1.2. It is clear that  $\beta_1 < 0$  in all 3 specifications. Model 3 is the fullest

**Table 1.2: OLS Estimates for Equation (1b)** <sup>21</sup>

<b>Independent Variables</b>	<b>Model 1</b>	<b>Model 2</b>	<b>Model 3</b>
Experience ( $\beta_1$ )	-.0016 *** (.00017)	-.0016 *** (.00017)	-.0033 *** (.0002)
Bachelors	-	-	-.0207 *** (.0031)
Masters	-	-	-.0352 *** (.0037)
PhD	-	-	-.0473 *** (.0065)
Year Dummies	-	X	X
<b>Obs</b>	3,790	3,790	3,790

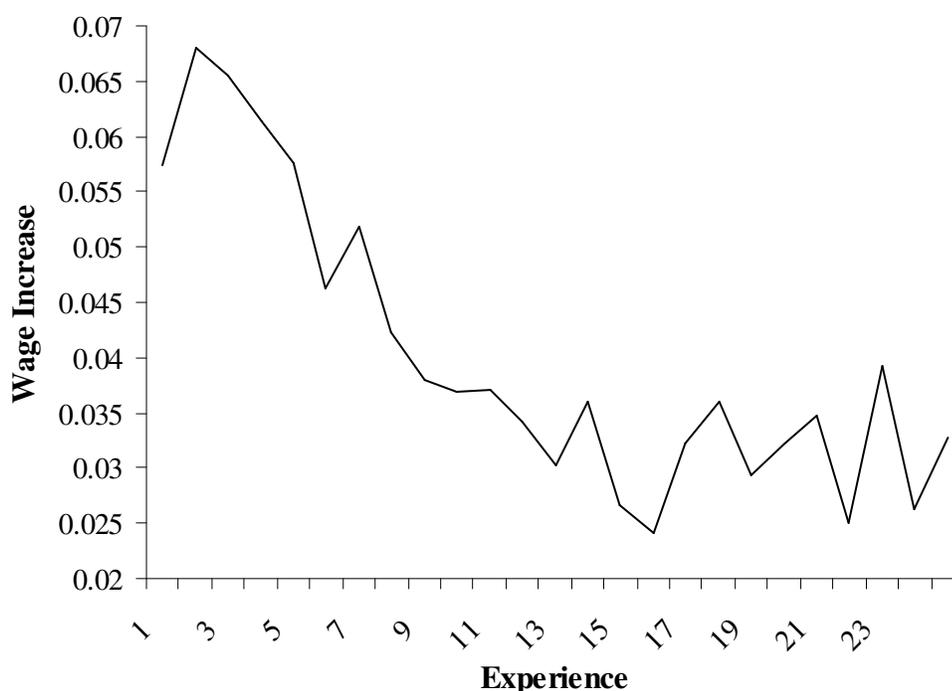
specification, controlling for observable skills and year dummies. The interpretation is that the prediction holds within observable subgroups of the data.<sup>22</sup>

For extra perspective, Figure 1.2 offers a graphical interpretation of the relationship between wage premium at promotion and experience. The plotted line is

<sup>21</sup> In all tables \*\*\* implies significance at the 1% level; \*\* implies significance at the 5% level; \* implies significance at the 10% level.

<sup>22</sup> Note that the coefficients on the education dummies in Model 3 of Table 1.2 are consistent with results reported in Devaro and Waldman (2006) analyzing the same data.

the difference between the *true* wage increase due to promotion in the data and the *predicted* wage increase if a promoted worker had *not* been promoted. Corollary 4.1 suggests that this difference should be largest early in a worker’s career and decrease with experience. This is clearly the case in Figure 1.2. Early in a worker’s career promotion leads to large wage increase relative to no promotion. As time passes, this difference shrinks.



**Figure 1.2: Wage Premium at Promotion as Function of Experience**

I next test the implication of Corollary 4.2. Corollary 4.2 predicts that the promotion rate should be higher for new hires than incumbents in the period after turnover. Formally,  $P(\text{Promotion in Period 3} \mid t_2^*=1) > P(\text{Promotion in Period 3} \mid t_2^*=0)$ . Such a test is easy to translate into a regression framework. Consider the

following OLS regression model for the sample of new hires and incumbents into a given level in period  $t$ :<sup>23</sup>

$$(2) \text{Promotion}_{it+1} = \alpha_1 + \beta_1 \text{NewHire}_{it} + \beta_2 \text{Level}_{it} + \beta_3 X_{it} + \epsilon_{it}$$

In the simplest case, when  $X_{it}$  is empty,  $\alpha_1$  is the promotion rate of incumbents and  $\beta_1$  is the difference in promotion rates between new hires and incumbents. More precisely,  $\beta_1$  is  $P(\text{Promotion in Period } 3 \mid t_2^*=1) - P(\text{Promotion in Period } 3 \mid t_2^*=0)$ . In this framework,  $\beta_1 > 0$  is a direct test of Corollary 4.2.

**Table 1.3: Effect of New Hire on Promotion in Subsequent Period, Levels 2-4**

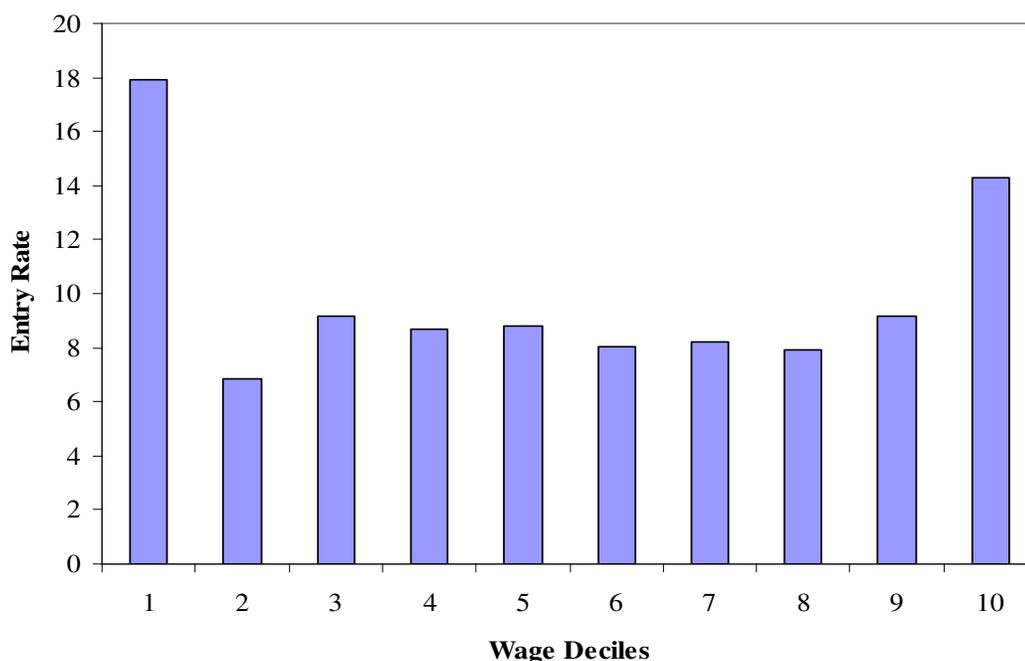
<b>Independent Variables</b>	<b>Model 1</b>	<b>Model 2</b>	<b>Model 3</b>
Promotion Rate for Incumbents ( $\alpha_1$ )	.1800	.1698	.1978
Effect of being New Hire ( $\beta_1$ )	.0478 *** (.0110)	.0529 *** (.0116)	.0637 *** (.0138)
Bachelors	-	-	.0304 * (.0180)
Masters	-	-	.0671 *** (.0208)
PhD	-	-	.0434 (.0383)
Potential Experience	-	-	-.0080 ** (.0040)
Potential Experience Squared	-	-	-.0078 (.0157)
Year Dummies	-	X	X
<b>Obs</b>	4,637	4,637	3,854

Table 1.3 presents OLS results for this regression for all three specifications of the  $X_{it}$  vector described above.  $\beta_1$  is strongly positive in all 3 specifications, with new

<sup>23</sup> I use OLS here because the independent variable of interest is discrete. Thus, we are simply comparing average promotion rates of new hires and incumbents. OLS suffices for such an exercise. Also, I drop level 1 from the analysis, as I can not identify “incumbents” into level 1.

hires having roughly a 33% greater chance of promotion in the full specification. Again, I interpret the full specification as implying the model holds within observable subgroups.

We now consider Corollary 4.3, which predicts that entry into a given job level should occur at the top and bottom of the wage distribution.<sup>24</sup> As a first pass, I examine starting wages for new hires without any controls. Figure 1.3 presents the average entry rate across Levels 2 - 4 at each decile of the wage distribution for the period 2 sample. The starkest pattern in Figure 1.3 is that entry rates spike at the 1<sup>st</sup> and 10<sup>th</sup> deciles. That is, workers are much more likely to enter a given level at either the highest decile or the lowest decile. Such a pattern is consistent with the two-sided entry implied by Corollary 4.3.



**Figure 1.3: Histogram of Average Entry Rates by Wage Decile, Levels 2-4**

<sup>24</sup> Note that for tests of Corollaries 4.3 and 4.4 I drop level 1 from the analysis. This is consistent with the theory where two-sided entry only applies to higher levels of the firm.

To test whether the pattern in Figure 1.3 is significant and robust to controls I use a semi-parametric regression to fit the relationship between starting wage and entry. I use the semi-parametric method because the relationship between wage and entry is non-linear. Standard linear regression and probit regression are not suited for such relationships. Further the semi-parametric regression allows me to control for  $X_{it}$  using standard parametric methods. Thus the control exercise is comparable to the other tests. Formally I estimate the following semi-parametric regression for each level:

$$(3) \text{Enter}_{it} = \alpha_1 + \beta_1 \text{Level}_{it} + G(\ln w_{it}) + \beta_2 X_{it} + \varepsilon_{it}$$

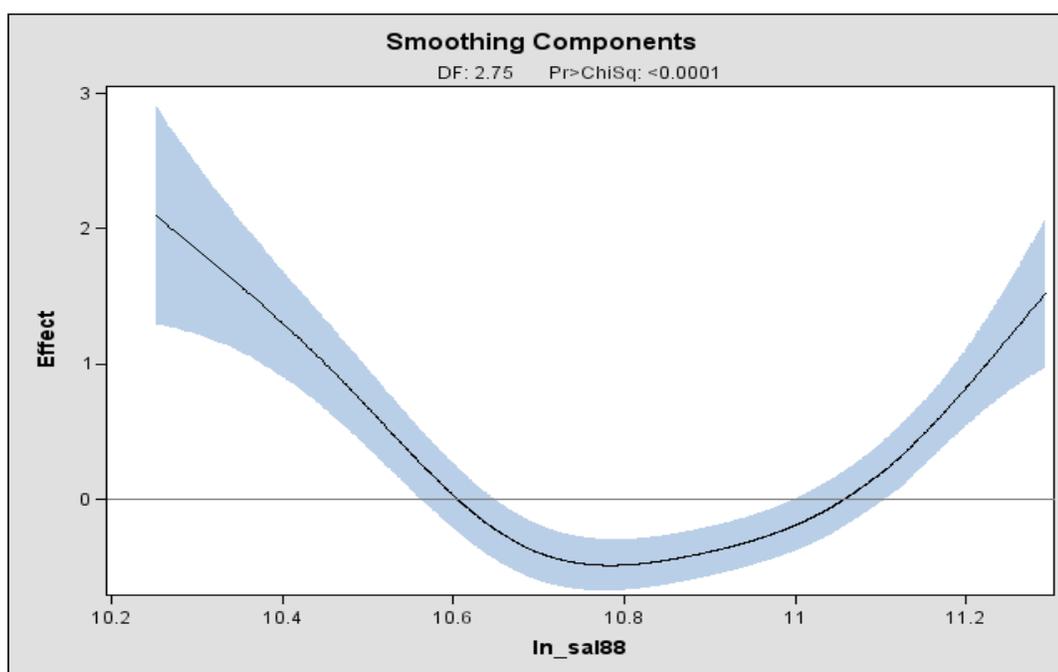
In equation (3), rather than estimate a specific functional relationship between entry and wage (i.e. linear, quadratic, etc...) as is done in parametric analysis, I estimate a smooth, flexible non-linear function  $G(\ln w_{it})$ . That is, the semi-parametric estimation does not impose a functional relationship on wage and entry. Rather it lets the data reveal the relationship  $G(\ln w_{it})$ . The function is estimated using local spline estimation.<sup>25</sup>

Figure 1.4 presents the results of the semi-parametric estimation for job level 3. The horizontal axis captures the effect of entry. The bold line captures the estimated relationship between entry and the log of starting wage controlling for the full  $X_{it}$  defined above. The U-shape implies that there is a large entry effect at the low end of the wage distribution in level 3, a small effect in the middle of the distribution, and a large effect at the top. The shaded area captures the confidence interval of the

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<sup>25</sup> For a given value of  $\ln w_{it}$ ,  $W$ , spline estimation predicts the probability of entry using the values of entry corresponding to the values of  $\ln w_{it}$  in a local neighborhood of  $W$ . Intuitively, just like parametric regression semi-parametric minimizes the sum of squared residuals. However it searches over a wider range of functions by doing so in a localized neighborhood of each point. This process allows for a flexible, non-linear relationship between the two variables. See Yatchew (1998).

curve. Note that even when accounting for the variability in the confidence intervals the U-shape relationship holds. Lastly, the p-value of  $<.0001$  on the reported Chi-Square statistic implies the U-shape relationship is strongly significant. All of these facts together suggest that the data is strongly consistent with two-sided entry. Further, I note that the relationship for levels 2 and 4 are qualitatively similar to level 3 (though weaker in level 4). See Appendix 1 for relevant figures. I also note that in an analysis not reported in the paper, I estimate a probit regression, regressing entry on a quartic in wage. The same U-shape relationship holds.



**Figure 1.4: Estimated Non-Parametric Relationship Between Entry and Starting Wage: Level 3**

**Note:** A few extreme observations from the tails were dropped to ensure smooth estimation.

Lastly, I consider the implications of Corollary 4.4. As discussed in the previous section, if we consider a simple extension of the model where  $T > 3$  and  $j > 2$ , Corollary 4.4 implies that new hires in a given year into a given level are more likely

(relative to incumbents) to reach high levels of the firm but also more likely to get stuck in low levels in their future progression up the hierarchy.

To measure a worker's future progression up the hierarchy I examine his job level in period  $t+n$  relative to the reference period  $t$ . For example, starting from 1972 I track a worker's job level in each year from, say, 1975-1979. When I do this for all workers in 1972 I have frequencies for future job level for each year in 1975-1979.

As a first pass, Table 1.4 presents the distribution of future level for new hires and incumbents in level 2 without controls.<sup>26</sup> I start at the 3 year horizon, as the two-sided pattern has not emerged at the 1 and 2 year horizon. The fact that the two-sided pattern does not immediately emerge is consistent with the extended version of the model discussed above. In such a model the two-sided pattern will not emerge until the enough entrants have been at the firm long enough to be promoted.

**Table 1.4: Distribution of Future Level for New Hires and Incumbents Who Entered Level 2 at 3, 4, 5, 6 and 7 year Horizons**

**Notes:** "N.H." stands for New Hire, "Inc" for Incumbent. The numbers in the table are calculated using all new hires and incumbents from the "period 2" sample over the years 1970-1979.

<b>Job</b>	<b><u>T+3</u></b>		<b><u>t+4</u></b>		<b><u>t+5</u></b>		<b><u>t+6</u></b>		<b><u>t+7</u></b>	
	<b>N.H.</b>	<b>Inc</b>								
2	39.2	33.6	34.0	24.8	31.3	19.9	25.8	14.3	24.4	11.5
3	49.1	57.6	36.9	53.0	31.7	48.0	29.2	47.0	26.9	43.6
4	11.7	8.7	29.2	22.2	36.6	31.7	44.0	38.1	47.2	44.1
5					0.4	0.3	1.0	0.5	1.5	0.7
6								0.2		0.2
<b>Avg</b>	2.7	2.8	3.0	3.0	3.1	3.1	3.2	3.2	3.3	3.3

<sup>26</sup> In the tests in this section I concentrate on new hires and incumbents in only level 2. I do this because the future promotion patterns of new hires into levels 3 and 4 spill over more heavily into levels 5-8, levels which have different dynamics. The distributions are conditional on not being demoted, as demotions do not occur in the theoretical model. Because demotions are so rare in the data, this is a very minor adjustment

To help interpret the table consider the “t+3” column. The 39.2 at the intersection of the “N.H.” sub-column and the “Level 2” row states that 39.2 percent of new hires into level 2 remain in level 2 three periods after entering the firm. The 33.6 in the “Inc” sub-column states that only 33.6 percent of incumbents into level 2 remain in level 2 three periods after being promoted into level 2. The relevant pattern in Table 1.4 is that at each point in time there is a higher proportion of new hires achieving extreme outcomes; more have been promoted to levels 4 and above, while, at the same time, more are stuck in level 2. For example, at the 7 year horizon roughly 24 percent of new hires are stuck in job 2, while only 12 percent of incumbents are. At the same time, roughly 49 percent of new hires have been promoted above level 4 while only 45 percent of incumbents have. This pattern is consistent with the two-sided entry predicted by the theory.

Although Table 1.4 offers strong evidence it does not offer a formal test. To test the results from Table 1.4, I use multinomial logit regression. I construct a 3-level categorical variable. The worker is in category 1 if he is in level 2 in period t+n, category 2 if he is in level 3 in period t+n, and category 3 if he is in level 4 or higher in period t+n. Denote the probability of being in category 1 in period t+n as  $\delta_{1,t+n}$ , and  $\delta_{2,t+n}$  and  $\delta_{3,t+n}$  for categories 2 and 3 respectively. I define category 2 as the reference category for the multinomial logit and estimate the following two equations.

$$(5a) \log(\delta_{1,t+n} / \delta_{2,t+n}) = \alpha_{1a} + \beta_{1a}\text{New\_Hire}_{it} + \beta_{3a}X_{it} + \varepsilon_{it}$$

$$(5b) \log(\delta_{3,t+n} / \delta_{2,t+n}) = \alpha_{1b} + \beta_{1b}\text{New\_Hire}_{it} + \beta_{3b}X_{it} + \varepsilon_{it}$$

$\beta_{1a}$  is the “new hire effect” on the odds of being in level 2 rather than level 3 in period t+n.  $\beta_{1b}$  is the “new hire effect” on the odds of being in level 4 or higher rather than level 3 in period t+n. In this context, the prediction of the theory is  $\beta_{1a}$  and  $\beta_{1b} > 0$ .

Table 1.5 presents the estimates of  $\beta_{1a}$  and  $\beta_{1b}$  for  $n=3, 4, 5, 6$  and  $7$ . The number in parenthesis is the p-value of the chi-square test for the significance of each parameter. To control for attrition, the results are presented for only the sample of workers who survive until  $t+7$ .

**Table 1.5: Multinomial Logit Parameter Estimates for Equations (5a) and (5b)**

**Note:** The table start with  $n=3$  because at  $n=1$  and  $2$  the two-sided pattern has not emerged

<b>Effect of Interest</b>	<b>t +3</b>	<b>T +4</b>	<b>t +5</b>	<b>t +6</b>	<b>t +7</b>
<b><u>Model 1: No Controls</u></b>					
New Hire Effect of being in Level 2 in $t+n$ ( $\beta_{1a}$ )	.1274 (.1226)	.3246 *** (.0004)	.4225 *** (<.0001)	.5142 *** (<.0001)	.5763 *** (<.0001)
New Hire Effect of being in Level 4+ in $t+n$ ( $\beta_{1b}$ )	.3245 ** (.0114)	.3311 *** (.0005)	.2769 *** (.0023)	.2976 *** (.0013)	.2786 *** (.0036)
<b>Obs</b>	807	807	807	807	807
<b><u>Model 2: Controls for Education and Experience</u></b>					
New Hire Effect of being in Level 2 in $t+n$ ( $\beta_{1a}$ )	.0260 (.8293)	.1542 (.2466)	.2535 * (.0811)	.4707 *** (.0033)	.5427 *** (.0040)
New Hire Effect of being in Level 4+ in $t+n$ ( $\beta_{1b}$ )	.3036 ** (.0332)	.3173 *** (.0038)	.2722 ** (.0159)	.2840 ** (.0188)	.2904 ** (.0260)
<b>Obs</b>	682	682	682	682	682

In the model with no controls we see the two-sided pattern emerge in  $t+3$  and strengthen as we get to  $t+7$ . When I control for observable skills the two-sided pattern does not emerge significantly until  $t+5$ . However by  $t+4$  it is relatively meaningful, and by  $t+7$  it is very strong. I interpret the parameters as evidence that in the long-run new hires are more likely than incumbents to achieve relatively “high” or “low” promotion outcomes. Such a pattern is consistent with the two-sided entry in the theoretical model.

The consistency of the model with a broad set of facts from the data suggests the existence of an underlying mechanism that gives rise to high ability workers entering the firm from the outside. In the data, new hires get promoted more quickly, enter at the top of the wage distribution, and climb to higher levels of the firm in the long run more frequently than incumbents. I believe the model offers a natural mechanism for such turnover; the incentive of high-ability workers to reveal their ability publicly. Further, the mechanism gives rise to other predictions that, a priori, would not necessarily seem consistent with high-ability turnover; two-sided entry and smaller wage premiums at promotion for older workers. I believe that bringing together all 4 disparate empirical findings under one framework is an important contribution to the literature on promotion dynamics.

## **1.6. Alternative Explanations**

In this section I briefly consider potential alternative explanations for the empirical findings presented above. I focus on models that can potentially tie turnover, or entry, to level of the hierarchy.

Chan (1996) considers a tournament model of promotion in which the inside firm fills vacancies with internal and external candidates. To ensure that internal workers have the proper incentive to choose high effort, Chan shows that the optimal promotion rule “favors” internal workers. As such, external hires must have higher general ability than workers internally assigned. Such a model could be consistent with the findings that new hires are more likely to be promoted in the period after entry, enter at a high wage and achieve more positive promotion outcomes in the long run. However, in Chan’s framework such predictions should weaken after controlling for observable skills. This is not the case in the data. Further, his framework can not accommodate two-sided entry. Waldman (2003) makes a similar argument to Chan.

Banerjee and Gaston (2004) consider a model of asymmetric information similar in spirit to my model. In their model a “raiding” firm receives a signal of ability from a worker at a competing firm and then attempts to bid the worker away. The inside firm can make a counter-offer to retain the worker, but counter-offers are costly. Banerjee and Gaston show that if the signal is sufficiently noisy and counter-offers sufficiently costly, then the inside firm will adopt a pooling wage policy in which it pays both high-ability workers and low-ability workers the same wage. As such, it will lose some of its high-ability workers to the “raiding” firm. Like Chan’s model, if such a model were embedded in a multi-level firm it could potentially explain the “high-end” of the turnover seen in the data, but it can not accommodate the two-sided entry.

While Chan’s model and the work by Banerjee and Gaston offer mechanisms that could induce “high end” turnover, Greenwald (1986) offers a natural explanation for “low-end” turnover. Greenwald argues that the act of firing a worker signals a worker as “low-ability” to the market. Thus, if the worker is to re-enter the market he must accept a low wage. Gibbons and Katz (1991) find evidence for this dynamic in the real world by comparing the starting wages for workers who are fired with workers who are exogenously laid off. This dynamic, however, can not account for the results associated with high-ability turnover seen in my data. Reality may in fact be some mix of the positive selection seen in my model and the adverse selection in Greenwald’s.

Finally, BGH (1994a) hypothesize that a screening-type model could explain the higher variance in future promotion outcomes of new hires compared to incumbents. If we consider a model in which the firm fills an open position by choosing between an external candidate and an internal candidate based on its beliefs about an unobserved ability parameter, it is reasonable to assume the firm will have

more precise beliefs about the ability of the internal candidate than the external candidate. Such precision would lead to less “mistakes” in job assignments and less variation in future promotion for the internal candidate compared to the external candidate. It is unclear, however, why such a model would have two-sided entry into the wage distribution for the external candidates. At the point of hiring, the firm’s beliefs should be the same for all new hires. As such they should receive similar wages. The model presented above has the nice feature of linking the larger variance in future promotion of new hires to two-sided entry into the wage distribution. Explanations that rely on more uncertain beliefs regarding new hires ability can not reconcile both.

Lastly I note that the literature offers a couple of alternative explanations for the finding that the wage premium due to promotion decreases with labor market experience. Bernhardt (1995) builds a T period model with asymmetric information in which job assignment is observed publicly. In equilibrium, if two ex-ante identical workers enter the market in the same period, the market believes the worker who is promoted at a younger age is of higher ability. It then follows that the wage jump upon promotion will be larger for young workers. Gibbons and Waldman’s (1999) model of symmetric learning could also be consistent with a shrinking wage premium at promotion. In such a model all firms in the market update their beliefs about a worker’s ability after observing each period’s production. As time passes, the variance of each firm’s posterior beliefs shrink which, in turn, causes the wage increase to decrease with experience. Neither model, however, offers a mechanism that *both* generates turnover *and* explains the decreasing wage increase at promotion.

In summary, there are several other models in the literature that can explain some of the findings in the data, but there is not a single model that can explain them all. A strength of my model, then, is that it can explain all four empirical findings.

## 1.7. Conclusion

The paper builds a model of turnover in which high ability workers leave their original firm to avoid the low wage of asymmetric information. When such a model is embedded in an environment of dynamic promotion, where job assignment is a signal to outside firms, the turnover decision of the worker becomes linked to his position in the promotion hierarchy, and generates movement between firms consistent with that from the personnel records of management in a US firm in the financial service industry. The paper's main contribution is that it offers a single explanation for a set of empirical findings about entry into the promotion hierarchy. Other potential explanations either fail to tie turnover to specific job levels within the firm, or only explain an isolated fact.

Along with explaining a set of empirical findings, I believe the paper makes an important theoretical contribution. In competitive labor markets where productivity is sensitive to unobservable skills, understanding how information is spread across firms is essential to understanding behavior. Clearly, firms and workers face very different incentives when it comes to the revealing of private information about ability. Firms can profit from keeping high-ability workers "hidden" from the market, while workers (especially high-ability workers) can profit from revealing their ability. I believe this paper offers a framework to think about this dynamic.

In particular, it offers an endogenous link between asymmetric information and symmetric information in the labor market. In most models of turnover a worker leaves his firm when a new piece of information enters the economy. Usually such information is modeled as a better wage offer from an outside firm; information exogenous to the worker. In the current model, the information is in possession of the worker and turnover arises from his incentive to share that information with other

firms. This then makes the market's ability to learn endogenous to the worker, and links asymmetric and symmetric information. Allowing the worker to partake in labor market "games," especially when analyzing high-ability labor markets, is realistic and potentially important to understanding micro level turnover behavior.

## APPENDIX 1

### I. Proof of Proposition 3.1:

The equilibrium concept I employ is a special case of the proper equilibrium. I start by defining this concept. The proper equilibrium refines the concept of trembling hand equilibrium by demanding that, when a player's hand trembles, he puts more weight on "mistakes" that have higher payoffs. I consider a specific case of the proper equilibrium in which the players only makes mistakes on  $\theta_L$  workers, the least costly mistake.

To formalize this idea, consider firm  $k$  which believes a given worker  $i$  has ability  $\Theta_i$ . Define  $\Theta_L$  as the minimum of the beliefs across all workers  $i$  on which it bids, and  $\Theta_H$  as the maximum. Denote the following mixed strategy as  $S_I'$ : Firm  $k$  bids  $S_k^*$  for all workers in  $(\Theta_L, \Theta_H]$ , and for workers with  $\Theta_i = \Theta_L$  bids  $S_k = S_k^*$  with probability  $[1 - \epsilon \int g(S) dS]$ , where  $g(S)$  is the distribution over possible actions in his action space, and puts  $\epsilon$  probability on all other actions, where  $\epsilon$  is arbitrarily small. Note that as  $\epsilon$  goes to 0  $S_I'$  converges to  $S_I^*$ . Define an analogous concept for all firms  $j \neq k$ ,  $S_j^*$ . The tuple  $[S_k^*, S_{-k}^*]$  is an equilibrium if for all firm  $j \neq k$   $S_j^*$  is a best response to  $S_{-j}^*$ , and  $S_k^*$  is a best response to  $S_{-k}^*$ .

I start at the end of the third period define the wages, and move backward.

Claim 3.1.1: The equilibrium wage in period 3 for workers with  $t^*=1$  is  $W_3 = \theta_i f(2)(1+S)$ .

Proof: It is a standard result of a 2-bidder, ascending, private-value auction that it is weakly dominant for players to set their reservation equal to their private value. Define  $R_I$  as the reservation of the inside firm, and  $R_{O1}$  and  $R_{O2}$  as the reservations of the two outside firms. It then follows that the triplet where each firm plays its valuation,  $[\theta_i f(2)(1+S), \theta_i f(2)(1+S), \theta_i f(2)]$ , is a Nash equilibrium. Because the strategies are weakly dominant, they are best responses to any other possible strategy profile. It follows immediately that the Nash equilibrium is also proper. Further, the proper equilibrium is unique because any other set of strategies must involve a weakly dominated strategy, and a proper equilibrium can not contain a weakly dominated strategy. Thus the inside firm wins the auction at  $W_3 = \theta_i f(2)(1+S)$ . QED

Claim 3.1.2: The equilibrium outcome of the auction in period 3 for workers with  $t^*=0$  is  $W_3 = \theta_i f(2)$ .

Proof: As shown below, in the unique equilibrium the period 2 wage perfectly reveals  $\theta_i$ . Thus information is perfect and symmetric for workers with  $t^*=0$ . By analogous logic it is then identical to Claim 3.1.1, the unique proper equilibrium involves all firms playing a reservation equal to their valuation. Thus the triplet  $[\theta_i f(2)(1+S), \theta_i f(2), \theta_i f(2)]$  is the unique equilibrium and the inside firm wins at  $\theta_i f(2)$ . QED

I now consider the equilibrium in period 2, given the wages in period 3. I start with workers who choose  $n_i = 0$ . I first solve for the reservation wages for all 3 firms, and then show that the inside firm will make a final offer greater than the reservation wage of the losing firm that maximizes profits.

Claim 3.1.3: In any proper equilibrium the inside firm is willing to stay in the auction until  $R_I^* = \theta_i f(1)(1+S) + S\theta_i f(2)$  for all workers.

Proof: First note that the inside firm values each worker at  $\theta_i f(1)(1+S) + \theta_i f(2)(1+S)$ . Given the period 3 wages above, his valuation in Period 2 is  $\theta_i f(1)(1+S) + S\theta_i f(2)$ . To show this strategy is weakly dominant, I proceed by contradiction. Let  $R_I$  denote any deviation from  $R_I^*$ . WLOG suppose  $R_{O1} = \max[R_{O1}, R_{O2}]$ .

Case 1: Suppose  $R_I > R_I^*$ . Suppose further  $R_{O1} < R_I^* < R_I$ . In this case, deviating to  $\theta_i f(1)(1+S) + S\theta_i f(2)$  changes nothing. Next suppose  $R_I^* < R_{O1} \leq R_I$ . In this case, I wins the worker at negative profit. Thus, it can profitably deviate to  $\theta_i f(1)(1+S) + S\theta_i f(2)$  and guarantee 0-profit. Finally suppose  $R_I < R_{O1}$ . Deviating to  $R_I^*$  changes nothing.

Case 2: Suppose  $R_I < R_I^*$ . Suppose further that  $R_I^* \geq R_{O1} > R_I$ . In this case, I loses the worker. By deviating to  $R_I^*$  it can win the worker at a (weakly) positive profit. Next suppose  $R_{O1} > R_I^* > R_I$ . Again I loses the worker and earns zero profit. By deviating to  $R_I^*$  it maintains 0-profit and is no worse off. Lastly suppose  $R_I > R_{O1}$ . Then I wins and earns positive profit. By deviating to  $R_I^*$  it maintains the profit, as the winning price,  $R_{O1}$ , is unchanged.

We conclude that  $R_I^* = \theta_i f(1)(1+S) + S\theta_i f(2)$  is a weakly dominant strategy. Thus any proper equilibrium must include  $R_I^*$ , as proper equilibria can not include weakly dominated strategies. QED

Claim 3.1.4: The unique proper equilibrium is  $[R_I, R_{O1}, R_{O2}] = [\theta_i f(1)(1+S) + S\theta_i f(2), \theta_L f(1), \theta_H f(1)]$  for all workers with  $n_i = 0$ .

Proof: Consider the mixed strategy in which the inside firm plays  $R_I^* = \theta_i f(1)(1+S) + S\theta_i f(2)$  for all workers in  $(\theta_L, \theta_H]$ , and for workers with  $\theta_i = \theta_L$  plays  $R_I = R_I^* = \theta_i f(1)(1+S) + S\theta_i f(2)$  with probability  $[1 - \varepsilon \int g(W_2) dW_2]$ , where  $g(W_2)$  is the distribution over possible wage offers and puts  $\varepsilon$  probability on all other offers, where  $\varepsilon$  is arbitrarily small.<sup>27</sup> Denote this strategy  $R_I'$  and note that as  $\varepsilon$  goes to 0 it converges to  $R_I^*$ .  $R_I'$  captures the idea that the inside firm will only make mistakes on the least costly worker. Further consider the mixed strategy in which the outside firm O1 plays  $R_{O1}^* = \theta_L f(1)$  with probability  $[1 - \varepsilon \int g(W_2) dW_2]$ , where  $g(W_2)$  is the distribution over possible wage offers and puts  $\varepsilon$  probability on all other offers, where  $\varepsilon$  is arbitrarily small. Denote this strategy as  $R_{O1}'$  and define an analogous strategy  $R_{O2}'$  for firm O2. To show  $[R_I, R_{O1}, R_{O2}] = [\theta_i f(1)(1+S) + S\theta_i f(2), \theta_L f(1), \theta_H f(1)]$  is the unique proper equilibrium I will show that  $R_{O1}^*$  is the *only* best response to  $R_I'$  and

<sup>27</sup> We limit the support to  $(0, (a_2 + b_2 \theta_H f(2))(1+S)]$ , i.e. no firm would offer a negative wage, or a wage higher than the best worker's productivity.

$R_{O2}'$ ,  $R_{O2}^*$  is the *only* best response to  $R_I'$  and  $R_{O1}'$ , and  $R_I^*$  is the *only* best response to  $R_{O1}'$  and  $R_{O2}'$ .

First I show  $[\theta_i f(1)(1+S) + S\theta_i f(2), \theta_L f(1), \theta_L f(1)]$  are best responses to each other. Consider first  $R_{O1}^*$ , given  $R_{O2}'$  and  $R_I'$ . Suppose O1 deviates to some  $R_{O1} > R_{O1}^*$ . It will still win all  $\theta_L$  workers at a positive profit when the inside firm and firm O2 “mistakenly” offer a wage in  $[0, \theta_L f(1)]$ . However, it will also win  $\theta_L$  workers at a negative profit when the inside firm and firm O2 “mistakenly” offer  $\theta_L$  workers a wage in  $(\theta_L f(1) + p\sigma_H, R_{O1}]$ . By staying at  $R_{O1}^*$  it is thus better off. Suppose O1 deviates to some  $R_{O1} < R_{O1}^*$ . It will lose all  $\theta_L$  workers on which it was earning positive profits when the inside firm and firm O2 “mistakenly” offers wages in  $(R_{O1}, \theta_L f(1)]$ . By staying at  $R_{O1}^*$  it is thus better off. We conclude that  $R_{O1}^*$  is a best response to  $R_{O2}'$  and  $R_I'$ . Because the problem is symmetric for O1 and O2, we also conclude that  $R_{O2}^*$  is a best response to  $R_{O1}'$  and  $R_I'$ . Finally, because  $R_I^*$  is a weakly dominant strategy it is a best response to  $R_{O1}^*$  and  $R_{O2}^*$ . We conclude that the triplet  $[R_I^*, R_{O1}^*, R_{O2}^*]$  is a proper equilibrium.

Now I show that no other set of strategies are part of a proper equilibrium. I first note that by Claim 3.1.3 we know that any proper equilibrium must have  $R_I = \theta_i f(1)(1+S) + S\theta_i f(2)$ . Thus I only need to show that any proper equilibrium must have  $R_{O1} = R_{O2} = \theta_L f(1)$  to conclude that  $[R_I, R_{O1}, R_{O2}] = [\theta_i f(1)(1+S) + S\theta_i f(2), \theta_L f(1), \theta_L f(1)]$  is unique.

I proceed by contradiction. WLOG, suppose the equilibrium is such that  $R_{O1} > R_{O2} \geq \theta_L f(1)$ . Firm O1 will win all  $\theta_L$  workers at a negative profit when the inside firm “mistakenly” offers  $\theta_L$  workers a wage in  $(R_{O2}, R_{O1}]$ . By deviating to some  $R_{O1} < R_{O2}$  it can be better off. Suppose the equilibrium is such that  $R_{O1} = R_{O2} > \theta_L f(1)$ . Each firm will win some proportion of  $\theta_L$  workers at a negative profit when the inside firm “mistakenly” offers  $\theta_L$  workers a wage in  $(\theta_L f(1), R_{O1}]$ . By deviating to  $\theta_L f(1)$  it will eliminate this possibility and be better off. WLOG, next suppose  $R_{O1} < R_{O2} \leq \theta_L f(1)$ . By deviating to  $\theta_L f(1)$  firm O1 will win all  $\theta_L$  workers at a positive profit when the inside firm “mistakenly” offers  $\theta_L$  workers a wage in  $(R_{O2}, \theta_L f(1)]$ , and be better off. Finally, suppose  $R_{O1} = R_{O2} < \theta_L f(1)$ . Either firm can deviate to  $\theta_L f(1)$  and win a proportion of  $\theta_L$  workers at a positive profit when the inside firm “mistakenly” offers  $\theta_L$  workers a wage in  $(R_{O2}, \theta_L f(1)]$ . Thus,  $R_{O1} = R_{O2} < \theta_L f(1)$  can not be part of an equilibrium

I conclude that the unique proper equilibrium is  $[R_I, R_{O1}, R_{O2}] = [\theta_i f(1)(1+S) + S\theta_i f(2), \theta_L f(1), \theta_L f(1)]$ . QED

We know that the unique equilibrium is  $[R_I, R_{O1}, R_{O2}] = [\theta_i f(1)(1+S) + S\theta_i f(2), \theta_L f(1), \theta_L f(1)]$ . However, as noted in the text, the winning firm in the auction may have an incentive to continue to raise the wage even after all other firms drop out. Claim 3.1.5 shows that the inside firm will find it optimal to raise the wage to  $W_2 = \theta_L f(1) + S\theta_i f(2)$  even though the two outside firms drop out at  $\theta_L f(1)$ .

Claim 3.1.5: In the unique proper equilibrium, the inside firm will win all workers with  $n_i = 0$  in Period 2 at a wage  $W_2 = \theta_L f(1) + S\theta_i f(2)$ .

Proof: Consider the inside firm's objective when choosing  $W_2$ . From Claim 3.1.3 the inside firm is willing to pay a total over Period 2 and 3 of  $\theta_i f(1)(1+S) + (1+S)\theta_i f(2)$  to retain a worker. Further it knows from Claims 3.1.1 and 3.1.4 that a worker who decides to turnover will earn  $W_2 + W_3 = \theta_L f(1) + \theta_i f(2)(1+S)$ . Because  $\theta_i f(1)(1+S) + (1+S)\theta_i f(2) > \theta_L f(1) + \theta_i f(2)(1+S)$ , the  $W_2$  that maximizes profits is such that  $W_2 + W_3 = \theta_L f(1) + \theta_i f(2)(1+S)$ . From Claim 3.1.2,  $W_3 = \theta_i f(2)$  for workers with  $n_i = 0$ . Thus the optimal Period 2 wage is exactly  $W_2 = \theta_L f(1) + \theta_i f(2)(1+S) - \theta_i f(2) = \theta_L f(1) + S\theta_i f(2)$ . Note that this wage perfectly reveals  $\theta_i$  for all workers. I conclude that in any separating equilibrium the inside firm retains the worker at  $W_2 = \theta_L f(1) + S\theta_i f(2)$ , and that  $W_2$  reveals each worker's ability to the outside firms in period 3. QED

Claim 3.1.6: The period 2 wage in the unique proper equilibrium must be perfectly revealing.

Proof: I proceed by contradiction. Suppose there is an equilibrium that is not perfectly revealing. Then there will be workers  $i$  and  $k$  such that  $\theta_i \neq \theta_k$  (WLOG assume  $\theta_i < \theta_k$ ) and period 2 wages  $W_2(\theta_i) = W_2(\theta_k)$ . In turn, the outside wage offer in Period 3,  $W_{30}$ , will be the same for  $\theta_i$  and  $\theta_k$ . We know that in order to maximize profits the firm must offer each worker a period 2 wage  $\theta_L f(1) + \theta_i f(2)(1+S) - W_{30}$ . But doing this implies  $W_2(\theta_i) \neq W_2(\theta_k)$ . This is a contradiction. QED

I now consider workers with  $n_i = 1$  in period 2:

Claim 3.1.7: For workers with  $n_i = 1$ , in the proper equilibrium, the networked firm will drop out of the bidding at  $\theta_i f(1)$  and the non-networked firm will drop out at  $\theta_L f(1)$ .

Proof: The logic for the non-networked firm is identical to Claims 3.1.4. For the networked firm, the logic is identical to 3.1.3. Because it has perfect information it is weakly dominant for him to play his valuation, which is  $\theta_i f(1)$  given the Period 3 wages. QED

Claim 3.1.8: For workers with  $n_i = 1$ , in the proper equilibrium, the inside firm will offer a wage of  $W_2 = \theta_i f(1) + S\theta_i f(2)$ .

Proof: The logic is identical to Claim 3.1.5. Given, Claim 3.1.7 and 3.1.1 the inside firm has to pay all workers with  $n_i = 1$  at least  $W_2 = \theta_i f(1) + S\theta_i f(2)$  to ensure they don't turnover. Thus it pays exactly that to maximize profits. QED

Given the Period 2 and 3 wages, I now consider the workers' networking decisions.

Claim 3.1.9: All workers with  $\theta_i \geq \theta_N = \theta_L + C/f(1)$  will choose  $n_i = 1$ .

Proof: If a worker networks he receives  $\theta_i f(1) + \theta_i f(2)(1+S) - C$ . If he does not network he receives  $\theta_L f(1) + \theta_i f(2)(1+S)$ . Thus, all workers with  $\theta_i \geq \theta_N = \theta_L + C/f(1)$  will network. QED

In summary, in the unique proper equilibrium in which the firm makes almost all mistakes on low-types:

- (1) All workers with  $\theta_i \geq \theta_N$  will choose  $n_i = 1$  and receive  $[W_2, W_3] = [\theta_i f(1) + S\theta_i f(2), \theta_i f(2)]$ .
- (2) All other workers choose  $n_i = 0$  and earn  $[W_2, W_3] = [\theta_L f(1) + S\theta_i f(2), \theta_i f(2)]$

## II. Proof that no worker networks in period 1

I first calculate a worker's expected lifetime income if he chooses  $n_i = 1$  in period 1.

In period 3, such a worker will receive  $(1+S)E[\theta_i f(2)]$  if he had turned over in period 2, and  $E[\theta_i f(2)]$  if he had not. In period 2, the inside firm, knowing that the worker can earn  $(1+S)E[\theta_i f(2)]$  if he turns over, will offer a wage of  $E[\theta_i f(1)] + S\theta_i f(2)$ . The outside firm at which he networked in period 1 will offer  $E[\theta_i f(1)]$  knowing it has to pay  $(1+S)E[\theta_i f(2)]$  in period 3 if it wins the worker. Given these wage offers, the worker will stay with the inside firm and earn  $E[\theta_i f(1)] + S\theta_i f(2)$  in period 2 and  $E[\theta_i f(2)]$  in period 3.

In period 1 the worker will produce  $E[\theta_i f(0)]$ . Given the period 2 and 3 wage outcomes above, the two competing firms at which he has networked know they can make  $\theta_i f(1)S$  in profits if they win the worker. Thus the wage in period 1 will be bid up to  $\theta_i f(0) + S\theta_i f(1)$ . The worker will randomize between the firms and earn a total expected lifetime income is equal to:

$$(1) E[\theta_i f(0) + \theta_i f(2)(1+S) + \theta_i f(2)(1+S)] - 2C$$

Now suppose he chooses  $n_i = 0$  in period 1, and chooses  $n_i$  optimally in period 2:

Given the wages solved for above, the worker earns an expected wage of  $E[\theta_L f(1) | \theta_i < \theta_N]P(\theta_i < \theta_N) + E[\theta_i f(1) | \theta_i > \theta_N]P(\theta_i > \theta_N) + (1+S)\theta_i f(2)$  over periods 2 and 3.

In period 1, all firms are completely ignorant of the worker's ability. They will expect him to produce  $E[\theta_i f(0)]$ . If they win him, however, they know they will earn  $E[\theta_i f(1)S] + P(\theta_i < \theta_N)[E[\theta_i f(1) | \theta_i < \theta_N] - \theta_L f(1)]$  in profits over periods 2 and 3. Thus they bid the wage up to  $E[\theta_i f(0) + S\theta_i f(1)] + P(\theta_i < \theta_N)[E[\theta_i f(1) | \theta_i < \theta_N] - \theta_L f(1)]$ . As a result his total lifetime income is:

$$(2) E[\theta_i f(0)] + P(\theta_i < \theta_N)[E[\theta_i f(1) | \theta_i < \theta_N] - \theta_L f(1)] + \theta_L f(1)P(\theta_i < \theta_N) + E[\theta_i f(1) | \theta_i > \theta_N]P(\theta_i > \theta_N) + S\theta_i f(1) + E[\theta_i f(2)(1+S)]$$

$$= E[\theta_i f(0)] + E[(1+S)\theta_i f(1) + E[\theta_i f(2)(1+S)]] > E[\theta_i f(0) + \theta_i f(2)(1+S) + \theta_i f(2)(1+S)] - 2C$$

The last line states that (2) > (1). We conclude no worker will network in period 1.  
QED

### III. Proof of Proposition 3.2:

The model analyzed is identical to that from Proposition 3.2 with the exception of the additive matching parameter. The logic of the equilibrium is identical, but the payoffs will change slightly, and some workers will turnover in Period 2.

Claim 3.2.1: The outcome of the auction in period 3 for workers with  $n_i = 1$  in period 2 is  $W_3 = \theta_i f(2)(1+S) + p\sigma_{iI}$  if  $t^*=1$  and  $W_3 = \theta_i f(2) + p\sigma_H$  if  $t^*=0$ .

Proof: The strategy of the inside firm and the networked firm is identical to Claim 3.1.1 with the addition of the match parameter. Each firm plays their valuation of the worker for period 3. The third firm, however, will form beliefs about  $\theta_i$  based on  $W_2$ . As will be shown below  $W_2 = \theta_i f(1) + S\theta_i f(2) + 2\sigma_{iO} - p\sigma_H$  for workers with  $n_i = 1$  and  $t^*=0$ . Because the third firm does not observe  $\sigma_{iO}$ ,  $W_2$  does not perfectly reveal  $\theta_i$ . Thus, or a given value of  $W_2$  the third firm has the following beliefs:

$$\begin{aligned}\theta_i &= (W_2 - (2\sigma_H - p\sigma_H))/(f(1) + Sf(2)) = \theta_i(W_2, \sigma_H) \quad \text{with probability } p \\ \theta_i &= (W_2 + p\sigma_H)/(f(1) + Sf(2)) = \theta_i(W_2, 0) \quad \text{with probability } 1-p\end{aligned}$$

Consider first workers with  $n_i = 1$  and  $t^*=0$ . Given that the minimum of the inside firm's and the networked firm's bids is at least  $\theta_i f(1)(1+S) + p\sigma_H$ , the third firm's valuation, there is no way that the third firm can win the worker at a profit. Thus the best it can do is earn 0-profit. Given the demands of the proper equilibrium, it must play a strategy as a function of  $W_2$ ,  $R_{O2}(W_2)$ , that earns 0-profit when the two informed firms make mistakes on  $\theta_N(\sigma_H)$  workers, the lowest ability worker with  $n_i=1$ . The strategy that achieves this is  $R_{O2}(W_2) = \theta_i(W_2, \sigma_H)f(1) + p\sigma_H$  because  $\theta_i(W_2, \sigma_H)f(1) \leq \theta_i$  ensures it can not win a worker at a negative profit.

For workers with  $n_i = 1$  and  $t^*=1$ ,  $W_2 = (1+S)\theta_i f(1) - p\sigma_H$ . Thus the third firm can perfectly infer  $\theta_i$  for such workers and will play  $R_{O2}(W_2) = \theta_i f(1) + p\sigma_H$ .

Combining the strategies, the equilibrium strategy triplet for  $n_i = 1$  and  $t^*=1$  is  $[\theta_i f(2)(1+S) + \sigma_{iI}, \theta_i f(2)(1+S) + p\sigma_H, \theta_i f(2) + p\sigma_H]$  for  $n_i = 1$  and  $t^*=0$  is  $[\theta_i f(2)(1+S) + \sigma_{iI}, \theta_i f(2) + p\sigma_H, \theta_i(W_2, \sigma_H)f(1) + p\sigma_H]$ . QED

Claim 3.2.2: The outcome of the auction in Period 3 for workers with,  $n_i = 0$  is  $W_3 = \theta_i f(2) + p\sigma_H$  if  $t^*=0$  and  $\theta_i f(2)(1+S) + p\sigma_{iI}$  if  $t^*=1$ .

Proof: For workers with  $n_i = 0$  all three firms have perfect information regarding  $\theta_i$ . The equilibrium strategy triplet is simply each firm playing its valuation:  $[\theta_i f(2)(1+S) + \sigma_{iI}, \theta_i f(2) + p\sigma_H, \theta_i f(2) + p\sigma_H]$  for  $t^*=0$  and  $[\theta_i f(2)(1+S) + \sigma_{iI}, \theta_i f(2)(1+S) + p\sigma_H, \theta_i f(2) + p\sigma_H]$  for  $t^*=1$ . QED

I now consider the ascending auction in Period 2 given the above wages in Period 3. I consider first the case of  $n_i = 0$ :

Claim 3.2.3: In any proper equilibrium the inside firm is willing to stay in the auction until  $R_1^* = \theta_i f(1)(1+S) + S\theta_i f(2) + 2\sigma_{iI} - p\sigma_H$ .

Proof: The logic is the same as Claim 3.1.3. Given the period 3 wages above the firm is now willing to pay  $\theta_i f(1)(1+S) + S\theta_i f(2) + 2\sigma_{iI} - p\sigma_H$  to keep a worker. QED

Claim 3.2.4: In the unique proper equilibrium  $(R_1', R_{O1}, R_{O2}) = (\theta_i f(1)(1+S) + S\theta_i f(2) + 2\sigma_{iI} - p\sigma_H, \theta_L f(1) + p\sigma_H + (1-p)p\sigma_H, \theta_L f(1) + p\sigma_H + (1-p)p\sigma_H)$ .

Proof: The logic is same as Claim 3.1.4. The outside firms know that the inside firm will make mistakes on  $\theta_L$  workers. Thus, it is optimal to choose a reservation in period 2 that exhausts the value of a  $\theta_L$  worker, given the period 3 wages above. A  $\theta_L$  worker is worth  $\theta_L f(1) + p\sigma_H + p[(1+S)\theta_L f(2) + \sigma_H]$ . The outside firms' expected wage in period 3 is  $p[(1+S)\theta_L f(2) + p\sigma_H]$ . Thus, it is willing to pay  $\theta_L f(1) + p\sigma_H + (1-p)p\sigma_H$ . QED

Claim 3.2.5: In the unique proper equilibrium, the inside firm will win all workers with  $n_i = 0$  in Period 2 at a wage  $W_2 = \theta_L f(1) + S\theta_i f(2) + p\sigma_H$ .

Proof: The logic is the same as claim 3.1.5. Given that  $\max[R_{O1}, R_{O2}] = \theta_L f(1) + p\sigma_H + (1-p)p\sigma_H$ , the inside firm knows that a worker who decides to turnover expects to earn  $\theta_L f(1) + p\sigma_H + (1-p)p\sigma_H + \theta_i f(2)(1+S) + pp\sigma_H = \theta_L f(1) + 2p\sigma_H + \theta_i f(2)(1+S)$ . Given the Period 3 wage of  $\theta_i f(1) + p\sigma_H$ , the firm must pay  $W_2 = \theta_L f(1) + S\theta_i f(2) + p\sigma_H$  to ensure the worker stays. QED

Claim 3.2.6: The period 2 wage in the unique proper equilibrium must be perfectly revealing.

Proof: This case is identical to Claim 3.2.6. The separating equilibrium always yields higher profits for the firm. QED

I now consider wages in period 2 for workers with  $n_i = 1$ :

Claim 3.2.7: For workers with  $n_i = 1$  and  $\sigma_{iI} = 0$  and  $\sigma_{iO} = \sigma_H$ , in the unique proper equilibrium, in period 2 the networked firm wins the worker at  $W_2 = \theta_i f(1)(1+S) - p\sigma_H$ .

Proof: The total value of the worker for periods 2 and 3 if he chooses the outside firm is  $\theta_i f(1) + (1+S)\theta_i f(2) + 2\sigma_H$  which, by assumption, is greater than the total value at the inside firm  $(1+S)\theta_i f(1) + (1+S)\theta_i f(2)$ . The worker chooses the period 2 wage that generates the most lifetime income. As such, the outside firm will drop out of the auction at a period 2 wage that generates lifetime income just equal to the worker's value at the inside firm and the worker will turn over. Such a period 2 wage,  $W_2$ , solves  $(1+S)\theta_i f(1) + (1+S)\theta_i f(2) = W_2 + (1+S)\theta_i f(2) + p\sigma_H$ . Algebra yields  $W_2 = \theta_i f(1)(1+S) - p\sigma_H$ . Lastly, because both strategies are weakly dominant, they are proper. Further because all other strategies are weakly dominated, they can not be proper. QED

Claim 3.2.8: For workers with  $n_i = 1$  and  $t^* = 0$ , in the unique equilibrium, the inside firm will offer a wage of  $\theta_i f(1) + S\theta_i f(2) + 2\sigma_{iO} - p\sigma_H$  in period 2.

Proof: Because this is a standard ascending value auction, it is weakly dominant for both firms to choose reservation wages in period 2 that exhaust the workers' total value, given the period 3 wages defined above. The total value at the inside firm  $(1+S)\theta_i f(1) + (1+S)\theta_i f(2) + 2\sigma_H$  is greater than the total value at the outside firm  $\theta_i f(1) + (1+S)\theta_i f(2) + 2\sigma_{iO}$ . Thus the inside firm will win all such workers. The optimal period 2 wage offer will be such that the worker is just indifferent between staying and leaving. Such a period 2 wage,  $W_2$ , solves  $\theta_i f(1) + (1+S)\theta_i f(2) + 2\sigma_{iO} = W_2 + \theta_i f(2) + p\sigma_H$ . Algebra yields  $W_2 = \theta_i f(1) + S\theta_i f(2) + 2\sigma_{iO} - p\sigma_H$ .

Lastly, because both strategies are weakly dominant, they are proper. Further because all other strategies are weakly dominated, they can not be proper. QED

Finally, given the above payoffs, I consider the worker's networking decision in Stage (1). There will be different cut-offs depending on  $\sigma_{iI}$ .

Claim 3.2.9: All workers with  $\sigma_{iI} = \sigma_H$  and  $\theta_i \geq \theta_N(\sigma_H) = \theta_L + C/f(1)$ , and all workers with  $\sigma_{iI} = 0$  and  $\theta_i \geq \theta_N(0) = \theta_L + C/f(1) + K$  choose  $n_i = 1$  where  $K = p(2\sigma_H - S\theta_N(0)f(1))/f(1) > 0$ .

Proof: When  $\sigma_{iI} = \sigma_H$ , if a worker networks he receives  $E(\theta_i f(1) - p\sigma_H + 2\sigma_{iO} + \theta_i f(2)(1+S) + p\sigma_H - C) = \theta_i f(1) + p\sigma_H + \theta_i f(2)(1+S) + p\sigma_H - C$ . If he does not network he receives  $\theta_i f(1) + p\sigma_H + \theta_i f(2)(1+S) + p\sigma_H$ . Thus, all worker with  $\sigma_{iI} = \sigma_H$  and  $\theta_i \geq \theta_N(\sigma_H) = \theta_L + C/f(1)$  will network.

When  $\sigma_{iI} = 0$ , if a worker networks he receives  $\theta_i f(1)(1+pS) - p\sigma_H + \theta_i f(2)(1+S) + p\sigma_H - C$ . If he does not network he receives  $\theta_i f(1) + p\sigma_H + \theta_i f(2)(1+S) + p\sigma_H$ . Thus, all worker with  $\sigma_{iI} = \sigma_H$  and  $\theta_i \geq \theta_N(0) = \theta_L/(1+pS) + (2p\sigma_H + C)/f(1)(1+pS)$  will network.  $\theta_N(0)$  can be re-written in terms of  $\theta_N(\sigma_H)$  as follows:  $\theta_N(0) = \theta_N(\sigma_H) + K$  where  $K = p(2\sigma_H - S\theta_N(0)f(1))/f(1)$  because the second term is always positive by assumption. QED

In summary, in equilibrium:

- (1) All workers with  $\theta_i \geq \theta_N(\sigma_{iI})$  will choose  $n_i = 1$ . If  $\sigma_{iO} = \sigma_H$  and  $\sigma_{iI} = 0$ , they turnover and  $[W_2, W_3] = [\theta_i f(1)(1+S) - p\sigma_H, \theta_i f(2)(1+S) + p\sigma_H]$ . Otherwise they stay with the inside firm and earn  $[W_2, W_3] = [\theta_i f(1) + S\theta_i f(2) + 2\sigma_{iO} - p\sigma_H, \theta_i f(2) + p\sigma_H]$ .
- (2) All other workers choose  $n_i = 0$  and earn  $[W_2, W_3] = [\theta_i f(1) + S\theta_i f(2) + p\sigma_H, \theta_i f(2) + p\sigma_H]$ .

#### IV. Proof of Proposition 4.1:

The logic of the model analyzed is identical to that from Proposition 3.3 with the addition of an extra job level. When the subscript  $j$  is used in the proof it is assumed to represent the optimal job assignment  $j$  from the employing firm's perspective. Formally  $j \operatorname{argmax} \{a_2 + b_2\theta_i f(t), a_1 + b_1\theta_i f(t)\}$ .

Claim 4.1.1: The outcome of the auction in period 3 for workers with  $n_i = 1$  is  $W_3 = (a_j + b_j\theta_i f(2))(1+S) + p\sigma_{iO}$  if  $t^*=1$  and  $W_3 = a_j + b_j\theta_i f(2) + p\sigma_H$  if  $t^*=0$ .

Proof: The logic is analogous Claim 3.2.1. For workers with  $n_i = 1$  and  $t^*=0$ ,  $W_2 = a_j + b_j\theta_i f(1) + 2\sigma_{iO} + S(a_j + b_j\theta_i f(2)) - p\sigma_H$  does not perfectly reveal  $\theta_i$  for the third firm. For such workers in a given job level  $j$ , the third firm will choose a reservation wage of  $a_j + b_j\theta_i(W_2, \sigma_H) + p\sigma_H$  where  $\theta_i(W_2, \sigma_H) = (W_2 - (2\sigma_H - p\sigma_H) - (1+S)a_j)/(b_j f(1) + b_j S f(2))$  is the inferred ability given  $W_2$  and  $\sigma_{iO} = \sigma_H$ . As in claim 3.2.1, the third firm chooses this strategy to ensure it does not win workers on which the other two firms make mistakes.

For workers with  $t^*=1$ ,  $W_2 = (a_j + b_j\theta_i f(1))(1+S) - p\sigma_H$  perfectly reveals  $\theta_i$  for the third firm. Thus it simply plays its valuation for period 3,  $a_j + b_j\theta_i f(2) + p\sigma_H$ .

Thus, the equilibrium strategy triplet for workers in job  $j$  is  $[(a_j + b_j\theta_i f(2))(1+S) + \sigma_{iL}, (a_j + b_j\theta_i f(2))(1+S) + p\sigma_H, (a_j + b_j\theta_i f(2)) + p\sigma_H]$  if  $t^*=1$  and  $[(a_j + b_j\theta_i f(2))(1+S) + \sigma_{iL}, a_j + b_j\theta_i f(2) + p\sigma_H, a_j + b_j\theta_i(W_2, \sigma_H) + p\sigma_H]$  if  $t^*=0$ . QED

Claim 4.1.2: The outcome of the auction in Period 3 for workers with,  $n_i = 0$  the outcome is  $W_3 = (a_j + b_j\theta_i f(2))(1+S) + p\sigma_{iO}$  if  $t^*=1$  and  $W_3 = a_j + b_j\theta_i f(2) + p\sigma_H$  if  $t^*=0$ .

Proof: The logic is the same as Claim 3.1.2, but now outside there are two jobs. The equilibrium strategy triplet for workers in job  $j$  is  $[(a_j + b_j\theta_i f(2))(1+S) + \sigma_{iL}, a_j + b_j\theta_i f(2) + p\sigma_H, a_j + b_j\theta_i f(2) + p\sigma_H]$  if  $t^*=0$  and  $[(a_j + b_j\theta_i f(2))(1+S) + \sigma_{iL}, (a_j + b_j\theta_i f(2))(1+S) + p\sigma_H, a_j + b_j\theta_i f(2) + p\sigma_H]$  if  $t^*=1$ . QED

Given the period 3 wages, I now consider the firm's optimal job assignment. The firm will promote all workers for which the marginal benefit is greater than the marginal cost.

Claim 4.1.3: For all workers in Period 3 the optimal promotion cut-off is  $\eta'/f(2)$ .

Proof: For workers with  $t^*=0$  I have:

$$\begin{aligned} MB &= (1+S)(a_2 + b_2\theta_i f(2) - a_1 - b_1\theta_i f(2)) \\ MC &= \max_2\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\} - \max\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\} \end{aligned}$$

$MC = 0$  for all  $\theta_i$ . The  $MB = 0$  only when  $\theta_i f(2) = \eta'$  and is greater than 0 for  $\theta_i$  greater than  $\theta_i f(2) = \eta'$  I conclude that workers  $\theta_i > \eta'/f(2)$  with  $t^*=0$  are promoted.

For workers with  $t^*=1$  I have:

$$\begin{aligned} MB &= (1+S)(a_2 + b_2\theta_i f(2) - a_1 - b_1\theta_i f(2)) \\ MC &= (1+S)\max_2\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\} - \max\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\} \end{aligned}$$

MC = 0 for all  $\theta_i$ . The MB = 0 only when  $\theta_i f(2) = \eta'$  and is greater than 0 for  $\theta_i$  greater than  $\theta_i f(2) = \eta'$ . I conclude that workers  $\theta_i > \eta' / f(2)$  with  $t^* = 0$  are promoted. QED

I now consider the ascending auction in Period 2 given the above wages in Period 3. I consider first the case of  $n_i = 0$ :

Claim 4.1.4: In any proper equilibrium the inside firm is willing to stay in the auction until  $R_1^* = (a_j + b_j \theta_i f(1))(1+S) + S(a_j + b_j \theta_i f(2)) + 2\sigma_{il}$ .

Proof: The logic is the same as Claim 3.1.3. Given the period 3 wages above the firm is now willing to pay  $(a_j + b_j \theta_i f(1))(1+S) + S(a_j + b_j \theta_i f(1)) + 2\sigma_{il} - p\sigma_H$  to keep a worker. QED

Claim 4.1.5: In the unique proper equilibrium  $(R_1', R_{O1}, R_{O2}) = (a_1 + b_1 \theta_i f(1))(1+S) + S(a_1 + b_1 \theta_i f(2)) + 2\sigma_{il}$ ,  $a_1 + b_1 \theta_L f(1) + p\sigma_H + (1-p)p\sigma_H$ ,  $a_1 + b_1 \theta_L f(1) + p\sigma_H + (1-p)p\sigma_H$  for workers in job 1 and  $(R_1', R_{O1}, R_{O2}) = (a_2 + b_2 \theta_i f(1))(1+S) + S(a_2 + b_2 \theta_i f(2)) + 2\sigma_{il}$ ,  $a_2 + b_2 \theta^* f(1) + p\sigma_H + (1-p)p\sigma_H$ ,  $a_2 + b_2 \theta^* f(1) + p\sigma_H + (1-p)p\sigma_H$  for workers in job 2.

Proof: The logic is same as Claim 3.1.4. The outside firms know that the inside firm will make mistakes on the lowest type workers in a given job level. Thus, they will choose a reservation wage in period 2 that exhausts the value of a  $\theta_L$  worker, given the period 3 wages above. It is willing to pay  $\theta_L f(1) + p\sigma_H + (1-p)p\sigma_H$ . For job 1 this reservation wage is  $(a_1 + b_1 \theta_L f(1)) + p\sigma_H + (1-p)p\sigma_H$ . For job 2 it is  $(a_2 + b_2 \theta^* f(1)) + p\sigma_H + (1-p)p\sigma_H$ . QED

Claim 4.1.6: In the unique proper equilibrium the inside firm will win all workers with  $n_i = 0$  in Period 2 in Job 1 at a wage  $W_2 = a_1 + b_1 \theta_L f(1) + S(a_j + b_j \theta_i f(2)) + p\sigma_H$ , and workers in Job 2 at a wage  $W_2 = a_2 + b_2 \theta^* f(1) + S(a_j + b_j \theta_i f(2)) + p\sigma_H$ .

Proof: The logic is the same as claim 3.1.5. Given Claim 4.1.6, the inside firm knows that a worker who decides to turnover from Job 1 expects to earn  $W_2 + W_3 = a_1 + b_1 \theta_L f(1) + (a_j + b_j \theta_i f(2))(1+S) + 2p\sigma_H$ . Given the Period 3 wage of  $a_j + b_j \theta_i f(2) + p\sigma_H$ , the firm find its optimal to offer  $W_2 = (a_1 + b_1 \theta_L f(1)) + S(a_j + b_j \theta_i f(2)) + p\sigma_H$  to ensure the workers stay. By similar reasoning, the inside firm will find it optimal to offer a worker in Job 2  $W_2 = (a_2 + b_2 \theta^* f(1)) + S(a_j + b_j \theta_i f(2)) + p\sigma_H$ . QED

Claim 4.1.7: The period 2 wage in the unique proper equilibrium must be perfectly revealing.

Proof: The logic is the same as Claim 3.1.6. I proceed by contradiction. Consider a worker in job 2 and suppose there is an equilibrium that is not perfectly revealing. Then there will be workers  $i$  and  $k$  such that  $\theta_i \neq \theta_k$  (WLOG assume  $\theta_i < \theta_k$ ) and period 2 wages  $W_2(\theta_i) = W_2(\theta_k)$ . In turn, the outside wage offer in Period 3,  $W_{3O}$ , will be the same for  $\theta_i$  and  $\theta_k$ . We know that in order to maximize profits the firm must offer each worker a period 2 wage  $a_2 + b_2 \theta^* f(1) + S(a_j + b_j \theta_i f(2)) + p\sigma_H - W_{3O}$ . But doing this implies  $W_2(\theta_i) \neq W_2(\theta_k)$ . This is a contradiction. The same logic holds for workers in Job 1. QED.

I now consider workers with  $n_i = 1$ :

Claim 4.1.8: For workers in Job  $j$  with  $n_i = 1$  and  $\sigma_{iI} = 0$  and  $\sigma_{iO} = \sigma_H$ , in the unique proper equilibrium, the networked firm wins at  $W_2 = (a_j + b_j\theta_i f(1))(1+S) - p\sigma_H$ .

Proof: The total value of the worker for periods 2 and 3 if he chooses the outside firm is  $a_j + b_j\theta_i f(1) + (1+S)(a_j + b_j\theta_i f(1)) + 2\sigma_H$  which, by assumption, is greater than the total value at the inside firm  $(1+S)(a_j + b_j\theta_i f(1)) + (1+S)(a_j + b_j\theta_i f(1))$ . Because this is a standard ascending value auction, it is weakly dominant for both firms to choose reservation wages in period 2 that exhaust the workers total value, given the period 3 wages defined above. The worker then chooses the period 2 wage that generates the most lifetime income. As such, the outside will win all such workers with a period 2 wage that generates lifetime income just equal to the worker's value at the inside firm. Such a period 2 wage,  $W_2$ , solves  $(1+S)(a_j + b_j\theta_i f(1)) + (1+S)(a_j + b_j\theta_i f(1)) = W_2 + (1+S)(a_j + b_j\theta_i f(1)) + p\sigma_H$ . Re-arranging yields  $W_2 = (a_j + b_j\theta_i f(1))(1+S) - p\sigma_H$ . Lastly, because both strategies are weakly dominant, they are proper. Further because all other strategies are weakly dominant, they can not be proper. QED

Claim 4.1.9: For workers in Job  $j$  with  $t^*=0$  in the unique proper equilibrium, the inside firm will offer a wage of  $W_2 = a_j + b_j\theta_i f(1) + 2\sigma_{iO} + S(a_j + b_j\theta_i f(2)) - p\sigma_H$ .

Proof: Because this is a standard ascending value auction, it is weakly dominant for both firms to choose reservation wages in period 2 that exhaust the workers total value, given the period 3 wages defined above. The total value at the inside firm  $(1+S)(a_j + b_j\theta_i f(2)) + (1+S)(a_j + b_j\theta_i f(2)) + 2\sigma_H$  is greater than the total value at the outside firm  $a_j + b_j\theta_i f(2) + (1+S)(a_j + b_j\theta_i f(2)) + 2\sigma_{iO}$ . Thus the inside firm will win all such workers. The optimal period 2 wage offer will be such that the worker is just indifferent between staying and leaving. Such a period 2 wage,  $W_2$ , solves  $a_j + b_j\theta_i f(2) + (1+S)(a_j + b_j\theta_i f(2)) + 2\sigma_{iO} = W_2 + a_j + b_j\theta_i f(2) + p\sigma_H$ . Algebra yields  $W_2 = a_j + b_j\theta_i f(2) + S(a_j + b_j\theta_i f(2)) + 2\sigma_{iO} - p\sigma_H$ . QED

Given the above payoffs in Period 2, I now consider the inside firm's optimal promotion rule. The firm will promote all workers for which the marginal benefit is greater than the marginal cost:

Claim 4.1.10: For all workers with  $n_i = 1$  the optimal promotion cut-off is  $\eta'/f(1)$ . For all workers with  $n_i = 0$ , the optimal promotion,  $\theta^*$ , is greater than  $\eta'/f(1)$ .

Proof: For workers with  $n_i = 1$  and  $t^*=0$ , the MB and MC of promotion of the marginal worker is:

$$\begin{aligned} MB &= (1+S)(a_2 + b_2\theta_i f(1)) - a_1 - b_1\theta_i f(1) \\ MC &= \max_2\{a_2 + b_2\theta_i f(1), a_1 + b_1\theta_i f(1)\} - \max\{a_2 + b_2\theta_i f(1), a_1 + b_1\theta_i f(1)\} + \\ &S[\max_3\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\} - \max\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\}] \end{aligned}$$

The RHS = 0 for all  $\theta_i$ . The LHS = 0 only when  $\theta_i f(1) = \eta'$  and is strictly > 0 above. I conclude  $\eta'/f(1)$  is the optimal promotion cut-off.

I now consider workers with  $n_i = 1$  who are new to the firm.

$$\begin{aligned} MB &= (a_2 + b_2\theta_i f(1) - a_1 - b_1\theta_i f(1)) \\ MC &= (1+S)[\max_2\{a_2 + b_2\theta_i f(1), a_1 + b_1\theta_i f(1)\} - \max\{a_2 + b_2\theta_i f(1), a_1 + b_1\theta_i f(1)\}] + \\ &S[\max_3\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\} - \max\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\}] \end{aligned}$$

I first that note that  $\max_t\{ \}$  resolves the maximum of the two argument given the optimal promotion decision in period  $t$ . As above the RHS = 0 for all  $\theta_i$  and the LHS=0 only when  $\theta_i f(1) = \eta'$  and is strictly  $> 0$  above. I conclude that  $\eta'/f(1)$  is the optimal promotion cut-off.

I now consider workers with  $n_i = 0$ . I define  $\theta^*$  as their promotion level:

$$\begin{aligned} MB &= (1+S)(a_2 + b_2\theta_i f(1) - a_1 - b_1\theta_i f(1)) \\ MC &= \max_2\{a_2 + b_2\theta^* f(1), a_1 + b_1\theta^* f(1)\} - a_1 + b_1\theta_L f(1) + \\ &S[\max_3\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\} - \max\{a_2 + b_2\theta_i f(2), a_1 + b_1\theta_i f(2)\}] \end{aligned}$$

The second term in MC goes to 0, but the first term is always positive. MB is only positive when  $\theta_i$  is such that  $\theta_i f(1) > \eta'$ . I conclude there exists a  $\theta^* > \eta'/f(1)$  such that  $MB > MC$  for all workers with  $\theta_i > \theta^*$ . QED

Finally, given the above payoffs and promotion rules, I consider the worker's networking decision in Stage (1). There will be different cut-offs depending on  $\sigma_{iL}$ .

**Claim 4.1.11:** All workers who would be assigned to Job 1 if  $n_i = 0$ ,  $\sigma_{iL} = \sigma_H$  and  $\theta_i \geq \theta_{N1}(\sigma_H) = \theta_L + C/f(1)b_1$ . All workers who would be assigned to Job 1 if  $n_i = 0$ ,  $\sigma_{iL} = 0$  and  $\theta_i \geq \theta_{N1}(0) = \theta_L/(1+pS) + [C + p((2-p)\sigma_H - Sa_1)]/(1+pS)f(1)b_1$  choose  $n_i = 1$ . For Job 2 I have  $\theta_{N2}(\sigma_H) = \theta_L + C/f(1)b_2$  and  $\theta_{N2}(0) = \theta_L/(1+pS) + [C + p((2-p)\sigma_H - Sa_2)]/(1+pS)f(1)b_2$ .

**Proof:** In Job 1 when  $\sigma_{iL} = \sigma_H$ , if a worker networks he receives  $a_1 + b_1\theta_{N1}(\sigma_H)f(1) + 2E(\sigma_{iO}) - p\sigma_H (1+S)\max\{(a_1 + b_1\theta_{N1}(\sigma_H)f(2), a_2 + b_2\theta_{N1}(\sigma_H)f(2))\} + p\sigma_H - C$ . If he does not network he receives  $a_1 + b_1\theta_L f(1) + p\sigma_H + (1+S)\max\{(a_1 + b_1\theta_{N1}(\sigma_H)f(2), a_2 + b_2\theta_{N1}(\sigma_H)f(2))\} + p\sigma_H$ . Thus, all workers in Job 1 with  $\sigma_{iL} = \sigma_H$  and  $\theta_i \geq \theta_N(\sigma_H) = \theta_L + C/f(1)b_1$  will network.

In Job 1 when  $\sigma_{iL} = 0$ , if a worker networks he receives  $(1+pS)(a_1 + b_1\theta_{N1}(0)f(1)) + (1+S)\max\{(a_1 + b_1\theta_{N1}(0)f(2), a_2 + b_2\theta_{N1}(0)f(2))\} - C$ . If he does not network he receives  $a_1 + b_1\theta_L f(1) + (1+S)\max\{(a_1 + b_1\theta_{N1}(0)f(2), a_2 + b_2\theta_{N1}(0)f(2))\} + 2p\sigma_H$ . Thus, all workers in Job 1 with  $\sigma_{iL} = \sigma_H$  and  $\theta_i \geq \theta_{N1}(0) = \theta_L/(1+pS) + [C + 2p\sigma_H - Sa_1]/(1+pS)f(1)b_1$  will network.  $\theta_N(0)$  can be re-written in terms of  $\theta_N(\sigma_H)$ , as follows:  $\theta_N(0) = \theta_N(\sigma_H) + p(2\sigma_H - S(a_j + b_j\theta_{N1}(0)f(1)))/f(1)b_j > \theta_N(\sigma_H)$  because the second term is always positive by assumption.

Similar logic for Job 2 yields  $\theta_{N2}(\sigma_H) = \theta^* + C/f(1)b_2$  and  $\theta_{N2}(0) = \theta^*/(1+pS) + [C+2p\sigma_H-Sa_2]/(1+pS)f(1)b_2$ . QED

In summary, in equilibrium:

- (1) All workers assigned to Job  $j$  in Period 2 with  $\theta_i \geq \theta_{Nj}(\sigma_{iL})$  will choose  $n_i = 1$ . If  $\sigma_{iO} = \sigma_H$  and  $\sigma_{iL} = 0$ , they turnover and receive  $[W_2, W_3] = [(1+S)(a_j + b_j\theta_i f(1), (1+S)\max\{a_1 + b_1\theta_i f(2), a_2 + b_2\theta_i f(2)\} + p\sigma_H]$ . Otherwise they stay with the inside firm and earn  $[W_2, W_3] = [a_j + b_j\theta_i f(1) + S\theta_i f(2) + 2\sigma_{iO} - p\sigma_H, \max\{a_1 + b_1\theta_i f(2), a_2 + b_2\theta_i f(2)\} + p\sigma_H]$ .
- (2) All other workers choose  $n_i = 0$  and stay with the inside firm and earn  $[W_2, W_3] = [a_j + b_j\theta_{\min} f(1) + S\theta_i f(2) + p\sigma_H, \max\{a_1 + b_1\theta_i f(2), a_2 + b_2\theta_i f(2)\} + p\sigma_H]$ .

## V. Proofs of Corollaries

As noted in the text, high ability turnover and two-sided turnover hold only for certain parameterizations of the model. High ability turn over holds if:

$$(B.1) \quad 0 < C < \max[(\theta' - \theta_L)f(1)b_1, (\theta_H - \theta^*)f(1)b_2].$$

Two-sided turnover holds if:

$$(B.2) \quad \theta^* < (\theta_H + \theta')/2 < \theta^{N2}(\sigma_H) \quad \text{which implies}$$

$$(B.3) \quad 0 < (\theta_H + \theta')/2 - \theta^*)f(1)b_2 < C.$$

(B.1) ensures that high ability networking occurs at both job levels. (B.3) ensures that the two-sided networking that occurs in Job 2 is such that the average ability worker in Job 2 is a worker who does not network. This assumption imposes a symmetry property on the two-sided turnover. Together they imply that  $C$  must be in the following range:

$$(B.4) \quad (\theta_H + \theta')/2 - \theta^*)f(1)b_2 < C < \max[(\theta' - \theta_L)f(1)b_1, (\theta_H - \theta^*)f(1)b_2]$$

We assume (B.1) and (B.3) hold in what follows.

### V1. Proof of Corollary 4.1:

I define the wage premium upon promotion at their current firm as the wage increase if promoted minus the wage increase if not promoted. Formally the premium for Period  $t$  is:

$$\Delta W_t(P^*) = ((W_t | P^*=1) - W_{t-1}) - ((W_t | P^*=0) - W_{t-1}) = (W_t | P^*=1) - (W_t | P^*=0).$$

For workers with  $n_i = 1$ , I note that symmetric competition forces the firm to pay them their outside option regardless of job assignment:

If  $t^*=0$  then we have:

$$W_{t=2} = \max\{y_2(\theta_i f(1)), y_1(\theta_i f(1))\} + S \max\{y_2(\theta_i f(2)), y_1(\theta_i f(2))\} + 2\sigma_{iO} - p\sigma_H.$$

$$W_{t=3} = \max\{y_2(\theta_i f(2)), y_1(\theta_i f(2))\} + p\sigma_H.$$

If  $t^*=1$  then we have:

$$W_{t=2} = (1+S) \max\{y_2(\theta_i f(1)), y_1(\theta_i f(1))\} - p\sigma_H.$$

$$W_{t=3} = (1+S) \max\{y_2(\theta_i f(2)), y_1(\theta_i f(2))\} + p\sigma_H.$$

As a consequence, the inside firm will have to pay all workers with  $n_i = 1$  the above wage whether they are promoted or not. Thus  $\Delta W_2(P^*) = \Delta W_3(P^*) = 0$  for workers with  $n_i = 1$ .

Workers with  $n_i = 0$  in Period 3, like workers with  $n_i = 1$ , are in a situation of symmetric information. Thus  $\Delta W_3(P^*) = 0$ . In Period 2 however we have:

$$\Delta W_2(P^*) = y_2(\theta^* f(1)) - y_1(\theta_L f(1)).$$

Thus, if we denote  $q$  as the probability a worker networks, we have:

$$\Delta W_2(P^*) = (1-q)(y_2(\theta^* f(1)) - y_1(\theta_L f(1))) > 0 = \Delta W_3(P^*). \text{ QED.}$$

Note on testing Corollary 4.1: When testing Corollary 4.1, I use the sample of workers who are not promoted to predict the wage change for workers who are promoted *if* they were not promoted, and then use this counterfactual prediction to construct the theoretical variable  $\Delta W_i(P^*)$ . Because the wage increase depends on ability, using such workers to predict the wage increase is biased. I control for this problem by transforming wages to logs in my empirical tests. Assume  $a_1 = 0$ . If we consider the empirical assumption that “period 2” consists of several worker years in the data, then for small  $S$  and  $p$  the wage change in year  $t+1$  for a worker in “period 2” who is not promoted is  $b_1 \theta_i f(t+1) - b_1 \theta_i f(t)$  if  $n_i = 1$  and  $b_1 \theta_L f(t+1) - b_1 \theta_L f(t)$  if  $n_i = 0$ . The wage change at the log level is then  $\ln[f(t+1)] - \ln[f(t)]$  which does not depend on  $\theta_i$ . The wage change for workers with  $n_i = 0$  who are promoted *if* they were not promoted is  $b_1 \theta_L f(t+1) - b_1 \theta_L f(t)$ . At the log level this change is *exactly*  $\ln[f(t+1)] - \ln[f(t)]$ , the wage change of workers not promoted, and there is no bias in the prediction. For workers with  $n_i = 1$  who are promoted the wage change *if* they were not promoted is  $b_1 \theta_i f(t+1) - b_1 \theta_i f(t)$ , which is *exactly*  $\ln[f(t+1)] - \ln[f(t)]$ , the wage change of workers not promoted, and there is no bias in the prediction.

VII. Proof of Corollary 4.2: From Proposition 4.1 I can deduce that the percent of workers promoted to Job 2 in Period 3. Because all workers are promoted efficiently in job 3, a worker is promoted to Job 2 in Period 3 if and only if:

$$\theta^{P1} = \eta' / f(1) > \theta_i \geq \eta' / f(2) = \theta^{P2}.$$

Thus I have:

$$P(\text{promotion in Period 3} | t^*=1) = P(\theta_i \geq \theta^{P2} | \theta_i < \theta^{P1}, t^*=1).$$

By Bayes rule I have:

$$\begin{aligned} &= P(\theta_i \geq \theta^{P2} \text{ and } t^*=1 | \theta^{P1} < \theta_i) / P(t^*=1 | \theta_i < \theta^{P1}) \\ &= \min\{Q * P(\theta^{P1} > \theta_i \geq \theta^{P2}) / [Q * P(\theta^{P1} > \theta_i \geq \theta^{N1}(0))], 1\} \\ &= \min\{(P(\theta^{P1} > \theta_i \geq \theta^{P2}) / P[\theta^{P1} > \theta_i \geq \theta^{N1}(0)]), 1\} \\ &\text{where } Q=(1-p)p. \end{aligned}$$

By Bayes rule I also have:

$$\begin{aligned} &P(\text{promotion in Period 3} | t^*=0, \theta_i < \theta^{P1}) \\ &= P(\theta_i \geq \theta^{P2} \text{ and } t^*=0 | \theta_i < \theta^{P1}) / P(t^*=0 | \theta_i < \theta^{P1}) \\ &= (1-Q) * P(\theta^{P1} > \theta_i \geq \theta^{P2}) / [(1-Q) * P(\theta^{P1} > \theta_i \geq \theta^{N1}(0)) + P(\theta_i < \theta^{N1}(0))] \\ &= P(\theta^{P1} > \theta_i \geq \theta^{P2}) / [P(\theta^{P1} > \theta_i \geq \theta^{N1}(0)) + P(\theta_i < \theta^{N1}(0)) / (1-Q)] \\ &< (P(\theta^{P1} > \theta_i \geq \theta^{P2}) / [P(\theta^{P1} > \theta_i \geq \theta^{N1}(0))]) \\ &\leq \min\{(P(\theta^{P1} > \theta_i \geq \theta^{P2}) / [P(\theta^{P1} > \theta_i \geq \theta^{N1}(0))]), 1\} \\ &= P(\text{promotion in Period 3} | t^*=1). \end{aligned}$$

QED

### VIII. Proof of Corollary 4.3:

First note that the wage distribution in Job 2 can be broken into the following 4 sections. Further, note that turnover ( $t^*=1$ ) occurs only in sections (1) and (4).

(1) For  $\theta_i$  such that  $\eta' < \theta_i < \theta^*$ :

$$\begin{aligned} W_2 &= (1+S)(a_2 + b_2\theta_i f(1)) - p\sigma_H \text{ if } t^*=1 \\ W_2 &= a_2 + b_2\theta_i f(1) + S(a_2 + b_2\theta_i f(2)) + 2\sigma_{iO} - p\sigma_H \text{ if } t^*=0. \end{aligned}$$

(2) For  $\theta_i$  such that  $\theta^* < \theta_i < \theta_{N2}(\sigma_H)$ :

$$W_2 = a_2 + b_2\theta^* f(1) + S(a_2 + b_2\theta_i f(2)) + p\sigma_H.$$

(3) For  $\theta_i$  such that  $\theta_{N2}(\sigma_H) < \theta_i < \theta_{N2}(0)$ :

$$\begin{aligned} W_2 &= a_2 + b_2\theta_i f(1) + S(a_2 + b_2\theta_i f(2)) + 2\sigma_{iO} - p\sigma_H \\ W_2 &= a_2 + b_2\theta^*(1) + S(a_2 + b_2\theta_i f(2)) + p\sigma_H \text{ if } \sigma_{iI} = 0. \end{aligned}$$

(4) For  $\theta_i$  such that  $\theta_{N2}(0) < \theta_i$ :

$$\begin{aligned} W_2 &= (1+S)(a_2 + b_2\theta_i f(1)) - p\sigma_H \text{ if } t^*=1 \\ W_2 &= a_2 + b_2\theta_i f(1) + S(a_2 + b_2\theta_i f(2)) + 2\sigma_{iO} - p\sigma_H \text{ if } t^*=0. \end{aligned}$$

I want to show that there is a range around the median wage in job 2,  $(W^M - \epsilon, W^M + \epsilon)$ , where turnover does not occur. To do this I show that at least  $.5 + \epsilon$  of the mass of the distribution is above the workers who enter in section (1) and at least  $.5 + \epsilon$  mass of the distribution is below workers who enter in section (4).

I first consider section (1). By the assumption above that  $\theta^* < (\theta_H + \theta')/2 < \theta_{N2}(\sigma_H)$ , it follows that at least  $.5 + \epsilon$  mass of the distribution is in section (2) –(4).

Further all the wages in (2)-(4) are strictly greater than the highest wage of a worker who enters in section (1). I conclude that at least  $.5 + \varepsilon$  mass of the distribution is above workers who enter in section (1).

Now I consider workers who enter in Section (4) of the distribution. Again, by the assumption above that  $\theta^* < (\theta_H + \theta')/2 < \theta^{N2}(\sigma_H)$ , it follows that at least  $.5 + \varepsilon$  mass of the distribution is in section (1) –(2). If  $(a_2 + b_2\theta_{N2}(0)f(1)) - a_2 + b_2\theta^*f(1) > Sb_2\theta_{N2}(0)(f(2) - f(1)) + 2p\sigma_H$  then all the wages in (1)-(2) are strictly less than the lowest wage of a worker who enters in section (4). From Proposition 4.1 we know that  $b_2\theta_{N2}(0)f(1) - b_2\theta^*f(1) > C$ , which then implies that all the wages in (1)-(2) are strictly less than the lowest wage of a worker who enters in section (4) if  $C > Sb_2\theta_{N2}(0)(f(2) - f(1)) + 2p\sigma_H$ . As  $p$  goes to zero, this clearly holds for small enough  $S$ .

#### IX. Proof of Corollary 4.4:

Define  $\theta_{2M} = (\theta_H + \eta'/f(1))/2$ , the overall mean of innate ability on Job 2. Define  $\theta' = \eta'/f(1)$ . Then we have:

$$(B.8) \ E((\theta_i - \theta_{2M})^2 \mid t^*=1, j=2) = \int_{\theta'}^{\theta^*} ((\theta_i - \theta_{2M})^2 f(\theta_i \mid t^*=1, j=2)) \\ + \int_{\theta_{N2}}^{\theta_H} ((\theta_i - \theta_{2M})^2 f(\theta_i \mid t^*=1, j=2)).$$

Define the analogous measure for incumbents as:

$$(B.9) \ E((\theta_i - \theta_{2M})^2 \mid t^*=0, j=2) = \int_{\theta'}^{\theta^*} ((\theta_i - \theta_{2M})^2 f(\theta_i \mid t^*=0, j=2)) \\ + \int_{\theta^*}^{\theta_{N2}} ((\theta_i - \theta_{2M})^2 f(\theta_i \mid t^*=0, j=2)) + \int_{\theta_{N2}}^{\theta_H} ((\theta_i - \theta_{2M})^2 f(\theta_i \mid t^*=0, j=2)).$$

Because  $\theta_i$  is uniform over all relevant ranges for both  $t^*=1$  and  $t^*=0$ , (B.8) and (B.9) reduce to (B.10) and (B.11) respectively:

$$(B.10) \ E((\theta_i - \theta_{2M})^2 \mid t^*=1, j=2) = zA + (1-z)D \\ \text{where } z = [(\theta^* - \eta'/f(1)) / (\theta^* - \eta'/f(1) + \theta_H - \theta_{N2}(0))]. \\ A = E((\theta_i - \theta_{2M})^2 \mid \theta' < \theta_i < \theta^*), \\ D = E((\theta_i - \theta_{2M})^2 \mid \theta^{N2}(0) < \theta_i \leq \theta_H).$$

$$(B.11) \ E((\theta_i - \theta_{2M})^2 \mid t^*=0, j=2) = qA + rB + (1-q-r)D \\ \text{where } q = [(\theta^* - \eta'/f(1)) / (\theta_H - \eta'/f(1))] \\ r = [(\theta^{N2}(0) - \theta^*) / (\theta_H - \eta'/f(1))] \\ B = E((\theta_i - \theta_{2M})^2 \mid \theta^* \leq \theta_i \leq \theta^{N2}(0)).$$

By assumption (B.2),  $\theta^* < (\theta_H + \eta'/f(1))/2 < \theta^{N2}(\sigma_H)$ , we know that  $A > D$  and  $B > D$ . Further we note that  $z/(1-z) = q/(1-q-r)$ . It then follows that  $zA + (1-z)D > qA + rB + (1-q-r)D$ . Or,  $E((\theta_i - \theta_{2M})^2 \mid t^*=1, j=2) > E((\theta_i - \theta_{2M})^2 \mid t^*=0, j=2)$ . QED

## X. Data Issues

In this section I address further issues that arise in interpreting the theoretical model empirically. Ideally, to test the model I would observe new hires at their previous employer. Because the dataset does not contain this information, I assume that workers who enter the firm in the data are coming from a competing firm in the same industry/occupation with a similar production environment. Given that management is a high-skilled, high wage profession, I think this is a reasonable assumption.

### Entry and Exit Dynamics at Higher Levels

In this subsection I look at entry and exit at higher levels of the firm. BGH find that 97% of employees are in levels 1-4, and that few people are promoted above level 4. Thus, it is possible that the promotion and entry/exit dynamics are quite different at higher levels. In Table A1.1 I examine these issues.<sup>28</sup> You will see that the promotion rates and exit rates remain fairly constant across all levels.

**Table A1.1: Promotion, Entry, and Exit Rates by Level**

<b>Rate</b>	<b>L 1</b>	<b>L 2</b>	<b>L 3</b>	<b>L 4</b>	<b>L 5</b>	<b>L 6</b>	<b>L 7</b>	<b>L 8</b>
Promotion	N/A	.2177	.1766	.1281	.1533	.2314	.1404	.0952
Entry	.4237	.1150	.0985	.0624	.0423	.0566	.0526	.0476
Exit	.1352	.1447	.1583	.1704	.1164	.2225	.1754	.1429

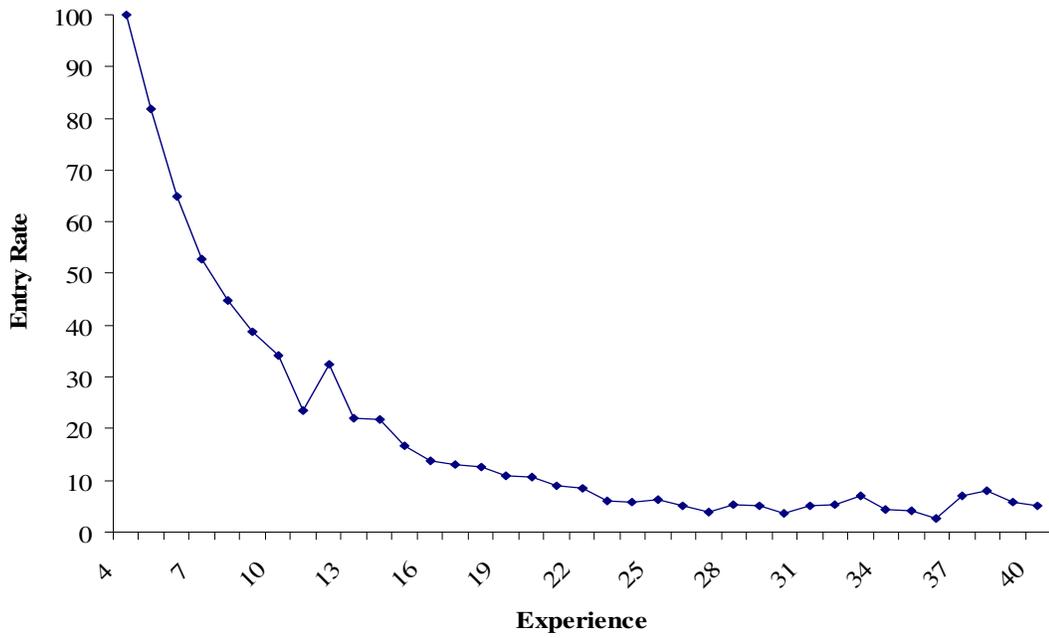
However, entry rates tend to decline a bit at higher levels. These numbers suggest that the patterns may indeed be a bit different at higher levels. Further, because there are so few workers at higher levels of the firm, the absolute numbers of new hires, leavers, and incumbents at Levels 5-8 are very small. Given the noise inherent in such small numbers, and that promotion may be different in Levels 5-8, I only use Levels 1-4 in the analysis in the paper.

### Entry Rates Over Time by Education Categories

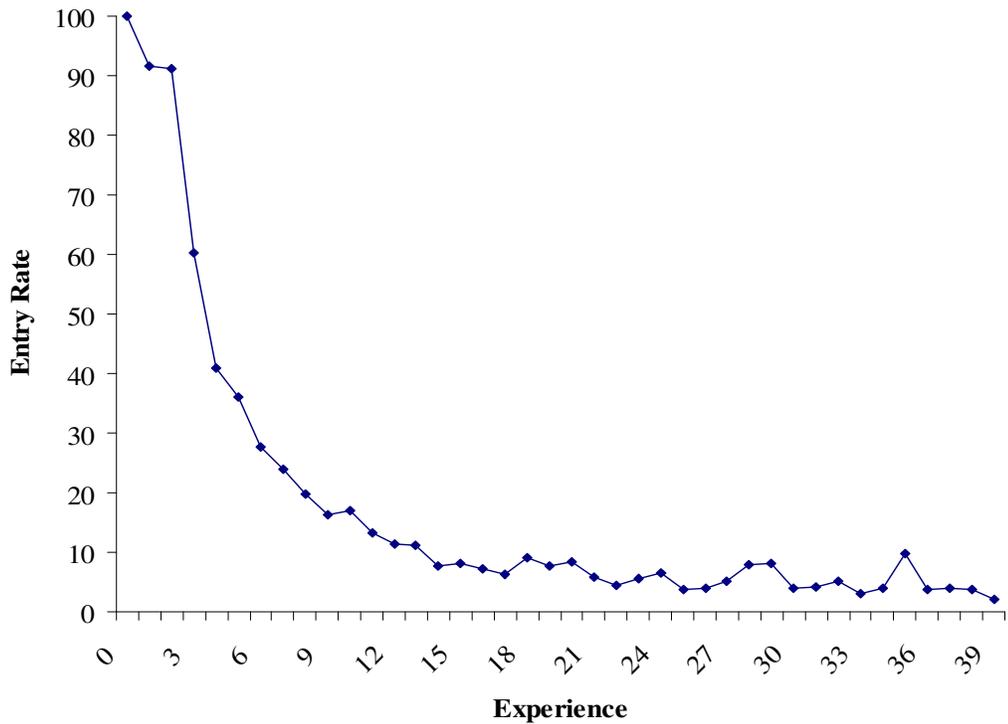
As noted in the text, in choosing my “window” for period 2 I examined entry rates over time as a measure of turnover activity. Figures A1.1 – A1.4 plot entry rates by potential experience for workers with a high school diploma, a bachelor’s, a master’s and a PhD respectively. The key pattern in each plot is that the entry rates are high early in a worker’s career and flatten out over time. In particular, note that entry rates flatten out at roughly 25 years for workers with a high school diploma, roughly 18 years for workers with a bachelor’s degree, roughly 15 years for workers with a master’s degree, and roughly 10 years for workers with a PhD.

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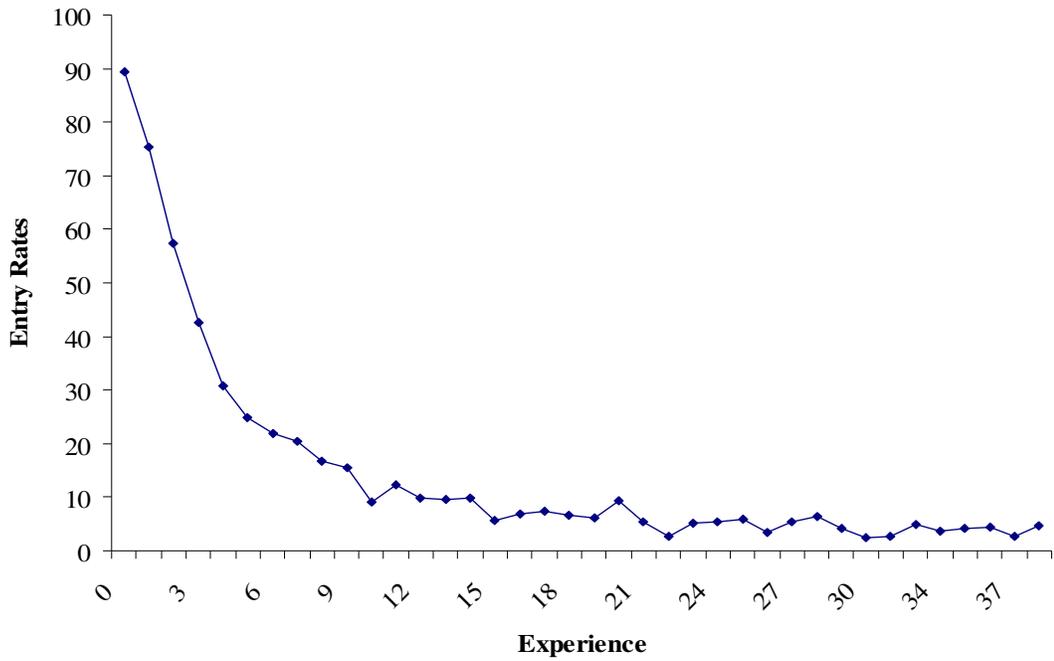
<sup>28</sup> The exit rates in Table 5 do not include workers 65 and older, as these workers have very high exit rates. We assume this is due to retirement, and do not want to include such exit in our analysis.



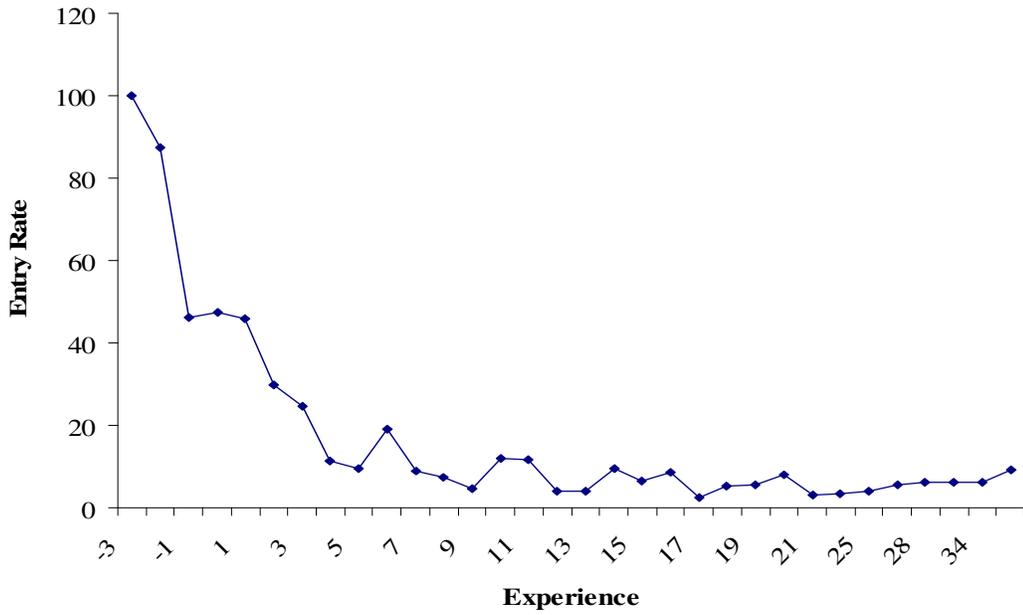
**Figure A1.1: Entry Rate by Potential Experience, High School Diploma**



**Figure A1.2: Entry Rate by Potential Experience, Bachelor's Degree**



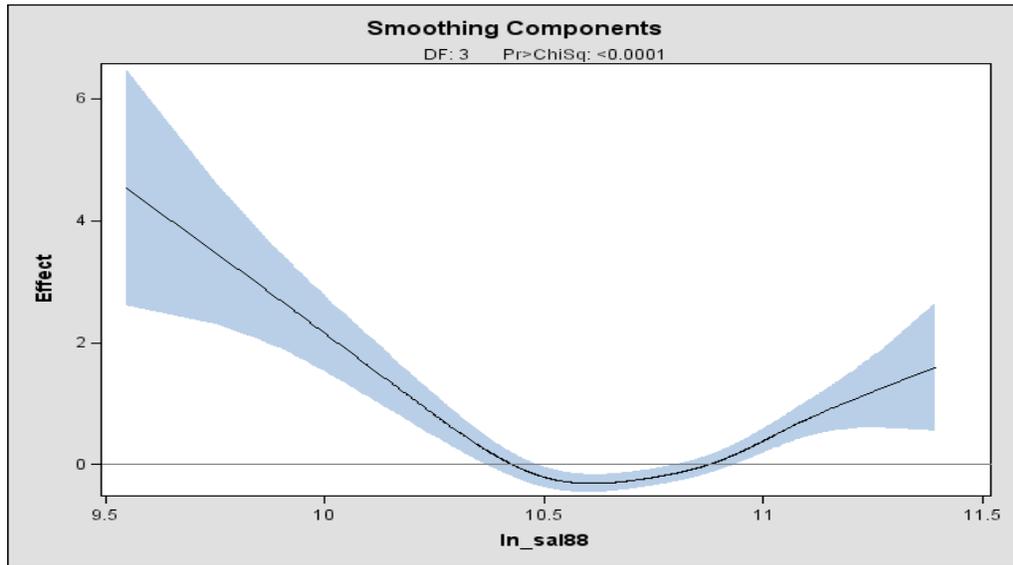
**Figure A1.3: Entry Rate by Potential Experience, Master's Degree**



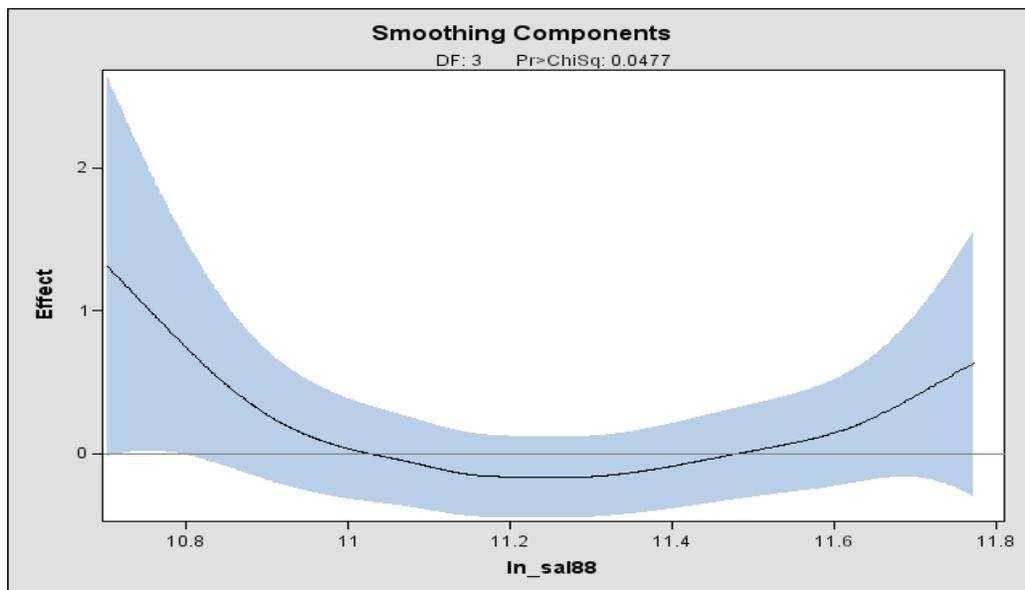
**Figure A1.4: Entry Rate by Potential Experience, PhD**

### Semi-Parametric estimation of Two-Sided Entry, Level 2 and 4

Below I present the semi-parametric analysis for levels 2 and 4. They are qualitatively the same as the analysis for level 3. The effect in level 4, though, is significantly more noisy. This is because the “period 2” sample restrictions slightly under-sample level 4.



**Figure A1.5: Estimated Non-Parametric Relationship Between Entry and Starting Wage: Level 2**



**Figure A1.6: Estimated Non-Parametric Relationship Between Entry and Starting Wage: Level 4**

Limiting the Sample and Observations With Missing Values for Education or Salary

Table A1.2 shows how the sample progresses as I impose the restrictions described in the text.

**Table A1.2: Effect of Sample Restrictions on Distribution of Job Level**

**Notes:** The “Period 2” No Missing column includes only observation with neither missing education nor missing salary information.

<b>Level</b>	<b>Full Sample</b>	<b>Basic Restrictions</b>	<b>Period 2 Sample</b>	<b>"Period 2", No Missing</b>
1	20.02	21.59	8.62	10.83
2	24.8	26.25	35.21	36.68
3	27.1	27.66	33.03	32.43
4	24.76	24.50	23.13	20.43
5	2.40	-		
6	0.77	-		
7	0.12	-		
8	0.04	-	0	0
<b>Obs</b>	46,930	38,296	20,633	16,037

Turning attention to Table A1.3, I now briefly discuss the “missing” sample in more detail. I compare the distributions of common variables across the missing sample and non-missing sample. I argue that the missing sample looks reasonably

**Table A1.3: Comparison of Missing and Non-Missing Period 2 Sample**

<b>Panel 1</b>							
<b>Level</b>	<b>Level</b>			<b>Tenure</b>		<b>Age</b>	
	<b>No Missing</b>	<b>Missing</b>		<b>No Missing</b>	<b>Missing</b>	<b>No Missing</b>	<b>Missing</b>
1	10.83	.91	Mean	4.73	4.25	35.2	40.0
2	36.68	30.07	Std Dev	3.03	3.22	4.8	9.1
3	32.05	36.37	Min	1	1	23	24
4	20.43	32.55	Max	17	17	48	71
<b>Obs</b>	16,037	4,596		16,037	3,736	16,035	4,477

<b>Panel 2</b>	
<b>Variable</b>	<b>Percent of missing obs from a whole record</b>
<b>Salary</b>	99.23
<b>Education</b>	88.78

similar to the non-missing sample, and thus there is little concern that the removal of the missing sample biases the analyses. Panel 1 indicates that the distributions look broadly similar. Panel 2 shows that almost all of the missing observations come from missing records. As such, there is very little “within” person information that could be used to impute the missings. As such, imputing the missing would be complex and noisy. As such I chose to simply drop them from my analysis.

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## **CHAPTER 2: PIECE-RATES, SALARY, PERFORMANCE AND JOB LEVEL**

### **2.1. Introduction**

Many employer/employee relationships are characterized by a classic agency problem: the employee can take actions that directly affect profits but are unobserved by the employer. When the employer can precisely measure the worker's output the solution to this problem is straightforward; the firm simply contracts directly on the worker's output to properly align his incentives (Lazear (1986)). When output is imperfectly measured, the employer is forced to use other means to motivate the worker such as the threat of firing (Shapiro and Stiglitz (1984)).<sup>29</sup> The goal of this paper is to use this simple agency framework to explore why bonuses are a larger component of compensation at higher job levels and salaries a larger component of compensation at lower job levels.

The motivation for applying the agency framework to bonus, salary and job level comes from two literatures. The literature on hierarchy and monitoring predicts that the cost of monitoring output should increase at lower levels of the firm (Rosen (1982), Qian (1994)), while the literature on pay-for-performance predicts that variation in the cost of monitoring should invoke variation in use of salary and piece-rates: "When it is costly to measure output, it is sometimes argued, workers are paid salaries. When monitoring costs are low, piece-rate payments is appropriate (Lazear (1986))."<sup>30</sup> These two predictions taken together suggest that piece-rates, or bonuses, should be more common at higher levels of the firm because monitoring output is less costly. Further, previous empirical work finds evidence that bonuses are in fact larger

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<sup>29</sup> Baker (1992) considers contracts which use performance measures that are imperfectly correlated with the firm's objective (profits). He shows that the stronger the correlation, the more effective the contract.

<sup>30</sup> Pendleton (2006) notes that the importance of monitoring costs is also widely recognized in the literature on profit-sharing and employee incentive plans.

and more common at higher levels of the firm (Gibbs (1995), Lambert et al. (1993), Lin (2005) Smeets and Warzynski (2006)). The goal of this paper is to build a formal model that captures this prediction and test its implications using personnel records.

The key assumption of the theoretical model is that the proportion of a worker's output that the firm can observe decreases at lower levels of the hierarchy. Further, the firm can only probabilistically monitor the unobserved proportion of output. By the logic discussed above, then, the firm will offer a wage contract that has a larger salary component at lower levels (efficiency wage solution), and a larger piece-rate, or bonus, component at higher levels. This variation in the use of bonus across job level gives rise to four testable implications: 1) measured performance should increase at higher job levels, 2) the probability of receiving a bonus should increase with performance 3) the absolute size of the bonus and the ratio of bonus to total compensation should increase at higher job levels and 4) the frequency of firing should decrease at higher job levels.

To test these implications I analyze personnel records from the management hierarchy of a firm in the US financial services industry (the same data analyzed by Baker et. al (1994)). The analysis examines variation in bonus and performance across job level both in the cross-section and within-person over time. The strength of the analysis is that it controls for both observable and unobservable worker skills, thereby ruling out many alternative explanations that rely on positive sorting on ability across job level (tournament models, learning models). I find that the data is consistent with the implications of the model.

My paper is not unique in its exploration of the determinants and effects of piece-rates. There is a significant amount of empirical work exploring such matters in the pay-for-performance literature. Two papers closely related to mine are Gibbons and Murphy (1992) and Lazear (2000). Gibbons and Murphy (1992) examine how

variation in long-term incentives over the course of a worker's career affects the firm's decision to use piece-rates. In particular, they argue that explicit incentives (i.e. "pay-for-performance" or piece-rates) should be less important early in a worker's career because "career concerns" serve as a sufficient implicit discipline device. The empirical implication is that pay-for-performance should vary systematically with experience. In a dataset that tracks CEO pay and firm performance, they find that CEO's closer to retirement are explicitly rewarded more for high-performance (extra return realized by shareholders) than CEO's early in their career.<sup>31</sup> My methodology is similar to Gibbons and Murphy with the main exception that I use job level rather than age to identify variation in the use of piece-rates. Unlike Gibbons and Murphy, Lazear (2000), focuses exclusively on the effects of piece-rates rather than the determinants. Using a unique dataset that tracks the personnel records of a firm over time that switches from salaries to piece-rate, Lazear is able to use a natural experiment framework to identify the effect of piece-rates.<sup>32</sup> He finds that piece-rates significantly increase the output and the ability of workers.

Four other empirical papers relevant for my work are Gibbs (1995), Lambert et al. (1993), Lin (2005), and Smeets and Warzynski (2006). Gibbs uses the same data as I do to investigate similar issues: performance, pay-raise, bonus and promotion. The focus of Gibbs' work, however, is *within* job-level incentives and promotion-based incentives, while the focus of my work is how incentives vary *across* job level (under the assumption that level is a proxy for monitoring cost). As such there is no overlap in our statistical models or results. Lambert et al. analyze data similar to mine (they have information on bonus, salary and level of rank) but asks different questions.

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<sup>31</sup> Beyond these two papers, there is a broad empirical literature on CEO compensation and incentives that addresses issues of pay-for-performance. For a full review of literature on executive compensation see Kevin Murphy's Chapter in the Handbook of Labor Economics (1999).

<sup>32</sup> In terms of empirical design, Hamilton, Nickerson, and Owan (2003) is another paper similar to Lazear (2000). However, they are more interested in team incentives.

The authors are predominantly interested in the broader implications of standard models of performance and promotion (tournament, managerial power, moral hazard) and do not explore the issue of performance and bonuses across level in depth. They do offer some simple descriptive statistics (consistent with my findings), but focus on other issues in their formal statistical tests. Lin's paper is closest to mine in terms of empirical methodology. Lin runs formal regressions, controlling for worker skills and performance, to identify the effect of job level on bonus. Consistent with my results, he finds that bonuses are larger at higher job levels. My paper is distinguished from Lin's in that it has a theoretical framework guiding its predictions. As such, it includes a set of secondary predictions about performance and job level that his does not. Further, my analysis examines bonus as a percent of total compensation, while his focuses solely on raw bonus size. Finally Smeets and Warzynski, using a unique data set, explore the relationship between span of control and compensation. In regression analysis that controls for worker characteristics and span, they find that bonuses are positively correlated with job level. Like Lin, they do not have a formal theoretical model guiding their predictions. Further they do not use a fixed-effect specification.

On the theoretical side of the pay-for-performance literature, Lazear (1986) is the paper most relevant to my work. Lazear tackles the salary vs. piece-rate dilemma in depth, walking through a series of different factors that may drive the decision of whether to implement a salary or a piece-rate. In particular he shows that as the cost of monitoring increases salaries become preferred to piece-rates. My model is formally different than his, but is in the same spirit. In my model, as monitoring becomes more difficult the firm shifts to salary. The interpretation of salary in my model, however, comes from the Shapiro and Stiglitz (1984) efficiency wage model. Shapiro and Stiglitz show that when effort is observed probabilistically, an optimal

response of the firm is to offer a salary and use the threat of firing. In my model, at lower levels of the firm a higher proportion of a worker's output is observed probabilistically (rather than perfectly). As such the efficiency wage salary a la Shapiro and Stiglitz becomes a bigger component of total compensation. Outside of these two key papers that contribute directly to ideas in my model, there is a large theoretical literature exploring moral hazard and wage contracts more generally.<sup>33</sup>

Finally, the central assumption of the paper that monitoring becomes more difficult at lower levels of the firm comes from the literature on monitoring in organizations. Rosen (1982) shows that managers at lower levels will be of lower ability. Further, Qian (1994), in an extension of Rosen's work, shows that (1) at lower levels of the hierarchy there will be more "loss of control" and (2) as the span of control gets larger (the number of workers that a manager monitors) the monitoring problem becomes more difficult. All of these results are consistent with the idea that monitoring will be more difficult at lower levels of hierarchies.<sup>34</sup>

The paper proceeds as follows: In section 2.2 I consider two basic models of moral hazard. In the first I assume output is observable and show that a bonus pay scheme induces efficient effort. In the second model I assume that output is not observable, but can be monitored probabilistically. I show that a salary scheme with the threat of firing is optimal. In section 2.3, I build a model of a multi-level firm in which monitoring output is more difficult at lower levels of the firm. The wage of the worker is a combination of the wage from the two models presented in section 2.2. This model yields testable implications for the data. In Section 2.4 I discuss the data, my empirical strategy, and show the results. Section 2.5 concludes.

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<sup>33</sup> A few classic papers from this literature include Holmstrom (1979), Lazear and Rosen (1981), Fama (1991), Baker, Jensen and Murphy (1988) and Baker (1992).

<sup>34</sup> The literature on moral hazard and teams suggests that monitoring is harder in teams (Holmstrom (1982)). If team production is more prevalent at lower job levels (something that has not been empirically examined), then this could be a second reason why the cost of monitoring is higher.

## 2.2. Perfect and Imperfect Monitoring

In this section I consider two simple models of pay and performance. The first assumes output is observable, and shows that a piece-rate wage is optimal. The second assumes that output is unobservable and shows that an efficiency wage solution similar to Shapiro and Stiglitz (1986) does better than a piece-rate. I use the definitions of piece-rate and salary presented by Lazear (1986). A piece-rate is a wage contract that depends on output. A salary is a wage contract that is independent of output. In terms of timing, a piece-rate wage has a component that is paid ex-post, and a salary does not. To be consistent with the data, I use the term “bonus” and piece-rate interchangeably. The main result of the section is that when monitoring is sufficiently difficult the efficiency wage induces lower effort in equilibrium than a piece-rate. This arises because the costly nature of monitoring in the unobservable case makes the return to effort lower. This result will be integral in deriving the testable predictions in Section III.

### A) Perfectly Observable Output: Piece-Rate Contract (Performance Bonus)<sup>35</sup>

Assume a worker’s output is equal to  $Y_i = b_i e_i$  where  $e_i$  is a worker’s effort (the harder he works, the more he produces) and  $b_i$  is a worker’s return to effort. Assume production is separable across workers and a worker has an outside option of  $R$ . Effort is continuous on the range  $[e_L, e_H]$ . I set  $e_L = 0$  and assume the cost of effort is  $C(e)$  where  $C(0)=0$ ,  $C'(e)$  and  $C''(e) > 0$ . Assume  $b_i$  is not known to the firm and, for simplicity, has a two point distribution; it equals  $b_L$  with probability  $1-p$  and  $b_H$  with probability  $p$ , where  $b_H > b_L \geq 1$ . A simple interpretation of  $b_i$  is that it is the worker’s intrinsic motivation. Some people have a stronger taste for effort than others. Assume

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<sup>35</sup> There are other payment schemes that depend on observing output and induce efficient effort (e.g. tournament models). For the sake of this paper I am concentrating only on piece-rate/bonus payment schemes.

workers are risk neutral and look to maximize wage less cost of effort. Finally, note that the efficient effort level in this environment, denoted  $e_i^*$ , is the effort level where the marginal benefit of effort equals its marginal cost. Formally  $e_i^*$  solves:  $b_i = C'(e_i^*)$ . Note that  $e_H^* > e_L^*$ . That is, workers with more motivation have a higher level of efficient effort.

When output is perfectly observable a piece-rate contract equal to  $W = Y_i - P$  will induce efficient effort, as it forces the worker to choose the effort level where the marginal return of his effort equals the marginal cost of his effort, which is precisely  $e_i^*$ . Such a piece-rate contract is optimal from the firm's perspective because, by inducing efficient effort, it maximizes the surplus between the worker and the firm.  $P$  is the amount of surplus the firm takes for itself and will be chosen such that  $R = Y_i - P$ , i.e. to ensure the participation constraint holds. Thus the contract allows the firm to make as much surplus as possible without losing the worker. We define this contract as a piece rate because  $W$  depends on  $Y$ .

An important point to note is that  $W = Y_i - P$  is not the only contract that induces efficient effort. Because the firm knows that all workers will choose at least  $e_L^*$ , it can offer the following contract,  $W = b_L e_L^* + (Y_i - b_L e_L^*)$ , where  $b_L e_L^*$  is a salary paid upfront (as it does not depend on  $Y_i$ ), and  $(Y_i - b_L e_L^*)$  is a "bonus" paid to workers who choose  $e_H^*$ . To ensure against shirking, the firm will fire all workers who choose any effort  $e_i < e_L^*$ . Such workers receive their outside option. This contract induces the same outcome as  $W = Y_i - P$ , but is more consistent with the data in that not every worker receives a bonus.<sup>36</sup>

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<sup>36</sup> In the empirical section it is shown that not every worker receives a bonus in the data. Thus, this second contract is more consistent with the data.

## B) Imperfectly Observable Output: Efficiency Wage

Now suppose that production is identical to the observable case, but that output is unobservable. Assume that there exists a (costless) monitoring technology that allows the firm to observe output with probability  $q$ , and that such output is not publicly verifiable.<sup>37</sup> As such, the firm can not implement a piece-rate wage. Further, it can not use the wage to induce higher-motivated workers to work hard. However there still exists an incentive to induce higher effort amongst all workers because effort creates surplus.

I consider a simple example of the efficiency wage solution proposed by Shapiro and Stiglitz (1984) in which the firm uses the threat of firing combined with a fixed salary to induce the worker to choose a higher effort.<sup>38</sup> Assume the firm offers a salary  $S$  and commits to firing the worker if he is caught shirking, i.e. he is caught expending effort less than a specified level in the contract,  $e'$ . Like Shapiro and Stiglitz, I assume that the worker can not pay a “bond” or “entrance fee” to the firm. It is this assumption that constrains the equilibrium to be inefficient. Shapiro and Stiglitz justify the assumption by arguing bonds (or entrance fees) would be too expensive, and by the fact that we do not see them in the real world. Carmichael (1990) offers an in depth discussion of the issue.

Under these assumptions, the worker's effort choice is as follows. If he chooses not to shirk,  $e \geq e'$ , he will receive  $S - C(e')$ . That is, he will receive the salary,  $S$ , with certainty but he will suffer the disutility of his effort. If he chooses  $e < e'$  he will expect to receive  $(1-q)S - c(e)$ . That is, by shirking the worker only receives the salary,  $S$ , with probability  $(1-q)$  but in return he suffers a lower disutility of effort.

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<sup>37</sup> Standard efficiency wage models assume effort is observed with probability  $q$  rather than output. To be consistent with the observable case, I assume the firm observes output rather than effort. Given that effort perfectly determines output in my model this is a matter of semantics.

<sup>38</sup> Shapiro and Stiglitz assume effort is discrete. I assume effort is continuous. This enrichment buys the result that the effort level induced by the efficiency wage is less than the efficient level of effort.

Thus to induce  $e = e'$  the lowest salary the firm can offer the worker and still retain his services equates these two quantities,  $S - C(e') = (1-q)S$ .<sup>39</sup> Re-arranging terms yields  $S(e') = C(e')/q$ . Thus, if the firm wants to induce a given effort level  $e'$  it simply offers a salary of  $S(e')$  and threatens to fire if it observes  $e < e'$ . The question now becomes which  $e'$  maximizes profits (surplus). Under this contract the firms' profits are expressed as:  $\pi = E(b_i)e - S(e) = E(b_i)e - C(e)/q$ . That is, profits are the expected output of the effort induced by the contract,  $E(b_i)e$ , minus the cost of inducing that output,  $S(e) = C(e)/q$ . Differentiating  $\pi$  by  $e$  and setting it equal to 0 implies the following first order condition characterized the profit-maximizing effort,  $e'$ :  $qE(b_i) = C'(e')$ . In order to eliminate trivial cases I assume  $E(b_i)e' > S(e') = C(e')/q > R$ . This ensures the firm and worker both find the contract better than no contract.

The key result of this section is the following: If monitoring is sufficiently difficult ( $q \leq (b_L/E(b_i))$ ), then the output induced by the efficient wage contract,  $b_i e'$ , will be lower than the output induced by the piece-rate contract discussed above,  $b_i e_i^*$ . For intuition of this result consider the first order conditions that define  $e'$  and  $e_i^*$  respectively:  $qE(b_i) = C'(e')$  and  $b_i = C'(e_i^*)$ . The economic interpretation of these two first order conditions is that the firm sets the marginal return of effort equal to the marginal cost of inducing that effort in the optimal contract. When monitoring is sufficiently difficult the marginal return to effort for the efficiency wage becomes strictly less than the marginal return to effort in the piece-rate scheme, which implies  $e' < e_i^*$  for all  $i$ . The intuition is that the imperfect nature of monitoring output erodes the return to effort. As such, inducing the efficient effort level ceases to be optimal.

In summary, because the firm can not observe output it is forced to pay a salary and use the threat of firing to motivate workers. This contract induces higher

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<sup>39</sup> I assume the worker decides to not shirk if he is indifferent between shirking and not shirking. Also, this relies on the fact that the worker choose  $e=0$  if he shirk

effort than a flat salary with *no threat* of firing, but still does not induce the efficient level of effort that a contract contingent on performance does. This result will be central in the testable predictions derived in Section 2.3.

### 2.3. Monitoring and Hierarchy

In this section I construct a model that combines the two simple models presented above. The model considers a  $j$  level hierarchy in which each worker in job  $j$  will be monitored by a worker in job  $j+1$ . Like previous models of hierarchy and monitoring (Rosen (1982), Qian (1994)), a worker's output is a function of the monitoring above him and his effort,  $e_{it}$ , which he chooses. The key feature of the model is that monitoring output becomes harder at lower levels of the firm. As such, at lower job levels salaries will become more important, bonuses less frequent, and output lower.

#### A) Environment

I start with a multi-level firm that consists of  $J$  levels. I assume each worker engages in multi-task production as presented in Holmstrom and Milgrom (1991). That is, each worker in job  $j$  faces a set of  $T$  tasks each of which is monitored by his manager in job  $j+1$ . The worker chooses an effort level  $e_{it}$  for each task  $t$  which generates output  $y_{it} = b_i e_{it}$ , where  $\mathbf{e}_i$  and  $\mathbf{y}_i$  will represent the  $T$ -dimensional vectors of effort and output. As above, I assume  $b_i$  is observed only by the worker and has a simple two-point distribution. Further I assume that the proportion of workers with  $b_i = b_H$  in level  $j$  is  $p_j$ . I assume that the manager at level  $j+1$  can only observe the output for  $a_j < T$  of a worker's tasks in level  $j$ .<sup>40</sup> The output of the other  $T - a_j$  tasks is

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<sup>40</sup> Previous papers that consider hierarchical production and monitoring (Rosen (1982), Qian (1994)) have assumed that the monitoring technology is discrete; each worker is either monitored fully or not at all. Time constraints then imply that only a proportion of workers will be monitored. In my model the

observed imperfectly with probability  $q$ . The idea is that due to time constraints the manager will only be able to monitor the most important tasks. The other tasks he will be able to monitor occasionally. For simplicity, I assume that the output at each task is separable. Formally, the output of a worker is  $Y_i = \sum_t y_{it} = \sum_t b_i e_{it}$ . To ensure that at least a minimal amount of output is observable at each level I assume that  $T(e_L^* - e^j) / (e_H^* - e^j) < a_j$  for all  $j$ . Finally, for simplicity I abstract away from dynamic promotion concerns. Although I am careful to address the issue in my empirical work

There are two key assumptions that will drive the results of the model. The first is that  $a_j$  decreases at lower levels. That is, managers are able to monitor fewer tasks at lower levels of the firm. This has several possible theoretical interpretations. As noted above, it could be due to lower ability managers at lower levels (Rosen (1982)), “loss of control” in monitoring, larger spans at lower levels (Qian (1994)), or even more team production at lower levels. Because the focus of the paper is not theoretical, I abstract away from the exact mechanics and simply assert that  $a_j$  decreases at lower levels. A direct consequence of this assumption is that total output for a given worker,  $Y_i$ , can be written as the sum of two components, output generated at observable tasks,  $b_i a_j e_i^o$ , and output generated at unobservable tasks,  $b_i (T - a_j) e_i^u$ :  $Y_i = b_i [a_j e_i^o + (T - a_j) e_i^u]$ .<sup>41</sup>

The second main assumption is that the costs of effort for different tasks are separable. Formally,  $C(\mathbf{e}_i) = \sum_t C(e_{it})$ . In a more general information environment in which the worker is risk averse and the contract is linear, Holmstrom and Milgrom (1991) analyze multi-task production when  $C(\mathbf{e}_i)$  is both separable and non-separable. The separable case simplifies the analysis significantly. The non-separable case (i.e

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monitoring of tasks is discrete. Each task is either monitored or not, and each worker faces some monitored and some un-monitored tasks.

<sup>41</sup> As will be shown, because the return to effort is the same across tasks, the worker will choose the same level of effort on all observable tasks and the same level of effort on all unobservable tasks in equilibrium.

assuming that tasks are either substitutes or compliments in the cost function) can lead to a wide set of implications depending on the observability of the tasks and the strength of compliment/substitute effect. To keep the analysis tractable and to focus on the intuition of the baseline models outlined in the previous section, I assume separable costs of effort across tasks. This assumption, along with the assumption that output at each task is separable, then implies that the worker's choices of  $e_{ij}$  and  $e_{ik}$  for any two tasks  $j$  and  $k$  are completely independent. To be consistent with the above notation on  $Y_i$  I will henceforth write  $C(\mathbf{e}_i)$  as the sum of the cost of total effort exerted on observable and unobservable tasks weighted by  $a_j$ ,  $C(\mathbf{e}_i) = [a_j C(e_{ij}^o) + (T - a_j) C(e_{ij}^u)]$ .

Given the assumptions above, the model is reduced to a simple, separable, two-dimensional choice problem: the worker must choose how hard to work on tasks with observable output and how hard to work on tasks with unobservable output. To help build intuition for the model, consider the following simple example: a worker faces two tasks, writing reports in the privacy of his office and presenting reports publicly to his superiors in a weekly meeting. The second task is observable while the first task is not. The assumption of separable costs of effort in this example implies that the skills/effort the worker uses to complete his writing task are different than the skills/effort he uses to complete his presentations. Thus if he decides to exert more effort writing he does not have to exert less effort on his presentations, nor should the extra time writing make the marginal cost of his effort on presentations higher. The other key assumption of the model,  $a_j$  increases at higher levels, is captured by the following scenario: suppose the worker is promoted to a higher level job where he must still write and present. But suppose that at this higher job level his writing is more highly scrutinized by his manager. Such a change would obviously affect the worker's effort choice on writing, but there is no obvious reason why it would affect his effort on presentations. It is this "comparative static" exercise that is the central

question of the paper. What happens when more of a worker's tasks become observable?

## B) Equilibrium

As noted above, the key to the model is that the choices of  $e_i^u$  and  $e_i^o$  are completely separable. This implies that the basic analysis from Section 2.2 applies to the observable and unobservable cases respectively; it will be optimal for the firm to contract directly on output for tasks in which it is observable, while it will be optimal to use an efficiency wage for tasks in which it is not. As such the optimal contract will be of the form  $S + a_j b_L e_L^* + a_j (Y_i^o - b_L e_L^*)$  where the first component,  $S$ , is the salary for unobserved output (with a minimum level of effort,  $e'$ , tied to it) and the second component,  $a_j b_L e_L^* + a_j (Y_i^o - b_L e_L^*)$ , is the bonus payment for observed output.

I start with the worker's choice of effort on tasks with observable output. Because the firm offers a piece rate, and because the cost of observable effort,  $a_j C(e_i^o)$ , is separable from unobservable effort, we know that just as in Section 2.2 the worker will choose  $e_i^*$  that solves,  $a_j b_i = a_j C'(e_i^*)$ . Workers with  $b_i = b_H$  will choose  $e_i^o = e_H^*$  while workers with  $b_i = b_L$  will choose  $e_i^o = e_L^*$ , where  $e_H^* > e_L^*$ . That is, more motivated workers will work harder on the observable dimensions.

Given the worker's optimal choice of effort on tasks with observable output, the firm will then use an efficiency wage approach to motivate workers on tasks with unobservable output. As in Section 2.2, the firm will choose a salary  $S(e_i^u)$  and a minimal amount of acceptable effort,  $e_i^u$ , to ensure workers do not shirk. Using logic similar to Section 2.2, this salary is exactly  $S(e_i^u) = [(T - a_j)C(e_i^u)/q - a_j b_L e_L^*]$ .<sup>42</sup> The

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<sup>42</sup> An important difference from the simple case in Section II, is that  $S(e_i^u)$  depends on  $a_j b_i e_i^*$ . However, because the firm can not distinguish high and low type workers, it can *not* offer two different salaries to the two different groups. Thus it will need to offer the higher of the two salaries,  $S(e_i^u) = (T -$

equilibrium salary will then be the salary that maximizes profits. In this case, profits for a given  $S(e_i^u)$  are  $(T-a_j)E(b_i)e_i^u - S(e_i^u)$ . That is, profits are the expected output induced by the salary minus the cost of the salary. The profit maximizing effort,  $e'$ , then solves the following first order condition  $qE(b_i) = C'(e')$ , the same first order condition from Section 2.2. Further, just as in Section 2.2 for sufficiently small  $q$  we have  $e' < e_L^* < e_H^*$ . That is, workers do not exert as much effort on unobserved dimensions as they do on observed dimensions because the costly nature of monitoring output on unobservable dimensions reduces its return.<sup>43</sup> Finally, for ease of notation I will refer to the equilibrium salary as  $S'$  rather than  $S(e')$ .

Putting together the optimal contracts on observable and unobservable dimensions, the equilibrium wage in job  $j$  will be  $W_{ij} = S_j' + a_j(b_i e_H^* - b_L e_L^*)$  for high output workers and  $W_{ij} = S_j'$  for low output workers, where  $S_j' = [(T-a_j)C(e')/q - a_j b_L e_L^*]$ . All workers will choose  $e_i^u = e'$ ,  $b_H$  workers will choose  $e_i^o = e_H^*$ , and  $b_L$  workers will choose  $e_i^o = e_L^*$ .

### C) Testable Implications for the Data

The above model builds a framework within which to think about piece-rates, salary, job level and performance (output). In this section I formalize four testable predictions of the model; performance should be better, bonuses should be larger and firings should be less frequent at higher levels of the firm. Further better performance should predict receipt of bonus and frequency of bonus receipt at a given level should serve as a proxy for prevalence of high-ability workers at a given level.

Job level and salary have direct constructs in the data. Piece-rate and output do not. In what follows I will use the variable “bonus” as the data representative of

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$a_j)C(e_i^u)/q - a_j b_L e_L^*$ , to ensure all workers do not shirk. Under the parameterization  $(T-a_j)E(b_i)e' > S(e') = (T-a_j)C(e')/q - a_j b_L e_L^* > 0$  such a salary is optimal. It is this case that is considered in the text.

<sup>43</sup> This result is similar in spirit to Baker (1992) which shows workers will work harder on measurable dimensions.

piece-rate, and the variable “performance rating” as the data representative of measured output. I define measured output as the output produced on perfectly monitored tasks *plus* output produced on the imperfectly monitored tasks when monitoring is successful. To be consistent with the fact that the performance measure in the data is one-dimensional, I will use a worker’s measured output *per task* as the theoretical analogue of “performance rating”. If we let  $y_i^m$  be worker  $i$ ’s measured output per task, then formally  $y_i^m = b_i[(T - a_j)e' + a_j e_i^*]/T$  for a worker who is monitored successfully and  $y_i^m = b_i e_i^*$  for a worker who is not. I now move on to the testable implications.

Implication 1: Consider a randomly chosen worker from job level  $j$  and one from job level  $j+1$ , where  $j+1$  is higher in the hierarchy. The expectation of  $y_i^m$  will be larger for the worker in level  $j+1$  than the worker in level  $j$ .

Proof: First note that a worker who is monitored successfully will have a measured output per task of  $b_i e_{ij} = b_i[(T - a_j)e' + a_j e_i^*]/T$ . A worker who is not monitored will have a measured output of  $b_i e_{ij} = b_i e_i^*$ . Thus expected measured output at level  $j$  is:

$$\begin{aligned} b_i e_{ij} &= b_i(q/T)[(T - a_j)e' + a_j E(e_i^*)] + b_i(1-q)E(e_i^*) \\ &= b_i(q/T)(T - a_j)e' + b_i((q/T)a_j + 1-q) E(e_i^*). \end{aligned}$$

We note that  $db_i e_{ij}/da_j = b_i(q/T)(E(e_i^*) - e') > 0$ . That is expected measured output per task is increasing in  $a_j$ . Because  $a_j$  is larger at level  $j+1$  than level  $j$ , it then follows that expected measured output per task is higher at level  $j + 1$ .

QED

Implication 1 states that the measured output will be higher at higher levels. This result arises for two reasons. First a larger proportion of a worker’s output is observed at higher levels. Second, as shown in the previous sub-section, when output

is observed and a piece-rate wage is used workers will work harder than when output is unobservable and rewarded via an efficiency wage. Combining these two ideas, we see that workers at higher levels of the firm have a larger incentive to work harder because more of their output is observable and fully rewarded. In terms of the data, if we assume that a performance rating is a proxy for measured output then the testable interpretation of implication 1 is that performance ratings should be higher at higher job levels.

The second implication tells us that performance and bonus should be linked.

Implication 2: The probability that a given worker  $i$  receives a bonus is (weakly) increasing in measured output per task,  $y_i^m$ .

Proof: In the model above there are only 4 possible outcomes for  $y_i^m$ :

$$b_i[(T - a_j)e' + a_j e_L^*]/T, b_i e_L^*, b_i[(T - a_j)e' + a_j e_H^*]/T, b_i e_H^*,$$

$$\text{where } [(T - a_j)e' + a_j e_L^*]/T < e_L^* < [(T - a_j)e' + a_j e_H^*]/T < e_H^*.$$

The probability of a receiving a bonus for each level of measured output is 0, 0, 1, 1 respectively. We conclude that the probability of bonus is weakly increasing in measured performance.

QED

Implication 2 tells us that the workers who produce more output are more likely to receive bonuses. This result comes directly out of the equilibrium of the model; only workers who produce the highest output will be rewarded with a bonus. Thus the probability of a bonus is increasing in output. Again, if we assume a performance rating is a proxy for measured output then implication 2 tells us that better performance ratings increase the likelihood of a bonus.

Implication 3: Consider a randomly chosen worker from job level  $j$  who has received a bonus and one from job level  $j+1$  who has received a bonus, where  $j+1$  is higher in the hierarchy. The absolute size of the bonus *and* the ratio of the bonus to total compensation (salary + bonus) will be larger for the worker in level  $j+1$ .

Proof: As shown above the wage for a worker in job  $j$  will be  $W_{ij} = S_j + a_j(b_i e_H^* - b_L e_L^*)$ . The first part of the statement is thus trivial. As  $j$  increases so does the bonus,  $a_j(b_i e_H^* - b_L e_L^*)$ .

The second part of the implication is non-trivial. The ratio of bonus to bonus plus salary for those receiving a bonus, BP, is thus:  $a_j(b_i e_H^* - b_L e_L^*) / [(T - a_j)C(e')/q + a_j(b_i e_H^* - b_L e_L^*)]$ . Differentiating BP by  $a_j$  using the quotient rule, yields that the sign of the  $\partial BP / \partial a_j$  will be the sign of the numerator, as the denominator is positive. The numerator is:

$$\begin{aligned} & (b_i e_H^* - b_L e_L^*)[(T - a_j)C(e')/q + a_j(b_i e_H^* - b_L e_L^*)] - (b_i e_H^* - b_L e_L^*)a_j(-C(e')/q + \\ & (b_i e_H^* - b_L e_L^*)) \\ & = (b_i e_H^* - b_L e_L^*) [C(e')/q] \\ & > 0 \end{aligned}$$

Thus the ratio is increasing in  $a_j$  and thus  $j$  and we conclude that the ratio of bonus to salary at level  $j+1$  is greater than at  $j$ .

QED

Implication 3 tells us that bonuses should increase in size at higher job levels. The first part of the implication follows immediately from the model. It states that bonuses will be larger in absolute terms at higher levels of the firm. This comes out of the fact that a larger proportion of output is observable at higher levels of the firm. The second part of the implication makes a slightly richer prediction regarding bonuses. It tells us that the *ratio* of the bonus relative to total compensation (salary

plus bonus) will be larger at higher levels. This captures the fact that bonuses are a bigger component of compensation at higher levels of the firm. In terms of the data, this is a stronger prediction than the first half of the implication because it rules out the possibility that larger bonuses at higher levels could simply be driven by higher overall compensation at higher levels.<sup>44</sup>

Implication 4 tells us how the frequency of bonus should vary across level.

Implication 4: Consider a randomly chosen worker from job level  $j$  and one from level job  $j+1$ , where  $j+1$  is higher in the hierarchy. The probability he receives a bonus is  $p_j$ .

Proof: The proof is straightforward. In the equilibrium of the model only  $b_H$  workers receive a bonus. Thus the likelihood of a random worker in job  $j$  receiving a bonus is exactly  $p_j$ , the probability that  $b_i = b_H$ .

QED

Implication 4 states that the frequency of receiving a bonus in a given job level is determined by the distribution of  $b_H$  workers in that level. As such, the frequency of receiving a bonus in a given level can be seen as a proxy for the concentration of high ability workers in a given level. As I will discuss in detail in Section 2.4, an important part of my empirical methodology is controlling for positive sorting on ability across job level. Implication 4 will, thus, be an important part of this methodology because it helps identify the prevalence of positive sorting; more frequent bonuses at higher levels will be evidence of positive sorting. I will discuss this issue in more detail in the empirical results section.

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<sup>44</sup> There is a strong correlation between level and overall compensation in the data.

Finally, despite the fact that the theory does not include firings in equilibrium, an intuitive idea from the model presented above (and an easy formal extension) is that firing should be more prevalent when output is harder to observe because it serves as an incentive mechanism in such circumstances. On the other hand, firing should be less prevalent when output is easy to observe because pay-for-performance compensation schemes (bonuses) rather than firing serve as the incentive mechanism. In the context of a multi-level firm where output is more difficult to observe at lower levels, it then follows that firing should be more prevalent at lower levels. As such, along with the three formal implications derived above, I also will explore this fourth more suggestive prediction in the empirical section.

In summary, in this section I derived four testable implications and one suggestive implication of the simple model built above. The model predicts that workers at higher levels should have higher measured performance, should receive larger bonuses in absolute and relative terms, and should be fired less often. All of these predictions are driven by the assumption that a larger proportion of worker's output is observable at higher levels of the firm. Further, it tells us that frequency of bonus serves as a proxy for the proportion of highly-motivated workers in a given job level. In the next section I explore these predictions using personnel records from a firm in the US financial services industry. The methodology I use is careful to distinguish the predictions of the model from alternative explanations that rely on positive sorting on ability across job levels.

#### **2.4. Empirical Strategy and Results**

In this section I explore the relationships between bonus, performance, firing and job level using personnel records from a firm in the US financial services industry. The focus of the section is on distinguishing the model from potential alternative

explanations that rely on positive sorting on ability across job level. Almost all models of hierarchy (Rosen (1982), Waldman (1984) for example) have the equilibrium feature that more talented, or able, workers sort to higher levels of the firm. As I discuss below, such models could in principle explain all of the implications outlined in the previous section without relying on variation in costs of monitoring. To separate my model from such explanations I first conduct a cross-sectional analysis that tests the implications of the theory while controlling for observable skills. I then conduct a longitudinal analysis which allows me to control for fixed unobserved skills as well as observable skills. Taken together the results suggest that there is significant variation in bonuses and performances across level that is independent of variation in ability, evidence consistent with the predictions of my model.

#### A) The Data: Key Variables and Sample Restrictions

The data I use in my analysis is the sample of white, male employees from the same dataset analyzed in Baker, Gibbs, and Holmstrom (1994a,b) (BGH hereafter). The data include yearly personnel records from 1969 to 1988 for all such managerial employees of a US firm in the financial services industry. The original sample analyzed by BGH included females and nonwhite males, for a total of 68,437 employee-years of data. The sample of white males has 50,556 employee-years. The dataset was originally constructed using raw data from the firm's personnel records.

The key feature of the data is that it allows researchers to identify the hierarchical structure of the firm and observe how workers move through it over time. The 8-level hierarchy was constructed by BGH by analyzing transitions between job titles. Figure 2.1 captures the organization structure of the firm.<sup>45</sup> The upward arrows capture promotion routes through the firm, and the size of the bubbles capture the size

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<sup>45</sup> This figure is taken from Baker, et al. (1994a).

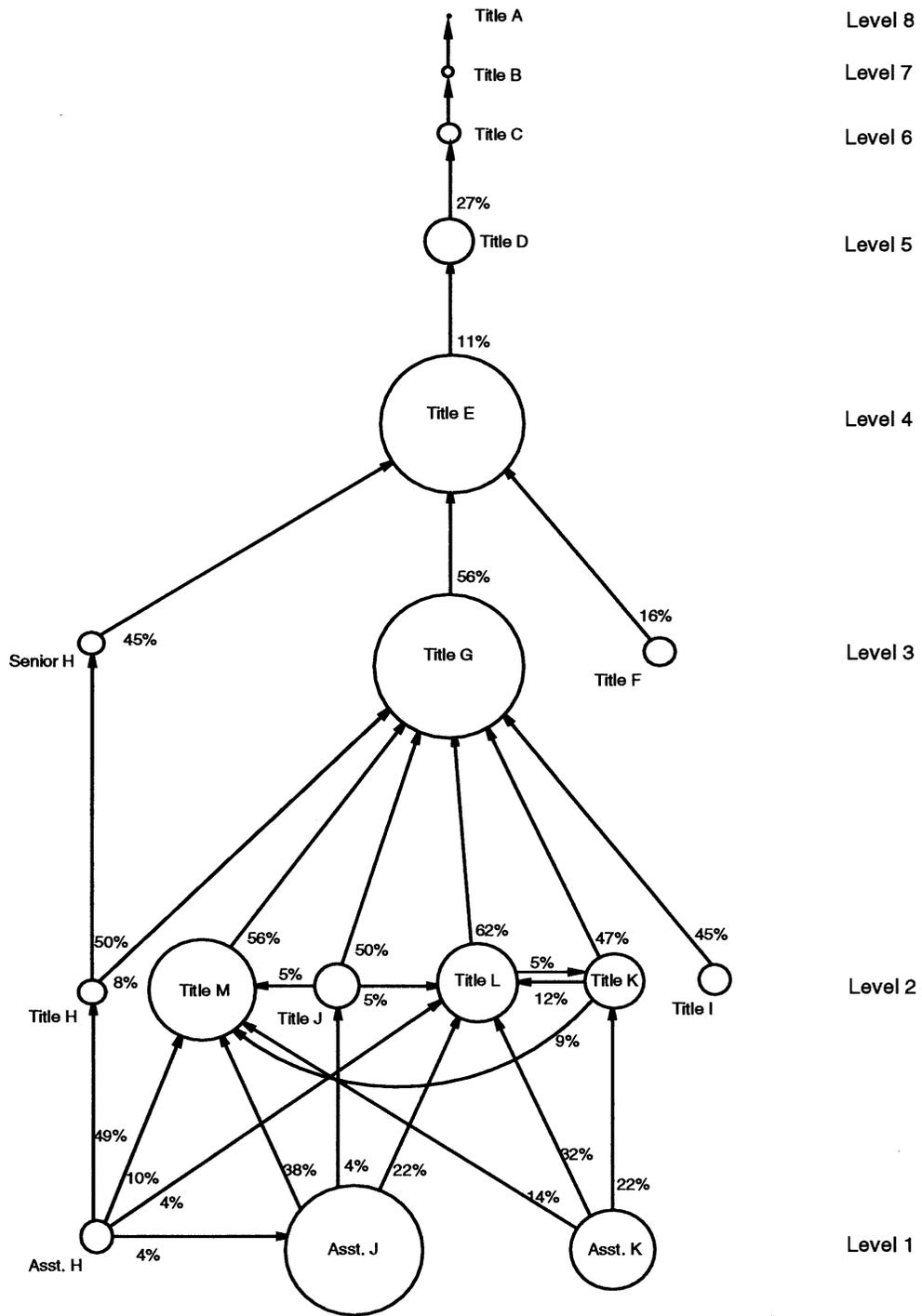


Figure 2.1: Organizational Structure of the Firm

of the job level. As BGH note, this structure is extremely stable over time. An important point for the analysis below is that levels 5 and higher have a distinctly different structure than levels 4 and lower. There are much fewer workers in the top 5 levels (roughly 96% of workers are in the first 4 levels) and the linkages across levels are more direct. The lower levels, on the other hand, are larger and have more complex linkages across job levels, suggesting supervision may be more difficult.

The key variables in my analysis are salary, bonus, performance, “fired”, job level, education and potential labor market experience. Salary is measured in 1988 dollars, deflated by the CPI. Information on bonuses is only available for 1981 to 1988. This fact will limit all analyses involving bonus to the eight-year window. Below I discuss in detail the implications of these restrictions for the sample. Performance is measured in the data using a relative performance rating. Each year each worker receives a rating of 1, 2, 3, 4, or 5 where 1 is the highest rating, 5 the lowest. The “fired” variable is constructed as I can not distinguish whether a worker’s exit was a voluntary quit, a fire, or a retirement. To keep things simple, in what follows I will define a fire as an exit that follows a period of relatively bad performance. More precisely, if a worker receives a performance rating of 3 or higher in period  $t$  and then exits the firm in period  $t + 1$ , he is said to have been “fired”. As you will see below, a performance rating of 3 or higher is relatively poor performance, as a performance rating of 2 is the median of the overall performance distribution. Education categories are constructed from an underlying “years of education” variable. Specifically, I construct dummy variables for high school graduate, bachelors, masters, and Ph.D.<sup>46</sup> Finally, following BGH, I construct potential labor market experience as:  $\text{experience} = \text{age} - \text{education years} - 6$ .

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<sup>46</sup> A high school graduate is defined as a worker with 12-15 years of education, a bachelor’s, 16-17 years of education., an MBA or other master’s degree holder, 18-19 years of education, a Ph.D., 20 or more years of education.

Because performance and bonus are the key variables in my analysis and they have not been widely analyzed in previous work using this dataset, Table 2.1 presents basic summary statistics. As is clear, bonuses occur relatively frequently. Over 25% of worker-periods involve bonus. The average size of bonuses

**Table 2.1: Summary Statistics of Bonus and Performance**

<b>Variable</b>	<b>Mean</b>	<b>Std Error</b>	<b>Min</b>	<b>Max</b>	<b>Obs</b>
Size of Bonus	10,049	16,438	141	296,128	5,441
Bonus, Percent of Comp	12.80	11.20	.22	194.00	5,441
Bonus Awarded	0.263	0.4403	0	1	20,679
Performance Rating	2.00	0.873	5	1	28,699
Rating of 1	0.293	0.4552	0	1	28,699
Ratng of 2	0.481	0.4996	0	1	28,699
Rating of 3,4 or 5	0.226	0.4181	0	1	28,699

(conditional on receiving one) is economically significant, at over \$10,000 in raw terms and 12.80 percent in relative terms. The frequency of the performance rating variable is harder to interpret because it is potentially relative. The important thing to note is that a rating of 2 is both the mode and the average. A rating of 1 is thus interpreted as high performance, while a rating of 3, 4, or 5 is low performance. In the regression analysis below I will continue to use this interpretation of the performance rating.

Finally, before moving onto the analysis I discuss concerns of bias that could arise when analyzing the sub-sample of the data that includes only non-missing bonus and performance information. Table A2.1 in the appendix shows the distributions for three samples of the data: the “Full Sample,” which includes all records in Levels 1-8 for the full sample window, 1969-1988, the “Bonus Sample,” which includes all

records from the window 1981-1988 with non-missing bonus information, and the “Bonus, Performance Sample,” which includes all records in the “Bonus Sample” with non-missing performance information. Table A2.1 makes clear that the distributions of level, performance, education and age (the key variables used in my analysis) are very similar across the three samples. That is, the restricted sample needed to analyze bonuses does not appear to bias the distributions of the key variables in any obvious way.

#### B) Methodology: Controlling for Sorting on Ability Across Job Level

In this section I present the basic methodology of the tests. The main goal of the methodology is to help distinguish the model from potential alternative explanations that rely on positive sorting on ability across level. At the end of the section I discuss briefly how I define the job levels for the empirical tests.

The literature on promotion dynamics offers two well-studied reasons why higher-ability workers should sort to higher job levels: skill is more valuable at higher job levels (Rosen (1982), Waldman (1984)) and promotion tournaments (Lazear and Rosen (1981)) should select the best workers upwards as “winners”. It is quite possible, then, that better performance, larger bonuses and less frequent firings at higher levels are driven by sorting on ability rather than variation in cost of monitoring. To properly test my model, then, it is essential that I control for such sorting on ability across job level. To do so I conduct both a cross-sectional analysis in which I control for observable worker characteristics and a longitudinal analysis in which I control for observable and fixed, unobservable worker characteristics.

I start with a discussion of the cross-sectional analysis. The key controls I use are worker’s education, total labor market experience, and on-the-job tenure (number of years at a given job level). Education and labor market experience are standard

proxies for worker skill.<sup>47</sup> On-the-job tenure is included because standard models of dynamic promotion (Gibbons and Waldman (1999, 2006)) suggest that it should be an important determinant of a worker’s relative standing at his job.<sup>48</sup> Further, previous cross-sectional work from the empirical literature on the Peter Principle (Peter and Hull (1969)) finds that on-the-job tenure is a significant determinant of worker’s outcomes.<sup>49</sup> Formally, for the outcome of interest for worker  $i$  at job  $j$  in year  $t$ ,  $Y_{ijt}$ , I estimate the following equation:

$$(1) Y_{ijt} = \beta_1 X_{it} + \beta_2 TEN_{ijt} + \beta_3 TEN_{ijt}^2 + L_j + A_i + e_{ijt}.$$

where  $X_{it}$  is a vector of observable skills,  $TEN_{ijt}$  is tenure at job  $j$ ,  $L_j$  is a fixed effect of level  $j$ , and  $A_i$  is the fixed effect of worker  $i$ .

The object of interest in equation (1) is  $L_j$ ; the effect of Level  $j$  on the outcome of interest,  $Y$  (in my case performance, bonus and firing). The cross-sectional analysis offers an unbiased estimate of  $L_j$  *only* under the assumption that  $A_i$  is orthogonal to  $L_j$ . That is, the cross-sectional estimates are only accurate if a worker’s fixed, unobserved skills are uncorrelated with job level. If such skills are in fact positively correlated with job level then the estimates of  $L_j$  in the cross-sectional analysis will be biased upwards. Such a correlation could arise if the promotion process selects workers with better unobservable skills (for example, intrinsic motivation) upwards, which, as noted above, is theoretically plausible.

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<sup>47</sup> The control for experience can also be seen as a control for different work incentives for older workers as found in Gibbons and Murphy (1992).

<sup>48</sup> Gibbons and Waldman (1999, 2006) show that high-ability workers will be promoted more quickly than low-ability workers. As such, in the cross-section, on-the-job tenure should be negatively correlated with performance, as it is a proxy for “failure at promotion.” When task specific capital is added to the Gibbons and Waldman framework, on-the-job tenure will also serve as a proxy for the accumulation of such capital.

<sup>49</sup> Medoff and Abraham (1980) find that subjective performance falls with tenure on the job, Lazear (1992) and Baker, et al. (1994a) find that wage falls with tenure, and Gibbs and Hendricks (2004) find that bonuses fall with tenure.

To address this possibility I also conduct a longitudinal analysis. The longitudinal analysis allows me to control for fixed unobserved skills, as well as observable skills and on-the-job-tenure, by looking at how bonus and performance differ across job level for *the same worker*. I implement the analysis by differencing the data on the dimension of job when on-the-job tenure is the same.<sup>50</sup> Formally, I estimate the following equation:

$$(2) Y_{ij+1t} - Y_{ijt-r} = \beta_1(X_{i+1t} - X_{it}) + (L_{j+1} - L_j) + (e_{ij+1t} - e_{ijt-r})$$

where r is the number of years spent at level j-1.

The dependent variable in equation (2) is the difference in the outcome of interest, Y, for worker i at job level j+1 and at job level j when  $T_{ij+1} = T_{ij}$ . That is, I compare outcomes *only* when on-the-job tenure is the same.<sup>51</sup> The important thing to note about equation (2) is that the fixed unobserved skills,  $A_i$ , drop out, thereby eliminating the concerns of bias present in the cross-sectional analysis. As such, even if  $A_i$  and  $e_{ijt}$  are not orthogonal (i.e. there is positive sorting on unobserved skills across job level) estimating (2) by OLS will yield an unbiased estimate of  $L_{j+1} - L_j$ . Another important thing to note is that on-the-job tenure also drops out of equation (2). This happens because Y is differenced on the dimension of level *controlling for tenure at the job level*. That is, the analysis examines differences in the outcome of interest across job level in the same year of tenure. As noted above, this is important because

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<sup>50</sup> The use of a first-difference estimator follows the traditional approach from the inter-sector wage differential literature (Krueger and Summers (1988), Katz and Gibbons (1992)) where first-difference estimators were used to estimate the effect of industry on wage while controlling for unobserved worker skills. It should also be noted that similar identification strategies are used in more complicated fixed-effect models, such as Abowd et al. (1999), in which identification of firm effects depends on movement of workers between firms.

<sup>51</sup> When the total number of years of tenure at adjacent job levels are unequal, observations for the high tenure years are dropped. For example, if worker i spends 5 years at job j and 4 years at job j-1, the dependent variables is only calculated for years 1-4 of tenure.

on-the-job tenure is a significant determinant of performance, particularly as workers move between job levels.

It is important to note that the longitudinal analysis itself is potentially biased because it relies exclusively on promoted workers. As Gibbons and Katz (1992) and Gibbons, et al. (2006) point out in their work on inter-sector wage differentials, first-difference estimators are biased if the movement of workers between sectors is endogenous to the variable of interest. Formally this could be captured in equation (2) if the worker effect is not fixed across job level, but rather interacts with job level. In such a world the estimates of  $L_j$  would be biased upward as they do not account for the interaction with ability. In my data, given that the performance measures are relative to job level, I believe the effect of such a dynamic is mitigated, but I can not rule out the possibility that the results are driven by such an endogenous sorting dynamic.

Finally, before moving onto the results, I discuss an important data point. In all of the analyses below I collapse the top 4 levels of the hierarchy (levels 5 and higher) into one category. I do this for two reasons. First, as noted above the sample sizes are very thin at the highest levels. As such, collapsing the cells gives the tests more strength. Second, if you remember from Figure 2.1 in subsection A, levels 5 and higher of the hierarchy have a distinctly different organizational structure than levels 1-4 (again, see Figure 2.1); there are much fewer workers and the chain of command is more direct. As such, the basic premise of the theory that supervision is easier at the top seems particularly plausible at levels 5 and higher. Thus collapsing them into one category, and interpreting that category as “top management” makes sense from a theoretical perspective. Note that I ran all of the analyses *without* collapsing the job levels and the qualitative results are the same.

### C) Cross Sectional Analysis: Controlling for Observable Skills

The theoretical section predicts that the size of the bonus and worker performance should increase and firings should decrease at higher levels of the firm. Further high performance should predict bonus receipt. In this section I explore the empirical support of these predictions in the cross-section. As noted above, I control for sorting on ability across job level by controlling for on-the-job tenure, education and general experience.

For each test, I report estimates from two specifications of equation (1) above. The first specification includes a full set of level dummies (where levels 5 and higher are collapsed into a single “level” as described above). The second specification includes only a dummy for levels 5 and higher. From a technical point of view, the second specification tests for a different pattern of statistical significance than the specification with the full set of job level dummies. In the full specification, the coefficients taken together inform us of the general trend in bonuses and performances at higher levels, but do not formally test if the highest levels are different than the lowest levels (the tests are only relative to job level 1, the excluded level). The second regression, with the dummy for levels 5 and higher, achieves exactly this by comparing the average bonus at levels 5 and up to that of levels 1-4. Finally, note that all tests include the controls described above although I do not present the coefficients in the tables for ease of exposition. At the end of the section, however, I briefly discuss their effects and how they relate to previous work that investigated similar relationships.

Before moving onto the regression analysis, I first present the averages of bonus, performance, and firing by job level with no controls. As you can see from Table 2.2 the average size of bonuses and the average worker performance increase

**Table 2.2: Average Bonuses, Performance Ratings, and Firings  
Across Job Level, No Controls**

<b>Var</b>	<b>L 1</b>	<b>L 2</b>	<b>L 3</b>	<b>L 4</b>	<b>L 5</b>	<b>L 6</b>	<b>L 7</b>	<b>L 8</b>
Raw Bonus	3,065	3,943	4,960	12,049	37,352	67,262	141,190	210,692
% Percent	8.17	9.236	9.47	14.98	28.90	35.77	39.14	38.00
Bonus Freq	0.112	0.164	0.281	0.401	0.364	0.247	0.467	0.5
<b>Obs</b>	3,427	4,655	5,896	5,906	542	215	30	8
Perf Rating	2.31	2.19	1.93	1.66	1.62	1.58	1	.
Rating = 1	0.241	0.186	0.275	0.443	0.46	0.609	1	.
Fired	.0579	.0548	.0422	.0267	.0078	.0451	0	
<b>Obs</b>	5,876	7,102	7,876	7,195	515	133	2	0

while the frequency of firings decrease at higher levels of the firm. It is this pattern that is the subject of the theory. It is interesting to note that the *frequency* of bonus receipt is larger at higher levels along with the size of bonuses. In the context of the theory above, the higher frequency of bonus at higher levels implies positive sorting on  $b_i$ . In the regression analysis that follows I attempt to control for such sorting to formally test whether the patterns on bonus, performance and firing are significant.

I start with the tests of Implications 1 and 2. Table 2.3 explores two basic questions: How does performance vary across job level? And does performance predict receipt of bonus? The second and third columns address the first question. Sub-column (1) of the “Performance Rating” column shows that the average performance rating is incrementally lower (i.e. performance is better) as we move up the hierarchy. Sub-column (2) shows that the average performance rating at the highest levels of the hierarchy is significantly lower than at the middle levels. That is, performance is significantly higher at the higher job levels. The third column presents a second way to look at performance. Rather than treating the performance rating measure as “continuous” (as the first two specifications implicitly do), it estimates a

linear probability model for the highest performance rating (rating=1). From sub-column (1) we see that the frequency of the highest rating increases at higher job levels. Sub-column (2) shows that the frequency of the highest rating is

**Table 2.3: Level Effects on Performance and the Effect of Performance on Bonus Received, Controlling for Observable Skills and Tenure<sup>52</sup>**

DepVar	Perf Rating		Perf Rating = 1		DepVar	Received Bonus	
	(1)	(2)	(1)	(2)		(1)	(2)
Level 2	-.058 *** (.019)		-.030 ** (.012)		Perf	-.12 *** (.006)	
Level 3	-.345 *** (.019)		.102 *** (.012)		Perf = 1		.254 *** (.012)
Level 4	-.707 *** (.020)		.310 *** (.013)		Perf = 2		.131 *** (.001)
Level 5 +	-.766 *** (.045)	-.320 *** (.044)	.379 *** (.029)	.207 *** (.028)			
<b>Obs</b>	12,621	12,621	12,621	12,621	<b>Obs</b>	12,621	12,621

significantly higher at the elite levels of management compared to the lower levels of management. Taken together, all four of the specifications present strong evidence that workers at higher levels of the firm have better performance. In the context of the theory, this behavior is driven by the fact that performance is easier to measure at the top levels, and thus more easily rewarded.

In the right-hand panel of Table 2.3, the “Received Bonus” column explores implication 2: does high performance predict receipt of bonus? Sub-column (1) treats performance rating in a “continuous” fashion and confirms that workers with higher performance (lower performance ratings) are more likely to receive a bonus. Sub column (2) treats performance rating in a discrete fashion and shows that workers with a rating = 1 or a rating = 2 (relatively high performance) are more likely than workers

<sup>52</sup> Note that because of small sample sizes and missing values in the performance rating, I do not have observations in levels 7 and 8.

with a rating of 3, 4 or 5 (relatively low performance) to receive a bonus. The effect of a rating=1 is particularly strong. These results show a strong relationship between performance and receipt of bonus, behavior consistent with the theoretical idea that bonuses are incentive devices.

I now move onto the results on bonuses presented in Table 2.4. The column entitled “Bonus” presents results from the regressions in which the raw size of bonus is the dependent variable. Sub-column (1) shows that the raw size of the bonus, controlling for observable skills, increases monotonically at higher levels of the firm. At the top levels bonuses are over \$45,000 more than at level 1. The coefficient on “Levels 5 +” in sub-column (2) tells us that not only does the bonus increase incrementally at each level, but it is significantly larger on average at elite levels of management compared to middle levels. Taken together, these coefficients make it clear that bonuses at higher levels are significantly larger than lower levels.

**Table 2.4: Level Effects on Bonus, Percent Bonus, and Receipt of Bonus, Controlling for Observable Skill and Tenure**

DepVar	Bonus		Percent Bonus		Receipt of Bonus	
	(1)	(2)	(1)	(2)	(1)	(2)
Level 2	799 (824)		1.31 ** (.65)		.071 *** (.0010)	
Level 3	1,606 ** (758)		1.46 *** (.60)		.204 *** (.0097)	
Level 4	8,150 *** (775)		6.81 *** (.61)		.348 *** (.0103)	
Level 5 +	45,716 *** (1,119)	40,010 *** (879)	23.72 *** (.88)	18.87 *** (.69)	.308 *** (.0185)	.081 *** (.0173)
<b>Obs</b>	5,052	5,052	5,052	5,052	20,699	20,699

The “Percent Bonus” column in Table 2.4 conducts the same analysis with the size of a bonus as a percent of total compensation (bonus plus salary) as the dependent variable. This analysis is a stronger test than the raw bonus analysis because percent

bonus controls for the strong upward trend in wages across levels present at the firm. The results in sub-column (1) show that the percentage of bonus relative to total compensation increases strongly as we move up levels, with bonuses at the highest 4 levels over 23 percentage points higher than in level 1. The results in sub-column (2) confirm that, like raw bonus, the percentage of bonus relative to total compensation is higher at elite levels of management than the middle levels. Again this test confirms that the upward pattern from sub-column (1) is significant.

The last column of Table 2.4 explores how the likelihood of bonus varies across level, while controlling for observable skills. As noted above, in the context of the theoretical model the frequency of bonus at a given level can be interpreted as a proxy for ability at that level. As such, more frequent bonuses at higher levels would suggest that more motivated workers are sorting to higher levels over time. Moving down sub-columns (1) and (2) this is exactly what occurs in the data. The frequency of receiving a bonus gets higher at higher levels. Thus, even after controlling for observable skills, there is strong evidence that sorting on ability is still prevalent. In the next section, I attempt to control for this using a first-difference analysis which controls for fixed, unobserved by the econometrician, worker skill.

The final cross-section regressions are presented in Table 2.5, which explores the frequency of firing across level. As noted in Section 2.3, the model suggests that firing should be more prevalent when output is harder to observe because it serves as an incentive mechanism in such circumstances. As such, because output is harder to observe at lower levels, firings should be more prevalent. Table 2.5 presents results of a linear probability model that estimates the effect of job level on two measures of “fired”. The first measure was discussed above in sub-section A). It defines a “fire” as an exit after a period of poor performance relative to the overall distribution. One potential problem with this measure is that, because bad performance is rarer at higher

levels of the firm, it builds in a bias against firing at higher levels. To protect against this bias I also use a second measure of “fired.” This measure defines a “fire” as an exit after a period of poor performance relative to the performance distribution of the worker’s most recent *job level*.<sup>53</sup> As you can in Table 2.5, the frequency of firing decreases significantly at higher levels, with workers at the top levels of management being almost 10 percentage points less likely to be fired for both measures. Such evidence is consistent with the suggestive prediction of the model.

**Table 2.5: Level Effects on Fired, Controlling for Observable Skills and Tenure**<sup>54</sup>

Dep Var	Fired (Measure 1)		Fired (Measure 2)	
	(1)	(2)	(1)	(2)
Level 2	-.0177 *** (.0069)		-.0177 *** (.0070)	
Level 3	-.0545 *** (.0067)		-.0544 *** (.0067)	
Level 4	-.0862 *** (.0071)		-.0860 *** (.0071)	
Level 5 +	-.0998 *** (.0158)	-.0430 *** (.015)	-.0882 *** (.0159)	-.0314 *** (.015)
<b>Obs</b>	12,469	12,469	12,469	12,469

Finally, as noted above, for the sake of presentation I did not report the coefficients of the covariates in the Tables 2.2 – Tables 2.5. I offer a brief summary of the results here. The coefficients for on-the-job tenure are perfectly consistent with the previous literature on the Peter Principle; worker’s performance, receipt of bonus, and size of bonus all decrease the longer they stay in a job. The coefficients on education and experience, however, were a bit anomalous. More educated and more experienced workers consistently had worse performance, bonus, and receipt of bonus

<sup>53</sup> In both cases “poor” performance is performance below the median level of the relevant distribution.

<sup>54</sup> Note that Levels 7 and 8 do not have estimates because all observations in those levels have missing performance information, and the “Fired” variable relies on performance information to be constructed.

outcomes controlling for tenure at the job. These results, though, may be consistent with the signaling model of Devaro and Waldman (2006) in which the critical level of ability required for promotion decreases with education. In such a world it would not be surprising for performance and bonus outcomes to also get worse with education.

In summary, in this section I have conducted a cross-sectional analysis that explores the implications of the theory presented in Section 2.3. I find evidence consistent with the theory. In particular, the compensation structure at higher levels is significantly different than at lower levels of the hierarchy. Workers at higher levels of the firm are more likely to receive higher performance ratings, receive larger bonuses and are less likely to get fired. Further there is evidence that the bonuses indeed play the role of an incentive mechanism. Workers who perform “better” are significantly more likely to receive bonuses. The analysis, however, can not rule out the possibility that the results are driven by positive sorting on an unobserved dimension, a possibility supported by the fact that bonus receipt is more likely at higher levels even after controlling for observable skills. In the next section I conduct a first-difference analysis to address this concern.

#### D) Longitudinal Analysis: Controlling for Unobserved Skills

In this section I report the results of a longitudinal analysis that explores how bonuses and performance vary across levels *within* a single worker’s career. As noted earlier the strength of the analysis is that, along with controlling for observable skills, it also controls for fixed unobserved worker skills. As such, it allows me to further distinguish my model from the sorting explanation discussed above.

Following the literature on inter-sector wage differentials, I conduct a first-difference analysis to estimate the effect of level on bonus and performance while controlling for fixed, unobserved skills (Krueger and Summers (1988), Katz and

Gibbons (1992)). Unlike the cross-sectional analysis, I do not include firings in the analysis because there is no usable variation within a person across level in the frequency of firing (i.e. no one is fired *twice* from different levels). Formally, I estimate equation (2) from sub-section B) for the following 5 measures: absolute size of bonus, percentage of bonus, frequency of bonus, performance, and frequency of high performance (performance rating =1).

Table 2.6 presents the results for bonus and performance. As you can see, the results on size of bonus and performance are consistent with the theory. As workers move up the promotion ladder their relative performance rating improves, they are more likely to achieve the highest performance rating, and they receive larger bonuses, controlling for number of years spent in the job level. In other words, the same identical worker in, say, his 3<sup>rd</sup> year at level 3 performs better and receives a larger bonus than he did in his 3<sup>rd</sup> year at level 2. This is quite strong evidence that workers perform better at higher levels of the firm as it controls for all fixed worker skills and tenure on the job. The one result in Table 2.6 inconsistent with the theory is that the frequency of bonus actually decreases at higher levels. This result is tension with the result from the cross-section analysis that bonuses are more frequent at higher levels.

**Table 2.6: Regression Results by Level for First-Difference Analysis, Controlling for Tenure at Job Level and Years to Promotion**

<b>Dep Var</b>	<b>Bonus</b>	<b>Percent Bonus</b>	<b>Received Bonus</b>	<b>Perform Rating</b>	<b>Perform Rating=1</b>
Level 3 – Level 2	265 (528)	.051 (.590)	.075 ** (.038)	-.261 *** (.062)	.190 *** (.047)
Level 4 – Level 3	1,602 *** (566)	.693 (.633)	-.144 *** (.039)	-.270 *** (.065)	.195 *** (.049)
Level 5 + – Level 4	9,122 *** (1,378)	3.881 *** (1.539)	-.558 *** (.076)	-.674 * (.378)	.367 * (.287)
<b>Obs</b>	1,300	1,300	2,237	1,076	1,076

One potential explanation of the result that is consistent with an enriched version of the model is that bonuses are rewarded for outstanding performance *relative* to workers in the same level. The theoretical model presented in Section 2.3 assumes bonuses are given for absolute performance, but one could imagine a simple extension in which bonuses are given for performance relative to the job level. That is, as workers get promoted they perform better in an absolute sense, but perform “worse” relative to the higher performance standards at the higher job level (Gibbons and Waldman (1999) discuss a related model). In such a world, when a given worker receive a bonus at a higher level it will still be larger but it is possible he will be less likely to receive that bonus.

In summary, the longitudinal evidence is broadly consistent with the predictions of the theory and the evidence found in the cross-sectional analysis: it suggests that workers perform better and receive larger bonuses at higher levels. Taken together, the two analyses offer strong evidence that better performance and larger bonuses at higher levels are driven by something other than positive sorting on ability. The potential explanation offered here is that monitoring of output is more difficult at lower job levels, making piece-rate/bonus payment schemes more difficult to implement. As such performance suffers, bonuses are smaller, and firings are needed to ensure minimal amount of output.

## **2.5. Conclusion**

In this paper I consider a new way to explore a classic question. Insights from the literature on hierarchies as monitoring devices suggest that monitoring should be more difficult at lower levels of the hierarchy. Coupled with the understanding from the pay-for-performance literature that piece-rates should be less effective when monitoring is costly, this insight tells us that compensation schemes should rely less

on bonuses and more on salaries at low level jobs. As a direct consequence, performance should be better at higher levels and firing should be less frequent. On the other hand, classic theories of promotion that imply positive sorting on ability across job level (human capital and learning, and tournament models) could also generate such empirical patterns. To distinguish my model from such competing theories I conduct two empirical analyses using personnel records of management of a firm in the US financial services industry. First, in a cross-sectional analysis I control for observable workers skills, worker experience, and tenure at the job level. I find the data is consistent with the implications of the model. Second, in a longitudinal analysis I control for fixed, unobserved skills as well as observable skills and on-the-job tenure, and find that most of the implications of the model continue to hold. I conclude that there is significant variation in bonus, performance and firing across job level that is *independent* of sorting on ability, and argue that variation in the cost of monitoring is likely the source of this variation.

APPENDIX 2

**Table A2.1: Distributions of Key Variables after Sample Restrictions**

<b>Level</b>	<b>L 1</b>	<b>L 2</b>	<b>L 3</b>	<b>L 4</b>	<b>L 5</b>	<b>L 6</b>	<b>L 7</b>	<b>L 8</b>
Full	20.02	24.82	27.08	24.76	2.40	0.77	0.12	0.04
Bonus Sample	16.57	22.51	28.51	28.56	2.62	1.04	0.15	0.04
Bonus, Perf Sample	15.25	23.43	29.80	29.45	1.48	0.60	.	.
<b>Performance</b>	<b>Rate=1</b>	<b>Rate=2</b>	<b>Rate=3</b>	<b>Rate=4</b>	<b>Rate=5</b>			
Full	29.32	48.11	18.15	2.21	2.21			.
Bonus Sample	28.15	49.16	21.29	1.29	0.01			.
Bonus, Perf Sample	28.15	49.16	21.29	1.39	0.01			.
<b>Education</b>	<b>HS</b>	<b>Bach</b>	<b>Masters</b>	<b>PHD</b>				
Full	35.28	39.67	21.72	3.34				.
Bonus Sample	34.74	39.11	22.42	3.73				.
Bonus, Perf Sample	36.52	39.54	20.56	3.38				.
<b>Age</b>	<b>Mean</b>	<b>Std Dev</b>	<b>Min</b>	<b>Max</b>				
Full	40.65	9.65	22	71				.
Bonus Sample	40.64	9.45	22	71				.
Bonus, Perf Sample	40.92	9.48	22	69				.
<b>Sample Size</b>	<b>Obs</b>							
Full	46,930							
Bonus Sample	20,679							
Bonus, Perf Sample	13,421							

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**CHAPTER 3:  
RISK OF DISCLOSURE IN SYNTHEHTIC DATA:  
RE-IDENTIFICATION EXPERIMENTS USING THE SIPP-PUF**

**3.1. Introduction**

Any agency that releases public data must protect the privacy of the individuals in the data from users with malicious intent (“intruders”, hereafter). Privacy and confidentiality can be compromised when intruders link a public database with sensitive information to an external file which identifies the individuals in the public file; an activity known in the literature as record linking or re-identification. To protect their data, agencies typically use techniques known as “masking”, which distort the underlying data in a manner sufficient to ensure re-identification is unlikely but not sufficient enough to destroy the analytic validity of the underlying data (Fuller (1993)). A problem with such masking techniques is that they demand that users of the data implement non-standard analytical techniques that are difficult to use in standard statistical software. Further, there is a fundamental tradeoff: the more data released the higher the risk of disclosing confidential data, however the higher the utility of the data for the public (Duncan, Stokes, Keller-Mcnulty (2003)).

The focus of this paper is the ability of a new masking technique, synthetic data, to eliminate the threat of re-identification, or record-linking. Synthetic data was originally proposed by Rubin (1993). His idea was that by using Bayesian methods to estimate the posterior predictive distribution of a data set, it is theoretically possible to create a synthetic data set that has the same properties as the original data set. The theory is a broader application of the technique of multiple imputation used in the completion of missing data (Rubin (1987)). The promise of the theory is threefold: it can maintain the analytical utility of the data, it does not demand special knowledge of non-standard statistical techniques from users, and it can mitigate confidentiality

concerns because it is completely synthetic. At this point, however, these benefits of synthetic data are still un-quantified.

As such, the goal of this paper is to quantify the risk of disclosure in synthetic data by conducting re-identification experiments on a uniquely rich synthetic data set: SIPP/SSA/IRD Public Use Beta File Version 4.1 (hereafter known as the SIPP synthetic beta file). In a long-standing research project joint with SSA and IRD, a group of economists at the US Census Bureau have been working on synthesizing an enriched version of the SIPP that contains both administrative earnings and benefit records provided by SSA and the IRS.<sup>55</sup> In terms of synthetic data projects, this is by far the most ambitious project attempted in the literature. The SIPP synthetic beta file attempts to maintain the univariate and bivariate relationships of hundreds of variables, both continuous and categorical, with unique and complex interdependent relationships. If synthetic data are to become a widespread, feasible method for protecting large-scale, public, micro-data, learning as much as we can about the SIPP project is important.

In addition to exploring the risk of disclosure in the SIPP synthetic beta file, a secondary goal of the paper is to compare the effectiveness of different re-identification techniques, in particular distance-based and probabilistic re-identification. Although there has not been sufficient research exploring which re-identification measures are most effective, the comparison between distance-based re-identification and probabilistic based record-linkage has been considered.<sup>56</sup> The literature has found that the two methods achieve similar results, with one anomalous experiment suggesting that mahalanobis-based distance linking is superior to probabilistic linking. In this paper I contribute to this work by comparing the results

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<sup>55</sup> Abowd, Stinson, and Benedetto (2006)

<sup>56</sup> Domingo-Ferrer and Torra (2001, 2002, 2003), Domingo-Ferrer, Abowd and Torra (2006), and Domingo-Ferrer, Torra, Sanz and Sebe (2006).

of mahalanobis distance and probabilistic linking. Unlike previous work, I find that probabilistic methods achieve better results. That is, they are more effective from the intruder's point of view. One difference between my study and previous experiments is the number of matching variables. Most experiments typically use a handful of variables (less than 20), while my experiments use 171 matching variables. In a set of experiments where I vary the number of matching variables, I find evidence that probabilistic-linking benefits more from extra matching variables than distance linking. When less than 20 matching variables were used some distance metrics outperform the probabilistic method, but as I add variables the probabilistic method becomes superior.<sup>57</sup>

Along with having direct implications for statistical agencies like the Census Bureau, the paper fits into a growing literature on record linking and confidentiality. As Feinberg (2006) discusses, with the expansion of the internet the threat against privacy and confidentiality is as important as ever. A sophisticated intruder can now use linking methods to gather huge amounts of private data about an individual. One particularly strong example of this possibility was noted by Feinberg: "41 graduate students in a computer security course at Johns Hopkins University... working with a strict requirement to use only legal, public sources of information... set out to vacuum up not just tidbits on citizens of Baltimore, but whole databases: death records, property tax information, campaign donations, occupational license registries. They then cleaned and linked the databases they had collected... Each group could spend no more than \$50.... Several groups managed to gather well over a million records, with hundreds of thousands of individuals represented in each database."<sup>58</sup> As such, continued understanding of re-identification is increasingly important.

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<sup>57</sup> Tendrick (1992) discusses how the probability of re-identification varies with the number of matching attributes, but within the context of different re-identification methods than explored here. He shows that as the number of matching variables gets large, re-identification should become exact.

<sup>58</sup> From "Personal Data for the Taking," by Tom Zeller Jr., *The New York Times*, May 18, 2005.

The paper proceeds as follows. Section 3.2 outlines the disclosure framework, describing the “intruder’s” behavior. Section 3.3 outlines the two re-identification methods used in the paper. Section 3.4 discusses the data I use in my experiments, how I implement the experiments, and the results. Section 3.5 concludes.

### **3.2. Disclosure Model**

In this section I formalize the disclosure framework. I follow the framework proposed by Domingo-Ferrer, Abowd and Torra (2006) and the notation originally used in Fellegi and Sunter (1969). Consider a publicly released micro data file  $F$ . The file has a set of variables, or attributes, which can be separated into the following four (non-mutually exclusive) groups: identifiers, quasi-identifiers, confidential attributes, and non-confidential attributes. Identifiers are variables that uniquely identify each record. This could be a SSN (social security number), for example. Quasi-identifiers are variables that can jointly identify *some* of the records uniquely. Age, sex and location could act in such a way if, for example, there is only one 99 year old male who lives in Ithaca, NY on the file. Confidential attributes are things like income, health, political affiliation, etc.... Finally, non-confidential attributes are things like sex and age. Note, that typically quasi-identifiers are non-confidential attributes.

In what follows, I assume that there exists an intruder who has access to an external file denoted,  $A$ , which is a sample of  $N$  records from some population  $P$ . Further, I assume he has access to a publicly released sample  $B$ , which consists of the same  $N$  records from  $P$  but has been “masked” by the agency which released it and contains some extra confidential variables not on  $A$ . It is these extra attributes that the intruder is interested in linking to the sample members in his  $A$  file.

I assume that  $A$  has a unique ID variable and a set of unmasked attributes (possibly non-confidential or confidential). I assume the agency has removed the ID

variable from B, but that it has a set of shared attributes that overlap with A, some of which have been masked. We will call the shared attributes Y if they are unmasked and Y' if masked, and assume there are L unmasked attributes and K masked attributes. As noted above, the risk of disclosure comes from the fact that B also contains some confidential attributes *not* on A. We call these attributes X. Thus, the goal of the intruder is to correctly match the common attributes from A to B so that he can learn X for the records in his file.<sup>59</sup>

In what follows I consider two ways in which he can do this: probabilistic re-identification on the shared attributes, and distance re-identification on the shared attributes. An important point to note in what follows is that the intruder knows there is exactly one correct match on B for every record on A. Thus, the intruder's goal is to select the *single* record he thinks is most likely to match.

### **3.3. Re-Identification Framework**

In this section I consider two matching techniques: probabilistic and distance matching. Before presenting the formal definitions of the two re-identification techniques, I discuss the measure of disclosure risk I use in my experiments. I argue that the probability that the closest or best-scoring record is a true match is the most relevant measure of disclosure risk for the disclosure framework discussed in Section 3.2. In what follows, I will continue with the notation introduced in Section 3.2.

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<sup>59</sup> Pass (1988), Yancey, Winkler and Creecy (2002), and Reiter (2005) all discuss similar frameworks in which risk of disclosure is estimated by modeling the behavior of an intruder who wants to link records from an external file to the file to be protected.

### A) Measure of Disclosure Risk

The probabilistic record linking framework is outlined in the seminal work by Fellegi and Sunter (1969).<sup>60</sup> In short, probabilistic matching starts by defining a *comparison vector* for each set of potentially matched records. The comparison vector captures the agreement pattern across all common attributes Y for two given records in the A and B file. The inference framework for declaring a match then treats the comparison vector as a random variable and relies on the conditional probability of two records being (or not being) a match given their comparison vector. If this conditional probability is high enough, then the record is declared a match, if it is low enough then the record is declared a non-match, and if it is in the “middle” then the decision is left to human judgment (clerical decision).<sup>61</sup> In practice, the comparison vector is mapped into a “matchscore.” Each attribute is given an agreement and disagreement weight depending on how well it distinguishes matches from non-matches. Under the assumption of conditional independence, these weights are then summed across all attributes to get the matchscore. The optimal decision rule is then expressed in terms of two thresholds for the matchscore. If the matchscore is above the upper threshold a match is declared, if below a non-match is declared, if in the middle it is left in the clerical region.

Distance matching has a slightly simpler framework. First, the user defines a metric that measures distance on a vector space (Euclidean distance for example). Then for each record in the A file, the distance between it and all potential matches in the B file are computed, treating each record as a vector. The closest record is then declared the match.

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<sup>60</sup> See Winkler (1995) and Winkler (2004) for a review of the record linkage literature and Jaro (1989) or Abowd and Vilhuber (2005) for applications.

<sup>61</sup> The key result from Fellegi and Sunter is that the optimal decision rule is the one that “minimizes the probability of failing to make a positive disposition.” That is, the optimal rule minimizes the clerical region.

As such, the decision rules for the two methods are quite different. The distance matching yields a single “best” candidate for a match, while probabilistic matching yields a set of records that are highly likely to be matches and a set that are highly likely to be non-matches. Given the disclosure framework described above, in which the intruder *knows* that there is exactly one match for every record, the threshold decision rule used in probabilistic matching is not very natural. If the intruder must ultimately choose one record as a match, he will want to choose the *single* record with the highest probability of being a match, rather than *a set* of records that cross a probabilistic threshold of being a match.

In what follows, then, I treat the matchscore generated by the probabilistic method as analogous to a distance metric. That is, instead of implementing a threshold decision rule as advised by Fellegi and Sunter’s work, I simply declare the record with the highest matchscore a match. Along with being more consistent with the intruder’s objective, this decision rule also makes the comparison between the probabilistic matching and the distance matching straightforward. For both methods I compute the same measure of risk of disclosure: the probability that the “closest” record (or the record with the highest matchscore) is the true match.

## B) Re-Identification Methods

I now formalize the measures used in the re-identification experiments. Before formally defining Mahalanobis distance, I first define some notation. Let  $\alpha$  be the  $K \times 1$  vector of the common attributes  $Y'$  that have been masked in the B file from an observation in the A file and let  $\beta$  be the analogue for the B file. These  $K$  attributes will be known as the matching attributes hereafter. Given this notation, we define a pair of potentially matched records as  $(\alpha, \beta)$  in the comparison space  $\Gamma$ . The Mahalanobis distance for the pair of records  $(\alpha, \beta)$  is defined as follows:

$$(1)d(\alpha, \beta) = (\alpha - \beta)' [\text{Var}(A) + \text{Var}(B) - 2\text{Cov}(A, B)]^{-1} (\alpha - \beta)$$

I consider 3 specific cases of the general Mahalanobis distance. In the first case we assume that the intruder can properly calculate the  $\text{Cov}(A, B)$ . We denote this distance MAHA1. In the second case we assume that the  $\text{Cov}(A, B) = 0$ . This is equivalent to assuming that we do not know how to link the observations across the file. We denote this distance MAHA2. Lastly we transform all of the matching variables in the A and B files to  $N(0,1)$  variables. Call the transformed file  $A'$  and  $B'$ . We then calculate the Euclidean distance ( $\text{Cov}(A, B) = I$ ) between  $A'$  and  $B'$ . We denote this metric NORM EUCL.

An important point to note is that each of the methods standardizes the distance in some way to ensure that distances across different attributes are comparable. The first two metrics use the inverse covariance matrix, and the last one transforms all attributes to the same scale of standard normal. Given these 3 definitions, the distance based re-identification proceeds simply by calculating the distance to every point in the A file for a given point in the B file, and declaring the closest a match.

I now formally define the matchscore from the probabilistic framework. The matchscore is a function of two things: the agreement pattern across the matching attributes of two records and the M and U probabilities for the matching attributes. First, I define the agreement rule for each matching attribute. Then I define the M and U probabilities.

Consider a pair of records  $(\alpha, \beta)$ . The agreement vector for the K matching attributes of a pair of records  $(\alpha, \beta)$ ,  $\gamma(\alpha, \beta)$ , is a  $K \times 1$  vector of 0's and 1's. If  $\alpha$  and  $\beta$  "agree" on attribute k then the agreement vector has a value of 1 in the  $k^{\text{th}}$

dimension. If  $\alpha$  and  $\beta$  “disagree” on attribute  $k$  then the agreement vector has a value of 0 in the  $k^{\text{th}}$  dimension. I define “agreement” as follows:

*Categorical Attribute:* Consider a *categorical* attribute  $k$  for two records  $\alpha$  and  $\beta$ . Let  $\alpha_k$  be the value of attribute  $k$  for record  $\alpha$ . Let  $\beta_k$  be the value of attribute  $k$  for record  $\beta$ . If  $\alpha_k = \beta_k$  then records  $\alpha$  and  $\beta$  are said to “agree” on attribute  $k$ . Otherwise they “disagree.”

*Continuous Variables:* Consider a *continuous* attribute  $k$  and two records  $\alpha$  and  $\beta$ . Let  $\alpha_k$  be the value of attribute  $k$  for record  $\alpha$ . Let  $\beta_k$  be the value of attribute  $k$  for record  $\beta$ . If  $(\alpha_k - \beta_k) / \alpha_k \leq p$  in  $(0,1)$  then records  $\alpha$  and  $\beta$  are said to “agree” on attribute  $k$ . Otherwise they “disagree.”

For categorical attributes agreement requires that the values of the attributes in the two files agree exactly. For continuous variables agreement demands that the values of the attributes in the two files are within  $p$  percent of each other (Jaro (1989)).

I now turn to the  $M$  and  $U$  probabilities. First, note that all possible matches in the  $A$  file and  $B$  file can be split into two groups:  $M$  and  $U$ . If a pair of records  $(\alpha, \beta)$  is a match then they are in the set  $M$ . If they are not a match then they are in the set  $U$ . Because all pairs of records  $(\alpha, \beta)$  are either a match or a non-match, sets  $M$  and  $U$  are exhaustive and mutually exclusive. Given these definitions we define the  $M$  and  $U$  probabilities for attribute  $k$  as follows:

$$M_k = P(\gamma_k(\alpha, \beta) = 1 \mid (\alpha, \beta) \text{ in } M)$$

$$U_k = P(\gamma_k(\alpha, \beta) = 0 \mid (\alpha, \beta) \text{ in } U)$$

In words,  $M_k$  is the probability that a pair of records  $(\alpha, \beta)$  agree on attribute  $k$ , given that the pair is a match.  $U_k$  is the probability that a pair of records  $(\alpha, \beta)$  agree on attribute  $k$ , given that the pair is *not* a match.

The matchscore,  $MS$ , is a function of the agreement vector and the agreement and disagreement weights (formally,  $\ln[M_k/U_k]$  and  $[\ln(1-M_k)/(1-U_k)]$ ) which determine the distinguishing power of each attribute. Formally we have:

$$(2) MS = \sum_k \gamma_k(\alpha, \beta) \ln\left(\frac{M_k}{U_k}\right) + (1 - \gamma_k(\alpha, \beta)) \ln\left(\frac{1 - M_k}{1 - U_k}\right)$$

In words, if the pair  $(\alpha, \beta)$  agree on an attribute then the matchscore increases by the agreement weight. If the pair  $(\alpha, \beta)$  disagree on an attribute then the matchscore increases by the disagreement weight. Because the agreement weight is typically positive ( $M_k > U_k$ ) and the disagreement weight is typically negative, agreement on an attribute increases the matchscore while disagreement decreases the matchscore. Further, variables that have more distinguishing power contribute more to the matchscore. In what follows, I will have to first compute  $M_k$  and  $U_k$  and then compute the matchscore for all possible pairs  $(\alpha, \beta)$ .

### 3.4. Data, Implementation and Results

In this section I discuss the data, the implementation of the experiments in SAS, and the results of the experiments. My experiments link 4 different synthetic data sets to the original, unperturbed data. I briefly discuss the methods used to generate the synthetic data, before discussing the results.

## A) Data

In my re-identification experiments I link 4 synthetic implicates back to the original underlying data file from which the synthetic files were generated. The original file is the “Gold Standard” SIPP file (Survey of Income and Program Participation). The Gold Standard file consists of all individuals who responded to the 1990-1993 and 1996 SIPP panels who would have been at least 18 years old as of January 1<sup>st</sup> 2000. The file includes standard SIPP demographic, economic, and public assistance information. The SIPP information was then linked to 4 data sources provided by Social Security Administration (SSA): SSA’s summary earnings record (SER), SSA’s detailed earning record (DER), SSA’s master beneficiary record (MBR), and the Cenus Numident.<sup>62</sup> The SER and DER provide longitudinal data on wage and salary, the MBR provides a longitudinal history of the type and amount of SSA benefits, and the Numident provides administrative birth and death dates. An individual was eligible to be included in the Gold Standard file if they met one major criterion. The individual must have been at least 15 years old at the time of the second wave of their SIPP survey. The Gold Standard file came into existence as part of a joint Census/SSA/IRS project aimed at improving the quality of the SIPP public use data file by linking administrative earnings and benefits information from SSA to SIPP variables.

The four synthetic implicates that I link to the Gold Standard file are a subset of 16 synthetic implicates that were created as a potential public use file and are now studied as SIPP synthetic beta file. The synthetic implicates attempt to replicate all univariate, bivariate and some multivariate relationships in the Gold Standard file, while masking all variables so that privacy is maintained. As noted in the

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<sup>62</sup> It should be noted that although SSA provided the Census Bureau with the earning records, IRS is technically the custodian of the data.

introduction, synthetic data is an application of the multiple imputation framework originally proposed by Rubin, and is created by estimating the posterior predictive distribution (PPD) of the original data, and taking draws from that distribution. The technique used to estimate the PPD of the SIPP and create the 16 synthetic implicates is known as sequential regression. The method proceeds as follows: First the data is broken into groups based on key demographic and economics variables referred to as by-variables. Within each group, for each variable  $Y_i$  that is masked, we estimate the PPD from a Bayesian linear regression model with  $Y_i$  as the dependent variable and  $X$  and all other  $Y_j, j \neq i$ , as independent variables. To introduce variation across the implicates, four different sets of by-variables were used, yielding 4 different “specifications” of the PPD. The first two sets of by-variables consist predominantly of demographic variables, while the second two consist predominantly of economic variables. For each of the specifications there were then 4 implicates generated for a total of 16 implicates.<sup>63</sup> Given that there is not that much variation across implicates within a specification and that the re-identification experiments are computationally expensive, in my experiments I link the Gold-Standard file to only 1 implicate from each specification, for a total of 4 implicates.

Finally, it is important to note that the experiments are not purely academic: they capture a *real world* risk of disclosure. Because the Gold Standard SIPP file is already in the public domain, once the SIPP synthetic beta file is released there exists the *real* possibility that an intruder could link it back to the Gold Standard and re-identify its source records. Further, because the SIPP synthetic beta file includes variables not previously available for the SIPP sample (the SSA/IRS administrative benefit and earnings data) there is a significant amount of *new* information exposed to intruders. Confirming that the risk of disclosure is small is thus very important.

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<sup>63</sup> See Abowd, Stinson, and Benedetto (2006) for further discussion of the synthetic implicates.

## B) Implementation

Using the notation above, the Gold Standard file plays the role of the A file, while the implicates of the SIPP synthetic beta file play the role of the B files in the re-identification tests. I use 173 common attributes across the two files (See Appendix 3 for list) in my experiments. Two variables are unmasked and thus identical across the two files: marital status and sex. I use these unmasked variables as blocking variables in my analysis.<sup>64</sup> The other 171 variables are used as matching variables. Relative to previous re-identification experiments in the literature, this is a large number.

The calculation of the re-identification rates proceeds in two main steps. First, I split the data into smaller, manageable blocks, and then calculate distances and probabilistic matchscore *within the blocks*. This is a standard procedure used in the literature to ease the computation burden (Jaro (1989), Abowd and Vilhuber (2005)) of the problem. The blocking is done as follows. I assume that the intruder will only consider possible matches amongst record pairs that share identical values on the blocking variables, marital status and sex, as he knows these variables are un-masked. This generates 8 blocking groups. Only records within the same blocking are compared.

The 8 blocking groups, however, are still relatively large (some have more than 70,000 observations). As such, to further ease the computation burden, I split the blocking groups into even smaller groups based on actual birth date. Because I use actual rather than synthetic birth date, this process forces the true match to be in the new, smaller blocking group. Formally, define the 8 blocking groups generated by interacting marital status and sex as G1-G8, and let them have  $N_{G1}, \dots, N_{G8}$  observations respectively. To split them into smaller groups, I first sort them by

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<sup>64</sup> Two other variables on the synthetic implicates were also left un-masked: “type of benefit at time of initial benefit receipt” and “type of benefit in April 2000.” They were not used in the experiments because they are not on the Gold Standard file, and thus not available to a potential intruder.

actual birth date (one of the strongest matching variables). If a group  $G_i$  has more than  $N_C$  observations it is then split into  $J = \text{ceiling}(N_{G_i}/N_C)$  smaller groups of size  $N_C$ , starting with the first observation and continuing until all  $N_{G_i}$  observations have been assigned to a group, with the last group containing the remaining  $N_{G_i} - (J-1)N_C$  observations.

The calculation of the distance metrics and the probabilistic matchscore are then implemented in SAS *within* the blocking groups. The calculation of the distance metrics is straightforward. All of the elements in equation (1) have direct matrix representations in SAS. Thus equation (1) can be computed directly using PROC IML in SAS. In order to calculate the matchscore I first must estimate the  $M_k$  and  $U_k$  for each matching attribute  $k$ . Because I know the true matches across the datasets, I can separate all possible matches  $(\alpha, \beta)$  into the sets  $M$  and  $U$  which exhaust the space of possible matches. Once I have done this, the  $M_k$  for a given attribute is estimated as the frequency of agreement for that attribute across all records  $(\alpha, \beta)$  in the set  $M$ , while the  $U_k$  for a given attribute is estimated as the frequency of agreement for that attribute across all records  $(\alpha, \beta)$  in the set  $U$ . With  $M_k$  and  $U_k$  estimated for all  $K$  matching variables I calculate the matchscore for all possible matches  $(\alpha, \beta)$  in a SAS data step. After the calculations are done I then search for the closest record in the case of the distance re-identification and the highest matchscore in the case of the probabilistic re-identification. In the next section I present the results. For a more detailed discussion of how the distance and probabilistic re-identification was implemented in SAS, see the appendix.

It is important to note that the estimates of the rates of re-identification generated by this method are *upper bounds* for the real rates that an intruder would achieve. Because the intruder would not have access to actual birth date, he would not be able to create the smaller blocking groups described above (he would only be able

to use the blocking groups based on marital status and sex). As such, by creating smaller blocking groups based on actual birth date I am eliminating many potential non-match records that the intruder would not be able to eliminate. This process will necessarily inflate my re-identification rates relative to the intruder's. Again, the only reason such a method is implemented is to decrease the computation time. If computing power was unconstrained, such a method would not be implemented.

Finally, because the parameter of interest is in fact the *true* rates of re-identification that an intruder could achieve, in the results that follow I am careful to translate the *upper bounds* generated by the smaller blocking groups into the true rates. To do this, I run several different experiments varying  $N_C$  to get a sense of how sensitive the upper bounds are to the size of the groups, which, in turn allows me to make inferences about the true re-identification rates.

### C) Results

I now present the results of the re-identification experiments. The parameter of interest, as noted above, is the probability that the closest or best scoring record is a true match, as it captures the risk that an intruder would successfully re-identify a record. In the experiments, it is estimated by the percent of closest or best scoring records that are a true match. As discussed above, because of the methods used, the estimates presented below are *upper bounds* of the parameter of interest. The discussion is careful to address this point.

The first concern is the overall risk of re-identification. As noted in the introduction, synthetic data is a relatively new masking technique. Thus, quantifying its ability to protect the true data is important. Table 3.1 presents the best estimates of the overall risk of re-identification using all 4 methods. The second through fourth columns in Table 3.1 present results for the three distance-based methods using the

full set of 171 matching variables. As you can see, using a maximum blocking group size of 10,500, the three distance-based methods have a very small probability of re-identification: Normalized Euclidean distance successfully re-identifies only 1.77 percent of records on average across implicates, while the first Mahalanobis metric re-identifies 1.09 percent of records on average across implicates and the second Mahalanobis metric re-identifies only .66 percent of records on average across implicates.<sup>65</sup> I conclude that the distance-based re-identification poses little threat.

We now turn to the probabilistic re-identification results. Because the probabilistic re-identification is more computationally burdensome than the distance-based re-identification, extending the maximum blocking group size to 10,500 while maintaining 171 matching variables would take an estimated 3 months of computation time.<sup>66</sup> As such, the last two columns of Table 3.1 present the re-identification rates when the maximum blocking group size is 1,050 and 2,100 respectively. As you can see, with a maximum blocking group size of 2,100 only 11.38 percent of the records were re-identified in implicate 1, while with a maximum blocking group size of 1,050 15.48 percent of records were identified (15.18 on average across implicates). Although these results are significantly larger than the results from the distance re-identification, they are very sensitive to maximum blocking group size. This sensitivity suggests that 11.38 and 15.48 are likely very “soft” upper bounds. That is, if the group size were pushed to what an intruder would face (groups of over 70,000 records for half of the data file), the rates of re-identification would fall dramatically.

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<sup>65</sup> Reiter (2003) shows that the proper point estimate for a parameter of interest in the case of multiple synthetic implicates is the average across implicates.

<sup>66</sup> This estimate is based on the following calculation: the computation time for the probabilistic re-identification routine when the maximum blocking group size is 1,050 is approximately 22 hours. Given that the computation time is quadratic in the maximum blocking group size, increasing the maximum blocking group size to 10,500 should increase the computation time by approximately 100 fold.

**Table 3.1: Percent of Records Successfully Re-Identified and Sensitivity of Upper Bound**

**Notes:** The rows “Implicate 1” – “Implicate 4” present the results for the four implicates respectively. Each column reports the re-identification results for one of the 4 methods (three distance-based, and one probabilistic). “Group Size” is the maximum blocking group size used in the experiment.

<b>Implicate</b>	<b>Maha1: Cov(A,B)≠0</b>		<b>Maha 2: Cov(A,B)=0</b>		<b>Normalized Eucl Dist</b>		<b>Probabilistic Matchscore</b>	
<b><u>171 Matching Variables</u></b>								
Implicate 1	1.00	3.02	0.63	1.88	1.68	5.24	11.38	15.48
Implicate 2	1.13	3.39	0.67	2.00	1.81	5.43	.	14.87
Implicate 3	1.07	3.21	0.65	1.95	1.83	5.49	.	14.98
Implicate 4	1.14	3.42	0.67	2.01	1.77	5.31	.	15.37
Average	1.09	3.26	0.66	1.96	1.77	5.37	.	15.18
<b>Group Size</b>	10,050	1,050	10,050	1,050	10,050	1,050	2,100	1,050
<b><u>18 Matching Variables</u></b>								
Implicate 1	.	.	.	.	.	.	0.20	1.55
<b>Group Size</b>	.	.	.	.	.	.	20,000	1,050

To explore the sensitivity of the upper bound further, the second panel of Table 3.1 presents results for the probabilistic re-identification with a limited set of matching variables (only 18), but with larger group sizes. The key result to notice is that when the maximum blocking group size changes from 1,050 to 20,000 the re-identification rates are almost 8 times smaller. If a similar relationship holds at 171 variables, an intruder would only be able to re-identify less than 2 percent of records. A second estimate of the sensitivity of the upper bounds comes from the distance matching results presented in columns 2-4 of Table 3.1. In particular, note that when the maximum blocking group size decreases from 10,500 to 1,050 the rates of re-identification roughly triple. If the same relationship holds for probabilistic matching then roughly only 5 percent of records would be successfully identified when the maximum blocking group size was pushed to 10,500.

In conclusion, the overall risk of re-identification seems very small. The synthetic implicates, while designed to maintain the statistical properties of the underlying data, appears to offer very little risk of re-identification (less than 2 percent for the distance based methods and less than 2-5 percent for the probabilistic methods). That said, an interesting secondary pattern appears to emerge in Table 3.1; when the maximum blocking group size is held constant and all 171 matching variables are used, the probabilistic re-identification seems to outperform the distance-based matching. Table 3.2 explores this issue further.

Table 3.2 presents the percent of records successfully re-identified for both distance-based and probabilistic methods when the maximum blocking group size is 1,050 and the full set of 171 matching variables are used. The “Ratio” column captures the magnitude difference in the success of the probabilistic matchscore

**Table 3.2: Percent of Records Successfully Re-Identified:  
Distance-Based vs. Probabilistic Re-Identification Methods**

**Notes:** The “Ratio” column presents the ratio of the percent of successfully re-identified records using the probabilistic matchscore and the percent of successfully re-identified records using the *best* performing distance metric. “Group Size” is the maximum blocking group size used in the experiment.

<b>Implicate</b>	<b>Maha: Cov(A,B)≠0</b>	<b>Maha: Cov(A,B)=0</b>	<b>Normalized Eucl Dist</b>	<b>Probabilistic Matchscore</b>	<b>Ratio</b>
Implicate 1	3.02	1.88	5.24	15.48	2.95
Implicate 2	3.39	2.00	5.43	14.87	2.74
Implicate 3	3.21	1.95	5.49	14.98	2.73
Implicate 4	3.42	2.01	5.31	15.37	2.89
Average	3.26	1.96	5.37	15.18	2.83
<b>Group Size</b>	1,050	1,050	1,050	1,050	

relative to the best performing distance metric (Normalized Euclidean Distance in this case). As is clear, the distance-metrics perform significantly worse. On average, the probabilistic methods successfully re-identify almost 3 times (2.83) as many records.

In the context of previous re-identification experiments which have found that the mahalanobis distance and the probabilistic matching perform similarly (Domingo-Ferrer and Torra (2001, 2002, 2003), Domingo-Ferrer, Abowd and Torra (2006), and Domingo-Ferrer, Torra, Sanz and Sebe (2006)), the results are quite striking. One major difference between the experiments presented here and previous experiments in the literature is the number of matching variables; most previous studies have used less than 20, while the results here are based on 171.

To address this issue, Table 3.3 explores the possibility that the results in Table 3.2 are driven by the number of matching variables, by presenting re-identification rates for each of the four methods for the cases of 10, 20, and 30 matching variables. Because there is minimal variation across implicates and computation is expensive, I conduct this experiment only on Implicate 1. The limited set of matching variables are chosen based on agreement weight,  $\ln(M_k/U_k)$ . I rank all 171 matching variables from largest to smallest agreement weight and then include the 10, 20, or 30 matching variables with the highest agreement weights. As you can see from Table 3.3, an

**Table 3.3: Percent of Records Successfully Re-Identified for Limited Sets of Matching Variables**

**Notes:** The “Ratio” column presents the ratio of the percent of successfully re-identified records using the probabilistic matchscore and the percent of successfully re-identified records using the *best* performing distance metric. “Group Size” is the maximum blocking group size used in the experiment.

<b>Implicate</b>	<b>Maha: Cov(A,B)≠0</b>	<b>Maha: Cov(A,B)=0</b>	<b>Normalized Eucl Dist</b>	<b>Probabilistic Matchscore</b>	<b>Ratio</b>
10 Match Vars	.68	.61	1.01	.40	.40
20 Match Vars	1.91	1.82	3.43	3.13	.91
30 Match Vars.	2.82	2.37	4.91	5.53	1.13
<b>Group Size</b>	1,050	1,050	1,050	1,050	1,050

interesting pattern emerges. When there are very few matching variables the distance metrics are much more competitive with the probabilistic matchscore. In fact the normalized Euclidean distance actually outperforms the probabilistic matchscore when only 10 and 20 matching variables are included. However, as more variables are added the probabilistic matchscore begins to perform better. By the time we get to 171 matching variables, as presented in Table 3.2, the probabilistic matchscore is almost 3 times as successful.

### **3.5. Conclusion**

In summary, the results of my experiments show that, overall, the synthetic data technique does a good job protecting the data. An intruder using the distance-based methods would identify no more than 2 percent of records. An intruder using probabilistic methods would identify less than 2-5 percent of the records. The most interesting result is the disparity across the two methods. When the maximum blocking group size is the same and 171 matching variables are used, the probabilistic matchscore successfully re-identifies almost 3 times as many as records as the distance-based methods, a result that goes against previous experiments that compare the two methods. That said, I find that the discrepancy between probabilistic re-identification and distance-based re-identification varies quite significantly with the number of matching variables used. Specifically, when there are very few matching variables (less than 20) the methods achieve similar results, with the distance methods sometimes finding more success. However, as I add more matching variables, the probabilistic method starts to dominate. Given that previous studies comparing the two methods use fewer matching variables, these second set of results seem to reconcile my findings with the broader literature.

## APPENDIX 3

In this appendix I discuss the software written in SAS used to do the probabilistic and distance-based re-identification. I start with the probabilistic software.

### A) Probabilistic Re-Identification Software in SAS

Let A\_FILE be the file to which the B\_FILE is matched. In my case the A\_FILE is the SIPP Gold Standard file. The B\_FILE is one of the synthetic implicates. These two files each have 263, 673 observations. Following the notation in the text, let  $\alpha$  be the K dimensional vector of matching variables for a single observation the A\_FILE, and let  $\beta$  be the K dimensional vector of matching variables for a single observation the B\_FILE.

Let BVARS be the set of blocking variables and MVARs be the set of K matching variables (MVARs\_1-MVAR\_K). In my applications K varies depending on the experiment and BVARS= MARITALSTAT and BLACK. As such the interaction of BVARS generates 8 blocks.

The software begins by splitting the A\_FILE and the B\_FILE into the 8 smaller files associated with the 8 blocking groups. Call the blocking groups from the A\_FILE and B\_FILE A\_1-A\_8 and B\_1-B\_8 respectively. Let  $N_g$  be the number of observation in each blocking group. Each pair of blocking groups (A\_g, B\_g) for g=1 to 8 are then fed into the probabilistic re-identification routine (PRR, hereafter).

Depending on the size of the A\_g and B\_g, the PRR will then split A\_g and B\_g into smaller files (chunks) to speed up the processing. In my application, if A\_g and B\_g have more than  $N_g > 1000$  observations, then they will be split into  $J = \text{ceiling}(N_g/1000)$  smaller chunks of 1000, with the last chunk having the remainder  $N_g - (J-1)1000$  observations. Call these J files A\_g\_j and B\_g\_j for j=1 to J. Before the files are split into smaller chunks, they are first sorted on IDVAR. This ensures that each observation in B\_g\_j will have its true match in the corresponding chunk A\_g\_j, and that the selection of the k chunks is independent of the MVARs.

Next, for each chunk  $j$ , the following two files are created:  $M\_AXB\_g\_j$  and  $U\_AXB\_g\_j$ .  $M\_AXB\_g\_j$  contains all possible true matches between  $A\_g\_j$  and  $B\_g\_j$  (1000 observations) while  $U\_AXB\_g\_j$  contains all possible non-match combinations between  $A\_g\_j$  and  $B\_g\_j$  (990,000 observations). More precisely, each observation in  $M\_AXB\_g\_j$  and  $U\_AXB\_g\_j$  is a combination of a record from  $A\_g\_j$  and a record  $B\_g\_j$ . Let  $IDVAR\_B$  be the value of the  $IDVAR$  for the record from  $B\_g\_j$  and let  $IDVAR\_A$  be the value of the  $IDVAR$  for the record from  $A\_g\_k$ . Each observation in  $M\_AXB\_g\_j$  and  $U\_AXB\_g\_j$  can then be written as the following vector:  $(IDVAR\_A, \alpha, IDVAR\_B, \beta)$ . Agreement variables,  $AGR\_1$ - $AGR\_K$ , for each of the  $K$  matching variables are then created. Consider a discrete matching variable,  $MVAR\_D$ . For each observation from  $M\_AXB\_j\_k$  or  $U\_AXB\_j\_k$ , if  $\alpha_D = \beta_D$  then  $AGR\_D = 1$ , else  $AGR\_D = 0$ . Consider a continuous matching variable,  $MVAR\_C$ . For each observation from  $M\_AXB\_j\_k$  or  $U\_AXB\_j\_k$ , if  $Abs[(\alpha_C - \beta_C)/\alpha_C] \leq .05$  then  $AGR\_C = 1$ , else  $AGR\_C = 0$ .

Given the agreement variables,  $AGR\_1$ - $AGR\_K$ , we now calculate the  $M$  and  $U$  probabilities. The  $M$ -probability for  $MVAR\_k$  for chunk  $j$  of block  $g$  is calculated as the frequency of  $AGR\_k = 1$  in  $M\_AXB\_g\_k$ . The  $U$ -probability for  $MVAR\_k$  for chunk  $j$  of block  $g$  is analogously calculated as the frequency of  $AGR\_k = 1$  in  $U\_AXB\_g\_j$ . Note that the  $M$  and  $U$  probabilities are the *exact* probabilities for chunk  $j$  of block  $g$ . The calculations are looped  $K$ -times to give us  $M$  and  $U$  probabilities for each of the  $K$  matching variables. Call them  $M\_PRB\_k$  and  $U\_PRB\_k$  for  $k=1$  to  $K$ .

After calculating the  $M$  and  $U$  probabilities, they are merged onto  $M\_AXB\_g\_j$  and  $U\_AXB\_g\_j$  which are then stacked on top of each other to create the file  $AXB\_g\_j$  which contains 1,000,000 observations: each record  $\beta$  in  $B\_g\_j$  is matched to all 1,000 records in  $A\_g\_k$ . In matrix notation we have:

$$\begin{matrix} [IDVAR\_B & \beta(1) & IDVAR\_A & \alpha(1) & M\_PRB\_1-M\_PRB\_k & U\_PRB\_1-U\_PRB\_k, \\ IDVAR\_B & \beta(1) & IDVAR\_A & \alpha(2) & M\_PRB\_1-M\_PRB\_k & U\_PRB\_1-U\_PRB\_k, \\ \dots & & & & & \\ IDVAR\_B & \beta(1) & IDVAR\_A & \alpha(N_g) & M\_PRB\_1-M\_PRB\_k & U\_PRB\_1-U\_PRB\_k, \\ \dots & & & & & \end{matrix}$$

IDVAR\_B  $\beta(N_g)$  IDVAR\_A  $\alpha(N_g)$  M\_PRB\_1-M\_PRB\_k U\_PRB\_1-U\_PRB\_k]  
 Where  $\beta(n)$  is the K-dimensional vector of matching variable for the  $n^{\text{th}}$  record in the B file and  $\alpha(n)$  is the analogue for the A file.

As is clear, the structure of AXB\_g\_j allows for the calculation of the matchscore for each potential match ( $\alpha(n)$ ,  $\beta(m)$ ) from A\_g\_j and B\_g\_j. After calculating matchscores, I sort AXB\_g\_j by IDVAR\_B and matchscore and output the records with the highest 3 matchscores for each observation  $\beta$  to the file FINAL\_g\_j. An observation in FINAL\_g\_j has the following vector structure:

(IDVAR\_B, IDVAR\_A1- IDVAR\_A3, MATCHSCORE1- MATCHSCORE3)

Where IDVAR\_Ap is the ID of the record from the A\_FILE that has the  $p^{\text{th}}$  highest matchscore out of all possible matches to the record IDVAR\_B, and MATCHSCOREp is the corresponding  $p^{\text{th}}$  highest matchscore.

This process loops through all J chunks for group g, generating J files, FINAL\_g\_1- FINAL\_g\_J, which are all stacked on top of each other to create the file FINAL\_g. FINAL\_g has the same structure as FINAL\_g\_j, but contains  $N_g$  observations.

The overall process then loops through all 8 blocks, creating FINAL, which has the same structure as FINAL\_g\_k but has all 263,673 observations in the original B\_FILE. FINAL can then be analyzed to estimate the probability of re-identification: the probability that the  $p^{\text{th}}$  highest matchscore is a true match.

Following is a step by step picture of the looping algorithm:

- (1) Input A\_FILE and B\_FILE
- (2) A\_FILE and B\_FILE split in G blocking groups. Submit block g to PRR.  $g = 1$ :
  - 1g) A\_g and B\_g split into J chunks to reduce processing time.  $j=1$ :
    - 1j) A\_g\_j and B\_g\_j joined together to create M\_AXB\_g\_j and U\_AXB\_g\_j.
    - 2j) Agreement definitions, M and U probabilities, and matchscores

- calculated for all possible match combinations between  $A_{g_j}$  and  $B_{g_j}$
- 3j) Highest 3 matchscores and corresponding IDVAR's to each record in  $B_{g_j}$  output to  $FINAL_{g_j}$ .
- 4j) If  $j < J$ , iterate  $j$  forward:  $j=j+1$  and return to step 1g). If  $j = J$  then stop.
- 2g) Stack  $FINAL_{g_j}$  for  $j=1$  to  $J$  to create  $FINAL_g$ .
- 3g) If  $g < G$ , iterate  $g$  forward:  $g=g+1$  and return to step 2). If  $g = G$ , then stop.
- 3) Stack  $FINAL_g$  for  $g=1$  to  $G$  to create  $FINAL$ , which contains the ID and matchscore for the records for A with the top 3 matchscores for a given record in B.

## B) Distance-Based Re-Identification Software in SAS

The  $A\_FILE$ , the  $B\_FILE$ , the  $BVARS$  and the  $MVARS$  are the same as the probabilistic case. Like the probabilistic software, the distance-based software begins by splitting the  $A\_FILE$  and the  $B\_FILE$  into the 8 smaller files associated with the 8 blocking groups. Call the blocking groups from the  $A\_FILE$  and  $B\_FILE$   $A_1$ - $A_8$  and  $B_1$ - $B_8$  respectively. Let  $N_g$  be the number of observation in each blocking group. Each pair of blocking groups ( $A_g, B_g$ ) for  $g=1$  to 8 are then fed into the distance-based re-identification routine (DRR, hereafter).

For the sake of processing time,  $A_g$  and  $B_g$  will then be split into smaller files (chunks). In my application, if  $A_g$  and  $B_g$  have more than  $N_g > 10,500$  observations, then they will be split into  $J = \text{ceiling}(N_g/1000)$  smaller chunks of 10,500, with the last chunk having the remainder  $N_g - (K-1)1000$  observations. Call these  $J$  files  $A_{g_j}$  and  $B_{g_j}$  for  $j=1$  to  $J$  and let  $N_{gj}$  be the number of observations in  $A_{g_j}$ . Before the files are split into smaller chunks, they are first sorted on IDVAR. This ensures that each observation in  $B_{g_j}$  will have its true match in the corresponding chunk  $A_{g_j}$ , and that the selection of the  $k$  chunks is independent of the  $MVARS$ .

Before inputting the chunks into the DBR, a few calculations are made. The covariance matrix for all of the matching variables in both the  $A\_FILE$  and  $B\_FILE$  are calculated, and the covariance *across* the  $A\_FILE$  and  $B\_FILE$  is calculated using PROC CORR. Also, a second set of matching variables,  $MVARS\_STD$  are created.

For a given matching variable  $k$ ,  $MVARS\_STD\_k = (MVARS\_STD\_k - MEAN\_k)/STDDEV\_k$ . That is,  $MVARS\_STD\_k$  is simply a transformed version of  $MVARS\_k$ . These variables will be used to calculate the standardized Euclidean distance, while covariance matrices will be needed to calculate the various Mahalanobis distance metrics.

After the necessary calculations and the splitting of the files are completed, each pair of chunks,  $(A\_g\_j, B\_g\_j)$ , are then iteratively input into the DBR. Each pair  $(A\_g\_j, B\_g\_j)$  are first read into matrix form in PROC IML. Define  $MVARS\_A$  as the  $N_{gj} \times K$  matrix of the matching variables for all  $N_{gj}$  records in  $A\_g\_j$ , and  $MVARS\_B$  as the same. Three covariance matrices are also read into PROC IML: Let  $COV\_A$  be the  $K \times K$  covariance of the  $K$  matching variables in the  $A\_FILE$ . Let  $COV\_B$  be the analogue for the  $B\_file$ . Let  $COV\_AB$  be the covariance between the  $A\_FILE$  and the  $B\_FILE$ .

The program then loops through the  $B\_g\_j$  one record at a time, computing the distance between a given record and *every* record in  $A\_g\_j$ , and then keeping the closest 3. More formally, define  $MVARS\_B_i$  as the  $i^{th}$  row of  $MVARS\_B$ . Define  $N\_MVARS\_B_i$  as a  $N_{gj} \times K$  matrix in which  $MVARS\_B_i$  is stacked on itself  $N_{gj}$  times. Define  $K\_1$  as a  $K \times 1$  matrix of 1's. The distance between a given record  $i$  in  $B\_g\_j$  and every record in  $A\_g\_j$  is then computed as follows:

$$D = (N\_MVARS\_B_i - MVARS\_A) \# INV\_COV \# (N\_MVARS\_B_i - MVARS\_A) * K\_1$$

where  $\#$  is matrix element-wise multiplication operator and  $INV\_COV = I_k$  for Euclidean Distance and  $INV\_COV = COV\_A + COV\_B$  and  $INV\_COV = COV\_A + COV\_B - 2 COV\_AB$  for the two mahalanobis distances respectively.

$D$  is a  $N_{gj} \times 1$  matrix which contains the distance between record  $i$  in  $B\_g\_j$  and *every* record in  $A\_g\_j$ . The ID and the distance of the closest three records are then saved. The program then loops through these calculations  $N_{gj}$  times, once for each observation in  $B\_g\_j$ , outputting the 3 closest records. As such for each chunk  $j$  of group  $g$  the final output file,  $FINAL\_g\_j$ , contains  $N_{gj}$  observations with the following vector structure:

$$FINAL\_g\_j = (IDVAR\_B, IDVAR\_A1-IDVAR\_A3, DISTANCE1-DISTANCE3)$$

After each chunk is processed through the DBR, the program then loops to the next chunk. In the end we have  $G \times J$  output files, FINAL\_g\_j, which are stacked on top of each other to create FINAL. FINAL contains one record for each observation in the B\_FILE, and the ID's of the three closest records in the A\_FILE

Following is a step by step picture of the looping algorithm:

(1) Input A\_FILE and B\_FILE

(2) A\_FILE and B\_FILE split in  $C = G \times J$  chunks. Submit chunk  $c$  to PRR.  $c = 1$ :

1c) A\_g\_j and B\_g\_j are read into matrices in PROC IML.

1n) Loop through B\_g\_j one record,  $n$ , at a time.  $n=1$ .

2n) Calculate the distance between record  $n$  in B\_g\_j and every record in A\_g\_j for all distance metrics. Output the 3 closest records from A\_g\_j to FINAL\_g\_j.

3n) If  $n < N_g$  iterate forward:  $n=n+1$  and return to step 1n). If  $n = N_g$  then stop.

2c) If  $c < C$ , iterate  $c$  forward:  $c=c+1$  and return to step 1g). If  $c = C$  then stop.

(3) Stack FINAL\_g\_j for  $g=1$  to  $G$  and  $j=1$  to  $J$  to create FINAL, which contains the ID and distance for the three closest records in A to a given record in B.

#### C) List of Blocking Variables

Marital Status

Male

#### D) List of Artificial Blocking Variables

Actual Birth date

#### E) List of Matching Variables

Weeks with pay: 1990-1999

Weeks worked part time: 1990-1999

Total annual hours worked: 1990-1999

Total family income: 1990-1999

Total personal income: 1990-1999

Total personal earnings: 1990-1999

Family annual poverty threshold: 1990-1999

Family total welfare benefit: 1990-1999

Family welfare participation: 1990-1999  
Public health assistance participation: 1990-1999  
Total benefit from health assistance: 1990-1999  
Health insurance coverage: 1990-1999  
Employer provided health insurance: 1990-1999.  
Total net worth  
Do you own home?  
Home equity  
Non-household wealth  
Marital history  
Marital history event 1 – Marital history event 7  
Age at marital history event 1 - Age at marital history event 7  
More than 3 marriages  
Number of kids under 18  
Survey Birth date  
Death date  
Death date non-missing  
Black  
Hispanic  
Five category education  
Eligible to be asked disability questions  
Disability limit work  
Disability prevents work  
Foreign born  
Time arrived in USA  
Occupation  
Valid occupation code  
Industry  
Valid industry code  
Eligibility for pension questions, age  
Eligibility for pension questions, employment  
Defined contribution plan  
Defined benefit plan  
Weight

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