LYNAH RINK

THE SCIENCE OF THE ICE

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EXECUTIVE SUMMARY

Ice hockey is probably the most popular sports team at Cornell, easily selling out tickets to every game at the start of every year. Although eager fans study the team, plays, opponents, and results, it is doubtful that they pay any attention to the science behind the very ice the game depends upon. Our project seeks to look into the ice at Lynah rink, specifically the heat transfer processes involved in making the ice, maintaining it, and removing it in the off-season. We investigated how much time it would take to prepare the surface for ice hockey, how much time it would take to resurface the ice during a game, and lastly how much time it would take to melt all the ice and remove it during the off-season. We used a one-dimensional geometry with sixteen sections to model the eight layers of ice needed for the rink. Two sections represented one layer, equal to 0.3175 cm. Layers and boundaries were turned on and off depending on the part of the problem that was being solved for. Our initial results showed that it takes about four hours to place eight layers, equivalent to 2.54 cm of ice down on the rink. Conversely, it takes about five minutes for the 2.54 cm of ice to melt with the concrete slab heated. The resurfacing process, needed to be complete in less than fifteen minutes, was found by our model to take about six minutes. This value is subject to change from variation in the heat transfer coefficient and rink temperature, but our analysis found that even high values for these parameters still allowed resurfacing in a maximum of eight minutes. Melting was the least complex situation to model, and we found that it took about five minutes to melt the ice with the concrete heated to 60°C(333.15K).

INTRODUCTION AND OVERVIEW

The average ice rink starts with a slab of concrete.[4] Pipes carrying chilled fluid run through (or in some cases, on top of) this slab. The chilled fluid consists of either salt brine or water mixed with antifreeze, both of which allow the temperature of the slab to get cold enough to freeze the water that will cover it.[4] Most rinks use the salt brine solution, to reduce the environmental impact in the event of a pipe leak.

To prepare the surface for ice hockey, the following procedure is implemented: First, a thin layer of water is sprayed on the surface, which allows the necessary markings for the game to be painted on without staining the concrete. This layer is often dyed white (as in Lynah Rink) so that the markings contrast better.[3] Further layers of water are added until the ice reaches about one inch (2.54 cm) in thickness. The temperature of the ice is maintained from 0 to 4°C, while the temperature of the building is often at 17°C(290 K), with an indoor humidity of about 30%.[3]

During the typical hockey game, the ice is scraped considerably by the ice skates worn by the players. Thus an ice resurfacing machine, called a zamboni, is used to smoothen the top layer of ice in between periods of the game.[4] While scraping off the top layer of ice, the zamboni simultaneously deposits a layer of hot water to make a new layer of smooth ice. The hot water loosens the crystal structure of the ice below it, and thus allows the new layer to form a solid bond with the previous layer, instead of a separate one that chips off easily. The temperature of this hot water was cited by one source to be about 60°C(333.15K), although the number varies slightly among different ice makers, who each have their own 'recipe' when it comes to ice-making.[1]

Most collegiate ice rinks maintain the rink even during the off-season. However some rinks, such as those municipally owned, melt it off in the summer months. This is usually due to expense of maintenance and lack of use. With the ice removed, the rink is often used to host other venues such as concerts, basketball games, circuses and ceremonies. In order to melt the ice, the chilled fluid in the pipes is replaced with hot fluid, and the melted water is eventually drained away.

We would like to model the three afore-mentioned areas of ice rink maintenance and answer some of the listed questions:

1. *Creation of the ice:* How long would it take to freeze a layer of water with the proper chilled fluid running through the concrete slab?
2. *Resurfacing of the ice:* How long does it take for hot water to freeze after deposition on an already frozen surface?
3. *Removal of the ice:* How long would it take to melt an ice layer with hot fluid running through the pipes in the concrete slab?

DESIGN OBJECTIVES

To translate the physical problem into a mathematical model, the assumptions and methodology described in the following sections was implemented. In summary, a 1D section of the ice was diagrammed and meshed using the structured mesh ability on COMSOL. The proper parameters were entered, such as density, heat capacity and ambient temperature of the rink. The modeling process using COMSOL was utilized to calculate the approximate time needed for freezing, maintaining, and melting the ice.

Assumptions

Although it seems easy, freezing and melting ice can be complicated, therefore in our model we had to make several assumptions. Density does change between ice and water however we assumed that with respect to time, temperature and position the density remains constant. In order to accurately apply latent heat of fusion during the phase change from liquid to ice, we assumed that ice melted over a range of temperatures from -0.5^{o}C to 0.5^{o}C (see Appendix A for full description). To simplify the model to one dimensional, we ignored any heat flux between the layers of ice. We also ignored the interface between the rink's boards and the ice.

Schematic

The total thickness of the ice is about one inch, or 2.54 cm. The total thickness is divided into eight layers, as seen in Figure 1, each with a thickness of 0.3175 cm. The fluid in the concrete is maintained at -21°C (252 K) for freezing purposes.

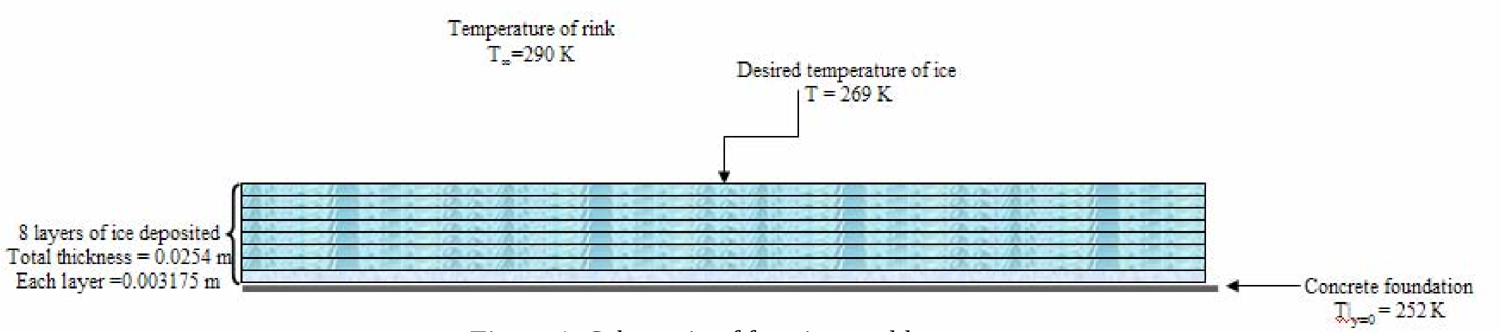


Figure 1: Schematic of freezing problem

Also shown is the fact that the ambient temperature of the rink is 17°C (290K).

For COMSOL, the schematic is modeled as a one-dimensional situation as shown in Figure 2.

Figure 2: Schematic as it would appear in COMSOL

There are sixteen sections, each representing half of one layer. This set-up was chosen in order to simplify the resurfacing stage. For the resurfacing, half of the topmost layer is removed and replaced with hot water.

RESULTS AND DISCUSSION

Initial Results

I. Creation of Ice

When the ice is initially made, each thin layer of water is deposited and frozen before the next thin layer is deposited on top of the previous one. This step is done 8-10 times for a total thickness of about 1 inch (2.54 cm). The reason for this is to obtain a smooth, thin surface. The first layer of water is deposited on the concrete at room temperature, 293 K (20 ºC). The following seven layers are deposited at a temperature of 303 K (60ºC). This is done so that it will melt a small amount of the layer below it and refreeze with it to create a strong layer of ice. However we assumed that the temperature of the first layer to be at room temperature, as it does not need to be as warm as the following layers. In Figure 3, the freezing time for the first layer is shown, and our results indicated that it takes about 28 seconds for the ice to form.

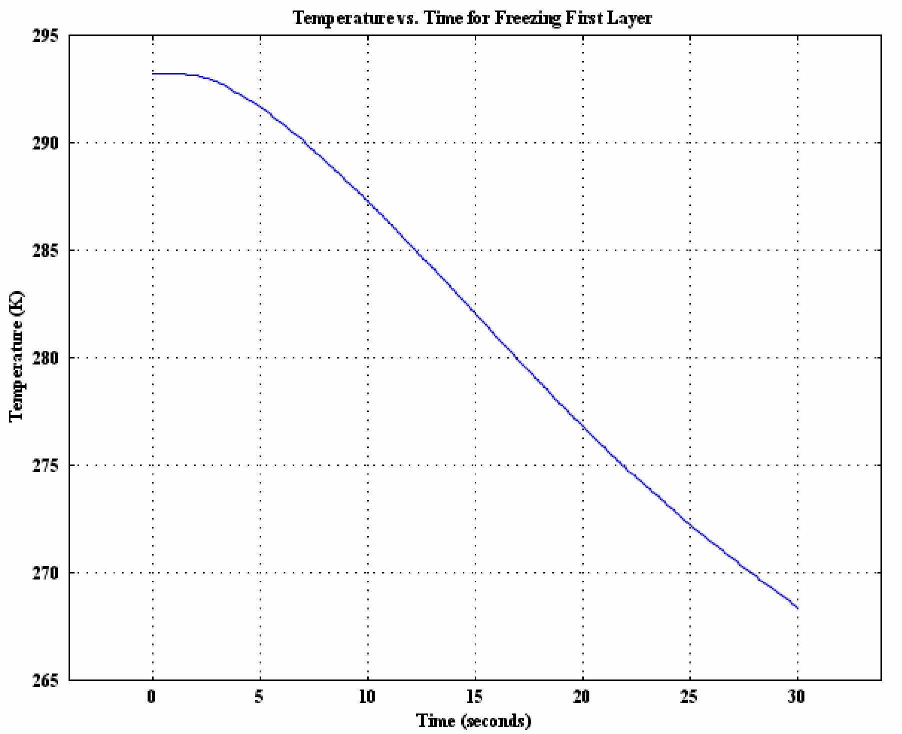


Figure 3: The graph of the final temperature vs time for the first layer of ice, graphed at x = 0.3174cm

To verify that the layer as frozen, the temperature of the topmost part of the layer was monitored. The logic behind this was simply this: the concrete slab is maintained at very cold temperatures, so the bottom of each layer will definitely freeze first. The topmost layer is exposed to the rink temperature and will freeze last. Thus, when the temperature of the topmost portion of the layer drops below 273 K, then the entire layer has frozen solid.

Once the top layer has frozen, we must then deposit the next layer, and continue until all the layers are added. Our results indicated that the topmost part of the final layer will freeze approximately 14000 seconds after the first layer was deposited. Thus it will take 3.89 hours to create the layers of ice that the players skate upon.

II. Resurfacing Ice

Due to the nicks and scrapes created in the ice by the skates, the rink is resurfaced between periods in the game. For this purpose, the zamboni machine scrapes off about one third of the top layer of ice, and deposits an equivalent amount of hot water at about 333.13 K (60°C).

Figure 4 shows the freezing profile after a fraction of the top layer is removed and the same thickness of hot water is added.

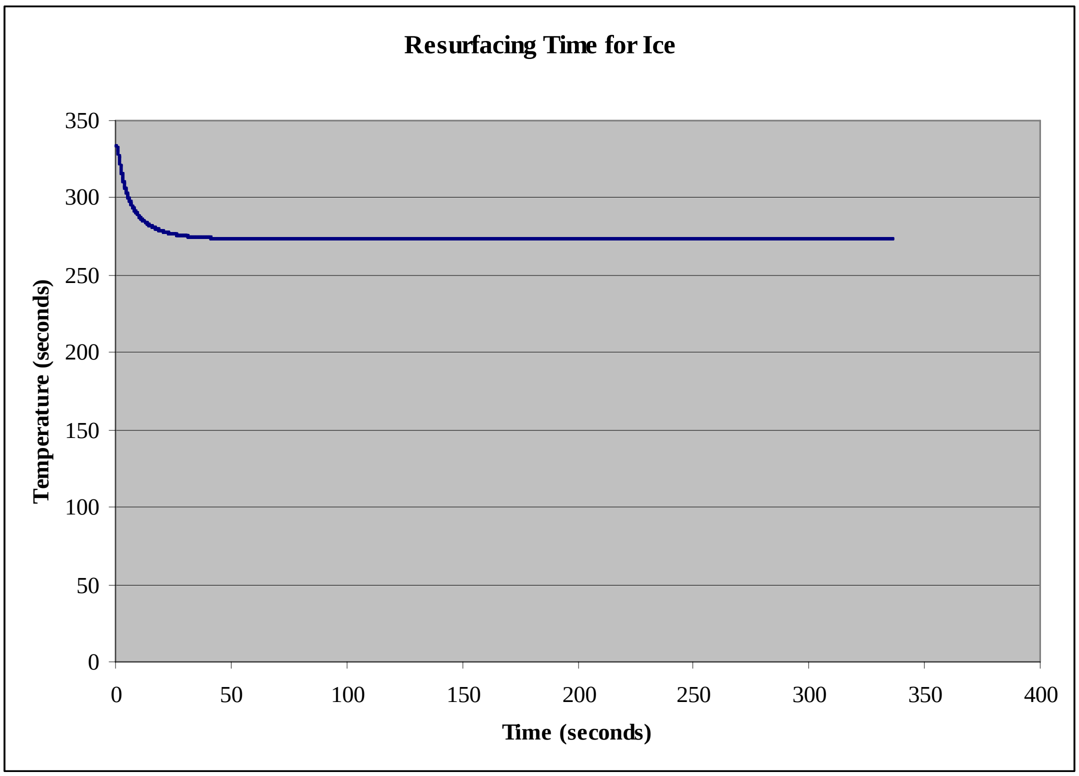


Figure 4: Time for the added water to reach below 273 K for the resurfacing process.

From these results, the resurfacing step will take 5.59 minutes. This is a reasonable answer, as the zamboni must traverse the entirety of the rink in under fifteen minutes.

III. Melting Ice Layers

Two scenarios were considered for the melting stage: one with water at room temperature (293.15 K) and one with water at a higher temperature (333.15 K). The results from both these scenarios are shown in Figures 5 and 6 respectively.

The water was considered to be completely melted when the temperature of the topmost layer was greater than 293.15 K. The logic to this decision was similar to that of the freezing stage. The bottom layers are exposed to the warmer temperatures and will thus thaw more quickly than the top layers.

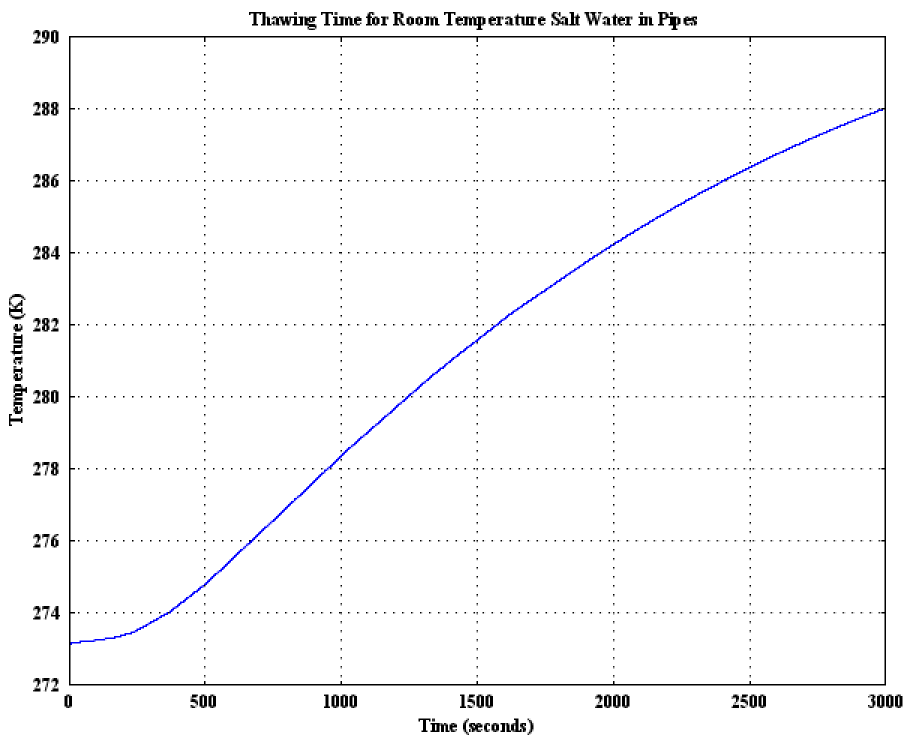


Figure 5: The graph of the final temperature vs time for the all of the ice to melt when at room temperature, graphed at y = 0.3174cm

If the pipes carry water at room temperature (293.15 K), the ice will thaw out in at most 8 minutes.

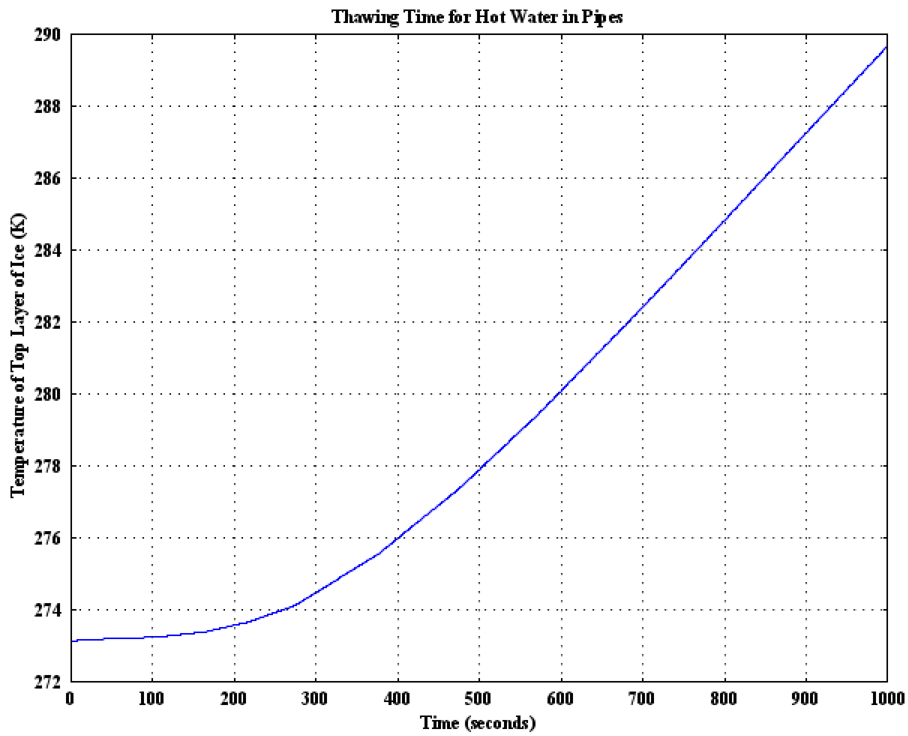


Figure 6: The graph of the final temperature vs time for the all of the ice to melt when the pipes are at boiling temperature, graphed at y = 0.3174cm

If the pipes carry hot water (333.15 K), all the ice will thaw in about 5 minutes.

SENSITIVITY ANALYSES

Introduction

Sensitivity analysis was done for different parameters to see how they affect the outcome of our model. The initial temperature of the water, the heat transfer coefficient, and the ambient temperature were all tested to determine their influence on freezing ice.

I. Sensitivity Analysis on T_{water}

In our model, there were several properties inputted that seemed subject to variability in certain scenarios. One such property was the temperature of the water being added. Water temperature is most likely the most important factor in the time necessary for all layers to freeze, and this analysis is required. To confirm the importance of initial water temperature in time for all layers to freeze, we ran simulations with a range of temperatures. For all simulations, we assumed that the initial layer would be put down at 293 K (20°C) with layers 2-8 being introduced at T = 20, 40, 60, or 80 degrees celcius. These results are summarized in Figure 7. We chose not to analyze the case of T = 100 for two reasons. First, it would be difficult to handle water this hot and maintain the water at this temperature while laying down the water. Second, this situation would introduce complexities because it is at the boundary temperature of a phase change.

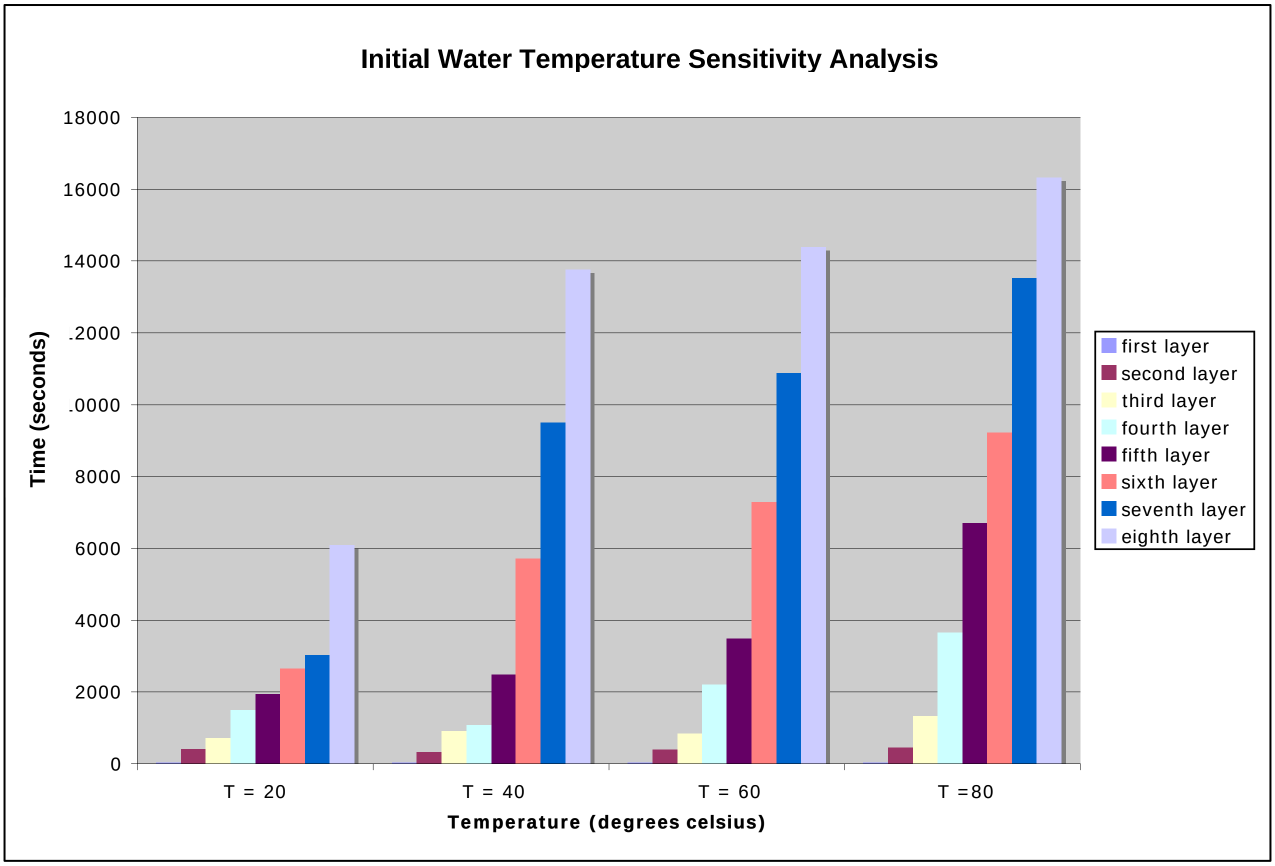


Figure 7: Freezing times for each layer dependent on initial water temperature

As expected, our analysis indicates that freezing times take longer when the temperature of the water put down on the rink is higher. However, it is unclear with only four values simulated whether there is some sort of linear relationship or possibly an exponential relationship. Intuitively, since there are seven consecutive layers (after the first) of water put down, the relationship would not be simply linear since the effects of higher temperature compound with each layer of water placed down on the rink.

II. Sensitivity Analysis on Heat Transfer Coefficient

Another factor that may change is the convective heat transfer coefficient (h). In the original calculations, h was found to have a value of about one. However, the value can change from such conditions as air movement resulting from players skating over the ice. Total freezing times were calculated and compared for the following values to see the effect: h = 0.5, 1, 2, and 4, and these results are summarized in Figure 8.

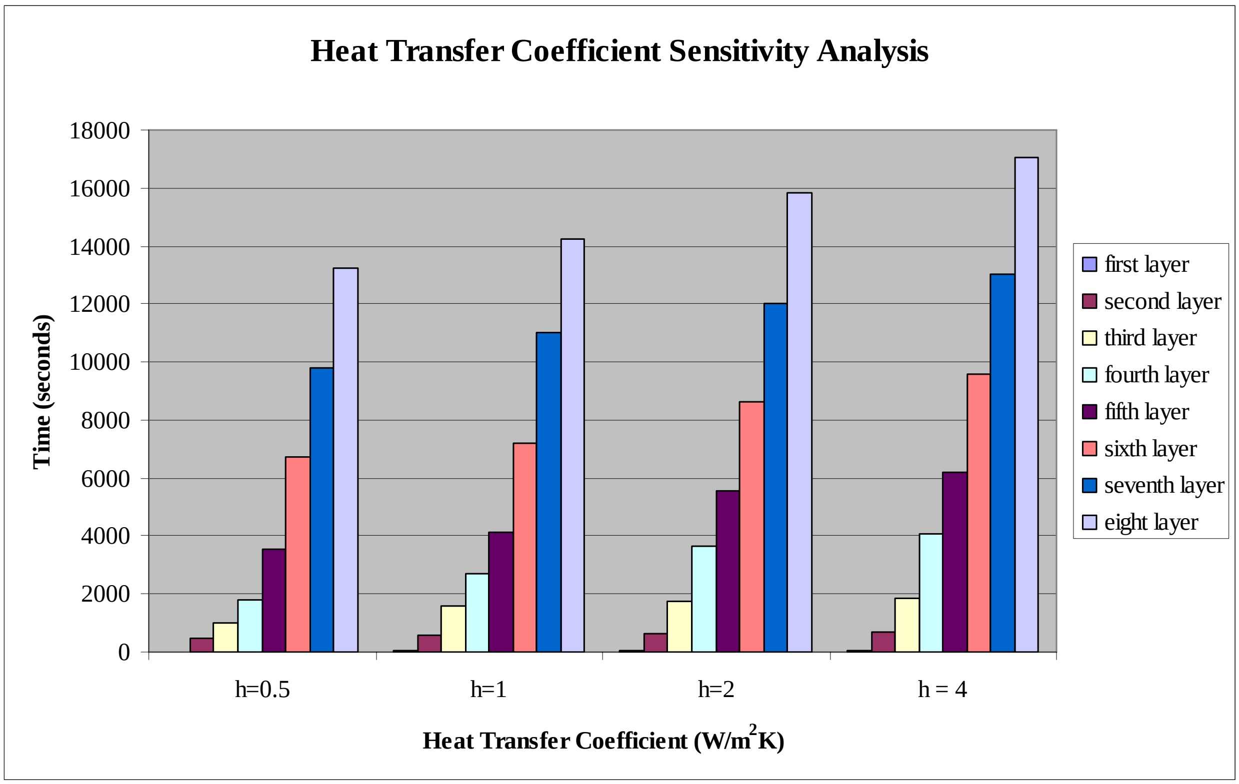


Figure 8: Freezing times for each layer dependant on heat transfer coefficient value

Our analysis indicated that as h values increased, the freezing time for each layer increased correspondingly.

Heat that is transferred between a surface and a moving fluid (liquid or air) is known as convection. Natural convection, the scenario here, is caused by buoyancy forces due to differences caused by temperature variations in the fluid. As each layer is added and cooled, the density changes in the boundary layer causes the fluid to rise and be replaced by cooler fluid, which in turn will be heated and rise.

Thus, the results of the sensitivity analysis make intuitive sense. The higher the value of h, the more the ice will be influenced by the temperature of the rink (17°C or 290K), resulting in a longer time to cool and freeze.

III. Sensitivity Analysis on Resurfacing as affected by Ambient Temperature

During the creation of the ice, the temperature of the rink was set at 290 K (17°C). Due to the absence of a large number of people in the rink during this process, this temperature values remains relatively constant. However, at a hockey game the presence of hundreds of fans will increase the rink temperature, which would affect the time to resurface the ice. While most rinks have expensive equipment in place to maintain a constant temperature, there will always be some variability. Figure 9 displays the impact of a variation in temperature (290, 293, 298, and 303 K) against the resurfacing time.

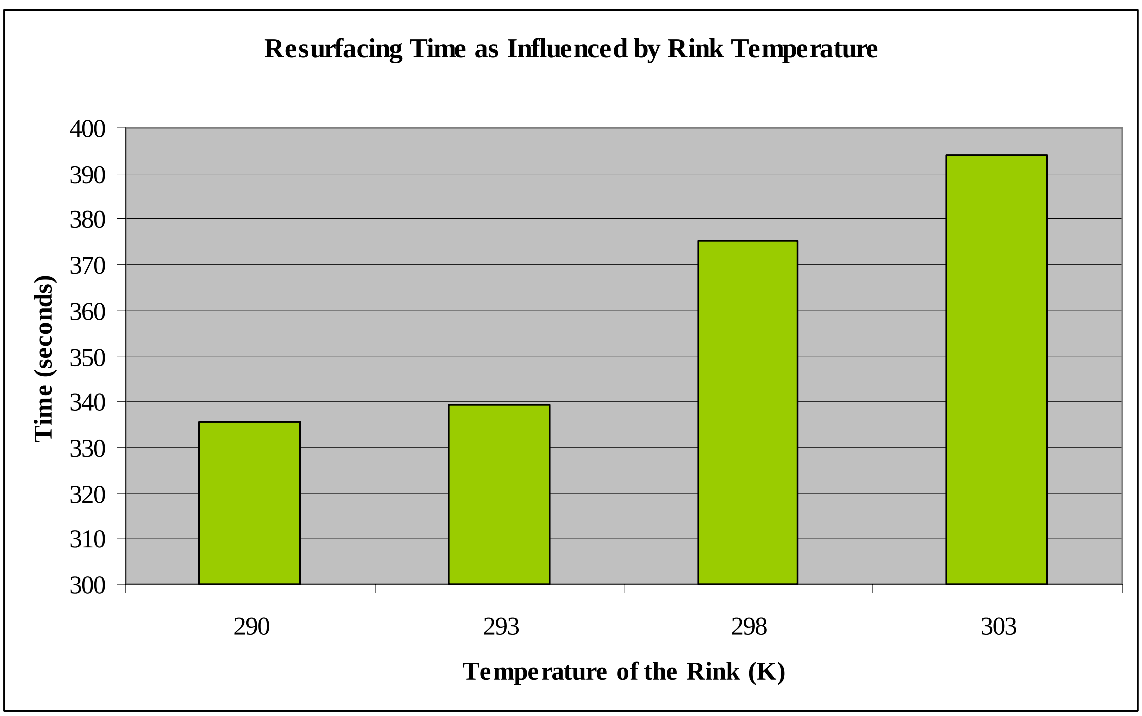


Figure 9:Resurfacing times for each layer as influenced by temperature of the rink

Our analysis shows that the temperature can reach a value of 303K and the resurfacing process can still occur in the allotted time of fifteen minutes. In practice, the temperature would never be allowed to reach this high above the set point.

IV. Sensitivity Analysis on Resurfacing as affected by Heat Transfer Coefficient

Another parameter that may alter the resurfacing time is the heat transfer coefficient. Again, when the ice is being created, this value may stay close to the calculated value of h = 1 W/m^2•K. However the effect of the audience, doors being opened and closed, and the zamboni driving over the ice may affect this value. Figure 10 shows the results of altering the heat transfer coefficient on the resurfacing time.

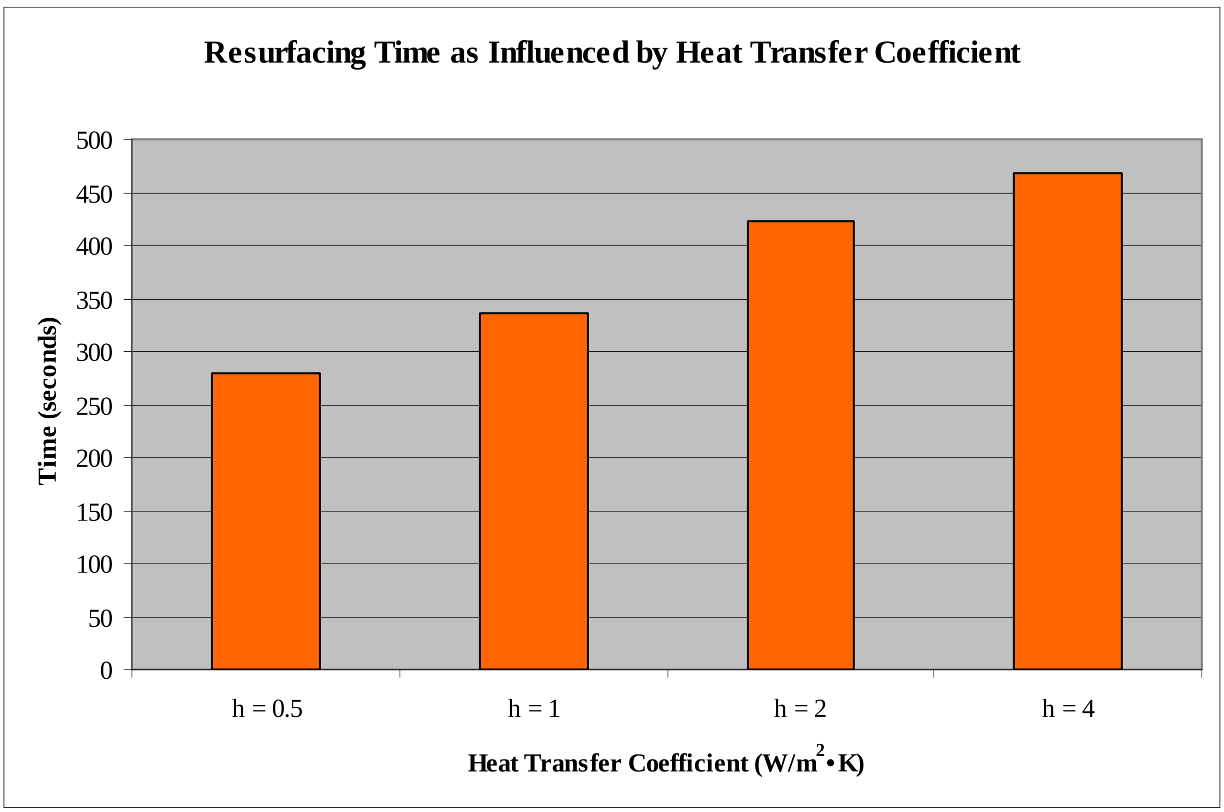


Figure 10: Freezing times for each layer as influenced by heat transfer coefficient

Even at a value of h=4, which is higher than what would be allowed realistically, the resurfacing time is 7.8 minutes. This is well under the allotted time of fifteen minutes, which shows that even under less than ideal conditions, the resurfacing step is still performable.

CONCLUSIONS AND DESIGN RECOMMENDATIONS

In this study, we were successfully able to model the structure of the ice in an ice rink. First, we looked at the time it would take to freeze the entire ice, all 8 layers successively. It was found to be approximately 14,000 seconds or 4 hours. In the real world, it usually takes an ice maker a day or two, but since we are assuming that each layer is added the instant the previous layer freezes, the time found from the model seems to be pretty realistic.

We then looked at the maintenance of the ice and the effects a Zamboni has on resurfacing.. It was assumed that the Zamboni scrapes 0.106 cm of ice off the top and redeposit it as hot water to freeze back as a smooth surface. The time required for the deposited hot water to freeze was found to be 5.59 minutes which is reasonable since the time between periods of a hockey game is 15 minutes, giving the ice resurfacer just enough time to drive over and resurface the whole rink.

The last thing we looked at was the time required to melt all the ice. For this part, we simulated it twice, once running room temperature water through the pipes and once running hot water through the pipes. When room temperature water was run though the pipes, it was found that it would take 8 minutes to melt the entirety of the ice. For hot water running through the pipes, it would take only 5 minutes to melt the ice. These numbers are somewhat low but are accounted for by the assumption that the concrete slab instantly changes its temperature to the temperature of the water running through it.

When creating the model, we tried to start as simple as possible. There are ways to complicate our model to make it more realistic. One thing that can be changed is to draw the geometry as 2D or 3D, making it axisymmetric and taking into account the effect of the walls. Also, we can do better measurements of the atmosphere of the arena to come up with a more accurate h-value. Certain things to consider are the effects of the fans in the arena or the flow caused by the doors being open. The sensitivity analysis performed on the h-value was able to determine some of these effects.

Our model was able to give us information on how to manage an ice rink more efficiently by minimizing freezing, resurfacing, and melting time. We can see from the sensitivity analysis that when freezing the ice, the additional water added for each layer should be as low as possible. However, this water must be hot enough to melt some of the layer below it. Therefore, the temperature currently used, 333.13 K, is the recommended temperature. Also, for a more efficient freezing and resurfacing process, the manager should try to minimize the h-value. This can be done by minimizing the air flow in the arena by ways such as keeping the doors closed and keeping the air ventilation system on low. One final recommendation when freezing and resurfacing is to keep the ambient air temperature as low as possible. A lower emissivity ceiling and lights that produce less heat are ways to accomplish a lower ambient air temperature.

Since the temperature of the water running through the pipes can be adjusted, when melting the ice as efficiently as possible, we are interested not only in the time, but the amount of energy that must be put into the process. The ice will melt a few minutes faster if the water through the pipes is heated to 333.13 K as opposed to using room temperature water. This does not justify the amount of energy and money needed to heat the water. Therefore, it is recommended that managers run room temperature water through the pipes under the ice when melting it.

There are few realistic constraints in this design. Economically, managing an ice rink can be very expensive, needing to maintain an optimum atmosphere in the arena. The freezing and resurfacing process is extremely costly where cold salt/brine water constantly needs to be pumped through the pipes under the ice. The melting process may also be expensive, but that is dependent upon how fast manager needs the ice to melt. Environmentally, many rinks are very friendly choosing to use salt/brine water as the fluid circulating in the pipes as opposed to other chemicals that are available to use. Salt/brine water may not be the most efficient liquid to use, but this way, a leak or burst in the pipes will not cause a great deal of environmental harm. There are no health and safety constraints since we are only concerned with water and ice.

APPENDIX A: MATHEMATICAL STATEMENT OF THE PROBLEM

Geometry or Schematic

Our problem is axis symmetric and an infinite slab. As a result we are able to model the problem in a 1D direction. Each layer of ice is 0.3175cm thick, and the problem was done eight times to simulate the eight layers of ice needs to make the playing surface for an ice hockey rink. (Refer to Figures 1 and 2) We determined how much time it takes for each layer to freeze and then the summation of the process. Additionally we evaluated the time involved in resurfacing the ice as well as the time to melt all the layers of ice.

Governing Equation

We start with the general heat transfer equation but since there is no fluid flow, the convective terms are eliminated, and since an axisymmetric geometry is used, terms with respect to the x-axis are also dropped. The heat generation term is neglected in the original problem statement, as it is considered insignificant compared to the heat lost. The simplified governing equation is as follows:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho c_{pa}\frac{\partial T}{\partial t}$$

The natural convection element on top of the ice to consider which is taken into account by: $h\left(T - T_{\infty}\right)$. The temperature in the ice rink, T_{∞} = 290K. The heat transfer coefficient (h) would be solved for by finding the Rayleigh number (Ra) and using that to find the Nusselt number (Nu). Nu would then be used to solve for h.

The Rayleigh number is equal to the Grashof number (Gr) multiplied by the Prantdl number (Pr).

$$Ra = Gr \times Pr$$

The Prandtl number is dependent of temperature and can be looked up on charts. The Grashof number can be determined by the relationship of several variables dependent on the temperature in the following equation:

$$Gr = \frac{\beta g \rho L^{3} \Delta T}{\mu^{2}}$$

The variables are: the thermodynamic property of the fluid(β), the acceleration due to gravity(g), the characteristic dimensions(L), the temperature difference between the surface and bulk fluid(T), and the viscosity of the fluid(μ).

Depending on the value of the Rayleigh number, the Nusselt number can be determined based on the following equation:

$$Nu_L = 0.54 Ra_L^{\frac{1}{4}} \qquad 10^5 < Ra_L < 2\times10^7$$

$$Nu_L = 0.14 Ra_L^{\frac{1}{3}} \qquad 2\times10^7 < Ra_L < 3\times10^{10}$$

Initial and Boundary Conditions

Initial Conditions

For the first layer added the initial temperature of the water layer being added is 293.15 K (room temperature).

First Layer:

$$T_0 = 293.15K$$

Last Seven Layers:

$$T\big|_{old} = T$$
$$T\big|_{new} = 333.15K$$

After the first layer of ice is added the initial temperature of the previously frozen layers of ice is changed to T, thus a function of the answer previously calculated in the first part of the problem. The new layer of water being added now has a temperature of 333.15K. This procedure is continued until the layers of ice are built up to the final thickness.

Boundary Conditions

The temperature of the boundary touching the concrete slab is 252K, and the ambient temperature is 290K.

$$T\big|_{x=0} = 252K$$
$$T_\infty = 290K$$

Heat flux exists on the boundary where water is being added with the value of 1 W/(m^2K). However there is no heat flux in the x-direction at x = 0 because of axis symmetry, and at x = ∞ due to an infinite slab

$$h\big|_{water\ boundary} = 1W/(m^2K)$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=\infty} = 0$$

Input Parameters

The density of the water was defined as $\rho = 1000\ kg/m^3$.[2]

Since we are dealing with water, the heat capacity (c_p) and thermal conductivity (k) both change in relation to temperature. We needed to define two functions, therm_cond(T) and heat_cap(T) and define values for the function over a set range of temperatures. For heat capacity, for instance, we used the known latent heat of water (334. kJ/kg) to solve for values of heat capacity in graph (Appendix C). The values used for the two parameters are below:

Table 2: The specific heat and thermal conductivity values used for the model

Temperature (K)	Specific Heat[2] (J/kgK)
250.00	2090.00
255.00	2090.00
260.00	2090.00
265.00	2090.00
270.00	2090.00
272.15	2090.00
272.31	216105.84
272.49	456873.66
272.65	670914.50
272.79	484246.20
272.97	244244.10
273.15	4217.00
275.00	4211.00
280.00	4198.00
285.00	4189.00
290.00	4184.00
295.00	4181.00
300.00	4179.00
305.00	4178.00
310.00	4178.00
315.00	4179.00
320.00	4180.00
325.00	4182.00
330.00	4184.00

Temperature (K)	Thermal Conductivity[2] (W/mK)
230.00	0.567
273.15	0.569
275.00	0.574
280.00	0.582
285.00	0.590
290.00	0.598
295.00	0.606
300.00	0.613
305.00	0.620
310.00	0.628
315.00	0.634
320.00	0.640
325.00	0.645
330.00	0.650

When dealing with the solution parameters with each layer, the solution from the previous layer is stored. Then the solver parameters are changed to solve the problem starting from the solution time of the last layer to a reasonable estimate of the time to freeze the next layer. The solution time from the last layer is the time that the ice layer reaches 273K. The solution time can be increased or decreased as needed. This procedure is repeated until the layers are built up to the final ice thickness

APPENDIX B: PROBLEM STATEMENT

Objective

Our objective in this study was to model the production, maintenance, and melting of a hockey ice rink. We are specifically looking at the time required to produce the ice, time for the Zamboni to resurface a single spot on the ice, and time required to melt all the ice by either running room temperature water or hot water through the pipes underneath the rink.

Problem Settings

Table 3: Problem Setting for the Model in COMSOL

Descriptor	***Type***	***Explanation***
Space Dimension	1D	Axisymmetry
Application Type	Heat Transfer	Only concerned with energy transfer. No mass transfer
Mode of Transfer	Conduction, Natural Convention	Heat transfer is influenced by conduction from concrete slab below, and natural convection from the temperature differences from the rink and the surface of the ice
Simulation Type	Transient	Interested in temperature changes over time

Solution Statement

We found that the time required to produce the ice, with its eight layers, was approximately 1.4 hours. The time required for the Zamboni to resurface a spot was found to be 42 minutes and the time to melt all the ice was either 8 minutes for room temperature water or 5 minutes for hot water running through the pipes.

Time Integration Statement

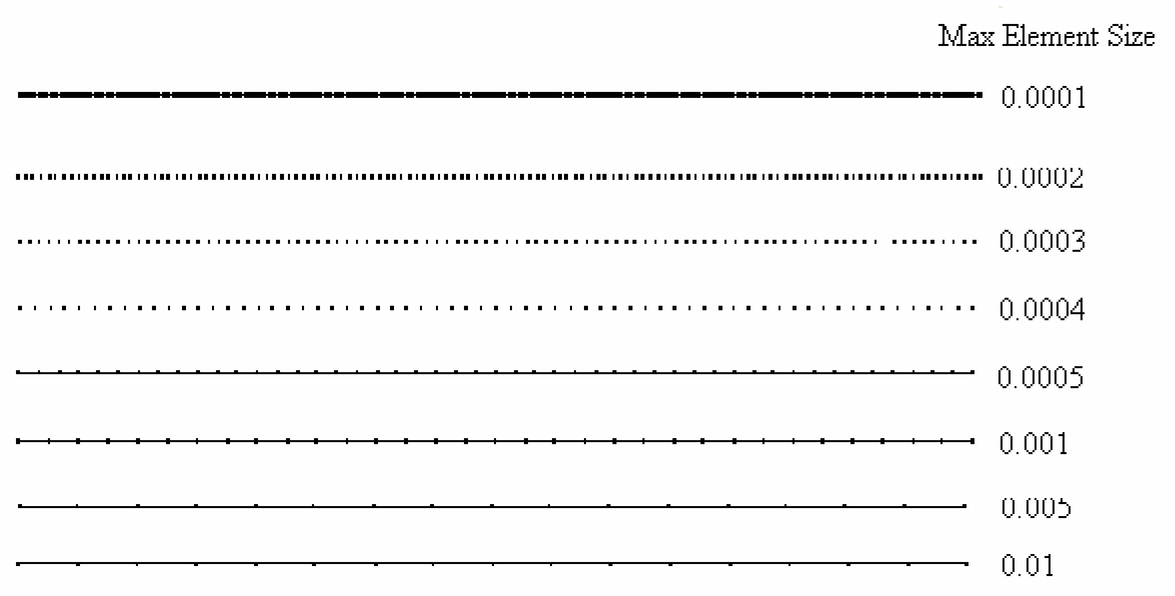
The time period and step were dependent upon the layer being frozen and determined from running to program and looking for the freezing time of each layer. For example, for the first layer, the time period was 0 to 30 seconds with a time step of 0.1 seconds. Looking at a plot of the uppermost portion of this first layer, we determined the time it took to freeze the whole layer (when the uppermost portion temperature dropped below 273 K). This time was found to be approximately 25.5 seconds. We then ran the model again for the next layers, shifting the time period from 25.5 seconds until an arbitrarily chosen future value. The solution from the previous solution at 25.5 seconds was stored and used for the next layer's run. This procedure of running, looking for the freezing time, and re-running the model was done for each layer.

Tolerance/Time Step

Initially, a general time step was used, which was determined by the software. However, this method showed a large variability in freezing times. This was believed to be the result of skipping over certain values during the freezing steps, when the values for heat capacity would be quite large. The time stepping was thus changed to manual mode, with an initial time step of 0.00010 and a maximum time step of 0.1. Although this significantly increased the time it took to solve each layer, the outputted results were much more consistent.

Plot of Element Mesh and Convergence

Element Mesh



Mesh Convergence

Mesh convergence was used to determine the total number of elements appropriate for our model to produce accurate results. The different meshes used with their respective statistics are displayed in the table below (Table 4). For each of these meshes, we are testing just the first layer.

Table 4: General Statistics of Different Meshes Modeled

Maximum Element Size	0.0001	0.0002	0.0003	0.0004	0.0005	0.001	0.005	0.01
Max El Size Scaling Factor	1	1	1	1	1	1	1	1
Element Growth Rate	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.3
# of Degrees of Freedom	65	33	25	17	13	9	5	5
Number of Mesh Points	257	129	97	65	49	33	17	17
# of Elements	256	128	96	64	58	32	16	16
# of Boundary Elements	17	17	17	17	17	17	17	17
Element Length Ratio	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

For the first layer, the program is allowed to run for each mesh from a time of 0 to 120 seconds with an increment of 0.1 sec. A plot of the average temperature for the whole layer at the end time vs. the element size is shown in Figure 11. It is evident that minimum element size is 0.0003 or 96 elements(Figure 11).

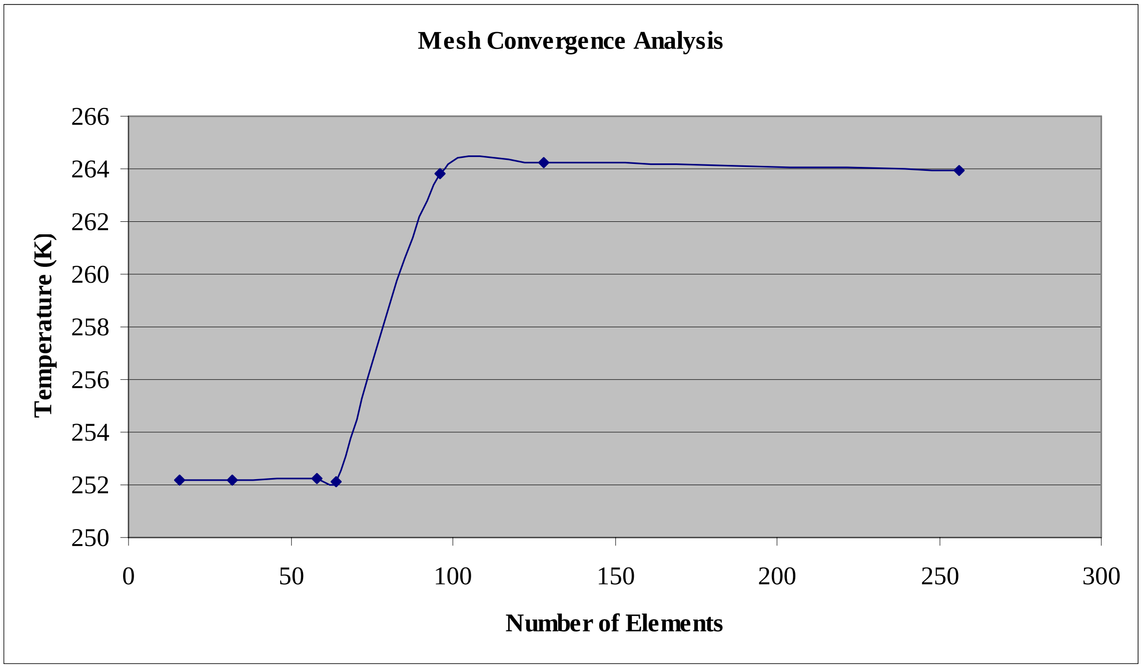


Figure 11: The final temperature of the first layer of ice as a function number of elements

APPENDIX C: CALCULATING APPARENT SPECIFIC HEAT

The latent heat for the transition from liquid to solid(ice) needs to be accounted for the heat capacity $c_p = \frac{d\left(H/m\right)}{dT}$, but as dT is technically 0, this gives ∞ for c_p. Instead we will plot c_p from T = -0.5°C to T = -0.5°C and will take the area under the curve to equal the latent heat, 333.944 kJ/kg.

The area under the curve is equal to:

$$\left[\frac{1}{2}(\Delta T)(c_{p\max} - c_{p\,ice})\right] + \left[\frac{1}{2}(\Delta T)(c_{p\max} - c_{p\,water})\right] + \left[(0.5)(c_{p\,water} - c_{p\,ice})\right] = 334955\ J/kg$$

$$\left[\frac{1}{2}(0.5°K)(c_{p\max} - 2090\ J/kgK)\right] + \left[\frac{1}{2}(0.5°K)(c_{p\max} - 4217\ J/kgK)\right]$$
$$+\left[(0.5)(4217\ J/kgK - 2090\ J/kgK)\right] = 334955\ J/kg$$

Using the area under the curve equations c_{pmax} can be calculated:

$$\boxed{c_{p\max} = 670914.5\ J/kgK}$$

Once c_{pmax} is found the slopes of the lines on each side of c_{pmax} can be calculated using the equation of a line and the specific heats of ice and water as the intercepts.

Water

$$m_{water} = \frac{c_{p\max} - c_{pwater}}{\Delta T}$$
$$m_{water} = 33348.75kg$$

Ice

$$m_{ice} = \frac{c_{p\max} - c_{pice}}{\Delta T}$$
$$m_{ice} = 334412.25kg$$

Using these equations the specific heats for the temperatures between 272.15K and 273.15K can calculated and used in the input parameters(Figure 12):

Water
$$c_p = m_{water}(T - 272.15K) + 2090J/kgK$$

Ice
$$c_p = m_{ice}(T - 273.15K) + 2090J/kgK$$

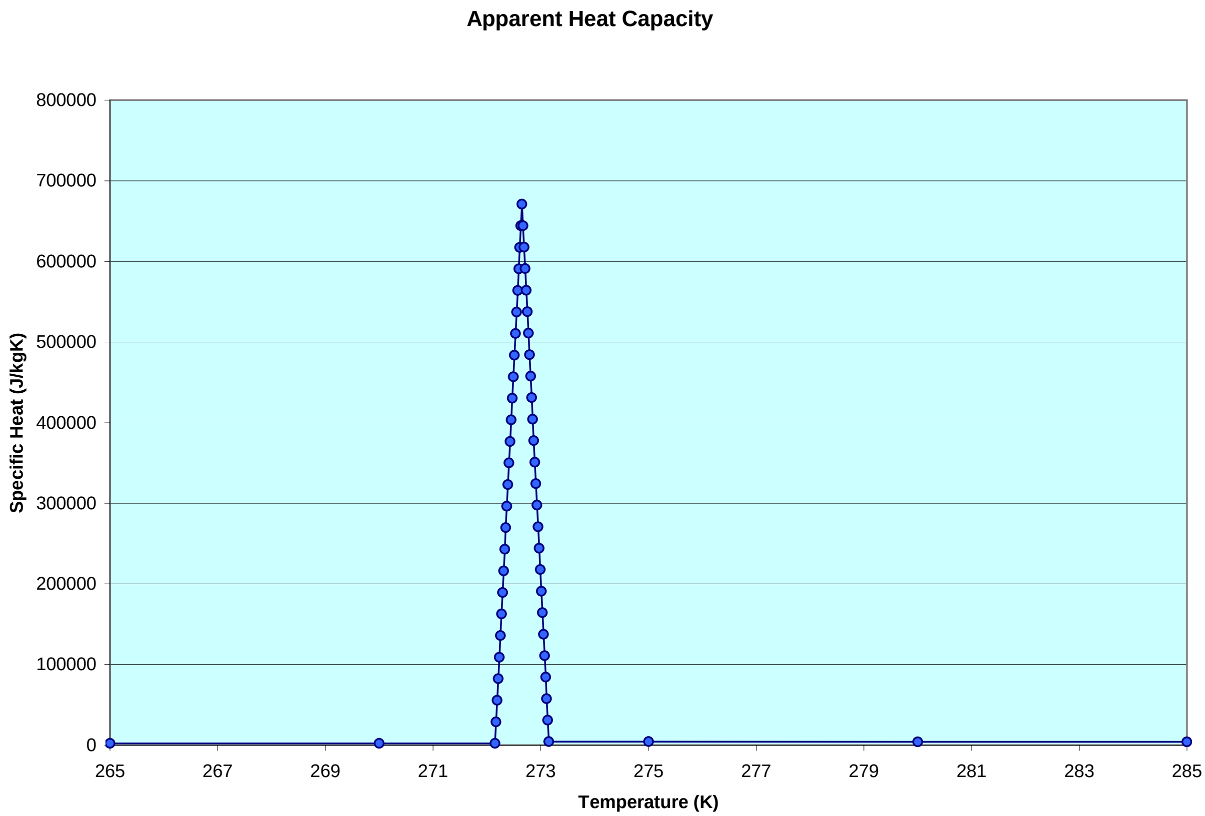


Figure 12 : The graph of apparent heat capacity for water.

The specific heat capacities from the graph were then added to the model as input parameters (Appendix A).

APPENDIX D: REFERENCES

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