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A SIGNALING/BONDING MODEL  
OF  
EMPLOYER FINANCE OF GENERAL TRAINING /

John H. Bishop  
Cornell University

Suk Kang  
Tokyo Metropolitan University

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Center for Advanced Human Resource Studies  
New York State School of Industrial and Labor Relations  
Cornell University  
Ithaca, NY 14851-0952  
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A SIGNALING/BONDING MODEL OF EMPLOYER FINANCE OF GENERAL TRAINING

I. INTRODUCTION

One of the central propositions of the human capital theory of on-the-job training is that workers pay for and receive all the benefits of general training. Since general training raises a worker's ability to be productive in other organizations as well as the one providing the training, the training firm must pay a wage commensurate with the trained worker's new higher level of productivity if they are to prevent the loss of their trained workers. Since the workers, not the firm, get the benefits of the training, "firms [will] provide general training only if they [do] not have to pay any of the costs" (Becker 1962 p. 13). Since the training is of value to prospective trainees, equilibrium in the training market requires that "employees pay for general on-the-job training by receiving wages below what could be received elsewhere" (Becker 1962 p. 13) in a job offering no training.

This paper challenges the general validity of these simple predictions. It begins in section 2 by presenting empirical evidence that (1) trainees often do not have to accept lower wage jobs in order to obtain training and (2) that employers often appear to be sharing the costs of general training with employees. In section 3 we expand and generalize Hashimoto's elegant theory of the sharing of the costs and benefits of specific training and show why with our modifications firms choose to offer front loaded compensation packages in which they appear to share the costs of general training with their employees. Employers share in the finance of general training for three reasons: (1) they have better access to capital markets than employees<sup>1</sup>, (2) turnover damages the reputation of the employee so workers ask employers <sup>to</sup> put up a bond at the initiation of the employment relationship so as to minimize their risk of involuntary separation and (3) the firm providing general training is better able to assess the success of that training than any other employer and this information assymetry effectively transforms skills that are technically general into skills that are behaviorally specific. In section 4, we examine the realism of the key assumptions of the theory--workers have limited access to capital markets and suffer severe long term damage if fired--which drive the predictions of the theory. The final section of the paper

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uses the theory to speculate on the reasons why employer training appears to be substantially heavier in Germany and Japan than in the US.

II. EMPIRICAL ANOMALIES

While the logic of Becker's prediction that employers will not and workers must pay for general on-the-job training seems impeccable, tests of it in large representative data sets are few and those that have been conducted have generally failed to confirm it. In <sup>an</sup> ~~a study of recent school leavers in~~ the National Longitudinal Survey, Parson's (1985, table 7.6) found that when <sup>young men</sup> ~~reported to be "very true"~~ "the skills [I am] learning would be valuable in getting a better job," was more reported than when the respondent reported the above statement was "not at all true" even when an extensive set of controls for schooling and academic achievement were included in the model. Another test of the Becker hypothesis can be conducted in the 1984 follow up of the High School and Beyond seniors. This survey contains the necessary data on the training received in the current or most recent job and an extensive array of worker characteristics that can be used to control for the skills and ability of the worker. Becker's prediction was tested by regressing the log of the deflated starting wage of the current or most recent job on indicators of the receipt of employer sponsored training while controlling for an array of background characteristics.<sup>2</sup> The coefficients on the training variables and statistics describing the overall fit of the model are presented in Table 1. Contrary to Becker's prediction, the jobs offering some training rather than none or which offer greater amounts of training paid higher starting wage rates even when a whole array of human capital characteristics were controlled. For females the positive effect of receiving training on the starting wage was statistically significant. Adding dummies for occupation and industry did not change the results appreciably.

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It could be argued, however, that these findings do not constitute a decisive refutation of the proposition that workers pay all of the costs of general training. Hiring decision makers are probably much better at assessing the ability of job candidates than econometricians whose only data is what appears on the High School and Beyond data tape. The positive

*calculated the # of hrs wage effect  
throughout*

Table 1  
Effect of Receipt of Training on a Worker's Starting Wage

	Males		Females		Means	
					White Males	White Females
Number of different types of training received on the job (range 1-5)	.022 (1.56)	.016 (1.05)	.040*** (3.40)	.030** (2.38)	.65	.78
Total Hours of Training (100's)	---	.007 (1.42)	---	.006 (.89)	1.85	.88
Total Hours Training Squared (10,000's)	---	-.00025** (2.24)	---	-.00016 (.93)	42.1	11.0
Dummies for Industry & Occupation Included	no	yes	no	yes		
R Square	.065	.131	.078	.121		

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The dependent variable is the log of the deflated starting wage of the current or most recent job. The sample is 1980 high school graduates who in 1984 were not attending college full time and do not have a military occupation. The background characteristics that are controlled include: time spent attending college during 1982 and 1983, work experience and vocational training in high school, high school GPA, grades in trade and business courses, test scores, attitudes toward work and school, number of extra curricular activities and leadership positions, self esteem, locus of control, deportment in high school, married, children, number of siblings, race, hispanic, parental income, parental education, parental occupation, urban, rural and 10 regional dummies and length of tenure on job and its square.

association between wages and training arises, it could be argued, because workers who are highly able (in ways not observed by the analyst) are both paid more and also recruited for jobs that require large amounts of training.

This phenomena is probably contributing to the positive association between training and starting wage rates, but to transform a large negative structural relationship into a statistically significant positive relationship, there would have to be very powerful sorting of more able job applicants into jobs with heavy training investments. If such a selection process were operating, access to training should depend on ability factors that are visible to the analyst as well as on factors that are not visible to the analyst. The results of a test of this hypothesis in the same High School and Beyond data are presented in Table 2. The dependent variables in the analysis were a dummy variable for having received some employer sponsored training and the trainee's estimate of the total number of hours that were spent in training. The ability proxies hypothesized to have a positive effect on the receipt of training were: test scores, GPA, grades in vocational courses, deportment in school, number of vocational courses, number of academic courses, hours spent doing homework, hours spent working for pay, number of leadership roles, having an internal locus of control (the belief that one controls one's own fate), an index of reading in high school, and a positive response to "do you enjoy working for pay." Variables hypothesized to have a negative effect on the receipt of training were: did not graduate, hours watching TV, and self reported study habit problems. The model estimated also included controls for tenure on the job and its square, race, hispanic, marital status, 5 variables describing college enrollment since high school, parental education, occupation and income, number of siblings, urban/rural dummies, and a set of ten dummies for region.

Despite reasonably large samples--1938 men and 2554 women--, the analysis offers only limited support for the hypothesized positive association between ability/productivity proxies and the receipt of training. Only two of 16 variables thought to proxy for ability/productivity had significant coefficients of the correct sign in two or more of the 4 regressions: average weekly hours worked for pay during the junior and senior year of high school and the intervening summer and the number of vocational courses taken in the final three years of high school. Of the 64 coefficients tested, only 31 had

Table 2  
Effect of Indicators of Learning Ability on Receipt of  
Employer Sponsored Training

	Female		Received Some Training		Hours of Training	
	Mean	S.D.	Men	Women	Men	Women
Test Score	51	8.7	-013 (1.19)	.004** (2.32)	-.7 (.34)	.82 (.80)
GPA	82	7.5	<i>WS</i> <del>-.004*</del> (1.91)	-.0026 (1.47)	4.3** (2.05)	<i>WS</i> <del>-1.86*</del> (1.74)
High Grades in Bus/Cler (0-1)	.47	.50	.005 (.17)	.033 (1.51)	-43.0 (1.55)	-2.0 (.15)
High Grades in Trade & Tech (0-1)	.07	.26	.012 (.45)	.014 (.35)	32.6 (1.11)	53.4** (2.39)
Good Department in School	-.38	2.90	-.003 (.84)	.0007 (.17)	-.6 (.15)	-1.3 (.54)
Vocational Courses	<i>As prev</i> 2.2	1.8	.003 (.52)	.116** (2.39)	4.3 (.67)	7.2* (1.82)
Academic Courses	10	2.8	<i>WS</i> .013*** (2.64)	-.003 (.79)	-5.2 (1.01)	4.8* (1.90)
Did Not Graduate	.01	.09	-.10 (.78)	.12 (.80)	9.9 (.71)	155.0* (1.8)
Wkly. Hours on Homework	3.5	2.5	-.001 (.20)	-.008** (1.96)	<i>WS</i> 1.96 (.36)	-.67 (.26)
Wkly. Hours in Jobs	<i>Strongly</i> 12.9	9.7	.003*** (2.75)	.005*** (4.35)	1.2 (.95)	1.4** (2.11)
Wkly. Hours Watching TV	21.5	11.8	.000 (.003)	-.001 (1.55)	.32 (.30)	-.006 (.01)
# of Leadership Roles	.69	1.02	-.004 (.13)	-.006 (.24)	10.4 (.37)	8.1 (.45)
Internal Locus of Control	-.04	.69	.02 (1.08)	.005 (.27)	45.3** (2.27)	-10.7 (1.03)
Study Habit Problems (0-4)	1.05	.95	.006 (.46)	.025** (2.04)	<i>WS</i> 19.4 (1.42)	1.2 (.18)
Reads a lot (0-3)	1.69	.88	-.004 (.31)	-.004 (.34)	-3.5 (.23)	-1.8 (.25)
Enjoys working for pay (0-1)	.92	.28	.064 (1.64)	-.054 (1.45)	63.7 (1.54)	-1.5 (.07)
R <sup>2</sup>			.071	.063	.069	.044
Mean of Dependent Variable			.39	.41	133	69
Standard Deviation			.49	.49	499	250

the hypothesized sign. Of the 11 coefficients significant at the 5 percent level on a two tail test, three had the wrong sign.

Of possibly the greatest significance is the failure of high school grades and test scores to have the expected effect on the receipt of training. Industrial psychologists have found strong positive relationships between success in training and test scores and a somewhat weaker positive relationship between training success and GPA (Hunter and Hirsh, 1988). Consequently, one would expect employers to seek out such workers when they fill jobs requiring a great deal of training. This does not appear to be the case in this sample for high school GPA had a negative effect on receipt of training in three of the four regressions. One coefficient was significantly positive but there were also two negative coefficients that were also significant (at the 10 percent level on a two tail test). Only one of the eight coefficients on dummies for good grades in vocational subjects was statistically significant. Half of the coefficients on test score were negative.

When filling jobs that require training, employers appear to try to economize on training costs by seeking out workers who have already received some training either on a job or in school. In other respects, however, their selections do not appear to be optimal for they fail to recruit the high school graduates with the strongest academic records and/or with high test scores. Parsons obtained similar results when he estimated models predicting which members of his sample had obtained "high learning" jobs. Given these findings, it is hard to imagine how selection on ability factors that are not proxied by High School and Beyond or NLS variables could be strong enough to transform a large negative effect of training on the starting wage in the true structural model into a positive association in this data set. *what seems more likely is that*

Even stronger evidence against Becker's predictions comes when we hold the individual constant (and therefore avoid the problem of unobservable ability variation across individuals) and look at patterns of change in productivity and wages during the first year on the job. Studies by industrial engineers of learning curves have found that in many jobs new hires are unable to make any significant contribution to output for weeks and often take a year or more to reach the productivity standard of an experienced worker (King 1964, Talbot and Ellis 1969). Wages, in contrast, exhibit a

remarkably flat profile in the first year on the job.

An explanation for these phenomena provided by human capital theory is that the training must be specific and the employer must be financing all of its costs. But standard models of the sharing of the costs of specific training do not predict that employers pay all of its costs and some of the new revisionist theories--Salop and Salop's (1976) adverse selection theory--predict that employers pay none of the costs of specific training.<sup>3</sup> The specific training explanation of the flat wage profile is especially suspect when to all outward appearances the training is largely general.

Studies of who pays the costs of apprenticeship training in three different nations--Germany, Great Britain, and the United States--all contradict the claim that employers will not provide general training if they have to pay some of its costs (Noll et al 1984; Ryan 1980; Jones 1985; Weiderhold-Fritz 1985). Despite the transferable character of the training and high turnover rates, these studies concluded that employers made large investments in general training that were not recovered during the apprenticeship. A welding apprenticeship program at a major U.S. shipyard was the subject of the first of these studies (Ryan 1980). The wage profile was quite flat--starting at \$3.99 and topping out at \$5.26 after about two years on the job--even though the investments in general training were very considerable. Inexperienced new hires spent 36 days in vestibule training before beginning work. During the first week following vestibule training, the trainee's output net of repair requirements was less than 10 percent of an experienced worker's output. Thirty-seven weeks after being hired it reached a level of 55 percent and at 60 weeks a level of 80 percent of an experienced worker's output. Despite the fact that the local economy was in deep recession, separation rates were extremely high: 10.8 percent per month for beginners and 6.3 percent per month for those with 12 to 24 months of tenure. The shipyard accounted for about one-fifth of the welding jobs in the area. When trained welders left the shipyard, they typically found better paying welding jobs at other local employers. This evidence clearly establishes that the shipbuilding company was contributing to the costs of general training.

The study of German apprenticeship training by the Bundersinstitut fur Berufsforschung found that in 1980 training costs ranged from a high of 25,200 DM per year for telecommunications technician apprentices to 2400 DM for

*a form of employer financing that ~~is~~ particularly promising that ~~is~~ not just specific but not \$1000 specific*

apprentice gardeners and averaged 10,300 DM or \$5668 per year at 1980 exchange rates. The apprentice's contribution to output, which was netted out to arrive at the above figure, averaged 6700 DM per year (Weiderhold-Fritz 1985). Jones's (1985) study of apprentice training in the engineering industry in Great Britain found that the employer's training costs were 1.31 times the annual payroll costs of a skilled worker and the apprentice's contribution to output (which was netted out in calculating the estimate of employer costs) was 1.26 times the payroll costs of a skilled worker. Thus even major upward revisions of these estimates of the apprentice's contribution to output would not change the basic conclusion that employers appear to be sharing the costs of general training.

In the section that follows we present a formal exposition of a theory of training and compensation packages that predicts the kinds of empirical findings just reviewed. The theory to be presented owes much to Hashimoto's (1981) elegant formulation of how workers and firms share the costs and benefits of investments that are specific to a match between worker and firm. Sorting effects, transfer costs, turnover events as signals, imperfect signaling of the outcomes of general training and differential access to capital markets are all incorporated into one model. Some of the important implications of the model is as follows:

- 0 Anything that contributes to the specificity of the match has the effect of lowering the second period wage below the worker's productivity in the firm and raising the first period wage by a compensating amount. Training in skills specific to the firm is one cause of specificity. Another four are identified: the damage to a worker's reputation from being fired or quitting, the adjustment costs of finding another job and adjusting to it, the improvement in the average productivity of the remaining workers that results from dismissing the least productive, and the sorting effect that results from the exit of those with the best alternative opportunities and those who dislike their current job.
- 0 When elasticities of labor supply are greater for new hires than for trained workers with more than a year or so of tenure at the firm, the time pattern of compensation will reflect the relative rate at which employers and workers discount future earnings and the wage elasticities of voluntary and involuntary turnover. Since the young workers who need general training the most have only limited access to capital markets, they discount the future much more heavily than their employer and as a result compensation tends to be front loaded.

- 0 When employers cannot accurately measure the amount and quality of general OJT that job applicants have received from other employers, workers tend to reduce their investment in general OJT and employers pay some of the incremental costs of investment in technically general OJT. The level of investment in general OJT that results and response to improvements in the quality of the signals of general OJT depends on both discount rates, the separation rate, the proportion of marginal investments in general OJT that are perceived by other employers and the response of turnover to marginal increases in the quality of general training that are not accurately perceived by other employers.
- 0 During the first year on a job a worker's productivity net of training costs grows much more rapidly than the wage even when training is completely general and employers therefore appear to be sharing the costs and benefits of general training.

While some of these results have appeared in earlier papers (eg. Parsons 1972; Feuer 1987), much of the recent wage growth literature appears to ignore the impact of differential access to capital markets, of specific human capital investments other than training and of signaling problems on wage growth and incentives to invest in training (Garen 1988). The purpose of this paper is to point out just how important these effects are by incorporating these factors in a formal model and then by reviewing empirical literature on transition costs and liquidity constraints to show that when reasonable assumptions are made about their magnitude, big changes occur in predicted rates of wage growth during the first year on the job.

### III. THEORY

The firm's training level and wage profile will be analyzed in a simple two period model. Training is assumed to produce two types of skills: general skills ( $g$ ) which are useful at other firms and specific skills ( $h$ ) which are productive only at the firm providing training. The cost of the training  $C(g,h)$  are incurred in the first period and the benefits are received in the second period.

There are two random elements in the model. The first is the utility that a worker can attain by leaving the firm at the beginning of the second period, and the second is the worker's second period productivity in this firm after the training is completed. We assume that wages and productivity in the two periods are the following.

	<u>Worker Productivity</u>	<u>Worker Utility</u>
1st period at the firm	$P$	$W^1$
2nd period at the firm	$P+g+h+\epsilon_0$	$W^2$
2nd period at other firms if quit		$U(g) - T + \epsilon$
2nd period at other firms if fired/laidoff		$U(g) - T - F_b + \epsilon$

where

$P$  is the worker's productivity without training

$g$  is the increment in productivity due to general training

$h$  is the increment in productivity at the firm due to specific training

$\epsilon_0$  is the random factor in productivity in this firm which captures one element of the quality of the match at the training firm

$W^1, W^2$  are the first and second period wages at the firm

$U(g)+\epsilon$  is the utility of the best alternative job if one leaves voluntarily. This depends on the amount of general skill and a random factor which measures from the worker's point of view the quality of the firm-worker match at the alternative firm relative to the match at the training firm.

$T$  is the costs of transition if the change in jobs is initiated by the worker: moving costs, reputational damage from having the quit signal on one's resume, lost income while waiting for the next job to start.

$F_b$  the additional transition costs imposed on the worker over and above  $T$ , if the exit is involuntary: the additional damage to the worker's reputation from being permanently laid off or fired rather than leaving voluntarily, the lost income due to the wait until another job is found.

Turnover decisions are made in two stages. At the end of the first period, the worker makes the first move by deciding whether to quit or to express an intention to stay. If the worker quits, he/she obtains a job which offers a utility level of  $U(g)+T+\epsilon$ . The worker but not his employer learns about  $\epsilon$  at the end of the first period.

Then the firm decides whether to keep or dismiss the worker by comparing the second period wage to the worker's productivity in the firm,  $P+g+h+\epsilon$ . If the worker's productivity is less than the second period wage, the firm will dismiss the worker. The random factor  $\epsilon_0$  is a measure of the quality of the firm-worker match at the current firm. If the worker is dismissed at this

stage he/she will be forced to look for work while unemployed and will incur an additional transition cost of  $F_b$ . Therefore, the worker's first stage decision will take into account the risk of involuntary turnover in the second stage.

At the beginning of the first period neither the worker nor the firm knows the worker's exact productivity in this firm and in other firms. The firm offers wage package  $(W^1, W^2)$  based on information obtained in the, interview and from references and the nature of uncertainties involved, i.e. the probability density function's of  $\epsilon_0$  and  $\epsilon$ . In the first period, the firm trains the worker, taking into account the possible loss of the investment due to a separation in the next period. Training investment takes two forms, investment in firm specific skills and general skills. General training increases the wage that the worker can obtain in alternative employment as well as his productivity in this firm. Workers accept the job offer from this firm if the wage package and training plan are generous enough to attract workers in a competitive labor market. In deciding, the worker takes account of possible gains or losses from a voluntary or involuntary separation. We assume the worker and the firm have the same prior distributions on the uncertainties surrounding the worker's productivity in this firm and worker's income opportunity outside the firm in the second period. Further, we assume that both the firm and the worker are risk neutral.

The firm's objective is to maximize the discounted sum of profit from the two periods by choosing wage rates in two periods,  $W^1$  and  $W^2$ , and an amount of general training,  $g$ , and specific training,  $h$ , subject to the constraint that the wage offer and amount of training are generous enough to attract new hires in a competitive labor market. The firm's expected profit maximization problem when  $\epsilon$  and  $\epsilon_0$  are independent is written as:

$$(1) \quad \text{Max} \quad P - C(g, h) - W^1 + \delta_a [\text{Pr}(S)\text{Pr}(K)(P+g+h+E(\epsilon_0|K)-W^2)]$$

$$g, h, W^1, W^2$$

Subject to the constraint

$$(2) \quad R \leq W^1 +$$

$$\delta_b [\text{Pr}(S)\text{Pr}(K)W^2 + (1-\text{Pr}(S))(U-T+E(\epsilon|Q)) + \text{Pr}(S)(1-\text{Pr}(K))(U-T+E(\epsilon|S)-F_b)]$$

$$\text{or } R \leq W^1 + \delta_b [\text{Pr}(S)\text{Pr}(K)\{W^2 - U + T - E(\epsilon|S) - \frac{1-\text{Pr}(K)}{\text{Pr}(K)}F_b\} + U - T]$$

where

$E(\epsilon_0|K)$  is the conditional expectation of  $\epsilon_0$  given that the firm wishes to keep the worker.

$E(\epsilon|Q)$  is the conditional expectation of  $\epsilon$  given that the worker quits the firm.

$E(\epsilon|S)$  is the conditional expectation of  $\epsilon$  given the worker wishes to stay in the firm.  $E(\epsilon|S) < 0$ .

$\delta_a$  and  $\delta_b$  are the discount factors of the firm and worker, respectively

$\text{Pr}(S)$  is the prior probability the worker is willing to stay with the firm

$\text{Pr}(K)$  is the prior probability the firm is willing to keep the worker

$R$  is the level of expected utility the worker can attain in the competitive labor market.

At the end of the first period, the worker learns what utility can be obtained by taking a job at another firm. Based on this information, the worker's decision rule is:

$$\text{STAY IF } \text{Pr}(K)W^2 + (1-\text{Pr}(K))(U-T-F_b+\epsilon) > U - T + \epsilon$$

The right hand side of the inequality is the utility level of the alternative job. It is affected by the amount of general training that other employers perceive the worker to have obtained,  $g$ , and the costs of making a voluntary transition,  $(T)$ , and the random term,  $(\epsilon)$ , which captures the worker's relative evaluation of two jobs. The left hand side of the inequality is the expected income if he/she wishes to stay at the firm. Note that the expected income of choosing to stay takes into account the risk of being fired or laid off and suffering the additional transition costs  $(F_b)$  that involuntary turnover imposes on the worker. The probability of a worker wishing to stay in the firm,  $\text{Pr}(S)$ , is:

$$(3) \quad \text{Pr}(S) = \text{Pr}\left(\epsilon \leq W^2 - U + T - \frac{1-\text{Pr}(K)}{\text{Pr}(K)}F_b\right) \\ = \Phi\left(W^2 - U + T - \frac{1-\text{Pr}(K)}{\text{Pr}(K)}F_b\right)$$

where  $\Phi$  is the cumulative density function of  $\epsilon$ .

Note that the argument for the cumulative density function,  $\Phi$ , contains the term  $-\frac{1-\text{Pr}(K)}{\text{Pr}(K)}F_b$ , which is minus the odds of being laid off or fired times the additional transition costs,  $F_b$ , that result from involuntary turnover. This implies that if a worker believes there is a high probability of being laid off or fired, he is more likely to quit.

By the end of the first period, the firm knows the worker's productivity in the second period and whether the worker wants to stay. It then lays off or fires the worker if  $P+g+h+\epsilon_0$  is less than the second period wage. Consequently,  $\text{Pr}(K)$  is written as

$$(4) \quad \text{Pr}(K) = \text{Pr}(P+g+h+\epsilon_0 > W^2) \\ = 1 - \Phi_0(W^2 - P - g - h)$$

where  $\Phi_0$  is the cumulative density function of  $\epsilon_0$ .

Denoting the probability density function of  $\epsilon$  and  $\epsilon_0$  by  $\phi$  and  $\phi_0$  the first order condition for the second period wage is written as:

$$(5) \quad 0 = \delta_a \left[ \frac{\partial \text{Pr}(S)}{\partial W^2} \cdot \text{Pr}(K)G_a - \text{Pr}(S)\text{Pr}(K) \right] + \delta_b [\text{Pr}(S)\text{Pr}(K) - \phi_0 \text{Pr}(S)G_b]$$

where  $G_a$  and  $G_b$  are defined as

$$Q_a = P + g + h + E(\epsilon_0|K) - W^2 > 0,$$

$$Q_b = W^2 - (U(g) - T - F_b + E(\epsilon|S)) > 0.$$

$$\frac{\partial \text{Pr}(S)}{\partial W^2} = \phi(1-v)$$

$$v = \frac{\phi_0 - 2F_b}{\text{Pr}(K)} = \frac{\gamma_k f_b}{\text{Pr}(K)}$$

$\gamma_k$  = the elasticity of the firm's keep rate,  $\text{Pr}(K)$ , with respect to the 2nd period wage times minus one.  $\gamma_k = \phi_0 W^2 / \text{Pr}(K) > 0$ .

$f_b = F_b/W^2$  is the ratio of the transition cost if fired to the 2nd period wage.

$Q_a$  is the firm's expected profit on workers who want to stay with the firm and which the firm wants to keep. Alternatively, it is the employer's share of 2nd period quasi-rents.  $Q_b$  is the gain the worker receives from not being dismissed or alternatively the worker's share of 2nd period quasi rents. It is the difference between the second period wage,  $W^2$ , and expected utility if

dismissed,  $(U(g) - T - F_b + E(\epsilon|S))$ . Note that  $\phi_o / \text{Pr}(K)^2$  is the derivative of the odds of being kept with respect to the second period wage. An increase in the second period wage has two effects on the worker's decision to stay. The direct effect increases the desire to stay. The second effect is that it raises the odds of being permanently laid off or fired and incurring the added transition costs  $F_b$ . While this second effect lowers the probability of staying, we may reasonably assume that the total effect of a wage increase on  $\text{Pr}(S)$  is positive, ( i.e.  $0 < v < 1$  ) because the elasticity of the keep rate,  $\gamma_k$ , is not likely to exceed 1 and the extra transition cost of an involuntary termination is probably less than 20 percent of the 2nd period wage.

The first order conditions for specific and general training (h and g) are given by (6) and (7).

$$(6) \quad C_h = \delta_a \text{Pr}(K) [\text{Pr}(S) + (\partial \text{Pr}(S) / \partial h) Q_a] + \delta_b \text{Pr}(S) \phi_o Q_b$$

$$\text{where } C_h = \partial C / \partial h, \quad \partial \text{Pr}(S) / \partial h = \phi v,$$

$$(7) \quad C_g = \delta_a \text{Pr}(K) [\text{Pr}(S) + (\partial \text{Pr}(S) / \partial g) Q_a] + \delta_b [(1 - \text{Pr}(K)) \text{Pr}(S) U_g + \phi_o \text{Pr}(S) Q_b]$$

$$\text{where } C_g = \partial C / \partial g, \quad U_g = \partial U / \partial g, \quad \partial \text{Pr}(S) / \partial g = \phi (-U_g + v)$$

These conditions can be more simply represented by:

$$(6') \quad C_h = \delta_a \text{Pr}(SK) [1 + \gamma_s v q_a] + \delta_b \text{Pr}(SK) \gamma_k q_b$$

$$(7') \quad C_g = \delta_a \text{Pr}(SK) [1 + \gamma_s (-U_g + v) q_a] + \delta_b [(1 - \text{Pr}(SK)) U_g + \text{Pr}(SK) \gamma_k q_b]$$

where

$\text{Pr}(SK) = \text{Pr}(S) \text{Pr}(K)$  is the probability the worker is at the firm in the second period

$q_a = Q_a / W^2$ , the ratio of the firm's quasi rent to the 2nd period wage

$q_b = Q_b / W^2$ , the ratio of the worker's quasi rent to the 2nd period wage

$\gamma_s$  = the elasticity of the worker's stay rate with respect to the 2nd period wage.  $\gamma_s = \phi (1 - v) W^2 / \text{Pr}(S) > 0$ .

Also the optimal wage in the first period,  $W^1$ , is determined so that the constraint (2) is binding. The first order conditions--(5), (6), (7) and (2) with equality constraint--characterize the optimal wage-training package the firm will offer. In what follows, we examine the economic implications of these conditions.

#### Choosing the Wage Structure

The understanding of what determines  $W^2$  will be aided by specifying the income opportunity outside the firm,  $U(g) + \epsilon$ , in more detail. We write  $U(g)$  in the following form:

$$(8) \quad U(g) = P + \hat{g}$$

where  $P$  is the productivity of the worker without the general training received in the first period and  $\hat{g}$  is the increment to the wage offer due to general training. Employers use the interview and the reputation of the previous employer to predict the true value of the general training. The estimate by other employers of the productivity gain due to the original firm's general training is  $\hat{g}$ .

Other potential employers cannot observe the exact amount of human capital that is produced by the training. The signal that provides information on the level of training contains a good deal of noise. Denoting the signal that other employers receive by  $\tilde{g}$ , we assume the following relation:

$$(9) \quad \tilde{g} = g + u$$

where  $u$  is a noise independent of  $g$ .

Given the signal,  $\tilde{g}$ , other firms predict the true level of general skill. Under the quadratic loss function, the best linear predictor of general skill,  $\hat{g}$  is:

$$(10) \quad \hat{g} = E(g|J) + \beta(\tilde{g} - E(g|J)) = E(g|J) + \beta[g - E(g|J)] + \beta u$$

where  $E(g|J)$  is the conditional mean of general human capital of the particular class of job seekers given information set  $J$ .  $J$  represents occupation, industry, and firm size of the previous job and background characteristics of the individual. Therefore,  $U_g$  is given by

$$(11) \quad U_g = \beta = \frac{\text{var}(g|J)}{\text{var}(g|J) + \text{var}(u)} < 1,$$

where  $\text{var}(g|J)$  is the conditional variance of  $g$  given  $J$  (see Leamer pp. 51-55). This implies that a unit increase of general skill results in less than proportional increases in other firms' wage offers.

Substituting (8) into the first order condition for  $W^2$ , rearranging terms and making use of the assumption that competitive equilibrium implies that the expected profit from hiring the marginal worker is zero, the optimal wage

check  
↓

rates for the two periods may be written as follows:

$$(12) W^2 = [P+h+g+E(\epsilon_0|K)] - \theta[h+(g-\hat{g})+T+F_b+E(\epsilon_0|K)-E(\epsilon|S)] - \frac{(\delta_a-\delta_b)W^2}{\delta_a\gamma_s+\delta_a\gamma_k}$$

$$(13) W^1 = P - C(g,h) + \delta_a\text{Pr}(S)\text{Pr}(K)\{\theta[h+(g-\hat{g})+T+F_b+E(\epsilon_0|K)-E(\epsilon|S)] + \frac{(\delta_a-\delta_b)W^2}{\delta_a\gamma_s+\delta_b\gamma_k}\}$$

where  $\theta = \frac{\delta_b\gamma_k}{\delta_a\gamma_s+\delta_b\gamma_k}$  is the employer's share of the costs of specific human capital investments and of quasi rents.

Equation (12) implies that the expected profit from the worker staying with the firm is positive. Since in long run equilibrium, competition among firms brings the expected profit of the firm to zero, the wage rate in the first period must be higher than the worker's productivity net of training cost by a compensating amount. Thus our model predicts that in the early stage of employment, productivity net of training cost grows faster than the wage rate. The firm's net profit is negative in the investment period but the loss is compensated in the second period when the firm receives the return from human capital investment.

The wage offer in the second period is the expected productivity of the worker,  $P+g+h+E(\epsilon_0|K)$ , less the second and third terms in (12). The expression in the second set of brackets is the difference (for those who are kept and want to stay) between the worker's productivity in the firm,  $P+g+h+E(\epsilon_0|K)$ , and the utility of the worker's best alternative job if he/she is laid off or fired,  $P+\hat{g}-T-F_b+E(\epsilon|S)$ . The second term indicates that given the value of  $\theta$ , the following factors reduce the firm's second period wage offer (and also raise the firm's first period wage offer):

- Transition costs if the exit is voluntary, (T)
- The additional transition costs of the worker resulting from being laid off or fired, ( $F_b$ )
- The difference between a worker's true general human capital,  $g$ , and other employer's perception of his general human capital,  $\hat{g}$ . (This could be positive or negative depending upon whether the firm provides more or less general training than is average for that occupation and industry) and
- The average unattractiveness of alternative employment for workers who want to stay, ( $-E(\epsilon|S)$ ).

Anything that raises productivity in the firm but does not raise it outside the firm, will raise the second period wage at the firm. The wage increase is smaller than the rise in productivity, so the firm's profit on the worker in the second period goes up. The two factors that will produce this effect are:

- Specific human capital,  $(h)$ ; and
- The firm's expected gain from having the option of dismissing less productive workers,  $(E(\epsilon_0|K))$ .

Also, other things being equal, second period wage offer declines if  $\theta$ , the employer's share of quasi rents, is increased. Factors that makes  $\theta$  larger are:

- The wage elasticity of the keep rate increases relative to the elasticity of the worker's willingness to stay at the firm. (ie.  $\gamma_k$  is large relative to  $\gamma_s$ ). This could be caused by  $\phi_0$  being large relative to  $\phi$  or by  $v$  being close to 1.
- High incremental transition costs when turnover is involuntary,  $(F_b)$ , result in workers becoming so fearful of dismissals that they prefer contracts in which employers finance a larger share of firm specific investments so as to reduce their risks of dismissal. If  $F_b$  is large relative to  $W^2$ ,  $v$  becomes larger.
- The worker's valuation of future earnings grows relative to the firm's valuation. (ie.  $\delta_b/\delta_a$  becomes larger).

Workers have poorer access to capital markets than employers and also tend to face higher marginal tax rates during the payoff period than the training period. The third term of (12) and (13) represents the effects of their consequent tendency to discount future returns more heavily than employers. Since the error term in the quit relationship does not have a degenerate distribution, the supply of trained labor is not infinitely elastic. Bloch (1979) cross section analysis of turnover in 49 manufacturing industries, for example, found wage elasticities of -1.3 for quit rates and .85 for the layoff rates when the lagged accession rate was included in the model. The supply of untrained labor, however, is assumed to be infinitely elastic at  $R$ . New hires take second period wages into account when evaluating the firm's job offer, however, so the decline in the elasticity of labor supply with the worker's tenure influences the wage structure only when the firm and its workers discount the future at different rates. The compensation

packages reflect the worker's preference for compensation now rather than later. Thus, the third term of (12) and (13) implies that the firm's second period wage will be reduced and the first period wage increased to the extent that:

- The firm's discount factor is large relative to the worker's discount factor, ( $\delta_a - \delta_b$  is large).
- The elasticities of the worker's stay rate,  $\gamma_s$ , and of the firm's keep rate,  $\gamma_k$ , with respect to the firm's second period wage are small.

### Choosing the Level of Training

The first order condition for specific capital, (6'), says that the marginal cost of investment in specific capital is equated to the marginal discounted revenue to the firm, the discount factor times the retention rate and the marginal increase in the stay rate resulting from the reduced probability of being terminated involuntarily times the share of the second period wage that is a quasi rent for the employer, ( $\delta_a \text{Pr}(SK)[1 + \gamma_k v q_a]$ ), plus the discounted marginal benefit to the worker of the specific training. The benefit of specific training to the worker is captured by the second term of (6'). The increased productivity makes the firm less likely to dismiss the worker. This effect is captured by  $\gamma_k$ , the elasticity of the keep rate with respect to the second period wage. In (6')  $\gamma_k$  is multiplied by  $q_b$ , the share of the second period wage that is a quasi rent for the worker.

The first order condition for general training, (7 or 7') characterizes the optimal amount of general training. The marginal cost of general training is equated to the discounted marginal revenue to the firm plus the discounted marginal benefit to the worker. The marginal revenue to the firm from general training has two elements. The first element is the marginal product of a dollar of expenditure on general training for the workers who are going to stay with the firm ( $\text{Pr}(S)\text{Pr}(K)$ ). The second element measures the loss the firm is likely to experience because with given  $W^2$ , quit rates will rise. The higher level of general skill implies better alternative income opportunities for the worker. For a given second period wage, quits will rise by  $\phi(U_g - v)$ . Per quit, the loss the firm experiences is  $\text{Pr}(K)G_a$ --the probability the firm wants to keep the worker times the quasi rent received by the firm from those

workers it keeps.

The marginal benefit of general training to the worker also has two elements. The first element is that, given he or she is leaving the firm, (voluntarily or involuntarily) general training increases the wage offer in other employment. The second element reflects the fact that the increased productivity makes the firm less likely to dismiss the worker. This benefits the worker, and the amount of the benefit is the worker's quasi rent ( $Q_b$ ) multiplied by the probability that the individual wants to stay, ( $\Pr(S)$ ). The worker benefit of reduced risks of dismissal tends to offset the loss the employer experiences from the quits that are induced by the rise in other firm's wage offers.<sup>4</sup>

Substituting the first order conditions for  $W^2$  and  $\beta = U_g$  and rearranging terms, the condition describing the equilibrium level of general human capital is:

$$(14) C_g = \delta_a \Pr(SK) \frac{(1-\beta)}{(1-v)} + \delta_b \beta + \delta_b \Pr(SK) \frac{(1-\beta)}{(1-v)} (\gamma_k q_b - v)$$

Equation (14) implies that investment in general OJT increases with the firm's and the worker's discount factor ( $\delta_a$  and  $\delta_b$ ) and the retention rate, and decreases if the marginal cost schedule shifts up. The derivative of  $C_g$  with respect to  $v$  is given by:

$$(15) \frac{\partial C_g}{\partial v} = (\delta_a - \delta_b [1 - \frac{\gamma_k - (q_b - \frac{f_b}{\Pr(K)})}{(1-v)}]) \frac{\Pr(SK)(1-\beta)\gamma_k}{\Pr(K)(1-v)} + [\delta_a + \delta_b(\gamma_k - v)] \frac{(1-\beta)\partial \Pr(SK)}{(1-v) \partial v}$$

Thus an increase in the cost of firing causes increased investment in both general and specific human capital primarily because of its tendency to reduce turnover.

If other firms fully perceive the quality of training provided by the firm ( $\beta=1$ ), the condition reduces to setting the marginal cost of training ( $C_g$ ) equal to  $\delta_b$ , the worker's discount factor. If other firms cannot perceive differentials in training quality ( $\beta = 0$ ), the condition becomes identical to that for specific human capital. The inability of other firms to perceive all of the firm to firm variations in the amount of general human capital has the effect of dividing the marginal returns to general human capital into two parts. The share of the marginal increase in skill that the worker is assured of getting whether or not he/she stays at the firm ( $\beta$ ) is discounted by the worker's rate of time preference. The share that is

perceived only by the firm that provides the training  $(1-\beta)$  is depreciated by the retention rate and discounted by the employer's internal rate of return. Improvements in the signals of general training (i.e. an increase in  $\beta$ ) will increase investment in general training if:

$$(16) \frac{\partial C_g}{\partial \beta} = \delta_b - \delta_a \frac{\text{Pr}(SK)}{(1-v)} - \delta_b \frac{\text{Pr}(SK)\gamma_k(q_b - \frac{f_b}{\text{Pr}(K)})}{(1-v)} + [\delta_a + \delta_b(\gamma_k - v)] \frac{(1-\beta)}{(1-v)} \frac{\partial \text{Pr}(SK)}{\partial \beta}$$

is greater than zero. This expression will tend to be greater than zero when new hires have high turnover rates, the worker's discount factor ( $\delta_b$ ) is not much smaller than the employer's discount factor ( $\delta_a$ ) and an increase in the visibility of the training to other employers does not lower the retention rate by very much. On the other hand, if turnover rates are not high, worker discount factors are a small fraction of employer discount factors and retention rates drop significantly when training becomes more visible, the inequality is likely to be reversed and an increase in the quality of the signals of skills learned will decrease investment in general OJT.

#### IV. The Magnitude of Transition Costs and the Prevalence of Liquidity Constrained Workers

In most matches between a worker and a firm there is a substantial difference between the average productivity of workers who stick with the firm and the expected utility of alternative employment of those who wish to stay at the firm but are nevertheless terminated involuntarily. This difference, the quasi rent associated with the match, is given by the expression:

$$(17) \text{ Quasi Rent} = [h + T + F_b + E(\epsilon_0|K) - E(\epsilon|S)] = Q_a + Q_b$$

By sharing these quasi rents, both parties try to induce the other to maintain the contract. The 2nd, 3rd, 4th and 5th terms inside the bracketed expression are often quite large. Even when training is entirely general ( $h=0$ ), this makes it optimal for the firm to pay wages which exceed productivity minus training costs in the first period and to offer a wage in the second period which is correspondingly lower than productivity in the second period. In effect, the firm pays part of the costs of general training and the rate of wage growth is considerably below the rate of growth of productivity net of training costs.

#### Transition Costs-Job Search and Reputations

The substantive importance of the points just made depends on just how

large the transition costs,  $T$  and  $F_b$ , are. The transition cost  $T$  is in part adjustment costs such as moving costs and the unhappiness resulting from losing friendships at work. A second reason why  $T$  is expected to be positive is that, for most workers, a quit damages one's reputation. A quit after a short time on the job is likely to be interpreted as a signal of problems that may recur, a lack of commitment to one's job or a high quit propensity. Hollenbeck and Smith's (1984) study of employer reactions to resumes found that the number of quits in the job history had a large negative effect on the rating assigned to the job applicant.<sup>5</sup>

The third term of the quasi rent expression,  $F_b$ , is the additional costs associated with involuntary terminations. Such terminations are very costly for the worker because (1) finding another job takes a great deal of time and is psychologically stressful and (2) a discharge does even more damage to a worker's reputation than a quit. Involuntarily terminated workers seldom have another job lined up so they immediately enter the ranks of the unemployed. Dynarski and Sheffrin (1987) have calculated that the expected length of a spell of unemployment was 10.3 weeks in 1980-81 for the household heads in the Panel Study of Income Dynamics data. Using 1974 CPS data, Clark and Summers (1979) calculated that if unemployed workers did not leave the labor force, it took on average 12.6 weeks for teenagers to find another job and 16.2 for those over 20 years of age to find another job. Blau and Robins' (1985) analysis of longitudinal data from the Employment Opportunity Pilot Projects found that it took on average 25 to 36 weeks for unemployed welfare recipients to find a job and 15 to 20 weeks for unemployed workers not on welfare to find a job.<sup>6</sup> If the termination is a dismissal or a layoff occurring after only a few months on the job, the individual may not be eligible for unemployment insurance. These costs are the natural consequences of involuntary turnover. They have not been generated by a bonding contract.

When they find another job, it typically pays less. In the National Longitudinal Survey, young men who changed employers between 1967 and 1973 subsequent to an involuntary separation experienced a 3 percent decline in their wage rate over the two year measurement period. For the mature men's sample the wage decline was 10 percent. These effects appear to persist for many years. Models were estimated in which dummies for a separation between 1969 and 1971 were used to predict wage growth during 1967-69 and 1971-73 as

well as for 1969-71. The workers who were involuntarily terminated between 1969 and 1971 experienced a sharp deceleration in their wage growth which persisted into 1971-73 (Bartel and Borjas 1981). Analyzing a five year time interval in PSID data, Ruhm (1987) found that involuntary terminations lowered the wage growth of male household heads by 13.6 percent but had no significant effect on the wage growth of female household heads. These wage reductions arise partly because the individual's specific human capital is now worthless, partly because of Lazear type bonding contracts (if they do indeed exist for young workers) and partly because quits and dismissals are signals which damage the worker's reputation.

The unemployment durations and wage reductions reported above are for all involuntarily terminated workers as a group. While no study reported separate estimates of the effects of discharges and layoffs, one suspects that those discharged experience longer spells of unemployment and bigger wage declines than those laid off. <sup>any</sup> Most employers contact a job candidate's previous employers prior to making a final hiring decision and are, therefore, <sup>may</sup> likely ~~to~~ find out about the discharge. If the job seeker does not include the employer who discharged him in his employment history, there is a long stretch of nonemployment that must somehow be explained. Discharged employees are reported to be 25 percent less productive than the workers who end up staying with a firm for a year or more (Bishop 1988). In some cases this productivity disadvantage is specific to the match, but it is difficult for other employers to assess whether that is the case so if they know a job applicant was fired by a previous employer, they are unlikely to hire him/her.

Since the costs of an involuntary termination are so severe, job seekers would be expected to prefer employers and employment contracts which minimize risks of discharge and layoff and which promise that bad recommendations will not be given. Promises not to give bad oral recommendations are not enforceable, however, so the worker's only recourse is to seek contracts which minimize the risk of dismissal and layoff. Seniority protection, grievance procedures and enforceable promises to dismiss a worker only after certain procedures are followed are one way to accomplish this but in nonunion settings there are always ways of forcing an unwanted employee out. A more reliable way of reducing the risk of dismissal and layoff is to have the employer put up a bond which is forfeited if the worker is laid off or

dismissed. Workers, therefore, prefer employment contracts containing a front loaded compensation package in which the employer <sup>in effect</sup> pays ~~for all of~~ the costs of specific training and contributes toward the costs of general training. There are, of course, countervailing forces such as the desire to reduce the number of trained employees who quit, so the form of the contract depends on how the various forces balance out.

The fourth and fifth terms of the bracketed expression capture the effect of sorting on the quasi rent.<sup>7</sup> As the worker and the firm learn more about the quality of the match, the unsuccessful matches tend to be terminated. The workers who discover that they do not like the job or that they have better opportunities elsewhere quit and the workers who are the least productive on the job are fired or induced to quit. Thus, even when training develops only general skills and there are no transition costs, the expectation of the difference between the productivity of workers who stick with the firm and their evaluation of the next best alternative--the quasi rent attached to the match--is considerably greater for long tenure workers than for recent hires.<sup>8</sup> Sorting's effect on average productivity has been estimated to be at least 2.6 percent between the fourth and seventeenth month on the job at small and medium sized non-union firms (Bishop 1988). Sorting is also generated by differences in tastes for the nonpecuniary features of a job and differences in alternative opportunities. Consequently, sorting probably causes quasi rents equal to 4 to 8 percent of yearly compensation. Transition costs probably generate quasi rents equal to another 5 or 10 percent of compensation.<sup>8</sup> The growth of these quasi rents during the first year causes wage growth to diverge from the growth of productivity net of training cost. If  $(T + F_b + E(\epsilon_0|K) - E(\epsilon|S))$  is 10 percent of compensation in the second period, a  $\theta$  (the employer's share of specific investments) of .5 implies that second period wages are reduced by 5 percent and a  $\theta$  of .8 implies they are reduced by 8 percent. If the investment and payoff periods are of equal length, wage growth will be  $\theta[1 + \delta_a \text{Pr}(S)\text{Pr}(K)](.10)$  less than the growth of productivity net of training costs. With a discount factor of .9, a retention ratio of .7, wages rise 8.1 percent less than productivity net of training costs when  $\theta$  is .5 and rise 13.1 percent less when  $\theta$  is .8. If the payoff period is twice as long as the investment period, the reduction in percentage growth is 11.2 percent when  $\theta$  is .5 and 18.3 percent when  $\theta$  is .8. Clearly,

transition costs and sorting effects can have significant effects on the time pattern of compensation in the first year or so of a job.

#### Liquidity Constraints

The second force tending to lower wage growth below the growth of productivity net of training costs is the fact that many workers are liquidity constrained while their employers are not. The young workers who have the greatest need for general training are the most likely to be constrained. Half of households headed by someone under the age of 25 have less than \$746 in financial assets and 19 percent have no financial assets at all. Half of households headed by someone between 25 and 34 have less than \$1514 in financial assets and 13 percent have none (Survey of Consumer Finances 1984). Subsidized or guaranteed student loans are not available to finance on-the-job training and banks will not lend money for this purpose without collateral. Borrowing against the equity in one's home is a possibility for some but only 34 percent of households with heads under the age of 35 own a home and many of the houses have been owned for only a short while, so the equity that can be borrowed against is small. Even with collateral, the loan will be at an interest rate that exceeds the interest rates charged businesses. Studies of the willingness of consumers to substitute consumption over time have all concluded that the intertemporal elasticity of substitution is no higher than one and most studies conclude it is .5 or below (Friend and Blume 1975; Hall 1988; Hubbard and Judd 1986). A substitution elasticity of .5 implies that reducing a liquidity constrained worker's wage by one half (in order to pay for general training) roughly quadruples the worker's marginal utility of consumption. Such a worker would be willing to give up four dollars of future income in return for one dollar of current income. The liquidity constraint phenomenon has no effect on the wage profile of jobs with no on-the-job training and which, therefore, have a flat productivity profile. Where significant general training is occurring, however, it comes very much into play and tends to result in an employment contract in which the employer shares the costs of general training.

To properly represent the liquidity constraint phenomenon just described, in a formal model, an intertemporal utility function should be maximized subject to a borrowing constraint. This, however, so complicates the presentation of the other results of the model, it was decided to make the

much simpler assumption that workers have access to the capital market but at a substantially higher interest rate than firms.

#### Progressive Taxation

In addition, the progressive nature of the personal income tax means that workers face higher marginal tax rates on the fruits of training investments than they are paying when they incur the costs of such investments. Firms, on the other hand, train continuously, so the marginal tax rates faced when the costs of training are incurred and expensed are no different from those faced during the payoff period.

These two factors result in firms being more willing than workers to trade off future earnings for present earnings. The compensation packages that result from the asymmetric access to capital markets and the progressive tax structure reflect the worker's strong preference for compensation now rather than later. If, for example,  $\delta_a = .9$ ,  $\delta_b = .75$  and both  $\gamma_a$  and  $\gamma_b = 1$ , the second period wage is reduced by 9.1 percent. If the two time periods are of equal length, the first period wage is increased by 5.7 percent and wage growth is reduced by 14.8 percent. If the wage elasticity of keep rates and stay rates is doubled to 2, then the wage growth effect is cut in half to 7.4 percent. If the payoff period is double the length of the investment period, the wage growth is reduced by 20.5 percent when wage elasticities,  $\gamma_a$  and  $\gamma_b$ , are 1 and 10.2 when wage elasticities are 2. In effect, firms offer new hires a loan that will be canceled if a separation occurs. Firms do not require repayment of the loan when separations occur for the same reasons that banks do not offer large unsecured loans without a government guarantee of payment. The administrative costs of obtaining repayment are extremely high and bankruptcy is a real option for someone with zero assets.

#### V. Summary and Conclusions

Becker's statement that the employee must always pay the full costs of general on-the-job training is apparently not necessarily true. A model has been presented in which employers find it in their interest to front load compensation and thereby share the costs of general training. This occurs because liquidity constraints prevent workers from financing their own general training without unacceptably large reductions in consumption and because sources of job-worker match specificity unrelated to training such as sorting, costs of finding a new job and the reputational damages of turnover results in

workers strongly preferring front loaded compensation packages.

If these conclusions are true, turnover becomes a much more important determinant of training investments than previously thought. In the standard model, a worker's propensity for turnover influences the amount of specific training supplied but not the amount of general training that is undertaken. If employers are financing some of the costs of general training, however, worker's with high turnover propensities are likely to find it hard to obtain jobs that offer general as well as specific training. The high rates of turnover of American youth probably help explain why investments in both specific and general on-the-job training are lower in this country than in Japan and Germany where turnover rates are considerably lower. Turnover rates are endogenous, however. If specific training were increased, they would go down. Mincer and Higuchi (1988) suggest that the heavy training investment in Japan is due to its high rate of productivity growth. While this is certainly a contributing factor, high rates of productivity growth are a consequence as well as a cause of training investment. One must look outside the turnover-specific training-productivity growth feedback loops to find the ultimate causes of the low levels of investment in training in the US relative to Japan and West Germany.

The theory outlined above suggests a number of possible causes for the discrepancies. The most obvious explanation of the heavier investment in training by Japanese corporations is the very low costs of capital they face (Japanese  $\delta_a$ 's are higher than American  $\delta_a$ 's). The fact that Japanese companies operating in the US spend more on training than American companies in the same industry is support for this hypothesis (Mincer and Higuchi 1988). A second possible explanation is that the Japanese workers are better educated and consequently faster learners (eg.  $C_g$  and  $C_h$  are lower in Japan).

A third potential cause of the difference is national differences in the variance of  $\epsilon$  and  $\epsilon_0$  or in the mean levels of  $T$  and  $F_b$  that result in major differences in turnover propensities. One of the most important determinants of these two variances is how well informed the employer and the worker are when the match is ~~is~~ first arranged. If they know alot about each other, their will be few surprises, so the variances of  $\epsilon$  and  $\epsilon_0$  will be small. Japanese companies invest much more in the selection of their blue collar employees than American companies (Rosenbaum and Kariya 1987; Koenig 1987). In

the US their are major institutional barriers to the free flow of information about job applicants--such as EEO testing guidelines, the reluctance of high schools to send out transcripts and the threat of suit if bad recommendations are given. This results in large variances for  $\epsilon$  and  $\epsilon_0$ , high turnover and reduced incentives to invest in both general and specific training.

Transition costs might also be higher in some countries than others. In Germany lay offs are not strickly based on seniority. Job performance is an important determinant of who is laid off, so laid off workers become stigmatised (ie  $F_b$  is high). The best Japanese employers hire straight out of high school and are said to discriminate against those with work experience. The reverse prevails in the US. Quitting is probably much less stigmatizing in the US than in Japan. If these characterizations are correct, lower turnover is the result and this in turn raises the payoff to employer investments in both specific and general training and this in turn contributes the high productivity growth rates of Japan and Germany.

An important reason why employers in Germany, Austria and Switzerland invest more heavily in employer training than in the US is their strong apprenticeship systems. The standardized curriculums and the proficiency exam at the end of the apprenticeship mean that the quality and nature of the training is well signaled to employers. The result is that the worker can count on benefiting from doing a good job in their apprenticeship even if the training employer does not keep them on. Since the future payoff is certain, German apprentices are willing to start out at a wage that is only about one-quarter of the wage they will be able to command at the end of the apprenticeship. If the apprentices were adults, they could not afford to accept so low a wage. They are teenagers, however, who by living at home are heavily subsidized by their parents. Consequently, the liquidity constraint that is such a barrier to heavy investments in general training in the US is much less of a problem in Germany.

## Footnotes

1. Becker clearly recognized the existence of liquidity constraints in his 1962 paper. "Since employer specific skills are part of the intangible assets or good will of firms and can be offered as collateral along with tangible assets, capital would be more readily available for specific than for general investments (p.42)." He did not, however, explicitly analyze how such constraints might influence the tenure profile of wages and thus induce employers to share the costs of general training. Parsons (1972) points out that "The worker's ...discount rate will affect the firm's choice of wage policies....It can be shown that firms will decrease the worker's share of specific investment as the workers discount the future more heavily (p.1129)."
2. Included in the array were: achievement test scores, GPA, dummies for an A or B grade in business and trade courses, an indicator of deportment in high school, the number of vocational credits, the number of academic credits, hours spent in homework, hours spent in jobs for pay, hours spent watching TV, the number of leadership roles, number of extracurricular activities, attitudes toward school work and working for pay, time spent reading for pleasure, a proxy for study habits, self esteem, internal locus of control, a dummy for dropped out during the senior year and the standard set of family background variables--parental income, education, and occupation and number of siblings. Also included among the controls were two dummy variables for handicapping conditions, dummies for currently in the active duty military or the reserves, and five variables describing college attendance since leaving high school. The only characteristics of the job included in the models were the number of different types of training received, total hours of such training, its square, total months in the job and its square.
3. In the adverse selection models of Salop and Salop (1976) and Nickell (1976) workers have information not available to firms on how likely they are to quit, so since turnover is costly, some employers attempt to attract those with low quit probabilities by imposing a hiring fee (through a below market starting wage) and raising the wage level in subsequent periods. The equilibrium wage pattern results in the worker paying all the costs and receiving all the benefits of investments in specific human capital and in wage rates which rise in step with gains in productivity net of training costs.
4. Studies of quit and layoff rates obtain wage elasticity estimates that range between 2 and .3 (Ehrenberg and Smith 1987; Bishop 1981 Chapter 8). This implies that the elasticities of stay and keep rates are even lower and that  $\delta_a \gamma_s (-U_g + v) q_a$  and  $\delta_b \gamma_k q_b$  are small.
5. The existence of these transition costs helps explain why so few jobseekers take stopgap jobs while they search for work and why so few new hires continue their job search after having accepted a job offer. Search theory attributes this behavior to increases in search costs when

one becomes employed. For some labor markets this explanation rings true, but in so many cases the unemployed worker spends so little time in active search, it is hard to see how having a 40 hour a week job can substantially interfere with search of such low intensity. It is possible, however, for reputational effects to operate in the opposite direction (ie. for  $T$  to be negative). Taking a very prestigious job [eg. an Assistant Professor position at MIT] may so enhance reputation and improve contacts that the distribution of job offers will shift up. Employers whose reputation is such that taking a job there enhances the worker's marketability can achieve target compensation levels,  $R$ , at lower cost and will typically find it optimal to backload their compensation package.

6. For most jobs, the firm's expected costs of recruiting and selecting a replacement if there is an unanticipated quit are considerably smaller than a worker's costs of finding another job if terminated involuntarily. When small and medium sized firms hire for a nonsupervisory position, they consider on average only nine applications, interview only five of the applicants and devote a total of only 10 hours of staff time to the task of filling one position. New positions are filled an average of 16 days after beginning the search. In 55 percent of the cases the firm had advance notice of the opening and so the job was not uncovered during much of the search.
7. The sorting effect may be thought of as the return to investment in information about the quality of the match. The firm learns about the trainability and productivity of the employee and the worker learns about conditions of work, the friendliness of coworkers and the quality of supervision and training and about his/her talent and taste for the work. After this information generates some separations, the employer and remaining workers receive a return on their investment in match specific knowledge.
8. Assuming a yearly  $\delta_a = .75$ , unemployment spells of one-third of a year and a UI replacement rate of .4, the ratio of the search costs resulting from an involuntary termination to the present discounted value of future wage payments is  $(1-.4)*.333/[1/(1-.75)] = .05$ . The wage reduction that results from an involuntary termination is in part due to the signal that it transmits. It would not be unreasonable for this signaling effect to lower the worker's wage in subsequent jobs by 5 percent.

Bibliography

- Bartel, Ann P. and Borjas, George J. "Wage Growth and Job Turnover." In Studies in Labor Markets, Sherwin Rosen, Ed., The University of Chicago Press, 1981, pp. 65-90.
- Becker, Gary. "Investment in Human Capital: A Theoretical Analysis." Journal of Political Economy, Vol. 70, No. 5 pt.2, October 1962, 9-50.
- Bishop, John. "The Recognition and Reward of Employee Performance." Journal of Labor Economics, Vol. 5, October 1987, pp. S36-S56.
- Bishop, John. "Match Quality, Turnover and Wage Growth." Center for Advanced Human Resource Studies Working Paper # 88-03, Cornell University, Ithaca, New York, 1988
- Bishop, John; Hollenbeck, Kevin; Kang, Suk and Wilke, Richard. Training and Human Capital Formation. Columbus: The National Center for Research in Vocational Education, The Ohio State University, 1985.
- Bishop, John and Kang, Suk. "On-the-Job Training/ Sorting: Theory and Evidence" National Center for Research in Vocational Education, Ohio State University, 1984.
- Blau, David M. and Robins, Philip K. "A Study of Turnover in Low Wage Labor Markets," February 1985, ASPE Project Report.
- Bloch, Farrell E. "Labor Turnover in U. S. Manufacturing Industries." Journal of Human Resources, Vol. 14, No. 2, Spring 1979, pp.236-46
- Clark, Kim and Summers, Lawrence. "Labor Market Dynamics and Unemployment: A Reconsideration." Brookings Papers on Economic Activity, The Brookings Institution, Washington, DC, 1979, p. 13-73.
- Dynarski, Mark and Sheffrin, Steven M. "New Evidence on the Cyclical Behavior of Unemployment Durations." In Kevin Lang and Jonathan S. Leonard (Eds.) Unemployment and the Structure of Labor Markets, 1987, pp. 164-185.
- Feuer, M., Glick, H., and Desai, A. "Is Firm-Sponsored Education Viable?" Journal of Economic Behavior and Organization, March 1987, Vol. 8, No. 1.
- Friend, Irwin and Blume, Marshal. "The Demand for Risky Assets." American Economic Review, Vol 65, December 1975, pp. 900-922.
- Glick, Henry A. and Feuer, Michael J. "Employer-Sponsored Training and the Governance of Specific Human Capital Investments." Quarterly Review of Economics and Business, Vol. 24, No. 2, Summer 1984.
- Hall, Robert. "Intertemporal Substitution in Consumption." Journal of

- Political Economy, Vol. 96, NO. 2, April 1988, pp. 339-357.
- Hashimoto, M. "Firm-Specific Human Capital as a Shared-Investment." American Economic Review, 71, no. 3, March 1981, pp. 475-482.
- Hashimoto, M., and Yu, B. "Specific Capital, Employment and Wage Rigidity." Bell Journal of Economics, 11, no. 2, 1980: 536-549.
- Hollenbeck, K., and Smith B. The Influence of Applicants' Education and Skills on Employability Assessments by Employers. Columbus: The National Center for Research in Vocational Education, The Ohio State University, 1984.
- Hubbard, R. Glenn and Judd, Kenneth L. "Liquidity Constraints, Fiscal Policy and Consumption." Brookings Papers on Economic Activity, 1986, # 1, pp. 1-60.
- Jones, Ian. "Trainee Pay and the Costs of Vocational Training in Britain and West Germany." Social Forces: Supplement on Youth Pay and Employers' Recruitment Practices for Young People in the Community, Commission of the European Communities, Directorate-Generale for Employment, Social Affairs and Education, 1985.
- King, S. D. M. Training within the Organization. London, 1964.
- Koenig, Richard. "Toyota takes Pains and Time, Filling Jobs at its Kentucky Plant." Wall Street Journal, December 1 1987, p. 1.
- Lazear, Edward P. "Agency, Earnings Profiles, Productivity and Hours Restrictions." American Economic Review, Vol. 71, September 1981, p. 606-620.
- Nickell, S. J. "Wage Structure and Quit Rates." International Economic Review, Vol. 17, February 1976, p. 191-203.
- Noll I., Beicht U., Boll G., Malcher W. and Wiederhold-Fritz S. Nettakosten der Betrieblichen Ber Ufsbildung Schriften Berufsbildungsforschung, Band 63. Beuth Verlag GMBH, Berlin, 1984.
- Parsons, Donald. "Specific Human Capital: An Application to Quit Rates and Layoff Rates." Journal of Political Economy, Vol. 80, No. 6, Nov/Dec 1972, pp. 1120-1141.
- Parsons, Donald. "Wage Determination in the Post Training Period." Center for Human Resource Research, Ohio State University, 1985.
- Rosenbaum, James and Kariya, Tobe. "Market and Institutional Mechanisms for the High School to Work Transition in Japan and the U.S. Paper given at the August 1987 American Sociological Association Meeting.
- Ruhm, Christopher J. "The Economic Consequences of Labor Mobility." Industrial and Labor Relations Review, Vol. 41, No. 1, October 1987,

p. 30-42.

Ryan, Paul. "The Costs of Job Training for a Transferable Skill." The British Journal of Industrial Relations, Vol. 18, # 3, 1980, pp. 334-352.

Salop, Joanne and Salop, Steven. "Self Selection and Turnover in the Labor Market." Quarterly Journal of Economics, Vol. 91, November 1976, p. 619-627.

"Survey of Consumer Finances, 1983." Federal Reserve Bulletin. Vol 70, September 1984, p. 686 and December 1984, p. 863.

Talbot, J. R. and Ellis, C. D. Analysis and Costing of Company Training. London, 1969.

Weiderhold-Fritz, Susanne. "Is There a Relationship between Cost of In-Company Vocational Training and the Offer of Training Places in the Federal Republic of Germany." Social Forces: Supplement on Youth Pay and Employers' Recruitment Practices for Young People in the Community, Commission of the European Communities, Directorate-Generale for Employment, Social Affairs and Education, 1985.

## APPENDIX. DERIVATION

Derivatives of Probabilitieswith respect to  $W^2$ :

$$\partial \text{Pr}(K) / \partial W^2 = -\phi_0 \quad \partial \text{Pr}(S) / \partial W^2 = \phi \cdot \left(1 - \frac{\phi_0}{\text{Pr}(K)^2} F_b\right) = \phi(1-v)$$

with respect to  $g$ :

$$\partial \text{Pr}(K) / \partial g = \phi_0 \quad \partial \text{Pr}(S) / \partial g = \phi \cdot \left(-U_g + \frac{\phi_0}{\text{Pr}(K)^2} F_b\right) = \phi(-U_g + v)$$

with respect to  $h$ :

$$\partial \text{Pr}(K) / \partial h = \phi_0 \quad \partial \text{Pr}(S) / \partial h = \phi \cdot \left(\frac{\phi_0}{\text{Pr}(K)^2} F_b\right) = \phi v$$

Derivatives of the Conditional Expectations $E(\epsilon_0|K)$ 

$$\text{Definition: } E(\epsilon_0|K) = \int_{W^2-P-g-h}^{\infty} t \phi_0(t) dt / \text{Pr}(K)$$

$$\begin{aligned} \partial E(\epsilon_0|K) / \partial W^2 &= \frac{-(W^2-P-g-h)\phi_0}{\text{Pr}(K)} - \frac{\int t \phi_0 dt}{\text{Pr}(K)^2} \cdot \frac{\partial \text{Pr}(K)}{\partial W^2} = \frac{\phi_0}{\text{Pr}(K)} [- (W^2-P-g-h) + E(\epsilon_0|K)] \\ &= \frac{\phi_0}{\text{Pr}(K)} \mathcal{Q}_a \end{aligned}$$

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$$\frac{\partial E(\epsilon_0|K)}{\partial W^2} = \frac{\phi_0}{\text{Pr}(K)} \mathcal{Q}_a \quad \frac{\partial E(\epsilon_0|K)}{\partial g} = - \frac{\phi_0}{\text{Pr}(K)} \mathcal{Q}_a \quad \frac{\partial E(\epsilon_0|K)}{\partial h} = - \frac{\phi_0}{\text{Pr}(K)} \mathcal{Q}_a$$


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 $E(\epsilon|S)$ 

$$\text{Definition: } E(\epsilon|S) = \int_{-\infty}^{W^2-U+T - \frac{1-\text{Pr}(K)}{\text{Pr}(K)} F_b} t \cdot \phi(t) dt / \text{Pr}(S)$$

$$\partial E(\epsilon|S) / \partial W^2 = \left\{ \frac{\{W^2-U+T - (1-\text{Pr}(K))/\text{Pr}(K) \cdot F_b\} \phi}{\text{Pr}(S)} - \frac{\int t \phi dt}{\text{Pr}(S)^2} \cdot \phi \right\} \cdot (1-v)$$

$$= \frac{\phi}{\Pr(S)} \{W^2 - U + T - (1 - \Pr(K)) / \Pr(K) \cdot F_b - E(\epsilon|S)\} \cdot (1-v) = \frac{\phi}{\Pr(S)} Q_b^* (1-v)$$

$$\frac{\partial E(\epsilon|S)}{\partial W^2} = \frac{\phi}{\Pr(S)} Q_b^* (1-v) \quad \frac{\partial E(\epsilon|S)}{\partial g} = \frac{\phi}{\Pr(S)} Q_b^* (-U_g + v) \quad \frac{\partial E(\epsilon|S)}{\partial h} = \frac{\phi}{\Pr(S)} Q_b^* \cdot v$$

Differentiation of (\*)  $\Pr(S)\Pr(K) \cdot G_a$

$$(G_a = \Gamma + g + h + E(\epsilon_0|K) - W^2)$$

$$\frac{\partial Q_a}{\partial W^2} = \frac{\phi_0}{\Pr(K)} Q_a - 1 \quad \frac{\partial Q_a}{\partial g} = 1 - \frac{\phi_0}{\Pr(K)} Q_a \quad \frac{\partial Q_a}{\partial h} = 1 - \frac{\phi_0}{\Pr(K)} Q_a$$

w.r.t.  $W^2$

$$\begin{aligned} & \{\partial \Pr(S) / \partial W^2 \cdot \Pr(K) + \partial \Pr(K) / \partial W^2 \cdot \Pr(S)\} Q_a + \Pr(S) \Pr(K) \cdot \frac{\partial Q_a}{\partial W^2} \\ &= \{\partial \Pr(S) / \partial W^2 \cdot \Pr(K) - \phi_0 \Pr(S)\} Q_a + \Pr(S) \cdot \Pr(K) \left( \frac{\phi_0}{\Pr(K)} Q_a - 1 \right) \\ &= \partial \Pr(S) / \partial W^2 \cdot \Pr(K) \cdot Q_a - \Pr(S) \cdot \Pr(K) \end{aligned}$$

w.r.t.  $g$

$$\partial \Pr(S) / \partial g \cdot \Pr(K) \cdot Q_a + \Pr(S) \cdot \Pr(K)$$

w.r.t.  $h$

$$\partial \Pr(S) / \partial h \cdot \Pr(K) \cdot Q_a + \Pr(S) \cdot \Pr(K)$$

Differentiation of (\*\*)  $\Pr(S)\Pr(K)Q_b^*$

$$(Q_b^* = W^2 - U + T - E(\epsilon|S) - (1 - \Pr(K)) / \Pr(K) \cdot F_b = Q_b - \frac{F_b}{\Pr(K)})$$

$$\frac{\partial Q_b^*}{\partial W^2} = 1 - \frac{\phi}{\Pr(S)} Q_b^* (1-v) - v = (1-v) \left( 1 - \frac{\phi}{\Pr(S)} Q_b^* \right)$$

$$\frac{\partial Q_b^*}{\partial g} = -U_g - \frac{\phi}{\Pr(S)} Q_b^* (-U_g + v) + v = (-U_g + v) \left( 1 - \frac{\phi}{\Pr(S)} Q_b^* \right)$$

$$\frac{\partial Q_b^*}{\partial h} = -\frac{\phi}{\Pr(S)} Q_b^* \cdot v + v = v \left( 1 - \frac{\phi}{\Pr(S)} Q_b^* \right)$$

w.r.t.  $W^2$

$$\begin{aligned}
& \{\partial \text{Pr}(S)/\partial W^2 \cdot \text{Pr}(K) + \partial \text{Pr}(K)/\partial W^2 \cdot \text{Pr}(S)\} \cdot Q_b^* + \text{Pr}(S)\text{Pr}(K) \cdot (1-v) \left(1 - \frac{\phi}{\text{Pr}(S)} Q_b^*\right) \\
&= \{\phi(1-v) \cdot \text{Pr}(K) + \partial \text{Pr}(K)/\partial W^2 \cdot \text{Pr}(S)\} \cdot Q_b^* + \text{Pr}(S)\text{Pr}(K) \cdot (1-v) \left(1 - \frac{\phi}{\text{Pr}(S)} Q_b^*\right) \\
&= \partial \text{Pr}(K)/\partial W^2 \cdot \text{Pr}(S) \cdot Q_b^* + \text{Pr}(S)\text{Pr}(K) \cdot (1-v) \\
&= -\phi_0 \text{Pr}(S) Q_b^* + \text{Pr}(S)\text{Pr}(K)(1-v) \\
&= -\phi_0 \text{Pr}(S) \left(Q_b - \frac{1}{\text{Pr}(K)} F_b\right) + \text{Pr}(S)\text{Pr}(K) - \text{Pr}(S)\text{Pr}(K)v \\
&= \text{Pr}(S)\text{Pr}(K) - \phi_0 \text{Pr}(S) Q_b
\end{aligned}$$

w.r.t. g

$$\phi_0 \text{Pr}(S) Q_b - \text{Pr}(S)\text{Pr}(K) U_g$$

w.r.t. h

$$\phi_0 \text{Pr}(S) Q_b$$

Since the objective function and constraint are given by

$$\text{Max } P - C(g, h) - W^1 + \delta_a \frac{[\text{Pr}(K)\text{Pr}(S)Q_a]}{(*)}$$

$$\text{s.t } R \leq W^1 + \delta_b \frac{[\text{Pr}(S)\text{Pr}(K)Q_b^* + U - T]}{(**)}$$

we can obtain (5'), (6'), and (7') by substituting the above results.

Derivation of (13')

Denote  $K = \text{Pr}(K)$ ,  $S = \text{Pr}(S)$ ,  $K' = \partial \text{Pr}(K)/\partial W^2$ ,  $S' = \partial \text{Pr}(S)/\partial W^2$ . The foc for  $W^2$  is written as

$$\delta_a [S' \cdot K \cdot Q_a - S \cdot K] + \delta_b [S \cdot K + K' \cdot S \cdot Q_b] = 0$$

$$Q_a = P + g + h + E(\epsilon_0 | K) - W^2 = X_1 - W^2, \quad X_1 = P + g + h + E(\epsilon_0 | K)$$

$$Q_b^* = W^2 - P - g + T - E(\epsilon | S) + F_b = W^2 + X_2, \quad X_2 = P + g - T + E(\epsilon | S) - F_b$$

Define  $\hat{S} = S'/S (>0)$  and  $\hat{K} = K'/K (<0)$ , and dividing through by  $S \cdot K$  the foc is rewritten as

$$\delta_a [\hat{S} \cdot (X_1 - W^2) - 1] + \delta_b [1 + \hat{K}(W^2 + X_2)] = 0$$

$$\Rightarrow W^2(\delta_a \hat{S} - \delta_b \hat{K}) = \delta_a \hat{S} \cdot X_1 - \delta_a + \delta_b + \delta_b \hat{K} \cdot X_2$$

$$\Rightarrow W^2 = X_1 - \theta(X_1 - X_2) - \frac{\delta_a - \delta_b}{\delta_a \hat{S} - \delta_b \hat{K}},$$

$$\text{where } \theta = \frac{-\delta_b \hat{K}}{\delta_a \hat{S} - \delta_b \hat{K}} = \frac{\delta_b \gamma_K}{\delta_a \gamma_S + \delta_b \gamma_K}, \text{ and}$$

$$\gamma_K = -\hat{K} \cdot W^2, \gamma_S = \hat{S} \cdot W^2 \text{ (elasticities)}$$

Also, using elasticities

$$\frac{\delta_a - \delta_b}{\delta_a \hat{S} - \delta_b \hat{K}} = \frac{\delta_a - \delta_b}{\delta_a \gamma_S + \delta_b \gamma_K} \cdot W^2, \text{ and}$$

$$X_1 - X_2 = (g - \hat{g}) + h + E(\epsilon_0 | K) + T - E(\epsilon | S) + F_b$$

Thus

$$(13') W^2 = P + g + h + E(\epsilon_0 | K) - \theta [(g - \hat{g}) + h + E(\epsilon_0 | K) + T - E(\epsilon | S) + F_b] - \frac{\delta_a - \delta_b}{\delta_a \gamma_S + \delta_b \gamma_K} \cdot W^2$$

#### Derivation of (14') and (15')

The foc for  $g$  and  $W^2$  are

$$(7') C_g = \delta_a [\phi \cdot (v - \beta) \cdot K \cdot Q_a + S \cdot K] + \delta_b [(1 - S \cdot K)\beta + \phi_0 S \cdot Q_b],$$

$$(5') 0 = \delta_a [\phi \cdot (1 - v) \cdot K \cdot Q_a - S \cdot K] + \delta_b [S \cdot K - \phi_0 \cdot S \cdot Q_b].$$

Adding RHS of (5') to (6') we obtain (14')

$$(14') C_g = \delta_a \phi (1 - \beta) \cdot K \cdot Q_a + \delta_b \{(1 - SK)\beta + SK\}.$$

An alternative expression is obtained by removing  $Q_a$  from (7') (This corresponds to old (14')). Multiply  $(v - \beta)/(1 - v)$  to (5') and subtract the result from (6').

$$(14'') C_g = \delta_a S \cdot K \cdot \left(\frac{1 - \beta}{1 - v}\right) + \delta_b \left\{ \beta + \phi_0 S \cdot Q_b \left(\frac{1 - \beta}{1 - v}\right) - \frac{v(1 - \beta)}{1 - v} SK \right\}.$$

Also, the expression for  $C_h$  can be modified by substituting the foc for  $W^2$ . Again addition of the RHS of (5') to the foc for  $h$  (6'):

$$(6') C_h = \delta_a [S \cdot K + \phi \cdot v \cdot K \cdot Q_a] + \delta_b \phi_0 S \cdot Q_b,$$

$$(5') \quad 0 = \delta_a [\phi \cdot (1-v) \cdot K \cdot Q_a - S \cdot K] + \delta_b [S \cdot K - \phi_0 \cdot S \cdot Q_b]$$

yields, .

$$(15') \quad C_h = \delta_a \phi \cdot K \cdot Q_a + \delta_b S \cdot K,$$

or removal of  $Q_a$  yields

$$(15'') \quad C_h = \delta_a S \cdot K \left( \frac{1}{1-v} \right) + \delta_b \left[ \underbrace{-S \cdot K \left( \frac{v}{1-v} \right) + \phi_0 S \cdot Q_b \left( \frac{1}{1-v} \right)} \right]$$

$$\parallel$$

$$\frac{\phi_0}{1-v} Pr(S) \cdot Q_b^*$$