RATIONAL RECONSTRUCTIONS OF SOCIETY

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The dissertation contains four stand-alone studies, chapters 2 through 5. In chapter 1, I highlight commonalities among the studies.

In chapter 2, I consider the principle of structural balance – "The friend of a friend is a friend, the enemy of a friend is an enemy, the friend of an enemy is an enemy, and the enemy of an enemy is a friend." I consider Harary's (1954) result that this principle can only be satisfied in a world consisting of two inimical friendship cliques. And I consider recent studies that show that when individuals in a structurally imbalanced world change ties one by one following the principle, they do not necessarily end up in a structurally balanced world. I prove that if multiple ties can be changed simultaneously, then a structurally balanced world is guaranteed.

In chapter 3, I consider Burt's (1992) argument of "structural holes" that unconnected parts of a social network are niches for brokerage. I consider Burt's suggestion that those aware of brokerage benefits end up occupying structurally advantaged network positions. I show how this statement crucially depends on the unawareness of these benefits by others. If everyone strives for structural holes, no one ends up with a structural advantage.

In chapter 4, I consider the extensive laboratory evidence on the relationship between the structure of small exchange networks and expected exchange rates. I consider a theory that reasonably predicts this relationship in a handful of networks. I show that if individuals add ties that increase expected earnings from exchange more than they cost and delete all other ties, then networks emerge that distribute exchange benefits equally.

In chapter 5, I consider the old immigrant assimilation model of a monotonic process. I consider recent work in the direction of an alternative model. I propose an alternative model that follows up on this work and adds minimal complexity to the old model. In this model, quite assimilated migrants further assimilate, while not so assimilated migrants reverse-assimilate. Using longitudinal survey data, I show that the model is empirically competitive.

In chapter 6, I propose four follow-up studies.

BIOGRAPHICAL SKETCH

Arnout van de Rijt studied Music Technology at the Utrecht School of the Arts and Sociology at Utrecht University.

To my parents

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1: COMMONALITIES AMONG THE FOUR STUDIES

The dissertation contains four self-contained studies, chapters 2 through 5. These studies are all motivated by two classic observations of modern society. The first is that its members are autonomous. Terms commonly associated with modernity are freedom, individualism, and rationality. We are legally backed and normatively pressured to do what is good for ourselves.

The second observation is that what we have autonomy over, what we possess and what we know, was not produced by our own hands and not experienced through our own eyes. We received it through production chains, exchange connections, and information channels. In modern society, then, people are independent in their life choices but interdependent in the consequences of these choices.

These two observations suggest a particular approach towards explaining modern society. Given observation one, an explanatory strategy of understanding a social phenomenon as the product of actions taken by autonomous actors should fare well. This strategy is called *methodological individualism*. My approach is methodologically individualist.

Given observations one and two, a tool for the analysis of situations with actors independently making decisions that have interdependent outcomes should be useful. Two such tools are *game theory* and *agent-based modeling*. My approach makes use of both tools.

Given observation two, how we are tied to our social environment should determine the distribution of wealth and knowledge in society. Extensive theoretical and empirical research on social networks has over the past decades established a number of *social network principles*, principles of how network structure translates into individual wealth and knowledge. I use these principles in my approach.

Given observations one and two, we have both the freedom and the incentive to selectively pick the interaction partners that we deem good for ourselves and to exclude others. In my approach, I assume that we attempt to maximize our wealth through the rational choice of interaction partners. I assume *optimization in the choice of social contacts*.

Figure 1.1 visually represents this derivation of my approach.

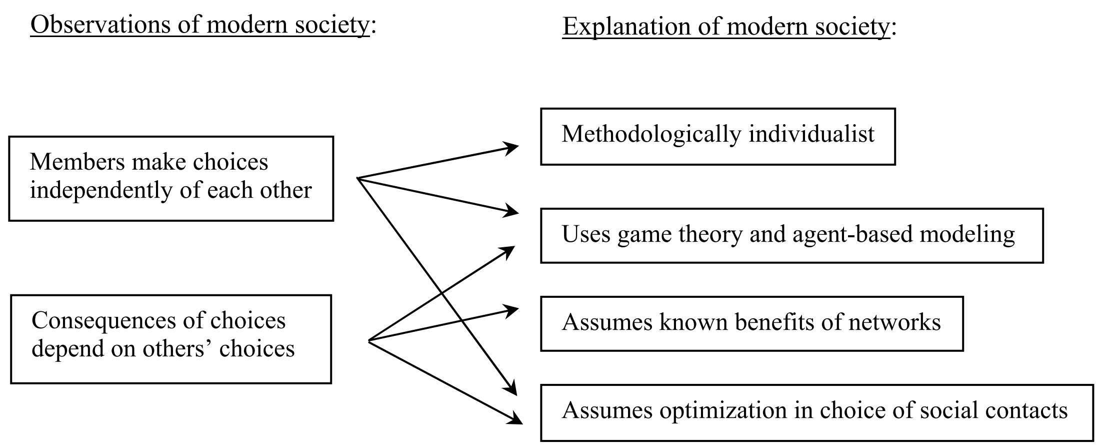

Figure 1.1. Derivation of the Explanatory Approach of the Dissertation

I apply this approach to the study of four types of members of modern society: Friends, traders, managers, and immigrants. For each, I ask two questions that sociologists typically ask, *the cohesion question* and the *inequality question*: How cohesive is society? How unequal is society? In order to answer these questions, I look at respectively the structure of the expected network and the distribution of wealth and information that it gives rise to. I obtain this expected network by taking the aforementioned approach. Making the individual the methodological unit of analysis, I use game theory and agent-based modeling to compute the network that we would expect to observe in modern society if its members constructed their personal networks such that they maximize the benefits that they derive from them. Here I preview the four applications.

Friends

It has been argued that we find structural imbalances among our friends uncomfortable (the social network principle). The so-called principle of structural balance lists the configurations of friendships that we feel comfortable with: 'The friend of a friend is a friend, the enemy of a friend is an enemy, the friend of an enemy is an enemy, and the enemy of an enemy is a friend.' In chapter 2, I compute how much friendship there would be (the cohesion question), and how comfortable each of us would feel about our friendships (the inequality question), if we individually (methodological individualism) made friends and enemies by maximizing the number of structurally balanced configurations (optimization in the choice of social contacts).

Managers

Ties that connect otherwise unconnected groups, thereby spanning so-called 'structural holes', benefit the broker (the social network principle). Managers in contemporary business schools learn how to strategically add 'non-redundant' ties that connected otherwise unconnected groups and delete 'redundant' ties that do not (optimization in the choice of social contacts). In chapter 3, I compute how dense and thus redundant in equilibrium (using game theory and agent-based modeling) managers' social networks would be (the cohesion question), and how structurally advantaged some managers would be over others (the inequality question), if they individually (methodological individualism) pursued structural holes.

Traders

How well people do in trade heavily depends on how they are connected to potential exchange partners (the social network principle). A branch of sociology that deals with bargaining outcomes in small networks provides methods for translating patterns of exchange relations into average profit. In chapter 4, I compute how dense in equilibrium (using game theory and agent-based modeling) exchange networks would be (the cohesion question), and how unequally the gains from trade would be distributed (the inequality question), if we individually (methodological individualism) initiated trade relations that increased our net profit and got rid of costly ones (optimization in the choice of social contacts).

Immigrants

The immigrant assimilation process is a process of network change. The dissimilated migrant has only co-ethnic friends, colleagues, and neighbors while the ethnicities of the contacts of the assimilated migrant represent their proportional presence in society. At the societal level this contrast translates into the metaphors ‘mosaic’, a society with segregated ethnic enclaves, and ‘melting pot’, a society with blending ethnic contacts, respectively. The social capital in the ties between migrants have been argued and shown to be of vital importance, providing them with the trust and information to survive in the host society (the social network principle). Immigrants are fully aware of these benefits (optimization in the choice of social contacts). In chapter 5, I compute how a migrant’s (methodological individualism) strategic decisions about who to work with, where to live, and who to be friends with determine overall levels of assimilation (the cohesion question), and how immigrant group differences in initial levels of financial, human, and social capital differentiate their members into assimilators and dissimilators (the inequality question).

In addition to these commonalities, chapters 2, 3, and 4 share a form of theory building. Simplistic assumptions about individuals are made that prevent the derivation of predictive hypotheses from the models but allow for the derivation of understandable, insight-generating theoretical results. In chapter 5, a different strategy is taken. A model of immigrant assimilation is built, not from a consistent set of simplistic assumptions, but by combining a number of unrelated hypotheses borrowed from the literature. Finally, in chapter 6, I draw on the various limitations and unanswered questions that I identified in the four discussion sections of chapters 2 through 5 to develop a research agenda.

CHAPTER 2: STRUCTURAL BALANCE IN EQUILIBRIUM

"The friend of a friend is a friend, the enemy of a friend is an enemy, the friend of an enemy is an enemy, and the enemy of an enemy is a friend." Heider (1946) argued and Jordan (1953) experimentally confirmed that we find situations that violate this *principle of structural balance* less pleasant than situations that do not.[1] Mathematical psychologist Frank Harary (1954:143-4) showed that the only two states of the world in which the principle applies to the relations between any three persons are global peace and bipolar war. Either everyone is friends, or people are divided up into two antagonistic groups. Representing persons as points, and relations between persons as lines that are either friendly, indicated by a '+', or inimical, indicated by a '–', these two states of the world can be visually represented as in figures 2.1 and 2.2 respectively.

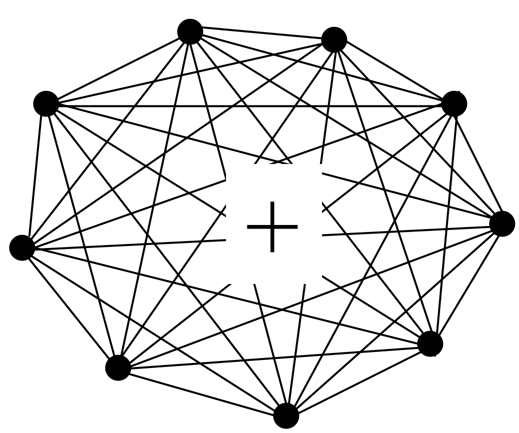

Figure 2.1. One Friendship Clique: Global Balance

[1] Heider's original "balance theory" is more general, allowing the third person to be an object, and liking to be unreciprocated.

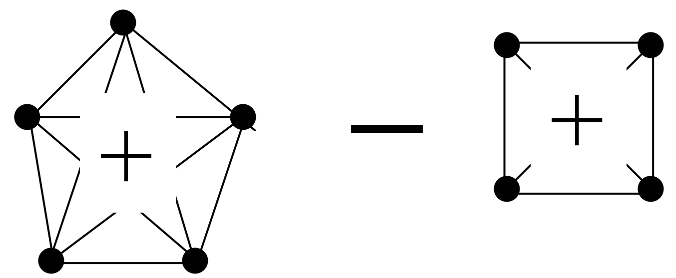

Figure 2.2. Two Antagonistic Friendship Cliques: Global Balance

Harary's result is an *efficiency* result. It tells us what the world would look like if we collectively managed to eliminate any cognitive constraint from structural imbalance. Methodologically individualist social science seeks results about the relationship between efficiency and *equilibrium*, the aggregation of individual action into collective outcomes (e.g. McClamrock 1991). For example, the first fundamental theorem of welfare economics states that buyers and suppliers in perfect markets in equilibrium trade efficiently. Individual striving for welfare maximization, as through some invisible hand, leads to collective welfare maximization. Recently, a literature has emerged that models such aggregation when utility is not derived from consumption or production bundles, but from network relations instead (e.g. Dutta & Jackson 2003).

Five studies have investigated the relationship between efficiency and equilibrium for the welfare principle of structural balance (Antal et al. 2005; Hummon & Doreian 2003; Kulakowsky et al. 2006; Kuwabara 2006; Wang & Thorngate 2003). In each of these studies, the unit of analysis has been the relation. Relations were swapped between states of friendship and enmity one by one, where changes that decreased imbalance in adjacent triads were more likely than changes that increased imbalance. Antal et al. (2005:7) discovered globally imbalanced (inefficient) networks in which no relational change could be made that decreased local balance (equilibrium). An example of such a network is three antagonistic groups (figure 2.3).

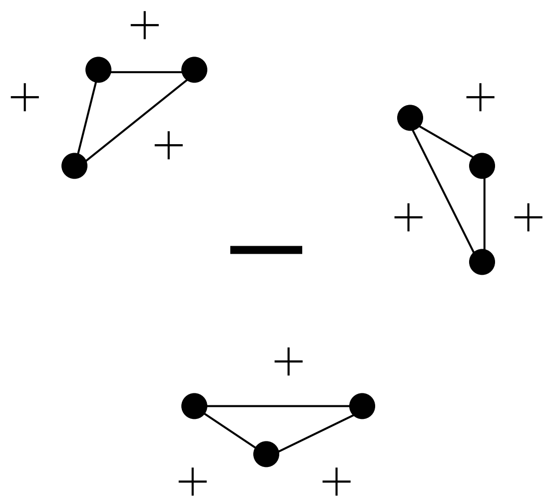

Figure 2.3. Three Antagonistic Friendship Cliques: Global Imbalance

Yet relations do not experience psychological imbalance and thus feel no need to restore it. The individuals that make up the relation – and that make up also other relations with other individuals – are the ones that perceive and restore balance. The relevant decision-making unit is the individual, not the relation. In previous models, agents were not maximizing their own welfare, but the welfare of an adjacent tie. It is not clear whether in these studies inefficient equilibria could be found simply because agents could only change one tie at a time, or because Harary’s globally balanced states are indeed not the only possible outcomes of individual welfare maximization.

Here I argue that global balance is nevertheless the inevitable end product of agents' sequential imbalance reductions. I do so by showing that, when the restriction of one tie change at a time is eliminated from agents' action space, in any imbalanced network some agent can always decrease local balance by making herself structurally equivalent to some other agent.

Let me reintroduce Harary's notation. "A *signed graph G*, or briefly an *s*-graph, consists of a set E of n points $P_1, P_2, \ldots, P_n$ together with two disjoint subsets L^+, L^- of the set of all unordered pairs of distinct points. The elements of the sets L^+, L^- are called *positive lines* and *negative lines* respectively", "A path is a collection of lines of the form $A_1A_2, A_2A_3, \ldots, A_{m-1}A_m$ where the points $A_1, A_2, \ldots, A_\mathrm{m}$ are distinct. The cycle $A_1A_2\ldots A_m$ of G consists of the path described above together with the line A_mA_1", "A *positive cycle* of an *s*-graph is one in which the number of negative lines is even; a *negative cycle* is not positive. An *s*-graph is in *balance* if all its cycles are positive" and "Two points A and B of G are adjacent if the line AB is in G. A *complete* graph is one in which each point is adjacent to every other point." (p. 143) Using this notation, he formulated the *structure theorem*:

Structure Theorem. "A complete *s*-graph is balanced if and only if its point set E is partitioned into two disjoint subsets E_1 and E_2, one of which may be empty, such that all lines between points of the same subset are positive and all lines between points of the two different subsets are negative." (p. 143)

I introduce a few additional definitions. Let us call cycles involving three points *triads*. Let $G^F = (F, L^{+F}, L^{-F})$ be the subgraph of a signed graph $G = (E, L^+, L^-)$ generated by a subset F of E where L^F is the set of lines in L with both points in F. Two *s*-graphs G_1 and G_2 with the same set of points E are *A-equivalent* if $G_1^F = G_2^F$ where $F = E \backslash A$. Denote by $t(G, A)$ the number of negative triads containing A in G. Two points A_i and A_j in G are *structurally equivalent* if $A_iA_k \in L^+ \Leftrightarrow A_jA_k \in L^+$ and

$A_iA_k \in L^- \Leftrightarrow A_jA_k \in L^-$ (Wasserman & Faust 1994:356).

Agency Theorem. For any imbalanced complete *s*-graph G_0 there exists a sequence of complete *s*-graphs $G_1, .., G_D$ and a sequence of points $x_0, .., x_{D-1}$ such that G_D is balanced and G_d and G_{d+1} are x_d-equivalent and $t(G_d, x_d) > t(G_{d+1}, x_d)$ for $d = 0, \ldots, D-1$.

Proof. Consider any line A_iA_j in a negative triad in G_d. Without loss of generality, assume $t(G_d, A_i) \geq t(G_d, A_j)$. Now construct G_{d+1} such that G_d and G_{d+1} are A_i-equivalent and A_i and A_j are structurally equivalent. By structural equivalence of A_i and A_j, $t(G_{d+1}, A_i) = t(G_{d+1}, A_j)$. Since all triads in G_{d+1} that connect both A_i and A_j are positive, while some were negative in G_d, and the sign of all triads in G_{d+1} that involved A_j but not A_i remained unchanged, $t(G_d, A_j) > t(G_{d+1}, A_j)$. It follows that $t(G_d, A_i) > t(G_{d+1}, A_i)$. By A_i-equivalence of G_d and G_{d+1}, triads in G_d and G_{d+1} that do not involve A_i are of the same sign, so that $\sum_k t(G_d, A_k) > \sum_k t(G_{d+1}, A_k)$. Therefore, for large enough D, $\sum_k t(G_D, A_k) = 0$, and hence G_D is balanced.

It is thus not only the case, as Harary showed, that *collective* imbalance minimization implies one or two opposed camps. Under the principle of structural balance, welfare-seeking agents *self-organize* to produce such a world. They are "either with us or against us." Classrooms should form maximally two peer groups (cf. Snijders 2001). International relations equilibrate towards Axis-Allies configurations, especially in the lead up to a war (Moore 1978).

The result is robust in three ways. First, it does not require restrictive rationality assumptions. It guarantees the existence of a sequence of local imbalance-reducing changes towards ultimate global balance from any starting network. Agents need not always maximize individual welfare to reach global balance. They may satisfice, imitate, or learn, and make mistakes. Also, they may be myopic – caring only about improved balance immediately after their action – or farsighted – anticipating subsequent actions by other agents. All that is required is that on average, agents reduce local imbalance. Second, Davis (1967) proposed an adjustment of the principle of structural balance in response to socio-metric data, where the triad consisting of three negative lines is considered positive rather than negative. With this alternative definition of triad positivity, the agency theorem can simply be proven by considering a sequence of mappings in which agents make enemies with everyone else. Third, the structure theorem has also been stated for incomplete graphs (Harary 1954:144). Hummon & Doreian (2003) and Wang and Thorngate (2003) accordingly give agents the option to eliminate lines. Clearly, agents can then always reach global balance by means of mere isolation.

A weakness is that the result currently relies on agents' ability to force friendships and enmities upon others. The result would be strengthened if it were shown in future work that a route to global balance exists even when sign changes require permission by the other agent in the respective relation.

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CHAPTER 3: STRUCTURAL HOLES IN EQUILIBRIUM

(with Vincent Buskens)

Introduction

Ideas about how positions in social networks are of use to people occupying them are abundant in sociology. The structure of one's social environment has been shown to matter in a number of ways. Balanced triads generate less cognitive inconsistency (Heider 1946; Krackhardt 1999). Job search through weak ties is more successful (Granovetter 1995 [1974]). Non-excludable trading parties have larger profit margins (Cook and Emerson 1978; Willer 1999). Dense structures and closure in networks facilitate trust (Coleman 1988; Raub and Weesie 1990; Buskens 2002; Burt 2005). And, ties between otherwise unconnected groups, that thereby span so-called "structural holes", benefit the broker (Burt 1992). The last example will be the focus of this paper.

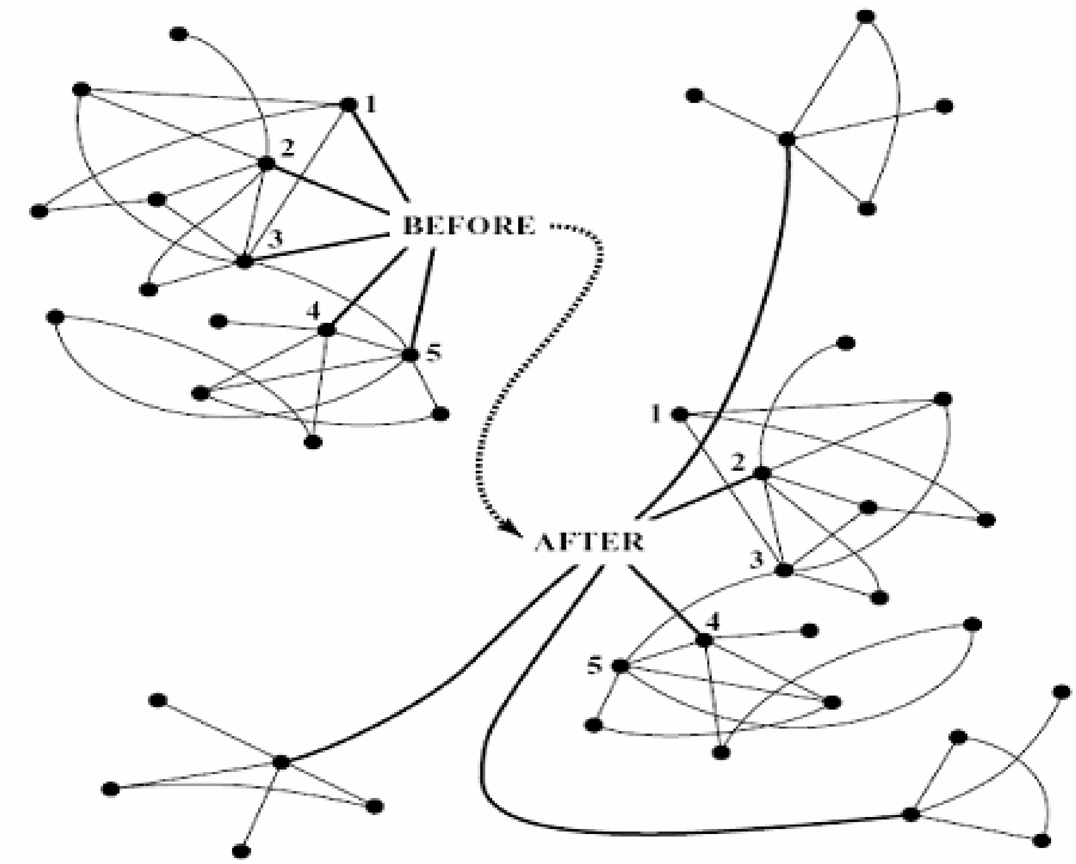
Those results constitute an important body of structural regularities, but from an agency perspective, they are only half the story. They enable us to understand consequences of certain network positions, but not how actors manage to reach these positions. The link from micro back to macro that completes the sociological explanation of real-world networks, from individual decisions on who to interact with to emergent global networks, is missing. Yet precisely in the individual preferences over network positions that social networks have been argued and shown to give rise to lies the very key to their explanation. If some networks are more beneficial than others, actors can be expected to modify the less beneficial ones to their advantage (Flap 2003, pp. 12–3). The network becomes a "device to be manipulated consciously for an actor's own ends" (Watts 1999, p. 495). People rebalance triads, breaking with friends whom other friends do not like. Traders actively seek out alternative trading parties as to enhance their bargaining positions.

By the mere non-atomic nature of networks, such actions will be interdependent – one can only broker as long as two brokees remain unconnected – and partly collective – it takes two to connect. Each choice to connect or disconnect then becomes contingent upon another's previous choice and precursor of a next. An initial network change may thus trigger non-trivial network evolution. Sociologists have only recently begun to explore this agency component of social network analysis in more detail (e.g., Doreian and Stokman 1997; Snijders 2001, 2005). In this paper, we will further develop theoretical insight into which networks can be expected to emerge when actors purposively choose their relationships. In particular, we develop a model in which actors strive for structural holes.

When Ronald Burt (1992) launched his idea of structural holes, he went beyond structuralism. He did not assume that actors would simply reap the fruits from structural advantages that they happen to have over others. He suggested the possibility that entrepreneurs, just as they can strategically put financial and human resources to work, exploit social resources and turn them into a profit: "You enter the structural hole between two players to broker the relationship between them" (Burt 1992, p. 34). Burt even went so far as to argue that social capital *implies* prior strategic networking: "I will treat motivation and opportunity as one and the same...a network rich in entrepreneurial opportunity surrounds a player motivated to be entrepreneurial. At the other extreme, a player innocent of entrepreneurial motive lives in a network devoid of entrepreneurial opportunity" (1992, p. 36). Burt's "structural entrepreneur personality index" (2005, p. 34) is an attempt to quantify this inclination to exploit social resources.

Nevertheless, the agency component in Burt's argument was never as fully developed as the structure component. Burt proposed a precise measure of structural disadvantage, the "constraint" formula (1992, p. 54), but the network dynamics he sketches in his book are instructions on how to unilaterally reduce one's score on the constraint measure (see figure 3.1). He thereby remained at the micro level, showing how an ego-network would change with structural holes as the driving force assuming cooperation and passivity on behalf of all alters. Thus neutralizing the interdependence between actors, he precluded possible cascades of subsequent network adaptations by other actors. Burt (2005, ch. 5) deals with some of these issues informally. Although his considerations are very insightful, they lack a strict theoretical deduction. For example, he speculates on the emergence of stable networks if network benefits are extended beyond brokerage to include closure, and if these benefits and also the costs of ties are heterogeneous among actors. We think that such speculations are unwarranted. We show that with much simpler assumptions, analysis is already challenging.

We will limit ourselves to formalizing network dynamics with a single type of benefit and with homogeneous actors. Clearly, these are strong, simplifying assumptions. Strategic networking will be more salient in some settings than in others, and in any particular setting not everybody will be equally interested in or able to occupy strong entrepreneurial positions. Still, our model provides an insightful benchmark that can straightforwardly be modified to accommodate more complex assumptions. In the discussion section, we indicate which assumptions are crucial for our findings and which can be relaxed without changing the substantive results.



Source: Figure 1.4 in Burt (1992)

Figure 3.1. Optimizing for Structural Holes

In Burt's typical example of a network after entrepreneurial activity (see figure 3.1), the majority of benefits are held by a single individual. However, in figure 3.1 no one else was granted an opportunity to add and delete ties. The environment around the focal actor was held fixed. We consider figure 3.1 a step in a series of network changes by interdependent actors who together equilibrate towards or around a certain stable end network. It is not obvious what this end network is. It is also not obvious that if everyone would like to follow the entrepreneur's example the structural advantage of the initial entrepreneur automatically disappears. Some economic studies of network dynamics in information and communication settings have identified the "star," a winner-take-all network in which a single entrepreneur receives and maintains all of the profit, as the unique stable network (Jackson and Wolinsky 1996; Bala and Goyal 2000; Goyal and Vega-Redondo 2007) even though all peripheral actors would like to be the central actor in these networks. In this paper, we show that the "star" is not stable if everyone tries to minimize his network constraint. By pursuing the following aims, we come to this conclusion.

Our first aim is to extend and improve the methodology for answering questions about which networks can be expected to emerge given that we know how actors benefit from network positions in a certain context. This involves the development of a model of network formation and a comparison of existing and new stability concepts indicating in which networks strategic actors will not make any further changes. In addition, we develop a tool to determine which of the stable networks are more or less likely to emerge. Our second aim is to apply this methodology to answer the question how empirically observed access and control benefits from brokerage would be distributed in equilibrium if actors strove to obtain these access and control benefits. Will a minority broker the rest and claim a majority of benefits, as Burt's informal treatment and economic studies suggest, or will everyone share the profits? Thus, we model network evolution not employing a stylized utility function, as previous economic studies have done, but instead assuming a relationship between network structure and profit that has solid empirical backing. Burt (1992) provides us with such an empirically tenable measure of access and control benefits, as we will argue in the next section. In this way, we build bridges between the sociological and economic literatures in this area.

We first review the structural-hole argument and the dynamic networks literature. Then we introduce a model of network entrepreneurship, explain the stability concepts we use, and identify two classes of stable networks. In both classes of networks, entrepreneurs share profits equally, but in the one more total profit is earned than in the other. Using simulation, we show that networks from this more efficient class – balanced complete bipartite networks – evolve with a much higher likelihood than the others. In these networks, everybody has a strong network position in terms of structural holes, but no one has a structural advantage over other actors. Our model – contrary to what existing economic models of network dynamics tell us – thus predicts a "network race" in which no one is able to maintain an advantaged position.

Structural Holes

Structural holes are "disconnections or nonequivalencies between players" and hence "entrepreneurial opportunities for information access, timing, referrals and control" (Burt 1992, p. 1−2). There exists a structural hole between two players if there is a potential for beneficial information flow between them. The word "disconnections" in the above definition refers to the absence of a tie or path through which the information can flow.

A network rich in structural holes thus contains many exploitable brokering opportunities: "The structural hole is an opportunity to broker the flow of information between people and to control the form of projects that bring together people from opposite sides of the hole" (1997, p. 340). The network entrepreneur recognizes these opportunities and places himself in the hole by initiating ties with both players. Just as the investment banker and the human resource manager generate returns from financial and human capital, so does the network entrepreneur seek profit in information structure. “When you take the opportunity to be the *tertius*, you are an entrepreneur in the literal sense of the word – a person who generates profit from being between others” (1992, p. 34). Occupying the hole and being essential to the information flow between the two, the entrepreneur can charge a brokering fee.

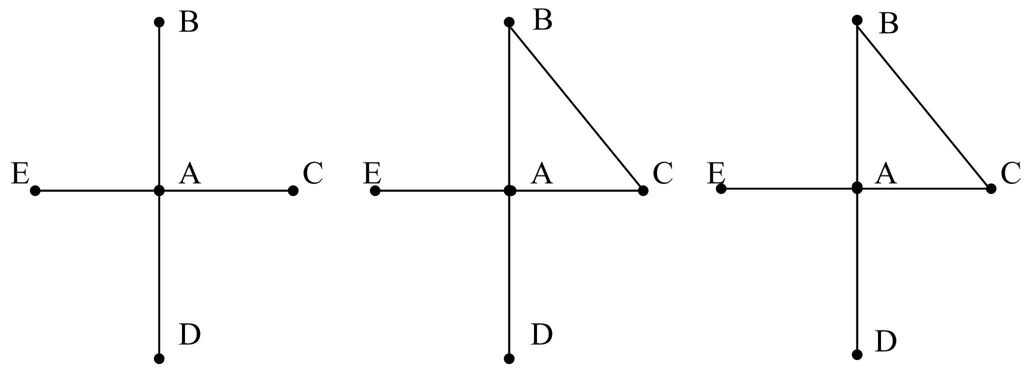
Burt introduced a formula for quantifying the benefits from spanning structural holes, the “constraint” measure. Entrepreneurial opportunities are considered constrained if there exists a feasible alternative road along which the information you are intending to broker can travel: “Contact *j* constrains your entrepreneurial opportunities to the extent that: (a) you’ve made a large investment of time and energy to reach *j*, and (b) *j* is surrounded by few structural holes with which you could negotiate to get a favorable return on the investment” (1992, p. 54).[1] The constraint measure c_i captures the extent to which this is the case for each contact *j* of actor *i*:

$$c_i \equiv \sum_{j \neq i} \left(p_{ij} + \sum_{k \neq i, k \neq j} p_{ik} p_{kj} \right)^2 \qquad (3.1)$$

where p_{ij} is the proportion of time *i* has invested in contact *j*. Burt assumes that an actor distributes his time equally over his contacts: If *i* is connected to *j*, $p_{ij} = 1/d_i$, where d_i is actor *i*’s degree. If *i* and *j* are not connected, $p_{ij} = 0$. The constraint measure c_i lies between 0 and 9/8 (see theorem 0 in the Appendix) and is 0 for isolates. This implies that isolates have the lowest constraint. It seems more plausible, though, that it is better to be connected in some way than not to be connected at all, and we think

[1] In recent work (e.g., Burt 2005), Burt multiplies his index by 100 and rounds to compare integer values of the index. We will use the original formulation in this paper. Of course, both formalizations are equivalent. As an alternative, we will show that our results stay the same if we use a *relative* version of the constraint formula as a measure of benefits, where an actor’s network benefits equals the reciprocal of his constraint score divided by the sum of all actors’ constraint scores, representing the idea that an entrepreneur wants a “better” position than the others in the network.

Burt did not intend isolates to be least constrained. Therefore, we additionally assume that $c_i = 2$ for isolates.[2]



$c_A = 4\,(\frac{1}{4} + 0)^2 = \frac{1}{4}$	$c_A = 2\,(\frac{1}{4} + 0)^2 + 2\,(\frac{1}{4} + \frac{1}{4}\,\frac{1}{2})^2 = 13/32$	$c_A = 3\,(\frac{1}{3} + 0)^2 = \frac{1}{3}$
$c_B = 1\,(1/1 + 0)^2 = 1$	$c_B = (\frac{1}{2} + \frac{1}{2}\,\frac{1}{4})^2 + (\frac{1}{2} + \frac{1}{2}\,\frac{1}{2})^2 = 61/64$	$c_B = 2\,(\frac{1}{2} + 0)^2 = \frac{1}{2}$
$c_C = 1\,(1/1 + 0)^2 = 1$	$c_C = (\frac{1}{2} + \frac{1}{2}\,\frac{1}{4})^2 + (\frac{1}{2} + \frac{1}{2}\,\frac{1}{2})^2 = 61/64$	$c_C = 1\,(1/1 + 0)^2 = 1$
$c_D = 1\,(1/1 + 0)^2 = 1$	$c_D = 1\,(1/1 + 0)^2 = 1$	$c_D = 1\,(1/1 + 0)^2 = 1$
$c_E = 1\,(1/1 + 0)^2 = 1$	$c_E = 1\,(1/1 + 0)^2 = 1$	$c_E = 1\,(1/1 + 0)^2 = 1$

Figure 3.2. The Constraint Measure; Three Example Networks

The higher the score on this measure c_i, the more structural opportunities are constrained, and as a result, the lower the network benefits. Consider as an example figure 3.2. In the network on the left, actor *A* is essential for all information flow. His constraint score is $c_A = 4\,(\frac{1}{4} + 0)^2 = \frac{1}{4}$. In the middle network, *B* and *C* can also communicate directly rather than through *A*. This constrains the relations between *A* and *B* and *A* and *C*. *A*'s constraint score is now $c_A = 2\,(\frac{1}{4} + 0)^2 + 2\,(\frac{1}{4} + \frac{1}{4}\,\frac{1}{2})^2 =$

[2] The precise value is irrelevant for the results of the model as long as the value is larger than 9/8. The consequence of this assumption is further, as we will see below, that actors always want to connect with isolates and isolates want this as well. This property is in correspondence with the more general property that we will derive that actors are always willing to connect as long as the connection does not create any closed triads.

13/32. In fact, a network without one of the two "redundant" ties with *B* and *C* would be better for *A* (on redundancy, see Burt 1992, p. 51). In the network on the right, *A*'s constraint is lower, namely $c_A = 3\ (⅓ + 0)^2 = ⅓ < 13/32$. Therefore, actor *A* is willing to give up his relation with *C* in the middle network of figure 3.2 and move to the network on the right.

The constraint formula has been found negatively related to a wide range of objective indicators of success (see Burt 2000, 2002, and 2005 for extensive reviews). Producer profit margins are larger for firms in buyer-supplier networks (Burt et al. 2002; Talmud 1994; Yasuda 1996). Jobs are more desirable (Bian 1994; Leenders and Gabbay 1999; Lin 1999; Lin, Cook, and Burt 2001). Salaries are higher (Burt 1997, 1998; Podolny and Baron 1997; Burt, Hogarth, and Michaud 2000; Mehra, Kilduff, and Brass 2001; Mizruchi and Sterns 2001). And negative correlations have been found with positive performance evaluations, peer reputations, promotions, and good ideas (Gabbay 1997; Burt 2001, 2004). Given that this evidence indicates a (negative) association between the constraint formula and network benefits, we use constraint as an (inverse) indicator for the utility actors can extract from a network.

Dynamic Networks

What the evidence above does *not* tell us is whether investments in brokerage relations pay off. Burt has suggested, but not shown, that those who remove redundant ties and add non-redundant ones will eventually obtain the returns on these investments. Current network manipulations that lead to an improved network position in the short run may trigger subsequent changes by others that could ultimately make oneself worse off. This temporal interdependence in decision-making is not trivial, and its examination requires an explicit model. Recently, some models have been proposed to examine such dynamics. There exist already many reviews of models of network dynamics. E.g., Weesie and Flap (1990), Doreian and Stokman (1997), Stokman and Doreian (2001), and Breiger, Carley, and Pattison (2003) provide extensive overviews in sociology. But also within economics and physics this line of research has been receiving more and more attention recently (see Dutta and Jackson 2003; Jackson 2004; Goyal 2006; Newman, Barabási, and Watts 2006).

We will not review this complete literature on dynamic networks here, but rather discuss a class of models that assume that actors optimize a utility function through choices in their ego networks (for a similar approach see also Robins, Pattison, and Woolcock 2005). We chose this modeling approach, because Burt's network entrepreneurs are precisely such optimizing actors, with brokerage benefits as network-derived utility. There exist two subclasses of models of this kind. First, there are equilibrium models in which actors simultaneously propose ties to other actors. They can select any combination of actors they wish. Actors derive utility from their positions in the resulting network. Stable networks are those that are induced by Nash equilibria, combinations of strategies that are each a best response to the strategies of the other actors. Gould (2002) proposed such a model to explain the emergence of status hierarchies. His model is special in the sense that actors are allowed to propose a *level* of attachment to all other actors. Utility depends on this level of attachment, the status of the actors that one connects to, and the extent to which attachment is reciprocated. A prominent instance of this type of model in economics is Bala and Goyal (2000), in which actors can unilaterally link to others and derive utility from the number of other actors they are directly or indirectly linked to.

In both Gould's model and that of Bala and Goyal, ties are directed (e.g., phone calls) and can be established without permission from the target actor.

However, Burt's ties are undirected. We will therefore assume that ties are formed only if both actors want to connect. This assumption is based on the idea that relevant information can be exchanged only if both actors agree to come together and talk. No one can force a tie upon anyone else. Myerson (1991, p. 448) proposed a model that uses this assumption. All actors simultaneously propose ties. Ties that are mutually proposed are formed, and actors derive utility from the resulting network. This type of model, which we will call the Gould-Myerson (GM) model, is attractive but has one important drawback: It shortcuts the network dynamics. Instead of representing network evolution as a continuous process in which one actor can react to changes by other actors elsewhere in the network, it assumes that all actors make their decisions simultaneously and that these decisions are binding. There are often numerous equilibria, and the lack of a specification of the network evolution process then leaves one unable to identify the network that is most likely to evolve. From an evolutionary viewpoint, many of these equilibria can hardly be considered stable networks. We will show later that we can identify a subset of these equilibria that have more appealing stability properties. These stability properties can be explained by describing adaptive processes through which actors solve coordination problems and reach specific equilibrium networks. The second subclass of actor-oriented models of network formation does specify such network evolution processes.

In sociology, Snijders (1996, 2001) develops methods for the statistical analysis of longitudinal network data. These models allow one to test hypotheses on the specific utility function that actors optimize when making changes to the network. Snijders' statistical model has been applied to, for example, the emergence and stability of friendship networks (Zeggelink, Stokman, and Van der Bunt 1996; Van der Bunt, Van Duijn, and Snijders 1999; Whitmeyer 2002). The model assumes that actors are randomly selected one by one and given the opportunity to remove or add one tie.

Actors then evaluate whether some change increases their utility. If so, they make that change; if not, they leave the network as it is. Originally, the model assumed that tie formation was unilateral, and thus each actor could force a tie upon some other actor. These ties were directed. In the latest versions of Snijders' models, one can also specify a model in which ties are undirected and formed only if both actors want that tie. This particular scenario was implied by Jackson and Wolinsky's (1996) concept of "pairwise stability, " and later developed by Watts (2001). A network is pairwise stable if no pair of actors wishes to connect and no single actor wishes to remove a tie. Sequential tie formation under mutual consent reaches stasis only once a pairwise stable network has formed. In this type of model, which we will call the Snijders-Watts (SW) model, actors are myopic; they do not consider what other actors will change after they make a change themselves. In the GM model, by contrast, the actors consider all other actors' possible actions in a setting in which everyone chooses all desired ties simultaneously and only once. We will show in the next section that the SW and GM models can usefully be unified despite these differences, namely through a single equilibrium concept that applies to both.

A Model of Network Entrepreneurship

Burt's (1992) network entrepreneurs are actors who change structure in pursuit of greater network-derived benefits. We can use the explicit formula that Burt used to quantify an actor's lack of network benefits, the aforementioned constraint measure. Through alterations of their network positions, actors maximize utility, which should be decreasing with the score of that individual on the constraint measure. The literature provides two candidate models for such network entrepreneurship, the GM model and the SW model. As we will show now, these models can be unified.

We first introduce the necessary notation. Let $n \geq 2$ be the number of actors, $N = \{1,2,...,n\}$ the set of actors, and X the n times n adjacency matrix with x_{ij} indicating the presence or absence of a connection between i and j. Actor i's degree is denoted by $d_i = \sum_j x_{ij}$.

In the GM model, each actor proposes whom he wants to be connected to. We use $s_i \in \{0,1\}^n$ to indicate a pure strategy of actor i in which s_{ij} indicates whether or not i proposes a link with j. Because actors cannot connect to themselves, $s_{ii} = 0$. The utility function $u_i(s)$ assigns a numerical value to each set of strategies $s = \{s_i \mid i \in N\}$. Gould considers Nash equilibrium as the stability concept.

Definition 1. A set of strategies $s^* = \{s_i^* \mid i \in N\}$ is a *Nash* equilibrium if $u_i(s^*) \geq u_i(s_i, s^*_{-i})$ for all i and s_i, where s^*_{-i} is the set of all strategies in s^* excluding the one of i.

Moreover, Burt's ties are non-reflexive, so $x_{ii} = 0$ for all i, they are undirected, so $x_{ij} = x_{ji}$ for all i and j, and they are unvalued, so $x_{ij} \in \{0, 1\}$. Let g^N denote the complete network of all non-reflexive, undirected, and unvalued connections ij. Let $g + ij$ denote network g with the tie ij added to it and $g - ij$ network g with tie ij removed. We say that the set of strategies s *induces* the network g if $ij \in g \Leftrightarrow s_{ij} = s_{ji} = 1$; i.e., only ties that are proposed by both actors are part of the network. The network constraint formula depends only on the network formed and not on the ties proposed. Therefore, we assume that proposing ties is costless. This implies that the utility function $u_i(s)$ is the same for combinations of strategies that induce the same network. Formally, the utility function has the property that $u_i(s') = u_i(s'')$ if s' and s'' induce the same network g. With some abuse of notation, we can also write $u_i(g)$ as the utility of a certain network g, given that it does not matter what strategies induce this network. Specifying the model for our purposes, actor i's utility is a decreasing function of the constraint measure c_i as indicated above, e.g., $u_i(g) = -c_i(g)$.[3]

Now, we define as a first stability concept the Nash network:

Definition 2. A network g^* is a *Nash* network if some s^* inducing g^* is a Nash equilibrium.

Nash network is a rather weak stability concept for undirected networks, since many networks that can hardly be considered stable are included. The problem is that if one actor does not propose a tie to another, the second actor has no incentive to propose a tie to the first, because a tie is formed only if it is proposed by both actors at the same time. No one can increase his utility through any proposal if no one else is proposing ties, which makes "nobody proposing any tie" a Nash equilibrium. The network induced by this set of strategies is the empty network, and this is a Nash network. Given our utility function, every *pair* of actors wants to initiate the first tie in the empty network, because isolates have the lowest possible utility. Thus, many Nash equilibria are due to trivial coordination problems.

In the SW model, such coordination problems do not exist. This model asks what an actor would change given the current status of the network if he were offered

[3] We make the strong assumption here that reducing network constraint is the only utility argument for all actors and that the utility derived from a lack of constraint is the same for all actors. We thereby neglect other utility arguments related to, e.g., closure (Burt 2005, ch. 3) and indirect brokerage (Burt 2007). Neither do we investigate the effects of heterogeneity among actors (see Burt 2005, ch. 1.4). We will consider these issues in more detail in the discussion section. Note that utility should be interpreted here as the extent to which an actor is expected to be able to extract benefits from the network. A network position is not considered beneficial in itself.

that possibility. An actor is allowed to delete a tie or add a tie with permission from the new contact; i.e., it does not make this other actor worse off. Stability is reached if no actor can profitably delete a tie or add an acceptable tie. Jackson and Wolinsky (1996) call this pairwise stability:

Definition 3. A network g is *pairwise stable* if both

(i) For all $ij \in g$, $u_i(g) \geq u_i(g - ij) \wedge u_j(g) \geq u_j(g - ij)$ and

(ii) For all $ij \notin g$, $u_i(g + ij) > u_i(g) \Rightarrow u_j(g + ij) < u_j(g)$.

Condition (i) states that no actor wants to sever a tie, and condition (ii) states that no pair of actors wishes to add a tie. On the one hand, pairwise stability has the advantage that pairs of actors can add ties if they both want to have a tie, which seems a clear indication that a network is unstable, since networks can be Nash networks despite this possibility. On the other hand, the disadvantage of pairwise stability is that it does not consider the simultaneous removal of multiple ties. If an actor would profit from removing two ties, this network could never be a Nash network in the GM game, because not proposing these two ties would have been a better strategy. Networks can be pairwise stable despite profitable deviations that involve multiple ties, even though such deviations arguably make a network less stable. Thus, Nash networks in the GM model and pairwise stable networks in the SW model both contain a subset of networks that could be considered unstable.

That makes the intersection of these two sets of networks a candidate for an appropriate set of stable networks. Goyal and Joshi (2006) introduced the concept of pairwise equilibrium or pairwise Nash in the GM model, which corresponds to the intersection of Nash networks and pairwise stable networks (see also Cálvo-Armengol

2004). Gilles et al. (2006) call the same stability concept *strong pairwise stability* using the SW model. In words, pairwise Nash networks are networks in which (i) no actor wants to delete a subset of his ties and (ii) no pair of actors wants to add a tie between them. But clearly, there is an asymmetry here: An actor can delete all his ties but add only one tie – if that tie does not make the other actor worse off.

Even more problematic is the assumption that actors do not simultaneously add and delete ties. For example, pairwise Nash implies that an actor does not contemplate improving his network position by replacing one contact with another. This, however, seems a rather straightforward change in a network. To resolve the problems of asymmetry and non-simultaneity in the deletion and addition of ties, we introduce our own stability concept, "unilateral stability." In our definition of this concept we use the concept of "unilateral obtainability":

Definition 4. A network *g*' is *unilaterally obtainable* from *g* by *i* through $S \subseteq N \setminus \{i\}$ if
(i) all ties that are in *g*' but were not in *g* involve actor *i* and an actor in *S*;
(ii) all ties that are not in *g*' but were in *g* involve actor *i*.

In other words, one network is unilaterally obtainable from another by a proposing actor *i* and through a subgroup *S*, if each tie that is added or deleted involves actor *i* and if each tie that is added involves also a member of *S*. Now, we can define our core stability concept.

Definition 5. A network *g* is *unilaterally stable* if for all *i*, $S \subseteq N \setminus \{i\}$, and *g*' unilaterally obtainable from *g* by *i* through *S*, $u_i(g') > u_i(g) \Rightarrow u_j(g') < u_j(g)$ for some $j \in S$.

In words, a network is called unilaterally stable if no actor *i* can change the ties that he is involved in himself such that two conditions are fulfilled: (i) *i* is strictly better off; (2) none of the actors in *S* to whom actor *i* proposes a new tie is worse off than in the original network. From the definitions, it is clear that all unilaterally stable networks are pairwise Nash, and all pairwise Nash networks are pairwise stable.

Unilateral stability unites the JWS and GM models as follows. It is a stability concept in the JWS model. It is conceptualized as convergence in a sequential process. It can also be formulated as a refinement of Nash equilibrium in the GM model, in which case we refer to "initiative proof" Nash equilibria (see AUTHOR 2006). Reformulated as such, it constitutes equilibrium in simultaneous decision-making.

Networks of Structural Entrepreneurs

We now identify networks to which no entrepreneur can profitably make any further change. Our first result states that two actors will always connect if they have no shared contacts.

Theorem 1. Adding a tie without creating closed triads is always beneficial for both actors involved in the new tie.

Proof. For purposes of legibility, we moved all proofs to the Appendix.

Theorem 1 establishes the unconditional benefits from brokerage. If an actor adds a tie without creating a closed triad, then this actor will be on the shortest path between the new contact and all the contacts the actor already had. And vice versa, the new partner comes to mediate the information the focal actor receives and passes this along to his old contacts. Scores on the constraint measures of both actors drop. The added value of an additional tie decreases as more ties are added because an actor has to distribute his time among more neighbors and can thus broker less information per pair of neighbors, but this marginal utility never becomes zero.

As we will see below, the reverse of theorem 1 is not true. Sometimes actors want to add ties that cause closed triads, and networks with triads can even be pairwise stable. We did not encounter any unilaterally stable network with closed triads, but we could not prove that they do not exist.

Corollary 1. The shortest path between any pair of actors in a pairwise stable network has length less than or equal to 2.

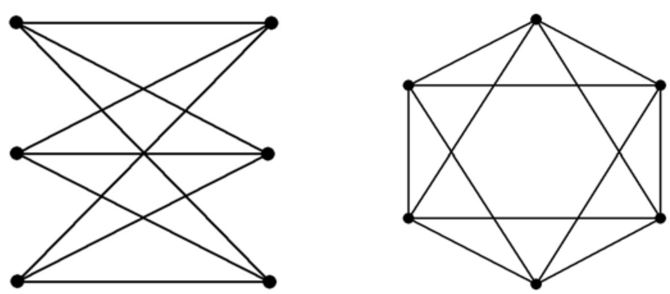
The shortest path between two actors can be of length 2 or less only if both actors are directly connected or can reach each other through a broker. If neither condition obtains for some pair of actors, then these actors can add a tie without creating a closed triad, which is profitable by theorem 1. Note that since pairwise stability is a weaker stability concept than unilateral stability, corollary 1 also holds for unilaterally stable networks.

Corollary 2. A network of disconnected parts cannot be pairwise stable.

A network of disconnected parts contains many brokerage opportunities. Every entrepreneur wants to add a tie to someone in another part because that tie will never create a closed triad. Such networks can therefore not be pairwise stable (and, consequently, not unilaterally stable, either). The following set of definitions describes a family of networks that includes important stable networks.

Definition 6.

- An *m-partite* network is a network in which the actors can be divided into *m* groups such that there are no ties within these groups.
- The *complete m-partite* network $K_{n_1,n_2,\ldots,n_m}$ is the *m*-partite network in which all the possible ties between the actors in the *m* groups, which have sizes n_1, n_2, …, n_m (see Wasserman and Faust 1994, p. 120). The complete 3-partite network $K_{2,2,2}$ (or Octahedron) is shown in figure 3.3.
- A *balanced m-partite network* is an *m*-partite network such that the difference between the number of actors in the largest group and the number of actors in the smallest group is at most 1.
- If $m = 2$, *m*-partite networks are called bipartite networks. The balanced complete bipartite network $K_{3,3}$ is shown in figure 3.3.
- If $m > 2$, *m*-partite networks are called multipartite networks.
- The complete bipartite network $K_{1,k}$ is also called the *k*-star.



$K_{3,3}$ $K_{2,2,2}$

Figure 3.3. Two Example Networks

Now we can formulate the following corollary of theorem 1.

Corollary 3. Pairwise stable networks that are bipartite networks are necessarily complete bipartite networks (otherwise some actors are at a distance greater than 2 and one can add ties without creating closed triads).

Corollary 3 says that in order for a bipartite network to be stable, it will have to be complete. Otherwise there would be a brokerage opportunity. As the reader can verify, eliminating any tie in the first network in figure 3.2 makes both actors involved in that tie worse off: Actor A would have constraint ⅓ instead of ¼, and the other actor would become an isolate. The next result states that the first network in figure 3.2 is *not* pairwise Nash, despite its completeness.

Theorem 2. A complete bipartite network of size *n* is pairwise Nash, unless it is a *k*-star with $k > 3$.

Theorem 2 identifies an important class of pairwise Nash networks, and thus also of pairwise stable networks.[4] As soon as both groups in a bipartite network consist of at least two actors and the network is complete, it is pairwise stable. The first network in figure 3.2 is a 4-star, and by theorem 2 it is not pairwise stable. The reason is that any two peripheral actors may wish to connect: B and C lower their constraint from 1 to 61/64 if they connect, even though this tie is creates a closed triad. This is a property of Burt's constraint formula. In some cases, when the broker has many ties and can therefore spend little time passing information, it is better to

[4] *All* bipartite networks, incomplete and complete, are Nash networks. Restricting oneself to the pairwise Nash networks excludes the incomplete bipartite networks. This avoids claiming that a network is stable only because two actors do not succeed in coordinating their proposals for a certain tie they both want to have.

establish a direct connection. This is what happens in this example network. B and C connect because A spends too much time with D and E. The middle network in figure 3.2 evolves. Given the argument for instability of larger stars, it is surprising that in other unbalanced complete bipartite networks actors in the larger groups never have an incentive to connect. If one broker can be too busy, then why cannot two?

Complete bipartite networks have another nice property, namely that they are efficient in the Pareto sense.

Definition 7. A network is *Pareto-efficient* if there is no other network such that no actor is worse off and at least one actor is better off.

If our actors could cooperate and enforce agreements – which we have assumed they cannot – then they would not be able to leave a Pareto-efficient network. There would always be some actor vetoing a transition. The following theorem tells us that the class of stable networks identified in theorem 2 is Pareto-efficient.

Theorem 3. Complete bipartite networks are Pareto-efficient.

We could not prove the reverse of theorem 3, namely that complete bipartite networks are the only efficient networks. We verified that there do not exist other Pareto-efficient networks that involve 8 or fewer actors.

We now specify a necessary and sufficient condition under which complete bipartite networks are not only pairwise stable but also unilaterally stable.

Theorem 4. A complete bipartite network is unilaterally stable if and only if it is balanced.

Theorems 3 and 4 together identify a class of networks that are efficient and stable under the strictest stability condition. In a balanced complete bipartite network, no actor can profit from deleting and adding permitted ties or be made better off in any network, whether unilaterally obtainable or non-obtainable, without another actor being made worse off. We were able to identify a second class of pairwise stable networks.

Theorem 5. All complete multipartite networks are pairwise stable if the groups are of equal size and contain more than one actor.

One example of a network that meets the requirement of theorem 5 is the six-actor complete 3-partite network $K_{2,2,2}$ (see figure 3.3). This network is neither unilaterally stable nor pairwise Nash. If an actor deletes two ties, then the actor's constraint drops from 9/16 to 1/2. The network is nevertheless pairwise stable because if an actor is allowed to delete only a single tie, he prefers to keep it. His constraint then increases to 43/72. Because the deletion of two ties makes an actor better off, this network is not a Nash equilibrium of this game using the GM-model. Therefore, this is also an indication that pairwise stability might be a bit too weak a stability concept. Moreover, the $K_{2,2,2}$ is extremely inefficient. All actors would fare better in the $K_{3,3}$, which gives each actor constraint ⅓. More generally, this class of complete multipartite networks consists of Pareto-inefficient pairwise stable networks.

Simulation

We have identified two main classes of pairwise stable networks to which structural entrepreneurship might give rise. First, complete bipartite networks in which there are no redundant ties, and, if Burt's constraint formula correctly quantifies brokerage benefits, they are Pareto-efficient. No alternative network makes one actor better off without making another worse off. Second, in the multipartite networks of theorem 5, all pairs are brokered, and in addition, the majority of pairs are directly connected. The latter networks contain many redundant ties and are consequently Pareto-inefficient. Every actor would fare better in the balanced complete bipartite network of the same size. Yet these multipartite networks are pairwise stable. No entrepreneur can profitably delete a single tie and no pair of actors can profitably add a single tie.

On the basis of our analysis, we could expect either type of network – or yet another – to arise in a world in which entrepreneurs pursue access and control benefits by changing ties one by one. Simulating such a world enables us to investigate which of the two types of networks is more likely to emerge. Such simulations also help us identify potential pairwise stable structures that the two classes of networks mentioned above do not cover.

The simulation we built executes the following steps:

1. Start from some network.
2. Randomly select one actor who is allowed either to delete a tie or to add a tie if he can count on the consent of the actor to whom he wants to add a tie.
3. The actor considers his ties in a random order. For each absent or present tie an actor considers, there are two possibilities:

a. The actor makes a mistake with probability $1 - (1 - \text{noise})^{1/(n-1)}$. In case of an absent tie, he adds it if adding increases his constraint and removes it if adding it would reduce his constraint. In case of a present ties, he removes the tie if this increases his constraint and keeps it if removing would decrease his constraint. Mistaken additions do not require consent. If a tie is proposed by mistake, it is always accepted. After any mistake, the simulation returns to step 2.

b. With probability $(1 - \text{noise})^{1/(n-1)}$, the actor does not make a mistake; if he has the tie and his constraint decreases when he would remove that tie he does so. otherwise not; while if his constraint decreases if he adds the tie, while he does not have it *and* the other actor agrees with the change (the other actor's constraint does not increase), then he will add the tie. After a tie change, the simulation returns to step 2. If the actor does not change this tie, step 3 is repeated until either the actor finds a tie that he can profitably change, or he makes a mistake, or he has considered all his ties without seeing any possibility to decrease his constraint. The noise-level is varied from 0 (no mistakes) to 0.3 in steps of 0.1.[5]

4. Repeat steps 2 and 3 until no actor can profitably delete or, with consent, add a tie, and every actor has reconsidered all his ties without making a mistake.

Networks that are formed after this simulation process stops are necessarily pairwise

[5] We chose the probability for a mistake such that an actor does *not* make a mistake with probability $((1 - \text{noise})^{1/(n-1)})^{(n-1)} = (1 - \text{noise})$ after considering all his ties. Thus, he makes a mistake with a probability "noise", which we will call the noise-level.

stable; otherwise they could not have passed step 4. We first ran such simulations starting with each of the 13,597 non-isomorphic networks of sizes 2 through 8 without noise. This allowed us to identify all pairwise stable networks for these network sizes, since simulations starting from pairwise stable networks without noise end instantly. We drew a sub-sample of networks stratified on density for network size *n* = 9 through 25. We decreased the number of networks per network density for larger network sizes, in order to have comparable numbers of networks per network size. In this way, we attempt to minimize bias toward networks of a particular density while keeping the set feasibly small (for a complete overview of the sampling procedure, see AUTHOR 2005). For each emerging pairwise stable network, we checked whether it was pairwise Nash or unilaterally stable as well.

Table 1 shows the numbers of stable networks by network size and stability concept. Figure 3.4 displays the Pentagon, the Wheel, and other pairwise stable networks that do not belong to one of the two general classes mentioned above. For *n* = 2, we have the connected pair (or 1-star) as the only pairwise stable network. For *n* = 3, the 2-star is the only pairwise stable network. For *n* = 4, the $K_{2,2}$ and the 3-star are the pairwise stable networks. For *n* = 5, there are two pairwise stable networks, the Pentagon (see figure 3.4) and the $K_{2,3}$, which are both unilaterally stable. For *n* = 6, there are four pairwise stable networks: the $K_{2,4}$ and the $K_{3,3}$ as well as the "Bag" (see figure 3.4) and the $K_{2,2,2}$. The $K_{2,2,2}$ is not pairwise Nash, and only the $K_{3,3}$ is unilaterally stable. For *n* = 7, there are three pairwise stable networks, the $K_{2,5}$, the $K_{3,4}$, and the $PS_2{}^1{}_{,3}{}^6$ (see figure 3.4, the name indicates the degree-distribution, i.e., there is one actor with two ties and six with three ties). The $K_{3,4}$ is also unilaterally stable. For *n* = 8, there are ten pairwise stable networks: the $K_{2,6}$, the $K_{3,5}$, the $K_{4,4}$, the Wheel (see figure 3.4), the $K_{2,2,2,2}$, and five other networks that can be found in figure 3.4. The three densest networks, including the $K_{2,2,2,2}$, are not pairwise Nash. The

Wheel is the second unilaterally stable network for $n = 8$, in addition to the $K_{4,4}$. It is a regular structure in which everyone has three ties and occupies a regularly equivalent (Wasserman and Faust 1994, pp. 473–4) position with all the others.

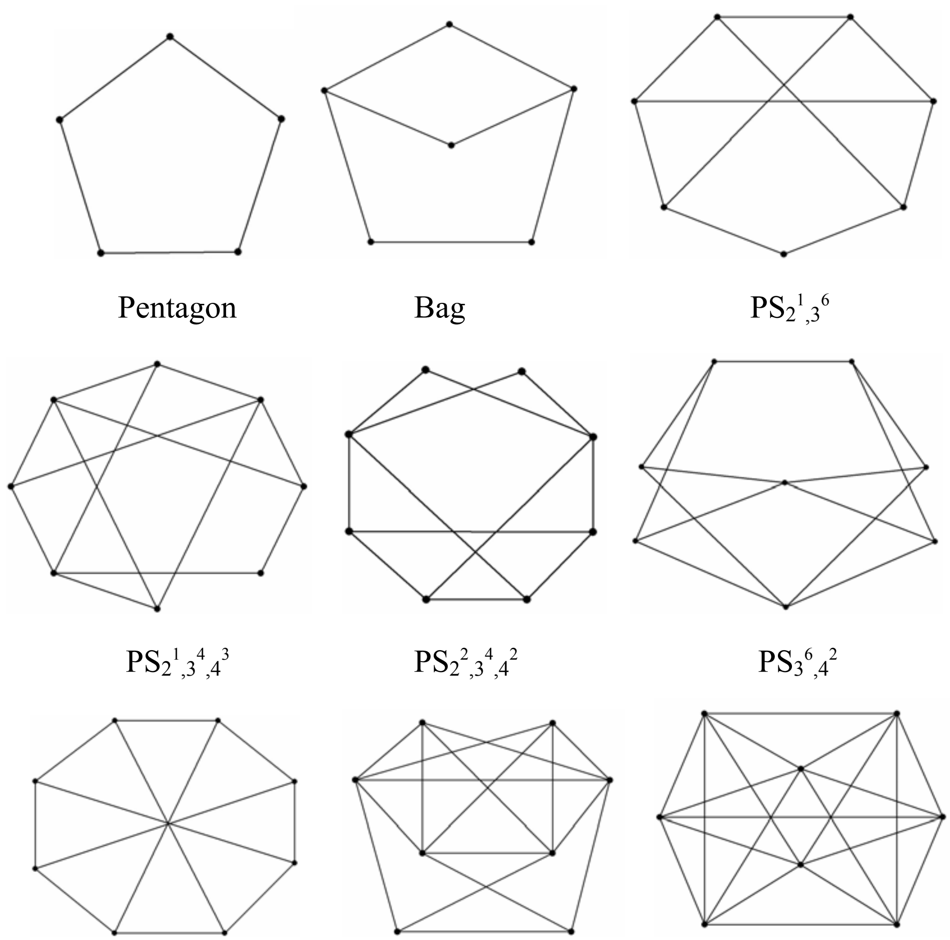


Figure 3.4. Other Pairwise Stable Networks

In addition, we checked which of the more than 12 million non-isomorphic networks of sizes 9 and 10 fulfilled a specific stability condition. In this way, we found 9 pairwise stable networks for $n = 9$ and 14 pairwise stable networks for $n = 10$.

The unilaterally stable structures for $n = 10$ are the $K_{5,5}$ and another network in which every actor has four ties. Finally, for sizes 11 through 25, we checked whether the networks that resulted from the simulations were pairwise Nash or unilaterally stable as well. The results are also summarized in table 1. For network sizes larger than 10, it is not guaranteed that every stable network has been found, because not all starting networks were considered.[6] In fact, we know from the analytical results that some networks that we did not find in the simulations are nevertheless pairwise stable. Still, these results show that the number of pairwise stable networks per network size is very small and increases only slowly with network size. There is no network size for which we identified more than two unilaterally stable networks, which suggests that the relative number of unilaterally stable networks increases even less with network size than the number of pairwise stable networks.

The additional unilaterally stable networks that we found fall into two classes that the simulation enabled us to discover. First, networks with a number of actors that is a multiple of 5, say $5m$, and that are generalizations of the Pentagon. The actors are divided into 5 equally sized groups and the groups are organized in a Pentagon. There are no ties within these groups but all actors are connected with all actors in the two neighboring groups around the Pentagon. The second network in figure 3.5 is the example with 10 actors. In these networks every actor has $2m$ ties, which makes them inefficient. By comparison, in the balanced complete bipartite network of the same size, which is also void of closed triads, an actor has approximately $2.5m$ ties and therefore a lower score on the constraint measure. The second additional class of unilaterally stable networks are networks with a number of actors that is a multiple of 8, say $8m$ actors. The actors are divided into 8 groups of size m, and these groups are

[6] Checking all structures for $n = 10$ took about 5 days with our software and computers, which implies that for $n = 11$ it would take about 500 days.

ordered along a circle. All actors are then connected to all actors in the two neighboring groups as well as to all actors in the group right across the circle. There are no ties within the groups. The third network in figure 3.5 is the example with 16 actors. These networks are generalizations of the Wheel in figure 3.4. Each actor has $3m$ ties in these networks, which is clearly inefficient if we compare it with the balanced complete bipartite network in which each actor has $4m$ ties. We found the cases $m = 1$ and $m = 2$ as results of simulations on which we report below, but not the 24-actor network with $m = 3$.[7]

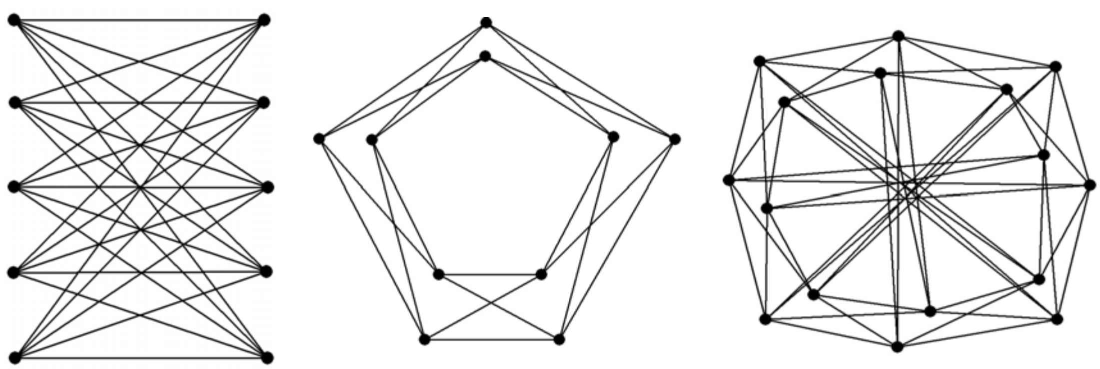
We varied the noise-level to investigate the extent to which our results depend on whether actors sometimes make mistakes in their decisions. We ran more noise-levels and more replications of the same starting network and noise-level for small networks to obtain more reliable estimates of the likelihoods to converge to a specific structure. To maintain feasibility we reduced the number of repetitions as well as the noise-levels for larger networks. One can also see from the results that variations with noise are smaller for larger networks. The complete overview is provided in table 2. Convergence to pairwise stability always occurs, and it does so reasonably fast although time to convergence increases quickly with noise. For $n = 25$, the maximum number of iterations to reach a pairwise stable network is 347 without noise, but the iterations exceed 3000 for $n = 16$ and a noise-level of 0.2.[8]

[7] The formal proof that these two classes of networks are unilaterally stable is even more laborious than the proof of theorem 4, because more different situations have to be distinguished. The proof is available from the authors, but not added to the appendix because it does not provide substantially new insights. We thank Jurjen Kamphorst for assistance in completing this proof.

[8] We did not extend the analyses to larger networks for three reasons. First, the patterns of the results are clear and change only gradually while network size is increasing. Therefore, increasing size to networks with 30 or even 40 actors would not lead to substantially new insights. Second, simulation time increases exponentially with network size, which makes running a considerable number of starting networks with 50 or more actors infeasible.

Table 3.1. Number of Stable Networks by Size and Criterion

n	Non-isomorphic networks	Connected	Pairwise Stable	Pairwise Nash	Unilaterally stable (degree distribution for non-bipartite networks)
2	2	1	1	1	1
3	4	2	1	1	1
4	11	6	2	2	1
5	34	21	2	2	(5 times 2) 2
6	156	112	4	3	1
7	1044	853	3	3	1
8	12346	11117	10	7	(8 times 3) 2
9	274668	261080	9	7	1
10	12.01e6	11.72e6	14	9	(10 times 4) 2
11	10.19e8	10.07e8	15	10	1
12	16.51e10	16.41e10	27	12	1
13	50.50e12	50.34e12	14	7	1
14	29.05e15	29.00e15	20	10	1
15	31.43e18	31.40e18	26	13	(15 times 6) 2
16	64.00e21	63.97e21	28	16	(16 times 6) 2
17	24.59e25	24.59e25	25	14	1
18	17.88e29	17.87e29	33	21	1
19	24.64e33	24.64e33	35	18	1
20	64.55e37	64.55e37	40	25	(20 times 8) 2
21	32.22e42	32.22e42	43	24	1
22	30.71e47	30.71e47	48	26	1
23	55.99e52	55.99e52	58	31	1
24	19.57e58	19.57e58	58	28	1
25	13.13e64	13.13e64	68	31	(25 times 10) 2



Balanced Complete Bipartite Generalized Pentagon Generalized Wheel

Figure 3.5. Three Classes of Unilaterally Stable Networks

Examining the entire range of *n* from size 2 through 25 in table 3, we can make several important observations. The number of pairwise stable networks increases as *n* increases, although not entirely monotonically. For any size, the number of pairwise stable networks is small compared to the total number of networks of that size. The balanced complete bipartite network is by far the most likely to emerge from the simulations. The less equal the group sizes of a complete bipartite network are, the less likely it emerges in the simulation. Table 3 shows the proportions of simulations from which the most equal and the second most equal complete bipartite networks emerge as pairwise stable networks. These two networks cover more than 90% of the resulting networks for $n > 9$ without noise and for all *n* except $n = 5$ and $n = 7$ if there is enough noise. If *n* is odd, the balanced complete bipartite network alone even accounts for over 80% of the resulting pairwise stable networks except for $n = 7$. There are no other pairwise stable networks that obtain in a large proportion of simulations (especially with noise) except for the Pentagon (19%) and the $PS_2{}^{1}{}_{,3}{}^{6}$ for $n = 7$ (32%). For $n \geq 8$, no other network occurs in more than 4% of the simulations although the complete bipartite networks that are two steps from balanced gain some territory for larger even-sized networks. The other unilaterally stable networks that are not bipartite do not emerge in substantially larger percentages than other pairwise stable networks (except for the Pentagon). The Wheel, for example, occurs in only 1% of the simulations for $n = 8$. Complete multipartite networks are obtained in only a negligible number of cases.

It turns out that adding noise to the dynamical process increases the likelihood that a network converges to a complete bipartite network, especially to a balanced complete bipartite network. This appears especially true for small networks.

Table 3.2. Simulation Design

Network size n	Number of repetition per network and per noise level	Noise-levels
2 – 3	4	0, 0.1, 0.2, and 0.3
4 – 5	250	0, 0.1, 0.2, and 0.3
6	25	0, 0.1, 0.2, and 0.3
7 – 8	4	0, 0.1, 0.2, and 0.3
9 – 16	2	0, 0.1, and 0.2
17 – 25	2	0 and 0.1

Table 3.3. Simulation Results

n	Number of starting networks	% Balanced complete bipartite ($K_{\lfloor n/2 \rfloor \lceil n/2 \rceil}$)				% $K_{\lfloor (n-2)/2 \rfloor \lceil (n+2)/2 \rceil}$				Occurrence of other networks at highest noise level
	noise	0	0.1	0.2	0.3	0	0.1	0.2	0.3	
2	2	1	1	1	1	n.a.	n.a.	n.a.	n.a.	no other
3	4	1	1	1	1	n.a.	n.a.	n.a.	n.a.	no other
4	11	.79	.83	.85	.87	.21	.17	.15	.13	no other
5	34	.80	.82	.82	.80	n.a.	n.a.	n.a.	n.a.	Pentagon: .20
6	156	.70	.76	.83	.84	.13	.10	.07	.06	Bag: .09
7	1044	.52	.62	.68	.68	.02	.01	.00	.00	PS$2^1{}_{,3}{}^6$: .32
8	12346	.61	.70	.80	.86	.12	.12	.10	.07	PS$2^1{}_{,3}{}^4{}_{,4}{}^3$, PS$3^6{}_{,4}{}^2$: .03
9	9292	.86	.92	.96	n.a.	.01	.01	.01	.01	PS$3^2{}_{,4}{}^7$: .02
10	10070	.68	.72	.73	n.a.	.24	.25	.26	n.a.	none > .01
11	10898	.91	.95	.97	n.a.	.03	.03	.02	n.a.	none > .01
12	10930	.61	.64	.70	n.a.	.33	.33	.30	n.a.	none > .01
13	5078	.88	.92	.96	n.a.	.07	.06	.04	n.a.	none > .01
14	5700	.57	.61	.70	n.a.	.35	.35	.30	n.a.	none > .01
15	6358	.86	.90	.94	n.a.	.07	.07	.05	n.a.	none > .01
16	7062	.58	.61	.63	n.a.	.35	.35	.36	n.a.	none > .01
17	2346	.86	.90	n.a.	n.a.	.09	.07	n.a.	n.a.	none > .01
18	2666	.55	.58	n.a.	n.a.	.39	.38	n.a.	n.a.	$K_{7,11}$: .01
19	3006	.85	.88	n.a.	n.a.	.10	.09	n.a.	n.a.	none > .01
20	3366	.53	.55	n.a.	n.a.	.42	.42	n.a.	n.a.	$K_{8,12}$: .01
21	3746	.84	.86	n.a.	n.a.	.13	.12	n.a.	n.a.	none > .01
22	4146	.52	.54	n.a.	n.a.	.41	.42	n.a.	n.a.	$K_{9,13}$: .02
23	4566	.82	.83	n.a.	n.a.	.14	.15	n.a.	n.a.	none > .01
24	5006	.50	.50	n.a.	n.a.	.43	.44	n.a.	n.a.	$K_{10,14}$: .04
25	5466	.80	.81	n.a.	n.a.	.16	.16	n.a.	n.a.	none > .01

Additional analyses show that it is very unlikely that our main results depend on the set of starting networks. Since we used all possible structures for $n < 9$, we reweighed our results by counting every network with the number of isomorphic structures that exist for this network.[9] In this way, we obtain statistics that resemble statistics for starting from a random network. It turns out that table 3 would hardly change despite such a rather drastic reweighing of cases. In addition, the correlation that does exist between the density of the starting networks and the density of the resulting networks is small for simulations without noise and completely disappears by adding noise.

Although only a bit more than half of the pairwise stable networks we discovered in the simulations are also pairwise Nash, it turns out that virtually all simulations (98.5% without noise, up to 99.8% with noise = 0.3) end in a pairwise Nash network. The dominant equilibrium is the balanced complete bipartite network – an efficient and egalitarian network. This result is robust throughout all analyzed network sizes and all noise levels.

Discussion

We have attempted to explicitly model Burt's network entrepreneurship: Optimize relationships in terms of brokerage opportunities, initiate relationships with others who are otherwise unconnected, and resolve relationships if they are redundant in terms of access and control benefits. More specifically, we have assumed that everyone tries to minimize his "network constraint," Burt's measure for brokerage. We then answered the question "What networks will evolve?"

In short, the answer is balanced complete bipartite networks. These networks

[9] Numbers of isomorphisms were determined using Nauty 2.2 (see McKay 1990).

consist of two groups of similar size with all inter-group ties and no intra-group ties present. Such networks meet the strongest stability criterion, most simulations generated such networks, and this result was robust across noise levels and network sizes. The balanced complete bipartite network strongly contrasts with the networks commonly depicted in the literature as outcomes of entrepreneurial activity. Burt's typical example of what a network looks like after entrepreneurial activity has taken place is that of one actor brokering multiple dense, otherwise separated groups (figure 4.1). Moreover, it contrasts with some economic models of network dynamics in information and communication settings that identify stars as the stable networks.

The difference between these networks is considerable not only in terms of structure, but also in the distribution of benefits among the entrepreneurs. Burt's single-broker structure and the star are both winner-take-all networks. Balanced complete bipartite networks, by contrast, are egalitarian. They benefit each entrepreneur equally.

Balanced complete bipartite networks have other interesting properties. No one really is a broker. Even though each entrepreneur attempts to occupy a brokering position, in these equilibria, two-step information flow between any two persons travels through at least ½ (n – 1) third parties. Thus, betweenness centrality (Freeman 1979; Wasserman and Faust 1994: 189-191) is not particularly high for any single actor. Moreover, although everybody has a low network constraint, which makes these networks efficient, no one has a substantial comparative advantage over anyone else.

The other three classes of stable networks that we identified, namely, symmetric multipartite networks, generalizations of the Pentagon, and generalizations of the Wheel, are all egalitarian as well. Every actor is equally well off. In addition, these networks are inefficient in terms of their network constraint. Especially the even-sized multipartite networks that are divided in n/2 groups of size 2 have numerous redundant relationships despite their pairwise stability. The generalizations of the Pentagon and the Wheel, by contrast, are inefficient due to sparseness. Balanced complete bipartite networks of the same size give each actor more non-redundant ties and no redundant ties.

Another property of balanced complete bipartite networks is that they are not stars. Economists have also recently modeled network dynamics as a process in which actors maximize information-based utility. Jackson and Wolinsky (1996), Bala and Goyal (2000), and Goyal and Vega-Redondo (2007), using three distinct utility functions, all find the star to be the dominant equilibrium network. Goyal and Vega-Redondo even contend that their utility function is a measurement for the richness of structural holes in someone's network. Their model does not use the constraint measure as proposed by Burt (2005). A theoretical reason for the difference between the two models is that control benefits are not subject to decay over longer paths in Goyal and Vega-Redondo's model. This implies that brokerage of indirectly received information is as valuable as brokerage of directly received information. By contrast, Burt's constraint measure implies that brokerage of information one receives indirectly is worthless, and only brokering directly received information creates value. Although both assumptions are quite extreme, we choose to insist strictly on using the constraint measure for three reasons. Actors who take indirect brokerage benefits into account must have information on the structure of the entire network. Their model is thus scope-limited to settings in which such information is readily available. The actors in our model need to know only which of their contacts are in contact with one another and which not and how many relations their direct contacts have in the network. Second, the constraint measure has empirically been shown to explain success (see the evidence discussed on page 6). And third, Burt (2007) demonstrates in a recent paper that the returns on indirect brokerage are in some contexts not visible at all, and if they are found, they are considerably smaller than returns on direct brokerage. This provides empirical evidence for an aspect of the constraint measure that is quite crucial for our results, namely that there is no negative effect of redundancy of information that travels over more than two actors. The other studies do not have a body of empirical evidence to back up their utility functions.

Still, that different formalizations of theoretically the same concept yield such different results begs the question, How robust are our results for changes in assumptions on the utility function? Three properties of the network constraint are crucial for our finding. First, network ties are in principle cheap, so if they are well chosen, one wants as many ties as possible. Second, closed triads are bad, so that you almost never want to create a tie if it closes a triad. Third, effects of actors that are at a distance larger than 2 are neglected, e.g., redundancy of information that travels over longer distances is not considered as causing a more constrained network position.

These three properties are maintained if one changes the utility function to be decreasing not in the absolute but in the relative constraint score, i.e. his absolute constraint divided by the sum of the absolute constraints of the other actors. This alternative measure would be applicable to settings in which benefits are arguably zero-sum, such as in Burt's example of promotions – if only one out of a pool of candidates can be promoted. We analyzed and simulated a model with this alternative utility function,. Results were very similar. Also under this utility function, the balanced complete bipartite networks are the dominant stable networks. They are pairwise stable and emerge in the majority of simulations.

In Burger and Buskens (2006), the utility function is reduced to its basic principles, namely, there are (marginally decreasing) benefits of having ties with others and costs for closed triads. This much more simple utility function, which also has the three properties mentioned above, led to complete bipartite networks as the most prominent class of networks to emerge and also with a higher likelihood for balanced complete bipartite networks.

In addition, Robins et al. (2005) find in a *stochastic* network evolution context that networks converge to the networks we also predict if they introduce a low probability that closed triads are formed and relatively small costs for having ties in general. Their figure 12 (Robins et al. 2005, p. 931) shows two representations of a complete bipartite network (the top one is balanced), which indicates that even with changed other parameters as long as triads are unlikely enough and direct relations are likely enough, also in this set-up complete bipartite networks emerge. Our conclusions would also not change, if we would add indirect constraint as is done by Burt (2007) – i.e., an actor's indirect constraint is the average of the constraints of his neighbors – we would still find the same networks to be stable because indirect constraint is optimized, given that each individual constraint is optimized.

The main results of our paper will change if we change one of the three crucial properties mentioned above. First, results change if we take into account redundancy over two steps; i.e., there is limited new information in multiple contacts if these are linked to many of the same third parties (see Reagans and Zuckerman 2006 for a detailed account of this type of redundancy). Complete bipartite networks are full of such redundancies over two steps and, therefore, unlikely to be stable if such redundancies are taken into account. Second, if relations are so expensive that actors do not want to connect to at least half of the group, clearly complete bipartite networks will no longer be stable. If this were the only change to the utility function, networks would remain to stabilize in bipartite structures, but these structures would not be complete. Third, stability results will – obviously – change if we do not assume that closed triads are costly but rather that actors value closed triads in a positive way. This will lead to either complete networks or, if direct ties are relatively expensive, to networks that segment into different complete sub-networks (see Burger and Buskens 2006).

An important next question is how we can test our results. The settings to which Burt's network entrepreneurship (and thus also our results) applies are competitive settings where non-redundant, first-hand information is important. We can imagine two examples of settings that fulfill these conditions to some extent. First, colleagues within firms in which the competition for promotion is very high (e.g., consider a firm as in Burt and Ronchi 2006 in which a large group of employees is trained in "structural-hole theory," we would expect that performance is increasing due to more efficient information flows, but the effects on promotion changes can expected to be less striking (as is also found by Burt and Ronchi) given that structural advantages over others are probably less pronounced). A second application can be represented by firms in competitive and innovative sectors in which well-chosen alliances with other firms are an important precondition for securing competitiveness within the sector. Testing the theory does not need to concentrate only on testing whether the ultimate stable networks emerge. Looking at the micro-level, our model also provides predictions for which relations are more likely to be established or broken than others. Using longitudinal network data one could investigate the extent to which the model predicts changes in the network even when a stable network is not yet formed. This would imply that one tests whether the network constraint has an effect on tie formation using statistical models such as developed by Snijders (2001, 2005).

To conclude, we want to emphasize that this article provides a benchmark for research on the emergence of networks. Using a combination of equilibrium and simulation analysis we have shown how one can derive stable networks, study the likelihood of the emergence of these networks, and thus how one can derive hypotheses on the structures that can be expected given specified network benefits. The theoretical methodology allows for many possible extensions. A seemingly obvious one would be adding explicit costs for maintaining ties, as is common in the literature. This would be particularly interesting if we assumed heterogeneity between actors in costs of "bridging" ties. Some actors might be "natural" entrepreneurs, while others do not have the inclination or courage to step up to strangers and build bridging ties, or they just do not observe these brokerage opportunities. Another way to include heterogeneity among actors into the model might be to assume that structural holes are not the only things that matter. In many settings, other competing incentives will be present, such as balance in friendship networks. As Burt (2005, ch. 5) notes in the last chapter of his recent book, stability might emerge in networks even with many brokerage opportunities still open, since a considerable number of actors are not interested in brokerage or are not able to observe these structural holes. Finally, considering the two types of social capital that Burt (2005, ch. 3) distinguishes, it may be fruitful to use a utility function that is a hybrid of a brokerage-based utility function and a closure-based utility function. One could then make the relative importance of closure a parameter and study the consequences for network stability. Sato (1997) has already taken a first step in this direction. Results would change, because complete bipartite networks do not include any closed triads. Likely, actors who care little about structural holes but a lot about friendship and trust end up in networks full of redundant ties and unexploited brokerage opportunities.

Appendix

Theorem 0. For the Burt constraint measure holds: $c_i \leq 9/8$ if $d_i > 0$ for all actors *i* in the network.

Proof. Recall that we can rewrite the constraint measure as:

$$c_i = \frac{1}{d_i^{\,2}} \sum_j \left[1 + \sum_k \frac{1}{d_k} \right]^2$$

where *j* is the index for neighbors of *i*, and *k* is the index for neighbors of *i* that are also connected to *j*. This constitutes the product of $\frac{1}{d_i^{\,2}}$ with a sum of squares of d_i numbers that are greater or equal to 1. Consider the sum of these numbers:

$$\left[d_i + \sum_j \sum_k \frac{1}{d_k} \right]$$

We rearrange the terms in this double summation realizing that for each neighbor *j* of *i*, the term $1/d_j$ is included exactly once for each common neighbor *k*. In addition, the number of neighbors that *j* shares with *i* is smaller than or equal to $d_j – 1$ *and* smaller than or equal to $d_i – 1$. Therefore,

$$\left[d_i + \sum_j \sum_k \frac{1}{d_k} \right] = \left[d_i + \sum_j \frac{\sum_k 1}{d_j} \right] \leq \left[d_i + \sum_j \frac{\min(d_i, d_j) - 1}{d_j} \right] \leq \left[d_i + (d_i - 1) \right] = 2d_i - 1$$

One maximizes a sum of squares of nonnegative numbers while holding the sum constant by assigning a value as close to 0 as possible to all elements but one and assigning the remainder to this last element. Since the sum of the d_i numbers is smaller than or equal to $2d_i - 1$ and d's are always larger or equal to 1, the maximum sum of squares is $d_i^2 + (d_i - 1) \cdot 1^2 = d_i^2 + d_i - 1$. Hence, we can write:

$$c_i = \frac{1}{d_i^2} \sum_j \left[1 + \sum_k \frac{1}{d_k} \right]^2 \leq \frac{d_i^2 + d_i - 1}{d_i^2} \leq \frac{5}{4}$$

For $d_i = 2$, we know that c_i reaches a maximum when the two neighbors are connected: $c_i = 9/8 < 5/4$. Since the previous formula tells us that for $d_i > 6$, c_i is strictly lower than 9/8, inspecting all 7-actor networks and not finding a value for c_i of at least 9/8 implies that 9/8 is indeed the maximum value for c_i. The argument for this last implication is that every network position for a focal actor with 6 or fewer neighbors that can occur will occur in a 7-actor network. In larger networks, this can only be complemented with neighbors who have more neighbors themselves outside the original seven actors, but this will only decrease the constraint of the focal actor. This completes the proof.

Theorem 1. Adding a tie without creating closed triads is always beneficial for both actors involved in the new tie.

Proof. We rewrite the constraint of actor i as:

$$c_i \equiv \frac{1}{d_i^2} \sum_j \left[1 + \sum_q \frac{1}{d_q} \right]^2$$

where d_i is the number of actors i is linked to, j is the index for neighbors of i, and q is the index for neighbors of i that are also connected to j. This can be done because $p_{ij} = 1/d_i$ for all neighbors j of i. Suppose that two actors i and r can add a tie without creating a closed triad. Neither before nor after tie addition are there any actors q who are connected to both *i* and *r*. Let c_i denote the network constraint of *i* before and c_i* after the initiation of the new tie, and let *j* continue to stand for the index of neighbors before tie addition. Then,

$$c_i^* \equiv \frac{1}{(d_i+1)^2}\left[1+\sum_j\left(1+\sum_q\frac{1}{d_q}\right)^2\right]$$

Using straightforward calculations, this implies that

$$c_i^* - c_i = \frac{1}{(d_i+1)^2}\left[1+\sum_j\left(1+\sum_q\frac{1}{d_q}\right)^2\right] - \frac{1}{d_i^2}\sum_j\left(1+\sum_q\frac{1}{d_q}\right)^2 = \frac{1}{(d_i+1)^2} - \frac{(d_i+1)^2-d_i^2}{d_i^2(d_i+1)^2}\sum_j\left(1+\sum_q\frac{1}{d_q}\right)^2$$

$$\leq \frac{1}{(d_i+1)^2} - \frac{(d_i+1)^2-d_i^2}{d_i(d_i+1)^2} = \frac{-d_i-1}{d_i(d_i+1)^2} = -\frac{1}{d_i(d_i+1)} < 0.$$

Thus, the addition of the new tie necessarily decreases actor *i*'s network constraint and hence increases his utility. Similarly, the constraint decreases for actor *r*. This completes the proof.

Theorem 2. A complete bipartite network of size *n* is pairwise Nash, unless it is a *k*-star with $k > 3$.

Proof. We proceed by showing that no change to such a network is profitable and feasible. Removing one or more ties is not an option in a $K_{k,l}$ because that would create a shortest path longer than 2, and hence it cannot be an improvement by corollary 1. Therefore, we need to consider only conditions under which group members create a tie within their group. Without loss of generality, assume $k \leq l$. The constraint in the complete bipartite network equals $\frac{1}{k}$ for actors in the group of size *l* and $\frac{1}{l}$ for actors in the group of size k. Creating a tie in the larger group of l actors changes the constraint of the two actors involved in that tie to

$$\frac{k}{(k+1)^2}\left[1+\frac{1}{(k+1)}\right]^2+\frac{1}{(k+1)^2}\left[1+\frac{k}{l}\right]^2=\frac{1}{(k+1)^2}\left[\frac{k(k+2)^2}{(k+1)^2}+\frac{(l+k)^2}{l^2}\right],$$

because these actors now have one common neighbor with all the actors in the group of size k and k common neighbors with each other. In order for the network to be pairwise Nash, this expression must be larger than $\frac{1}{k}$, or

$$\left[\frac{k(k+2)^2}{(k+1)^2}+\frac{(l+k)^2}{l^2}\right]>\frac{(k+1)^2}{k}\Leftrightarrow k^2l^2(k+2)^2+k(k+1)^2(k+l)^2>(k+1)^4l^2 \quad (3.2)$$
$$\Leftrightarrow k(k+1)^2(k+l)^2>(2k^2+4k+1)l^2.$$

Thus, if $k = 1$, $4(l+1)^2>7l^2\Leftrightarrow l<4$ should hold. Therefore, stars are stable only if there are fewer than 4 peripheral actors. If $k > 1$, then the inequality above is always implied by $k(k+1)^2>2k^2+4k+1\Leftrightarrow k^3-3k-1>0$, and this condition is always fulfilled for $k > 1$.

The same expression should hold for actors in the small group, but then with k and l reversed:

$$\left[\frac{l(l+2)^2}{(l+1)^2}+\frac{(l+k)^2}{k^2}\right]>\frac{(l+1)^2}{l} \quad (3.3)$$

Inequality (3.3) is satisfied for any $l > 1$, $l \geq k \geq 1$, by reasons of symmetry, because (3.2) holds for all $k > 1$. Note that the case $l = k = 1$ is irrelevant because no tie can be added. This completes the proof.

Theorem 3. Complete bipartite networks are Pareto-efficient.

Proof. Consider an actor i from the smaller group of $k \le l$ actors. The network constraint of i can be lower in another network than in the focal complete bipartite network only if he has more than l ties in that other network. This is so because the minimal constraint one can have with l ties, namely in the absence of closed triads, is

$$c_i \equiv \frac{1}{d_i^{\,2}} \sum_j \left[1 + \sum_q \frac{1}{d_q}\right]^2 = \frac{1}{l^2} l = \frac{1}{l}.$$

Let $a \le k - 1$ be this additional number of ties of actor i, let j be the index for neighbors of i in the new network, and q the index for actors that i and j share as neighbors, let π_j indicate the proportion ties of j with other neighbors of i out of all ties of j, and $\overline{\pi}_j$ the average of all $l + a$ proportions π_j. Then, for i to have a lower network constraint in the new network, the following inequality must hold:

$$\frac{1}{l} > \frac{1}{(l+a)^2} \sum_j \left[1 + \sum_q \frac{1}{d_q}\right]^2 \ge \frac{1}{(l+a)^2} \sum_j \left[1 + \sum_q \frac{1}{d_q}\right] = \frac{1}{(l+a)^2} \sum_j \left[1 + \sum_q \frac{1}{d_j}\right]$$

$$= \frac{1}{(l+a)^2} \sum_j \left[1 + \pi_j\right] = \frac{1}{(l+a)} \left[1 + \overline{\pi}_j\right] \Rightarrow \pi_j < \frac{a}{l} \text{ for some } j.$$

The least obvious step in the derivation above is the first equality, which is implied by the fact that for each j the number of times $1/d_j$ should be added due to closed triads is equal to number of common neighbors j has with i.

Note that for each j, in order to be at least as well off in the new network as in the complete bipartite network considered, his degree d_j must be at least k. Only $k - a$

– 1 of *j*'s connections can be to actors whom actor *i* is not connected to, thereby excluding *i* himself. For each *j*, π_j may therefore be no less than *a*/*k*:

$$\pi_j \geq \frac{d_j - k + a}{d_j} \geq \frac{a}{k} \geq \frac{a}{l} \text{ for all } j.$$

We have reached a contradiction. Thus, to lower *i*'s network constraint, at least one actor *j* must be given fewer than *k* neighbors, and this actor is consequently strictly worse off in the new network than in the complete bipartite network considered.

A potential Pareto-improvement must therefore leave the network constraints of all actors with *l* ties unchanged, give them precisely *l* ties and no closed triads. However, this can be done only in the complete bipartite network considered, or in the case of *k* = *l*, in another complete bipartite network with two groups of size *k*. This renders the assumed Pareto-improvement impossible and completes the proof.

Theorem 4. A complete bipartite network is unilaterally stable if and only if it is balanced.

Proof. *If*. Consider an actor *i* from the group of *k* actors. We know from the proof of theorem 3 that we cannot make this actor better off without letting one of his neighbors have a degree lower than *k*. But leaving the ties that do not involve actor *i* unchanged, all his neighbors have at least degree *k*. Actor *i* can therefore not lower his constraint by only changing his own ties. In the even case, in which *k* = *l*, by symmetry, this impossibility of unilateral improvement extends to actors of the group of size *l*. The single remaining possibility for unilateral improvement is therefore a permitted decrement of the constraint of an actor *i* from the group of *l* actors in the odd case, in which $k = l - 1$. Let $0 \leq b_k \leq k$ be the number of ties actor i has with actors from the group of size k in the new network, and let $0 \leq b_l \leq k$ be the number of ties he has with actors from the group of size l. Again, leave the ties that do not involve actor i unchanged. Because these ties constitute the balanced complete bipartite network with $k = l - 1$ actors in each group, the following inequality must hold:

$$\frac{1}{k} > \frac{1}{\left(b_k + b_l\right)^2}\left(b_k\left[1 + \frac{b_l}{k+1}\right]^2 + b_l\left[1 + \frac{b_k}{k+1}\right]^2\right) = \frac{b_k\left(b_l + k + 1\right)^2 + b_l\left(b_k + k + 1\right)^2}{\left(b_k + b_l\right)^2\left(k+1\right)^2} \Leftrightarrow$$

$$b_l^2\left(k+1\right)^2 + 2b_kb_l\left(k+1\right)^2 + b_k^2\left(k+1\right)^2 > b_kk\left(k+1\right)^2 + 2b_kb_lk\left(k+1\right) + b_kb_l^2k + b_lk\left(k+1\right)^2 + 2b_kb_lk\left(k+1\right) + b_lb_k^2k \Leftrightarrow$$

$$b_l^2\left(k+1\right)^2 - 2b_kb_l\left(k-1\right)\left(k+1\right) + b_k^2\left(k+1\right)^2 > b_kk\left(k+1\right)^2 + b_kb_l^2k + b_lk\left(k+1\right)^2 + b_lb_k^2k \Leftrightarrow$$

$$\left[b_k\left(b_k - k\right) + b_l\left(b_l - k\right)\right]\left(k+1\right)^2 > b_kb_l\left[\left(b_k + b_l\right)k + 2\left(k-1\right)\left(k+1\right)\right].$$

The left-hand side of this last inequality is never strictly positive, and the right-hand side is never strictly negative. Hence, it cannot be satisfied.

Only if. If $l - k > 1$, an actor from the larger group of l actors can delete all his ties with actors from the smaller group of k actors and add $l - 1$ ties to the other actors from the larger group of l actors. By doing so, she decreases her constraint from $\frac{1}{k}$ to $\frac{1}{l-1}$. By permitting this change, the $l - 1$ actors see their constraint fall from $\frac{1}{k}$ to $\frac{1}{k+1}$. This completes the proof.

Theorem 5. All complete multipartite networks are pairwise stable if the groups are of equal size and contain more than one actor.

Proof. Let $n_2 = n / m > 1$ be the size of each group. Then for a complete m-partite network with equal groups of n_2 size with constraint c_i, the following inequality should hold such that no one wants to severe a tie to obtain a network with constraint c_i*

(note that we need only one equation because all actors have automorphically equivalent positions).

$$c_i - c_i^* =$$

$$\frac{1}{(n-n_2)^2}(n-n_2)\left[1+\frac{n-2n_2}{n-n_2}\right]^2 - \frac{1}{(n-n_2-1)^2}\left[(n-2n_2)\left[1+\frac{n-2n_2-1}{n-n_2}\right]^2 + (n_2-1)\left[1+\frac{n-2n_2}{n-n_2}\right]^2\right] < 0 \Leftrightarrow$$

$$\frac{(2n-3n_2)^2}{(n-n_2)^3} - \frac{1}{(n-n_2-1)^2}\left[\frac{(n-2n_2)(2n-3n_2-1)^2 + (n_2-1)(2n-3n_2)^2}{(n-n_2)^2}\right] < 0 \Leftrightarrow$$

$$\frac{(2n-3n_2)^2}{(n-n_2)^3} - \frac{1}{(n-n_2-1)^2}\left[\frac{(n-n_2-1)(2n-3n_2)^2 - (n-2n_2)(4n-6n_2-1)}{(n-n_2)^2}\right] < 0 \Leftrightarrow$$

$$\frac{(n-2n_2)(4n-6n_2-1)}{(n-n_2-1)^2(n-n_2)^2} - \frac{(2n-3n_2)^2}{(n-n_2-1)(n-n_2)^3} < 0 \Leftrightarrow$$

$$(n-2n_2)(4n-6n_2-1)(n-n_2) - (n-n_2-1)(2n-3n_2)^2 < 0 \Leftrightarrow$$

$$4(n-2n_2)(n-n_2)(n-\frac{3}{2}n_2-\frac{1}{4}) - 4(n-n_2-1)(n-\frac{3}{2}n_2)^2 < 0$$

which is always true because

$$(n-2n_2)(n-n_2) < (n-\frac{3}{2}n_2)^2 \text{ and } (n-\frac{3}{2}n_2-\frac{1}{4}) < (n-n_2-1) \text{ if } n_2 \geq 2.$$

For no actor to benefit from adding any of his equivalent potential ties in this complete n/n_2-bipartite network with equal-sized groups, the following inequality must hold:

$$\frac{1}{(n-n_2)^2}(n-n_2)\left[1+\frac{n-2n_2}{n-n_2}\right]^2 - \frac{1}{(n-n_2+1)^2}\left[(n-n_2)\left[1+\frac{1}{n-n_2+1}+\frac{n-2n_2}{n-n_2}\right]^2 + \left[1+\frac{n-n_2}{n-n_2}\right]^2\right] < 0 \Leftrightarrow$$

$$\frac{(2n-3n_2)^2}{(n-n_2)^3} - \frac{[(n-n_2+1)(2n-3n_2)+(n-n_2)]^2 + 4(n-n_2+1)^2(n-n_2)}{(n-n_2+1)^4(n-n_2)} < 0 \Leftrightarrow$$

$$(2n-3n_2)^2(n-n_2+1)^4-\left([(n-n_2+1)(2n-3n_2)+(n-n_2)]^2+4(n-n_2+1)^2(n-n_2)\right)(n-n_2)^2<0 \Leftrightarrow$$

$$x^4(7-6n_2)+x^3(12-18n_2)+x^2(4-16n_2)-4xn_2+2x^3n_2^2+5x^2n_2^2+4xn_2^2+n_2^2<0, \text{ where } x=n-n_2.$$

Since $n_2 < x = n - n_2$, the equation above is implied by (replacing n_2^2 by xn_2):

$$x^4(7-4n_2)+x^3(12-13n_2)+x^2(4-12n_2)-3xn_2<0,$$

which holds because $n_2 \geq 2$ and $x > 0$. This completes the proof.

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CHAPTER 4: EXCHANGE NETWORKS IN EQUILIBRIUM

(with Vincent Buskens)

Introduction

How well people do in social or economic exchange depends heavily on how they are connected to potential exchange partners. This conclusion draws a branch of sociology that seeks to answer the question at what rate two trading parties exchange if they are embedded in a network of exchange relations (e.g., Willer 1999).

The conclusion is important. It makes relaxing the classical exchange assumption that every trader is connected to every other trader worth the cost of the theoretical complexity that it adds. However, this edge over classical exchange theory immediately dematerializes once the assumption is replaced by one of a different exchange network. Any fixed alternative network that sociological exchange scholars impose is as arbitrary as the complete network.

Our aim is to investigate what deals traders close in equilibrium when networks are variable. Instead of forcing exchange relations upon the profit-seeking actors, we allow them to choose trading partners freely. Costs of entertaining a trade relation may prevent them from forming the complete network.

We will vary these costs between 0 and maximum profit. We stay within the framework of sociological exchange networks. This will allow us to use the information it has acquired in laboratory research over the past thirty years for an informed guess on expected profit splits in incomplete networks. It will, on the other hand, limit us theoretically to the particular experimental exchange setting it considers. Let us take a look at this setting to clarify how limiting this framework precisely is.

Subjects are positioned behind a networked computer. Subjects occupy each position in the network precisely once. The time they occupy a position is called a period. The number of periods thus equals the number of positions in the network under study. A period consists of several rounds. Each round ends either because no further exchanges are possible or because time is up. During a round, a subject can make offers to other subjects he is connected to in the network ('neighbors') as long as he has not closed a deal with a neighbor. A subject can also accept an offer from a neighbor, or confirm a neighbor's acceptance of an offer he made himself. An offer is a proposed division of 24 profit points. Whenever an offer is accepted and confirmed, the two subjects exchange, that is, they receive the share of the profit they agreed upon. They are temporarily removed from the network and must wait till the next round to continue bargaining. Thus, each subject can close a deal with at most one neighbor in the network, which has been called the "one-exchange rule."

Before C and D exchange   After C and D exchange

Figure 4.1. The 4-Line Network

Consider a network called the 4-Line (see Figure 4.1). A node represents a position and an edge a relation in which exchange might take place. In the 4-Line, B and C can exchange with one another or with A resp. D, while A and D can only exchange with one actor, B and C resp. Table 1 shows an imaginary bargaining sequence for the 4-Line in the experimental setting outlined above.

Table 4.1. Imaginary Bargaining Sequence for the 4-Line

Round	From	To	Kept	Offered	Type	Time
1	A	B	16	8	offer	0:07
1	C	B	17	7	offer	0:08
1	B	C	15	9	offer	0:12
1	D	C	10	14	offer	0:16
1	C	D	14	10	accept	0:19
1	D	C	10	14	confirm	0:21
1	C	D	14	10	exchange	0:21
1	A	B	12	12	offer	0:23
1	B	A	15	9	offer	0:26
1	A	B	12	12	offer	0:28
1	B	A	14	10	offer	0:32
1	A	B	12	12	offer	0:35
1	B	A	12	12	accept	0:40
1	A	B	12	12	confirm	0:42
1	B	A	12	12	exchange	0:42
2	C	D	14	10	offer	0:05

At the beginning of the sequence, the network looks like the one left in Figure 4.1. The first accepted and confirmed offer is the (10, 14) division proposed by D to C in the 17th second. After their exchange the only remaining potential exchange is one taking place between A and B. The network now looks like the one right in Figure 4.1, and is essentially a dyad with two isolates. The exchange between A and B occurs at time 0:42 after a number of offers and counteroffers. The first bid in round 2 is a (14, 10) division proposed by the C to D.

Let us point to two important differences between this exchange setting and typical exchange settings studied in economics. First, no actor can sell or buy multiple products or services. Second, buyers and sellers are not distinguished. Everyone can trade with everyone. An example would be a monogamous homosexual relationship market. Trade relations are potential relationships, only one of which can be realized at a time. The exchange rate then reflects the balance of power in the relationship. (For an application of exchange theory to intimate relationships, see Van de Rijt and Macy 2006).

Our strategy will be to provide the actors with all information on how much they would earn from exchange if they added or deleted a tie. Then we let them maximize profit by adding and deleting ties and see what exchange network has emerged when the dust clears. The distribution of exchange benefits in this equilibrium network will be the answer to our research question. This strategy thus requires information on expected earnings and an equilibrium concept.

In order for the earnings information to be of decent quality and unambiguous, we need a method that translates the host of experimental findings on earnings into a single-valued estimate for every position in every network, and does so following a theoretically underpinned principle of exchange. We employ Sequential Power-Dependence Theory (SPDT), developed by Buskens and Van de Rijt (2006), who adjust Cook and Yamagishi (1992)'s implementation of Emerson's (1962) exchange principle of equidependence, show that the method is empirically competitive with other existing methods, and prove that its predictions satisfy uniqueness and existence. The equi-dependence principle corresponds to the Nash bargaining solution (Nash

1950) from cooperative bargaining theory when profit is equated with utility and expected earnings in alternative relationships are equated with disagreement points; Exchange occurs when the difference between proposed share and disagreement point for actor 1 equals that difference for actor 2. We review SPDT in the next section.

In order to determine equilibrium given these predicted earnings, we employ two equilibrium concepts: Pairwise stability (Jackson and Wolinsky 1996) and unilateral stability (Van de Rijt and Buskens 2007). Networks are stable if none of n actors can instantly gain from changing t ties without any of his new contacts incurring an instant loss. For pairwise stability, $t = 1$. For unilateral stability, $t = n - 1$.

Specification of Exchange Rates

Let $X = (x_{ij})$ be an $n \times n$ symmetric network, with $x_{ij} = x_{ji} = 1$ if actors i and j can exchange, 0 otherwise. Let $N(X) = \{\{i, j\} | x_{ij} = 1\}$ be the set of ties in network X, where ties are two-element subsets of the set of all actors, and let $N = |N(X)|$ be the cardinality of this set. $X \backslash \{i, j\}$ is the network X with the ith and jth column and row deleted, i.e., with i and j and all their relations removed from the network.

$v_i(X)$ is the value of network X to actor i, which is defined as i's expected value across all his relations. If i is expected to earn all of the 24 points in every experiment, $v_i(X) = 1$, while if he is expected to earn 12 points on average, $v_i(X) = ½$.[1] Let p_{ij} be the probability that i and j are the first to exchange. Because only actors that have a link can exchange, $p_{ij} = 0$ if $\{i, j\} \notin N(X)$. As long as the network is not empty it is assumed that at each point in time exactly one tie exchanges, or $\sum_i \sum_{j \neq i} = 1$. $p_{kl}^{ij} = p_{kl} / (1 - p_{ij})$ denotes the conditional probability that k and l are the first to exchange if i and j are not. Define $p_{kl}^{ij} = 0$ if $p_{ij} = 1$, which corresponds below with i and j dividing the profit pool equally if they have to exchange with each other. It

[1] The rather arbitrary profit pool of 24 points is transformed here to a 0-1 scale via the value function.

follows that $v_i(X) = v_j(X) = ½$ if $n = 2$ and $x_{ij} = x_{ji} = 1$ (a dyad). Furthermore, $v_i(X) = 0$ for any actor i without any ties. Define s_{ij}^{i} as the share an actor i obtains if he is involved in the first exchange with actor j. Because the profit pool is always distributed in its entirety, $s_{ij}^{j} = 1 - s_{ij}^{i}$. If i is not involved in the first exchange, but instead j and k are, then he receives what he can obtain in $X \backslash \{j, k\}$. For the example of the 4-Line, in the situation after C and D have exchanged, right in Figure 4.1, the 4-Line has been transformed into a dyad. A and B then earn ½ each. The value of a network to an actor i can thus be recursively defined as

$$v_i(X) = \sum_{j \neq i} p_{ij} s_{ij}^{i} + \sum_{j \neq i} \sum_{k \neq i, k \neq j} p_{jk} v_i(X \backslash \{j,k\}) \tag{4.1}$$

where the values for p_{ij} are given. Now, suppose that for a certain network size n, we know each value v_i for each actor i of each network of smaller size. Then, in order to be able to calculate the values of a network of size n for all actors in that network, what remains to be done is computing the s_{ij}. Consider the case that i and j exchange first. Now, the equidependence principle (Emerson 1962) is applied that each actor gets his alternative payoff plus half of the remainder of the profit pool (this remainder might be a negative amount). More precisely, i's alternative payoff in an exchange with j is what i would have gotten if i and j would not have been the first to exchange. This is what i gets in the case of each of the other relations exchanging first, weighted by the corresponding probabilities of first exchange. In a formula, i's expected payoffs if he does not start to exchange with j equals

$$\sum_{k \neq i, j} p_{ik}^{ij} s_{ik}^{i} + \sum_{k \neq i, j} \sum_{l \neq i, k} p_{kl}^{ij} v_i(X \backslash \{k,l\}) \tag{4.2}$$

Here the first term indicates the alternatives payoffs if i exchanges with someone else in the first opportunity and the second term indicates what i expects to receive if he does not exchange first. A similar formula holds for *j* and the equidependence principle then implies that the difference between the payoffs in a first exchange between *i* and *j* should be the same as the difference between their expected exchanges if they would not exchange with each other. Thus,

$$s_{ij}^{i} - s_{ij}^{j} = \sum_{k \neq i,j} p_{ik}^{ij} \left(s_{ik}^{i} - v_j \left(X \setminus \{i,k\} \right) \right) + \sum_{k \neq i,j} p_{jk}^{ij} \left(v_i \left(X \setminus \{i,k\} \right) - s_{jk}^{j} \right) \tag{4.3}$$
$$+ \sum_{k \neq i,j} \sum_{l \neq i,j,k} p_{kl}^{ij} \left[v_i \left(X \setminus \{k,l\} \right) - v_j \left(X \setminus \{k,l\} \right) \right]$$

or

$$2s_{ij}^{i} = 1 + \sum_{k \neq i,j} p_{ik}^{ij} \left(s_{ik}^{i} - v_j \left(X \setminus \{i,k\} \right) \right) + \sum_{k \neq i,j} p_{jk}^{ij} \left(v_i \left(X \setminus \{i,k\} \right) - s_{jk}^{j} \right) \tag{4.4}$$
$$+ \sum_{k \neq i,j} \sum_{l \neq i,j,k} p_{kl}^{ij} \left[v_i \left(X \setminus \{k,l\} \right) - v_j \left(X \setminus \{k,l\} \right) \right]$$

The right-hand side of equation (4.4) consists of three parts. The first part indicates how much *i* and *j* would obtain if *i* and someone else than *j* were the first to exchange. The second part indicates what *i* and *j* would obtain if *j* and someone else than *i* were the first to exchange. And the third part indicates what *i* and *j* would obtain if two other actors exchanged first. So the difference in the profit between *i* and *j* when they exchange first is assumed equal to the difference between what *i* and *j* earn when they do not exchange first. The small rearrangement in equation (4.4) uses the fact that the proportions the two actors obtain sum to 1. A similar equation holds for every exchange relation $\{i, j\}$ in the network *X*. These *N* equations include *N* unknown variables s_{ij}^{i} when the outcomes of exchanges for smaller networks are already known. The corresponding system of linear equations *As* = *c* has *A* as N x N matrix of coefficients and c as vector of constants. Buskens and Van de Rijt (2006) show that for parameters p_{ij} and given network values in smaller networks $v_i(X\backslash\{i, j\})$, the system $As = c$ always has a unique solution, and this solution satisfies $0 \leq s_{ij}^{i} \leq 1$ for all $\{i, j\}$ in $N(X)$.

As an example we will take a look at the so-called 'Box-Stem' (see Figure 4.2). This network contains six actors, A through F, and six ties: $N(X) = \{\{A, B\}, \{A, C\}, \{B, D\}, \{C, D\}, \{D, E\}, \{E, F\}\}$. Assume we have already calculated the values of networks with 4 actors. If A and B exchange first they obtain s_{AB} and $1 - s_{AB}$. Then we would have a 4-Line (see Figure 4.1) left, in which actors C through F earn 7/16, 9/16, 9/16, and 7/16, in that order. These are the six entries in the first row of Table 2. We repeat this procedure for all other rows in a similar manner. This leads to the following list of exchange possibilities.

Table 4.2. Profit Matrix for the Box-Stem

First exchange	A	B	C	D	E	F
AB	s_{AB}	$1 - s_{AB}$	7/16	9/16	9/16	7/16
AC	s_{AC}	7/16	$1 - s_{AC}$	9/16	9/16	7/16
BD	1/2	s_{BD}	1/2	$1 - s_{BD}$	1/2	1/2
CD	1/2	1/2	s_{CD}	$1 - s_{CD}$	1/2	1/2
DE	1	0	0	s_{DE}	$1 - s_{DE}$	0
EF	1/2	1/2	1/2	1/2	s_{EF}	$1 - s_{EF}$

Now we derive the six equidependence equations. We assume for this example that all relations are equally likely to be the first to be used (the existence and uniqueness of the solution do not depend on that assumption). For the exchange between A and B should hold (subtracting the payoffs for A and B in rows 2 through 6 above)

$$2s_{AB} = 1 + 1/5((s_{AC} - 7/16) + (½ - s_{BD}) + (½ - ½) + (0 - 1) + (½ - ½)) \tag{4.5}$$

or

$$2s_{AB} - 1/5s_{AC} + 1/5s_{BD} = 13/16 \tag{4.6}$$

Similarly, the other five equations can be derived. Then the solution to this system of six linear equations in six unknowns can be calculated: $s_{AB} = 0.65$, $s_{AC} = 0.65$, $s_{BD} = 0.39$, $s_{CD} = 0.39$, $s_{DE} = 0.51$, and $s_{EF} = 0.60$. These values eliminate all unknowns from Table 2, so that the weighted column averages are now the expected profit shares of the six actors. Since it was assumed that all dyads are equally likely to be first, all the weights (p_{ij}) are equal to 1/6. For actor *A* we have

$$v_A = 1/6(0.65 + 0.65 + 0.5 + 0.5 + 1 + 0.5) = 0.63 \tag{4.7}$$

Similarly, we obtain $v_B = 0.34$, $v_C = 0.34$, $v_D = 0.55$, $v_E = 0.53$, and $v_F = 0.33$ for the other actors.

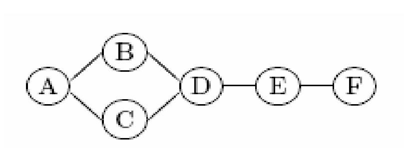

Figure 4.2. The Box-Stem Network

Analysis Strategy

We are interested in the networks that are pairwise or unilaterally stable if actors expect SPDT earnings in their exchange relations and pay a maintenance cost for each relation they entertain. We vary this maintenance cost between 0 and ½. Values above ½ are uninteresting because dyadic costs then outweigh dyadic benefits, so that only the empty network can be stable. Due to computational constraints, we limit analysis to all small networks, with between 2 and 8 actors.

Let us review the two equilibrium concepts in more detail. We identify those networks in which no actor wants to delete a tie and no pair of actors wants to add a tie. Such networks are referred to as pairwise stable (Jackson and Wolinsky, 1996). An actor wants to delete a tie if he receives a larger profit in the network without it. This may depend on how costly it is to maintain such a tie. A pair of actors wants to add a tie if at least one of the actors is strictly better of adding this tie and the other actor is not worse off by adding this tie. E.g., in the so-called 3-Line (A connected to B and B connected to C) with no cost for maintaining ties, a dyad and an isolate is not stable, because one actor in the dyad would be better off by connecting to the isolate. And the isolate would not object because he earns zero in either network. As soon as there is some cost $0 < c \leq ½$ for maintaining ties, the isolate vetoes a proposed connection. He would be paying maintenance cost for a tie that brings no profit.

In addition, we check which of the pairwise stable networks also fulfill the stronger equilibrium concept unilateral stability (equivalent to initiative-proof Nash in the related Myerson game, see Van de Rijt and Buskens, 2007). The stronger stability concept considers a network stable only if no actor can propose a new configuration of his own ties to which none of his new contacts object. If some actor can delete some ties and add some ties to other actors that agree to add those ties, i.e. they are at least as well off in the new network as in the original one, then a network is not stable. E.g., consider a triangle (closed triad) and an isolate. As we will see below this network is pairwise stable in some cost range. However, this network will never be unilaterally stable, because an actor in the triangle can propose to remove the two links to the other actors in the triangle and add a link to the isolate. The isolate will not object to this proposal. Although they have no 'veto right', also the two other actors will profit from this hange. They lose one tie and gain profit. If we use the term 'stable' below without the qualification pairwise or unilateral, we mean that both stability concepts apply. Which networks are pairwise stable for a given cost range is determined as follows. The upper bound of the cost range in which the network is stable is the minimum loss for an actor over all possible tie deletions. The lower bound is the maximum gain of the least interested actor across all tie additions. Clearly if the interval is empty, i.e. the upper bound is lower than the lower bound, the network is unstable for all possible costs. Given the fact that for pairwise stability we always consider either only tie deletion or only tie addition, the range of values for which a network is pairwise stable is always one connected interval (possibly empty). On the contrary, unilateral stability considers tie addition and tie removal simultaneously, which implies that every unilateral deviation from a network has its own lower bound and upper bound for which the actors want such change. This can also lead to a stability cost range that consists of multiple disconnected intervals. We identified the only two such cases for small networks.

Stability Results

Our first result is that few networks are stable. Of all 13,597 networks with 2 − 8 actors, only 210 are stable. Moreover, most of the networks that are stable under only one of the two stability concepts are stable for a small cost range. If we consider pairwise stable networks that are stable in a cost range of at least 1/12 (or two points in the experimental setting), only 59 stable networks remain. We will focus on this particular subset, to make all results fit in a 1-page table. Networks with a broader cost range can be considered more stable; A random utility component exceeding 1/12 would destabilize any of the omitted networks. Complete data on the stability ranges under SEDT and EVT are available from the authors. 8 of the 59 stable networks are empty, which are pairwise stable as well as unilaterally stable for any tie cost larger or equal to ½. Table 3 displays all 51 non-empty networks that for costs in an interval of at least 1/12, or 2 profit points in the experiment, are pairwise stable. For these networks, it lists the name, the lower diagonal of the adjacency matrix of the network, the lower and upper bound of the cost interval within which it is pairwise stable, and the lower and upper bound of the cost interval within which it is unilaterally stable. 34 of these pairwise stable networks are also unilaterally stable within some cost interval.

Second, the efficiency of stable networks is positively related to the size of the interval within which they are stable. The complete network, the most inefficient network for costs between 0 and 1/2, is never stable for costs greater than some critical value, but always stable if costs are 0. For any size, a network consisting of dyads with at most one isolate, the most efficient network for costs between 0 and 1/2, is stable if costs are 0.042 or higher. This is because the addition of a tie with an isolate creates a 3-Line in which the isolate again earns zero, and the addition of a tie between two dyads creates a 4-Line in which the middle actors earn only marginally more than in two dyads. The positive relationship between efficiency and stability is also reflected in the negative relationship between density and tie cost.

Table 4.3. Stable Networks

	Lower diagonal network matrix	Lower bound pairwise	Upper bound pairwise	Lower bound unilateral	Upper bound unilateral
Dyad	1	$-\infty$	.50000	$-\infty$	.50000
Dyad, isolate	001	.00000	.50000	.00000	.50000
Triangle	111	$-\infty$	.33333	$-\infty$	.16667
Two dyads	100001	.04167	.50000	.04167	.50000
Triangle, isolate	001011	.19643	.33333	not unilaterally stable	
Square	110011	.03333	.20833	.03333	.20833
Full square	111111	$-\infty$	.13333	$-\infty$	.13333
Two dyads, isolate	0010000001	.04167	.50000	.0625	.50000
Square, isolate	0010100011	.03333	.20833	not unilaterally stable	
Triangle, dyad	1000010011	-.01255	.33333	.03419	.16667
Pentagon	1100100011	.10936	.22759	.10936	.20000
Full square, isolate	0010110111	.00855	.13333	not unilaterally stable	
Full pentagon	1111111111	$-\infty$	.16923	$-\infty$	.10000
Three dyads	100001000000001	.04167	.50000	.04167	.50000
Triangle, dyad, isolate	001000000100011	.19643	.33333	not unilaterally stable	
Pentagon, isolate	001010001000011	.13084	.22759	not unilaterally stable	
Square, dyad	100001001000011	.03333	.20833	.03333	.20833
Hexagon	110000010100110	.07937	.23439	.07937	.22222
3,3-full bipartite[1]	110100011101110	.02908	.12179	.02908	.12179
Two triangles	111000000100011	.10606	.33333	not unilaterally stable	
Full square, dyad	100001001100111	.02644	.13333	.02644	.13333
Full hexagon	111111111111111	$-\infty$	.06883	$-\infty$	.06883
Three dyads, isolate	001000000100000000001	.04167	.50000	.06250	.50000
Square, dyad, isolate	001000000100010000011	.03333	.20833	not unilaterally stable	
Hexagon, isolate	001010000000101000110	.07937	.23439	not unilaterally stable	
3,3-full bipartite, isolate[1]	001010010000111001110	.02908	.12179	not unilaterally stable	
Triangle, two dyads	100001000000001000011	.04167	.33333	.04167	.16667
Pentagon, dyad	100001001000010000011	.10936	.22759	.10936	.20000
Two triangles, isolate	001011000000001000011	.19643	.33333	not unilaterally stable	
Full square, dyad, isolate	001000000100011000111	.02644	.13333	not unilaterally stable	
Heptagon	110000000101010001010	.08200	.22823	.08200	.21429
Square, triangle	111000000100010000011	.03333	.20833	.07930	.16667
Full square, triangle	111000000100011000111	-.00526	.13333	.10244	.11985
Full pentagon, dyad	100001001100111001111	-.01250	.16923	.03159	.10000
Full heptagon	111111111111111111111	$-\infty$	.10689	$-\infty$	.07143
Four dyads	1000010000000010000000000001	.04167	.50000	.04167	.50000
Triangle, two dyads, isolate	0010000001000000000010000011	.19643	.33333	not unilaterally stable	
Pentagon, dyad, isolate	0010000001000100000100000011	.13084	.22759	not unilaterally stable	
Square, two dyads	1000010000000010000100000011	.04167	.20833	.04167	.20833
Heptagon, isolate	0010100000000010010100001010	.09347	.22823	not unilaterally stable	
Hexagon, dyad	1000010010000000001010000110	.07937	.23439	.07937	.22222
3,3-full bipartite, dyad[1]	1000010010001000001110001110	.02908	.12179	.02908	.12179
Two squares	1100000001000100000110110000	.03647	.20833	.03647	.20833
Octagon	1100000001000100100100010010	.08353	.23087	.08353	.21667
Cube[1]	1101000000011010101100011100	.04512	.13564	.04512	.13564
Two triangles, dyad	1000010011000000000010000011	.10606	.33333	not unilaterally stable	
Full square, two dyads[1]	1000010000000010000110000111	.04167	.13333	not unilaterally stable	
Pentagon, triangle	1110000001000100000100000011	.10936	.22759	.10936	.16667
Square, full square	1100110000000010000110000111	.03333	.13333	.03333	.13333
Two full squares	1111110000000010000110000111	.02021	.13333	.02021	.13333
Full pentagon, triangle	1110000001000110001110001111	.06719	.16923	not unilaterally stable	
Full octagon	1111111111111111111111111111	$-\infty$	.04385	$-\infty$	.04385

No other networks than the ones in this table are pairwise stable for cost intervals of at least 1/14.

[1] These networks are pairwise stable in an interval smaller than 1/10.

Our third result is that all stable networks are egalitarian: Profit is split equally. In the literature on exchange networks, these networks are called ‘equal-power’. This equality can be traced back to the symmetric structure of the networks; Actors within components have automorphically equivalent positions. Any profit prediction method that gives automorphically equivalent actors the same profit will make such networks equal-power. SPDT does so because such actors are obviously equi-dependent.

It is instructive to look at some networks to get an idea why automorphism is so common-place in stable networks. Recall that the cost range within which a pairwise network is stable has as lower bound the maximum gain of the least interested actor across all tie additions and as upper bound the minimum loss for an actor over all possible tie deletions. For the ‘cycle’ networks and the dyads—both symmetric—it is clear that the loss of any tie decreases the benefits of the networks for each of the actors in the network quite dramatically. This implies that the upper bound is pretty high. The gains that can be made in these networks by adding a tie on the other hand are rather low. This leaves a considerable range of tie costs for which these networks are stable. The full networks—another class of symmetric networks—are stable because there the lower bound of the interval does not exist (or equals minus infinity) because no one can add a tie. Therefore, these networks are pairwise stable as long as the gain from adding the last tie in the network outweighs its cost.

One reason for why many asymmetric networks are unstable for most or all costs of ties is that the profits for actors who add a tie to a symmetric network are mostly smaller than the externalities they create for the other actors. Consider the square. If two actors add a tie we obtain the D-Box. Because the new tie creates the possibility for inefficient exchange patterns, namely the two actors with the new tie exchange, the expected total profit of the new network is smaller than in the square. This implies that there has to be a net loss and that if tie cost is low enough to add this tie in the D-Box, these cost are certainly low enough for the other two actors to add the last tie and make the symmetric full square in which everything is even again and the expected total profit is also optimal again. Similarly, if ties become too costly to cover the advantage of being in the square rather than being one of the peripheral actors in the 4-Line, the square will disintegrate. Following a similar argument, if the 4-Line is better than the square for the peripheral actors, then being in a dyad is certainly better for the middle actors in the 4-Line than being in the 4-Line. This is why asymmetric networks like the D-Box and the 4-Line are not stable. These arguments become more complex in the larger networks we considered. Non-automorphically equivalent actors might coexist in larger stable networks, but these networks are stable only for small cost ranges. We do not know to what extent these conclusions generalize for large networks.

Also in the disconnected stable structures, components contain positions that are all automorphically equivalent. Stability of such structures involves two conditions. First, each subnetwork should be stable itself, thus the stability range can be at most the intersection of the stability ranges of the components. Second, the actors in the two components should not have an incentive to add a tie between them. Because of the automorphic equivalence, only one tie needs to be considered between each pair of subnetworks for understanding stability. For two components with an even or two components with an odd number of actors, the arguments are similar as above. With separate components everybody is obtaining a similar proportion of the resource pool. Adding a tie mostly hurts the not involved actors more than it helps the involved actors. This implies that whenever ties are cheap enough to provide an incentive to connect subnetworks, other actors will tend to subsequently also have incentives to try to get back to a more symmetric structure. If costs on the other hand are just high enough for the connection to be no longer profitable, actors also do not have an incentive to remove ties. A two-component network, one with an even number of actors and one with an odd number of actors, is sometimes even stable for zero tie cost, because the actor in the odd component does not want to connect at all with the other component. The reason is that the addition of this tie is so much more advantageous for the connecting actor in the component with an even number of actors that the expected outcome for the connecting actor in the odd-sized component is decreased as a result of this new connection with a relatively high power actor. These networks are not unilaterally stable for zero cost because (at least in the networks of small *n* that we consider here) one actor of the even-sized component wants to connect with all actors in the odd-sized component making himself and the actors in the odd-sized component better off.

Equilibrium Selection

We have identified stable networks for all costs. For many costs, multiple equilibria exist. This raises the question which equilibrium is more likely. We conclude by providing a partial answer to that question.

We start from a given network structure and give actors the opportunity to add and delete ties. We are interested in what network will ultimately evolve. We choose a setup in which an actor is randomly chosen. This actor is allowed to change one tie. We assume that the actor maximizes utility across all tie changes that can count on approval or don’t need approval. He does not propose a tie that makes the relevant other worse off. Thus, one could say that these actors are myopically rational actors and think all other actors are as well; They choose the best option possible, but disregard subsequent changes by other actors, and expect other actors to do so as well in their evaluations of tie proposals. We continue to choose random actors until no more actor wants to remove a tie or can add a beneficial tie with consent of the other actor in this tie. If this process ends, it ends in a pairwise stable network. We run the process starting from all 13,597 possible different network structures with 2 to 8 actors. All runs converged to a pairwise stable network. The simulation was completely deterministic except for the order of actors that could change a tie. To estimate the likelihood of convergence to a certain network we weighted the outcomes proportionally to the number of isomorphisms of the starting network. We used eight different values of tie costs: 0, 1/24, 1/12, 1/8, 1/6, 5/24, 1/4, 7/24, and 1/3. For tie costs higher than 1/3, a network consisting of dyads and possibly an isolate is the only stable network given the size of the network.

If tie costs are 0, pairwise stable networks other than the complete network exist only for network sizes 5 and 7. These incomplete networks hardly ever evolve. The full network loses its dominance if cost becomes higher, but the precise moment depends on the size of the network. The full network already loses its dominance at tie costs 1/24 for sizes 3, 4, and 8. For sizes 6 and 7 this is 1/12, and 1/8 for size 5. For certain intermediate cost ranges, no networks are very dominant. At a certain cost level (for most sizes around 1/8), cycles (pentagon, hexagon etc.) or combinations of cycles, e.g., squares and triads, become the most dominant networks. As can be expected, the density of the emerging networks is strongly negatively related to tie cost. The correlation between tie cost and density of the emerging networks is about -0.84. For network sizes 2 through 8, at and beyond tie cost 5/24, the (efficient) networks with dyads and possibly one isolate are the most dominant networks.

Discussion

We have sought an answer to the question how exchange parties trade in equilibrium when the available trade relations are not fixed as in classical exchange theory but variable. We proceeded by having profit-seeking actors add and delete costly ties after giving them the information on earnings increases and decreases from such changes to the network. This information came from a profit prediction method, SPDT, that (1) was backed up by extensive experimental investigations on how actors in small networks exchange, (2) provided unique predictions on who earns how much in any relation in any network, (3) was grounded in theoretical work on exchange. We employed stability concepts from the recently emerged literature on dynamic networks to determine equilibrium. We found that very few networks are stable, that in these stable networks actors always split the profit pool equally, and that they often do not trade efficiently. This inefficiency comes from the fact that too many costly exchange relations are entertained. The traders would all make more net profit in a network with fewer exchange relations. The equality result could be traced back to automorphic equivalence between trading parties in stable networks. Emerson's principle of equi-dependence, on which the SPDT method is based, assigns equal profits to automorphically equivalent parties. One related study of exchange network evolution has been carried out that draws a similar conclusion with respect to exchange outcomes in changing networks. Bonacich (2001) has actors move on a checkerboard rather than add and delete ties, and has them satisfice rather than maximize, but also finds that in equilibrium payoff differentials are small. The current study is limited in at least three important ways. First, we have given actors an incredible amount of information, or, equivalently, an incredible ability to compute the marginal earnings impact from tie changes. It is unclear how the results change when, instead, actors apply simple heuristic rules and make mistakes. Second, we have only analyzed the stability for very small groups of traders. No group size-independent analytical results were established. Perhaps a simpler prediction method would allow for such results. Third, the results are scope-limited to setting in which buyers and suppliers are not distinguished and no trader can engage in more than one transaction. The results nevertheless suggest that for inequality to arise in the terms of exchange, traders must also be unequal in their ability to change the network structure. They must be monopolists keeping entrants out by temporarily undercutting prices, lobbyists for protectionist policies, or possessors of superior naval technology allowing them to broker and capture the gains from trade between geographically disconnected parties.

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CHAPTER 5: THE MATTHEW EFFECT IN IMMIGRANT ASSIMILATION

Introduction

In the research cycle, a simple model persists until a slightly more complicated model that makes theoretical sense empirically outperforms it. For a social *process*, the simplest model one can try to fit is one with a constant rate. On a chart with as horizontal axis time and as vertical axis the focal social phenomenon, this model produces a *straight line*. It has been repeatedly claimed that the conceptualization of the immigrant adjustment process as a straight line is flawed: "“Straight-line” theory…is much less successful in accounting for the experience of non-European origin groups." (Waldinger & Gilbertson 1994:432; quotes in original) "These anomalies immediately question the applicability of straight-line assimilation." (Zhou 1997:72) "One cannot but…see that the process of "becoming American" has been far from a uniform or straight-line march" (Rumbaut 1997:488; quotes in original)

The straight-line model is commonly attributed to classical scholarship on assimilation, sometimes called "the canonical account" (e.g., Nee & Alba 1997:827). A collection of early sociological works are said to share a common view of assimilation, namely that of a monotonic process during which the immigrant or immigrant group, as life- or calendar time progresses or subsequent generations replace preceding ones, further and further adjusts towards greater and greater assimilation. The debate does not revolve around the quantitative feature of linearity. It can be a "bumpy line" (Gans 1992:44). Some even contend that "…none of the scholars whose names are associated with assimilation theory (e.g., Park, Frazier, Myrdal, Gordon) posited that the path of ethnic change was linear..." (Hirschman

1991:180) And, without a quantifiable definition of assimilation, the linearity feature of the straight-line model is not even falsifiable. Contested, rather, is the qualitative model property of an irreversible course with only one possible end point, complete assimilation. It is this property against which much evidence has been presented, taken as null hypothesis in a multitude of empirical studies.

These studies, of which I mention a few, show that in some 'domain' of assimilation and during some period or over the course of some generations, certain immigrants do not assimilate or even 'reverse' assimilate (Portes & Stepick 1993:8). In their classic study of New York City neighborhoods, Glazer & Moynihan (1963) describe how the Italians, Irish, Jews, and Puerto Ricans do not melt. More recently, Alba et al. (1999) show that length of stay is not related and U.S. citizenship inversely related to immigrants' odds of suburbanization (p. 457), which Park & Burgess (1924) identified "…as a key step in a process of spatial, or residential, assimilation" (p. 447). Portes, Haller & Guarnizo (2002) find that "long periods of residence in the United States increase the probability of engaging in…transnational entrepreneurship…" which is "…contrary to what may be expected from an assimilationist perspective" (p. 289). Guarnizo, Portes & Haller (2003) find no significant effect of U.S. citizenship and a positive effect of length of stay on engagement in political activities that involve the sending nation (p. 1234), which is at odds with "…orthodox theoretical approaches…" which predict "…that the longer immigrants live and are socialized into the ways of the host society, the greater the likelihood of their becoming thoroughly absorbed in it." (p. 1215) Perreira et al. (2006) state that "The straight-line hypothesis predicts that ethnic differences in high school dropout rates, within and across immigrant generations, will diminish over time" (p. 512) but find that "…for every race-ethnicity except Hispanic, the first generation drops out at significantly lower rates than third and higher generations." (p. 522) The question that these studies

thus raise is 'If not a straight line, then what model does describe assimilation?'
In response to the counter-evidence, theorists have first removed the inevitability from the argument. In what has been called 'segmented assimilation theory', Portes & Zhou (1993) consider straight-line assimilation one out of a number of possible trajectories that immigrants may follow. In their 'new assimilation theory', Alba & Nee (2003) conclude that "…the assumption of inevitability assumes away what requires explanation..." (p. 38) and instead regard assimilation as "…something that frequently enough happens to people when they make other plans." (p. 282)

They have then proceeded to take two more important steps in the direction of an alternative model. Scholars have identified qualitative system behaviors, or "modes of assimilation" (Portes & Zhou 1993), that are commonly observed but that the old model cannot exhibit. Examples are long-term states of segregation in "immigrant enclaves" (Wilson & Portes 1980:295), forms of reverse assimilation such as "reactive ethnicity" (Portes & Rumbaut 2001:186) and "downward assimilation" (Portes & Rumbaut 2001:59), and forms of "selective assimilation" (Portes & Zhou 1993:90) such as assimilation without loss of ethnic identity or gain in economic status.

They have also identified a host of mechanisms that appear to drive the assimilation dynamic. Alba & Nee (2003:38-57) organize "mechanisms of assimilation" into four categories, namely "purposive action", "network mechanisms", "forms of capital", and "institutional mechanisms". I will follow up on their effort and bring together from dispersed sources a range of mechanisms of assimilation of the first three categories.

I attempt to take one further step. I propose an alternative model of assimilation that (1) incorporates these mechanisms of assimilation, (2) can exhibit the aforementioned modes of assimilation, (3) adds minimal complexity to the straight-line model, and (4) is empirically competitive with the straight-line model. This alternative model, as I then show, has the following characteristic qualitative property, namely that the quite-assimilated assimilate further while the not-so-assimilated reverse-assimilate, a property that, in the context of academic prestige was dubbed *Matthew Effect* (Merton 1968:57).

I will proceed as follows. I define terms, overview mechanisms of assimilation, show how the hypotheses associated with the mechanisms are of a common, shared form, synthesize the hypotheses into a model, derive the existence of a Matthew Effect, show that the model allows for the various modes of assimilation, outline the testing strategy, test the various hypotheses and the Matthew Effect using panel data, and discuss limitations and some implications for contemporary debates and perspectives on assimilation.

Definitions

When an individual migrant or ethnic group arrives in a host country, it is potentially ethnically distinguishable in a number of ways. With time, these distinctions can remain, fade, or intensify. I borrow the definition of assimilation from Alba & Nee (2003), namely "the attenuation of distinction based on ethnic origin" (pp. 30-1). Each way in which either an ethnic group or an individual can be distinct I will call a domain. Let a_{ik} denote the level of assimilation of individual $i = 1, 2, \ldots I$ in domain $k = 1, 2, \ldots K$. $a_{ik} \rightarrow -\propto$ indicates minimal assimilation or maximal distinction, and $a_{ik} \rightarrow \propto$ indicates maximal assimilation or minimal distinction. I consider eight domains ($K = 8$): marriage, language, residence, workplace, career, friendship, culture, and identity.

Table 5.1. Domains of Assimilation

Domain	*Ethnic group*	*Individual migrant*	*Operationalization*
Household	Homogamy	Ethnicity of household	Proportion household members who speak English/French
Language	Monolingualism	Fluency in official language	English/French language skills
Residence	Residential segregation	Ethnicity of neighbors	Proportion neighbors from another country of origin
Workplace	Workplace segregation	Ethnicity of colleagues	Proportion cross-ethnic colleagues
Career	Income Disadvantage	Income Disadvantage	Log annual individual income from all sources
Friendship	Homophily	Ethnicity of friends	Proportion cross-ethnic friends
Culture	Ethnic traditions are practiced	Practices ethnic traditions	Does not find important to carry on ethnic traditions
Identity	Ethnic identity	Identification with ethnic group	Does not feel close to ethnic group

Table 5.2. Hypotheses by Source and Target Domains

Domain	Household	Language	Residence	Workplace	Career	Friendship	Culture	Identity
Household		exposure	interaction	interaction & referral & trust		interaction	influence	identification
Language	facilitation		facilitation	facilitation	skill	facilitation		
Residence	interaction	exposure		interaction & referral & trust	job prospects	interaction	influence	identification
Workplace	interaction	exposure	interaction		job prospects	interaction	influence	identification
Career			affordability					
Friendship	interaction	exposure	interaction	interaction & referral & trust			influence	identification
Culture	selection		selection	selection		selection		
Identity	inclusion		inclusion	inclusion		inclusion		

Note: Rows are source domains and columns are target domains.

Table 1 overviews the eight domains for two levels of analysis, the level of the individual migrant and the level of the ethnic group. A member of an ethnic group distinguishes herself in the household domain by sharing a household with members of her own ethnic group. An ethnic group distinguishes itself in the marriage domain when strong homogamy makes many households mono-ethnic. A member of an ethnic group distinguishes herself in the language domain by speaking the official language of the host country poorly. An ethnic group distinguishes itself in the language domain when it exhibits strong monolingualism. A member of an ethnic group distinguishes herself in the residence domain by having co-ethnic neighbors only. An ethnic group distinguishes itself in the residence domain when its members live segregated. A member of an ethnic group distinguishes herself in the workplace domain if all her colleagues co-ethnic. An ethnic group distinguishes itself in the workplace domain when there are high levels of workplace segregation. A member of an ethnic group distinguishes herself in the career domain when she earns less. An ethnic group distinguishes itself in the career domain when its members earn less than the mean income. A member of an ethnic group distinguishes herself in the friendship domain if all her friends are co-ethnic. An ethnic group distinguishes itself in the friendship domain if it exhibits strong homophily. A member of an ethnic group distinguishes herself in the culture domain if she carries on the traditions associated with her ethnic group. An ethnic group distinguishes itself in the culture domain if its traditions are practiced. A member of an ethnic group distinguishes herself in the identity domain if she identifies with the ethnic group. An ethnic group distinguishes itself in the identity domain if it exhibits a strong ethnic identity.

A member of an ethnic group has lost her distinction entirely when she does not share her household with members of her own ethnic group, when she speaks the official language fluently, when her neighbors, colleagues, and friends are all of a different ethnicity, when she earns a top-income, when she does not uphold the cultural values of her group, and when she does not identify with her ethnic group. An ethnic group has lost its distinction entirely when its members have as spouses, friends, colleagues, and neighbors members from other groups, when they speak the official language fluently, when they all earn top incomes, when they do not uphold the cultural values of the group, and when they do not identify with the ethnic group. One could argue that minimal distinction should occur instead when scores on domains correspond to those of an average member of society, not when they reach one of the extremes. For example, those with top incomes would then be less assimilated then those with nationally average incomes, and Chinese with no Chinese friends less than Chinese with one Chinese friend. Lacking those empirical averages for most domains and opting for simplicity, I place the point of minimal distinction at the extreme.

The domains are neither exhaustive nor mutually exclusive, but do reflect existing categorizations. The domains household, friendship, residence, and workplace match the first four categories of Bogardus' social distance scale, namely people's willingness to accept out-group members as close relative by marriage, close personal friend, neighbors on the same street, and co-workers in the same occupation (e.g. Babbie 1992). Assimilation along the domains residence, workplace, and friendship are perhaps what Gordon (1964:79) referred to as "structural assimilation," assimilation along the culture domain "acculturation", assimilation along the identity domain "ethnic identification", assimilation along the household domain "intermarriage", assimilation along the language domain "language acquisition", and assimilation along the career domain what Chiswick (1978) referred to as "economic assimilation". Note however, that while Gordon took as reference and end point of assimilation the time-specific and arguably arbitrary "middle-class cultural patterns of, largely, white Protestant, Anglo-Saxon origins," (p. 72) the reference here is simply the out-group. Thus, in principle, everyone is considered an immigrant, and members of all classes, religions, and ethnicities can be dissimilated. The out-group as reference is implied by the Alba & Nee definition: If one looks like members of other ethnic groups, distinction is lost. Note also that Gordon considered the domains to be sequential stages in the assimilation process, whereas I allow the assimilation process to involve simultaneous changes, forward or backward, in multiple domains.

I will express all model dynamics in terms of one conception of time, lifetime. Let *t* denote lifetime and $\dot{a}_{ik} = \partial a_{ik} / \partial t$ the rate of assimilation for *i* in domain *k*. The straight-line hypothesis has also been cashed out in terms of calendar time as well as in terms of generations. We can analyze calendar time dynamics by considering individual-specific time zeros, and we can analyze generational dynamics by ensuring that the time zeros of members of later generations come after the time zeros of members of earlier generations.

Hypotheses

The hypotheses I bring together from the immigration literature will all be of the following form: For any individual *i*, the rate of assimilation $\dot{a}_{ik}$ in some target domain *k* positively depends on the level of assimilation a_{il} in some source domain $l \neq k$. Table 2 lists all hypotheses and gives the corresponding target domain, source domain, and social mechanism.

Van Tubergen & Kalmijn (2005) discuss a strand of research that hypothesizes that exposure to the official language of the host country eases an immigrant's acquisition of this language. The more frequent the contact with those who do not speak the ethnic language, the faster conversation skills improve. The 'exposure hypothesis' indeed fits the specified form when the target domain is language and the source domain marriage, residence, workplace, or friendship.

The converse of the exposure hypothesis has also been stated (Chiswick & Miller 1995; Espinosa & Massey 1997). Language skills allow conversations one could otherwise not have and thus facilitate cross-ethnic contact. The 'facilitation hypothesis' has as target domain marriage, residence, workplace, or friendship, and as source domain language.

Alba & Logan (1993) hypothesize that "…members of minority groups...convert socio-economic and assimilation progress into residential gain." (p. 1390) The desire to live in a mixed neighborhood is often not sufficient. Because of housing price differences between ethnic and mixed neighborhoods, it must be complemented with the socio-economic means. The hypothesis stems from theoretical work on residential segregation by Massey (1985), and has come to be labeled the 'spatial assimilation hypothesis.' Here I label each hypothesis by the associated social mechanism, hence 'affordability hypothesis'. It can be made to fit the specified form by making residence the target domain and career the source domain.

Sanders, Nee, and Sernau (2002) argue that immigrants' "...reliance on social ties...facilitates job hunting in the wider domain of the economy, where prospective employers may be of any ethnicity." (p. 281) Conversely, "...ethnic networks provide sources of information about...sources of jobs inside the community." (Portes & Rumbaut 1996:86) By definition, those who work in the 'ethnic economy' (Light & Karageorgis 1994) tend to be co-ethnic and those who work in the mixed economy of a different ethnicity. Information about jobs in the ethnic economy thus flows in through co-ethnic social ties and information about jobs in the mixed economy through cross-ethnic social ties. The 'referral hypothesis' (Nee, Sanders, and Sernau 1994; Sanders et al. 2002) fits the proposed form when the target domain is workplace and the source domain is marriage, residence, or friendship.

Portes and colleagues (e.g., Portes & Sensenbrenner 1993) show how trust in ethnic ties allows fellow members of an ethnic group to provide startup funds for a business in the ethnic economy in the absence of a contract or formal guarantee. The 'trust hypothesis' is based on this alternative mechanism through which assimilation in the workplace domain is contingent upon assimilation in the marriage, residential, and friendship domains.

The monetary gains from trust in ethnic ties suggests another target domain, namely 'career', with an associated negative cross-domain effect, but this effect has in the past been subject to much debate (e.g., the back-and-forth in December 1987 issue of ASR). Sanders and Nee (1987) take elements from classical theories of assimilation and segmented labor market theory to counter-formulate what they call the "ecological hypothesis" (p. 745) that there is a mobility cost to economic and residential segregation. Advanced capitalist societies are structured such that jobs in ethnic firms located in ethnic neighborhoods tend to be in the secondary labor market thereby lacking opportunities for promotion and advancement. The ecological hypothesis thus has as source domains residence and workplace and as target domain career and operates through a "job prospects" mechanism. Whether or not the ecological hypothesis counteracts the trust hypothesis that concerns the career domain sufficiently to make the cross-domain effects positive is an empirical question.

Kalmijn and Flap (2001) speak of a "supply-side perspective" as a collection of theories that argue "that the social contexts in which people participate, mold their networks by shaping the pool from which they draw their contacts…" (p. 1290) The argument goes that people who share common characteristics tend to run into one another more readily, namely at meeting places that are associated with those characteristics (Blau & Schwartz 1984). Marriage, friends, the neighborhood, and the workplace are all typical meeting places. One's spouse and one's friends introduce one to third parties who are all potential future friends, romantic partners, neighbors, or colleagues. Similarly, one meets romantic partners, friends, neighbors, and colleagues in the neighborhood and at work. The extent to which these meeting places are populated by co-ethnics determines how likely future spouses, friends, colleagues, and neighbors are co-ethnic. The ‘interaction’ hypothesis thus has as target and source domains marriage, residence, workplace, and friendship.

According to social identity theory (Tajfel & Turner 1979), people form an identity by classifying themselves as member or non-member of groups. They are more likely to identify with groups whose other members they interact with. The ‘identification hypothesis’ thus has as target domain identity and as source domain marriage, residence, workplace, and friendship.

One consequence of this identification is the inclusion of members in and exclusion of non-members from one’s social network. For the case of ‘residence’, this hypothesis can be found in the immigration literature as the ‘place stratification hypothesis’, which traces residential segregation back to exclusion on the basis of group membership (Denton & Massey 1989; Logan & Alba 1993). This ‘inclusion’ hypothesis has as target domains marriage, residence, workplace, and friendship, and as source domain ‘identity’.

Two mechanisms underlie the pervasive tendency for people to interact with similar others: ‘selection’ and ‘influence’ (e.g., Macy et al. 2002). Selection is the choice of interaction partners on the basis of similarity. When cultural similarity is considered, the selection hypothesis has as target domains marriage, residence, workplace, and friendship, and as source domain ‘culture.’ For the case of ‘residence’, the selection mechanism is found in literature on the link between residential preferences and segregation (e.g., Schelling 1971). Influence concerns the opposite causality and is the tendency of people to grow similar to interaction partners. When cultural similarity is considered, the influence hypothesis has as target domain culture and as source domains marriage, residence, workplace, and friendship.

And lastly, human capital theory (e.g., Becker 1984) predicts that language skills, being a form of human capital, enhance career advancement (Chiswick & Miller 1995). Assimilation in the career domain will proceed faster if language skills are better. The ‘skill’ hypothesis has as target domain career and as source domain language.

Derivation of the Matthew Effect

It has been argued that in model building in the social sciences a procedure of stepwise increases in complexity is desirable (e.g., Lindenberg 1992). The classic model of assimilation, the straight line, is the simplest possible model for a social process. Let us take its formalization, a constant rate, α_k, as baseline. This baseline has been repeatedly argued and shown to fail to capture the essence of the assimilation process. I therefore increase complexity minimally in a way that has maximal theoretical and empirical backing. Let $\boldsymbol{B}$ be a KxK matrix of coefficients. I maximize backing by adding terms that do not represent new hypotheses but rather the hypotheses discussed afore, which have not yet been rendered obsolete in the process of scientific inquiry. The added complexity is minimal because the hypotheses all fit a common form so that I only have to add one type of formal term to the constant rate equation for assimilation in domain k, namely i’s level of assimilation in target domain l, a_{il}, multiplied by a coefficient β_{kl}:

$$\dot{a}_{ik} = \alpha_k + \sum_{l=1}^{K} \beta_{kl} a_{il} \qquad (5.1)$$

I choose to introduce this term through addition, and not, for example, multiplication. This choice is arbitrary, though convenient because it makes the system of differential equations linear, and thus solvable in closed form (e.g., Strogatz 1994:123-44). In fact, the dynamical system of K equations of type (5.1) has been well studied across the physical sciences. The solution vector $a_i(t)$ is given by

$$a_i(t) = \sum_{m=1}^{K} c_m e^{\lambda_m t} v_m \tag{5.2}$$

Here, c is a vector of constants and λ_m is the m-th eigenvalue with corresponding eigenvector v_m. If the coefficients β_{kl} are zero, the summation in equation (5.1) simply drops out and we obtain the classical straight-line model of assimilation, with for each domain only a (positive) constant rate α_k. And, adding an error term, we would get Herbert Gans' "bumpy line" (1992:44). If, however, the coefficients β_{kl} are instead positive, conform the assimilation hypotheses we reviewed, then the Perron-Frobenius theorem says that for some n, λ_n is real positive and strictly larger than all other eigenvalues, and that the corresponding eigenvector v_n has non-zero entries of the same sign. As inspection of equation (5.2) establishes, this means that as t becomes large, the term $c_n e^{\lambda_n t} v_n$ comes to dominate in magnitude, as it is asymptotically approached by $a_i(t)$. Since $c_n e^{\lambda_n t} v_n$ grows exponentially with t, the system has only two attractors, namely maximal assimilation in all domains, $a_i \rightarrow \propto$, and minimal assimilation in all domains, $a_i \rightarrow -\propto$, depending on whether c_n is positive or negative. After sufficient time has elapsed, the immigrant finds herself on a solid trajectory of either ever increasing or ever decreasing assimilation. This is the Matthew Effect that Merton spoke of in the context of status in science – the famous becoming more famous and the fameless less famous, later borrowed to refer to increasing income inequality – the rich becoming richer and the poor poorer –, to increasing educational inequality (Stanovich 1986), and musical taste (Salganik, Dodds, and Watts 2006). Which of the two trajectories the immigrant will follow depends on initial conditions. Immigrants who start off rather assimilated further assimilate. Immigrants who start off not so assimilated reverse assimilate. This endogenously-arrived-at dichotomization of immigrants into assimilators and reverse assimilators on the basis of initial assimilation levels appears to fit exogenously-arrived-at dichotomizations on the basis of initial assimilation levels from the literature, such as the one between new and old immigrants (Massey 1995) and high and low capital migrants (Nee et al. 2002).

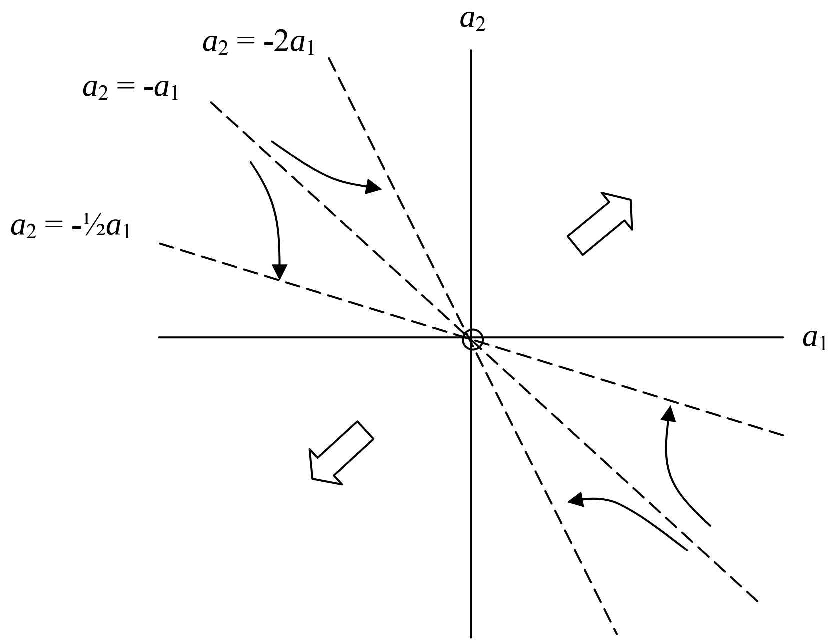
Modes of Assimilation

The Matthew Effect tells us what will ultimately happen to the immigrant under the model assumptions. It concerns the system behavior for sufficiently large *t*. The ultimate mode of assimilation is either assimilation in all domains or reverse assimilation in all domains. We have, however, not defined the unit of time, and thus left open the possibility that the Matthew Effect sets in postmortem. It is therefore informative to look at possible system behavior early on, for small *t*, before the eigenvector with largest eigenvalue starts to dominate and the Matthew Effect sets in.

These early assimilation dynamics cannot at any time involve assimilation in all domains ($\dot{a}_i > 0$) or reverse assimilation in all domains ($\dot{a}_i < 0$), because these states are self-perpetuating; once assimilation or reverse assimilation in all domains has set in, it cannot stop. They must therefore involve “selective assimilation” (Portes & Zhou 1993:90) where assimilation occurs in some domains but not in others. For example, no economic progress is made or no ethnic identity is lost.

We can visualize the early and ultimate assimilation dynamics together in a two-dimensional phase portrait if we reduce the number of domains *K* from eight to two. Figure 5.1 shows a phase portrait with on the horizontal axis domain 1, say, structural assimilation, and on the vertical axis domain 2, say, acculturation, and for arbitrary parameters values $\alpha = 0$, and $B = 2 - \boldsymbol{I}$. The thick arrows represent ultimate assimilation dynamics. Once the immigrant finds herself in the area northeast of the lines $a_2 = -2a_1$ and $a_2 = -\frac{1}{2}a_1$, she monotonically assimilates in both domains. Similarly, once the immigrant finds herself in the area southwest of the lines $a_2 = -2a_1$ and $a_2 = -\frac{1}{2}a_1$, she monotonically reverse assimilates in both domains. Before she arrives on either trajectory, which because of the Matthew Effect is inevitable, she may find herself temporarily in the area between the two lines, where she assimilates in one domain and reverse assimilates in the other. The thin arrows represent these early assimilation dynamics. If she starts northeast of the line $a_2 = -a_1$, these early assimilation dynamics will eventually bring her to the assimilation trajectory. If she starts southwest of the line $a_2 = -a_1$, she will drift towards the reverse assimilation trajectory.

The modes of assimilation considered so far concerned individual migrants, within their lifetime. The model of assimilation developed here also allows for various modes of assimilation that have in the literature been specified for generations or ethnic groups, modes that the straight-line model did not allow. "Downward assimilation" (Portes & Rumbaut 2001:59) occurs when a second-generation immigrant becomes economically less successful than her parents. There are several ways this dynamic can be accommodated in the model, for example when $\dot{a}_i > 0$ for the parents while the child finds herself selectively or reverse assimilating. If this were the case for enough first generation parents and second-generation children, we have an instantiation of the phenomenon of "second generation decline" (Gans 1992).



Note: $K = 2$, $\alpha = 0$, and $B = 2$ - ***I***.

Figure 5.1. Phase Portrait of the Assimilation Model

Testing Strategy

I seek evidence that speaks for or against the hypotheses and for or against my claim that the Matthew Effect is empirically competitive with the straight line. The purest test is a direct estimation of the parameters in equation (5.1) as coefficients in a statistical model that is as similar as possible to the theoretical model developed afore. Each coefficient β_{kl} and associated t value would then address a separate hypothesis from table 2. t values below –1.96 are votes against both the straight line and the Matthew Effect, t values between –1.96 and +1.96 are votes for the straight line, and t values above +1.96 are votes for the Matthew Effect. Moreover, plugging the estimated coefficients back into equation (5.1), one could do equilibrium analysis to see whether there is either a single assimilation equilibrium (straight line), or an assimilation ánd a reverse assimilation equilibrium (Matthew Effect), or some other set of equilibria.

A strong candidate for such a statistical model is the Euler approximation of equation (5.1) with an error term added:

$$a_{ik,t+1} = \alpha_k + (\beta_{kk} + 1)a_{ik,t} + \sum_{l \neq k}^{K} \beta_{kl} a_{il,t} + \varepsilon_{ik} \tag{5.3}$$

Conveniently, equation (5.3) is a regression equation with as dependent variable the target domain *k* at time *t*+1, as independent variables the target and source domains *k* and *l* at time *t*, and as estimable coefficients the theoretical model parameters. Needed to estimate equation (5.3) for each domain are measures of all eight domains at the beginning and end of some period of the immigrant's stay in the host country. It does not matter what period, because the theoretical model assumes the assimilation process to be ahistoric – *t* does not appear at the right-hand side in equation (5.1).

The use of more than one observation of assimilation per individual necessitates data with that information, which is scarce due to the higher collection costs. In the past, scholars lacking repeated measurements had to address the problem by making comparisons between more and less recent migrants (e.g. table 9.1 in Massey et al. 1987:257) or by trusting respondents' ability to retrospect (e.g. table 3 in Sanders et al. 2002:298-9). Fortunately, recently, a number of panel studies of assimilation were undertaken. For my analysis, I draw on the first two waves of the

Longitudinal Survey of Immigrants to Canada (LSIC).[1]

The target population of the LSIC is all immigrants aged 15+ who entered Canada between October 1, 2000 and September 30, 2001 from abroad with legal 'landed immigrant' status. The sampling frame is an administrative database from Citizenship and Immigration Canada of all landed immigrants. This excludes those who landed from within Canada, who may have spent considerable time in Canada before landing. Although the ahistoric nature of the model makes it irrelevant what period out of the life of the immigrant is analyzed, the fact that all sampled immigrants are starters avoids certain omitted time variable biases.

Immigrants were interviewed six months, two years, and four years after landing. The numbers of respondents for the first and second wave are respectively 14,356 and 9,332. Weights constructed by Statistics Canada, who collected the data, were designed to compensate for biases from non-response at both waves (see chapter 12 of the Wave 2 User Guide). I will use these weights throughout the analysis. Note that the resulting rounding in head counts will make some cells not add up to their row or column totals.

These interviews were conducted face-to-face or by telephone in one of 15 languages. Validity and reliability of responses were augmented in two ways. Computer-assisted interview technology automatically detected real time for various inconsistencies and errors in the responses, with previous questions being asked again until consistent. Also, Statistics Canada assured respondents' beforehand that "…your answers will be kept strictly confidential. They will be added to answers from many other immigrants and then studied." (p. 8 of LSIC wave 1 questionnaire). It enforced this promise by not releasing a PUMF, by having me undergo a security screening,

[1] The research and analysis are based on data from Statistics Canada and the opinions expressed do not represent the views of Statistics Canada.

and by making me analyze the data in one of Canada's Research Data Centers so that it could double-check that I did not accidentally reveal respondent-identifying information. For this reason, I can here not display minimum and maximum values or cross-tabulations with cell counts below 10.

For the intended regression equations, I need data at each wave on the level of assimilation of the respondent in each of the eight domains. The scaling of each variable is irrelevant, because the theoretical model leaves it unspecified, as long as higher scores represent higher levels of assimilation. The operationalizations of the eight domains are shown in the last column of table 1. For the *household* domain, I use a measure on the proportion of the household members who speak English and French. The existence of two official languages in Canada requires a decision on which measure(s) to use and how. In the analysis I will present, I use the measurement of French language skills for residents of Quebec and the measurement of English language skills otherwise. I have tried alternative strategies, such as separate models for Quebec and the rest of Canada, or models with both language measures included as separate variables, and these models do not alter the conclusions I will draw later. I would have preferred a measure of the actual ethnicity or country of origin of household members, but the LSIC questionnaire asks for spouse's country of origin at wave 1 and for spouse's ethnicity at wave 2, so that there is no wave-constant measurement of either. Fortunately, for all other structural assimilation variables, I do have wave-constant measures of ethnicity available to me.

For the *language* domain, I use the question "How well can you speak English/French?" Answer categories are "cannot speak this language", "poorly", "fairly well", "well", and "very well". The computer skipped the question if English/French was both the first language of the respondent and the language spoken most often at home, which cases I treated as "very well". Consistent with the household measure, I consider French language skills for Quebec residents and English language skills for others. I considered responses equidistant and standardized them, recoding the five answer categories as respectively 0, ¼, ½, ¾, and 1.

For the *residence* domain, complete residential histories since landing for the first three digits of the postal code are available. I matched these with 2001 Canadian Census proportions neighborhood residents whose country of origin matches that of the respondent for those postal code areas. Note that this leads to a violation of the i.i.d. assumption. To correct for this violation I cluster cases by postal code and compute robust standard errors (Huber 1967). This reduces T values for most effects slightly.

For the *workplace* domain, I use the question "How many of your co-workers are of the same ethnic or cultural group as you?" Note that this question leaves it up to the respondent to define what her or his ethnic or cultural group is. An advantage of this question is that it measures assimilation in a way that is meaningful to the respondent. A drawback is that it permits a risk that assimilated migrants consider their ethnicity to be Canadian. Possible answers were "all of them", "most of them", "some of them", and "none of them", which I coded as 0, 1/3, 2/3, and 1 respectively. The question was asked for respondents who ever worked since landing (wave 1) or since wave 1 (wave 2) and were not self-employed. I consider other respondents segregated in the workplace and assign them score 0. Whether or not this is the correct way to deal with these cases is debatable. Therefore, in analyses not reported on here I treated such cases as missing and these do not alter the conclusions I will draw later.

For the *career* domain, I measure the total personal income from all sources since landing (wave 1) or since wave 1 (wave 2). I follow the convention of taking the logarithm after imposing an artificial lower bound of 1 dollar.

For the *friendship* domain, I use the question "How many of your friends are of the same ethnic or cultural group as you?” Again, the question leaves it up to the respondent to define what her or his ethnic or cultural group is. Possible answers were “all of them”, “most of them”, “half of them”, “some of them”, and “none of them”, which I coded as 0, ¼, ½, ¾, and 1 respectively. Note the extra category in comparison to the workplace measure. The question was asked for respondents who made friends since landing (wave 1) or new friends since wave 1 (wave 2). For those without friends at wave 1, I made the same decision and ran the same robustness checks as for the colleague measure. The wave 2 measure concerns new friends. If wave 2 friends replaced the wave 1 friends, we could treat the answers as reflecting the entire Canadian friendship network. However, if they were added to the old friends, our best guess of the ethnic composition would be the average of the wave 1 and wave 2 measurements. The structure of the questionnaire allows for both possibilities. In separate analyses not shown here I tried both operationalizations of friendship network composition, as well as analyses excluding these special cases, and found results to be robust. The results that are presented here are based on the assumption that new friends replace old friends. If there were no new wave 2 friends, I used the wave 1 measurement for wave 2 as well.

For the *culture* domain, the LSIC contains no behavioral measures of engagement in cultural traditions, so I employ the attitudinal question “How important is it for you to carry on the values and traditions of your ethnic or cultural group or your homeland?” Answer categories are “very important” (0), “important” (1/3), “not very important” (2/3) and “not important at all.” (1)

For the *identity* domain, I use a measure of perceived closeness to the ethnic group: “When you think of others in Canada from the same ethnic or cultural group as yourself, how close would you say you feel to that group as a whole?” Answers ranged from “very close” (0), to “close” (1/3), to “not very close” (2/3), to “not close at all" (1).

Some of these variables are of ordinal, others of interval measurement level. I report results from OLS regression models, but I did run parallel ordered and multinomial logistic regression models to verify that conclusions were robust across model specifications. Differences in the means of these eight variables at waves 1 and 2 indicate assimilation patterns for the 2001 cohort of Canadian immigrants as a whole. The straight line permits no negative differences, while the Matthew Effect permits any differences.

In the intended regression models, omitted variable bias may lead to a false assessment of the relative empirical tenability of the Matthew Effect with respect to the straight line. To reduce this risk, I control in all models for a number of variables. These are age, church attendance, and marital status, gender, domestic college degree, foreign college degree, immigration class, and Western, muslim, Asian, and Hispanic countries of origin. The first three are wave-specific measures, the others wave-constant.

Table 3 displays the means and standard deviations of all variables. I cannot report the exact numbers of missing observations per variable, because many of them are below 10 and thus shielded by Statistics Canada. Missing values reduce the sample from 9,332 to 8,575. I report on analyses of this smaller sample of list-wise deleted cases. It would not be correct to impute the domain variables, because each is also used as dependent variable. I ran models with imputed control variables and found no deviating results. As mentioned before, I can also not report minimum and maximum values.

All empirical domain variables are bounded, while none of the theoretical variables are. I deal with this discrepancy in two ways. First, I do not interpret intra-domain effects, because strong ceiling effects are operating. Cross-domain effects do not have this problem. Note that all hypotheses concern cross-domain effects; they are placed in off-diagonal cells in table 2. Second, in the equilibrium analysis I analyze the system behavior given a matrix ***B*** with the estimated cross-domain effects as fixed off-diagonal cells and varying inter-domain effects on the diagonal.

Table 5.3. Means and Standard Deviations

		Wave 1		Wave 2	
Domain	Variable	Mean	Standard Deviation	Mean	Standard Deviation
Household	Proportion household members who speak English/French	.676	.251	.849	.244
Language	English/French language skills	.675	.322	.740	.301
Residence	Proportion neighbors from another country of origin	.961	.053	.963	.052
Workplace	Proportion cross-ethnic colleagues	.313	.383	.462	.396
Career	Log annual individual income from all sources	5.32	4.21	7.02	4.33
Friendship	Proportion cross-ethnic friends	.316	.315	.396	.295
Culture	Does not finds important to carry on ethnic traditions	.276	.245	.264	.231
Identity	Does not feel close to ethnic group	.354	.265	.343	.249
	Age	35.0	12.0	36.6	12.0
	Single	.240	.427	.222	.416
	Attends religious services	.144	.351	.160	.367
	Male	.497	.500		
	Holds college degree from foreign country	.648	.478		
	Western	.061	.239		
	Muslim	.236	.425		
	Asian	.588	.492		
	Hispanic	.059	.235		
	Immigrant Class "Family"	.263	.440		
	Immigrant Class "Independent"	.670	.470		
	Immigrant Class "Business"	.061	.239		
	Immigrant Class "Refugee/Other"	.005	.074		

Note: I = 8,575

The estimated cross-domain effects will speak to the empirical tenability of the hypotheses, but not to the mechanisms through which they operate. At the end of the results section, I present some descriptive evidence concerning these mechanisms.

Results

The differences between the wave 1 and wave 2 means of the domain variables in table 3 represent the change over 1½ years in the levels of assimilation of the average migrant of the 2001 cohort of immigrants to Canada. While any negative change would violate the straight-line model's prediction of monotonous positive change, the Matthew-Effect model is not falsifiable at the group level. The table shows that the cohort as a whole assimilates along the household, language, workplace, career, and friendship domains. No significant assimilation occurs along the residence domain. Reverse assimilation is visible in the culture and identity domains.

Table 4 shows the estimated theoretical parameters and the associated *t* values (both multiplied by 100 for a better visual assessment of the relative sizes), R^2, number of clusters, and number of observations from the eight corresponding linear regression models.

Table 5.4. Results from Eight OLS Regression Models

Wave 1↓ 2→	Household		Language		Residence		Workplace		Career		Friendship		Culture		Identity	
	100 x		100 x		100 x		100 x		100 x		100 x		100 x		100 x	
	par	*t*(*par*)	*par*	*t*(*par*)	*par*	*t*(*par*)	*par*	*t*(*par*)	*par*	*t*(*par*)	*par*	*t*(*par*)	*par*	*t*(*par*)	*par*	*t*(*par*)
β_{kl}																
Household			2	179	0	6	4	242	29	140	2	136	5	354	3	250
Language	21	1304			0	101	15	879	68	339	14	1052	-3	-308	-1	-88
Residence	31	441	37	513			36	459	-163	-112	73	1060	8	125	7	105
Workplace	1	102	-0	-12	0	247			167	1190	1	131	-0	-19	1	97
Career	-0	-247	-0	-76	-0	-222	0	185			-0	-81	0	65	0	54
Friendship	2	227	4	484	0	87	3	225	-3	-16			2	232	5	402
Culture	0	20	-1	-87	-0	-62	0	29	30	161	1	98			6	492
Identity	0	18	2	290	0	191	0	25	-24	-135	7	625	4	293		
$\beta_{kk}+1$	18	1229	57	4845	77	2860	39	3126	245	1578	26	2252	22	1744	15	1264
α_k	14	212	1	21	21	853	-8	-101	623	432	-51	-797	6	93	16	260
R^2		.26		.57		.70		.31		.24		.31		.10		.7
# clusters		894		894		894		894		894		894		894		894
# migrants (*I*)		8,575		8,575		8,575		8,575		8,575		8,575		8,575		8,575

Note: Effects of control variables are omitted

Table 5.5. Hypothesis Testing

Domain	Household	Language	Residence	Workplace	Career	Friendship	Culture	Identity
Household		0	0	+		0	+	+
Language	+		0	+	+	+		
Residence	+	+		+	0	+	0	0
Workplace	0	0	+		+	0	0	0
Career			–					
Friendship	+	+	0	+			+	+
Culture	0		0	0		0		
Identity	0		0	0		+		

Note: – = falsified; 0 = undecided; + = confirmed

Of all 56 cross-domain effects, 3 have *t* values below –1.96, 31 have *t* values between –1.96 and +1.96, while 22 have *t* values above +1.96. We could count these as 3 votes against both models, 31 votes for the straight-line model, and 22 votes for the Matthew-Effect model.

This test has some drawbacks. Strictly speaking, the Matthew Effect does not require every single cross-domain effect to be positive. Also, the competitive edge of the Matthew Effect over the straight-line increases with sample size. An alternative assessment of the competitiveness of the Matthew Effect with the straight line is an answer to the question whether the cross-domain coefficients in table 4 give rise to a straight-line dynamic, a Matthew Effect dynamic, or some other dynamic. The answer should come from the eigenvalues and eigenvectors of the parameter matrix ***B***. I constructed a matrix ***B*** with as off-diagonal entries the estimated cross-domain effects and with zeros on the diagonal. The eigenvalue with largest real part is then strictly positive, namely .146, and the corresponding eigenvector [.039 .026 .001 .073 .993 .062 .032 .048] as well. As I discussed in the section "Derivation of the Matthew effect" this gives rise to a Matthew effect dynamic. This conclusion continues to hold for any sufficiently large intra-domain effects on the diagonal of **B**. For strongly negative intra-domain effects, the dominant eigenvalue becomes negative and the system dynamics drift towards an intermediate state of assimilation along all domains. This dynamic would be predicted by neither the straight-line nor the Matthew Effect.

Although all of these hypotheses are borrowed from the literature, for many this is one of the few tests if not the first with longitudinal data. Table 5 displays the results of the hypothesis testing. A '–' signifies a *t* value below –1.96, a '0' a *t* value between –1.96 and +1.96, and a '+' a *t* value above +1.96. 1 hypothesis was falsified, 20 had an undecided test result, and 19 were confirmed. These counts are different from the counts for the cross-domain effects because for some cross-domain effects no hypotheses were formulated. We can express the results also in terms of "mechanisms of assimilation" (Alba & Nee 2003:38-57). The hypothesis associated with the affordability mechanism was rejected. Tests of hypotheses that operated through the attraction mechanism were all undecided. For each of the mechanisms exposure, interaction, referral, trust, influence, identification, skill, job prospects, and inclusion, some hypothesis that operates through that mechanism was confirmed and none rejected.

The evidence for and against these mechanisms is indirect. Hypotheses that were formulated on the bases of their operation through these mechanisms were tested, but table 4 does not speak to the question if they indeed operate through these mechanisms. It is possible that confirmations are mistakenly attributed to these mechanisms and that falsifications are due to counter-acting other mechanisms. Here I provide some complementary descriptive evidence that is suggestive of some of these mechanisms.

The interaction mechanism associates assimilation along the target friendship domain with levels of assimilation in the household, residence, and workplace domains, through the functioning of the household, the neighborhood, and the workplace as meeting places where new friends are made. If the interaction mechanism is indeed the mechanism that drives these cross-domain effects, then when asked where they acquired these new friends, respondents should in large numbers mention these meeting places. Of all 7,385 respondents who reported new friends at wave 1, 4% said they acquired some of these through spouse's work, 9% through their children's school, 29% in their neighborhood, and 37% at work. Of all 8,076 respondents who reported new friends at wave 2, 5% said they acquired some of these through spouse's work, 10% through their children's school, 38% in their neighborhood, and 55% at work.

The exposure mechanism associates assimilation along the language domain with the level of assimilation in the workplace through the opportunity for practicing language skills that a cross-ethnic workplace provides. If this effect indeed operates through the exposure mechanism, then at least it would have to be true that migrants much more often speak the official language with their co-workers in the mixed economy. Of all 435 respondents whose co-workers were all co-ethnic at wave 1, 34% said they spoke to their co-workers in English (French for Quebec), while of all 1,072 respondents with no co-ethnic co-workers 95% said that they spoke English with their co-workers. Of all 538 respondents whose co-workers were all co-ethnic at wave 2, 35% said they spoke to their co-workers in English, while of all 1,788 respondents with no co-ethnic co-workers 95% said that they spoke English with their co-workers.

The referral mechanism associates assimilation along the workplace domain with the level of assimilation in the friendship domain through the opportunity for referrals to mixed economy jobs that cross-ethnic friends provide more than co-ethnic friends. If this effect indeed operates through the referrals mechanism, then at least it would have to be true that referrals to mixed economy jobs less often come from co-ethnic friends than ethnic economy jobs do. Of all 740 respondents who at wave 1 had only cross-ethnic supervisors and said they found their job through a referral by a friend, 79% said this friend was co-ethnic, while of all 551 respondents who at wave 1 had some co-ethnic supervisor and said they found their job through a referral by a friend, 94% said this friend was co-ethnic. Similarly, at wave 2, of all 758 respondents who had only cross-ethnic supervisors and said they found their job through a referral by a friend, 72% said this friend was co-ethnic, while of all 506 respondents who had some co-ethnic supervisor and said they found their job through a referral by a friend, 95% said this friend was co-ethnic.

Discussion

I have synthesized into an alternative model theoretical work that followed repeated falsification of the standard model of immigrant assimilation and assessed its empirical competitiveness. In the process, I have provided evidence from panel data for and against a series of hypotheses and associated mechanisms from this theoretical work. I finish by proposing ways in which limitations to the current study can be addressed in future work and by relating the study to contemporary perspectives, debates, and public policy.

A first limitation is the mismatch between the unbounded assimilation variables in the theoretical model and the bounded assimilation variables in the empirical model. This prevented me from using the estimates from the regression models as diagonal entries in the matrix $\boldsymbol{B}$, thereby necessitating an assumption about these entries to investigate the observed assimilation dynamics and thus weakening the confidence in the evidence for the existence of a Matthew Effect. Transforming the empirical variables so that they become unbounded is not an option, because respondents with as scores on the bounded assimilation variables the extreme values 0 and 1 could not be given a finite score. The only option in future work is to leave the empirical variables bounded, and to instead bound the theoretical variables. This would make the dynamical system nonlinear and complicate system analysis. An example is a model consisting of a logistic map (e.g., May 1976) for each target domain, the rate of each map being a function of the source domains. Following the principle of stepwise complexity, I here decided to add minimal complexity and stay as close to existing models and theory, thereby sacrificing the match between empirical and theoretical model. I did, however, estimate an alternative empirical model that has a weaker link with theory but allows for a test of the existence of a Matthew Effect without needing any assumptions about within-domain effects. The eight domains are represented by dichotomous variables with as only values “high assimilation” and “low assimilation” and the model regresses the logit of each target domain at wave 2 additively on the target and source domains at wave 1. The Matthew Effect is found back in this alternative model in three ways. First, of the 56 cross-domain effects, 32 are significantly positive, 24 are insignificant, and 0 are significantly negative. Second, calling a situation in which one is highly assimilated in *m* domains an “*m*-state”, the model then yields the following vector of $K + 1$ probabilities with which *m*-states at *t* reproduce themselves at $t + 1$: [.159 .064 .035 .026 .026 .031 .047 .081 .172]. Thus, the 0-state (low assimilation in all domains) and the 8-state (high assimilation in all domains) reproduce themselves with a higher probability than other states: They are more stable. Third, the Markov chain with 2^8 x 2^8 transition matrix that the parameters in this logit give rise to has a 2^8-entry steady state vector in which m-states have the following nine average probabilities: [.032 .025 .028 .042 .069 .100 .156 .235 .316]. Again, the probabilities follow a bimodal distribution over *m*-states, albeit skewed.

A second limitation is the confinement of the empirical analysis to the first years of first-generation immigrants. The fact that the LSIC samples the same period in the same calendar year for the same generation prevents the interference of period and calendar time effects. The cost is that it prevents me from testing the theoretical model’s assumption that assimilation is ahistoric. The generalizability of the Matthew Effect to other periods and generations can be assessed through a replication of this study using other longitudinal surveys that were recently conducted or are currently underway. The third wave of the LSIC allows for a test for later years in the experience of first-generation immigrants and the Children of Immigrants

Longitudinal Study a test for second-generation immigrants.

Third, I have not investigated the system dynamics at the group level beyond a demonstration that the model allows for some of the modes of assimilation that have been identified in past work for groups and generations. In the current model, the assimilation of individual migrants occurs independently. At the group level, the Matthew Effect, pushing immigrants to one of two extremes, produces a tendency toward bimodal distributions of assimilation variables. This tendency has been found back in empirical studies. For example, Esser (1987) noted such bimodality in the ethnic friendship network compositions of first- and second-generation Turkish and Yugoslavian immigrants to Germany. There are two reasons to suspect that the Matthew Effect at the level of the individual migrant scales up to the level of the ethnic group if the assimilation of migrants is made interdependent. The first reason is that some domains of assimilation for migrant *i* cannot change unless some domains of migrant *j* change in the same direction. For example, by marrying someone of another ethnicity one assimilates in the household domain, and so does partner. A similar logic applies to the other structural assimilation domains: residence, workplace, and friendship. This logic caused Schelling (1971) to find that small deviations from assimilation in the residence domain for some migrants could cause reverse assimilation in that domain for all other migrants; One migrant cannot move without reducing the proportion of co-ethnic neighbors for some stayers. The second reason is that many of the mechanisms that underlie the hypotheses about cross-domain effects *within* migrants are equally valid for hypotheses about cross-domain effects *between* migrants. For example, if the language skills of some migrants improve, then through the exposure mechanisms the language skills of other migrants improve as well. The significant effect of English language skills on the proportion of members who speak English in the household and its marginally significant converse effect provide some evidence for the existence of such cross-domain, cross-migrant effects. If the domains across migrants are mathematically related in the same way the domains within migrants are, then the Perron-Frobenius theorem equally applies to the population of migrants; after sufficient time, either all migrants are assimilating in all domains, or all migrants are reverse assimilating in all domains. This suggests that the snowball effects that Massey et al. (1987) and Palloni et al. (2001) find in the entry phase of migration, where one migrant increases the chance of a second migrant, may generalize to the adjustment phase: The more migrants make the step from segregation to integration, the more likely subsequent migrants follow. See Esser (1985, 2003) for a model of interdependent assimilation that allows for a similar threshold dynamic.

Fourth, the model developed here makes multiple assimilation experiences possible for any migrant. Which one she will undergo depends on initial conditions, as shown in the section 'modes of assimilation'. I have not investigated what determines these initial conditions. One candidate factor is relative group size. Following the logic of Blau & Schwartz (1984), large groups will by random chance equip their members with more co-ethnic neighbors, friends, household members, and colleagues. This makes it more likely they start off on a reverse assimilation trajectory than members of smaller groups. In multi-level models not shown here I found a significantly negative effect of group size at the national level on both the initial level of assimilation and on the change between wave 1 and wave 2. The Chinese, forming the largest immigrant group in Canada, start off most segregated and more likely reverse assimilate than members of other groups.

The study speaks to contemporary perspectives, debates, and public policy in a number of ways. First, the only rejected hypothesis was what has been called the 'spatial assimilation hypothesis', operating through an affordability mechanism. This hypothesis has been critiqued earlier in the literature (e.g., Logan & Molotch 1987, Denton & Massey 1989, Crowder & South 2005). Scholars have argued that even with better incomes, racial discrimination in the housing market prevents economically assimilated migrants from moving out of ethnically segregated neighborhoods. The rejection of the hypothesis in the current study supports this critique.

Second, while confirmation was found for the 'ecological' hypothesis with residence as its source domain, an insignificantly negative effect was found for the ecological hypothesis with workplace as its source domain. This speaks to the aforementioned ethnic enclave debate by suggesting that ethnic segregation hurts in the residence domain, but may benefit in the workplace domain. According to the empirical analysis, most mobile are migrants who work with co-ethnics but live with cross-ethnics.

Third, a more recent debate in the immigration literature is one between two contrasting perspectives on the adaptation process that some have dubbed 'assimilationist' vs. 'transnationalist' (e.g., Portes, Haller & Guarnizo 2002, Guarnizo, Portes & Haller 2003, Waldinger & Fitzgerald 2004). The debate centers around the question whether length of stay and mobility in the host country reduce respectively increase the chance of engagement in political or economic activities that involve the homeland. The theoretical model advocated here leaves the effect of length of stay undetermined because it in principle allows for two ultimate assimilation courses, further and further assimilation or further and further reverse assimilation. Average ethnic identification and cultural distinction were found to increase over time in the empirical model. As for the effect of mobility, the theoretical model says that those with more resources are on the one hand more able to engage in transnational activities but on the other hand are less likely to identify with the homeland. However, no significant effect of income on identification was found in the empirical model.

Lastly, if immigrant assimilation along certain domains is politically desirable (typically, structural assimilation) or undesirable (typically, cultural assimilation), and tax dollars can be used to speed up or slow down assimilation along certain domains, then the present study suggests that these exhibit non-constant marginal returns. Minimally and maximally assimilated migrants will not move unless pushed away from equilibrium along multiple domains. Targeted intervention that involves multiple domains of few migrants, such as sponsored family adoptions of migrants, with which European governments have experimented, are then more effective. By contrast, the semi-assimilated can be steered rather cheaply. For them, general policies that involve a single domain for many migrants, such as mandatory language programs, subsidies for schools with minority representation, or sponsoring of ethnic organizations, are more likely to work. But according to the model developed here, any policy that is aimed at permanent selective assimilation is doomed to fail. Structural assimilation in combination with the continued practicing of cultural traditions and strong ethnic identification is a (potentially very long, but nevertheless strictly) temporal phenomenon. Ultimately the Matthew Effect takes over, either reversing course along the structural domains or setting in motion a process of cultural detachment and fading ethnic identification.

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CHAPTER 6: FUTURE RESEARCH

In this last chapter, I briefly sketch four follow-up studies. Follow-up study 1 tests the behavioral assumption that the models of chapters 2, 3, and 4 share in a laboratory experiment. Follow-up study 2 consists of additional tests of the assimilation model of chapter 5 using different surveys. Follow-up studies 3 and 4 serve to illustrate two strategies for exploiting the promise of Internet data. In both illustrations, the objective is a test of the operation of the selection and influence processes mentioned in chapters 2 and 5. Follow-up study 3 is an online game experiment. Follow-up study 4 uses continuous-time data on the co-evolution of networks and behavior from actual online environments.

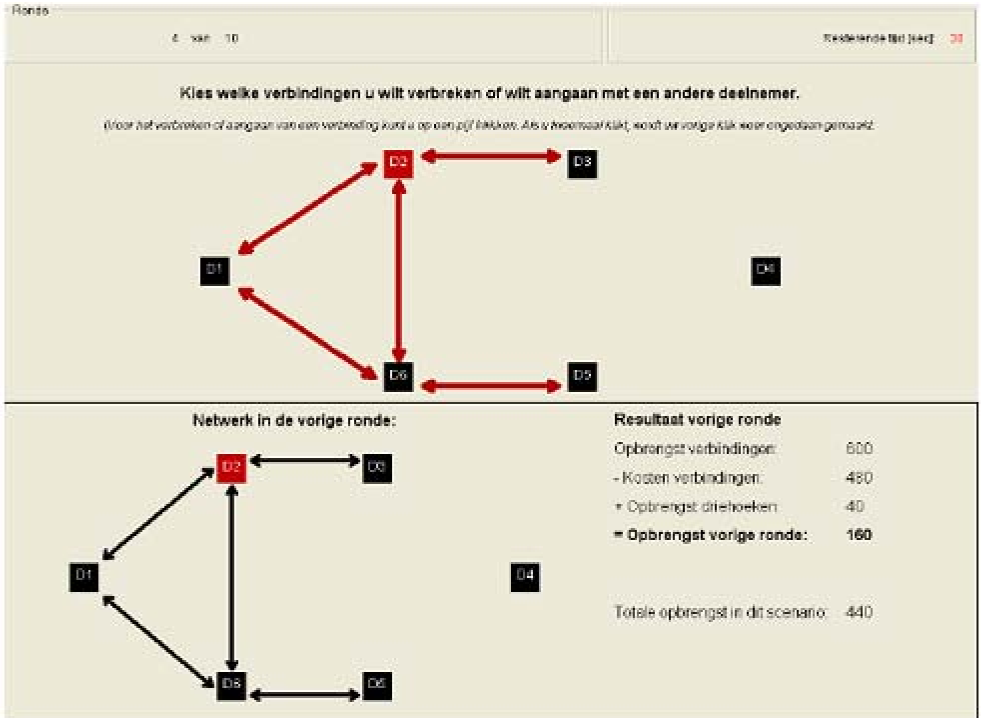
The theory in this dissertation has been of two types and therefore gives rise to distinct avenues of future research. The type of theory in chapters 2, 3, and 4 has been simplistic. The aim was to derive formal and computational results that are insightful, rather than predictive. Strong assumptions were made that we could know from the onset would never hold in any particular empirical instance. Actors were only interested in one type of network-derived benefit: Structural balance, structural holes, or gains from trade. They knew all other actors and whom those were connected to. And they were perfectly able to process all that information as to arrive at a utility-maximizing choice. In chapter 5, by contract, the theoretical model had a large empirical scope. Although the hypotheses were borrowed from theories from across the literature rather than deduced from a unified theory, the model extends to any time during the assimilation process of any immigrant of any generation in any Western country. The assimilation model is immediately applicable to contemporary immigration.

These distinctions in type of theory give rise to different avenues for future research. There is little use in testing the equilibrium predictions of chapters 2, 3, and 4 in the field, because no empirical setting falls within the strong scope conditions of any of the models in these chapters. Useful to test is only the behavioral assumption, under the artificially controlled satisfaction in the laboratory of all scope conditions that would be violated in natural environments. This is proposed study 1 below. All other welcome follow-up investigations of these chapters are of a purely theoretical nature. They should ask if and how the main result alters if what scope conditions are modified. These robustness checks were already extensively discussed in the discussion sections of the three respective chapters.

Follow-up Study 1. Testing the Behavioral Assumption of Chapters 2, 3, and 4

I borrow from Burger & Buskens (2007) an experimental environment that imposes all scope conditions of the strategic network formation models of chapters 2, 3, and 4. The environment consists of a number of networked computers equaling the number of agents in the model. Subjects see the entire network on a screen and click links to add and delete relations with other subjects. A screenshot is displayed in figure 6.1.

Subjects are informed on precisely how network configurations translate into profit that they receive at the end of the experiment. Cumulative profit and is continuously visible in the bottom-right corner of the screen. This profit function can be varied to match the profit functions of chapters 2, 3, and 4. In each chapter it was assumed that actors would add ties that increase utility and delete ties that decrease utility in the short run. The proposed experiment directly assesses how many of the link changes made by subjects violate this behavioral assumption. The experiment also assesses if the deviations from the assumed behavior prevent the predicted equilibrium network from emerging.



Source: Figure 3 in Burger & Buskens (2007)

Figure 6.1. Screenshot of a Networking Experiment

The assimilation model developed in chapter 5 turned out to be predictive. It was found empirically competitive with the standard model of assimilation. However, its test remained limited to the first two years of first-generation immigrants to Canada. The scope of the model extends to all stages of all generations of any Western country. In follow-up study 2, I propose additional tests for other years of other generations in other countries.

Follow-up Study 2. Additional Tests of the Assimilation Model of Chapter 5

Three recently collected data-sources allow for these additional tests of the model. First, during the summer of 2007 the third wave of the LSIC becomes available, allowing for a test of the model for later stages in the assimilation process, until four years after entry. Comparing model estimates for the wave 1-->2 transition with those for the wave 2-->3 transition, I can then assess if the process is indeed ahistoric, which would be the case if the coefficient matrix **B** does not change significantly from the first transition to the next. The third wave also allows me to obtain within-subject effect estimates, comparing for each immigrant the wave 1-->2 transition with the wave 2-->3 transition, getting rid of all potential time-constant omitted variable bias. Second, the Children of Immigrants Longitudinal Study allows for a test of the assimilation model for second-generation immigrants. Third, the New Immigrant Survey has yielded data comparable to the LSIC for contemporary immigrants to the United States. The first two waves will be available Fall 2007. The foci of these surveys on different aspects of the assimilation process may restrict me in the number of domains for which I can run the regression models.

Homophily is the tendency for people to interact with those with similar identities. As mentioned in chapter 2, a host of social-psychological theories from the mid-20th century attributed this to striving for the elimination of psychic discomfort with imbalanced situations in which interaction partners have different identities. As mentioned in chapter 5, there are two fundamental mechanisms through which such elimination can take place: Either one changes identity or one changes interaction partner. The former is called influence, the latter selection.

Online environments provide a new testing ground that improves on offline environments in a number of ways. All aspects of every instance of behavior are recorded without human error and for free. They allow for the study of groups whose members are not geographically proximate, or who do not even know where the others are. And online games can be designed that constitute experimental environments in which many subjects participate gladly and for free. Follow-up studies 3 and 4 exemplify these new opportunities by outlining two strategies for testing the operation of selection and influence mechanisms in the creation of homophily.

Follow-up study 3: Online Experiment to Test Selection and Influence

People spend increasing amounts of time in online environments where their visible identity is reduced to a colorful nametag. Online name formation is highly interactive: Chatters change their name to broadcast a continuous message to or about someone else in the chatroom and gamers form clans with associated name formats. In some games, names are given, in others, names can be changed at any time. In some games, players can choose their partner, in others, they are randomly matched. I investigate if in these anonymous environments, patterns of identity form through the same mechanisms that underlie identity formation in offline environments, namely through the dual processes of selection - people choose interaction partners they identify with - and influence - people come to identify with partners they interact with a lot. To this end I propose an online game that can be played in four conditions, with or without name changing, and with or without partner selection.

The game is fun so that subjects like to play and create data. The game involves cooperation among two players. A player is re-matched after the game and the game is repeated until she logs off. Players can log on to the game environment like any other game environment: They get a username and password. Usernames are randomly assigned to four conditions. In condition 1, players obtain a fixed player name and are randomly matched with another player. In condition 2, players obtain a fixed player name and select their partner from a list. In condition 3, players obtain a variable player name that they can continuously update through simple mouse clicks and are randomly matched with another partner. In condition 4, players obtain a variable player name and select their partner from a list. Conditions 2 and 4 allow for selection, conditions 3 and 4 for influence. The conditions are expected to yield different homophily levels, which can with this design be traced back to the underlying mechanisms. My own experience playing games online is that homophily in player names is easy to spot with the naked eye and even with a not so sophisticated algorithm. If the pilot shows that this is not the case, one could decide to simplify identity formation to the choice of pre-selected simple shapes and colors.

Follow-Up Study 4: Using Twitter Data to Test Selection and Influence

Twitter is "a global community of friends and strangers answering one simple question: *What are you doing?*" (http://twitter.com/) Participants use phone, instant messaging, or web talk. Twitter is an example of an online environment in which identities and networks are continuously recorded. Users post a message and select an audience. Only the audience sees the message. Currently, the audience data are not available. The analysis I propose assumes that these data have become available. If an influence mechanism is operating, then the contents of messages from one user should increasingly contain words that are used in messages from other users with the focal user in their audience. If a selection mechanism is operating, then the users should change their interaction partners, choosing to receive only messages with contents similar to one's own messages. Since many Twitter messages are sent every second, these data are of enormous precision. Moreover, there is little measurement error because recording occurs electronically and automatically.

I plan to execute some of these follow-up studies myself in the coming years.

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