Measuring Movement of Incomes

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Abstract

We consider a small set of elementary properties for income movement measures, and show that there is essentially only one measure that satisfies all of these properties: the per capita aggregate change in log-incomes. We demonstrate next that this movement-mobility measure has a number of appealing descriptive and normative properties, and provide a formal generalization of our basic characterization theorem drawing from the related literature on poverty measurement. Finally, we present here an empirical application in which we show by using our new measure that there has been a broad-based increase in income movement in the United States between the 1970s and 1980s.

Introduction

Income mobility analysis is concerned with measuring the extent of changes in economic status of individuals from one time period to another. Such an analysis is dynamic by nature, and can take place in either intergenerational or intragenerational contexts. While in the former case one usually focuses on how dissimilar are the incomes of the parents from those of their offspring, in an intragenerational setting the focus is typically on the income stability of the individuals.

Most analyses of intragenerational income mobility concentrate on the variation of the income shares or rank orders of the individuals throughout the time period under examination.[1] Such studies are obviously of importance, for they are intimately linked to the notion of 'lifetime income inequality'. On the other hand, only a small number of researchers have chosen to focus instead on the aggregate variation of the incomes of the individuals (see e.g. Cowell 1985; Fields and Ok 1996; and Mitra and Ok 1998). Such an analysis, henceforth called the analysis of *income movement*, gives direct information with regard to the income flux that takes place in the society. A measure of income movement, therefore, identifies how unstable the incomes of the individuals have been throughout a given time period. Since income instability may cause economic insecurity, and clearly corresponds to a particular aspect of the economic conception of 'income mobility', measures of income movement are useful complements to the traditional measures of relative income mobility.

Measuring all incomes in real terms, we view aggregate income movement as being built up from the real income movements of individuals; this is the basic premise behind the notion of income movement. Thus, if the incomes of all individuals stay the same through time, then—and only then—we say that there is no income movement in this society. Moreover, the larger is the income change of any one constituent individual, holding others' incomes constant, the larger is aggregate income movement. Consequently, we view income growth as an integral part of the notion of aggregate income movement. In particular, since there is no natural upper bound on income changes, a given society's income movement can in principle increase without limit. This observation marks clearly the difference of the notion of aggregate income movement from other facets of income mobility: the idea of 'maximum (or perfect) mobility' that applies to other mobility concepts does not apply here.

Given this conceptualization of the movement of incomes, our task is to find ways of measuring and aggregating individual income changes. In this paper we adopt an axiomatic approach in developing such a method of measuring aggregate income movement. In accordance with the basic premise noted above, our axioms are formulated with respect to the income changes of the income recipients in the economy (as opposed to changes in ranks, quantiles or income shares). As such, of the various axiomatic approaches to mobility analysis in the literature, our work is most closely related to the approach suggested by Cowell (1985).[2]

We begin our analysis by presenting a set of four elementary properties for income movement measures. While two of these properties (scale invariance and symmetry) are quite primitive and are satisfied by virtually all mobility measures in use, one of them (multiplicative path separability) is a new consistency postulate which appears especially compelling for measures of movement. Our final postulate is, on the other hand, a standard additive separability property, a counterpart of which is widely used in the analysis of poverty measurement: subgroup decomposability. Loosely speaking, this postulate requires that the overall level of income movement be computed as a particular weighted average of the movement levels of the subpopulations of the society. Since it allows one to determine precisely the contribution of any given subgroup of the society to the aggregate income variation, subgroup decomposability becomes useful in empirical applications. In this respect, the advantages of mobility measures that satisfy this property are identical to those of subgroup decomposable poverty measures; cf. Foster *et al.* (1984).

The key properties in the above list of axioms are subgroup decomposability and multiplicative path separability. When these are combined with the other two axioms, it turns out that there is essentially only one measure that satisfies the aforementioned axioms: the per capita aggregate change in log- incomes. This is a particularly simple (descriptive) income movement measure which provides a formal justification for concentrating on individuals' log- incomes in mobility analyses. Moreover, it also admits an interesting normative interpretation in terms of the Bernoullian social utility function, and is additively decomposable into two parts, one attributable to transfer mobility and the other to growth mobility.

Given that subgroup decomposability is a very demanding property, we next explore the possibility of relaxing this property by following the lead of Foster and Shorrocks (1991). The result is a characterization of the class of all income movement measures that satisfy our three basic axioms (other than subgroup decomposability) along with three additional postulates. All of these additional properties are strictly weaker than subgroup decomposability, and are straightforward reflections of the corresponding properties used in the context of inequality and poverty measurement. Unfortunately, while the resulting class of movement measures has an interesting mathematical structure, it is at the same time inconveniently large. What is more, it is not clear how one may go about refining this class further to obtain an operational set of movement measures (other than directly imposing the subgroup decomposability property).

In the final section of the paper, we turn to an illustration of how our subgroup-decomposable income movement measure can be applied. For this purpose, data are drawn from the Panel Study of Income Dynamics (PSID) in the United States to compare the extent of aggregate income movement between the 1970s and the 1980s, overall and for various demographic subgroups. We find a broad-based increase over time in income movement in the United States. Incidentally, this observation complements the earlier findings of others, who demonstrated that relative mobility in the United States has been unchanged or falling over the same period of time. We conclude that, while there was no significant difference in the income decile/quintile changes in the 1970s and the 1980s (so rank orders and relative incomes were equally stable in these time periods), there was considerably more income flux in the 1980s than in the 1970s (so *absolute* incomes were more unstable in the 1980s).

I. Basic Properties of Movement Measures

We consider R^n_{++} as the space of all income distributions with population size $n \geq 1$. Thus, $x = (x_1, \dots, x_n) \in R^n_{++}$ represents an income distribution where x_i is the level of income of the *i*th individual at a given point in time. For any x and y in R^n_{++}, by $x \to y$ we mean that the income distribution of the society has evolved from x to y (i.e. the *i*th agent's income has changed from x_i to y_i for all *i*) in a given amount of time.

An *income movement measure* (for an $n-$person population) is defined as any function $m_n: R^{2n}_{++} \to R_+$ that is continuous and surjective. Continuity is a weak regularity condition which is hardly objectionable given the usual vagaries of income data. Surjectivity of m_n is also a reasonable requirement since, as we shall see shortly, we consider income growth as a basic component of aggregate income variation. Consequently, it is natural that we be able to trace the entire range of a 'well-behaved' movement index m_n by varying individual income changes from zero to infinity. At any rate, this assumption is adopted for convenience, and it can be relaxed with ease.

Let $//_n$ denote the class of all income movement measures on R^{2n}_{++}, and define $// \equiv \prod_{n=t}^{\infty} //_n$. Since we wish to study the movement measurement problem for populations of arbitrary sizes, we take as the primitives of the following axioms the sequences $\{m_n\}$ in $//$.

Scale-invariant movement measures are those that are invariant to changes in scale (like doubling all incomes). Formally, they satisfy the following axiom:

Scale Invariance: For all $x, y \in R^n_{++}$ and $\lambda > 0, m_n(\lambda x, \lambda y) = m_n(x, y)$.

Scale invariance introduces a degree of coherence into the measurement exercise by ensuring a consistent evaluation of movement when all incomes are *simultaneously* scaled up or down. As noted by Shorrocks (1993), the justification for this property is reminiscent of the justification of relative income *inequality* measures. In particular, scale-invariant measures avoid the problem of adjusting currencies in the case of inter-country mobility comparisons. Perhaps for this reason, most commonly used mobility measures (whether they target income movement or not) are scale-invariant.

The next property of movement measures is a *symmetry* requirement, which states that the transformations $x \to y$ and $y \to x$ are equally mobile:

Symmetry: For all $x, y \in R^n_{++}, m_n(x, y) = m_n(y, x)$.

The symmetry property is unexceptionable if one does not distinguish between 'good' and 'bad' movement of incomes. Consider the two processes $x \equiv (1,2) \to (2,3) \equiv y$ and$(2,3) \to (1,2)$. In both of these processes, the rank orders of the individuals are the same and the absolute values of the changes in relative incomes are identical. Therefore, traditional mobility measures, such as the rank correlation (Schiller 1977) and the correlation coefficient (McCall 1973), would declare these two processes equally mobile. (Indeed, these two indices (along with many others) are symmetric.) In the context of the present analysis, on the other hand, these two transformations are deemed ‘equally mobile’ for a different reason: each individual in these processes is subject to the same monetary income change. Therefore, since when saying ‘movement of incomes’ we do not distinguish for the moment between moving *up* and *down*, we posit that $m_2(x,y) = m_2(y,x)$.

Of course, $(1,2) \rightarrow (2,3)$ is welfaristically a more desirable transformation than $(2,3) \rightarrow (1,2)$. It is important to note, however, that the symmetry axiom takes the welfare evaluations based on Pareto optimality (or economic growth) out of mobility measurement analysis. In the case of mobility measures that are based on relative incomes or ranks, this is no cause for concern. Such measures would simply contend that there is no welfare difference between the transformations $x \rightarrow y$ and $y \rightarrow x$ that arise from the fluidity of the income shares or ranks of individuals; the welfare comparison between these two transformations must be performed on other grounds. In contrast to such measures, however, movement measures that are based on absolute incomes must view income growth/contraction as one of the basic sources of aggregate income movement. Consequently, one may argue that it would be desirable to consider (ethical) movement measures which would actually distinguish between the transformations $x \rightarrow y$ and $y \rightarrow x$, and which would declare the movement that takes place in the former (latter) process welfare enhancing (reducing). Yet, this is not a reason to abandon the symmetry axiom. One may, after all, wish to know the extent of income flux without attributing to it any welfare connotations. Moreover, it turns out that some interesting welfaristic measures can be readily obtained by *directionalizing* the symmetric movement measures. We shall in fact introduce one such directional movement measure in the next section.

We now turn to our next axiom, and note it would be desirable to have a movement measure that relates the income flux of population subgroups to the income movement of the entire population in an operational way. Indeed, just as in the case of poverty analysis, mobility studies might benefit from breaking down the population into certain subgroups on the basis of ethnic, occupational and/or geographical origin, and then attempting to find out the contribution of these subgroups to the total movement in the population (see Section IV). Therefore, the movement measures that satisfy the following *additivity* axiom (familiar from the theory of poverty measurement; cf. Foster *et al.* 1984) are of interest:

Subgroup Decomposability: Partition the population into $J \in \{1, \dots, n\}$ subgroups and let n_j stand for the number of persons in subgroup j. For any $j = 1, \dots, J$ and $x^j, y^j \in R_{++}^{n_j}$,

$$m_n\big((x^1, \dots, x^J), (y^1, \dots, y^J)\big) = \sum_{j=1}^{J} \left(\frac{n_j}{n}\right) m_{n_j}\big(x^j, y^j\big)$$

While subgroup decomposability is a widely used property in theories of inequality and poverty measurement, to our knowledge it has never been considered in the context of income mobility measurement. This is because a vast majority of mobility studies are conducted in terms of the movement of relative incomes and/or ranks of the individuals, and therefore they cannot be expected to satisfy subgroup decomposability.[3] When one's focus is shifted to absolute income changes, however, it is entirely possible to devise movement indices that compute the overall movement level as a weighted average of the subgroup movement levels where weights are chosen to be the population shares of the subgroups. Notice that subgroup decomposability is after all nothing but a strong form of the basic premise that aggregate income movement is obtained simply by aggregating individual levels of movement.

As we shall see in Section IV, subgroup decomposability is a property that is particularly useful in empirical applications. One should nevertheless note that subgroup decomposability is a technically demanding axiom. In Section III, therefore, we shall explore the possibility of replacing this axiom with a weaker separability property along the lines of the work of Foster and Shorrocks (1991).

To introduce the final property we shall discuss in this section, consider an arbitrary income distribution $x \in R_{+}^{n}$ and suppose that all agents in the population experience a particular path of income change in the time period under study, namely, a fixed rate of income increase. We thus have $x \to \alpha x$ for some $\alpha > 1$. Now suppose we are provided with a further observation point in the time interval originally specified; we thus have two consecutive time periods to measure the mobility path of the population. Assume that in the first time interval we observe the process $x \to \beta x$ for some $1 \leq \beta \leq \alpha$. In the second observation period, we shall then observe $\beta x \to \alpha x$. In effect, we decompose the process $x \to \alpha x$ into two subprocesses, $x \to \beta x$ and $\beta x \to \alpha x$. It seems quite reasonable then that the sum of the movement levels found in the two subprocesses should be equal to the total mobility of the mother process. This would, after all, introduce a time independence feature to the measurement study with respect to the particular (multiplicative) income change paths under consideration. We thus postulate the following property:

Multiplicative Path Separability: For any $x \in R_{++}^{n}, \alpha \geq 1$ and $\beta \in [1, \alpha]$,

$$m_n(x, \alpha x) = m_n(x, \beta x) + m_n(\beta x, \alpha x)$$

Multiplicative path separability is a weak separability requirement that is posited for a very specific (and simple) process of income changes. If all incomes in an economy grow β-fold between times 0 and 1 while they are growing (α/β)-fold between times 1 and 2, then one would presumably want to use a movement measure that would reflect that incomes have, in fact, grown exactly α-fold between times 0 and 2. Measures that satisfy multiplicative path separability are designed precisely to meet this requirement.

II. Two Measures of Income Movement

Characterization

Our first result is that the basic properties introduced above imply a unique income mobility measure up to a positive linear transformation:

Proposition 1. An $\{m_n\} \in //$ satisfies scale invariance, symmetry, subgroup decomposability and multiplicative path separability if, and only if, there exists a constant $c > 0$ such that

$$m_n(x,y) = c\left(\frac{1}{n}\sum_{i=1}^{n}|\log y_i - \log x_i|\right) \quad \text{for all } x,y \in R_{++}^n, \qquad n \geq 1$$

Although our main interest in this paper is in symmetric indices, it is instructive to note how easily Proposition 1 can be modified to yield an analogous characterization of a directional movement index. We say that a continuous function $d_n: R_{++}^n \to R$ is a *directional movement measure* if, for all $x, y \in R_{++}^n$ and all $\alpha > 1$, the following two properties hold:

$$d_n(x,y) = -d_n(y,x) \quad \text{and} \quad d_n(x,\alpha x) > d_n(x,x)$$

The first property simply says that if $x \to y$ is deemed to exhibit a ‘good’ movement overall, then $y \to x$ must be viewed as depicting a ‘bad’ movement in the aggregate. This property is clearly the defining feature of a directional movement index. The second property, on the other hand, is a natural requirement that forces a directional movement index to view a proportional increase in everybody’s incomes as a ‘good’ movement.

We denote the set of all directional movement measures defined on R_{++}^{2n} by $\mathcal{D}_n$, and define $\mathcal{D} \equiv \prod_{n=1}^{\infty} \mathcal{D}_n$. The properties considered in the previous subsection are defined for directional movement measures in a straightforward manner. The following result is a straightforward analogue of Proposition 1:

Proposition 2. A $\{d_n\} \in \mathcal{D}$ satisfies scale invariance, subgroup decomposability and multiplicative path separability if, and only if, there exists a constant $c > 0$ such that

$$d_n(x, y) = c\left(\frac{1}{n}\sum_{i=1}^{n}(\log y_i - \log x_i)\right) \quad \text{for all } \ x, y \in R_{++}^{n}, \qquad n \geq 1$$

The elementary proofs of these results are found in the Appendix.

Interpretation

Proposition 1 provides an axiomatic justification for the movement measure $\{m_n^*\} \in //$ defined as

$$m_n^*(x, y) = \frac{1}{n}\sum_{i=1}^{n}|\log y_i - \log x_i|$$

for all $x, y \in R_{++}^{n}$ and $n \geq 1$. Apart from its formal support articulated in Proposition 1, $\{m_n^*\}$ has also an interesting interpretation. If one chooses the social utility function for income as $a \to$

$\log a$ and adopts utilitarianism in computing social welfare, then $m_n^*(x, y)$ corresponds to the per capita aggregate change in the individual social utility levels that has been experienced during the process $x \to y$.[4, 5]

Consider, for instance, the transformations $x = (2, 1000) \to (3{,}1000) = y$ and $x = (2, 1000) \to (2{,}1001) = z$. Which of these two processes exhibits more income movement? If one focuses only on the absolute values of changes in personal incomes (as, for example, the movement measure introduced in Fields and Ok 1996 does), then one would conclude that both processes exhibit an equal amount of income movement. On the other hand, if one wishes to rank the movement contents of these transformations on the basis of changes in individual welfare levels, then the law of diminishing returns would argue for $x \to y$ depicting strictly more movement than $x \to z$. This is precisely what $\{m_n^*\}$ would do: $m_n^*(x, y) > m_n^*(x, z)$. Moreover, all directional movement measures introduced in Proposition 2 argue that the movement observed in $x \to y$ is more desirable than that observed in $x \to z$. This again accords with our intuition.

Remark 1. The above interpretation of $\{m_n^*\}$ brings up a natural generalization. The notion of aggregating social utilities is extensively used in the literature on income inequality where the social utility function for income is assumed to have constant elasticity, and hence is of the form

$$U^\sigma(a) = \begin{cases} \dfrac{a^{1-\sigma}}{1-\sigma}, & 0 \leq \sigma \neq 1 \\ \log a, & \sigma = 1 \end{cases} \qquad \text{for all} \quad a > 0$$

Choosing U^σ as the social utility function and adopting utilitarianism in aggregating individual levels of social utility yields the subgroup-decomposable movement measure $\{m_n^\sigma \in //\}$, defined as

$$m_n^\sigma(x,y) = \frac{1}{n}\sum_{i=1}^{n}|U^\sigma(y_i) - U^\sigma(x_i)| \qquad \text{for all} \quad x,y \in R_{++}^n \ \text{ and } \ n \geq 1$$

The class $\Sigma \equiv \left\{\{m_n^\sigma\}: 1 \geq \sigma \geq 0\right\}$ then emerges as an appealing one-parameter class of income movement measures.

It is clear that the only *scale-invariant* member of Σ is $\{m_n^1\} = \{m_n^*\}$, and the only *translation-invariant* member of Σ is $\{m_n^0\}$, which is precisely the measure characterized in Fields and Ok (1996). The rest of the elements of Σ may be thought of as compromise measures of aggregate income variation.

Remark 2. Another useful property of $\{m_n^*\}$ is that it additively disaggregates into two components which can be interpreted as *total social utility growth* and *total social utility transfer*. The first component is easily defined for a process $x \to y$ with a non-negative level of economic growth, i.e. with $\Sigma y_i \geq \Sigma x_i$. Adopting the utilitarian conventions outlined above, the per capita social welfare change that comes about during $x \to y$ is $(\Sigma \log y_i \geq \Sigma \log x_i)/n$. Turning to the second component, total social utility transfer can be thought of as twice the social utility lost by the losers. To see this more clearly, let us define the set of all individuals whose income decreased during the process as $L \equiv \{i: x_i > y_i\}$. L is then the set of all losers'. Of course, the social utility loss of an $i \in L$ is $\log x_i - \log y_i$ so that the total social utility lost by 'losers' is $\sum_{i \in L}(\log x_i - \log y_i)$. Yet, because the economy grew, every util lost by a loser is gained by a winner, and thus one may think 'as if' $\sum_{i \in L}(\log x_i - \log y_i)$ amount of social utility is transferred from losers to winners.[6] Consequently, $2\sum_{i \in L}(\log x_i - \log y_i /n)$ can be thought of as the per capita total social utility that has 'changed hands'. Given this interpretation, we have the following decomposition for a growing economy: For all $x, y \in R_{++}^n$ and $n \geq 1$,

$$m_n^*(x,y) = \frac{1}{n}\sum_{i=1}^{n}(\log y_i - \log x_i) + \frac{2}{n}\sum_{i\in L}(\log x_i - \log y_i)$$

(An analogous decomposition holds also in the case of a contracting economy.) The sources of aggregate income variation can thus be gauged by disaggregating $\{m_n^*\}$ into two components, one attributable to economic growth and one attributable to transfer of incomes. This is reminiscent of decomposing total social mobility into exchange and structural mobility components, a problem that is widely studied in the related sociological literature.[7]

Remark 3. It is important to note that the very notion of 'income mobility' is really multidimensional, and therefore no single movement index can be expected to measure all aspects of mobility satisfactorily, $\{m_n^*\}$ is no exception. While it offers a meaningful way of assessing the aggregate fluctuations of individual incomes through time, it is, for instance, insensitive to the re-rankings of individuals. To illustrate, consider the simple transformation $x \equiv (10,20) \rightarrow (10+\varepsilon, 20=\varepsilon) \equiv y(\varepsilon), 0 \leq \varepsilon \leq 10$. In accordance with our intuition, $m_2^*(x, y(\varepsilon))$ is a *strictly increasing* function of ε on $[0,5]$. Interestingly, this particular movement of incomes would almost entirely be missed by any mobility measure based on quantile transition matrices (like the widely used immobility ratio). Indeed, since the transition matrix associated with the process $x \rightarrow y(\varepsilon)$ is simply

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

for all $0 \leq \varepsilon < 5$ (where the first (second) group consists of the richer (poorer) individual), any such measure would declare the process $x \rightarrow y(\varepsilon)$ 'completely immobile' for all $0 \leq \varepsilon < 5$.[8] On the other hand, $m_2^*(x, y(\varepsilon))$ is a *continuous* function of ε on $[0,5]$, although one may actually wish to have a jump discontinuity at $\varepsilon = 5$; for the initially poorer individual is no longer the poorest at $\varepsilon = 5$: a rank reversal has taken place. This would, in turn, be picked up by the transition matrix approach, since the mobility matrix associated with $x \to y(\varepsilon)$ is

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

for all $5 < \varepsilon \leq 10$.

The upshot is that the mobility measurement technique we propose here and the standard approach based on quantile transition matrices and on relative incomes are essentially complementary. While the former concentrates on individual relative income movements, the latter measures the extent of re-rankings (usually in terms of quantiles, deciles, etc.). Mobility analyses that focus on aggregate income variation, therefore, would be enriched if both methodologies were simultaneously employed.[9]

Remark 4. The multidimensionality of the notion of mobility makes it impossible to devise a fundamental axiom that would unify all axiomatic inquiries that pertain to mobility measurement. This contrasts sharply with the theory of inequality measurement in which the Pigou-Dalton transfer principle plays a central role. However, it is conceivable that a fundamental axiom can be devised with respect to specific facets of mobility such as income movement. In the present conceptualization, we contend that a fundamental axiom would be one that preconditions aggregate (directional or nondirectional) movement to be built up from the real income changes of individuals. (This property is formalized as the 'weak decomposability' axiom below.) At the very least, this premise, a strong version of which is the property of subgroup decomposability, is the starting point of the present work.

The literature provides three alternative axioms as candidates for a fundamental axiom of movement mobility: monotonicity (Shorrocks 1978a), the axiom of diagonalizing switches (Atkinson 1981), and monotonicity in distance (Cowell 1985). The first two of these axioms are formulated in terms of transition matrices, and thus are not immediately applicable to our framework. Yet, as shown in Fields and Ok (1996, Sections 5.2-5.4), $\{m_n^*\}$, and all measures in the class Σ, satisfy suitable reformulations of these three axioms.[10]

III. A Generalization: Subgroup-Consistent Measures of Income Movement

As noted earlier, while subgroup decomposability is quite desirable for empirical applications, it is a technically demanding property which postulates an additive structure for indices in a rather *ad hoc* manner. It is therefore not surprising that this axiom is weakened to *subgroup consistency* in the literature on axiomatic poverty measurement (Foster and Shorrocks 1991). In this section, we shall explore the implications of such a weakening for income movement measures.

Subgroup decomposability encompasses three distinct properties of movement measures.

Weak Decomposability: For all $x, y \in R_{++}^n$,

$$m_n(x, y) = G_n\big(m_1(x_1, y_1), \dots, m_1(x_n, y_n)\big)$$

for some symmetric and strictly increasing $G_n: R_+^n \to R_+, n \geq 2$.

Replication Invariance: For all $x, y \in R_{++}^n$, and $k = 1,2, \dots$

$$m_n(x, y) = m_{kn}\big((x, \dots x), (y, \dots, y)\big)$$

Subgroup Consistency: For all $x, y, z, w \in R_{++}^{n_1}$ and $x^*, y^*, z^*, w^* \in R_{++}^{n_2}$

$$m_{n_1}(x, y) > m_{n_1}(z, w) \quad \textit{and} \quad m_{n_2}(x^*, y^*) \geq m_{n_2}(z^*, w^*)$$

imply

$$m_{n_1+n_2}\big((x, x^*), (y, y^*)\big) > m_{n_1+n_2}\big((z, z^*), (w, w^*)\big)$$

Weak decomposability is an axiom of nonpaternalism which asserts that total movement is a monotonic function of the *individual* income variations (cf. Cowell 1985). (The symmetry of this function guarantees the impartial treatment of individuals; the interpretation of strict monotonicity is straightforward.) As we have noted above, this axiom reflects the fundamental premise behind the notion of ‘aggregate income movement’. Replication invariance, on the other hand, is again a requirement, the analogues of which are widely used in the literatures on inequality and poverty measurement; it simply preconditions a movement index to view the aggregate income variation in *per capita* terms. Finally, subgroup consistency is a compelling property which posits that overall income movement should increase whenever a subgroup of the population experiences a rise in movement keeping the movement level of the rest of the society unaltered.

It is clear that a subgroup-decomposable movement index necessarily satisfies weak decomposability, replication invariance and subgroup consistency. The converse, however, is evidently invalid. It is thus of interest to see how Proposition 1 would be modified if one replaced subgroup decomposability by these three properties. This query leads us to the following observation:

Proposition 3. An $\{m_n^*\} \in //$ satisfies scale invariance, symmetry, weak decomposability, replication invariance, subgroup consistency and multiplicative path separability if, and only if, there exists a strictly increasing and continuous function $F: R_+ \to R_+$ such that

$$m_n(x,y) = F\left(\frac{1}{n}\sum_{i=1}^{n} F^{-1}(|\log y_i - \log x_i|)\right) \quad \text{for all } x,y \in R_{++}^n, \qquad n \geq 1$$

Proposition 3 shows that replacing subgroup decomposability with weak decomposability, replication invariance and subgroup consistency expands considerably the class of movement measures captured by Proposition 1. The resulting class has an interesting structure; it contains, for instance, all the movement measures in $\left\{\{m_{n,\alpha}\} \in //: \alpha > 0\right\}$ where

$$m_{n,\alpha}(x,y) = \left(\frac{1}{n}\sum_{i=1}^{n}|\log y_i - \log x_i|^{\alpha}\right)^{1/\alpha}$$

for all $x, y \in R^n_{++}$. (This subclass is clearly reminiscent of the widely used P_α– class of poverty indices.) In particular, $\{m_{n,1}\} = \{m^*_n\}$; that is, $\{m_{n,1}\}$ is precisely the measure that is characterized by Proposition 1 (with $c = 1$).

Unfortunately, the class of measures characterized in Proposition 3 is too large to be useful in applications. It is clear that one must use an additional property for movement measures to 'refine' this class further in order to obtain some applicable measures. Subgroup decomposability is, for instance, one such property. The imposition of it 'refines' this class to a singleton consisting only of $\{m^*_n\}$. It is not readily apparent, however, which other properties can be used for this purpose. In particular, how one may improve upon Proposition 3 to provide a complete characterization of the class $\left\{\{m_{n,\alpha}\}: \alpha > 0\right\}$ is at present an open problem.[11]

IV. An Empirical Application

To illustrate how the movement measure $\{m^*_n\}$ can be applied, we consider the case of income mobility in the United States, where the common contention is that relative income mobility either did not change or fell between the 1970s and the 1980s. While Hungerford (1993) and Gittleman and Joyce (1995, 1996) maintain that the mobility rates were 'rather stable' in this period, Buchinsky and Hunt (1996) reported a 'sharp decrease' in wage and earnings mobility over time from 1979 to 1991. The works of Moffitt and Gottschalk (1995) and Gittleman *et al*

(1997), on the other hand, report a 'slight fall' in earnings mobility in the same period.[12] (See Gottschalk 1997 for a survey.)

It is important to note that *all* of these concentrate either on the movement of the income ranks of the individuals or on the movement of their relative incomes. In particular, it is not possible to learn anything from these studies with regard to the comparison of the directional or nondirectional movement of absolute incomes (as measured in dollars) between 1970s and 1980s. In this section we shall report our findings with regard to this particular aspect of income mobility by using the measurement methodology we have developed in the preceding sections.

We use precisely the same extract from the Michigan Panel Study of Income Dynamics that Hungerford used; we thank him for making this data set available to us.[13] Our main finding is that the extent of movement of absolute incomes, as measured by $\{m_n^*\}$, *increased* in the United States from 0.498 during the 1969-76 period to 0.528 in the 1979-86 period—an increase of 6%, which is statistically significant at conventional levels. Perhaps even more interestingly, as measured by any directional movement measure of Proposition 2, we find that the aggregate change in welfare during 1969-76 was significantly positive, while the aggregate change in welfare during 1979-86 was significantly negative.[14]

These findings complement those of the earlier papers cited above. While the aggregate quintile order changes and the movement of relative incomes were (slightly) larger in the 1970s than in the 1980s, there was more income flux overall (and within quintiles) in the 1980s than in the 1970s. Moreover, as also suggested by many other authors, we contend that the income movement observed in the 1969-76 period was welfaristically more desirable than that observed in the 1979-86 period.

We have also calculated the income movement for different demographic groups in the population; these are displayed in columns (1) and (2) of Table 1. Our main finding on this regard is that the income flux increased between the 1970s and the 1980s for all but the two middle educational groups. (For these two, the value of $\{m_n^*\}$ remained roughly the same.) This tells us that the increase in income movement in the United States was not confined to certain demographic groups, but rather was widespread throughout the US population.

Insert Table 1 Here

Finally, by using the subgroup decomposability property of $\{m_n^*\}$, we are able to decompose the aggregate (absolute) income movement in each decade as follows. Take the log-dollar movements in Table 1 for the different demographic groups; multiply them by the weighted fraction of persons in group j in the total sample ('weighted' to reflect the non-random selection of individuals in the US population into the PSID); and normalize them so that the weights sum to 100% for each demographic cut. (These weights appear in columns (3) and (4) of Table 1.) The change in log-dollar movement in the whole sample is then decomposed for each separate partition in column (5).

Looking at the last column of Table 1, our findings can be summarized as follows:

1. More of the increase in income movement is accounted for by males than by females, even though females constituted a somewhat larger share of the (weighted) PSID sample than did males.

2. More of the increase in income movement is accounted for by nonwhites than by whites—despite the fact that the (weighted) share of nonwhites in the PSID sample was only 14%.
3. The increase in income movement is accounted for entirely by persons with high school educations or above. High school dropouts actually contributed to a decrease in flux between the 1970s and 1980s.
4. More of the increase in income movement is accounted for by young adults than by prime-age adults.

Finally, we note that, as measured by any directional movement measure of Proposition 2, the aggregate change in welfare during 1969-76 was positive for both whites and nonwhites, while the aggregate change in welfare during 1979-86 was negative for both of these subgroups.

V. Conclusion

In this paper, our main focus was a particular aspect of the multi-faceted notion of income mobility: the movement (flux) of absolute incomes. We considered a small set of elementary properties for income movement measures, and showed that there is essentially only one measure that satisfies all of these properties: *the per capita aggregate change in log-incomes*. We demonstrated next that this movement-mobility measure has a number of appealing descriptive and normative properties, and provided a formal generalization of our basic characterization theorem drawing from the related literature on poverty measurement. Finally, we presented an empirical application in which we showed, by using our new measure, that there was a broad-based increase in income movement in the United States between the 1970s and 1980s.

Appendix

Proof of Proposition 1

Since sufficiency can be readily verified, we focus only on necessity. Take any $\{m_n\} \in$ $//$ which satisfies the hypotheses of the proposition. By subgroup decomposability, we must have

$$m_n(x, y) = \sum_{i=1}^{n} \left(\frac{1}{n} m_1(x_i, y_i)\right) \quad \text{for all } x, y \in \mathbf{R}^n_{++}, \qquad n \geq 1.$$

But by relativity, $m_1(a, b) = m_1(1, b/a)$ for all $, b > 0$, so that we have

$$m_n(x, y) = \sum_{i=1}^{n} \left(\frac{1}{n} f\left(\frac{y_i}{x_i}\right)\right) \quad \text{for all } x, y \in \mathbf{R}^n_{++}, \qquad n \geq 1,$$

where $f(a) = m_1(1, a)$ for all $a > 0$. By continuity of m_1, f is also continuous on R_{++}. On the other hand, multiplicative path separability implies that $m_1(1, ab) = m_1(1, a) + m_1(a, ab)$ for all $a, b > 1$; that is, $f(ab) = f(a) \; + f(b)$ for all $a, b > 1$. This is Cauchy's third functional equation on the restricted domain $(1, \infty)$, the unique solution of which is well known to be $f(a) = c \log a$ for all $a > 1$, where c is an arbitrary positive constant. Moreover, by symmetry and scale invariance, for any $a \in (0,1)$ we have

$$f(a) = m_1(1, a) = m_1(a, 1) = m_1(1, 1/a) = f(1/a) = -c \log a$$

But, by continuity of f, $f(1) = 0$ holds, and therefore, we may conclude that there exists a $c > 0$ such that $f(a) = c|\log a|$ for all $a > 0$.

Proof of Proposition 2

The proof of this result is analogous to that of Proposition 1, and is thus omitted.

Proof of Proposition 3

Since the proof of Proposition 3 is similar to that of Proposition 1 of Foster and Shorrocks (1991), we shall provide here only a sketch of the basic argument. For this, we need

Lemma A. Let $\{m_n\} \in //$ be a subgroup-decomposable movement measure such that

$$m_n(x, y) = G_n(m_1(x_1, y_1), \ldots, m_1(x_n, y_n)) \qquad \text{for all } x, y \in \mathbf{R}^n_{++},$$

for some symmetric and strictly increasing $G_n: R^n_+ \to R_+, n \geq 2$. For any $u, u^* \in R^{n_1}_{++}$ and $v, v^* \in R^{n_2}_{++}$

(i) $G_{n_1}(u) \geq G_{n_1}(u^*)$ if and only if $G_{n_1+n_2}(u, v) \geq G_{n_1+n_2}(u^*, v)$,
(ii) $G_{n_1+n_2}(u, v) \geq G_{n_1+n_2}(u^*, v)$ if and only if $G_{n_1+n_2}(u, v^*) \geq G_{n_1+n_2}(u^*, v^*)$.

Proof. (ii) immediately follows from (i). To see (i), notice that, by surjectivity of m_1 we may choose $x, y, z, w \in R^{n_1}_{++}$ and $x^*, y^* \in R^{n_2}_{++}$ such that $u_i = m_1(x_i, y_i), u_i^* = m_1(z_i, w_i)$ and $v_j = m_1(x_j^*, y_j^*)$ for all $i = 1, \ldots, n_1$ and $j = 1, \ldots, n_2$. Let $n = n_1 + n_2$. Since, by subgroup consistency, $m_{n_1}(x, y) > m_{n_1}(z, w)$ implies $m_n\big((x, x^*), (y, y^*)\big) > m_n\big((z, x^*), (w, y^*)\big)$ it follows that

$$G_{n_1}(u) > G_{n_1}(u^*) \Rightarrow G_n(u, v) > G_n(u^*, v)$$

To show that the converse of this observation also holds, let $m_n\big((x, x^*), (y, y^*)\big) = G_n(u, v) > G_n(u^*, v) = m_n\big((z, x^*), (w, y^*)\big)$. By the previous observation, $G_{n_1}(u) \geq G_{n_1}(u^*)$ must be the case. In fact, this inequality must hold strictly, for if $G_{n_1}(u) = G_{n_1}(u^*)$, then $m_{n_1}(x, y) = m_{n_1}(z, w)$, and by subgroup consistency and symmetry of G_{n+n_1} we obtain the following contradiction:

$$\begin{aligned} G_{n+n_1}(u, v, u^*) &= m_{n+n_1}((x, x^*, z), (y, y^*, w)) > m_{n+n_1}((z, x^*, x), (w, y^*, y)) \\ &= G_{n+n_1}(u^*, v, u) \\ &= G_{n+n_1}(u, v, u^*). \end{aligned}$$

The sufficiency part of Proposition 3 can be readily verified; we prove only the necessity part. Let $\{m_n\} \in //$ satisfy the hypotheses of the proposition. By weak decom- posability and continuity of m_n,

$$m_n(x, y) = G_n(m_1(x_1, y_1), \ldots, m_1(x_n, y_n)) \qquad \text{for all } x, y \in \mathbf{R}^n_{++}$$

for some symmetric, strictly increasing and continuous $G_n: R^n_+ \to R_+, n \geq 2$. By Lemma A(ii), G_n induces an independent ordering in the sense of Debreu (1960) for any $n \geq 2$. Moreover, this ordering is obviously essential (again in the sense of Debreu 1960), since G_n is not a constant function by hypothesis. Therefore, by Debreu's additive representation theorem (Debreu 1960, Theorem 3), there exist strictly increasing and continuous functions $F_n: R_+ \to R_+, n \geq 2$ and $g_n: R_+ \to R_+, n \geq 2$ such that $G_n(u) = F_n(\Sigma g_n(u_i))$ for all $u \in R^n_+, n \geq 3$.

By Lemma A(i), we have

$$\begin{aligned} G_2(u) \geq G_2(u^*) &\Leftrightarrow G_3(u, a) \geq G_3(u^*, a) \\ &\Leftrightarrow g_3(u_1) + g_3(u_2) \geq g_3(u_1^*) + g_3(u_2^*) \end{aligned}$$

for any $u, u^* \in R^2_{++}$ and $a > 0$. Therefore, defining $g_2 \equiv g_3$, we can conclude that $G_n(u) = F_n(\Sigma g_n(u_i))$ for all $u \in R^n_+$, for some strictly increasing and continuous functions $F_n: R_+ \to R_+$ and $g_n: R_+ \to R_+, n \geq 2$. But then, where $h_n = n g_n$ for all $n \geq 2$, we can write

$$G_n(u) = F_n\left(\frac{1}{n}\sum_{i=1}^{n} h_n(u_i)\right) \quad \text{for all } u \in \mathbf{R}^n_+, \qquad n \geq 2.$$

Furthermore, one can show by replication invariance that we must have $F_n = F_{n'}$ and $h_n = h_{n'}$ for all $n, n' \geq 2$. (This step is identical to the derivation of equation (21) from (14) in Foster and Shorrocks 1991, pp. 694-5.) So, $G_n(u) = F_n(\Sigma h(u_i)/n)$ for all $u \in R_+^n, n \geq 2$ for some strictly increasing and continuous $F: R_+ \to R_+$ and $h: R_+ \to R_+$; that is, by weak decomposability,

$$m_n(x, y) = F\left(\frac{1}{n}\sum_{i=1}^{n} h(m_1(x_i, y_i))\right) \quad \text{for all } x, y \in \mathbf{R}_{++}^n, \qquad n \geq 2.$$

But by replication invariance,

$$m_n(a\mathbf{1}_n, b\mathbf{1}_n) = F(h(m_1(a, b))) = m_1(a, b)$$

for all $a, b > 0$ and $n > 2$. Thus, since $\{m_1(a, b): a, b > 0\} = R_+$ by surjectivity of m_1 we have $F \circ h = I$; i.e. $h = F^{-1}$. Moreover, that $m_1(a, b) = |\log b - \log a|$ can easily be established as in the proof of Proposition 1.

Notes

1. By ‘relative income’, we mean the individual’s share of the total income.
2. Among other axiomatic approaches to mobility measurement are Chakravarty *et al*. (1985); King (1983); Geweke *et al*. (1986); Shorrocks (1978a, 1993); and Van de Gaer *et al.* (1998).
3. Examples include the correlation coefficient, the fraction of people changing more than one income quantile (Lillard and Willis 1978), Hart’s index (Hart 1981), Bartholomew’s index (Bartholomew 1982), Shorrocks’ index for two periods (Shorrocks, 1978b) and the Maasoumi-Zandvakili index for two periods (Maasoumi and Zandvakili 1986).
4. The notion of *social utility* is commonly taken in welfare economics either as the social income evaluation function of the social planner acting as a social norm, or as the utility of the representative agent of the society.
5. This particular interpretation of $\{m_n^*\}$ is analogous to that of the Watts index of poverty; see Atkinson (1987) and Zheng (1993).
6. The notion of a ‘utility transfer’ is well recognized to be problematic, for it envisages the cardinal full comparability of preferences. This is, however, less of an issue for the present analysis, for our interpretation of $\{m_n^*\}$ is already based on the premiss of representing individual preferences by a single social utility function for income. We stress that the issue here is one of ‘interpretation’; a nonutilitarian (or an adherent of risk-loving behaviour) would understandably disregard the conclusions based on this interpretation. Indeed, Proposition 1 allows $\{m_n^*\}$ to be treated simply as a ‘descriptive’ measure of income mobility with no underlying welfare content.

7. See, among others, Bartholomew (1982) and Markandya (1982, 1984).

8. Similarly, the correlation coefficient (which is a relative mobility measure) deems the extent of mobility *independent* of the value of ε: $\mathrm{Cor}\big(x, y(\varepsilon)\big) = 1$ for all $\varepsilon \in [0,5)$. So, the correlation coefficient too (like all other measures that are based on relative incomes) fails to capture the ‘movement’ aspect of mobility satisfactorily.

9. This is analogous to using the head count and poverty gap measures together to get a more complete picture of the extent of economic poverty. The head count ratio is reminiscent of the immobility ratio (or any other trace-based index) in that it is insensitive to income movements within the poor class while being discontinuous at the poverty line. On the other hand, the poverty gap measure resembles $\{m_n^*\}$, for it is a continuous measure which is not particularly sensitive to income movements that carry individuals above the poverty line.

10. While the discussion in Fields and Ok (1996) is limited to $\{m_n^0\}$, the arguments are easily seen to extend to all members of Σ. As a caveat, however, we note that our approach is consistent with Atkinson’s diagonalizing switches axiom only when a diagonalizing switch occurs on the diagonal (see the discussion in Fields and Ok 1996, pp. 364-5.)

11. We note that the notion of compatibility introduced by Foster and Shorrocks (1991) to characterize the P_α– class is of no help in characterizing $\Big\{\{m_{n,\alpha}\}: \alpha > 0\Big\}$. Indeed, in contrast with what Foster and Shorrocks establish for poverty indices (see their Proposition 7), one can show that there does *not* exist a translation invariant movement measure that is compatible (see their definition on p. 704) with a measure of the form characterized in Proposition 3.

12. Hungerford's conclusion was based on three measures applied to data from the Michigan PSID: the lambda asymmetric stastistic, Cramer's V, and the contingency coefficient. Gittleman and Joyce used the extent of movement across earnings quintiles in successive rounds of the Current Population Survey, and the correlation coefficient between income in one year and income in the next. Buchinsky and Hunt based their analysis on the average quintile jump and the mean of the reciprocal exit times in the National Longitudinal Survey of Youth. Finally, both Moffitt and Gottschalk and Gittleman *et al*. calculated the sum of the off-diagonal elements in quintile transition matrices for the PSID. It should also be noted that all of these studies except Hungerford use earnings (or wages) as the basis for the analysis, while our present focus is on *income* mobility. As an anonymous referee rightly observed, this distinction is important because the relationship between income and earnings mobility is not transparent. See Shorrocks (1981) and Jarvis and Jenkins (1998) for a discussion of this issue.
13. Hungerford's unit of observation is the individual, weighted by the PSID sampling weights. Individuals are assigned the real total income (including income from cash transfers) of the families to which they belong. To be included in these calculations for the 1970s, the individual must have had a valid income observation in 1969 and 1976; for the 1980s, income observations must have been available for 1979 and 1986. All income figures are in 1982-4 dollars.
14. Several other observations of this nature are reported in a companion paper (Fields *et al.* 1998), which also contains an appendix detailing the analysis of the statistical significance of these results.

Table 1

TABLE 1
LOG-DOLLAR MOVEMENT, TOTAL AND BY SUBGROUP, IN THE UNITED STATES IN THE 1970s AND 1980s

	Level of log-dollar movement		Weights		Decomposition of change in log-dollar movement
	1970s (1)	1980s (2)	1970s (3)	1980s (4)	(5)
Entire sample	0.498	0.528			
Sex of the individual					
Male	0.475	0.516	0.478	0.475	0.018
Female	0.519	0.539	0.523	0.526	0.012
Race of the individual					
White	0.489	0.511	0.861	0.850	0.013
Nonwhite	0.549	0.625	0.139	0.150	0.018
Education of the head of household					
Less than high school	0.473	0.516	0.531	0.367	−0.062
High school graduate	0.516	0.517	0.268	0.320	0.027
Some college	0.560	0.562	0.092	0.153	0.035
College graduate	0.514	0.547	0.110	0.161	0.032
Age of the individual in the base year					
Under 22	0.560	0.622	0.473	0.387	−0.024
22–29	0.487	0.544	0.123	0.177	0.036
30 and over	0.428	0.439	0.404	0.437	0.019

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