

Measuring Inequality Change in an Economy With  
Income Growth  
Reply

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When an economy achieves income growth, it typically will experience a change in inequality. The purpose of my paper ‘Measuring Inequality Change in an Economy With Income Growth’ was to discuss ways of determining whether inequality increases or decreases in the event of Lorenz curves crossing.

The challenge was to respond to an unmet need: the development of a non-circular justification for the choice of a particular Lorenz-consistent inequality index which is well-behaved in the context of various kinds of economic growth. The types of growth I called ‘high income sector enrichment’ and ‘low income sector enrichment’ posed no problem. What I tried to justify is the behavior of inequality when growth is of the ‘high income sector enlargement’ type. I offered one such justification leading to a new kind of Lorenz-consistent index and argued that while the standard indices might be justified by some rationalization of the inverted-U pattern, they had not yet been so justified.

To progress on this problem, I considered how inequality might be expected to behave in the case of two incomes wherein  $\phi\%$  of the population is in the higher-income sector and  $100-\phi\%$  of the population is in the lower-income sector. My Axiom 6 (A6) was: ‘Numerical equality is greatest when the population is divided evenly between two groups; numerical inequality is greater the further is the actual division from 50:50’.

Moore has taken up the challenge and offered a different justification. He argues for use of the Gini coefficient, noting that it can be interpreted as ‘one-half of the expected difference of the incomes of two people chosen randomly (with replacement) from the society, with the difference expressed as a percentage of the mean income of the society’.

In the interval between the acceptance of my paper for publication and the drafting of Moore’s comment, I came up with yet another justification for the way inequality changes as the

high-income sector expands. Let  $S_i$  = the number of individuals in an economy who are in the same group. Then each of the  $n_H$  individuals in the high income group has  $n_H - 1$  individuals like him and each of the  $n_L$  individuals in the low income group has  $n_L - 1$  individuals like him. The total number of similarities is thus

$$S = n_H(n_H - 1) + n_L(n_L - 1)$$

In this formulation, inequality varies inversely with the number of similarities. The index  $S = n_H(n_H - 1) + n_L(n_L - 1)$  is minimized when the population is divided 50:50 between sectors, and hence the number of dissimilarities is maximized at 50:50. If we take this index of dissimilarities as our index of inequality, we then find an interior maximum at 50:50.

These alternative formulations result in different paths for inequality as the high-income sector enlarges to employ more people. The considerations I proposed in my paper, as embodied in A6, imply a U-shaped pattern with an interior minimum at 50:50. Moore's thinking, which led him to the Gini coefficient, implies an inverted-U pattern with a turning point determined by the behavior of the Gini coefficient itself (his table 1). The index  $S = n_H(n_H - 1) + n_L(n_L - 1)$  has an inverted-U with a turning point at 50:50. Thus, the choice of an inequality index is consequential.

Moore analyzes the process I had used as an example, namely, the sequence [1,1,1,1,1,4] to [1,1,1,1,4,4] to [1,1,1,4,4,4] to [1,1,4,4,4,4] to [1,4,4,4,4,4]. He points out, correctly, that as high-income sector enlargement occurs in the move from the first of these distributions to the second, each of the remaining poor has an inequality experience twice as often. This contributes to a rise in inequality.

Let me continue the argument he started. He might have gone to say: Then, from [1,1,1,1,1,4] to [1,1,1,4,4,4], each of the poor will have an inequality experience one and a half

times as often'. And so on. So, this aspect of inequality increases but at a decreasing rate, i.e., the change in inequality associated with this component decreases. Fine, but: if this component pushes inequality up, what else operates to pull inequality down (so that it reaches an interior maximum, as Moore asserts it does)? And: why is the peak *not* at 50:50? Moore doesn't say.

The latter question raises the issue of symmetry. I plead not guilty to confusing symmetry with numerical equality. Rather, I asserted that my notion of numerical inequality is symmetric. In retrospect, it would have been better to have kept the symmetry aspect of numerical inequality and the U aspect separate. Let me do that now.

Axiom 6 stated: 'Numerical equality is greatest when the population is divided evenly between two groups; numerical inequality is greater the further is the actual division from 50:50'. Suppose I make the argument in two parts:

*A6'*. Numerical equality is *greatest* when  $f\%$  of the population is in the high-income group and  $1 - f\%$  of the population is in the lower-income group; numerical inequality is greater the further is the actual division from  $f$ .

*A6''*. The turning point occurs when  $f = 1/2$ .

Moore would replace 'greatest' by 'smallest' in the first sentence of *A6'* and 'greater' by 'smaller' in the second, resulting in:

A6\*. Numerical equality is *smallest* when  $f\%$  of the population is in the high-income group and  $100 - f\%$  of the population is in the lower-income group; numerical inequality is smaller the further is the actual division from  $f$ .

This is not circular and it is defensible in the terms he used. As for A6", Moore would find this unacceptable. He would instead use the Gini to determine the turning point. If this is not circular, it is very close to it. Clearly, the issue is a matter of perception, not one that can be resolved by deductive argumentation - which has been my point all along.

I would conclude with a question for Moore and others who advocate the use of the Gini on conceptual (as opposed to practical) grounds: why is  $[1,1,1,1,4,4]$  more unequal than  $[1,1,1,1,1,4]$  or  $[1,1,1,4,4,4]$ ? A circular answer would be: because the Gini coefficient says it is. Please, can we have a non-circular answer?