# Inequality in Dual Economy Models

Gary S. Fields

Cornell University

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Ever since the pathbreaking work of Kuznets (1955), economists have been concerned with the question of how inequality should respond to income growth in a two-sector economy. How does inequality change when those in a particular sector (here termed ‘high-income’ and ‘low-income’) get richer? When the high-income sector gets larger and incorporates a larger share of the population? The present paper adds a new perspective to the debate about the effects of the enrichment and enlargement of various sectors on (relative) income inequality.

In a two-sector model, overall inequality may be regarded as a function of inequality within the two sectors and inequality between them. When analysing the between-sector effects, most but not all authors have taken the simplifying line of assuming perfect equality within sectors; see the citations in footnote 2 below. All of these authors have reached the conclusion that in the case of zero within-sector inequality, as the share of the population in the modern sector increases, inequality first increases, then reaches an interior maximum, and finally decreases - the so-called ‘inverted-U pattern’.

Although the inverted-U shape is plausible, other shapes are plausible also, as has been argued philosophically (Temkin, 1986) and experimentally (Amiel and Cowell, 1992); see also Fields (1987). The present paper adds to this discussion by introducing two new concepts—‘elitism of the rich’ and ‘isolation of the poor’. These concepts may inform people’s views about inequality quite generally and can help underpin non-inverted-U shapes in dual economy models,

## I. Elitism of the Rich and Isolation of the Poor

Let there be $n$ income recipients ('persons'), the total number of whom is fixed. Assume that a proportion $\phi$ of these persons each have income $y_H$ and the rest $(I - \phi)$ each have income $y_L (< y_H)$. Define $\phi = n_H/n$ and $\Theta = Y_H/Y_L \, \forall\phi \in (0, I), \Theta = I$ for $\phi = 0. I$. The term 'increase in $\Theta$' shall be understood as signifying an increase in $y_H/y_L \, \forall\phi \in (0, I)$.

In the two-income world, income growth can take place by increasing $y_H, y_L,$ or $\phi$ or by some combination of these. *Ceteris paribus*, an increase in high- income-sector-enrichment') may be said to increase inequality and an increase in $Y_H$ ('low-income-sector enrichment') to reduce inequality. Indeed, these judgements are required by Lorenz consistency. The two preceding judgements imply that inequality is an increasing function of $\Theta$.

What happens to inequality if $y_H$ and $y_L$, and hence $\Theta$, remain constant while $\phi$ increases ('high-income sector enlargement', the term connoting the enlargement of the high-income sector to employ or otherwise provide high incomes to more people)? I now introduce two concepts having the status of 'primitives' in the sense of Sen (1973). These notions—'elitism of the rich' and 'isolation of the poor'—seem to underlie many people's notions about what inequality is.

*Elitism of the rich* is the following idea. In a population of size $n$, let $n_H$ people have a high income (£$y_H$ each) and the rest have low incomes (£$y_L$each). Suppose $n_H = 1$ initially. The one rich person may be said to enjoy a very elite position, and in that sense the economy may be said to have a high degree of elitism of the rich.

Suppose now that a second person becomes equally rich, all others' incomes remaining unchanged. We might regard the two rich persons, taken together, as less elite than one person was when he alone was rich, perhaps because each of the rich now has to share his elitist position with someone else. Elitism of the rich might therefore be said to fall. Now suppose a third person is enriched. Elitism of the rich might be thought to fall further, but not by as much as when the second person was enriched. And so on: the larger the fraction rich, the smaller is elitism of the rich and the smaller is the change in elitism of the rich for a given increase in size of the high income group. Ultimately when the last person is made rich, elitism of the rich is eliminated.

Elitism of the rich also varies with $y_H/y_L$, the relative income ratio. Suppose that in an economy with $n_H(> 0)$ persons receiving incomes $y_H$ and $n_L(> 0)$ receiving $y_L < y_H, y_H$ increases, other things constant. Elitism of the rich might be said to have increased due to such a change. The same is true if $y_L$ decreases.

We have defined $\phi = n_H/n$ and $\Theta = Y_H/Y_L \, \forall \phi \in (0, I), \Theta = I$ for $\phi = 0. I$. The considerations in the preceding paragraphs suggest that elitism of the rich can be defined as a function of $\phi$ and $\Theta$ as follows:

ER. 1. $ER = h(\phi, \Theta), \phi \in (O, I]$
ER. 2. $ER(I, \cdot) = 0$
ER. 3. $h(\cdot)$ decreasing in $\phi$
ER. 4. $\Delta ER = ER(n_H/n) - ER[(n_H - 1)/n]$ decreasing in $\phi$
ER. 5. $h(\cdot)$ increasing in $\Theta$

*Isolation of the poor* is a reciprocal notion to elitism of the rich. Suppose everyone in the population is poor. In this case, the poor are not isolated from the rich, because there are no rich, so there is no isolation of the poor. Now let one person escape poverty. Those who remain poor constitute a group which is isolated from the rich, so isolation of the poor is created. As more and more persons attain high incomes, those who are left behind may be regarded, as a group, as increasingly isolated. Isolation of the poor may thus be viewed as increasing at an increasing rate as the high-income group expands. When just one person is poor, that one person may be thought to be *very* isolated from everyone else, and isolation of the poor may be thought to reach its maximum value on the domain $[0, I)$ for given $y_H$ and $y_L$. Furthermore, any increase in $y_H$ or reduction in $y_L$ holding the numbers in the two groups $n_H$ and $n_L$ constant may reasonably be regarded as raising the extent of isolation of the poor.

These considerations suggest that isolation of the poor be defined as a function of $\phi$ and $\Theta$ as follows:

IP. 1. $IP = i(\phi, \Theta), \phi \in [O, I)$
IP. 2. $IP(I,\cdot) = 0$
IP. 3. $i(\cdot)$ increasing in $\phi$
IP. 4. $\Delta IP = IP(n_H/n) - IP[(n_H - 1)/n]$ increasing in $\phi$
IP. 5. $i(\cdot)$ increasing in $\Theta$

## II. Inequality and Dualistic Economic Growth: Possible Patterns

From these concepts of elitism of the rich and isolation of the poor, we may derive various inequality patterns depending on the weight given to each. Those observers who wish to view inequality solely in terms of elitism of the rich would regard inequality as *falling* continuously on the interval $\phi \in [O, I)$ for any given $\Theta$. The higher is $\Theta$, the higher is inequality. Others may view inequality solely in terms of isolation of the poor. These observers see inequality as *rising* continuously on the interval $\phi \in [O, I)$ for any given $\Theta$, while a higher $\Theta$ implies more inequality.

For those observers whose perceptions of inequality consist of *both* elements, elitism of the rich and isolation of the poor, how might these notions be combined on their common domain, the open interval (0, I)? First, we need to express elitism of the rich and isolation of the poor in comparable units, since thus far they are unit-free. The case can be made that elitism of the rich and isolation of the poor are more than just inverse notions. They might also be viewed as comparable in scale in a perfectly reciprocal way, such that for any $\phi, ER(\phi) = IP(I - \phi)$. Such an assumption shall be termed the *scale comparability property* and denoted by $SC$.

One plausible way of combining elitism of the rich and isolation of the poor, and thereby to link inequality to $\phi$, would be to give equal weight to each. For a given $\Theta$, the simplest such mixing function, defined on the open interval (0, I), the common domain of $ER(\cdot)$ and $IP(\cdot)$, is

$$I(\phi, \Theta) = (\mathrm{ER} + \mathrm{IP})/2$$

Such an equally-weigh ted function is said to have the *EW property*. But one may object to equal weighting, preferring to give strictly positive but not equal weights to elitism of the rich and to isolation of the poor. This may be accomplished by using a linear mixing function.

$$I(\cdot) = wEP(\cdot) + (I - w)IP(\cdot), w > 0, \quad w \neq I - w$$

denoted the *unequally-weighted mixing property UW*.

Define the *U class* to be those $I(\cdot)$ rankings which are U-shaped as $\phi$ varies on the open interval (0, I) for a given $\Theta$, and which lie on higher contours for higher $\Theta$; denote those which also are symmetric with a unique minimum at $\phi = \frac{1}{2}$ as the *symmetric U class*. We have two results:

Proposition I. *Properties ER, IP, SC, and EW generate a ranking which is a member of the symmetric U class.*

*Proof.* $I = \mathrm{ER}(\phi, \Theta) + \mathrm{IP}(\phi, \Theta) = \mathrm{ER}(\phi, \Theta) + \mathrm{ER}(\mathrm{I} - \phi, \Theta)$ which by the convexity of $ER$ falls as $\phi \uparrow \frac{1}{2}$ and rises thereafter.

Proposition 2. *Properties ER, IP, SC, and UW generate a ranking which is a member of the U class but not the symmetric U class.*

*Proof.* $I = w\mathrm{ER}(\phi, \Theta) + (\mathrm{I} - \mathrm{w})\mathrm{IP}(\phi, \Theta) = \mathrm{wER}(\phi, \Theta) + (\mathrm{I} - \mathrm{w})\mathrm{ER}(\mathrm{I} - \phi, \Theta)$

$$= w[ER(\phi, \Theta) - ER(I - \phi, \Theta)] + \mathrm{ER}(\mathrm{I} - \phi, \Theta)$$

The first term reaches a minimum at $\phi = \frac{1}{2}$. But because the second term $\mathrm{ER}(\mathrm{I} - \phi, \Theta)$ is increasing in $\phi$ throughout, the sum must be increasing at $\phi = \frac{1}{2}$, which rules out symmetry.

The axiomatic set *ER*, *IP*, *SC*, and *UW* produces an ordering whereby $I(\cdot)$ falls and then rises with $\phi$ on (0, I). Given $I(\cdot) = wER(\cdot) + (I - w)IP(\cdot)$, we have as special cases monotonically decreasing inequality for $w = I$ and monotonically increasing inequality for $w = 0$.

These three inequality patterns - monotonically decreasing, monotonically increasing, and U-shaped as $\phi$ varies for a given $\Theta$—are the *only* patterns permitted by the preceding axiomatic set. This poses a problem for those observers who see inequality as rising up to some point, reaching an interior maximum, and then falling. Specifically:

Proposition 3. *The inverted-U pattern cannot be generated from the ER, IP, SC, and EW or UW properties.*

To generate the inverted-U pattern, a different justification is needed.

## III. Continuity

The analysis so far has been on the open interval (0, I), the common domain of $ER(\cdot)$ and $IP(\cdot)$. Lorenz-consistency requires that inequality be minimised at the end-points $\phi = 0$, and $\phi = 1$, where everybody has the same income. The natural normalisation axiom to adapt is that the most equal points have no inequality:

*Normalisation Axiom* $(N)$. $I(Y/n, Y/n, \ldots, Y/n) = 0$ for all total income amounts $Y$.

At the ends $\phi = 0$ and $\phi = 1$, all three patterns discussed in Section II display a jump: the monotonically decreasing class jumps when the first person attains a high income, the monotonically decreasing class when the last person attains a high income, and the U-shaped class under both circumstances. I shall how demonstrate that despite these jumps, continuity is preserved.

Given the Lorenz axioms of income homogeneity and population homogeneity, we may confine our specification of inequality orderings to the $\phi, \Theta$ domain for inequality analysis is the positive quadrant for $\phi$ strictly between 0 and I, plus the points $(\phi = 0, \Theta = \mathrm{I})$ and $(\phi = I, \Theta = \mathrm{I})$. This is illustrated by the shaded area in Fig. i, plus the two corners.

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Insert Figure 1 Here

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Now imagine that starting from perfect equality $Y_0$ (the lower left corner), we add e to one person's income. This entails a small vertical change in $\Theta$ *and* a small horizontal change in $\phi$. The new distribution (call it $Y_e$) lies close to the original distribution $Y_0$. The inequality function $I(\phi, \Theta)$ is continuous at $Y_0$ if, when $Y_e$ gets arbitrarily close to $Y_0, I(Y_e)$ gets arbitrarily close to $I, Y_0$. This must be the case—Lorenz-consistency assures it—which means that the three inequality orderings discussed in Section II are *continuous on the left*. By an analogous argument, these orderings are *continuous on the right*. All are obviously continuous on the interior. They are therefore continuous throughout. Continuity is *not* violated.

Why, then, the *appearance* of discontinuity? Starting from a situation of perfect equality, when one person acquires a high income, $\phi$ and $\Theta$ both increase, e.g. a change in the income distribution from $[I, I, I, I, I, I]$ to $[I, I, I, I, I, 6]$ increases $\phi$ by $\frac{1}{6}$ and $\Theta$ by 500%. It is perfectly consistent with continuity, the Lorenz axioms, and other axioms for $I(\cdot)$ to rise by a lot in response to a large increase in $\Theta$.

Let us define an inequality index to be *practically discontinuous on the left* if, for given $y_H$ and $Y_L$.

$$(i) \quad \lim_{\phi \downarrow 0} I(\phi, \Theta) \neq I(0, I)$$

*practically discontinuous on the right* if, for given $y_H$ and $Y_L$.

$$(ii) \quad \lim_{\phi \uparrow 1} I(\phi, \Theta) \neq I(I, I)$$

and *practically discontinuous* if it is practically discontinuous on the left or on the right, i.e. if (i) or (ii). Those observers who object to the practically discontinuous inequality notions must base their objections on something other than violation of continuity.

## IV. Inequality Measures and Inequality

In a just-published paper, Anand and Kanbur (1993) examined the properties of five commonly-used inequality measures, here denoted $C$, all of which are Lorenz-consistent: $C$ = {Theil's entropy index, Theil's second measure, coefficient of variation squared, Atkinson index, Gini coefficient, non-overlapping case}.

They also examined the log variance, which is not Lorenz-consistent. Given a high-income sector and a low-income sector with fixed within-sector distributions $f_H$ and $f_L$ respectively and with the high-income sector comprising a variable fraction of the population, $\phi$, they showed: (i) Each of the six inequality measures is a quadratic form in $\phi$ or in a continuous monotonic transformation of $\phi$ such as $\log \phi$; (ii) when the distribution within the high- income sector is more unequal than the distribution within the low-income sector, inequality in the economy either (a) increases monotonically on $\phi \in [0, I]$ or (b) increases up to some point $\phi^*$ and then decreases.

Given Anand and Kanbur's findings, it follows immediately that when there is no within-sector inequality each of these six inequality measures starts at zero, increases continuously to an interior maximum at $\phi^*$, and then decreases to zero.[2] We therefore have:

Proposition 4. *The inequality measures in C are Lorenz-consistent and follow an inverted-U-pattern in high-income sector enlargement growth. The log variance, although not Lorenz-consistent, also follows an inverted-U pattern.*

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[2] This had been shown earlier for the no-within-sector-inequality case by Knight (1976) and Fields (1979) for the Gini coefficient and was implied in the work of Swamy (1967) and Robinson (1976) for the coefficient of variation and the log variance respectively. More recently, Kakwani (1988) has shown that if the high- income and low-income sectors have the same Lorenz curve as each other, then the inverted-U shape holds for the income share of the poorest $p$%, the generalised entropy class, and the Atkinson index.

Propositions 3 and 4 lead directly to:

Proposition 5. *The following inequality measures are incompatible with ER, IP, SC, and EW or UW: Theil's entropy index, Theil's second measure, the coefficient of variation squared, the Atkinson index for finite e, the Gini coefficient (non-overlapping case), and the log variance.*

Four additional remarks about existing inequality measures bear mention:

1. Although each of the six measures produces an inverted-U pattern, these measures turn at different places. This means that these measures are *not* ordinally equivalent $\phi \in [0, I]$.
2. The five inequality measures in $C$ are asymmetric, in that they turn for values of $\phi \neq \frac{1}{2}$ details may be obtained from the author upon request.
3. The log variance, on the other hand, turns at $\phi = \frac{1}{2}$ but is not Lorenz- consistent. Therefore:
4. None of the six commonly-used inequality measures is both Lorenz- consistent and symmetric. For those who adhere to the Lorenz criteria and who also believe that inequality *should* increase until half the population is in the high-income group and decrease thereafter, this means that the six inequality measures considered here do *not* represent their views.

Proposition 5 is *not* an impossibility result. There exist inequality measures which are Lorenz-consistent and which have the U-pattern in the interior of (0, I). The general class of such measures is the class of representations, real-valued or otherwise, of the Lorenz properties along with *ER*, *IP*, *SC,* and *EW* or *UW*. Representations of the form $f(\Theta) \cdot g(\phi)$, where $f(\cdot)$ is increasing in $\Theta$ and $g(\cdot)$ is U-shaped in $\phi$, have the required characteristics. An example of such a function is:

where

$$I = (\Theta - \mathrm{I})^{\alpha} \left[K + \frac{1}{4} - \phi(I - \phi)\right]^{1-\alpha},$$

$$\Theta = y_H/y_L \ \ if\ \phi \in (0, I);$$

$$\Theta = \mathrm{I} \ \ \text{if} \ \ \phi = 0, \mathrm{I}; 0 < \alpha < \mathrm{I} \ \ \text{and} \ \ \mathrm{K} > 0$$ [3]

The proof that $I$ fulfils the required properties is omitted for space reasons.

The index $I$ is but one example of a real-valued function with the desired properties. There are many other possible representations, e.g. $I' = (\Theta^2 - \mathrm{I})^{\alpha}\left[K + \frac{1}{4} - \phi(I - \phi)\right]^{1-\alpha}$. It remains to explore the properties of various alternatives and determine their relative merits.

## V. Concluding Remarks

Several tasks are left to future work. One is to develop further the class of inequality measures consistent with the U shape, with the goal of choosing which of the various possible measures have desirable properties on the two- income domain. Another is to expand the domain to allow for intra-group inequality and for more than two groups, so that the new measures can be taken to actual data on countries' economic growth experience. A third is to extend these notions of inequality to the question of welfare comparisons, in order to specify when an increase in inequality is large enough to outweigh an income gain and render economic growth welfare-decreasing.

[3] $\mathrm{K} > 0$ guarantees that $\mathrm{I} > 0$ for all $\phi$, in particular, at the interior minimum.

Figure 1

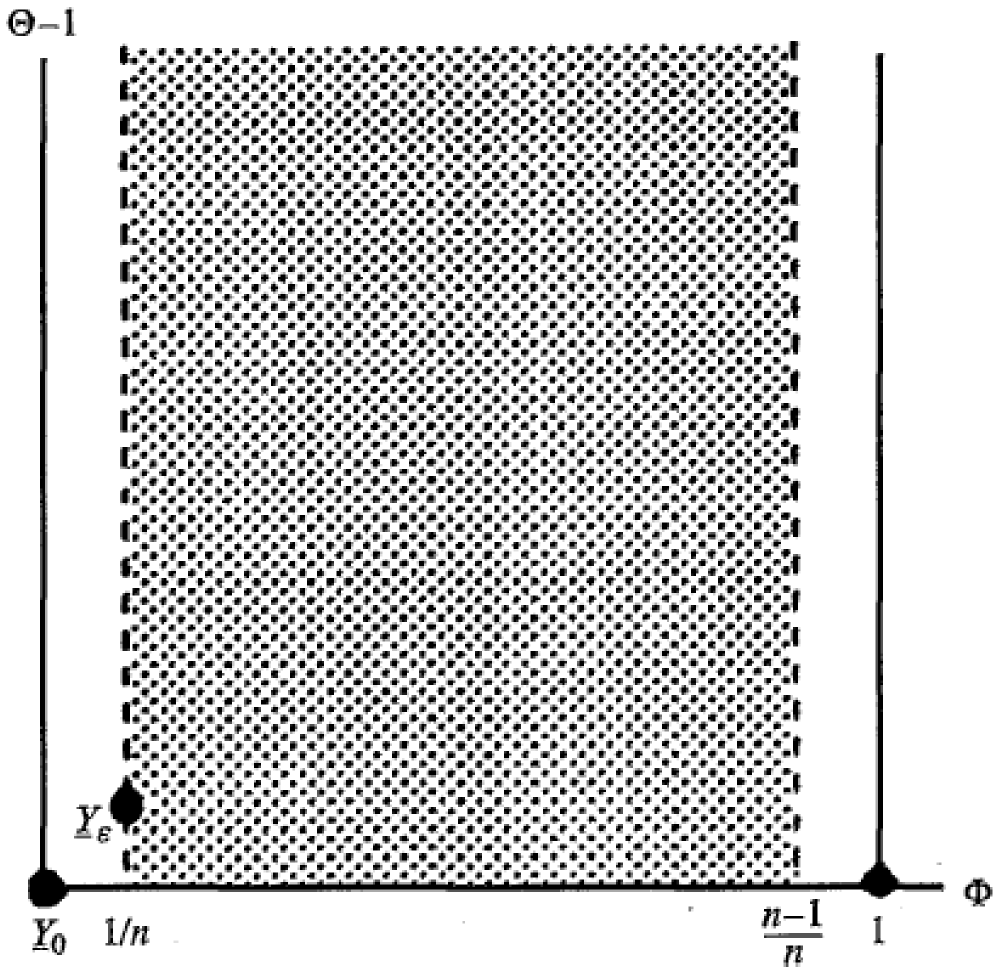


Fig. 1. The domain of inequality analysis in the two income world.

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