Toward a Model of Education and Labor Markets in

Labor Surplus Economies

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Introduction

This model is intended to describe the essential relationships between the demand for and supply of education and the demand for and supply of educated workers. The terms "education" and "training" will be used interchangeably throughout, since the proposed model is a general one designed to apply both to traditional education and to specialized training for such occupations as agricultural and veterinary workers, teachers, the skilled trades, and the like. The terms "educated," "trained," and "skilled" will also be used synonymously.

If the model is to be meaningful, it must possess two basic characteristics. First, it must be consistent with the historical facts of labor surplus economies. Second, it must suggest qualitative, and hopefully quantitative, factors to be considered by policy-makers in formulating educational and labor market policies consistent with national objectives.

Work of this nature must progress through three definite phases. First is the formulation of the model. Next comes the solution of the model, which is used to describe the historical time paths of interesting magnitudes and to suggest optimal paths for the control variables for planning purposes. Finally, as much empirical evidence as possible is needed to make the study operationally meaningful for planners.

I am here concerned only with phase one: a statement of the model. I hope to formulate the basic relationships, including the essential institutional facts of life. As will become evident, such an exercise leads to a conceptually straightforward but mathematically complex model. An analytical solution may prove to be impossible. Perhaps computer simulations of the basic relationships are all that can be found. In any event, a clear statement of the model is a prerequisite for further study.

A General Description of the Model

There are in fact two models. In the historical model, no optimal behavior is postulated to have occurred; society is not assumed to have been maximizing national income, social welfare, or anything else. Rather, the course of the economy and the educational system are assumed to have been determined by the interaction of the endogenous responses of individuals and communities to such variables as the demand for and supply of training facilities and workers of different skill categories. It is assumed that the labor surplus economy was initially in disequilibrium characterized by an excess demand for education. The solution of the historical model, if one exists, is an equation that defines historical time paths of important labor market and educational variables as the economy adjusts toward equilibrium. This equation can also be used to project the future course of the economy and the eventual equilibrium if the existing relationships are assumed to hold indefinitely. If, however, the supply of schooling facilities is controlled by the government for the purpose of maximizing social welfare, then it may be possible to solve the planning model for an optimal path of the supply of schooling facilities. The solution, if one exists, will allow the calculation of the time paths of such magnitudes as the total

supply of training spaces, the fraction of the labor force that is trained, the employment rate, and the national income.

The Historical Model

The model assumes that there are two categories of labor, educated and uneducated; two types of jobs, skilled and unskilled; and one produced good with a numéraire price of one. The aggregate level of output depends upon the employment of these two types of labor and upon the quantity of capital and a disembodied level of technology, both of which are assumed to grow exogenously and are neutral with respect to the two categories of labor. The number of new entrants to the labor force is assumed to grow exogenously, and a constant exogenous fraction of the existing labor force is assumed to drop out due to death or retirement. It is assumed that the aggregate production function is homogeneous of degree less than one, which implies diminishing returns to scale and a linear expansion path.

Uneducated workers can work only in unskilled jobs. Educated workers can choose to work in skilled or unskilled jobs. Their choice is assumed to depend, on the margin, upon the wage rates and probabilities of employment in skilled and unskilled jobs.

Education can be acquired instantaneously, but there may be a positive money cost to the trainee. It is assumed that the only persons eligible to receive training are new entrants to the labor force; adult education is ruled out. The number of new entrants who demand education is assumed to depend on the margin, upon the wage rates, and upon the probabilities of employment in skilled and unskilled jobs. This decision is assumed to be as rational as possible in a world of incomplete information.

The supply of educational spaces has two components: central government facilities and community-financed facilities. In the historical model, both supplies are endogenous, depending on the difference between total demand and total supply. Since central governments in labor surplus economies command far greater resources than localities, it is assumed that endogenous responses of central governments are much quicker.

The dynamics of the labor market must reflect institutional realities. For simplicity, wages for skilled and unskilled jobs are assumed to have grown at the same constant exogenous rate, so that the percentage wage differential remains constant. These wages are assumed always to lie above the market-clearing wage. Workers are employed until the marginal product of the last worker hired equals the wage. Consequently, there will always be unemployment. It is assumed that, once a worker obtains a job, that job is his for life. Furthermore, employers hire according to educational attainment, so that educated workers in the unskilled labor market are hired first. In the relevant range, it is assumed that educated workers can immediately obtain an unskilled job for life. Within a skill category, the labor market operates completely randomly, so that the probability of obtaining a job is the ratio of hires to job-seekers. Implicitly, this assumes that there are no vintage effects and therefore that workers trained today are identical with workers trained earlier, skills do not depreciate with disuse or appreciate with experience, older workers are not discriminated against due to a shorter expected lifetime on the job, technical change is disembodied, and there are no quality or ability differences within a skill category.

The social cost of education is equal to the quantity of resources devoted to the maintenance and operation of the education establishment. No output is forgone as a result of having a smaller labor force since training occurs instantaneously. Consumption is defined as the

difference between output and the social cost of training. Social welfare is presumed to depend

positively on consumption, the employment rate, and the fraction of the labor force that has

training. Since these arguments are determined by the model, social welfare is determined

implicitly once the values of all other magnitudes are known.

The Planning Model

The planning model is identical with the historical model, except that the supply of

government schooling facilities no longer follows a fixed endogenous relationship but rather is

under the control of the government. It is the objective of the government to maximize social

utility, defined as the present value of future social welfare, by appropriate choice of the control

variable. The time paths of all other magnitudes are determined implicitly from the social

welfare and government training facility time paths.

A Formal Statement of the Model

The factors of production are capital and two categories of labor, skilled and unskilled.

The quantity of capital available to the economy is determined exogenously. Aggregate output is

given by the production function

$$Q = Q(E_{uu} + p^E_{eu}, E_{es})e^{\theta t}, p > 1$$

where E_{uu} and E_{eu} are respectively the employment of uneducated and educated workers in

unskilled jobs, p is an index of productivity of E_{eu} relative to E_{uu}, E_{es} is the employment of

educated workers in skilled jobs, and θ is an index of the technological level of the economy,

including capital utilization.* Technical change and capital use are neutral with respect to the

different categories of labor. The production function is assumed to be homogeneous of degree

less than one, or

$$Q(cE_{uu} + cpE_{eu}, cE_{es})e^{\theta t} = c^k(E_{uu} + pE_{eu}, E_{es})e^{\theta t}, k > 1,$$

where c is a scale factor. The fact that $\theta < k < 1$ implies that there are diminishing returns to

scale of employment in skilled and unskilled jobs. The homogeneity of the production function

ensures a linear expansion path.**

Wage and Employment Determination

Wages for both groups of workers are set exogenously above the market-clearing rate.

The unskilled wage (W_u) is always equal to some fraction α of the skilled wage (W_s).*** The

unskilled wage and the skilled wage are both assumed to be growing at the same constant rate

(w). Educated and uneducated workers are employed in skilled and unskilled jobs respectively

until the point where the wage for each is equal to the respective marginal product, $Q_E es$ and

Q_{Euu}. The wage for a worker in an unskilled job is assumed to be the Maine whether or not the

worker is educated. Since educated workers are p times more productive in unskilled jobs and

are available at the same wage, they are naturally preferred by employers, and employers hire as

many educated workers as they can for unskilled jobs. These relations may be expressed

algebraically as

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* An index of all symbols used in this paper can be found on the last page.
** See for instance J. M. Henderson and R. E. Quandt, <u>Microeconomic Theory: A Mathematical Approach</u> (New York: McGraw-Hill, 1971) for a proof of this assertion.
*** Throughout this discussion, a bar (-) over a variable denotes that it is set by public policy, a hat (Λ) denotes a rate of growth, and a dot (.) denotes a time derivative.

$$w_u = \bar{w}_u, w_s = \bar{w}_s, w_u = \alpha w_s, \alpha < 1$$

$$\hat{w}_u = \hat{w}_s = w$$

$$w_u = Q_{E_{uu}}, w_s = Q_{E_{es}}$$

$$E_{eu} = L_{eu}$$

Labor Force

The total labor force (L) is assumed to grow at some exogenous constant rate ($\lambda - \delta$) where λ is the gross addition rate and δ the gross dropout rate due to death or retirement. At any point in time, the labor force is composed of three groups: uneducated workers working or seeking work in unskilled jobs (L_{uu}), educated workers working or seeking work in skilled jobs (L_{es}), and educated workers working in unskilled jobs (L_{eu}).

$$L = L_{uu} + L_{es} + L_{eu}$$

Of these workers, the number employed in skilled jobs is E_s and the number employed in unskilled jobs is E_u.

Uneducated workers can work only in unskilled jobs. Educated workers, however, can choose between skilled and unskilled jobs, for which they are overqualified. Since it is assumed that a worker never loses a job he holds, no educated worker who already holds a high-paying skilled job will be interested in a lower-paying unskilled job.[*] However, potential members of

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[*] Nonpecuniary preference for the unskilled job will not motivate a skilled worker to move. Assuming that the nonpecuniary benefits of the job are constant over time, such a worker has already revealed his preference for an uncertain skilled job to an unskilled job with certainty. Once he has a certain skilled job, he can now prefer the unskilled job only if uncertainty is preferred to certainty to such an extent that it outweighs the gain in utility from higher expected income. I assume this is not the case.

the unskilled work force include the newly educated (S_e), old educated workers who are currently unemployed ($L_{es} - E_{es}$), and educated workers working in unskilled jobs (L_{eu}), all of whom are free to choose between an unskilled job with certainty and a chance at a skilled job without certainty.

It is hypothesized that, at any point in time, the net change in the number of educated workers working in unskilled jobs (L_{eu}) is some fraction β of the potential number of overqualified workers minus the number who die or drop out of the labor force. This relation may be expressed as

$$L_{eu} = \beta\{s_e + (L_{es} - E_{es}) + L_{eu}\} - \delta L_{eu}$$

β is a fraction between -1 and $+1$ and is a negative function of the percentage difference between the present value of expected future income from an uncertain skilled job ($V_{es}\,*$) as compared with a certain unskilled job ($V_{eu}\,*$). That is,

$$\beta = \beta(V_{es}\,* - V_{eu}\,*/V_{es}\,*), -1 < \beta < 1.$$

Figure 7.1 illustrates the hypothesized shape of this function.

Insert Figure 1

γ is the fraction of potential overqualified workers who would choose unskilled jobs if the expected incomes were identical.

At any point in time, the number of educated workers in the skilled labor force is the original number plus the total number trained, minus those who have dropped out of the labor force and minus the gross number who have entered the unskilled labor force. This may be expressed as

$$L_{es} = L_s(0) + \int_0^t s_e \, dt - \int_0^t \delta L_{es} \, dt - \int_0^t (L_{eu} + \delta L_{eu}) \, dt.$$

Differentiating (10) with respect to time gives the net change in the number of educated workers

in the skilled labor force at any point in time:

$$\dot{L}_{es} = s_e - \delta L_{es} - \left(\dot{L}_{eu} + \delta L_{eu} \right).$$

That is, the change in the number of educated workers in the skilled labor force is the number of

new school-leavers minus the number who drop out of the labor force less the gross number who

enter the unskilled labor force.

Demand for Education

Education is demanded by individuals who respond to private costs and benefits. To the

extent that education is demanded by society, this is reflected in the supply of government

schooling facilities.

At any point in time, the number of new entrants to the labor force is λL. The fraction of

new entrants who-demand education $(D_e / \lambda L)$ is some positive function of the present value of

the expected percentage income differential of educated workers as compared to uneducated, or

$$(D_e / \lambda L) = \Psi(V_e * - V_u * / V_u *)$$

Insert Figure 2

ϵ is the fraction of new entrants who would demand education if the expected present values

were indentical.

Supply of Education

The total supply of schooling spaces (S_e) is the sum of the central government supply

(S_{e_G}) and the local community supply (S_{e_C})

$$S_e = S_{e_G} + S_{e_C} *$$

Both the government supply and community supply depend on the difference between total

demand and total supply. At any point in time, the number of new community spaces being built

is some fraction η of the difference between demand and supply

$$\dot{S}_{e_c} = \eta\{D_e - S_e\}, 0 < \eta < 1$$

and the number of new government spaces being built is some fraction μ

$$S_{e_G} = \mu\{D_e - S_e\}, 0 < \mu < 1.$$

Ordinarily, μ is larger than η, reflecting the greater availability of resources to the central

government. The numbers μ and η are coefficients of adjustment and vary positively with

society's taste for education, positively with the level of income, positively with the private rate

of return to education, and negatively with the cost of constructing and operating schools. η

varies positively and μ negatively with the amount of subsidies granted by the central

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* The symbol S_e is used for both supply of education and the number of students. This assumes that demand for training always exceeds the supply, so that school facilities are always fully utilized. There is probably a strong ratchet that prevents the closing of existing facilities. The model is proposed only for labor surplus economies in which a situation of excess demand for education will prevail until equilibrium.

government to communities for the construction and operation of schools and μ varies negatively with the amount of the bursaries granted to students by the central government.

(14) and (15) can be combined to give

$$\dot{S}_e = (\eta + \mu)\{D_e - S_e\}.$$

In order to avoid cycles of overbuilding, closing of facilities, and so on, it is assumed that

$$(\eta + \mu) < 1$$

Labor Market Dynamics

As noted previously, it is assumed that, once a worker obtains a job, that job is his for life. Therefore, hiring takes place for two purposes: net employment creation and replacement of labor force dropouts. Educated workers are hired preferentially for unskilled jobs. This may either be because educated workers are more productive than uneducated workers or because employers prefer better-educated workers ceteris paribus. All workers within a skill category are homogeneous, and the labor market operates randomly.

Looking first at the market for unskilled jobs, the number of uneducated workers hired is the difference between total unskilled hires and gross hires of educated workers. The total number of unskilled hires is the number of new unskilled jobs available ($\dot{E}_u$) plus the replacement demand, $\delta(E_{uu} + E_{eu})$, minus the gross hires of educated workers for unskilled jobs, ($\dot{L}_{eu} + \delta L_{eu}$). We assume that there will be fewer educated job-seekers in the unskilled market than there are unskilled jobs, so that all educated job-seekers in the unskilled market are immediately employed. Therefore, $E_{eu} = L_{eu}$, and the number of uneducated workers being

hired is $(\dot{E}_u + \delta E_{uu} - \dot{L}_{eu})$. The number of uneducated job-seekers is the number unemployed

$(L_{uu} - E_{uu})$ Thus, the probability of an unemployed uneducated worker becoming employed is

$$P_u = \frac{\dot{E}_u + \delta E_{uu} - \dot{L}_{eu}}{L_{uu} - E_{uu}}$$

At any point in time, the cohort deciding whether or not to demand education is assumed

to have accurate knowledge only of the current labor market situation. It is further assumed that

the individual takes the current probability of gaining employment and projects that probability

into the future. On the basis of such a calculation, the change in the projected probability of

being employed for an uneducated worker is today's probability of becoming employed projected

into the future multiplied by the projected probability of being unemployed, or

$$\dot{\phi}_u^* = P_u^*(1 - \phi_u^*),$$

where the *'s denote projections based on current figures rather than the unknown actual

probabilities. Solving this differential equation yields an expected probability of being employed

$$\phi_u^* = 1 - e^{-P_u * t}$$

Turning now to the market for skilled jobs, the analysis is similar. The number of new

hires is replacement demand (δE_{es}) plus new jobs $(\dot{E}_S)$. The number of job-seekers is the

educated labor force in the skilled market (L_{es}) minus the number employed (E_{es}). Therefore,

$$P_S = \frac{\dot{E}_S + \delta E_{es}}{L_{es} - E_{es}}$$

As above,

$$\dot{\phi}_s^* = P_s * (1 - \phi_s^*),$$

which solves to

$$\phi_s^* = 1 - e^{-P_s * t}$$

These projected probabilities of employment, ϕ_u^* and ϕ_s^*, are used in the calculation of expected present values of various alternatives.

Expected Present Values

The expected present value of future income for an uneducated worker is

$$V_u \ast = \int_0^\infty W_u(t)\, \phi_u^*(t)\{1 - \Delta(t)\}e^{-rt}dt.$$

$\Delta(t)$ is the probability of having died or dropped out of the labor force at time t; $\{1 - \Delta(t)\}$ is the probability of being in the labor force, r is the discount rate. Substituting and integrating, this solves to

$$V_u \ast = W_u(0)\frac{-P_u \ast}{(w - \delta - r)(w - \delta - r - P_u \ast)}$$

The expected present value of future income for an educated worker is the present value of expected income if he is educated less the present value of the private costs of education. The present value of expected lifetime income for an educated worker ($V_e \ast$) is the expected value of lifetime income if he works in a skilled job ($V_{es} \ast$) times the expected probability of working in a skilled job plus the expected value of lifetime income if he works in an unskilled job times the expected probability of working in an unskilled job. The fraction of skilled workers choosing to work in unskilled jobs is β. (See equation (9).) Therefore, the expected present value of future income for an educated worker is

$$V_e \ast = -PC_e + (1 - \beta)V_{es} \ast + \beta V_{eu} \ast, \quad \text{where}$$

$$V_{es} \ast = W_s(0)\frac{-P_s \ast}{(w - \delta - r)(w - \delta - r - P_s \ast)}$$

$$\text{and} \quad V_{eu} *= W_u(0)\frac{1}{(w - \delta - r)}$$

and PC_e is the present value of the private cost of education.

Social Welfare Function

The social welfare (SW) is a positive function of consumption, the employment rate, and

the fraction of the labor force that is educated:

$$SW = f(C, E/L, L_e/L)$$

Consumption is the difference between aggregate output Q and the resources used in producing

education (cS_e), where c is the unit cost of educating an individual and S_e is the total number

being educated:

$$C == Q - cS_e$$

Social welfare is determined after all other magnitudes in the system.

The Social Utility Function

Social utility (U) is simply the present discounted value of future social welfare

$$U = \int_0^\infty SW(t)e^{-rt}dt$$

taking the current time as time zero. Care should be exercised to avoid confusing the time origin

for the historical model (independence or some other arbitrary date) with the time origin for the

planning model (the present). Bearing this change of time origin in mind, the planning model and

historical models are formally identical except that the adjustment function for central

government schooling spaces, equation (15), is replaced by

(15') Se is controlled in order to maximize U as given by (31). Equation (16) and relation

(17) are in this case no longer relevant and should be dropped.

Thoughts on Simplification

At the cost of less realism, the model can be simplified in order to isolate the time paths

of the educational and labor market variables. Essentially, these simplifications hold the

aggregate size of the economy constant, thereby isolating the variables of greatest interest.

The size of the labor force can be held constant by equating the gross addition rate λ to

the gross withdrawal rate δ. The constant labor force has the advantage of focusing attention on

time derivatives without worrying about rates of growth.

Another simplification is to hold output constant. If θ, the combined effect of neutral

capital utilization and disembodied technical change, is zero, and, if educated workers are no

more productive in unskilled jobs than uneducated workers, i.e., $p = 0$, then changes in output

can result only from changes in the quantity of labor employed. If the rate of growth of wages,

w, is set equal to zero, then marginal products remain constant, so total employment will be

unchanged.

This is perhaps an extreme model, since there is no economic motivation for additional

education. If a greater share of the labor force becomes educated, society gains no extra output

and in fact sacrifices a greater share of its resources in order to educate its young. There remain

two possible noneconomic motivations for an expanding educational sector. First, society may

derive a sufficient gain in social welfare from having a larger fraction of its populace educated to compensate for reduced consumption of other things. Second, private demands may perpetuate a cycle in which a greater fraction of new entrants are educated and the probability of an uneducated entrant obtaining a job becomes smaller, so the private demand increases, supply increases in partial response to demand, and the cycle continues ad infinitum. Thus, although total employment would remain the same, the educational composition of the labor force would be changing. The interesting questions are under what circumstances this cycle would occur and how should government control the supply of schooling in order to maximize social utility.

An alternative extreme model is one that ignores the private demand for education, ignores any gain in social utility from greater education, and considers only economically motivated social demand. In this case, it may be assumed that output would rise, the greater the number of workers in unskilled jobs who are educated. Such a model would effectively treat the growth of community self-help schools as unexplainable and would evaluate the social cost of the resources expended on community schools as zero, assuming that these resources are supplied only for the purpose of constructing and operating such a school and would not be supplied otherwise. The interesting questions again are under what circumstances the supply of schooling would increase and how should government control the supply in order to maximize social utility.

Notation

C	Aggregate consumption
D_e	Demand for education
E	Aggregate employment
E_s	Employment in skilled jobs
E_u	Employment in unskilled jobs
E_{es}	Employment of educated workers in skilled jobs
E_{eu}	Employment of educated workers in unskilled jobs
E_{uu}	Employment of uneducated workers in unskilled jobs
L	Total labor force
L_s	Labor force for skilled jobs
L_u	Labor force for unskilled jobs
L_{es}	Educated workers in labor force for skilled jobs
L_{eu}	Educated workers in labor force for unskilled jobs
L_{uu}	Uneducated workers in labor force for unskilled jobs
P_s	Probability of a skilled worker's becoming employed
$P_s *$	Expected probability of an unemployed skilled worker's becoming employed in the future
P_u	Probability of an uneducated worker's becoming employed
$P_u *$	Expected probability of an unemployed unskilled worker's becoming employed in the future
PC_e	Present value of private costs of education
Q	Aggregate output
$Q_{E_{es}}$	Marginal product of educated worker in a skilled job
$Q_{E_{uu}}$	Marginal product of an uneducated worker in an unskilled job
S_e	Total supply of educational facilities
S_{e_c}	Supply of community-financed education
S_{e_g}	Supply of government-financed education
SW	Social welfare

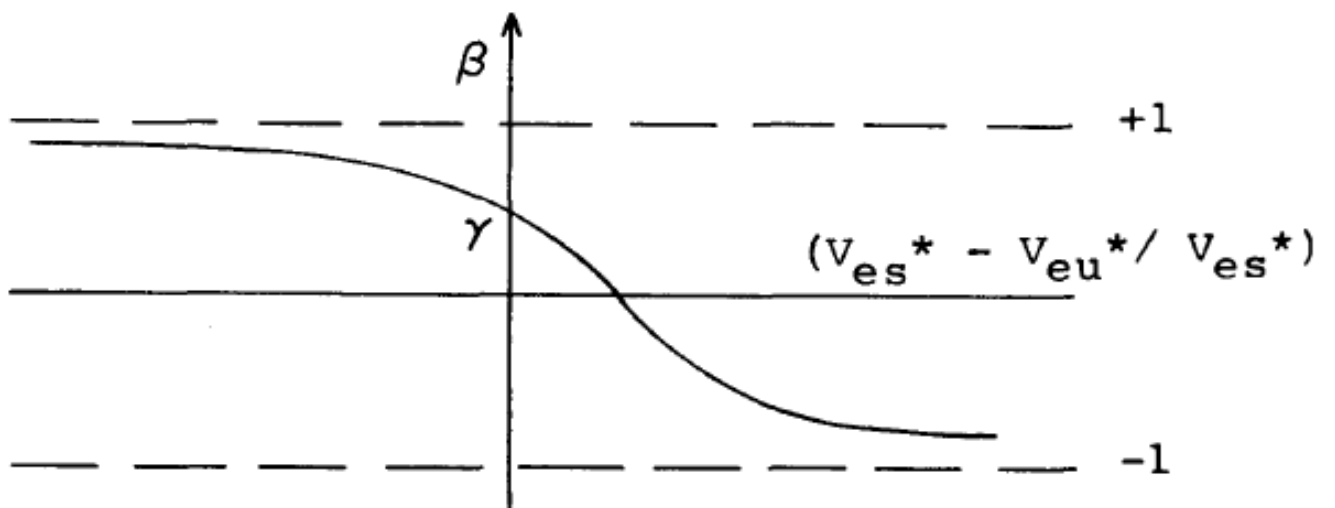|	
U	Social utility
$V_e *$	Expected present value of future income of an educated worker
$V_{es} *$	Expected present value of future income of an educated worker in a skilled job
$V_{eu} *$	Expected present value of future income of an educated worker in an unskilled job
$V_u *$	Expected present value of future income of (unemployed) uneducated worker
W_s	Wage rate for workers in skilled jobs
W_u	Wage rate for workers in unskilled jobs
c	Per pupil resource cost of education
k	Degree of homogeneity of the production function
p	Productivity of educated worker working in an unskilled job relative to an uneducated worker in that same job
r	Discount rate, both private and social
t	Time
w	Rate of growth of wages
α	Ratio of unskilled wage to skilled wage
β	Fraction of educated workers not presently employed in skilled jobs who choose to work in the unskilled labor force
γ	$\beta(0)$
δ	Probability of dying or retiring from the labor force
Δ	Probability of being already dead or retired from the labor force
ϵ	$\Psi(0)$
η	Coefficient of adjustment for community schools
λ	Gross addition rate to labor force
μ	Coefficient of adjustment for government schools
θ	Index of level of technology and capital utilization
$\phi_s *$	Expected probability of an unemployed skilled worker's being employed in the future
$\phi_u *$	Expected probability of an unemployed unskilled worker's being employed in the future
Ψ	Education demand function

Figure 1

FIGURE 7.1

Figure 2

FIGURE 7.2