Absenteeism and the Overtime Decision

By RONALD G. EHRENBERG*

Upon reading the congressional hearing on the *Overtime Pay Penalty Act of 1964*, one cannot fail to be impressed by the emphasis that management places on absenteeism as a primary cause of overtime. The argument given is basically quite simple: Large firms, it is claimed, attempt to account for absenteeism by hiring standby workers; however because of the stochastic nature of the absentee rate, it is impossible for them to have replacements always available. Hence overtime must be worked by existing employees in order to meet production schedules. One concludes from this argument that the randomness of absenteeism is the cause of overtime. If the absentee rate were known with certainty, then management could take account of it without recourse to additional overtime.[1]

In this note we challenge this conclusion and argue that a rational economic response to a certainty absentee rate involves *increasing* the amount of overtime worked per man, while the effect on the level of employment is ambiguous. Furthermore we claim that a stochastic absentee rate leads to a larger optimal employment stock and, in at least one special case, on average, to a *smaller* amount of overtime worked per man than in the certainty absentee rate case. Crucial to our argument is the observation that many of the labor costs which we classify as "fixed" must be paid by the employer even when an employee is absent, but overtime hourly wage payments need not be made to absentees.

Section I presents the structure and assumptions of a simple static model, similar to that found in Sherwin Rosen's paper, when absenteeism is zero. The next section generalizes this model to include a certainty absentee rate. In Section III, we consider the case of a stochastic absentee rate and compare the optimal solution with the results of the previous section. The final section briefly summarizes the results.

I

Given the level of output to be provided, the level of technology, and the flow of capital services, a neoclassical production function can be inverted to determine a unique required flow of labor services. The firm decision problem is then to choose that combination of men and hours per man which will produce that flow, and which will minimize its labor costs. Symbolically, the problem is to:

$$\text{minimize } w_1M + (r+q)TM + w_2M\overline{H} + w_3M(H-\overline{H}) \tag{1}$$

* Department of economics, Northwestern University. Without implicating them for what remains, I am grateful to Dale Mortensen and George Delehanty for comments on an earlier draft. Finis Welch suggested the approach taken in Figure 1, that both generalized the results and simplified the presentation of the original note, which was based solely on the Cobb-Douglas function. Research was supported by a grant from the Manpower Administration of the U.S. Department of Labor under the Manpower Development and Training Act of 1962, as amended.

[1] This conclusion and the argument that follows neglect the indivisibilities inherent in small firms, union rules concerning the existence of stand-by workers, and the heterogeneity and scarcity of skilled labor. While all these factors tend to limit the employment of standby workers they lead us to expect that overtime is positively related to the absentee rate.

$$(2) \quad \text{subject to } L = F(M, H)$$

where (a) $F_1, F_2 > 0$

$$\text{(b) } (2F_1F_2F_{12} - F_1^2F_{22} - F_2^2F_{11}) - (2F_1F_2^2/M) > 0$$

Here w_1 represents those employment costs per man which are fixed in the sense of being independent of the exact number of hours that each employee works. These include the costs for such items as paid vacations, paid holidays, private welfare and insurance plans, and many legally required insurance payments.[2] Some of these costs are annual, others monthly, still others weekly; the assumption here simply being that the employer imputes them to himself on a weekly basis. The next term represents what Rosen and M. Ishaq Nadiri have called the "user cost of labor." T represents the once-over turnover and investment cost per man of hiring and training workers. If these costs are financed by borrowing, they must be discounted by the interest rate, r, and also adjusted for expected replacement costs by the quit rate, q. Assuming that equilibrium occurs in the overtime region, the wage costs per man are the wage rate, w_2, times the maximum number of hours per man payable at straight-time wages, $\bar{H}$, plus the overtime wage rate, w_3, times the number of overtime hours per man, $(H-\bar{H})$.

The constraint (2) asserts that the flow of labor services, L, is a function of the number of men employed, (M), and the number of hours per man, (H). For a number of reasons discussed by Martin Feldstein, it is inappropriate to specify the labor input as being equal to the number of man-hours worked, (MH). Here we assume only that the marginal contribution to labor services of each input is positive over the relevant region, and that the necessary condition (2b) for the optimizing problem to have a solution is met. This condition requires that the marginal rate of substitution of men for hours be a decreasing function of hours, the usual convex isoquant assumption.[3]

Upon minimizing (1) subject to (2), these assumptions lead to an equilibrium combination of men and hours (M^*, H^*) which is a function of all of the parameters in the model. In particular, it is important for later use to note that an increase in the component of cost independent of hours ($w_0 = w_1 + (r+q)T$) increases the marginal cost of labor through additional employment relative to the marginal cost of labor through added hours per worker. Consequently a substitution of overtime hours for employment would occur.

$$(3) \qquad \partial H^*/\partial w_0 > 0, \quad \partial M^*/\partial w_0 < 0$$

II

Suppose we now introduce a certainty absentee rate into the model. That is, the firm knows that at any given day only the fraction "a" of its employees will be in attendance. Then its appropriate labor input function becomes

$$(2') \qquad L = F(aM, H).$$

The inclusion of an absentee rate also modifies the cost function, but in a nonsymmetric way. In particular, the capitalized turnover costs must be paid regardless of whether an employee works on any given day (or week). Similarly many of the fixed employment costs such as health insurance, pension coverage, vacation pay, and unemployment compensation insurance are independent of the

[2] That the magnitude of these costs is not small, can be seen by consulting *B.L.S.* publications such as Bulletin 1428.

[3] Actually condition (2b) is stronger than the requirement of diminishing marginal rate of substitution between factors because a constant budget cost curve is not linear when viewed in (M, H) space. See Rosen (p. 515) for a graphical illustration of this point.

employee's attendance on any particular day.[4] For simplicity we initially assume that they are all independent of attendance. Finally wage costs, for the most part, are paid only to workers actually working, hence the appropriate cost function becomes

$$(1') \quad [w_1 + (r+q)T]M + w_2 aM\overline{H} + w_3 aM(H - \overline{H}).$$

Through a simple transformation of variables, it is easy to see directly what the effect of the certainty absentee rate is on the equilibrium values of men and hours. Let $A = aM$, the number of employees actually working on a given day. Then rewriting (1′) and (2′) the firm seeks to

$$(1'') \quad \text{minimize } [(w_1 + (r+q)T)/a]A + w_2 A\overline{H} + w_3 A(H - \overline{H})$$

$$(2'') \quad \text{subject to } L = F(A, H)$$

Obviously, in terms of the optimal A, H combination, a decrease in a (an increase in the absentee rate $1-a$) has the same effect as an increase in any of the other components of costs that are independent of hours, w_0. That is, an increase in absenteeism increases the marginal cost of labor through additional workers in attendance, relative to the marginal cost of labor through additional overtime. Consequently

$$(3') \quad \frac{\partial H^*}{\partial(1-a)} > 0, \qquad \frac{\partial A^*}{\partial(1-a)} < 0$$

An increase in absenteeism causes a substitution of additional overtime per man for workers in attendance. In general, however, we cannot predict the effect on the equilibrium employment stock. Since $A = aM$ we know that

$$(4) \quad \frac{\partial M^*}{\partial(1-a)} = \frac{1}{a}\,\frac{\partial A}{\partial(1-a)} + \frac{M}{a}$$

While an increase in absenteeism (a decrease in the attendance rate) causes a substitution effect which tends directly to decrease employment (the first term in (4)), it also causes a scale effect since more employees are now required to attain a given level of workers in attendance on any day. Because the substitution and scale effects work in opposite directions, it is impossible to predict what the net effect of a certainty absentee rate is on the number of employees.

Note that the above results will continue to hold even if some of the fixed employment costs, such as daily travel expenses, need not be paid to absentees. Similarly they will hold if some (or all) absentees receive sick leave payments. All that we require is that absentees do *not* receive any pay for overtime hours that they may have been scheduled to work and that some of the employment costs which are independent of hours worked are also independent of attendance.[5]

III

Instead of being known with certainty, we now assume that the attendance rate is a random variable with a probability density $p(a)$ that is symmetric[6] around the mean value, $E(a)$. As in the recent works of Kenneth Smith and Michael Rothschild, we assume that the firm faces a two-stage decision process. The employer

[4] This last statement should be qualified. Often no holiday pay is received unless the employee works the days directly before and after a paid holiday. Similarly unless a minimum number of days are worked, the employee is ineligible for vacation pay and pension credit. Also unemployment compensation insurance costs are man-hour (not man) related unless the employee's annual income is above a certain level. Finally, if the employee is "fired for cause," all of these obligations cease.

[5] This proposition which can be rigorously proved has been omitted for brevity. For notational simplicity we ignore these generalizations in what follows since they in no way effect the results of the next section.

[6] Obviously we also require $P_{(a)} \geq 0$,

$$\int_0^1 P(a) = 1, \qquad \int_0^1 aP(a) = E(a).$$

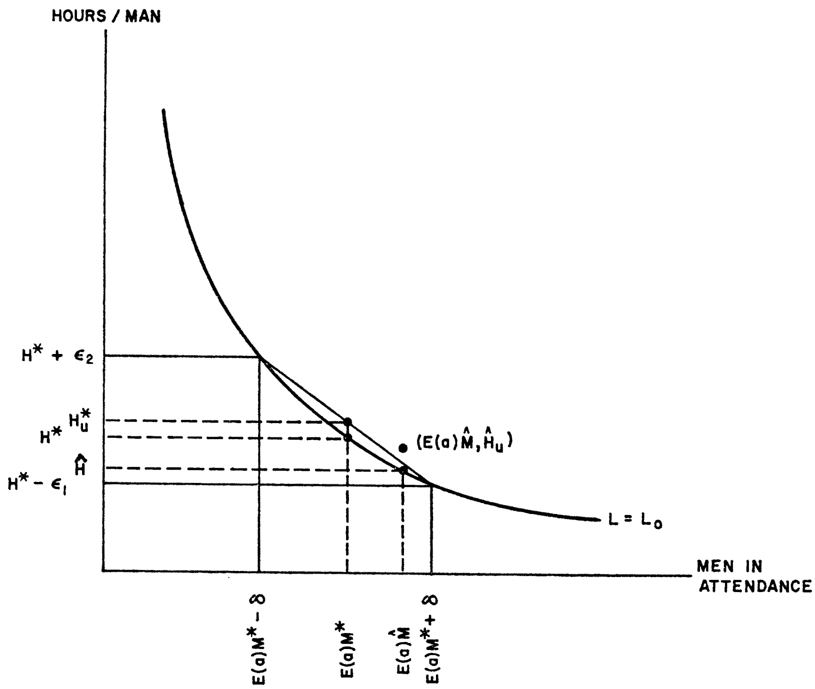

FIGURE 1

must choose the employment stock, M, each period before the actual realized value of the attendance rate is known. Once the attendance rate is observed, then hours per man, H, is determined from the labor services requirement (2′) in order to meet production schedules. The two-stage process seems particularly relevant in this context, in light of our discussion in the opening paragraph.

It is easy to illustrate graphically, that due to the assumption of diminishing marginal rate of substitution between workers in attendance and hours per worker, that random fluctuations of absenteeism about a given mean level serves to *increase* the optimal stock of employees.

Referring to Figure 1, the contour $L = L_0$ indicates the various combinations of workers in attendance and hours per worker which yield the required level of labor services, L_0. Suppose that when absenteeism was nonstochastic that the equilibrium was given by the combination $(E(a)M^*, H^*)$. Now if the attendance rate is stochastic, symmetric fluctuations in worker attendance about $E(a)M^*$ (of size δ for example) will require compensating fluctuations in hours per worker. However, because of the assumed diminishing marginal rate of substitution, negative deviations in attendance require more than proportional increases in hours, while positive deviations require less than proportional increases (in Figure 1, $\epsilon_2 > \epsilon_1$). As a result, if the attendance rate is distributed symmetric about its mean, average hours associated with M^* in the stochastic case (H_u^*) will exceed the level of the certainty case (H^*). If we assume that the objective of the firm is to minimize the expected value of labor costs, then since $(E(a)M^*, H^*)$ represented the point at which the extensive (employee) marginal labor cost was equated to the intensive (hours/worker) marginal labor cost, we know that $(E(a)M^*, H^*)$ cannot be an equilibrium point. Effectively the intensive marginal cost has increased relative to the extensive marginal cost; for a given number of men, a larger number of hours per man are required on average. Consequently, it is optimal to *increase* the number of employees to some level $\hat{M} > M^*$.

Associated with this employment level ($\hat{M}$), there is an optimal level of the expected value of hours per man ($\hat{H}_u$). Due to the stochastic absentee rate and assumption of diminishing marginal rate of substitution between factors, $\hat{H}_u$ is greater than the level of hours per man that would be associated with $\hat{M}$ in the nonstochastic case ($\hat{H}$). While it is certainly true that $\hat{H} < H^*$ and $\hat{H}_u < H^*_u$, the relevant comparison is between $\hat{H}_u$ and H^*. That is, would we observe on average, a greater or smaller level of overtime per man in the stochastic or nonstochastic case? Figure 1 does not give us sufficient information to answer this question and indeed we have been unable to compare these terms for the general class of labor input functions in (2). It appears that the comparison will depend upon properties of the labor input function, such as the elasticity of substitution between the factors, which determine the shape of the isoquant $L = L_0$.

If we consider the special case of the Cobb-Douglas labor input function, we can however uniquely determine the relationship between equilibrium overtime hours per man in the certainty and stochastic cases. That is, we assume

$$(2''') \qquad L = (aM)^\alpha H^\beta, \quad \alpha > \beta^7$$

In addition to its analytic convenience, we may also justify this function's use because it has been employed with success in recent empirical work by Martin Feldstein and Nadiri and Rosen. For this particular function, the solution for equilibrium hours in the case of a certainty absentee rate of $E(a)$ becomes

$$(5) \qquad H^*_c = \left[\frac{w_0 + (w_2 - w_3)\overline{H}E(a)}{w_3 E(a)}\right] \cdot \left[\frac{\beta}{\alpha - \beta}\right]$$

In the stochastic case, the firm knows that once it chooses an employment level M, hours per man will be uniquely determined by the value of the absentee rate that actual obtains in the period, i.e.

$$(6) \qquad H = L^{1/\beta}(aM)^{-\alpha/\beta}$$

We assume that the firm will attempt to choose M, conditional on the value of H in (6), to minimize the expected value of its labor costs. Symbolically, substituting (6) into (1′), the firm seeks to

$$(7) \qquad \underset{M}{\text{minimize}}\; E[w_0 M + (w_2 - w_3)\overline{H}aM + w_3 L^{1/\beta}(aM)^{(\beta-\alpha)/\beta}]$$

The necessary condition for this unconstrained minimization problem is that

$$(8) \qquad E\left[w_0 + (w_2 - w_3)\overline{H}a + \left(\frac{\beta - \alpha}{\beta}\right) w_3 L^{1/\beta} a^{(\beta-\alpha)/\beta} M^{-\alpha/\beta}\right] = 0$$

or that

$$(9) \qquad M^{-\alpha/\beta} = \frac{[w_0 + (w_2 - w_3)E(a)\overline{H}]}{w_3 L^{1/\beta} E[a^{(\beta-\alpha)/\beta}]}\left(\frac{\beta}{\alpha - \beta}\right)$$

Substituting (9) into (6) when the attendance rate takes its mean value $E(a)$ determines the optimal expected level of hours per man in the stochastic case

$$(5') \qquad H^*_u = \left[\frac{w_0 + (w_2 - w_3)E(a)\overline{H}}{w_3 E[a^{(\beta-\alpha)/\beta}]}\right][E(a)]^{-\alpha/\beta} \cdot \left(\frac{\beta}{\alpha - \beta}\right)$$

The optimal expected level of hours per man in the certainty (H^*_c)and stochastic (H^*_u) cases may now be directly compared. Dividing (5) by (5′) yields that

[7] The requirement $\alpha > \beta$ is the second order necessary condition corresponding to (2b) in the general case. This condition will be crucial to what follows.

$$(10)\qquad \frac{H_c^*}{H_u^*} = \frac{E[a^{(\beta-\alpha)/\beta}]}{[E(a)]^{(\beta-\alpha)/\beta}}$$

Since $\alpha > \beta$ is a necessary condition for our solution in (5) to be a relative minimum, we can let $\alpha = K\beta$, where $K > 1$, and obtain

$$\frac{H_c^*}{H_u^*} = \frac{E(a^{1-K})}{[E(a)]^{1-K}}, \quad \text{or rewriting}$$

$$(11')\qquad \left(\frac{H_u^*}{H_c^*}\right)^{1/(K-1)} = \frac{[E(a^{1-K})]^{1/(1-K)}}{E(a)}$$

Using Holder's inequality, it can be shown that the right-hand side of (11′) is always less than unity and hence[8]

$$(12)\qquad H_u^* < H_c^*.$$

In this case the expected level of overtime per man is *less* than when the absence rate is known with certainty.

IV

Summarizing our results briefly, contrary to popular belief, it is not always the stochastic nature of absenteeism which is responsible for increased overtime hours per man above the zero absentee level. This is due to the fact that a certainty absentee rate modifies the labor cost function in a nonsymmetric way so as to increase the marginal cost of labor purchased through additional workers relative to the marginal cost of labor purchased through increased hours per man. Although a certainty absentee rate causes this substitution of hours per worker for workers, the net effect on the employment stock is ambiguous since the scale effect of increased absenteeism tends to increase employment.

A stochastic absentee rate tends to increase the optimal employment stock above the certainty absentee rate level. While the effect on the expected level of hours per man has not been determined in general, for the special case of the Cobb-Douglas labor input function it is shown that the optimal level decreases. That is, observed overtime hours per man would be lower in the stochastic than nonstochastic case.

[8] See G. H. Hardy, et al., (pp. 134–45) for a statement and proof of Holder's inequality and relevant corollaries. In particular it is shown that for t any real numbers, $[E(x^t)]^{1/t}$ is a monotonically increasing function of t. Note that if we define variability more generally in terms of a "mean preserving spread," then M. Rothschild and J. Stiglitz have shown that for any convex function $f(a)$, the expected value $E(f(a))$ rises when the variability increases. Since a^{1-K} is convex for $K > 1$, it immediately follows from (11′) that an increase in the variability of absenteeism decreases the optimal level of overtime per man.

REFERENCES

M. S. Feldstein, "Specification of the Labor Input in the Aggregate Production Function," *Rev. Econ. Stud.*, Oct. 1967, *34*, 375–86.

G. H. Hardy, J. E. Littlewood, and G. Polya, *Inequalities*, Cambridge 1934, pp. 134–45.

M. I. Nadiri and S. Rosen, "Interrelated Factor Demand Functions," *Amer. Econ. Rev.*, Sept. 1969, *59*, 457–71.

S. Rosen, "Short-run Employment Variations on Class-I Railroads in the U.S., 1947–63," *Econometrica*, Oct. 1968, *36*, 511–29.

M. Rothschild, "Changing Demand: Its Costs and Consequences." Paper presented at the December 1968 meetings of the Econometric Society.

——— **and J. Stiglitz**, "Increasing Risk: A Definition and Its Economic Consequences," Cowles Foundation Discussion Paper No. 275, 1969.

K. Smith, "The Effect of Uncertainty on Monopoly Price, Capital Stock and Utilization of Capital," *J. Econ. Theor.*, June 1969, *1*, 48–59.

U.S. Bureau of Labor Statistics, *Employer Expenditures for Selected Supplementary Compensation Practices, for Production and Related Workers, Manufacturing Industries 1962*, Bull. 1428, Washington 1965.

U.S. Congress, hearings before Subcommittee on Labor of the House of Representatives, *Overtime Pay Penalty Act of 1964*, 88th Cong., 2nd Sess., Washington 1964.