

# Challenges Facing “Pay-What-You-Can-Afford” Tuitions at Public Universities

Gary Fethke\*

## ABSTRACT

Pay-What-You-Can-Afford (PWYCA) tuition structures are a suggested way to offset declines in state appropriations to public higher education, without limiting access for qualified low-income students. The concept implements high tuition, high aid tuitions routinely employed at private universities, effectively replacing private endowment income with state appropriations. The idea involves setting resident tuitions at non-resident rates minus the state appropriation per resident, and then using the subsequent incremental tuition revenue to subsidize low-income residents. This fairness-based approach in public higher education ignores: i) how non-resident tuitions are determined; ii) the welfare effects of introducing inefficient distortions in relative demand patterns; and iii) the likely accompanying decline in state appropriations. Here, the tuition-setting rules associated with PWYCA are derived, rather than being imposed, as solutions to a welfare-maximizing model in which non-residents pay fully-allocated costs and state appropriations are endogenous. University of Michigan budget, enrollment, and tuition data are used to illustrate the implications of selecting alternative high tuition, high aid tuition structures. Because demand-side inefficiencies are introduced, both welfare and, importantly, the state appropriation decline with implementation of PWYCA. While the decline in welfare is modest, the redistribution of value among residents, non-residents, and state taxpayers is substantial.

**Key Words:** Pay-What-You-Can-Afford tuitions, Tuition subsidies, High tuition-high aid, Non-resident full-cost tuitions, Appropriations to higher education, Higher education finance, Tuition structure efficiency

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## I. Introduction

While a few selective public universities have access to significant private support, most have just two primary sources of revenue, tuition revenue and state appropriations. Over the past several decades, there has occurred well documented increases in tuition revenue accompanied by decreases in the levels of state funding; overall, the combined level of support has decreased at a moderate rate; see College Board (2016) and SHEEO (2016). With declining state funding and rising tuition, a key challenge becomes how to provide access to public higher education for lower-income students. Consequently, much attention is directed to identifying ways of setting tuitions and determining the enrollment mix that will bring in sufficient revenue to replace state support, without eliminating access for qualified, lower-income resident students.

Approaches to accommodate access are increasingly based on charging higher tuitions to those with identified ability to pay, while providing high aid to those with low ability to pay; this means-tested approach is often called “high tuition, high aid.” While these practices might be motivated as a way to reduce income inequality<sup>1</sup>, their usual stated rationale is to broaden the background, diversity, and cultural mix of the student body. There are many ways to implement this approach when setting tuitions. It is regular practice, for example, to “set aside” a proportion of increased tuition revenue as a quid pro quo to support grants and subsidies for low-

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<sup>1</sup> As put by economist, Robert Solow: “You don’t have to be a card-carrying utilitarian to believe that taking a dollar from a random rich person and giving it to a random poor person would lead to a better social state.”([https://www.researchgate.net/publication/263245613\\_The\\_One\\_Percent](https://www.researchgate.net/publication/263245613_The_One_Percent)). Accessed August 9, 2017. While Solow is presumably not suggesting that changing relative prices is the best way to reduce inequality, this perception of social justice influences much of the current tuition-setting agenda, and is a guiding principle of high tuition, high aid.

income students; a frequently mentioned set-aside rate is 20 percent.<sup>2</sup> Tuition waivers for low-pay residents are common: University of California System’s Blue and Gold Opportunity Plan; Indiana’s 21<sup>st</sup> Century Scholars Program; the University of Michigan’s recent commitment to provide a four-year tuition waiver for qualified resident students with family income below the state median (\$65,000); and New York’s Excelsior Scholarship, which offers “free tuition” for four years to students whose family income does not exceed \$125,000.<sup>3</sup> As many as twenty states are currently considering not charging tuition at community colleges

( <https://www.universitybusiness.com/news/free-college-keeps-growing-arkansas-indiana-montana-and-rhode-island-join>), with California recently waiving community-college tuition for full-time, first-year students(<http://money.cnn.com/2017/10/16/pf/college/california-free-community-college/index.html>).

One articulated version of high tuition, high aid is called: “Pay What You Can Afford (PWYCA).”<sup>4</sup> This approach suggests specific rules for determining resident tuition and allocating the state appropriation: i) set **all** tuitions at non-resident rates; ii) allocate the state appropriation uniformly to every resident; and iii) use the associated incremental tuition revenue to subsidize those with limited ability to pay. With the incremental revenue being used to reduce

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<sup>2</sup> For examples of state set-aside programs, see: <https://www.legis.iowa.gov/docs/publications/SD/15858.pdf>. There has developed a reluctance by some legislators and full fee-paying students to supporting tuition set-back programs, which have been referred to as “a 20 percent backdoor secret tax” or even as a form of “theft”; see <https://www.texastribune.org/2011/02/09/texas-legislators-look-for-end-to-tuition-set-aside/>.

<sup>3</sup> University of California Blue and Gold Opportunity Plan (<http://admission.universityofcalifornia.edu/paying-for-uc/glossary/blue-and-gold/>); 21<sup>st</sup> Century Scholars Program Indiana (<https://secure.in.gov/21stcenturyscholars/2525.htm>); the University of Michigan’s tuition waiver plan (<https://www.michigandaily.com/section/administration/university-guarantees-free-tuition-students-family-income-less-65000/>); and the New York Excelsior Scholarship (<https://www.ny.gov/programs/tuition-free-degree-program-excelsior-scholarship>)

<sup>4</sup> The concept has been promoted by Robert Birgeneau, chancellor of UC Berkeley (2004-13), and Mary Sue Coleman, president of the University of Michigan (2003-14), who currently are co-presidents of the Lincoln Project on Higher Education supported by the American Academy of Arts and Sciences; see <https://www.amacad.org/content/Research/researchproject.aspx?d=929>.

tuition for lower-income students, the concept is therefore not being suggested as way to increase university net revenue; it is offered as a way to enhance access for public universities that can attract large numbers of non-resident students. The concept is suggested as being specifically applicable to the University of Michigan (UM), where the claim is made, in 2016, that PWYCA will generate \$90 M of incremental resident tuition revenue, which can be used as a cross subsidy; see the discussion provided by the American Academy of Arts and Sciences (2016).<sup>5</sup>

Recognizing that the structures of tuition and the level of state support are the results of decisions by legislatures, universities, and students, the PWYCA approach is incomplete. First, no attempt is made to determine enrollment responses to the suggested change in the tuition structure; presumably, there will be decreases in high-pay enrollments and increases in low-pay enrollments; these adjustments will differentially affect budget allocations across programs. Second, there is no discussion of what motivates the setting of non-resident tuitions, and whether there are associated and ignored interactions in the demand and cost structure with resident tuitions. Third, there is the problematic assumption that the state appropriation will not adjust to the way the tuition structure is altered. But, why will legislatures provide the same funding if the university increases its net tuition revenue? Is the presumed answer that additional tuition revenue will not be captured by the university but, instead, is used only to subsidize low-income students consistent with legislative preferences?<sup>6</sup>

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<sup>5</sup> The median family income for students at the University of Michigan is \$154,000, which is the highest reported among major public universities surveyed by the New York Times; 66 percent of the students at UM come from families with incomes in the top 20 percent and 3.6 percent come from the bottom 20 percent; see <https://www.nytimes.com/interactive/projects/college-mobility/university-of-michigan-ann-arbor>; accessed August 6, 2017.

<sup>6</sup> Milton Friedman (1968) argues that subsidies to public higher education can be best explained as the result of lobbying by special interest groups (“pleading”). In particular, if those currently receiving a taxpayer-supported subsidy are required to pay higher tuitions, will they accept paying the same taxes? Ronald Ehrenberg argues, for example, that “there’s always (a) sort of fear that if you raise tuition you’re going to make state appropriations go away even faster.” (<https://www.insidehighered.com/news/2009/04/01/new-strategy-wisconsin>)

This paper adapts a model developed by Burer and Fethke (2016) and extended by Fethke (2017) to represent welfares of students, a public university, and a state legislature to examine situations like high tuition, high aid. The motivation is to examine the use of tuitions as prices in the economic sense, where their role is to ration the supply of higher education service among different academic programs; also see Nerlove (1972). I measure social welfare by the monetary value students receive after paying tuition and fees (student consumers' surplus) minus the state appropriation. Thus, the main idea is to identify tuition structures and associated appropriations that are as economically efficient as possible, given the mitigating presence of fixed-costs. The university is constrained to breaking even; non-resident students are required to pay at least the fully-allocated costs of their programs; and the state appropriation is endogenous. The model is calibrated to replicate standard public university budget templates, and the numerical representations permit the asking of "What If?" type questions.

One prominent solution to the model develops where non-residents pay **exactly** fully-allocated costs and residents pay fully-allocated costs minus a per-resident appropriation; thus, the derived tuition-setting rules are those suggested de novo by PWYCA. This result implies that PWYCA tuition-setting rules are consistent with welfare maximization, but only when the appropriation is endogenous. The model is applied to University of Michigan data.<sup>7</sup> When full-cost restrictions are placed on non-resident tuitions, the primary empirical implication is that increases in average resident tuition are *always* accompanied by decreases in other revenues. In a welfare-maximizing setting, constraints that make the tuition structure less efficient always lead

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<sup>7</sup> Fethke (2017) considers related issues and applications for the University of Iowa and Iowa State University, where it also appears that the tuition structure is determined such that non-residents pay more than fully-allocated costs and therefore subsidize both resident students and state taxpayers.

to reductions in the optimal appropriation. Simply put, when the appropriation and the tuition structure are endogenous and non-residents are required to pay at least fully-allocated costs, there is no incremental revenue to redistribute.

Section II provides numerical solutions for six scenarios: i) a demand and cost calibrated base-level case that replicates UM 2013-2014 budget allocations; ii) pay-what-you-can afford numerical examples; iii) a symmetric-preference case with non-residents paying fully-allocated costs; iv) asymmetric-preferences, where residents' welfare is favored, v) an enrollment cap imposed on non-residents, and vi) unrestricted tuitions. Section III contains a summary. Section IV (the appendix) presents the optimization model and provides solutions for particular specifications of constrained tuition structures.

## II. Numerical representations of tuition structures

**Base Case:** Table 1 presents a calibrated demand and cost structure for determining tuitions and enrollments for the University of Michigan (UM), using budget allocations, tuitions, and enrollments for FY 2013-14.<sup>8 9</sup> Demand curves for multiple programs and two residency statuses are calibrated assuming that the initial tuition elasticity of demand is  $-.5$ ; see the

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<sup>8</sup> These data represent budget allocations for the General Education Fund and exclude revenues and expenditures that are independently generated at the college level; thus they exclude private donations and external program fees and expenditures. At UM, with an endowment of nearly \$10 Billion, the exclusion of private donations is an important omission, since the annual flow of endowment income exceeds the state appropriation. Conceptually, the major proportion of private donations- upwards of 60 percent- support fixed expenditures and thus can be netted out; donations that support variable expenditures can be handled by treating them equivalently to the state appropriation. Vedder and Strehle (2016) urge the use of endowment income to reduce tuition, and this will occur in the suggested formulation.

<sup>9</sup> Historical programmatic budget allocations are available at University of Michigan (2016). For the ten-year period 2007-2016, real net tuition per FTE for all public universities in Michigan grew at a compound average annual rate of 4.4percent, while the real appropriation's growth rate was  $-2.0$  percent. The geometric mean of the ratio of tuition revenue to the appropriation per FTE was 2.1 percent. These rates were computed by the author using data from SHEEO, which is accessible at: <http://www.sheeo.org/projects/shef-percentE2percent80percent94-state-higher-education-finance>; accessed, June 26, 2017.

appendix for the parameter values and the detailed calibration procedure.<sup>10</sup> The resulting tuitions and enrollments are those provided in Table 1. With demand curves calibrated, net consumer surplus (the monetary value students receive after tuition is paid) can be measured for each program. I assume that the budget allocations to the academic programs are variable expenditures, with constant marginal costs calculated for each program. Fixed costs are given by shared-service expenditures.<sup>11</sup>

The allocated budgets ranked from largest to smallest are presented in Column 2. The variable expenditures allocated to all programs, \$993M, plus fixed costs, \$504 M, equals the sum of tuition revenue, \$1.218 M, and the state appropriation, \$279 M. Enrollments (measured by winter term student head counts) are presented in Columns 3 and 4. Using list tuitions for each program, the reported tuitions in Table 1 are normalized values such that average of the enrollment-weighted tuitions matches the reported value of \$29,589. Tuitions for each program are presented in Columns 5 and 6. For example, the annual resident and non-resident tuitions in the College of Literature, Arts and Sciences (LA&S) are \$14,308 and \$45,213, respectively. In Column 7, the non-resident-resident tuition ratio is presented for each program, with an enrollment-weighted average of 2.9. The average tuition gap between non-resident and resident

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<sup>10</sup> The consensus estimate of demand responsiveness in higher education is that a \$1,000 change in tuition is associated with a 3-5 percentage decrease in enrollment (Kane 2006). Thus, a \$1,000 increase in average UM tuition implies a percentage increase of 3 percent and a tuition elasticity between -1 and -.6. Recent estimates by Deming and Walters (2017), using data dominated by non-selective public institutions from 1990 to 2013, find that raising tuition has no discernable effect on enrollment. The presence of vertical demand curves obviates any need to discuss the rationing role of tuition.

<sup>11</sup> The primary motivation is to calibrate and simulate the implications changing demand and cost parameter in the context of familiar budget templates. The simple linear specifications of demand and cost functions permit derivation of explicit, easy-to-understand, and intuitively appealing, tuition-setting rules. All subsequent comparisons using the optimization model are measured relative to the base-level expenditures, enrollments, and tuitions, as given in Table 1. Since base-level parameter estimates exactly replicate the budget data, they permit comparisons of alternative tuition structures that can be assessed relative to a familiar base budget, enrollment, and tuition structure.

tuition of \$29,746 significantly exceeds the average appropriation per resident of \$12,130.

Welfare of \$939M is measured by total consumers' surplus minus the state appropriation.

### **Table 1 Here**

From these calculations, it appears that non-resident tuition revenue and the state appropriation are used to accommodate reduced resident tuitions.<sup>12</sup> The contributions toward covering fixed costs are: -\$160 M for residents, \$385 M for non-residents, and \$279 M for the appropriation. If resident tuitions are set at marginal costs, without any change in the appropriation, the non-resident contribution will decline from \$385 M to \$225 M. There is considerable variation in total contribution among the colleges, with LA&S contributing \$140.1 M, while Medicine contributes -\$23.3 M. Cross subsidies are apparent in the base-case scenario, basically non-residents subsidize residents, and the large, primarily undergraduate, colleges subsidize graduate professional programs.

**Pay-What-You-Can-Afford examples:** PWYCA assumes that non-resident tuition revenue and the state appropriation are given, and then sets resident tuitions equal to non-resident tuitions minus the per-resident allocation of the given appropriation. By having some residents pay more, the idea is to generate tuition revenue to subsidize low-income students. This case can be illustrated using the UM cost and demand parameters. The enrollment solution for tuition-paying residents is 14,792, with every tuition-paying resident receiving a subsidy of \$18,869, exhausting the \$279.1 M appropriation; see the appendix for the calculation details. The resulting incremental net tuition revenue exceeds fixed costs by \$171.9 M. The idea is to use this

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<sup>12</sup> Groen and White (2004) empirically examine several politically-motivated reasons they suggest for treating non-residents differentially, and they conclude from these specifications that public universities over-charge non-residents.



net revenue to subsidize low-income students. For example, using enrollment-weighted marginal cost of \$24,126, this additional revenue can support 7,127 low-income residents; this action requires some sort of rationing decision to determine which low-income students are admitted. The key assumption, however, is that the added tuition revenue is used to accommodate low-income students to meet the break-even outcome.

To address the enrollment rationing issue, a more complete version of the PWYCA structure is computed, and the budgetary implications are presented in Table 2. Here, the value students receive after paying subsidized tuition is the criteria used for rationing enrollment. Two groups of residents are depicted: high-pay residents, who are charged full tuitions, and low-pay residents, who face discounted tuitions. For a given appropriation, there exists a rate of tuition discount to low-pay residents that permits the university to break even. In Table 2, the discount rate and resident enrollments are determined such that the appropriation is determined at its base-level amount.<sup>13</sup> The solution is found by determining the rate discount and high-pay resident tuitions such that: i) the legislative budget constraint binds for the given appropriation, and ii) high-pay resident tuitions are charged non-resident tuitions for each program minus a uniform appropriation per resident. When these conditions hold, resident enrollments and welfare can be determined for every program; see the appendix for details.

Under tuition rationing, resident enrollment increases from the base-level enrollment of 22,319 to 27,198, with the uniform discount rate assigned to low-pay resident tuitions being 72 percent. Here, 8,420 residents pay high tuitions, while 18,778 residents pay discounted tuitions. For example, in LA&S, non-residents pay \$45,213, high-pay residents pay \$34,951, and low-pay

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<sup>13</sup> It's plausible that the legislature expresses a preference tradeoff between the rate of tuition discount to low-pay residents and the size of the appropriation, with a higher rate of tuition discount being associated with a larger appropriation.

residents pay \$9,799, implying a discount of 72 percent.<sup>14</sup> The overall average tuition declines from \$29,812 in the base case to \$29,224. The budget allocations in Table 2 use the base-level marginal cost estimates and the calculated PWYCA enrollments. For example, marginal cost in LA&S is \$18,695, and calculated PWYCA total enrollment is 20,780, implying a budget allocation of \$388.5 M. Since both high- and low-pay tuitions are pushed further away from marginal costs, welfare declines by 5 percent from its base level. If the appropriation is determined to be lower than the base level, the achievable rate of discount to low-pay residents is reduced. For example, if the appropriation is reduced from \$279.1 M to \$104.3 M, the discount rate declines from 72 percent to 50 percent, even as welfare increases from \$880.8 M to \$943.1 M (6 percent). The welfare increase results because subsidizing low-pay residents, while arguably a fair thing to do, distorts relative demands and reduces economic efficiency; a reduced discount implies higher *total* welfare.

### **Table 2 Here**

The efficacy of all high tuition-high aid programs depends on the assumption that enrollment demand for high-pay residents is relatively inelastic. In calculating demand parameters, it is assumed that the initial elasticity of demand for every resident program is  $-0.5$ . If the initial program elasticities are instead assumed to be  $-0.25$ , the appropriation sustaining rate of discount increases to 105 percent. Then, 52 percent of the residents pay full resident

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<sup>14</sup> In the latest Department of Education data (2012-13), which is collected by the Education Writers Association and presented in Tuition Tracker <http://www.ewa.org/tuition-tracker>, about 16 percent of UM admitted freshmen are classified “low-income.” The 288 admitted students with family income in  $[0, \$30,000]$  reportedly paid about 29 percent of the reported sticker price.

tuition, while 48 percent of the residents pay no tuition and receive a subsidy of \$2,062, which can be used to pay other expenses.

This particular implementation of PWYCA determines resident enrollment when non-resident tuitions and the appropriation are given. When the appropriation is instead considered to be endogenous, the university's budget constraint provides budgetary discipline.<sup>15</sup> With an endogenous appropriation, if there is an increase in non-resident tuition revenue, resident tuition revenue and the state appropriation needed for the university to breakeven will decline. When allowed to adjust to PWYCA tuitions, the appropriation declines from \$279.1 M to \$82.8 M; the discount rate declines from 72 percent to 46 percent; and welfare increases from \$880.8 M to \$944.5 M.<sup>16</sup> The recent increases in non-resident and international student enrollments at major flagship public universities are indeed associated with declines in state taxation capacity.<sup>17</sup> Presumptions made in determining tuitions that take both non-resident tuitions and the state appropriation as given are conceptually problematic, and both are removed in the next section.

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<sup>15</sup> The university break-even constraint provides budgetary discipline in the sense that it ensures the participation of the legislature in a principal-agent context. Specifically, the purpose proposed here for the state appropriations is to counter inefficient tuition structures that arise in the presence of fixed costs. In the absence of effective competition, the break-even requirement imposed by the legislature prohibits the university from setting tuitions that exceed average costs. The PWYCA rules, in contrast, allow the university to capture and then discretionarily use incremental net revenues; effectively, this action violates the break-even requirement, and it raises the basic question of whether it's reasonable to take the state appropriation as exogenous.

<sup>16</sup> This solution is a local optimum that holds for discount rates to low-pay residents between 35 percent and 55 percent; see the appendix for a discussion of the PWYCA case with an endogenous appropriation.

<sup>17</sup> Bound, et al. (2016) provide convincing evidence for the period 1996-2012 that high-quality, research-intensive public universities (for example, UC Berkeley, University of Michigan, University of Colorado, and University of Iowa) are able to attract non-resident and international enrollments (primarily from China) of sufficient quality to offset declines in state support. They argue that there is an elastic supply of international students of sufficient quality willing to pay non-resident rates. A related investigation by Jaquette and Curs (2015) for the period 2002-2012 also supports the claim that distinctive research universities possess a financial incentive to recruit non-residents. Alternatively, Rizzo and Ehrenberg (2004) provide evidence for the period 1988-2000 that non-residents are attracted primarily for quality rather than financial reasons; their results, however, might reflect a sample period featuring more generous state support.

**Non-residents pay fully-allocated costs:** The basic motivation for PWYCA is to increase tuition revenue for high-pay residents and use it to subsidize low-pay residents. There is no discussion of how to determine non-resident tuitions. One approach, with considerable legal backing at the state level, is to argue that non-residents be required to pay at least the full cost of their education.<sup>18</sup> A model that includes these legally-imposed constraints is presented in the appendix. The objective is to maximize the weighted value students receive from a public education net of tuition and fees and the state appropriation (consumers' surplus net of the appropriation), subject to several constraints. The first constraint is the university break-even condition, which requires the sum of net tuition revenue and the state appropriation to equal fixed expenditures. The second constraints are legally-imposed requirements that non-residents pay at least fully-allocated costs. The third constraint restricts the total amount of the state appropriation. The model's parameters are: maximum willingness to pay and substitution parameters that define program-level demand curves, constant marginal costs, shared fixed costs, an exogenous limit imposed on the state appropriation, and the relative weight assigned to non-residents' welfare in the objective. Prominent features of the tuition-setting structure are derived analytically. To accommodate those cases that lack closed-form solutions, the model is solved numerically using the UM data.

**Symmetric weighting of welfare with non-residents paying at least fully-allocated costs:** When student welfares are equally weighted in the objective but non-residents are required to pay at least fully-allocated cost, a unique solution to the optimization model occurs

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<sup>18</sup> State codes in Florida, Iowa, Illinois, and West Virginia, for example, require that non-residents pay at least the full cost of their educations. Guidelines in many states for determining non-resident tuition often use the full cost of undergraduate education as a reference point; see state codes for the Pennsylvania State System (<http://www.passhe.edu/inside/policies/Pages/Board-of-Governors-Policies.aspx>) and the Florida System ([http://www.leg.state.fl.us/Statutes/index.cfm?App\\_mode=Display\\_Statute&URL=1000-](http://www.leg.state.fl.us/Statutes/index.cfm?App_mode=Display_Statute&URL=1000-)).

where the tuition-setting rules are those imposed de nova by PWYCA. Specifically, the following results maximize constrained welfare: i) resident tuitions equal fully-allocated costs minus the endogenously determined appropriation per resident; ii) non-resident tuitions exactly equal fully-allocated costs; and iii) the optimal appropriation is less than its exogenous (base-level) amount. Since the intent of the appropriation is to offset fixed costs and to facilitate efficient pricing, once non-residents pay full cost, it is not efficient to charge them more. Once non-residents are charged no more than full cost, the endogenously determined state appropriation declines and resident tuitions increase.

In Table 3, using the UM data, the solution indicates that resident tuitions equal fully-allocated costs net of the optimal appropriation per resident, which is \$13,938, as compared to the base amount of \$29,746. However, the intended increase in net resident tuition revenue associated with PWYCA is accompanied by decreases in both non-resident net tuition revenue and the appropriation. The optimal appropriation of \$264 M is less than the base amount of \$279 M. Total enrollment declines from 41,158 to 38,721, and there is a \$62 M increase in welfare; increases in non-resident and legislative welfare exceed the decrease in resident welfare. Generally, non-resident tuitions decrease and resident tuitions increase from the base case. For example, resident and non-resident tuitions in LA&S are \$17,771 (\$14,308) and \$31,709 (\$45,213), respectively, with base rates in parentheses. These numerical results support the notion that non-residents are being charged more than fully-allocated costs in the base-case. If they weren't, as previously noted, there is no basis to argue for PWYCA.

The claim made for PWYCA tuitions is that incremental resident tuition revenue will result that can be used to subsidize low-income residents. With an endogenous appropriation and non-resident tuitions equaling fully-allocated cost, the break-even implies that net tuition

revenue plus the appropriation equal fixed cost. Effectively, there is no slack revenue to offset tuitions for low-income students. There is an increase in resident tuition revenue of \$24 M, but this gain is offset by declines of -\$103 M in non-resident revenue and -\$15 M in the appropriation. With the decline in resident enrollments more than offsetting non-resident enrollment gains, there is a corresponding drop in total expenditures. Declines in both total tuition revenue and the appropriation are not the intent of PWYCA, which assumes there are no endogenous changes in either non-resident tuitions or the state appropriation. Relaxing these restrictions, however, implies that reductions in non-resident tuition revenue and the appropriation will ensure continued satisfaction of the university break-even condition, effectively eliminating the cross subsidy of low-income residents.

### **Table 3 Here**

Even with increases in fixed cost, the constraints on non-resident tuitions will bind for every program. For example, an increase in fixed cost of \$25 M reduces total enrollment by 70, reduces welfare by \$26.8 M, and causes the legislative budget constraint to now bind at \$279.1 M. When the legislative budget binds, the PWYCA rule for determining resident tuition will not hold. Here, the results are quantitatively insensitive to this (“small”) change in fixed cost. The average tuition gap between resident and non-resident tuition becomes \$14,692, and presents a narrow range across the programs of \$14,677 to \$14,702.

In extending this case, I believe that high tuition, high aid pricing formulations used in private universities can be similarly described by replacing state support with endowment

income. Plausibly, similar issues of tuition revenues crowding out private donations arise if ever higher sticker prices paid by some are used to subsidize the low tuitions of others.<sup>19</sup>

**Asymmetric preferences:** Reduced the weighting of non-resident preferences in the objective implies that non-residents are considered as a revenue source that can be used to subsidize low-income residents. In Table 4 non-residents' welfare is valued at 75 percent of residents'; otherwise, the optimization problem remains unchanged. Here, increases in tuition revenues are more than offset by associated decreases in the appropriation, with total revenue declining by -\$50.3 M. Compared to the symmetric preference case, average non-resident tuition increases to \$45,041 (\$37,176). Resident and non-resident tuitions in the LA&S are \$18,915 (\$17,771) and \$45,252 (\$31,709). In three colleges (LA&S, Business, and Engineering), non-residents now pay tuitions that exceed fully-allocated costs, with the legal constraints binding for the remaining programs. By inefficiently increasing non-resident tuition, the appropriation required to meet the break-even constraint declines from \$264 to \$123 M. In effect, non-resident tuition revenue "crowds out" state appropriations, reducing total revenue. Welfare declines from \$1,001 M to \$760 M (-24 percent).

#### **Table 4 Here**

Full privatization (complete elimination of the appropriation) occurs when non-residents' welfare is weighted at 63 percent of residents' welfare. Average resident tuition is \$24,468 (\$21,309) and average non-resident tuition is \$51,732 (\$37,176), again with the symmetric-case in parenthesis. Two sources of inefficiency are apparent: requiring non-residents to pay more

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<sup>19</sup> A common complaint I've heard from other deans of professional schools is that successful private fundraising is often "taxed" by offsetting reductions in centrally-allocated budgets. Others have indicated to me that large increases in list tuitions at public universities do indeed put off some private donors. These antidotal claims suggest to me that all sources of revenue should best be considered as endogenous.

than fully-allocated costs, and the discounting non-resident welfare in the objective. While doing so can eliminate state funding, the efficiency loss is considerable.

**A cap placed on non-resident enrollment:** Some states impose a cap on non-resident enrollments. A recent example is the California Legislature’s decision in 2017 to cap UC’s non-resident enrollment at 18 percent; see: <https://www.usnews.com/news/best-states/california/articles/2017-05-18/uc-regents-to-consider-audit-out-of-state-enrollment-cap>.

Clearly, the UM does not similarly discourage non-residents and international students, with base-case non-resident enrollments being 44 percent of the total.<sup>20</sup>

When a non-resident enrollment cap of 50 percent is imposed, the UM results are presented in Table 5. With no other legislative mandates (except the standard break-even requirement), the cap expands non-resident enrollment to the 50 percent limit, and average non-resident tuition declines from the base-case level of \$46,218 to \$37,461. Average resident tuition increases from \$16,472 to \$20,330, accompanied by a decrease in resident enrollment from 23,009 to 19,752 (-14 percent). The resulting average tuition subsidy per resident becomes \$17,131, compared to \$29,746 in the base case. Actually, imposing a cap on non-residents leads to a more efficient enrollment profile than either the base case or the case where non-resident pay fully-allocated costs. Welfare of \$998.8 M under the cap exceeds the base-case amount of \$938.7 M by 6 percent, and the appropriation constraint binds at \$279.1 M. These results are consistent with non-residents being inefficiently overcharged in both the base case and the legally-constrained case.<sup>21</sup> If the maximum appropriation is reduced by 10 percent and the cap

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<sup>20</sup> Then president of the UM, Mary Sue Coleman, and prominent business leaders, such as Domino’s Pizza CEO Patrick Dole, actively encouraged increasing non-resident enrollment in the state; see [http://www.mlive.com/education/index.ssf/2012/09/public\\_universities\\_eye\\_out-of.html](http://www.mlive.com/education/index.ssf/2012/09/public_universities_eye_out-of.html)

<sup>21</sup> One interpretation is that implicit caps on non-resident enrollments have been ignored at some UC universities in response to cuts in state appropriations; this practice has led from 5 percent non-resident enrollments at UC Berkeley



on non-resident enrollment exceeds 55 percent, non-resident enrollment will increase by 2.8 percent and resident enrollment will decrease by 16 percent.<sup>22</sup> Here, no effective restrictions are placed on non-resident tuitions. The final tuition structure scenario will consider this case in greater detail.

### **Table 5 Here**

**An unrestricted tuition structure:** A final comparative case develops when all constraints on non-resident tuitions are either eliminated, or they fail to bind.<sup>23</sup> As shown in the appendix, the defining operational feature for the unrestricted case is that the ratio of enrollments between any two programs is **proportionately** efficient; this implies, for example, that an increase in shared fixed cost will leave the enrollment ratios between any two programs unchanged. As presented in Table 6, absent residency restrictions, average resident tuition is \$24,403 (\$16,472), average non-resident tuition is \$33,399 (\$46,218), and welfare is \$1,008 M (\$939 M), with base-level amounts in parentheses. This case presents the highest value of social welfare. Since neither fixed costs nor the appropriation change for this case, the break-even condition implies that net tuition revenue does not change from the base case; it remains at \$225 M. There is a decline in total tuition revenue of \$122 M, accompanied by an equivalent decrease in resident enrollment (-27 percent) that offsets the increase in non-resident enrollment (13 percent).

### **Table 6 Here**

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ten years ago to over 20 percent in 2017. In contrast to Michigan, some UC universities may face capacity constraints.

<sup>22</sup> Bound et al. (2016) find that a 10 percent reduction in the state appropriation leads to a 17 percent increase in foreign enrollment at “most resource intensive public universities.”

<sup>23</sup> Burer and Fethke (2016) develop an unrestricted tuition structure case when optimal subsidies per enrollment are determined for every program. Basically, larger subsidies are assigned to programs that exhibit higher net willingness to pay.

The challenges faced by implementation of PWYCA tuitions are apparent by considering the unrestricted case. If the appropriation is intended to facilitate efficient pricing, imposing restrictions on the tuition structure does just the opposite: tuitions are inefficiently increased for non-residents and high-pay residents, while tuitions for low-pay residents are inefficiently decreased. In attempting to achieve a “fairer” tuition structure, there is a sacrifice of economic efficiency and an associated reduction in the optimal appropriation. With unrestricted tuitions, it is optimal to increase the state appropriation as long as it remains less than fixed costs.

### **III. Summary**

There exists an inverse relationship between tuition revenues and state appropriations that has persisted in U.S. public higher education for nearly three decades, with declines in state appropriations basically matched by increases in tuition revenue. Does the state appropriation crowd out private support, or does private support in the form of non-resident tuition crowd out the state appropriation? One explanation is that exogenous decreases in state capacity and rising autonomous levels of enrollments prompted increases in net tuition revenue that allow public universities to meet their fixed expenditures. Ad hoc tuition-setting formulations are often proposed as fair ways to increase tuition revenue, while continuing to provide enhanced access for low-income residents. These equity-based structures shift the payment burden from state taxpayers onto non-residents and high-pay residents. Often, they assume that changes in the tuition structure can be introduced without prompting offsetting changes in either state support or non-resident tuition revenue. The counter claim, which is considered here, is that higher net tuition revenues lead to lower appropriations, with legislatures permitting tuition revenue to crowd out taxpayer support.

For the Pay-What-You-Can Afford application I consider, a resident tuition-setting rule is imposed on high-pay residents and discounted low-pay resident tuitions are determined to facilitate their access. The results, which require both that non-residents are charged more than full costs and that the state appropriation is given, do imply additional low-pay enrollments and fewer high-pay enrollments. However, since the resulting tuition structure distorts relative demands and increases the gaps between tuitions and marginal costs, there is a loss in general welfare.

There are three identified problems with PWYCA: i) no criteria is specified for determining non-resident tuitions, ii) no account is taken of the inefficient distortions in relative demand, and iii) no recognition is paid to the plausible endogeneity of state appropriations. To address these issues, I develop a model where both the tuition structure and the state appropriation are endogenous, with non-residents required to pay at least fully-allocated costs. A closed-form solution occurs where all non-residents are charged fully-allocated costs, resident tuitions equal non-resident tuitions net of a uniform level of state support, and the legislature's budget does not bind. Thus, the PWYCA tuitions setting rules are compatible with a standard welfare maximization formulation for determining the tuition structure, but only where non-residents pay no more than fully-allocated costs and the appropriation is endogenously determined. This solution is the one selected by direct optimization using the UM data.

I argue here that increases in net tuitions revenues are overstated in the usual high tuition, high aid specifications because adjustments in state appropriations are not allowed. With endogenous appropriations, all the examples I present indicate that state appropriations will decline when there are inefficient increases in tuition revenues. This result is a straightforward implications of the welfare maximization objective, constrained by the break-even requirement

and by restrictions placed on non-resident tuitions. State appropriations are reduced even more if non-residents' welfare is weighted less than residents' welfare in setting tuitions.

#### IV. Appendix

The optimization problem is:<sup>24</sup>

$$\max_{T,E,S} [\sum_i E_{i1}^2 / 2b_{i1} + \delta \sum_i E_{i2}^2 / 2b_{i2} - S] \dots \text{Welfare} \quad (1)$$

s.t.

$$E_{ij} = a_{ij} - b_{ij}T_{ij} \dots \text{Demand curves, } i = 1, 2, \dots, n \text{ and } j = 1, 2 \quad (2)$$

$$\sum_{ij} (T_{ij} - c_i)E_{ij} + S = F \dots \text{University break-even requirement} \quad (3)$$

$$M - S \geq 0 \dots \text{State appropriation budget constraint} \quad (4)$$

Representing residents by "1" and non-residents by "2", the objective (1) is to select tuitions,  $T_{ij}$ , enrollments,  $E_{ij}$ , and the appropriation,  $S$ , to maximize consumers' surplus net of the state appropriation; consumers' surplus associated with each program's linear demand curve is:

$E_{ij}^2 / 2b_{ij}$ . Non-residents may receive less weight in the objective than residents, which occurs

when  $0 \leq \delta < 1$ . The demand curves (2) display positively defined intercept and slope

parameters,  $a_{ij}$  and  $b_{ij}$ , respectively. The cost expressions feature constant marginal cost,  $c_i$ ,

and shared fixed costs,  $F$ . The break-even constraint (3) implies that net tuition revenue plus the

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<sup>24</sup> The basic formulation extends Fethke (2017) and Burer and Fethke (2016).

taxpayer appropriation equals shared fixed cost. The legislative budget constraint (4) has  $M \leq F$  externally determined.

**A tuition structure unrestricted by residency requirements:** With preferences treated symmetrically ( $\delta = 1$ ), enrollments determined from (1) - (4) satisfy:

$$(1 + \lambda)(a_{ij}/b_{ij} - c_i) - (1 + 2\lambda)E_{ij}/b_{ij} = 0 \text{ for } i = 1, 2, \dots, n \text{ and } j = 1, 2. \quad (5)$$

To ensure positive enrollments in all programs, it is assumed that  $a_{ij}/b_{ij} - c_i > 0$ . With  $M \leq F$ , the legislative budget constraint will bind at the optimal solution. It can be represented as:

$$\theta x^2 - \theta x + F - M = 0, \text{ where } x \equiv (1 + \lambda)/(1 + 2\lambda) \text{ and } \theta \equiv \sum_{ij} b_{ij} (a_{ij}/b_{ij} - c_i)^2. \text{ Here, define}$$

$$\sum_{ij} E_{ij} \equiv \sum_{i=1}^n \sum_{j=1}^2 E_{ij}. \text{ The optimal "degree of efficiency," which is associated with the larger root of}$$

the above expression, is:

$$x^* = [1 + \sqrt{1 - 4(F - M)/\theta}]/2, \text{ with } 1/2 \leq x^* \leq 1. \quad (6)$$

When the appropriation equals fixed cost,  $M = F$ , tuitions equal marginal costs. When  $M < F$ , the budget restriction rules out the first-best solution. Economic efficiency, and thus welfare, are increasing in  $M$ . This result implies that both the break-even constraint and legislative budget constraint will bind when the tuition structure is unrestricted. Inserting (6) into (5) provides enrollments when there are no restrictions on the tuition structure:

$$E_{ij}^* = x^* b_{ij} (a_{ij}/b_{ij} - c_i), \text{ for } i = 1, 2, \dots, n \text{ and } j = 1, 2. \quad (7)$$

For any two programs, (7) implies that the optimal ratio of enrollments equals those occurring when tuitions equal marginal costs for any level of net fixed cost,  $F - M$ .

**Non-residents pay fully-allocated costs:** The unrestricted model (1)-(4) can be amended to include the common legal restriction that non-residents pay at least fully-allocated costs:

$$T_{i2} - c_i - F / \sum_{ij} E_{ij} \geq 0 \dots \text{Non-residents pay at least fully-allocated cost} \quad (8)$$

Using the break-even condition to eliminate  $S$ , and the demand curves to eliminate  $T_{ij}$ , the problem, with  $\delta = 1$ , reduces to:

$$\max_{E_{i1}, E_{i2}} \sum_i (a_{i1} / b_{i1} - c_i) E_{i1} - \sum_i E_{i1}^2 / 2b_{i1} + \sum_i (a_{i2} / b_{i2} - c_i) E_{i2} - \sum_i E_{i2}^2 / 2b_{i2} - F \quad (9)$$

s.t.

$$M - F + \sum_i (a_{i1} / b_{i1} - c_i) E_{i1} - \sum_i E_{i1}^2 / b_{i1} + \sum_i (a_{i2} / b_{i2} - c_i) E_{i2} - \sum_i E_{i2}^2 / b_{i2} \geq 0, \quad (10)$$

and

$$(a_{i2} / b_{i2} - E_{i2} / b_{i2} - c_i - F / \sum_{ij} E_{ij}) \geq 0 \text{ for } i=1,2,\dots,n \text{ and } j=1,2. \quad (11)$$

Since the objective (9) is concave and the constraints (10) and (11) are convex, there exists a unique solution. The associated Lagrangian expression is:

$$\begin{aligned} \max_{E_{ij}, \lambda, \mu_{i2}} L(E_{ij}, \lambda, \mu_{i1}, \mu_{i2}, \dots) = & \sum_{i,j} (a_{ij} / b_{ij} - c_i) E_{ij} - \sum_i E_{i1}^2 / 2b_{i1} - \sum_i E_{i2}^2 / 2b_{i2} \\ & + \lambda [M - F + \sum_{ij} (a_{ij} / b_{ij} - c_i) E_{ij} - \sum_{ij} E_{ij}^2 / b_{ij}] + \sum_i \mu_{i2} [(a_{i2} / b_{i2} - c_i - E_{i2} / b_{i2}) \sum_{ij} E_{ij} - F] \end{aligned} \quad (12)$$

The KKT conditions are:

$$\frac{\partial L}{\partial E_{i1}} = (1 + \lambda)(a_{i1} / b_{i1} - c_i) - (1 + 2\lambda)E_{i1} / b_{i1} + \sum_i \mu_{i2} (a_{i2} / b_{i2} - c_i - E_{i2} / b_{i2}) \leq 0, \quad (13)$$

$$\frac{\partial L}{\partial E_{i2}} = (1 + \lambda)(a_{i2} / b_{i2} - c_i) - (1 + 2\lambda)E_{i2} / b_{i2} + \sum_i \mu_{i2} (a_{i2} / b_{i2} - c_i - E_{i2} / b_{i2}) \quad (14)$$

$$-(\mu_{i2} / b_{i2}) \sum_{ij} E_{ij} \leq 0,$$

$$\frac{\partial L}{\partial \lambda} = M - F + \sum_{ij} (a_{ij} / b_{ij} - c_i - E_{ij} / b_{ij}) E_{ij} \geq 0, \quad (15)$$

and

$$\frac{\partial L}{\partial \mu_{i2}} = a_{i2}/b_{i2} - E_{i2}/b_{i2} - c_i - F / \sum_{ij} E_{ij} \geq 0 \text{ for } i = 1, 2, \dots, n \text{ and } j = 1, 2. \quad (16)$$

These expressions are used to illustrate features of optimal solutions.

When the legislative budget constraint does not bind,  $S < M$ , and the legal constraints bind for every non-resident program, then  $\lambda = 0$  and  $\mu_{i2} > 0$  for  $i = 1, 2, \dots, n$ . Key features of the tuitions-setting rules for this case can be illustrated. Specifically, tuitions and the shadow prices are described by the solution of (13) and (16). These conditions are:

$$T_{i1} = c_i - \mu F / \sum_{ij} E_{ij}, \quad (17)$$

where  $\mu \equiv \sum_i \mu_{i2}$ , and

$$T_{i2} = c_i + F / \sum_{ij} E_{ij} \text{ for } i = 1, 2, \dots, n \text{ and } j = 1, 2. \quad (18)$$

Thus, resident tuitions are proportional to fully-allocated cost, and non-resident tuitions equal fully-allocated cost. With positive enrollments in all programs (the presumed case), inserting (17) and (18) into the break-even condition (3) implies:

$$\mu = (S / F) \sum_{ij} E_{ij} / \sum_i E_{i1} - 1 \quad (19)$$

Together, (17) and (19) indicate:

$$T_{i1} = c_i + F / \sum_{ij} E_{ij} - S / \sum_i E_{i1} \text{ for } i = 1, 2, \dots, n \text{ and } j = 1, 2. \quad (20)$$

Resident tuitions equal fully-allocated costs minus the appropriation per resident enrollment.

These conditions are those assigned de novo by PWYCA, with the exception that here the state appropriation is endogenous. Substituting (17) and (20) into the breakeven condition (3), it is apparent that there is no incremental revenue to use to cross subsidize low-income students.

An expression for the subsidy in terms of enrollments is derived from (14) and (19).

Using (18) and (19), (14) reduces to the following:

$$\mu = (S / \sum_i E_{i1})(b_2 / \sum_{ij} E_{ij}), \text{ with } b_2 \equiv \sum_i b_{i2}. \quad (21)$$

Using (19) and (21):

$$S = \sum_i E_{i1} / [\sum_{ij} E_{ij} / F - b_2 / \sum_{ij} E_{ij}]. \quad (22)$$

For a given total enrollment, the optimal subsidy is increasing in resident enrollment. If no restrictions are placed on either the tuition structure or the legislative budget, then  $S = F$  and tuitions equal marginal costs. The appropriation is positive if:  $(\sum_{ij} E_{ij})^2 > b_2 F$ .

Using (17), (20), (22) and the demand curves (2), resident and non-resident enrollments depend on total enrollment as follows:

$$\sum_i E_{i1} = a_1 - \sum_i b_{i1} c_i - b_1 F / \sum_{ij} E_{ij} + b_1 / [\sum_{ij} E_{ij} / F - b_2 / \sum_{ij} E_{ij}], \text{ with } a_1 \equiv \sum_i a_{i1}, \quad (23)$$

and

$$\sum_i E_{i2} = a_2 - \sum_i b_{i2} c_i - b_2 F / \sum_{ij} E_{ij}, \text{ with } a_2 \equiv \sum_i a_{i2}. \quad (24)$$

The solution for total enrollment involves solving the following polynomial:

$$\sum_{ij} E_{ij}^4 - (a - \sum_{ij} b_{ij} c_i) \sum_{ij} E_{ij}^3 + (a - \sum_{ij} b_{ij} c_i) b_2 F \sum_{ij} E_{ij} - b b_2 F^2 = 0, \quad (25)$$

Using the UM parameter values, (25) it can be demonstrated that (25) has one positive real root, which is 38,721.

**Pay-What-You-Can-Afford Tuitions:** Residents, denoted by “1”, are charged tuitions that equal non-resident tuition minus a uniform subsidy per resident:



$$T_{i1} = \bar{T}_{i2} - \bar{S} / \sum_i E_{i1} \text{ for } i = 1, 2, \dots, n. \quad (26)$$

Here, non-resident tuitions and the appropriation are given. Using resident demand curves and summing yields:

$$\sum_i a_i - \sum_i E_{i1} = \sum_i b_{i1} \bar{T}_{i2} - \bar{S} \sum_i b_{i1} / \sum_i E_{i1}. \quad (27)$$

The solution for total resident enrollment is the positive root of (27):

$$\sum_i E_{i1} = (1/2)(a_1 - \sum_i b_{i1} \bar{T}_{i2}) + (1/2)[(a_1 - \sum_i b_{i1} \bar{T}_{i2})^2 + 4b_1 \bar{S}]^{1/2} \quad (28)$$

Given (28), resident tuitions are determined using (26). The incremental tuition revenue assigned to subsidize low-income residents is:

$$I = \sum_i (\bar{T}_{i2} - c_i)(E_{i1} + \bar{E}_{i2}) - F \geq 0. \quad (29)$$

If non-resident tuitions in each program equal fully allocate costs, then  $I = 0$ , regardless of the value of the appropriation. Thus, (29) implies that PWYCA tuitions require non-residents be charged more than full cost. Using the weighted average marginal cost,  $c$ , the number of residents that can be accommodated at no tuition is given by  $I/c$ . For the UM parameters, resident enrollment using (28) is 14,792, and  $I = \$171.9$  M. Dividing this amount by the weighted average of marginal costs, \$24,126, implies that 7,127 qualified low-income residents can be admitted tuition free, with a total enrollment of 40,068. These values are reported in the main text.

The PWYCA structure in the text is a restricted version of (1)-(4). Here, high-pay residents are denoted by “1”; low-pay residents are denoted by “2”; and non-residents are

denoted by “3”. High-pay resident tuitions are set to equal non-resident tuitions minus the endogenous state appropriation per **total** resident enrollment:

$$T_{i1} = \bar{T}_{i3} - S / \sum_{ij} E_{ij}, \text{ where } i = 1, \dots, n \text{ and } j = 1, 2. \quad (30)$$

Low-pay residents face discounted tuitions:

$$T_{i2} = kT_{i1}, \quad (31)$$

with  $0 \leq k \leq 1$  a scalar applied to all programs. Demand curves for low-pay residents lie below high-pay resident tuitions:

$$E_{i2} = a_{i1} - b_{i1}T_{i2} - E_{i1}, \text{ where } i = 1, \dots, n. \quad (32)$$

Substituting for S, using (3), the representation for the legislative budget constraint (4) is:

$$F - \sum_i (T_{i3} - c_i) \bar{E}_{i3} - \sum_{ij} (T_{ij} - c_i) E_{ij} \leq M. \quad (33)$$

Welfare (total consumer surplus minus the appropriation) is:

$$W = \sum_{ij} E_{ij}^2 / 2b_{ij} - S. \quad (34)$$

If the legislative budget constraint binds, then  $\bar{S} = M$ . Substituting (30) into (31) and also into the high- and low-pay resident demand curves provides expressions that depend on high-pay resident tuitions,  $T_{i1}$ , and the scalar, k. Solutions for resident tuitions, resident enrollments, and thus welfare, involve finding values for k and high-pay resident tuitions that satisfy (30) and (33). These n+1 restrictions, in turn, determine the values of resident enrollments and welfare. Using UM demand and cost parameters, Table 2 presents solutions for this case, which is found using the Excel Solver routine.

If  $S \leq M$  is endogenous, the welfare criteria (34) comes into play. The problem of maximizing (34) subject to (30) and (33) is not a convex problem. It is possible, however, to find the local solution that is presented in the text.

**Determination of the demand and cost parameters:** The calculations feature demand and cost parameter estimates using UM data for FY 2013-14. Here, the intercepts are  $a_{ij} = E_{ij} + b_{ij}T_{ij}$ , and the slopes are  $b_{ij} = -\varepsilon_{ij}E_{ij}/T_{ij}$ . Given base-level tuitions, enrollments, and values of demand elasticities of  $\varepsilon_{ij} = -0.5$ , the demand parameters are calculated for each program.<sup>25</sup> Marginal costs are base-level budget allocations to each college divided by total college enrollments.

### Appendix Table of Parameter Values Here

#### References

American Academy of Arts and Sciences. 2016. “Public Research Universities: Understanding the Financial Model”; accessible at: [https://www.amacad.org/multimedia/pdfs/publications/researchpapersmonographs/PublicResearchUniv\\_FinancialModel.pdf](https://www.amacad.org/multimedia/pdfs/publications/researchpapersmonographs/PublicResearchUniv_FinancialModel.pdf); accessed April 19, 2017.

Bound, John, Breno Braga, Gaurav Khanna, and Sarah Turner. 2016. “A Passage to America: University Funding and International Students,” National Bureau of Economic Research, Working Paper 22981, December; accessible at: <http://www.nber.org/papers/w22981>; accessed July 18, 2017.

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<sup>25</sup> Alternatively, linear demand curves can be constructed by specifying a priori the maximum willingness to pay,  $a_{ij}/b_{ij}$ , for each program. Then,  $b_{ij} = E_{ij}/(a_{ij}/b_{ij} - T_{ij})$  and  $a_{ij} = E_{ij} + b_{ij}T_{ij}$ . Here, the implied elasticities for each program can be backed out.

Burer, S., and G. Fethke. 2016. “Nearly-efficient Tuitions and Subsidies in American Public Higher Education, *Economics of Education Review*, 39, 182-97.

College Board. 2016. Trends in College Pricing, October; accessible at: [https://trends.collegeboard.org/sites/default/files/2016-trends-college-pricing-web\\_0.pdf](https://trends.collegeboard.org/sites/default/files/2016-trends-college-pricing-web_0.pdf); accessed June 13, 2017.

Deming, David J. and Christopher R. Walters, 2017. “The Impact of Price Caps and Spending Cuts on U.S. Postsecondary Attainment,” National Bureau of Economic Research Working Paper No. 23736, August.

Education Writers Association, Tuition Tracker; accessible at: <http://www.tuitiontracker.org/>; accessed October 10, 2017.

Fethke, Gary. 2017. “Efficiency and Equity Implications of Charging Non-residents Full-Cost Tuitions,” *Contemporary Economic Policy*, Vol. 35, Issue 4, 603-614.

Friedman, Milton. 1968. “The Higher Schooling in America,” *The Public Interest* 11 (spring): 108-112.

Groen, Jeffrey A. and Michelle J. White. 2004. “In-State versus Out-of-State Students: The Divergence of Interest between Public Universities and State Governments,” *Journal of Public Economics*, V88 (9-10, Aug): 1793-1814.

Jaquette, Ozan and Bradley R. Curs. 2015. “Creating the Out-of-State University: Do Public Universities Increase Non-resident Freshman Enrollment in Response to Declining State Appropriations?” *Res. High Educ.* 56:535-565.

Kane, T. J. 2006. “Public Intervention in Postsecondary Education,” Chapter 23 in: E. Hanushek and F. Welch (Eds.) “Handbook on the Economics of Education,” Elsevier/North Holland, Amsterdam, Vol 2, pp. 1369-1401.

Nerlove, Marc. 1972. "On Tuition and the Costs of Higher Education: Prolegomena to Conceptual Framework." *Journal of Political Economy*, Vol. 80, No. 3, Part 2: Investment in Education: The Equity-Efficiency Quandary (May - June), pp. S178-S218.

Rizzo, Michael J. and Ronald G. Ehrenberg. 2004. "Resident and Non-resident Tuition and Enrollment at Flagship State Universities," in College Choices: The Economics of Where to Go, When to go, and How to Pay For It, C. Hoxby, editor.

State Higher Education Executive Officers (SHEEO). 2016. The State Higher Education Finance Report FY 2016; accessible at: <http://www.sheeo.org/projects/shef-percentE2percent80percent94-state-higher-education-finance>; accessed June 13, 2017.

University of Michigan. 2016. "2016-17 Budget Detail"; accessible at: [http://obp.umich.edu/wp-content/uploads/pubdata/budget/greybookdetail\\_fy17\\_allcamp.pdf](http://obp.umich.edu/wp-content/uploads/pubdata/budget/greybookdetail_fy17_allcamp.pdf); accessed April 19, 2017.

Vedder, Richard and Justin Strehle, "Why Not Use Endowments to Lower Tuition Costs?" *Minding the Campus*, April 13, 2016; accessible at: <http://www.mindingthecampus.org/2016/04/why-not-use-endowments-to-lower-tuition-costs/>; accessed October 10, 2017.

Table 1

## Base-Case Allocations, Enrollments, and Tuitions for the UM, FY 2013-14

<b>UM FY 2013-14 Colleges</b>	<b>Budget Allocations</b>	<b>Resident Enrollment</b>	<b>Non-Res Enrollment</b>	<b>Resident Tuition</b>	<b>Non-Res Tuition</b>	<b>Tuition Ratio</b>
Lit, Arts & Sci (LA&S)	\$353,946,452	11,714	7,220	\$14,308	\$45,213	3.16
Engineering	\$174,083,728	4,570	3,765	\$15,326	\$45,478	2.97
Business	\$92,801,175	1,207	2,012	\$15,189	\$45,983	3.03
Medicine	\$76,528,291	679	455	\$33,878	\$53,134	1.57
Law	\$42,740,687	226	791	\$55,437	\$58,699	1.06
Public Health	\$33,118,706	466	472	\$27,177	\$44,926	1.65
Music	\$32,309,833	381	669	\$14,889	\$45,822	3.08
Dental	\$31,524,830	386	247	\$25,837	\$40,419	1.56
Social Work	\$20,213,974	319	143	\$26,203	\$41,998	1.60
Nursing	\$18,026,117	735	160	\$14,308	\$45,213	3.16
Education	\$17,731,456	327	198	\$16,157	\$48,412	3.00
Architec&Urban Plan	\$17,038,697	270	371	\$14,308	\$45,213	3.16
Information	\$16,263,371	137	289	\$16,157	\$48,412	3.00
Pharmacy	\$13,015,776	285	142	\$25,046	\$41,853	1.67
Kinesiology	\$12,840,886	549	373	\$16,157	\$48,412	3.00
Nat Resources	\$11,252,254	113	180	\$14,308	\$45,213	3.16
Art&Design	\$10,894,277	279	286	\$14,308	\$45,213	3.16
Public Policy	\$10,154,297	117	179	\$16,157	\$48,412	3.00
Rackman (Graduate)	\$8,482,279	249	197	\$21,833	\$44,926	2.06
Totals/Averages	\$992,967,086	23,009	18,149	\$16,472	\$46,218	2.90
Fixed Cost	\$503,949,649					
Total Cost	\$1,496,916,735					
Tuition and Fees	\$1,217,808,035					
State Appropriation	\$279,108,700					
Total Revenue	\$1,496,916,735					
Average Tuition	\$29,589					
Total Enrollment	41,158					
Welfare	\$938,699,335					

**Note:** Table 1 contains actual base-level allocations, enrollments and (compatible) tuitions for the UM FY 2013-14. Demand curves are calibrated for each program assuming initial tuition elasticities of demand are:  $\varepsilon_{ij} = -0.5$ .

With this assumption, demand curves,  $E_{ij} = a_{ij} - b_{ij}T_{ij}$ , with  $b_{ij} = -\varepsilon_{ij}E_{ij} / T_{ij}$  and  $a_{ij} = b_{ij}T_{ij}$ , are determined that use base-level tuitions and enrollments. The resulting demand and cost parameters are presented in the appendix.

Table 2

## Pay-What-You-Can Afford Resident Tuitions

<b>UM Programs</b>	<b>Budget Allocations</b>	<b>NonRes Enroll</b>	<b>Res 1 Enroll</b>	<b>Res 2 Enroll</b>	<b>NonRes Tuition</b>	<b>Res 1 Tuition</b>	<b>Res 2 Tuition</b>
Lit, Arts &Sci	\$388,453,976	7,220	3,264	10,296	\$45,213	\$34,951	\$9,799
Engineering	\$191,063,799	3,765	1,605	3,778	\$45,478	\$35,216	\$9,873
Business	\$98,728,319	2,012	391	1,021	\$45,983	\$35,721	\$10,015
Medicine	\$91,310,844	455	589	309	\$53,134	\$42,872	\$12,019
Law	\$46,326,349	791	240	71	\$58,699	\$48,437	\$13,580
Public Health	\$38,403,596	472	402	214	\$44,926	\$34,664	\$9,718
Music	\$37,991,305	247	354	162	\$40,419	\$30,157	\$8,455
Dental	\$34,246,680	669	117	327	\$45,822	\$35,560	\$9,969
Social Work	\$24,822,877	143	285	139	\$41,998	\$31,736	\$8,898
Nursing	\$19,597,983	198	104	278	\$48,412	\$38,150	\$10,696
Education	\$18,169,678	371	75	237	\$45,213	\$34,951	\$9,799
Arc.&Urban	\$17,147,312	289	44	116	\$48,412	\$38,150	\$10,696
Information	\$20,358,930	160	205	646	\$45,213	\$34,951	\$9,799
Pharmacy	\$15,823,416	142	248	129	\$41,853	\$31,591	\$8,857
Kinesiology	\$14,133,110	373	175	466	\$48,412	\$38,150	\$10,696
Nat Resources	\$11,936,109	180	31	99	\$45,213	\$34,951	\$9,799
Art&Design	\$11,742,027	286	78	245	\$45,213	\$34,951	\$9,799
Public Policy	\$10,832,634	179	37	99	\$48,412	\$38,150	\$10,696
Rackman	\$9,796,145	197	176	142	\$44,926	\$34,664	\$9,718
Total/Average	\$1,100 M	18,149	8,420	18,778	\$46,217	\$35,693	\$9,899
Fixed Cost	\$503.4 M						
Total Cost	\$1,604 M						
Tuition Rev	\$1,325 M						
Appropriation	\$279.1 M						
Total Revenue	\$1,604 M						
Total Enroll	45,347						
Welfare	\$889.8 M						

**Note:** Non-resident enrollments, tuitions, and the appropriation are the base-level amounts presented in Table 1. Resident enrollments are determined by finding enrollments for high-pay residents and the discount scalar for low-pay residents that solve the university budget constraint. Using the UM cost and demand parameters, the Excel Solver routine is used to compute the values for resident tuitions and enrollments that are presented in Table 2.

Table 3

## Allocations, Enrollments, and Tuitions When Non-residents Pay

## Fully-Allocated Costs

<b>UM Colleges</b>	<b>Budget Allocations</b>	<b>Resident Enrollments</b>	<b>Nonresident Enrollments</b>	<b>Resident Tuitions</b>	<b>Nonresident Tuitions</b>	<b>Tuition Ratios</b>
Lit, Arts & Sci	\$347,609,340	10,297	8,298	\$17,771	\$31,709	1.78
Engineering	\$169,654,267	3,879	4,244	\$19,963	\$33,901	1.70
Business	\$80,844,227	702	2,103	\$27,907	\$41,844	1.50
Medicine	\$46,515,291	351	338	\$66,564	\$80,500	1.21
Law	\$45,004,273	255	816	\$41,104	\$55,041	1.34
Public Health	\$30,306,514	404	454	\$34,385	\$48,323	1.41
Music	\$26,877,751	190	684	\$29,848	\$43,786	1.47
Dental	\$19,543,683	214	179	\$48,882	\$62,817	1.29
Social Work	\$14,685,409	218	118	\$42,830	\$56,768	1.33
Nursing	\$15,915,955	609	181	\$19,218	\$33,156	1.73
Education	\$12,137,032	158	201	\$32,853	\$46,789	1.42
Architec&Urban Plan	\$14,805,389	163	394	\$25,655	\$39,596	1.54
Information	\$12,531,975	48	281	\$37,253	\$51,192	1.37
Pharmacy	\$12,147,620	259	139	\$29,561	\$43,497	1.47
Kinesiology	\$14,738,681	603	456	\$13,005	\$26,942	2.07
Nat Resources	\$7,263,852	21	168	\$37,481	\$51,419	1.37
Art&Design	\$10,920,681	240	327	\$18,358	\$32,297	1.76
Public Policy	\$8,083,875	55	181	\$33,383	\$47,320	1.42
Rackman (Graduate)	\$9,425,664	270	225	\$18,091	\$32,034	1.77
Totals	\$899,011,477	18,935	19,786	\$21,309	\$37,176	1.74
Tuition Revenue	\$1,139,049,204					
Average Tuition	\$29,417.16					
Total Enrollment	38,721					
Optimal Appropriation	\$263,911,922					
Appro Res Enroll	\$13,938					
Fixed Cost	\$503,949,649					
Welfare	\$1,000,677,102					

**Note:** In the model presented in the appendix, the state appropriation and enrollments are endogenously determined, and analytical solutions are presented. The calculations presented in Table 3 use these solutions and the UM cost and demand parameters.



Table 4

Budget Allocations, Enrollments, and Tuitions with Non-resident Welfare Valued at  
75 percent of Resident Welfare

	<b>Budget</b>	<b>Resident</b>	<b>Non-Res</b>	<b>Resident</b>	<b>Non-Res</b>
<b>UM Colleges</b>	<b>Allocations</b>	<b>Enrollment</b>	<b>Enrollment</b>	<b>Tuition</b>	<b>Tuition</b>
Lit, Arts &Sci	\$323,111,181	9,828	7,456	\$18,915	\$42,252
Engineering	\$157,330,644	3,713	3,820	\$21,074	\$44,154
Business	\$73,888,345	657	1,906	\$29,039	\$50,816
Medicine	\$45,491,123	340	334	\$67,719	\$81,340
Law	\$44,669,894	253	810	\$42,231	\$55,881
Public Health	\$29,814,614	395	450	\$35,496	\$49,163
Music	\$26,237,909	175	678	\$30,994	\$44,626
Dental	\$19,006,472	206	176	\$49,982	\$63,657
Social Work	\$14,318,680	211	116	\$43,972	\$57,608
Nursing	\$15,304,534	580	180	\$20,342	\$33,996
Education	\$11,707,523	147	200	\$33,940	\$47,629
Architec&Urban	\$14,437,887	153	391	\$26,755	\$40,436
Information	\$12,253,486	43	278	\$38,383	\$52,032
Pharmacy	\$11,910,900	253	138	\$30,676	\$44,337
Kinesiology	\$14,426,854	583	452	\$14,132	\$27,782
Nat Resources	\$7,028,745	17	166	\$38,608	\$52,259
Art&Design	\$10,650,827	228	324	\$19,521	\$33,137
Public Policy	\$7,890,687	51	179	\$34,510	\$48,160
Rackman	\$9,268,795	264	223	\$19,215	\$32,873
Totals/Averages	\$848,749,102	18,095	18,278	\$22,456	\$45,041
Fixed Costs	\$503,949,649				
Total Cost	\$1,352,698,751				
Tuition and Fees	\$1,229,609,586				
Appropriation	\$123,089,165				
Total Revenue	\$1,352,698,751				
Average Tuition	\$33,805				
Total Enrollment	36,373				
Welfare	\$760,040,824				

**Note:** The results presented in Table are developed by direct optimization, using Excel's Solver routine. The solution determines which of the legal constraints are relaxed.

Table 5

## An Enrollment Cap of 50 Percent Imposed on Non-residents

<b>UM Colleges</b>	<b>Budget Allocations</b>	<b>Resident Enrollments</b>	<b>Non-Res Enrollments</b>	<b>Resident Tuitions</b>	<b>Non-Res Tuitions</b>	<b>Tuition Differential</b>
Lit, Arts &Sci	\$355,735,460	10,768	8,262	\$16,619	\$32,167	\$15,548
Engineering	\$172,498,779	4,029	4,230	\$18,953	\$34,249	\$15,296
Business	\$82,971,944	772	2,106	\$26,143	\$41,673	\$15,530
Medicine	\$47,204,969	355	345	\$66,210	\$78,941	\$12,730
Law	\$43,888,250	241	803	\$48,036	\$56,885	\$8,850
Public Health	\$30,294,236	399	459	\$35,008	\$47,373	\$12,365
Music	\$27,742,445	215	687	\$27,861	\$43,420	\$15,559
Dental	\$20,357,485	220	188	\$48,008	\$59,606	\$11,598
Social Work	\$14,937,091	220	122	\$42,536	\$54,440	\$11,905
Nursing	\$16,561,373	642	181	\$17,946	\$33,513	\$15,567
Education	\$12,793,302	178	201	\$30,928	\$46,813	\$15,885
Architec&Urban	\$15,272,583	180	395	\$23,869	\$39,421	\$15,552
Information	\$12,938,803	57	282	\$34,976	\$50,856	\$15,880
Pharmacy	\$12,128,434	256	141	\$30,056	\$42,208	\$12,152
Kinesiology	\$14,727,752	608	449	\$12,683	\$28,564	\$15,881
Nat Resources	\$7,765,958	32	170	\$34,737	\$50,293	\$15,557
Art&Design	\$11,120,809	251	326	\$17,159	\$32,712	\$15,553
Public Policy	\$8,329,058	62	181	\$31,417	\$47,305	\$15,888
Rackman	\$9,339,892	267	224	\$18,740	\$32,403	\$13,663
Totals	\$916,608,622	19,752	19,752	\$20,330	\$37,461	\$17,131
Fixed Cost	\$503,949,649					
Total Cost	\$1,420,558,271					
Tuition Revenue	\$1,141,449,571					
Appropriation	\$279,108,700					
Total Revenue	\$1,420,558,271					
Average Tuition	\$28,895					
Total Enrollment	39,503					
Welfare	\$998,829,361					

**Note:** Total non-resident enrollment is constrained to not equal 50 percent of total enrollment; otherwise, there are no other residency constraints. The solution is found by direct optimization, using Excel's Solver routine.

Table 6

## Budget Allocations, Enrollments, and Tuitions for an Unrestricted Tuition Structure

<b>UM Colleges</b>	<b>Optimal Allocations</b>	<b>Resident Enrollment</b>	<b>Non-Res Enrollment</b>	<b>Resident Tuition</b>	<b>Non-Res Tuition</b>	<b>Tuition Ratio</b>
Lit, Arts & Sci	\$331,053,815	9,122	8,587	\$20,640	\$28,088	1.36
Engineering	\$163,729,750	3,441	4,399	\$22,902	\$30,169	1.32
Business	\$80,926,932	612	2,195	\$30,174	\$37,595	1.25
Medicine	\$45,663,648	315	362	\$70,229	\$74,869	1.07
Law	\$44,705,425	233	831	\$52,011	\$52,797	1.02
Public Health	\$29,835,076	364	481	\$39,021	\$43,299	1.11
Music	\$27,072,779	164	716	\$31,888	\$39,343	1.23
Dental	\$19,480,007	190	201	\$52,028	\$55,543	1.07
Social Work	\$14,171,046	195	129	\$46,553	\$50,360	1.08
Nursing	\$14,624,821	538	188	\$21,971	\$29,420	1.34
Education	\$11,699,273	137	210	\$34,955	\$42,729	1.22
Architec&Urban	\$14,707,752	142	411	\$27,894	\$35,343	1.27
Information	\$12,751,645	40	294	\$39,004	\$46,778	1.20
Pharmacy	\$11,643,754	234	148	\$34,070	\$38,120	1.12
Kinesiology	\$13,996,067	540	465	\$16,702	\$24,476	1.47
Nat Resources	\$7,466,578	16	178	\$38,767	\$46,215	1.19
Art&Design	\$10,613,565	212	338	\$21,181	\$28,630	1.35
Public Policy	\$8,088,243	47	189	\$35,443	\$43,217	1.22
Rackman	\$9,075,064	244	233	\$22,753	\$28,318	1.24
<b>Totals</b>	<b>\$871,305,240</b>	<b>16,785</b>	<b>20,556</b>	<b>\$24,404</b>	<b>\$33,077.0</b>	<b>1.36</b>
Tuition and Fees	\$1,096 M					
Average Tuition	\$29,355					
Total Enrollment	37,341					
Net Tuition and Fees	\$224.5 M					
Appropriation	\$279,1 M					
Fixed Cost	\$503,9 M					
Welfare	\$1,007.8 M					

**Note:** Only the university break-even constraint and the legislative budget constraint are included here; there are no further restrictions placed on the structure of tuition. This case yields analytical solutions for tuitions, enrollments and the appropriation that are “quasi efficient” in the sense that they are the best that can be achieved under the requirement that the university must at most break even.

## Appendix

### Cost and Demand Parameters for the UM

<b>UM</b>	<b>Marginal</b>	<b>Resident</b>	<b>Resident</b>	<b>Nonresident</b>	<b>Nonresident</b>
<b>Colleges</b>	<b>Cost</b>	<b>Slope</b>	<b>Intercept</b>	<b>Slope</b>	<b>Intercept</b>
Lit, Arts & Sci	\$18,694	0.409	17,571	0.080	10,830
Engineering	\$20,886	0.149	6,855	0.041	5,648
Business	\$28,829	0.040	1,811	0.022	3,018
Medicine	\$67,485	0.010	1,019	0.004	683
Law	\$42,026	0.002	339	0.007	1,187
Public Health	\$35,308	0.009	699	0.005	708
Music	\$30,771	0.013	572	0.007	1,004
Dental	\$49,802	0.007	579	0.003	371
Social Work	\$43,753	0.006	479	0.002	215
Nursing	\$20,141	0.026	1,103	0.002	240
Education	\$33,774	0.010	491	0.002	297
Architec&Urban	\$26,581	0.009	405	0.004	557
Information	\$38,177	0.004	206	0.003	434
Pharmacy	\$30,482	0.006	428	0.002	213
Kinesiology	\$13,927	0.017	824	0.004	560
Nat Resources	\$38,404	0.004	170	0.002	270
Art&Design	\$19,282	0.010	419	0.003	429
Public Policy	\$34,305	0.004	176	0.002	269
Rackman	\$19,019	0.006	374	0.002	296
<b>Totals</b>	<b>\$24,126</b>	<b>0.740</b>	<b>34,514</b>	<b>0.197</b>	<b>27,224</b>

**Notes:** Constant marginal costs for each program are developed by assuming that college budget allocations are variable costs and then dividing these allocations by total college enrollments. When combined with actual enrollments and tuitions for FY 3013-14, these parameter values replicate the base level budget allocations for that year.