# New and Improved Manipulation Algorithms for MEMS Arrays and Vibratory Parts Feeders: What Programmable Vector Fields Can (and Cannot) Do — Part II

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**Abstract:** *This paper explores how to use programmable vector fields to control flexible planar parts feeders. For a description of these devices and their actuation technology, see our companion paper, Part I [2]. When a part is placed on our devices, the programmed vector field induces a force and moment upon it. Over time, the part may come to rest in a dynamic equilibrium state. By chaining together sequences of vector fields, the equilibria may be cascaded to obtain a desired final state. By analyzing and constraining the equilibria of programmable vector fields, we can generate and execute plans to orient and sort parts. These plans require no sensing.*

*This paper describes new manipulation algorithms using the tools developed in Part I [2]. In particular, we improve existing planning algorithms by a quadratic factor, and the plan-length by a linear factor. Using our new and improved strategies, we show how to simultaneously orient and pose any part, without sensing, from an arbitrary initial configuration. We relax earlier dynamic and mechanical assumptions to obtain more robust and flexible strategies.*

*Finally, we consider parts feeders that can only implement a very limited "vocabulary" of vector fields (as opposed to the pixel-wise programmability assumed above). We show how to plan and execute parts-posing and orienting strategies for these devices, but with a significant increase in planning complexity and some sacrifice in completeness guarantees. We discuss the tradeoff between mechanical complexity and planning complexity.*

## 1 Introduction

This paper continues our exploration of programmable vector fields to control flexible planar parts feeders. For a description of these devices and their actuation technology, see our companion paper, Part I [2].

In Part I, we observed that planar parts feeders often exploit exotic actuation technologies such as arrayed, microfabricated motion pixels [5, 6] or transverse vibrating plates [1]. This motivates our research into novel and powerful control strategies. When a part is placed on our devices, the programmed vector field induces a force and moment upon it. Over time, the part may come to rest in a dynamic equilibrium state. By chaining together sequences of vector fields, the equilibria may be cascaded to obtain a desired final state. Our research goal is to develop a science base for manipulation by analyzing and constraining the equilibria of programmable vector fields.

Our companion paper (Part I [2]) describes our experimental apparatus, a technique for analyzing our devices called *equilibrium analysis*, lower bounds (i.e., impossibility results) on what the devices *cannot* do, and results on a classification of control strategies yielding design criteria by which well-behaved manipulation strategies may be developed. This paper (Part II) describes new manipulation algorithms using the tools developed in Part I. In particular, we improve existing planning algorithms by a quadratic factor, show how to simultaneously orient and pose a part, and we relax earlier dynamic and mechanical assumptions to obtain more robust and flexible strategies.

In Part I, we asked *Which vector fields are suitable for manipulation strategies?* To answer, we analyzed the equilibrium structure of general programmable vector fields. We then proposed the fields with *potential* as those which induce stable equilibria in all parts. In addition, such fields can be composed by pointwise addition, temporal switching, or "morphing" to obtain more complex, but nevertheless stable strategies. As a reality check, we demonstrated the existence of fields *without* potential that induce pathological behavior.

Previous results on array manipulation strategies may be formalized using our *equilibrium analysis.* In [5] we proposed a family of control strategies called *squeeze patterns* and a planning algorithm to uniquely orient a part. Squeeze patterns have potential; this observation yields an $E = O(n^2)$ upper bound on the number of equilibria of a planar part with $n$ vertices. This results in an $O(E^2) = O(n^4)$ planning algorithm under certain dynamic and mechanical assumptions. In Part I [2] we argued that the bound on equilibria appears tight, resulting in a very high planning and execution complexity.

Using our equilibrium analysis, in Part I, we introduced *radial* fields, which satisfy our stability property. Radial fields can be combined with squeeze fields. We show this has several benefits: (1) The number of equilibria drops to $E = O(n)$. (2) The planning complexity drops to $O(E^2) = O(n^2)$. (3) throughout the strategy execution, every part rotates about one fixed, unique point (after the first step). This means that (4) we can dispense with one critical assumption (called 2PHASE in [5]): we no longer need assume that the transitional and rotational motions induced by the array interact in a "quasi-static" and "sequential" manner.

The exotic planar parts-feeders described in Part I [2] — for example, the M-CHIP (*manipulation chip*), a massively parallel array of programmable micro-motion pixels — bring up several key practical issues. First, the radial strategies employed by our improved algorithms and analysis require significant mechanical and control complexity — even though they require no sensing. While we believe such mechanisms are feasible to build using the silicon MEMS (microelectromechanical systems) technologies we advocate, it is undeniable that no such device exists yet. (The M-CHIPS will have pixel-wise programmability, but the first generation will probably not have sufficient resolution to implement highly accurate radial strategies). For this reason, in Part II we introduce and analyze strategies composed of field sequences that we know are implementable using current (microscopic or macroscopic) technology. Each strategy is a sequence of pairs of squeezes satisfying certain "orthogonality" properties. Under these assumptions, we can ensure (a) equilibrium stability, (b) relaxed mechanical and dynamical assumptions (the same as (4), above) and (c) complexity and completeness guarantees. The framework is quite general, and applies to any set of primitive operations satisfying certain "finite equilibrium" properties (which we define) — hence it has broad applicability to a wide range of devices. In particular, we view the restricted class of fields as a *vocabulary* and their rules of composition as a *grammar*, resulting in a "language" of manipulation strategies. Under our grammar, the resulting strategies are guaranteed to be well-behaved.

Finally, both our radial strategies and our *finite manipulation grammar* have the following advantage over previous manipulation algorithms for programmable vector fields: previous algorithms such as [5] guarantee to uniquely orient a part, but the transitional position of the part is unknown

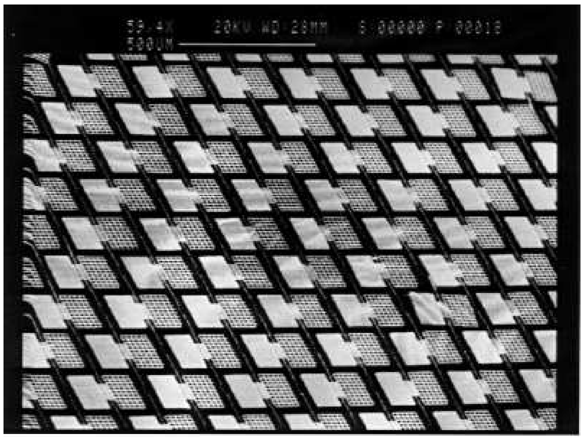

Figure 1: A prototype M-Chip fabricated in 1995. A large unidirectional actuator array (scanning electron microscopy). Each actuator is $180 \times 240\,\mu m^2$ in size. Detail from a 1 $\mathrm{in}^2$ array with more than 11,000 actuators. For more pictures on device design and fabrication see URL `http://www.cs.cornell.edu/Info/People/karl/MicroActuators`.

at the strategy's termination. Both of our new algorithms guarantee to position the part uniquely in translation *as well as* orientation space.

However, the complexity and completeness guarantees we obtain for manipulation grammars are considerably weaker than for the ideal radial strategies. For radial strategies, we show that *any* connected, planar part can be oriented within the complexity bounds above. Under the simplified "manipulation grammar," our planner is guaranteed to find a strategy if one exists (If one does not exist, the planner will signal this). However, it is not known whether there exists a strategy for every part. Moreover, the planning algorithm is exponential instead of merely quadratic.

This result illustrates a tradeoff between mechanical complexity (the dexterity and controllability of field elements) and planning complexity (the computational difficulty of synthesizing a strategy). If one is willing to build a device capable of radial fields, then one reaps great benefits in planning and execution speed. On the other hand, we can still plan for simpler devices, but the plan synthesis is more expensive, and we lose some completeness properties.

Finally, the desire to implement complicated fields raises the question of control uncertainty. We close Part II by describing how families of potential functions can be used to represent control uncertainty, and analyzed for their impact on equilibria.

## 2 Experimental Apparatus

We investigate actuation technologies that are capable of generating planar, programmable force fields. These devices have only recently been invented and are described in more detail in Part I [2]. (1) Advances in microfabrication based on VLSI technology have made possible the fabrication of micro actuator arrays, consisting of many thousands of individual silicon "fingers" on an area of one square inch (Figure 1). (2) Transverse vibrations of a horizontal plate generate a force field on the plate. The vibrational patterns can be programmed by changing the frequency, or by employing clamps as programmable fixtures that create various vibratory nodes (Figure 2).

We have built and tested prototype micro actuator arrays and vibratory plate parts feeders. For a more detailed description of device design and experimental results please refer to [4] and [1],

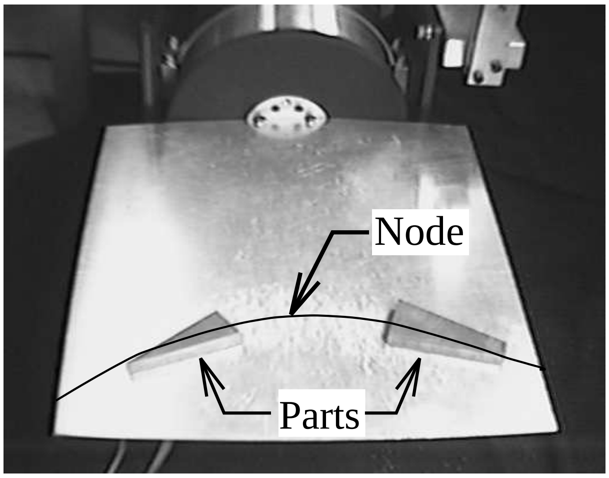


Figure 2: Vibratory parts feeder: the aluminum plate exhibits a vibratory minimum. Parts reach equilibrium along this node line. See also URL `http://www.cs.cornell.edu/Info/People/karl/VibratoryAlign`. Reproduced with permission from [1].

respectively.

# 3 Equilibrium Analysis for Programmable Vector Fields

## 3.1 Planning (Review)

For the generation of manipulation plans with programmable vector fields it is essential to be able to predict the motion of a part in the field. Particularly important is determining the stable equilibrium poses a part can reach in which all forces and moments are balanced. This *equilibrium analysis* was described in our previous paper [5], where we presented a theory of manipulation for programmable vector fields, and an algorithm that generates manipulation plans to orient polygonal parts without sensor feedback using a sequence of *squeeze fields*.

We now briefly review the algorithm in [5], since the tools developed there are essential to understanding our improved results. The algorithm is based on the observation that any connected polygonal part, when placed into a force vector field with a simple squeeze pattern, aligns with the field in a finite set of orientations in which the part is in equilibrium (Figure 3a). As in [5] we make the following assumptions:

SIMPLICITY: The moving part $P$ can be treated as a flat polygon.

BILATERAL SYMMETRY: We have the following simple actuator control scheme available: We can divide the array by a straight line $l$ such that all motion pixels on either side of $l$ push normally towards $l$.

DENSITY: The generated forces can be described by a two-dimensional vector field. This means that the individual microactuators are dense compared to the size of the moving part. (We will discuss later how to relax this assumption.)

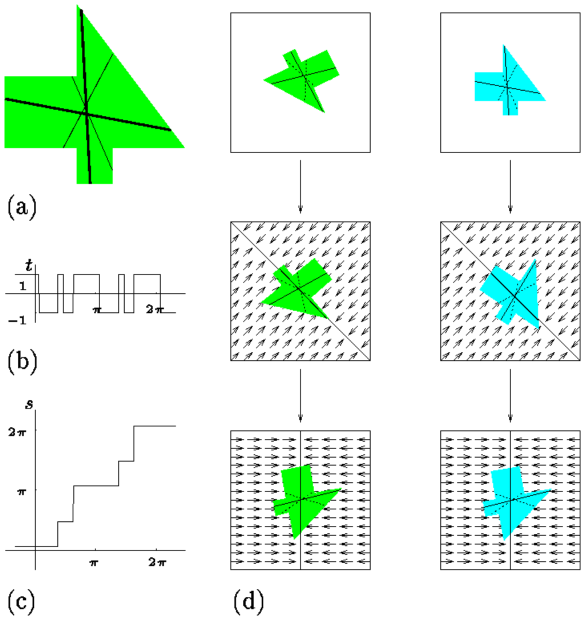


Figure 3: (a) Polygonal part. Stable (thick line) and metastable (thin line) medians are also shown. (b) Turn function. (c) Squeeze function. (d) Alignment strategy for two arbitrary initial configurations. See URL http://www.cs.cornell.edu/Info/People/karl/Cinema for an animated simulation.

Assuming quasi-static motion, a small object will move perpendicularly towards the line $l$ and come to rest there. We are interested in the motion of an arbitrarily shaped part $P$. Let us call $P_1$, $P_2$ the regions of $P$ that lie to the left and to the right of $l$, respectively, and $C_1$, $C_2$ their centers of gravity. In a rest position both translational and rotational forces must be in equilibrium. We get the following two conditions:

**I** : The areas $P_1$ and $P_2$ must be equal.

**II** : The vector $C_2 - C_1$ must be normal to $l$.

**Definition 1** *A* bisector *of a polygon $P$ is a straight line that divides $P$ into two parts of equal size.*

In [5] we made another assumption.

2Phase: The motion of $P$ has two phases: (1) Pure translation towards $l$ until condition **I** is satisfied. (2) Motion until condition **II** is satisfied without violating condition **I**.

Relaxing this assumption is one of the key results of this paper.

**Definition 2** *Let $\theta$ be the orientation of a connected polygon $P$ on an actuator array, and let us assume that condition* **I** *holds. The* turn function $t : \theta \rightarrow \{-1, 0, 1\}$ *describes the instantaneous rotational motion of $P$. $t(\theta) = 1$ if $P$ will turn counterclockwise, $t(\theta) = -1$ if $P$ will turn clockwise, and $t(\theta) = 0$ if $P$ is in equilibrium.*

This definition immediately implies the following lemma:

**Lemma 3** [5] *Let $P$ be a polygon with orientation $\theta$ on an actuator array such that conditions* **I** *and* **II** *hold. $P$ is stable if $t(\theta) = 0$, $t(\theta+) \leq 0$, and $t(\theta-) \geq 0$. Otherwise $P$ is metastable.*

Using this lemma we can identify all stable orientations, which allows us to construct the squeeze function of $P$ in analogy to Goldberg [10]:

**Lemma 4** [5] *Let $P$ be a polygonal part on an actuator array $\mathcal{A}$ such that assumptions* Simplicity, Bilateral Symmetry, Density, *and* 2Phase *hold. Given the turn function $t$ of $P$, its corresponding squeeze function $s : S^1 \to S^1$ is constructed as follows:*

1. *All stable orientations $\theta$ map identically to $\theta$.*
2. *All metastable orientations map (by convention) to the nearest counterclockwise stable orientation.*
3. *All orientations $\theta$ with $t(\theta) = 1$ $(-1)$ map to the nearest counterclockwise (clockwise) stable orientation.*

*Then $s$ describes the transition of $P$ induced by $\mathcal{A}$.*

### 3.2 Equilibrium analysis

Consider again a simple squeeze pattern as described in Section 3.1. In [5] we outlined how to determine the orientations $\theta_i$ in which a given part achieves stable equilibrium. This proof can be extended to show that stable equilibria *always exist* as long as the contact areas have finite size, and that for simply-connected parts the equilibria are *discrete*. The proof also gives a polynomial upper bound for the number of possible equilibria.

In Section 3.1, assumption 2Phase allowed us to determine successive equilibrium positions in a sequence of squeezes, by a quasi-static analysis that decouples translational and rotational motion of the moving part. For any part, this obtains a *unique* equilibrium (after several steps). If 2Phase is relaxed, we obtain a dynamic manipulation problem, in which we must determine the equilibria $(x, \theta)$ given by the part orientation $\theta$ and the offset $x$ of its center of gravity from the squeeze line. A stable equilibrium is a $(x_i, \theta_i)$ pair in $\mathbb{R} \times S^1$ that acts as an *attractor* (the $x$ offset in an equilibrium is, surprisingly, usually not 0). Again, we can obtain these $(x_i, \theta_i)$ equilibrium pairs as outlined in [5].

### 3.3 Complexity

Considering $(x_i, \theta_i)$ equilibrium pairs has another advantage. We can show that, even without 2Phase, after two successive, orthogonal squeezes, the set of stable poses of any part can be reduced from $\mathcal{C} = \mathbb{R}^2 \times S^1$ to a *finite* subset ($\mathcal{C}$ is the configuration space of part $P$). Subsequent squeezes will maintain the finiteness of the state space. This will significantly reduce the complexity of a task-level motion planner. Hence if assumption 2Phase is relaxed, this idea still enables us to simplify the general motion planning problem (as stated e.g. in [12]) to that of Erdmann and Mason [9]. Alternatively, relaxing assumption 2Phase raises the complexity from the "linear" planning scheme of Goldberg [10] to the forward-chaining searches of Erdmann and Mason [9], or Donald [8].

Summary: In Section 3.1 we reviewed reasonable assumptions under which there exist efficient (polynomial time) algorithms to compute manipulation plans, and we showed that the generated plans are polynomial in the part complexity (i.e. its number of vertices). Below we investigate how these bounds change when our assumptions are relaxed.

In Section 5 we will present new manipulation algorithms that relax the 2Phase assumption. These new algorithms have the advantage that the parts are not only uniquely oriented — in addition, their final translational position is unique, too. The first algorithm uses radial vector fields to generate *linear-size* alignment plans, and enjoys a *quadratic* improvement in the planning time. The second algorithm broadens the scope of our alignment plans to a "language" of more general force vector fields, which can be used even with devices that have only a limited "vocabulary" of programming for their vector fields, such as the vibratory plate device described in Section 2 and Part I [2].

# 4 Potential Fields (Review)

**Radial fields** A *radial field* is a vector field whose forces are directed towards a specific center point. As a specific example, consider the *unit radial field* $R$ which is defined (in polar coordinates) by $R(r,\theta) = r/||r||$ for $r \neq 0$, and $R(0,\theta) = 0$. Note that $R$ has a discontinuity at the origin. A smooth radial field can be defined for example by $R'(r,\theta) = -r^2$. R and R' clearly have a potential: $U(r,\theta) = r$, and $U'(r,\theta) = \frac{1}{3}r^3$, respectively.

**Morphing and combining vector fields.** Our strategies from Section 3 have *switch points* in time where the vector field changes discontinuously (Figure 3). This is because we have shown that after one squeeze, for every part, the equilibrium is in general non-unique, and hence subsequent squeezes are needed to disambiguate its pose. Therefore this switch is necessary for strategies with squeeze patterns.

One may ask whether for another class of potential field strategies, *unique* equilibria may be obtained without discrete switching. We believe that *continuously varying* vector fields of the form $(1-t)F + t\,G$, where $t \in [0,1]$ represents time, and $F$ and $G$ are squeezes, may lead to vector fields that have this property (see Part II [3] for progress in that direction). Here "+" denotes point-wise addition of vector fields, and we will write "$F \rightsquigarrow G$" for the resulting continuously varying field.

Let us formalize the previous paragraphs. If $F$ is a vector field (in this case a squeeze pattern) that is applied to move part $P$, we define the *equilibrium set* $E_P(F)$ as the subset of $\mathcal{C}$ for which $P$ is in equilibrium. Let us write $F * G$ for a strategy that first applies vector field $F$, and then vector field $G$ to move part $P$. $F + G$ can be understood as applying $F$ and $G$ simultaneously. We have shown that in general $E_P(F)$ is not finite, but for two *orthogonal* squeezes $F$ and $G$, the discrete switching strategy $F * G$ yields a finite equilibrium set $E_P(F * G)$. Furthermore, for some parts the equilibrium is unique up to symmetry.

We will explore the interesting relationship between equilibria in simple vector fields $E_P(F)$ or $E_P(G)$, combined fields $E_P(F+G)$, discretely switched fields $E_P(F*G)$, and continuously varying fields $E_P(F \rightsquigarrow G)$. For example, one may ask whether there exists a strategy with combined vector fields, or continuously varying fields, that, in just one step, reaches the same equilibrium as a discretely switched strategy requiring multiple steps. Finally, let $F_1 * F_2 * \cdots * F_k$ be a sequence of squeeze fields guaranteed to uniquely orient a part $P$ under assumption 2Phase. We will investigate how continuously varying strategies such as $F_1 \rightsquigarrow F_2 \rightsquigarrow \cdots \rightsquigarrow F_k$ can be employed to dynamically achieve the same equilibria even when 2Phase is relaxed. Research in this area could lead to a *theory of parallel distributed manipulation* that describes *spatially distributed* manipulation tasks that can be *parallelized over time* by superposition of controls.

# 5 New and Improved Manipulation Algorithms

The part alignment strategies in Section 3.1 have *switch points* in time where the vector field changes discontinuously (Figure 3). We can denote such a *switched strategy* by $F_1 * F_2 * \cdots * F_n$, where the $F_i$ are vector fields. In Section 3.1 we recalled that a strategy to align a polygonal part with $n$ vertices may need up to $O(n^2)$ switches, and require $O(n^4)$ time in planning. To improve these bounds, we now consider a broader class of vector fields including simple squeeze patterns, radial, and combined fields as described in Section 4.

In Section 5.1 we show how, by using radial and combined vector fields, we can significantly reduce the complexity of the plans from that of Section 3. In Section 5.2 we describe a general planning algorithm that works with a limited "grammar" of vector fields (and yields, correspondingly, less favorable complexity bounds).

## 5.1 Radial Strategies

Consider a connected polygonal part $P$ in an ideal radial vector field $R$ as described in Section 4.

**Proposition 5** *For each polygonal part $P$ in a radial field $R$ there exists a unique* **pivot point** *$v$ such that $P$ is in stable equilibrium iff $v$ coincides with the center of $R$, $(0,0)$. Surprisingly, $v$ need not be the center of mass.*

*Proof:* Consider the translational forces (but not the moments) acting on $P$ in the radial field $R$. To do this, let us separate $R$ into its $x$ and $y$ components, $R_x$ and $R_y$, such that $R = (R_x, R_y)$. Assume for now that the orientation of $P$ is fixed. If $P$ is placed at a sufficiently large negative $x$ coordinate, the force induced by $R_x$ on $P$ will be in the positive $x$ direction. Symmetrically, placing $P$ at a sufficiently large positive $x$ coordinate will cause a force in the negative $x$ direction. We claim that, by translating $P$ rigidly with increasing $x$ coordinate, this force decreases continuously and strictly monotone, and hence has a unique root. To verify this claim, consider a small area patch $\wp(t)$ of $P$, where $\wp(t) = \wp(0) \ominus (z_0 - t\hat{x})$ with $z_0$ the initial position of the patch, and $\ominus$ the Minkowski Difference defined by $A \ominus z = \{a - z | a \in A\}$ for any $A \subseteq \mathbb{R}^2$. The force in $x$ direction is $\displaystyle\int_{\wp(t)} R_x dA$. This force decreases continuously and strictly monotone with $t$, because $R_x$ is strictly monotone and continuous.

A similar argument applies for the $y$ direction. We conclude that for $P$ in a given fixed orientation, there exists a unique position for $P$ such that it is in force equilibrium.

Now consider $P$ at two different orientations $\theta_i$ and $\theta_j$. We claim that the corresponding pivot points $v_i$ and $v_j$ for these orientations must be identical. To see this, suppose $v_i \neq v_j$. Then, due to the rotational symmetry of $R$, $P$ would also be in force equilibrium at orientation $\theta_i$ when $v_j$ coincides with the center of $R$. This contradicts the uniqueness of $v_i$ for a given part orientation $\theta_i$ as proven above.

Finally, we claim that $P$ is in moment equilibrium when $v$ coincides with the center of $R$. Assume that there were a nonzero moment $m$. Due to the symmetry of $R$, this moment would be constant for any orientation $\theta$ of $P$ so long as the pivot point $v$ remains at the center of $R$. Hence during a full rotation of $P$ about $v$, the vector field $R$ would do non-zero work. This contradicts the fact that $R$ is a potential field, in which the work along all closed paths is zero. Hence we conclude that the $P$ is in equilibrium iff the unique point $v$ coincides with the center of $R$. $\square$

Now assume $R$ is combined with a simple squeeze pattern $S$, which is scaled by a factor $\delta > 0$, resulting in $R + \delta S$. The squeeze component $S$ of this field will cause the part to align with the squeeze, similarly to the strategies in Section 3.1. But note that the radial component $R$ keeps the part centered in the force field. Hence, by keeping $R$ sufficiently large ($\delta$ small), we can assume that the pivot point of $P$ remains within an $\epsilon$-ball of the center of $R$. This implies that assumption 2PHASE is no longer necessary. Moreover, $\epsilon$ can be arbitrarily small by an appropriate choice of $\delta$.

**Proposition 6** *For a connected polygon with $n$ vertices there are at most $O(n)$ stable equilibria in a field of the form $R + \delta S$ if $\delta$ is sufficiently small.*

*Proof:* First assume $\delta = 0$, so Proposition 5 applies, that is, the part is in equilibrium iff a specific point $v$ of $P$ coincides with the center of $R$. Now consider a ray from $v$, $w(0)$. Assume w.l.o.g. that $v$ is not a vertex of $P$, and that $w(0)$ intersects the edges $S(0) = \{e_1, \cdots, e_k\}$ of $P$ in general position, $1 \leq k \leq n$. Parameterize the ray $w(\cdot)$ by its angle $\theta$ to obtain $w(\theta)$. As $\theta$ sweeps from 0 to $2\pi$, each edge of $P$ will enter and leave the *crossing structure* $S(\theta)$ exactly once. $S(\theta)$ is updated at *critical angles* where $v(\theta)$ intersects a vertex of $P$. Since there are $n$ vertices, there are $O(n)$ critical angles, and hence $O(n)$ changes to $S(\theta)$ overall. Hence, since between critical angles $S(\theta)$ is constant, we see that $S(\theta)$ takes on $O(n)$ distinct values. Now place the squeeze line $l$ to coincide with $w(\theta)$. For a given crossing structure $S(\theta) \cup S(\theta + \pi)$, satisfying conditions **I** and **II** as defined in Section 3 devolves to solving two algebraic equations of fixed low-degree (see [5]). This implies that between any two adjacent critical values there are only a fixed number of orientations of $l$ (given by $w(\theta)$) that satisfy conditions **I** and **II**. Hence, the overall number of orientations satisfying **I** and **II** is $O(n)$.

If $\delta > 0$ the part $P$ will be perturbed, so that the conditions for Proposition 5 are only approximately met. However, the center of rotation (COR) of $P$ is guaranteed to lie in some small region about $v$. As $\delta \to 0$, this region shrinks to the point $v$. To see this, place $P$ at some arbitrary configuration $z_0$ in the squeeze field $\delta S$. The lateral force on $P_0 = P(z_0)$ is $f_0 = \int_{P_0} \delta S \, dA$. $f_0$ is bounded above by $f_0 \leq \delta A|S|$, where $|S|$ denotes the magnitude of the squeeze field, and $A$ is the area of $P$. Hence, the perturbation introduced by $S$ can be kept arbitrarily small. Thus the COR is constrained to lie in a region $C(v, \delta)$ about $v$. This region can be made arbitrarily small by choosing $\delta$ small enough. We conclude that the number of equilibria in a field $R + \delta S$ is bounded by $O(n)$, for sufficiently small $\delta$. □

In analogy to Section 3.1 we define the turn function $t : S^1 \to S^1$, which describes how the part will turn under a squeeze pattern, and hence yields the stable equilibrium configurations. Given the turn function $t$ we can construct the corresponding squeeze function $s$ as described in Section 3.1. With $s$ as the input for Goldberg's alignment planner, we obtain plans for unique part alignment (and positioning) of length $O(n)$. They can be computed in time $O(n^2)$.

The result is a plan for parts positioning of the form $R + \delta S_1 \ * \cdots * \ R + \delta S_{O(n)}$. Compared to the old algorithm in Section 3.1 it improves the plan length by a factor of $n$, and the planning complexity is reduced by a factor of $n^2$. The planner is complete: For any polygonal part, there exists a plan of the form $*_i(R + \delta S_i)$. Moreover, the algorithm is guaranteed to find a plan for any input part.

## 5.2 Manipulation Grammar

The development of devices that generate programmable vector fields is still in an early stage. The existing prototype devices exhibit only a limited range of programmability. For example, the prototype MEMS arrays described in Section 2 currently have actuators in only four different directions, and the actuators are controllable row-wise. Arrays with individually addressable actuators at various orientations are possible (see [6, 5, 11]) but require significant development effort. There are also limitations on the resolution of the devices given by fabrication constraints. For the vibrating plate device from Section 2 the fields are even more constrained by the vibrational modes of the plate.

We are interested in the capabilities of these constrained systems. In this section we will give an algorithm that decides whether a part can be uniquely positioned with a given set of vector fields, and it gives an optimal-length plan if one exists. If we think of these vector fields as a vocabulary, we obtain a language of manipulation plans. We are interested in those expressions in the language that correspond to a plan for unique posing of the part.

The elements of our "manipulation grammar" are (sequences of) vector fields that bring the part into a *finite* set of possible equilibrium positions. From Section 5.1 we know that combined radial-squeeze patterns $R + \delta S$ have this property. However, there are simpler fields that also have this finiteness property, for example two combined non-parallel squeezes $F + G$, or a sequence of two orthogonal squeezes $F * F_{\perp}$. That is: For any polygonal part $P$, either of these examples is guaranteed to *always* reduce $P$ to a *finite* set of equilibria in its C-space $\mathcal{C} = \mathbb{R}^2 \times S^1$.

Fix a part $P$. Assume that our manipulation grammar consists of $f$ fields $F_i$, and that there are at most $k$ different equilibria for $P$ in each field $F_i$. Then we can construct a transition table of size $f^2k$ that describes how the part moves when the field $F_i$ is applied. This table can be constructed either by a dynamical analysis similar to Section 5.1, or by simulation. The time to construct this table is $O(f^2ks(n))$, where $s(n)$ is the complexity for analysis or simulation, which will typically depend on the complexity of the part (with $n$ vertices). Using the table, we can search for a plan. Finding a plan, or deciding that it exists, requires time $O(2^{f^2k}s(n))$. Hence, as in [9], for any part we can decide whether it can be uniquely posed using the field vocabulary $\{F_i\}$ but (a) the planning time is exponential and (b) we do not know the class of parts that can be oriented by $\{F_i\}$. However, the resulting plans are optimal in size.

This result illustrates a tradeoff between mechanical complexity (the dexterity and controllability of field elements) and planning complexity (the computational difficulty of synthesizing a strategy). If one is willing to build a device capable of radial fields, then one reaps great benefits in planning and execution speed. On the other hand, we can still plan for simpler devices, but the plan synthesis is more expensive, and we lose some completeness properties.

# 6 Conclusions and Future Work

**Uncertainty.** In practice, the force vector field as well as parts geometry will not be exact and exhibit tolerances [7]. This is particularly important at micro scale. Within the framework of potential fields, we can express this uncertainty by considering not one single potential function $U_P$, but rather *families of potentials* that correspond to different values within the uncertainty range. Bounds on part and force tolerances will correspond to limits on the variation within these function families. An investigation of these limits will allow us to obtain upper error bounds for manipulation tasks under which a specific strategy will still achieve its goal.

**Output sensitivity.** We have seen in Sections 5.1 and 5.2 that the efficiency of planning and executing manipulation plans crucially depends on the number of equilibria configurations. In practice, we have found that there are almost no parts with more than two distinct equilibria. If this observation can be supported by an statistical analysis of part shapes, it could lead to extremely good expected bounds on plan length and planning time, even for the less powerful strategies with manipulation grammars.

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## References

[1] K.-F. Böhringer, V. Bhatt, and K. Y. Goldberg. Sensorless manipulation using transverse vibrations of a plate. In *Proc. IEEE Int. Conf. on Robotics and Automation (ICRA)*, Nagoya, Japan, May 1995. URL http://www.cs.cornell.edu/Info/People/karl/VibratoryAlign.

[2] K.-F. Böhringer, B. R. Donald, and N. C. MacDonald. What programmable vector fields can (and cannot) do: Part I — classification and lower bounds for MEMS arrays and vibratory parts feeders. Technical report, Cornell University, Robotics and Vision Laboratory, Ithaca, NY, Oct. 1995. URL http://www.cs.cornell.edu/Info/People/karl/MicroManipulation.

[3] K.-F. Böhringer, B. R. Donald, and N. C. MacDonald. What programmable vector fields can (and cannot) do: Part II — new and improved manipulation algorithms for MEMS arrays and vibratory parts feeders. Technical report, Cornell University, Robotics and Vision Laboratory, Ithaca, NY, Oct. 1995. URL http://www.cs.cornell.edu/Info/People/karl/MicroManipulation.

[4] K.-F. Böhringer, B. R. Donald, and N. C. MacDonald. Single-crystal silicon actuator arrays for micro manipulation. In *Proc. IEEE Workshop on Micro Electro Mechanical Systems (MEMS)*, San Diego, CA, Feb. 1996. Submitted for review. URL http://www.cs.cornell.edu/Info/People/karl/MicroActuators.

[5] K.-F. Böhringer, B. R. Donald, R. Mihailovich, and N. C. MacDonald. Sensorless manipulation using massively parallel microfabricated actuator arrays. In *Proc. IEEE Int. Conf. on Robotics and Automation (ICRA)*, pages 826–833, San Diego, CA, May 1994. URL http://www.cs.cornell.edu /Info/People/karl/MicroManipulation.

[6] K.-F. Böhringer, B. R. Donald, R. Mihailovich, and N. C. MacDonald. A theory of manipulation and control for microfabricated actuator arrays. In *Proc. IEEE Workshop on Micro Electro Mechanical Systems (MEMS)*, pages 102–107, Oiso, Japan, Jan. 1994. URL http://www.cs.cornell.edu /Info/People/karl/MicroActuators.

[7] B. R. Donald. *Error Detection and Recovery in Robotics*, volume 336 of *Lecture Notes in Computer Science*. Springer Verlag, Berlin, 1989.

[8] B. R. Donald. The complexity of planar compliant motion planning with uncertainty. *Algorithmica*, 5(3):353–382, 1990.

[9] M. A. Erdmann and M. T. Mason. An exploration of sensorless manipulation. *IEEE Journal of Robotics and Automation*, 4(4), Aug. 1988.

[10] K. Y. Goldberg. Orienting polygonal parts without sensing. *Algorithmica*, 10(2/3/4):201–225, August/September/October 1993.

[11] W. Liu and P. Will. Parts manipulation on an intelligent motion surface. In *IROS*, Pittsburgh, PA, 1995.

[12] T. Lozano-Pérez, M. Mason, and R. Taylor. Automatic synthesis of fine-motion strategies for robots. *Int. Journal of Robotics Research*, 3(1), 1984.