Employee Stock Option Valuation with an Early Exercise Boundary

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Many companies are recognizing that the Black-Scholes formula is inappropriate for employee stock options (ESOs) and are moving toward lattice models for accounting or decision-making purposes. In the most influential of these models, the assumption is that employees exercise voluntarily when the stock price reaches a fixed multiple of the strike price, effectively introducing a "horizontal" exercise boundary into the lattice. In practice, however, employees make a trade-off between intrinsic value captured and the opportunity cost of time value forgone. The model proposed here explicitly recognizes and accounts for this reality and is intuitively appealing, easily implemented, and compliant with U.S. accounting standards.

Another assumption that Lehman made in valuing Mr. Fuld's pay in 2000 was that he would exercise the options two and a half years after receiving them. But Mr. Fuld had a history of holding onto his options until they were about to expire. Indeed, all 2.6 million options that Mr. Fuld has exercised since November 2003 were in the last year of their life when he did so.

—Patrick McGeehan (2006)

The need to value employee stock options (ESOs) for accounting and economic purposes makes the modeling of employee early exercise behavior relevant for corporate financial officers, security analysts, and those involved in determining option awards. Early exercise by employees reduces the cost of the ESO to the company in relation to an otherwise identical European option (which can be exercised only at the end of its life), so ESO-pricing models must make assumptions to describe in what circumstances an employee will exercise an option before expiration. Examples such as the one in the *New York Times* quotation by McGeehan illustrate the importance of identifying which factors drive early exercise decisions and of making reasonable estimates of the numerical value of those drivers.

A majority of companies have previously used variants of the Black-Scholes option-pricing model to value ESOs, but more and more companies have recognized that these formulas are inappropriate for

ESOs and are moving toward *lattice* models, such as that proposed by Hull and White (2004; henceforth, HW).[1] This trend can be expected to increase because under current financial accounting rules, any company that adopts a lattice model is not permitted subsequently to revert to a Black- Scholes model.[2]

In a much cited and influential paper, HW proposed a *barrier option lattice model* for ESOs that assumes employees exercise voluntarily whenever the stock price reaches a fixed multiple, *M,* of the strike price. Their model is flexible, easily understood and interpreted, and widely used. Conceptually, it introduces into the ESO a "horizontal" boundary; voluntary early exercise occurs if the stock price reaches this barrier, Empirical evidence (e.g., Huddart and Lang 1996) is consistent with the implications of financial theory that employees require the stock to be at a relatively high multiple of the strike price to induce voluntary exercise early in the ESO life, but later in the option life, employees will exercise voluntarily at a relatively low multiple of the strike price.[3] This behavior has implications not only for the appropriateness of the HW model but also for the empirical estimation of the *M* parameter from observations of historical early exercise: Two identical companies with identical employee characteristics could value identical ESO awards significantly differently when using the HW model simply because one company previously enjoyed more rapid stock price growth than the other; that is, the company with the rapid stock price growth experienced a historically higher M in voluntary exercise observations than did the other company.

We propose an alternative model of the voluntary exercise boundary. Our model assumes that employees exercise voluntarily whenever the "moneyness" of the option reaches a fixed *proportion,* u, of its remaining Black-Scholes value. The result is an intuitively appealing "downward-sloping" voluntary exercise boundary, which requires the stock to be at a relatively high multiple of the strike price to induce voluntary exercise early in the ESO life but allows voluntary exercise at relatively low multiples of strike price later in the option life. Our μ model shares the practical simplicity of the M model and can be used to value ESOs in circumstances when the M model can be used. Like HW's *M* model, it readily encompasses such ESO characteristics as vesting restrictions, forfeiture, and forced early exercise as a result of employment termination. Unlike the HW model, our model takes into account throughout the ESO life the early exercise trade-off between *intrinsic value captured* and the opportunity cost of *option value forgone.* Moreover, the empirical estimation of the u parameter from observations of historical early exercise is potentially less susceptible than the estimate for the M model to bias caused by atypical stock price histories.

An approach still used by many practitioners (as exemplified in the *New York Times* quotation) is simply to take an estimate of the ESO's expected life and input this as the option maturity in the Black-

Scholes equation. A lattice model variant of this approach is to assume mat employees exercise voluntarily if a fixed maximum anticipated life, L, is reached. Conceptually, this approach introduces a "vertical" voluntary exercise boundary into the ESO. This L model is a third heuristic (rule-of-thumb) approach to the modeling of the voluntary early exercise boundary that we use for comparison with the performance of the M model and our proposed μ model.[4]

A commonly practiced approach to valuing ESOs is to take the average of the parameters determined from a heterogeneous population to model a "representative investor." This approach is conceptually incorrect for the L model and the M model (and causes potentially significant bias in the *M* model). The reason is that prices produced by the *Stock Option Valuation with any Early Exercise Boundary* L model and M model are concave in, respectively, L and M, so the model price based on the average parameter of the population is greater than the average of the model prices found when each parameter is used.

In contrast, in the absence of vesting restrictions, forfeiture, and employment termination-related exercise, the μ model has a simple closed-form solution that is linear in μ. We show that the grant-date value of an ESO when the employee exercises early if (and only if) she captures a fixed proportion (μ) of the remaining Black- Scholes value is equal to the same proportion (μ) of the grant-date Black-Scholes value of an otherwise identical European option. Even in the presence of vesting restrictions and employment termination, the μ-model lattice gives option prices that are nearly linear in *μ;* therefore, the use of a representative investor approach is potentially justified for the μ model—even for option grants for heterogeneous populations of employees.

Our analysis also reveals how the particular *price history* of a stock can systematically bias the valuations that an *L* or *M* model produces when historical data are used to estimate parameters for the models. The reason is as follows: Employees of a company that has experienced high stock returns will tend to have voluntarily exercised options relatively early in the ESO life but at stock prices representing relatively high multiples of the strike price. The L's measured in this scenario will be low, and the measured M's will be high. If these low L's are used as inputs to value new ESO grants, they are likely to underestimate the true cost of the options. If the high M's are used, they will overestimate ESO value. Conversely, a company that has had sluggish stock returns will have experienced voluntary option exercises relatively late in the term and at relatively low multiples of the strike price, with corresponding consequences if those data are used to estimate parameters for an L model or an M model.[5] In contrast, Bettis, Bizjak, and Lemmon (2005; henceforth, BBL) found that variation in *Ratio,* which is their measure of the proportion of the remaining Black-Scholes value captured at early exercise (our μ) across years, is much smaller than the variation in the degree of early exercise. Their result provides further motivation for our proposed pricing model.

We show how any one of the L, M, and u models may, depending on the particular historical data of voluntary early exercises used to calculate parameter inputs, give the highest or lowest estimate of ESO price. In general, however, no objectively "correct" option price exists against which to measure each model's output for accuracy. We adopted a rational model of voluntary early exercise behavior (used in Carpenter 1998 and BBL) to serve as a benchmark.[6] This model is intuitively appealing, has been well studied in the literature, and has been shown to adequately describe qualitative and quantitative characteristics of empirical data. It uses the binomial lattice to depict the voluntary early exercise decision for an ESO held by a utility-maximizing employee whose outside wealth is invested in an optimal portfolio of stock and bonds. At every node in the lattice, the risk-averse employee compares the expected utility from immediate exercise with the expected utility from waiting for at least one more period. Voluntary exercise is triggered if or when utility from immediate exercise is expected to be the greater— which enabled us *to* obtain the voluntary exercise barrier for an ESO held by that employee and, hence, to calculate an *objective* value for the ESO.

Furthermore, we generated the full probability distribution of possible individual "price histories" within the lattice and calculated a full distribution of what L, M, and u an analyst could potentially measure from the associated voluntary early exercises. Using these parameter values in the respective heuristic models, we could then analyze those models' accuracy relative to the objective option value.

Heuristic Models of a Voluntary Early Exercise Boundary

To compare the models, we use the following notation: A company's stock price S evolves with lognormal returns over time t in the interval [0,T] with volatility σ. An ESO is issued at time $t = 0$ with maturity T years. The option has strike price X and is issued at the money. We normalize the initial stock price so that S_0 is equal to X, which is equal to 1. The risk-free interest rate is r. The stock price process may have a drift reflecting a positive risk premium (we use 12 percent drift in our examples), but all valuation of stock options is performed under risk-neutral probabilities. The stock has a constant dividend yield d. We denote the Black- Scholes model value of a European call option with these same parameters as $B\{S, X, r, d, \sigma, T, t)$ but suppress certain parameter arguments when convenient. The employee's employment may terminate for exogenous reasons at any time with probability q per year.[7] The option may have a vesting date, v, which would be less than T—that is, $v < T$. If employment terminates before time *v,* the option is forfeited. If employment terminates between vesting and expiry, the option is exercised (in the money) or forfeited {out of the money).[8]

We study each option valuation model here in the context of a standard binomial lattice model that can easily be adapted to encompass state-contingent σ, *r* (e.g., term structures of volatility and interest rates), *d, q,* and *v* (e.g., performance vesting). The option can be exercised voluntarily by the employee at any time *t* in the interval *[v,T].*

The three models differ only in the modeling of a voluntary early exercise boundary. As the stock price evolves over time, voluntary early exercise occurs if it reaches the boundary. An *L* model has a vertical barrier at $t = L$. An M model has a horizontal barrier at $S = M$. As we show, a μ model has a downward-sloping barrier converging to $S = 1$ at $t = T$.

"Adjusted Maturity" Approach (an *L* Model). The lattice version of this model effectively assumes that at time $t = L$ (if the option has not already been forfeited or exercised early as a result of termination), an employee will voluntarily exercise (if $S > 1$) or the option will lapse (if $S < 1$). In the absence of vesting restrictions and employee termination, the ESO price obtained from an *L* model, *A(L),* is simply the Black-Scholes value, *B(L),* with modified option life L.

HW "Multiple of Strike Price" Approach (an *M* Model). HW assumed that the employee will voluntarily exercise early whenever the stock price reaches a fixed multiple, M, of the option strike price. This assumption implies a simple horizontal boundary at S = M. The presence of a vesting date, *v,* means that exercise cannot occur at times before *v* even if $S > M$ at those times; so, the boundary is vertical at $t = v$, implying that "pent-up" exercise activity may occur at $t = v$ at stock prices strictly greater than M. At expiry, $t = T$, all in-the-money options will be exercised, so the boundary is vertical at $t = T$, corresponding to exercises that occur at prices between 1 and M. Exercises that occur at times *v* or *T* (or at any time as a result of employee termination) are necessarily constrained; therefore, they should not be included as "voluntary exercises" in data from which the analyst is making estimates of M.

Clearly, M is greater than 1 because only in-the-money options are exercised. HW performed illustrative option valuations with M in the range of 1.2 to 3.0. Several studies have calculated the ratio of stock price to strike price, S/X, for data from observed early exercises: BBL's sample of corporate insiders has a median of 2.57 and a mean of 3.55. Carpenter's (1998) sample of executives has a median of 2.47, a mean of 2.75, and a standard deviation of 1.42. Huddart and Lang (1996) analyzed a much broader sample of lower-level employees than Carpenter or BBL analyzed and found a median of 1.6 and a mean of 2.2. Institutional Shareholder Services (ISS), a corporate governance and proxy-voting services firm, uses the HW model to compare option awards among companies (ISS was acquired by

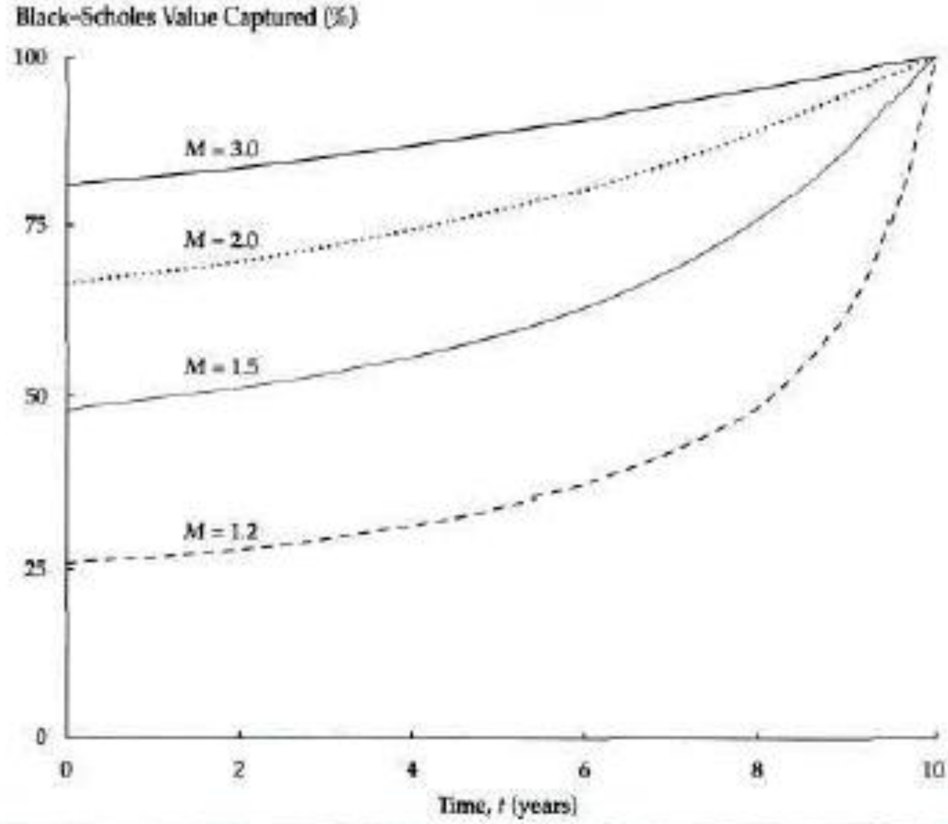
RiskMetrics Group in January 2007). To all awards, the firm applies a uniform assumption of M = 2, which the firm states is the median exercise ratio for S&P 1500 Index companies.

We denote by *H(M)* the HW model price of an ESO with voluntary exercise boundary multiple M. Prices are concave, increasing in M, and rapidly approaching the Black-Scholes value as M increases beyond about 4.0.

Before developing our own model of voluntary exercise based on the proportion of remaining option value captured, we note at this point what proportion of Black-Scholes value captured by an M policy implies. For fixed *M,* exercise of an option captures a dollar value of M - 1 , so the proportion of remaining Black-Scholes value captured at time *t* is simply *(M-1)/B(M,T-t).* For example, with parameter values S0 = X = 1, *T* = 10, *v* = 0, *d* = 0, *q* = 0, r = 5 percent, and a = 40 percent, Figure 1 shows that an employee with a voluntary exercise boundary at, say, M = 2 would capture only 65 percent of the remaining option value if the stock price reached that level early in the option life. Yet, for later exercises, less time value would remain and the proportion of remaining option value captured would approach 100 percent as *t* approached maturity. An employee with an exercise policy of M = 1.2 could initially capture as little as one-quarter of the remaining Black-Scholes value.

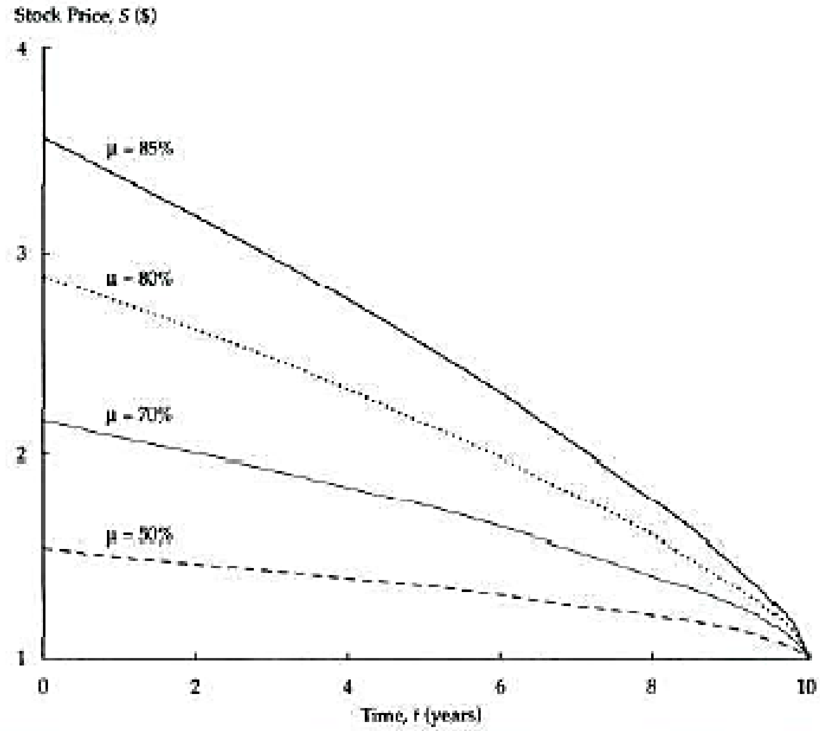
"Proportion of Option Value Captured" Approach (a μ, Model). As illustrated in Figure 2, the exercise boundary for an employee who exercises whenever he can capture a fixed proportion μ of the remaining Black-Scholes value of the ESO is downward sloping, which reflects the intuition that the employee requires a lower moneyness to trigger exercise later in the option life than earlier.[9] The presence of a vesting date truncates the boundary, introducing a vertical portion at *t* = *v* and implying that pent-up exercise activity can occur at *t* = *v* at stock prices strictly greater than that implied by a μ policy. Exercises that occur at times *v* or *T* (or at any time as a result of employee termination) are necessarily constrained, so they should not be included as voluntary exercises in data the analyst is using to make estimates of μ.

Figure 1. Proportion of Remaining Black–Scholes Value Captured by Early Exercise at Stock Price Multiple *M*, as a Function of Time of Exercise



Note: Option parameter values are $S_0 = X = 1$, $T = 10$, $v = 0$, $d = 0$, $q = 0$, $r = 5$ percent, and $\sigma = 40$ percent.

Figure 2. Exercise Boundary for ESO When Employee Has a Fixed μ Policy for Early Exercise



Note: Option parameter values are $S_0 = X = 1$, $T = 10$, $v = 0$, $d = 0$, $q = 0$, $r = 5$ percent, and $\sigma = 40$ percent.

In a database of 140,000 option exercises by corporate executives at almost 4,000 companies during the 1996-2002 period, BBL's most striking finding was that "conditional on exercise, executives in our sample capture a large fraction of the remaining Black-Scholes option value" (p. 447). They denote this fraction *"Ratio,"* which was 90 percent at the median and 84 percent at the mean for their whole sample.

We use K(μ) to denote the grant-date cost of an ESO with continuous dividend yield when the voluntary exercise policy is described by the fixed proportion n of the remaining Black-Scholes value of the option that exercise would capture for the employee.

- **Proposition:** In the absence of vesting restrictions and employment termination, K(μ) is equal to the same proportion (μ) of the Black- Scholes value of a similar European option:

(1)

$$K(\mu) = \mu B(1,T).$$

- **Proof:** The grant-date value of a number μ=(0,1) of at-the-money European call options is μB(1,T). These options have the same value whether they can be sold before maturity or not. In particular, suppose they will be sold if or when the stock reaches a price S > 1 and at a time *t < T th*at satisfies the following equation:

(2)

$$S - 1 = \mu B(S, T - t).$$

Note that because μB (S,T -t*)*→ 0 as t→*T,* such a price and time are inevitably reached unless the option expires out of the money. The proceeds from such a sale would be the option's remaining Black-Scholes value, μB(S,T - t), which, because of Equation 2, equals S - 1 , the option's moneyness. But this package replicates precisely the payoffs to one option that automatically exercises and pays S - 1 at time *t* whenever S - 1 = μB(S,T *-t)*—that is, the very definition of an ESO with an exercise barrier described by our u model, Equation 2. Because the payoffs are identical, the grant-date value of the ESO is also μB(l,T), the Black-Scholes value of a number *μ* of similar European call options.

The existence of a closed-form solution enabled us to benchmark the accuracy of our lattice model. With the parameters $S_0 = X =$ \$1, r = 5 percent, T = 10, d = 0, q = 0, v = 0, σ = 40 percent, and μ = 85 percent, we ran the lattice model with 2,500 steps and obtained the option price of \$0.51332, correct to within 0.4 percent of the closed-form model price of \$0.51132. Figure 3 illustrates model prices obtained for this option for a range of μ's. Also shown is the same option but with q = 3 percent and v = *2* years, and the result is well approximated as a linear function, particularly when μ is in the empirically "reasonable" range of 60-90 percent.

Comparison of *L, M,* and μ Models

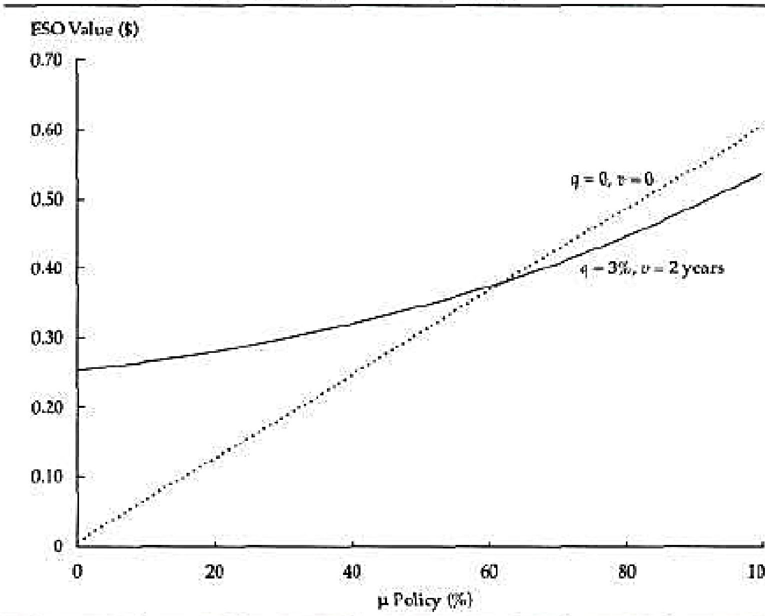
To carry out our comparison of how the three model types perform in valuing ESOs, we begin with how the use of a "representative investor" approach affects the models' performances. We then use a numerical example to illustrate the potentially different ordering of prices obtained from the models, present a more general analysis, and measure the accuracy of the model outputs against an objective option value.

Calculation of Model Prices for a Representative Investor. ESO models commonly use the notion of a representative investor, whose characteristics (e.g., the parameter L or M) are then assumed to hold for all the employees to whom a particular option tranche is granted. This approach produces biases in the L model and the M model, however, because they are concave in, respectively, *L* and M.

In the context of an *L* model, for heterogeneous employees (i.e., those with different L's), the model price of the average (i.e., representative) employee is *strictly greater than* the average of the model prices of each employee. The price of a four-year option and an eight-year option combined is *not* the same as the price of two six-year options, even though many ESO-pricing methodologies implicitly assume that it is.

This same effect is even more striking, quantitatively, when employee behaviors are described by M. Consider the following scenario: The parameters are S0 = X = 1, *r* = 5 percent, *T* = 10, *d* = 0, *q* = 0, *v* = 0, and a = 40 percent. Employee A has a voluntary exercise multiple of M = 1.5, and Employee B has a voluntary exercise multiple of M = 3.5. The M-model price of a representative employee with a voluntary exercise multiple of *M* = (1.5 + 3.5)/2 = 2.5 is $0.50069, whereas the average of the two employees' M-model prices is (0.31167 + 0.55358)/2 = $0.43262. Therefore, in this case, a representative investor approach leads to overvaluation of the options by a factor of 0.50069/0.43262 - 1.1573 (i.e., the value is 16 percent "too high"). Representative investor models are biased upward when used to value ESOs in an *L* model or an M model.

Figure 3. ESO Value by Probability of Termination and Vesting Date, as a Function of μ Policy for Early Exercise



Note: Option parameter values are $S_0 = X = 1$, $T = 10$, $d = 0$, $r = 5$ percent, and $\sigma = 40$ percent.

In contrast, and to the extent that the prices obtained from a u model are linear in u, the u-model price of a representative employee is unbiased. Even when vesting restrictions and termination are present (e.g., with q = 3 percent and $v = 2$ years, as in Figure 3), the bias induced by taking the model price of the mean of voluntary exercise factors is not significant. Hence, a representative investor approach is potentially justified for the µ model, even for option grants among heterogeneous populations of employees. Nevertheless, for consistency with the *L* and M models, in our subsequent calculations, we calculate the *mean of model prices* rather than the model price of mean parameters.

Relative Prices Obtained from the Models.

In analyzing the performance of the three heuristic models, we wish *to* understand how they compare when we use data from the same source. The main intuitions are first illustrated with a simple numerical example:

Consider Company X and Company Y, which are *{ex ante)* identical companies with constant volatility of 40 percent per year and identical expected returns; the risk-free rate is 5 percent per year. Each company issued a single at-the-money *10-year ESO* when its (normalized) stock price was $1.

Company X's stock price enjoyed rapid growth, and the ESO was exercised after 1.89 years when the stock price was $2.90.

Company Y's stock price experienced more sluggish growth, and the ESO was exercised after 9.87 years when the stock price was $1.45.

These examples exhibit the well-known empirical regularity that employees require a higher stock price to voluntarily exercise early in the life of the ESO than they require later in the option life. In fact, these two voluntary exercises were generated by simulation of the early exercise decisions of *identical* rational risk-averse investors (from a well-known utility-based model that we introduce in the next subsection). The investors' endogenously determined exercise policies imply an *objective (ex ante)* "true" option valuation of $0.5214 for ESOs issued by Companies X and Y (normalized to an at-the-money strike price of $1).

An analyst who is trying to value new grants of stock options for Companies X and Y might turn to these historical exercise data to determine inputs into either an *L* model or an *M* model. If the analyst uses the realized lives of the options, 1.96 and 9.76 years, respectively, in an L model, the value ascribed to further ESO grants (normalized to an at-the- money strike price of $1) will be $0.25 for Company X or $0.60 for Company Y. If the analyst uses the realized exercise multiples of the options, 3.02 and 1.63, respectively, in an *M* model, the value ascribed *to further* ESO grants will be $0.53 for Company X or $0.29 for Company Y. New option grants *objectively* have identical values for Companies X and Y, yet an

L model values Company X's options "low" and Company Y's options "high" whereas an *M* model does the reverse.

In comparison, if the analyst takes the realized times and stock prices at voluntary option exercise to calculate the percentage of the remaining Black- Scholes value captured for each company and uses that information as inputs into a u, model, the value ascribed to further option grants will be \$0.50 for Company X and \$0.59 for Company Y.[10]

More generally, the realized times and stock prices of *any* observed voluntary option exercise can be used to define an L policy (L = *t),* an M policy (M = S), and a n policy [μ = (S -1)/*B(S,T-t)].* Thus, three alternative heuristic model valuations can be calculated—namely, Λ(L), *H(M),* and K(μ). But we want to discover which model will give the highest or lowest estimate and in which circumstances.

Figure 4 shows the combinations of times and stock prices (expressed as multiples of strike price) at which observed voluntary option exercise data will yield A(L) = *H(M), H(M)* = K(μ), and K(μ) = Λ(L). These three curves allow us *to* identify regions where the resulting model prices will be differently ordered and confirm the intuition of the motivating example just given. Note, in particular, that voluntary exercise observations occurring substantially early in the life of the option and at all but the very lowest stock prices cause the M model to yield the highest valuations. Conversely, exercise data occurring substantially late in the life *of* the option and at lower stock prices cause the M model to yield the lowest valuations. Valuations using historical exercise observations will generally have many more data points than in our example, but a particular company can have experienced only one stock price history, so the resulting exercise data are necessarily drawn from that stock price path and are vulnerable to the biases described.

Performance of Heuristic Models against an Objective Option Value: We have shown how the analyst can take real historical observations of voluntary exercises within a company, calculate the implied L, M, and μ, and in this way, calculate the heuristic model prices for further ESO awards. Determining which model gives the highest or lowest valuation for any given dataset is easy, and all permutations of ranking are possible. Because each company has necessarily experienced only one price history, however, we cannot know the "true" exercise policy that has generated the early exercises we have observed; thus, we have no benchmark against which to measure the accuracy of one model versus the accuracy of the others.

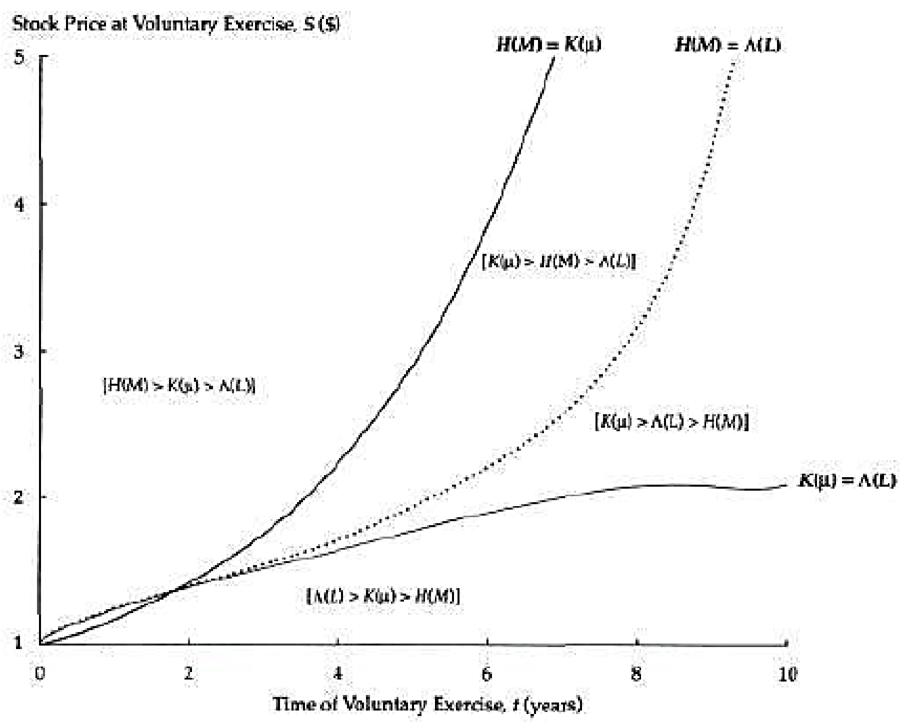
To address this issue, we introduce now a well-known utility-based model of rational early exercise that was used by, among others, Carpenter (1998) and BBL. The model uses a traditional binomial lattice to describe the possible price paths of the underlying stock and calculates the voluntary exercise decisions of a rational utility-maximizing employee. The employee has a power utility function

(3)

$$U(w) = \frac{w^{1-A}}{1-A},$$

where w is terminal wealth and *A* is the risk-aversion parameter.

Figure 4. Early Exercise Observations (t, S) That If Used to Generate Estimates for L, M, and μ Would Produce Model Prices $\Lambda(L) = H(M)$, $H(M) = K(\mu)$, and $K(\mu) = \Lambda(L)$



Notes: Regions in between the curves are labeled with the relative ordering of model prices when voluntary exercise observations from those regions were used to generate estimates for L, M, and μ. Option parameter values are $S_0 = X = 1$, $T = 10$, $v = 0$, $d = 0$, $q = 0$, $r = 5$ percent, and $\sigma = 40$ percent.

The employee has initial outside wealth *x* that is invested dynamically in the optimal Merton (1971) portfolio of stocks and bonds.[11] The lattice can also take into account the effects of employment termination, dividends, and vesting characteristics. At every node in the lattice, the employee compares the expected utility from exercising the stock option with the expected utility from waiting at least one more period to exercise.

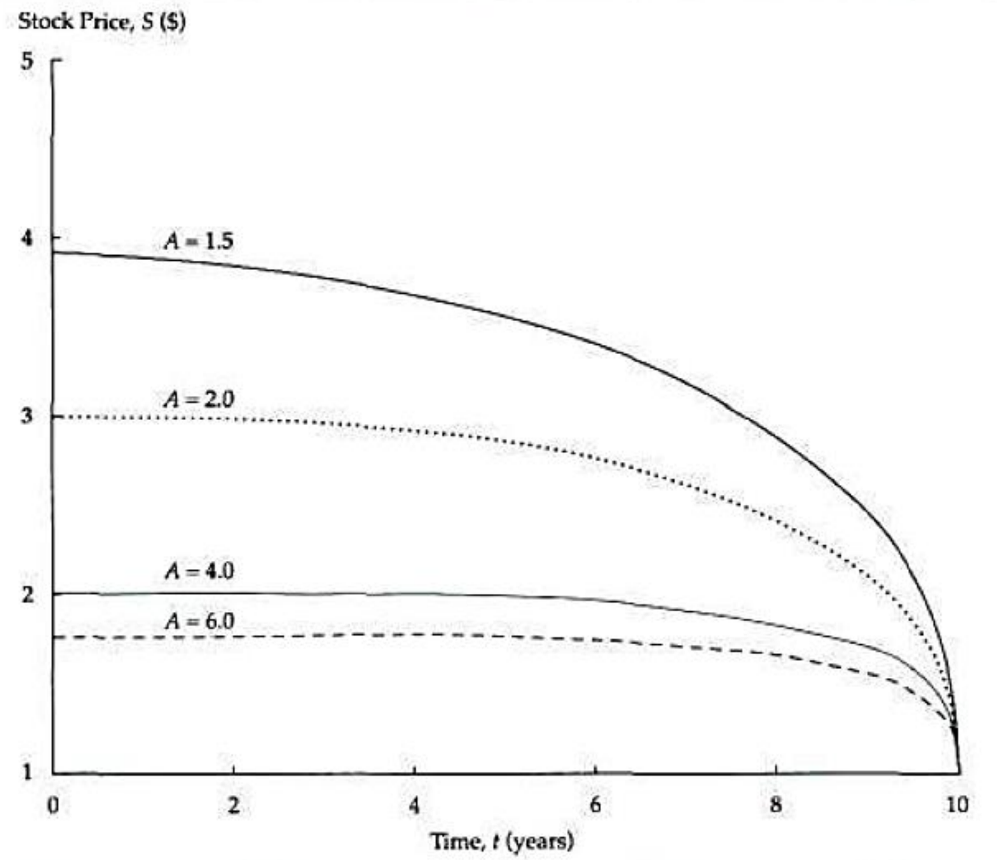
We have followed the approach in Carpenter (1998) and BBL to derive the endogenously determined exercise policy—a voluntary early exercise boundary—permitting us to determine objectively and unambiguously the grant-date value of the ESO held by this employee. We emphasize that our purpose for the utility-based model is *not* to come up with option values for real data. Indeed, practical implementation of the Carpenter model requires parameters that are unobservable. Rather, by making explicit assumptions about these parameters, we can use the Carpenter model to come up with objective option values for a known, albeit theoretical, early exercise policy. It also allows us to consider the exercise behavior under this known policy for all potential stock price paths—a luxury not afforded by real data because each stock necessarily has only one price history. Thus, we can use the model to generate the full probability distribution of possible price histories and calculate a full distribution of what L, M, and μ the analyst could potentially measure from the associated voluntary early exercises. Using these parameter values in their respective heuristic models, we then analyze their accuracy relative to the objective option value.

Table 1 reports results for a range of risk-aversion parameters and option volatilities. The base-case example of risk-aversion parameter $A = 2.0$ and initial outside wealth $x = 2.1$ is the combination of parameters identified by BBL as most closely explaining the overall early exercise behaviors in their extensive dataset. For this base-case employee, we used the lattice to determine the voluntary early exercise boundary for the ESO— that is, the stock price (expressed as a multiple of strike price) that would induce voluntary early exercise at the given time. As shown in Figure 5, the exercise boundary is downward sloping, which is typical of those illustrated in Carpenter, Stanton, and Wallace (2007). This boundary implies an exercise multiple of about M = 3.0 early in the option's life. The boundary is concave and decreasing over time and reaches M = 1.0 at ESO maturity.

Each possible voluntary early exercise point on the boundary was used to generate model valuations $\Lambda(L)$, $H(M)$, and $K(\mu)$, as described previously. These price estimates confirm the intuition gained from comparing the exercise boundary in Figure 5 with the regions in Figure 4: The boundary is such that for voluntary exercises at small t, the M model gives the highest price whereas the L model gives the lowest. And the ordering of the prices changes as we move later in the exercise boundary and cross successively into the different regions of Figure 4.

Within the lattice, conditional on voluntary exercise occurring, we calculated the conditional probability that exercise will occur for every point on the boundary. Weighting the model prices at each boundary point with the conditional probability of voluntary exercise occurring at that boundary point, we calculated the expected value of the model prices and report in Table 1 their percentage deviations from the objective option value (in this case, \$0.5214) as $E[\Lambda(L)]$%error, *E[H(M)]% error,* and $E[K(\mu)]$% error. The results indicate that if the analyst had a dataset of exercises by this employee from *all* *possible price histories* and used each exercise to estimate model prices, the means of the L-model and M-model price estimates would undervalue the ESO by 15.7 percent and 11.6 percent, respectively, as shown in the shaded column of Panel A of Table 1. The mean of the n-model price estimates would overvalue the ESO by 3.0 percent.

Figure 5. Voluntary Exercise Boundaries for Utility-Maximizing Employee with Power Utility Function for Various Risk-Aversion Parameters



Notes: ESO parameter values are $S_0 = X = 1$, $T = 10$, $v = 0$, $d = 0$, $q = 0$, $r = 5$ percent, and $\sigma = 40$ percent. Initial outside wealth is $x = 2.1$.

Table 1. ESO Value for Varying Risk Aversion and Volatility

A. Varying risk aversion, A

Measure of deviation	$A = 0.5$	$A = 1.0$	$A = 1.5$	$A = 2.0$	$A = 3.0$	$A = 4.0$	$A = 5.0$	$A = 6.0$	$A = 7.0$	$A = 8.0$
ESO value	\$0.6015	\$0.5790	\$0.5475	\$0.5214	\$0.4768	\$0.4411	\$0.4119	\$0.3870	\$0.3649	\$0.3464
$E[H(M)]$%error	−25.5%	−20.7%	−15.0%	−11.6%	−6.9%	−4.9%	−3.7%	−2.9%	−2.3%	−1.9%
$E[K(\mu)]$%error	0.0	1.6	2.6	3.0	3.4	3.7	3.9	4.0	4.0	4.1
$E[\Lambda(L)]$%error	0.0	−7.1	−12.6	−15.7	−19.0	−20.2	−20.7	−20.8	−20.7	−20.4
$H(M)$%MSD	38.2	35.4	29.7	26.4	19.7	16.5	14.3	12.7	11.4	10.5
$K(\mu)$%MSD	0.0	2.9	5.4	7.2	9.9	12.0	13.6	14.9	16.1	16.9
$\Lambda(L)$%MSD	0.4	16.2	24.2	28.9	34.6	38.2	40.7	42.8	45.0	46.6

B. Varying volatility, σ

Measure of deviation	$\sigma = 25\%$	$\sigma = 30\%$	$\sigma = 35\%$	$\sigma = 40\%$	$\sigma = 45\%$	$\sigma = 50\%$	$\sigma = 60\%$	$\sigma = 70\%$	$\sigma = 80\%$	$\sigma = 85\%$	$\sigma = 90\%$	$\sigma = 100\%$
ESO value	\$0.4845	\$0.4914	\$0.5048	\$0.5214	\$0.5375	\$0.5525	\$0.5740	\$0.5896	\$0.5993	\$0.6027	\$0.6054	\$0.6082
$E[H(M)]$%error	−18.5%	−15.6%	−13.0%	−11.6%	−8.5%	−6.1%	−4.3%	−2.6%	−1.7%	−1.3%	−0.8%	−0.5%
$E[K(\mu)]$%error	0.2	1.9	2.6	3.0	3.0	3.0	3.1	2.8	2.4	2.2	1.8	1.4
$E[\Lambda(L)]$%error	−2.4	−9.9	−13.5	−15.7	−17.9	−19.7	−21.8	−23.6	−25.0	−25.7	−26.2	−27.0
$H(M)$%MSD	30.4	29.9	27.1	26.4	21.7	17.3	15.2	11.4	10.1	8.6	6.5	5.4
$K(\mu)$%MSD	1.1	4.6	6.0	7.2	7.9	8.5	9.8	10.0	9.9	9.6	9.2	8.4
$\Lambda(L)$%MSD	8.0	20.3	25.5	28.9	31.6	33.8	37.1	39.3	41.0	41.7	42.0	42.8

Notes: ESO parameter values are $S_0 = X = 1$, $T = 10$ years, $r = 5$ percent, $d = 0$, $q = 0$, $w = 0$, stock price drift = 12 percent, and $x = 2.1$. In the base case, $A = 2.0$ and $\sigma = 40$ percent.

Because exercise data are in reality often drawn from a single price history of the company, a more relevant measure of the expected accuracy of each model may be the mean squared deviation (about the objective ESO value) of all possible model estimates drawn from single points on the boundary. We report the mean squared deviations of these estimates for the L, M, and n models, respectively, as shown in Panel A of Table 1 for the base-case parameters, as *Λ(L)%MSD* = 28.9 percent, *H(M)%MSD* = 26.4 percent, and *K(μ)%MSD = 72* percent. By this metric, and for these reasonable parameter values, the u model is by far the most reliable; the *L* and *M* models are vulnerable to potentially large valuation errors, but the μ model is quite stable even when exercise data are from stock price histories exhibiting particularly rapid or slow growth.

Panel A of Table 1 also shows that as the risk-aversion parameter increases, the M model becomes less negatively biased and its mean squared deviation of estimates shrinks. As illustrated in Figure 5, increasing the risk-aversion parameter *decreases* the stock price required to trigger voluntary exercise. In extreme examples of risk aversion, the employee would be so risk averse as to be willing to exercise at comparatively low stock prices even at the very beginning of the term, implying early dissipation of substantial amounts of Black-Scholes value. The exercise boundary of such an employee would necessarily be close to horizontal, and Panel A shows that the M model would perform better than our μ model at extreme risk aversions—those above about *A* = 6. Keeping the outside wealth fixed (as we have done), BBL showed that increasing *A* by 50 percent (i.e., moving from *A = 2* to *A* = 3) significantly reduces the ability of the power utility model to explain the data. So, increasing *A* by 300 percent (i.e., moving from *A = 2* to *A* = 6) is indeed an extreme assumption.

Panel B of Table 1| shows that as volatility increases, the performance of the µ model remains respectable and that of the M model becomes less negatively biased and its mean squared deviation of estimates decreases. Eventually, when volatility reaches about 85 percent, the mean squared deviation of estimates of the M model becomes lower than that of the u model, but that volatility assumption is extraordinarily high for the 10-year duration of an ESO. An artifact of the Carpenter (1998) model is that at very high volatilities, the voluntary exercise boundary can eventually become hump shaped (i.e., increasing and then decreasing), as illustrated in Figure 6.[12] As a result, the horizontal *M* boundary can induce smaller estimation errors than can the monotonically decreasing boundary of a µ model.

For reasonable specifications of employee risk-aversion parameters and ESO characteristics, our u. model performs significantly better than the *M* model. Although combinations of risk-aversion parameters and ESO characteristics can be constructed to make the voluntary exercise barrier sufficiently horizontal (or even humped) for the M boundary to induce smaller errors than can the decreasing boundary of a µ model, doing so requires assumptions about employee risk aversion and volatility of stock returns that do not fit our understanding of the empirical data.

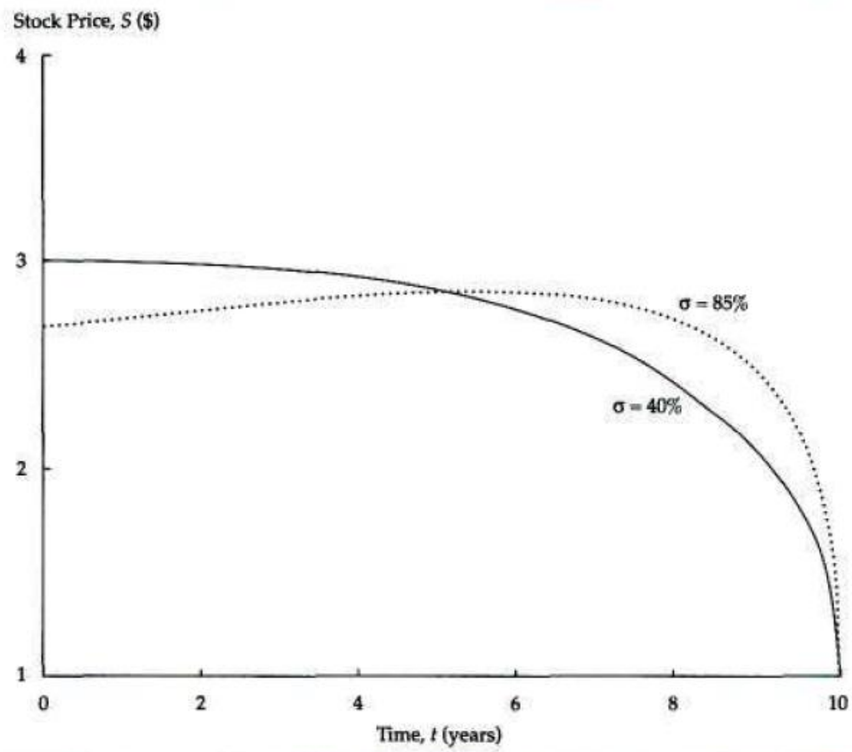
Conclusion

Empirical evidence and financial theory suggest that employees considering exercise of an ESO make a trade-off between option value captured and time value forgone. We have proposed a new ESO-pricing lattice model that explicitly recognizes and accounts for this reality. We demonstrated the properties of our u model and showed why it is less prone to bias than an *L* or *M* model when parameter inputs are calculated from historical observations of voluntary exercise behaviors.

Using a well-known power utility model of employee exercise behavior (Carpenter 1998) as a benchmark confirmed our intuitions. Depending on the stock price path, L, M, and u estimated from a single voluntary exercise can lead to any of the three heuristic models yielding the lowest or highest ESO price estimate. On average, however, across all possible price paths, *the* µ *model most accurately approximates the objective ESO values* generated by the Carpenter model. Because the µ values determined from past exercise observations are less sensitive to the particular historical stock price path traveled than are L and M values, estimates from the u model have much smaller mean squared deviations from objective ESO values for reasonable parameters of risk aversion and option specifications. The *L* model (or modified Black-Scholes model) is particularly prone to significantly underestimating option values. The M model may be adequate for practical purposes in some circumstances, but it is potentially vulnerable to significant bias whenever available historical data come from a period of strong or weak stock returns.

The μ model is intuitively appealing, easily implemented, and compliant with Statement of Financial Accounting Standards No. 123 (revised), *Share-Based Payment* Whereas the HW model requires the assumption that voluntary early exercise occurs at some fixed multiple of strike price, our model assumes that voluntary exercise occurs when the employee can capture some fixed proportion of the remaining option value. When appropriate, those parameters can be estimated from historical data. In this regard, we showed why the μ model may perform more consistently than the HW model.

Figure 6. Voluntary Exercise Boundaries for Utility-Maximizing Employee with Power Utility Function for Two Volatility Levels



Note: $A = 2.0$, $x = 2.1$, $T = 10$, $v = 0$, $d = 0$, $q = 0$, and $r = 5$ percent.

Our results have management implications for compensation committees and consultants who need to understand the potential economic cost of ESO awards to executives and employees. The findings are also relevant to the work of practitioners who must select ESO valuation models for accounting disclosure purposes. Academic researchers and other users of financial statements will also find our results important for understanding the sensitivity of disclosed figures to the choice of valuation model and to the estimation of parameter values—which are themselves dependent on the chosen historical dataset.

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[1] A lattice model divides time between grant and option expiration into N discrete periods. At each time *n,* possible changes in the underlying stock price between times n and n+ 1 are captured by two (or three) branches. This process is iterated until every possible path between $n = 0$ and $n = N$ is mapped. Probabilities are men estimated along every stock price path. The stock option payoff outcomes and probabilities flow backward through the tree until a fair value of the option today is calculated. A simple lattice model for options is the binomial option-pricing model.

2 Statement of Financial Accounting Standards No. 123 (revised), Share-Based Payment (FAS 123R), makes compulsory the expensing of ESOs in the United States as of fecal years commencing after 15 June 2005. Similar rules have been implemented by the International Accounting Standards Board and the Canadian Accounting Standards Board. FAS 123R permits the use of modified Black-Scholes methodologies but also envisages the use of

lattice models; the standard specifically cites the HW model and illustrates its use with numerical examples. U.S. SEC Staff Accounting Bulletin No. 107 also refers to the HW model.

[3] See Huddart (1994); Carpenter (1998); Detemple and Sundaresan (1999).

[4] Some practitioners using a Black-Scholes approach do not distinguish between voluntary early exercise and termination-related exercise *or* forfeiture. Rather, they combine these factors into a single expected-life estimate. We do make the distinction here, however, to ensure comparability of the L model with the M and u models.

5 In confirmation of this intuition, Carpenter (1998) found that "at firms with strong overall stock price performance, options are exercised deeper in the money" (p. 139). Bettis, Bizjak, and Lemmon (2005) found "a steady increase in the degree of early exercise paralleling the increase in stock market valuations" (p. 457). Yet, they also state that "corresponding to the decline in the stock market, we observe options being exercised later" (p. 457).

[6] Carpenter's model is an extension of earlier work by Huddart (1994) and Kulatilaka and Marcus (1994).

7 . Adapting the lattice to make quit rate q endogenous is a simple matter, as in, for example, Cuny and Jorion (1995).

[8] FAS 123R distinguishes between preventing and postvesting terminations. The assumption of a positive quit rate postvesting permanently reduces the accounting value of each option granted. Anticipated prevesting termination, however, is ignored in calculating the accounting value for each option granted. Rather, anticipated prevesting termination is taken into account for accounting purposes by reducing the number of stock options that are initially recognized to the number expected to vest. The resulting total anticipated accounting cost is the same as if prevesting termination were taken into account in the valuation of each option. The initial expensing effect is precisely the same as if the model value took into account a positive quit rate and did not adjust the number of options downward. The subtle difference is that FAS 123R requires annual adjustment of the accounting expense, in which companies must adjust the originally anticipated number of options that will vest to the actual number of options that will vest. In this article, we are interested in comparing the total anticipated accounting cost (which corresponds to the expected economic cost), so we do not distinguish between pre- and postvesting termination. A trivial adjustment to our model gives the correct FAS 123R treatment.

[9] The boundary satisfies the equation $S - 1 = \mu B\{S,T-1)$. For computational simplicity, we measured the full Black-Scholes option value over the remaining term. Alternatively, the lattice itself could be used to determine the remaining option value.

[10] The illustrative "past transactions" in this example were generated by the lattice model, for which we already know what stock return volatility to use. With real data, estimates of μ require estimates of the volatility parameter. BBL are silent, however, about which method they used to calculate volatility for their Ratio statistic. Several approaches are possible—namely, historical volatility measured in the period prior to exercise, implied volatility from comparable traded options or LEAFS (long-term equity anticipation securities), or the realized volatility of stock returns in the period after the exercise transaction until the option's original maturity date.

[11] Carpenter (1998) generalized the power utility approach of Huddart (1994), who assumed that outside wealth is invested only in the risk-free asset.

[12] Carpenter et al. (2007) noted this possibility for certain parameters of risk aversion and volatility