

# Investment Value Depends on Investment Value

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## Abstract

Users of discounted-cash flow models for estimating real estate investment values encounter the following problem: the unknown value being estimated depends on inputs to the model that rely on the unknown value. The models, therefore, produce biased estimates of investment value unless iterative or simultaneous solutions are found. The market value literature addresses this problem by invoking simultaneous solutions. Parallel approaches for estimating investment value are hampered by complications resulting from the need to incorporate income tax effects. As shown in this paper, serious estimation bias results from first-run solution models in modern real estate text books and commercially available computer programs. Closed-form solutions for a variety of investment value models are obtainable to remove the estimation bias.

## Investment Value Depends on Investment Value

Mortgage-equity models used in real estate appraisal practice have the general form of Equation (1):

$$V = V_m + V_E, \quad (1)$$

where  $V$  is the estimated market value of the property,  $V_m$  is the value of the mortgage position, and  $V_E$  is the value of the equity position. If  $V_m$  is assumed to be the product of the loan-to-value ratio and the market value of the property, then  $V_m$  is a function of  $V$ . Because debt-service payments are deducted from net operating income to obtain before-tax cash flow,  $V_E$  also is a function of  $V$ . Even when  $V = V_E$ ,  $V_E$  depends on  $V$  if the estimated sale price at the end of the holding period is derived from an extrapolation of  $V$ . Inputting approximations for  $V$  on the right side of Equation (1) results in biased estimates of  $V$  on the left side. These simultaneous valuation problems are resolved by applying models which explicitly recognize that market value depends on market value.

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Estimating investment value adds another dimension to simultaneous valuation. During the holding period, statutory depreciation allowances are taken. Depreciation allowances are a function of the depreciable bases of properties, which in turn depend on  $V$ . In addition, capital gain taxes are a function of adjusted basis, which are a function of depreciation allowances that depend on  $V$ .

Under a standard set of assumptions, the investment value model for real estate, financed with equity and debt, involves a solution to an equation with the following seven  $V$ s on the right side:

1.  $V$  to determine  $V_m$  when  $V_m$  is estimated using the loan-to-value ratio.<sup>1</sup>
2.  $V$  to determine the periodic principal amortization and mortgage interest.
3.  $V$  to determine the remaining mortgage balance.
4.  $V$  to determine the sale price when the sale price is estimated from an extrapolation of  $V$ .<sup>2</sup>
5.  $V$  to determine depreciation allowances.
6.  $V$  to determine the capital gains tax.

Simultaneous solutions to market value models are documented in the literature and are part of appraisal practice. Simultaneous solutions to investment value models for real estate, however, are not fully developed. This paper presents a family of investment value models with closed-form solutions for a single  $V$  among multiple  $V$ s that may be part of investment value estimation. These models close a gap in the real estate literature. We show that a serious bias results from applications of standard discounted cash flow models which rely on approximations for  $V$  on the right side of the equation. The bias is removed by solving iteratively for  $V$  or by using a closed-form solution.

### ***Literature***

In their real estate finance and investment text book, Brueggeman and Fisher (1993, pp. 456-459) show a straightforward approach to simultaneously solving for market value. Another version of a simultaneous market value model is applied as common practice

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<sup>1</sup> The alternative for specifying  $V_m$  commonly followed in mortgage lending is to rely on a debt coverage ratio formulation which eliminates this  $V$  from the right side, as discussed later in this paper.

<sup>2</sup> The terminal period sale price may be estimated by capitalizing the net operating income in period  $N + 1$  by the 'going out' capitalization rate. This often used procedure removes this  $V$  from the right side.

in hotel appraisals (Mellon, 1983 and Rushmore, 1990). The evolution of before-tax discounted cash flow models with simultaneous solutions is traced by Cannaday and Colwell (1986), who evaluate the constraints in several models including their own model and those of Lusht and Zerbst (1980) and Fisher and Lusht (1981).

Recent additions to the literature extend market value models to include taxes so that investment value may be determined simultaneously. Randel and Johnson (1992) identify depreciation allowance as endogenous and propose a simple formula that makes tax savings from depreciation depend on value. Their model ignores the effect of depreciation on the adjusted basis of the property in the reversion. Adding a term that makes the adjusted basis a function of value resolves this problem. Also, the model is not extended beyond all-equity financing. Additional terms are needed to account for either fully-amortized or other forms of mortgage financing.

The adjusted basis is included in the simultaneous investment model of Kare and Rosenberg (1993) who show how the maximum purchase price of a property may be determined from the partial derivative of net present values with respect to purchase price. The calculation of the slope coefficient is simplified by several limiting assumptions: the values of the mortgage, the interest payments, and the sale price are all treated as exogenous. Jaffe and Sirmans (1995, pp. 351-52) offer a similar model for determining the maximum purchase price with all-equity financing.

In summary, the literature contains closed-form solutions for  $V$  in before-tax valuation models and some advancement toward finding closed-form solutions in after-tax valuation models for real estate. The models presented in this paper represent the next step along the path to resolving the simultaneous valuation problem when an investment is financed with mortgage debt and the analysis is performed after-tax.

## ***Model***

The symbols listed below are used in the presentation of the investment value models.

$V$	After-tax investment value
$r$	After-tax required rate of return on equity
$n$	Holding period, indexed by $j$
NOI	Net operating income (usually a vector of yearly projections)
NOIR	Net operating income in year $n+1$
NOIS	Stabilized net operating income
DCR	Debt-coverage ratio
$i$	Before-tax mortgage interest rate
$m$	Mortgage term
$M$	Loan-to-value ratio
$B$	Proportion of $V$ allocated to depreciable improvements
$L$	Life of depreciable improvements
$t$	Tax rate on ordinary income

R	Capitalization rate used to determine terminal sale price (going-out cap rate)
app	Appreciation rate of V to determine terminal sale price
se	Selling expenses as a percent of sale price
tg	Tax rate on capital gain

The model relies on the following limiting assumptions:

1. The holding period (n) is less than both the life of the depreciable improvements (L) and the mortgage term (m).
2. The tax rates on ordinary income (t) and capital gain (tg) are constant during the holding period.
3. The mortgage is fully amortized with annual payments and the mortgage terms, M, m, i, and DCR are determined exogenously. These terms do not vary during the holding period.<sup>3</sup>
4. The depreciation allowance is calculated using the prevailing statutory life and method.

### All-Equity Financing

Equation (2a) is an investment value equation in which the depreciation allowance, sale price, and capital gain are a function of V and the property is entirely financed with equity capital.

$$(2a) \quad Value := \left( \sum_{j=1}^n \frac{NOI_j(1-t)}{(r+1)^j} \right) + \frac{\left(1 - \frac{1}{(r+1)^n}\right) tBV}{rL} + \frac{V(app+1)^n(1-se) - \left( V(app+1)^n(1-se) - V + \frac{nVB}{L} \right) tg}{(r+1)^n}$$

The first term of the equation is the present value of the after-tax NOIs for periods j = 1...n. The second term is the present value of the tax shield from depreciation, and the last term is the present value of the after-tax reversion.

Equation (2b) is identical to Equation (2a) except the terminal sale price is estimated by capitalizing NOIR instead of assuming an appreciation rate during the holding period.

$$(2b) \quad Value := \left( \sum_{j=1}^n \frac{NOI_j(1-t)}{(r+1)^j} \right) + \frac{\left(1 - \frac{1}{(r+1)^n}\right) tBV}{rL} + \frac{\frac{NOIR(1-se)}{R} - \left( \frac{NOIR(1-se)}{R} - V + \frac{nVB}{L} \right) tg}{(r+1)^n}$$

Terminal sale prices are estimated as in Equation (2a) throughout the remainder of this paper.

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<sup>3</sup> The use of yearly rather than monthly payments significantly simplifies the closed-form solutions and it is conservative, yielding a smaller V than using monthly payments. Our simulations show that the difference between V with monthly amortization of the mortgage and V with annual amortization of the mortgage is considerably less than one percent.

## Debt Financing

Cannaday and Colwell (1986) identify four before-tax valuation models that include debt financing: Ellwood, Lusht-Zerbst, Fisher-Lusht, and Cannaday-Colwell. Each model has a different approach to valuing the mortgage position. In the Ellwood model,  $V_m$  is the product of the assumed loan-to-value ratio,  $M$ , and  $V$ . The Lusht-Zerbst model specifies the mortgage loan amount as

$$V_m = \text{NOI} / (\text{DCR}) f \quad (3)$$

Where

$$\begin{aligned} \text{DCR} &= \text{Debt-coverage ratio} \\ f &= \text{Mortgage constant} \end{aligned}$$

In the Fisher-Lusht model, the loan amount is found by capitalizing NOI using a rate derived from a modified band-of-investment equation, as follows:

$$V_m = \text{NOI} / [(y [1-m] + mf) / m] \quad (4)$$

where  $y$  is the before-tax equity yield rate. Finally, Cannaday and Colwell (1986) present a model in which the value of the mortgage position is related to the equity dividend rate (EDR) in the following way:

$$V_m = [V (\text{EDR}) - \text{NOI}] / (\text{EDR} - f) \quad (5)$$

Because most mortgage lenders determine loan amounts either from loan-to-value ratios or from debt-coverage ratios, only expanded versions of the Ellwood and Lusht-Zerbst models are presented below. Other models may be expanded in the same way.

Equation (6) restates Equation (2a) by adding the economic and tax effects of debt financing with a fully amortized mortgage and valuation of the mortgage position following Ellwood. The equation contains six terms. The first term is  $V_m$ . The next term is the present value of the after-tax NOIs, as in Equation (2a). The third term is the present value of debt service paid during the holding period and the fourth term is the present value of the tax shield from payment of the interest portion of debt service. The fifth term is the present value of the depreciation tax shield. The final term is the present value of reversion, which accounts for payment of the remaining mortgage balance and the capital gain tax.

$$\begin{aligned}
 \text{Value} := & VM + \left( \sum_{j=1}^n \frac{NOI_j(1-t)}{(r+1)^j} \right) - \frac{\left(1 - \frac{1}{(r+1)^n}\right) iMV}{r \left(1 - \frac{1}{(1+i)^m}\right)} + \left( \sum_{j=1}^n \frac{ti \left(1 - \frac{(1+i)^{(j-1)} - 1}{(1+i)^m - 1}\right) MV}{(r+1)^j} \right) + \frac{\left(1 - \frac{1}{(r+1)^n}\right) tBV}{rL} \\
 & + \frac{V_{(app+1)^n(1-se)} - \left(1 - \frac{(1+i)^n - 1}{(1+i)^m - 1}\right) MV - \left(V_{(app+1)^n(1-se)} - V + \frac{nVB}{L}\right) g}{(r+1)^n}
 \end{aligned} \tag{6}$$

The Lusht-Zerbst model is expanded in Equation (7). The six terms in Equation (7) are interpreted identically to those in Equation (6). This equation is:

$$\begin{aligned}
 \text{Value} := & \frac{NOIS}{DCRf} + \left( \sum_{j=1}^n \frac{NOI_j(1-t)}{(r+1)^j} \right) - \frac{\left(1 - \frac{1}{(r+1)^n}\right) NOIS}{rDCR} + \left( \sum_{j=1}^n \frac{ti(1-P_j)NOIS}{DCRf(r+1)^j} \right) + \frac{\left(1 - \frac{1}{(r+1)^n}\right) tBV}{rL} \\
 & + \frac{V_{(app+1)^n(1-se)} - \frac{(1-P_n)NOIS}{DCRf} - \left(V_{(app+1)^n(1-se)} - V + \frac{nVB}{L}\right) g}{(r+1)^n}
 \end{aligned} \tag{7}$$

where  $f$  is the mortgage constant.

### **Implementing Simultaneous Models**

Real estate textbooks and other investment literature present similar approaches to extend to the case of an investment financed with debt and the analysis is performed on an after-tax basis. The standard DCF models presented in the five books incorporate the effects of debt and taxes, but rely on approximations for  $V$  on the right side. In addition, the DCF model examples are structured to emphasize decision measures, such as net present value and internal rate of return outputs, and minimize the importance of using investment value for the preparation of list prices and bids.

Table 2 compares the textbook solutions from a single run of the DCF model to the simultaneous solution generated with the same inputs.

Estimation bias is present in all cases. In three of the five cases, the difference is greater than five percent and in one case the difference is greater than 10 percent. Most of the  $V$ s are less than the simultaneous solutions for  $V$ .

Simultaneous solutions actually represent shortcuts to iterative solutions because the unique investment values from the simultaneous solutions will emerge from an iterative process that it begins with a standard DCF application. Once a solution for  $V$  is

found from the first iteration of the DCF model, it is entered to perform mortgage, depreciation, and reversion calculation which are combined to generate a second iterative solution for V. Repeating this process results in convergence to the simultaneous solution.

The problems with the iterative process are:

1. Iterations involve time-consuming steps. Thirty to fifty iterations generally

**Table 1**  
**Valuation Models from Selected Real Estate Investment Texts**

Textbook	Brueggeman and Fisher (1993)	Ford (1994)	Greer and Farrell (1993)	Jaffe and Sirmans (1995)	Plattner (1988)		
<b>Model Inputs &amp; Outputs</b>	pp. 456-463	pp. 316-324, 359-364	pp. 136-144	pp. 255-262	p. 352	pp. 344-345	pp. 61-67, 176-182
<b>Type of Model</b>	Simultaneous Market Value	Standard DCF Investment Value	Standard DCF Investment Value	Standard DCF Investment Value	Simultaneous Equity Investment Value	Standard DCF Investment Value	Standard DCF Investment Value
<b>Total Acquisition Cost</b>	NA	\$8,500,000	\$367,450	\$1,300,000	NA	\$400,000	\$350,000
NO <sub>1</sub>	\$500,000	\$903,050	\$44,255	\$117,000	\$47,600	\$45,176	\$46,976
NO <sub>2</sub>	515,000	928,985	46,468	124,000	47,600	47,217	48,180
NO <sub>3</sub>	530,450	955,439	48,791	132,000	47,600	49,348	49,422
NO <sub>4</sub>	546,364	973,790	51,231	139,000	47,600	51,571	50,694
NO <sub>5</sub>	562,754	1,010,822	53,792	148,000	47,600	53,890	51,994
NO <sub>6</sub>		1,052,459	56,482				
NO <sub>7</sub>			59,306				
NO <sub>8</sub>			62,271				
NO <sub>9</sub>			65,385				
NO <sub>10</sub>			68,654				
<b>Equity</b>							
Required Rate (%)	15	13	23.05	10	12	11	20
Holding Period (Yrs)	5	5	10	6	5	5	5
Tax Rate - Income (%)	28	NA	28	28	28	28	28
Tax Rate - Gain (%)	28	NA	28	28	28	28	28
<b>Mortgage</b>							
Loan Amount	NA	\$5,950,000	\$261,000	\$840,000	NA	\$300,000	\$280,000
Loan-to-Value (%)	75	70	NA	NA	NA	75	80
Rate (%)	12	10	10	12	NA	12	12
Term (Yrs)	25	20	interest only	25	NA	20	30
Monthly Amortization	Yes	Yes	NA	Yes	NA	Yes	Yes
Prepayment Penalty	No	No	No	No	NA	6%	NA
<b>Depreciation</b>							
Basis (%)	80.2815	85	95.5891	80	80	85	80
Life (Yrs)	31.5	31.5	27.5	27.5	27.5	39	27.5
Method	SL	SL	SL	SL	SL	SL	SL
<b>Reversions</b>							
Dollar Amount	NA	NA	NA	\$1,800,000	NA	\$480,000	NA
Appreciation Rate (%)	3	3	5	NA	1.2 (V)	NA	NA
Cap Rate (%)	NA	11	NA	NA	NA	NA	13.0857
Expenses (%)	0	0	5	5	0	6	5
<b>Notes on Inputs</b>	Before- and after-tax examples given. After-tax IRR computed (11.84%) but no after tax valuation	Reversion is average of appreciation and capped income	Depreciable basis adjustment made	Gross rent multiplier of 5.47115 used to estimate sales price	None	Acquisition costs amortized over five years	None
<b>Value</b>							
Equity		\$2,773,376	\$106,450	\$540,418		\$100,214	\$72,261
Mortgage		\$5,950,000	\$261,000	\$840,000		\$300,000	\$280,000
Total	\$5,071,381	\$8,723,376	\$367,450	\$1,380,418	\$358,472	\$400,214	\$352,261

are required to achieve convergence to the simultaneous solution with a tolerance of one dollar.

2. Inputting errors often compound in an iterative model. The first round of iterations may lead to an answer that is much different from the simultaneous solution.

**Table 2**  
**Comparison of Solutions for Investment Value**

<b>Text Book</b>	<b>First-Run Solution from DCF Model</b>	<b>Simultaneous Solution</b>	<b>Percent Difference</b>
Brueggeman and Fisher (1993)	\$8,723,376	\$9,340,358	6.6%
Ford (1994)	367,450	402,051	8.6%
Greer and Farrell (1993)	1,380,408	1,602,979	13.9%
Jaffe and Sirmans (1995)	400,214	418,613	4.4%
Plattner (1988)	352,261	347,550	1.4%

## Conclusions

This paper presents closed-form solutions for several investment value models that incorporate the effects of mortgage financing and income taxes. The strength of closed-form solutions comes from eliminating the need for 'apriori' estimates of the unknown solution (i.e.,  $V$ ). Rather than specify  $V$  exogenously to produce a single-run result as is standard practice, closed-form or iterative solutions may be found that do not rely on  $V$ . Both approaches eliminate bias in the estimate of  $V$ .

We advocate the use of simultaneous DCF models to estimate the investment values of real estate, recognizing that investment value depends on investment value. Examples in the real estate literature should stress the importance of estimating investment value without bias and more strongly emphasize the importance of investment value for preparing bids and list prices.



## Sources

- Brueggeman, William B. and Jeffrey D. Fisher, *Real Estate Finance and Investments* 9<sup>th</sup> ed. (Homewood, IL: Richard D. Irwin, 1993).
- Cannaday, Roger E. and Peter F. Colwell, "Real Estate Valuation Models: Lender and Equity Investor Criteria," *AREUEA Journal* 14 (Summer 1986) : 316-337.
- deRoos, Jan A. and Stephen Rushmore, "Investment Values of Lodging Property: Modeling the Effects of Income Taxes and Alternative Lender Criteria," working paper, Cornell University School of Hotel Administration, 1995.
- Ellwood, L.W. *Ellwood Tables for Real Estate Appraising and Financing*. (L.W. Ellwood, 1959).
- Fisher, Jeffrey D. and Kenneth M. Lusht, "Mortgage Equity Analysis with a Debt Coverage Constraint," *The Real Estate Appraiser and Analyst* 47 (No. 3 1981): 5-12.
- Ford, Deborah Ann, *Fundamentals of Real Estate Investment for Decision Makers* (Minneapolis, MN: West Publishing Company, 1994).
- Greer, Gaylon E. and Michael Farrell, *Investment Analysis for Real Estate Decision* (Dearborn Financial Publishing, 1992.)
- Jaffe, Austin J. and C.F. Sirmans, *Fundamentals of Real Estate Investment* 3rd ed. (Englewood Cliffs, N.J. Prentice Hall, Inc., 1995).
- Kare, Dilip and Sidney Rosenberg, "An Analytical Model to Determining the Maximum Purchase Price of a Real Property." Working paper, University of North Florida, 1993.
- Lusht, Kenneth M. and Robert H. Zerbst, "Valuing Income Property in an Inflationary Environment," *The Real Estate Appraiser and Analyst* 46 (No. 4 1980) : 11-17.
- Maple V Release 3 for Windows (Waterloo Maple Software distributed by Math Soft Inc., Cambridge, MA, 1995).
- Mellen, Suzanne R. "Simultaneous Valuation: A New Capitalization Technique for Hotel and Other Income Properties," *The Appraisal Journal* 51 (April 1983): 165-189.

Plattner, Robert H., *Real Estate Investment Analysis and Management* (Columbus, OH: Merrill Publishing Company, 1988).

Randel, Paul H. and I. Richard Johnson, "Do Real Estate Investments Meet Your Return Expectations," *Real Estate Finance* 8 (Winter 1992) : 63-70.

Rushmore, Stephen, "Hotel Valuation Techniques," *The Real Estate Finance Journal* 6 (Summer 1990) : 50-57.