INTEREST RATE PARITY WITH CREDIT RISK: IMPLICATIONS FOR CARRY TRADES

A Dissertation

Presented to the Faculty of the Graduate School

of Cornell University

In Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

by

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May 2020

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Cornell University 2020

The incredible profitability of the carry trade over the past six decades constitutes a puzzle for interest rate parity. Contrary to recent behavioral or friction-based approaches that explain deviations from traditional interest rate parity, I examine the effect of foreign sovereign credit risk and associated sharp currency devaluations on interest rate parity. To ensure that the theoretical implications apply generally, the setting is a continuous time arbitrage pricing model driven by Levy processes. I derive the statements of covered and uncovered interest rate parity under credit risk. The model produces novel measures of sovereign credit risk and carry trade profitability – most notably, forward-implied default intensities and the difference of same-maturity futures and forward prices. Empirically, introducing credit risk into the statement of covered interest rate parity makes pricing errors vanish for Mexico and the G10 countries: The profitability of both the covered and uncovered carry trade are fully accounted for by a modest allowance for credit risk and currency devaluation. I find mixed results for a carry trade trading system whose long/short position is determined by an estimate of the risk neutral expected return to the carry trade.

BIOGRAPHICAL SKETCH

Toby graduated from the University of Washington in 2015 with bachelors degrees in mathematics and finance. He began his doctoral studies at Cornell University in fall of the same year.

ACKNOWLEDGEMENTS

I thank Professor Jarrad Harford for getting me into graduate school, and the finance faculty at Cornell for tolerating me once I arrived. I especially thank my advisor Professor Robert Jarrow for his contagious enthusiasm and inspiration. My most fruitful and excited days were always those following our meetings.

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Interest rate parity addresses one simple question: How can two countries simultaneously offer different interest rates, without offering extraordinary opportunities for investment? The speculator, hoping to profit from this interest rate differential, may borrow in the low interest rate currency[1], immediately convert the proceeds into the corresponding quantity of high interest rate currency, and lend the quantity at the high interest rate. At maturity, the speculator collects interest and principal in the high interest rate currency, converts the full amount to the low interest rate currency, and pays back principal and interest. If the exchange rate has not changed from initiation to termination of the trade, the speculator earns exactly the interest rate differential on each unit of borrowed currency; if the high interest rate currency appreciates (depreciates), the speculator earns more (less) than the interest rate differential. Uncovered interest rate parity states that, on average, this sequence of trades should make zero expected returns. This trade, of course, is the carry trade, and has been one of the most persistently profitable trading strategies of the last six decades.[2]

The exchange rate uncertainty in the carry trade is easily hedged by the purchase of the appropriate number of currency forward contracts; the remaining payoff is then deterministic. Since this covered carry trade requires zero initial investment and only produces a payoff at termination of the trade, this payoff should equal zero, or the market presents riskless profit opportunities for the well-financed arbitrageur. This is the statement of covered interest rate parity. As covered interest rate parity is based on arbitrage arguments[3], it would be a surprise if forward

[1]For convenience, I refer by "low interest rate currency" to the currency of the country which offers the lower interest rate.

[2]The profitability has been heavily documented – Engel (1996) is an extensive survey.

[3]Uncovered interest rate parity follows from the statement that expected returns to currency speculation should be zero. Not only is this premise less convincing a basis than arbitrage arguments, it is in fact theoretically inconsistent with a risk neutral market, in which expected returns to currency speculation are proportional to the covariance of the exchange rate with

prices deviated substantially from those implied by covered interest rate parity – this deviation is precisely what Du, Tepper, and Verdelhan (2018) and others document in the recent decade.

I find that foreign sovereign credit risk explains the profitability of both the covered and uncovered carry trade. I present an arbitrage pricing model of foreign exchange with credit risk, and derive a credit-risky version of covered interest rate parity, under which forward contracts are strictly more valuable[4] relative to standard covered interest rate parity. Under credit-risky CIP, currency forward prices contain information about sovereign default intensities: Deviations of standard CIP forward prices from market forward prices map straightforwardly to default intensities. Since larger (negative) deviations map to larger foreign default intensities, I find support for the recent notion that the US is a global provider of safe assets (Jiang, Krishnamurthy, and Lustig (2018), and references therein). I find that moderate credit risk is capable of producing the persistent underpricing of forward contracts by standard CIP in our sample for Mexico and the G10 countries. The model produces explicit formulas for conditional and unconditional expected returns to the uncovered carry trade, where – here, and for the remainder of the paper – the conditioning event is no default; in particular, conditional payoffs overstate unconditional payoffs. I examine carry trade payoffs for the US/Mexico pair: Empirically, unconditional payoffs are vanishingly small, and conditional payoffs are large and positive – in other words, the peso problem fully explains the returns to the carry trade.

domestic interest rates. I find, in agreement with literature, that this quantity is empirically small (e.g. Frenkel and Razin (1980)).

[4]Denominated in US dollars.

I. Carry Trades with Credit Risk

Foreign sovereign credit risk muddles the implications of interest rate parity for the exchange rate. For concreteness, define the US to be domestic and Mexico to be foreign. Let R denote the recovery rate on Mexican bonds, e.g. $R = 0.6$ means 40% of face value was lost during the carry trade due to default; $R = 1$ means Mexico did not default during the carry trade. Note that R is a random variable whose value is not known until the carry trade is complete. Let $P(0,T)$ be the price in USD of a T-maturity zero coupon bond issued by the US government; let $P_f(0,T)$ be the price in MXN of the corresponding bond issued by the Mexican government. Let S_t be the spot MXN/USD rate (e.g. $S_0 = 0.05$ means 1 Mexican peso is worth 0.05 US dollars at time 0). Let $F(0,T)$ be the forward price of MXN/USD (i.e. at time 0, traders can enter into contracts that pay $S_T - F(0,T)$ USD at time T).

Observe from Table 1 that when there is no credit risk[5], uncovered interest rate parity produces a statement about the expected value of the foreign exchange rate: Setting the expectation of the carry trade payoff equal to zero produces

$$E(S_T) = S_0 \frac{P_f(0,T)}{P(0,T)}.$$

Allow for credit risk, and uncovered interest rate parity produces instead a joint statement about the recovery process and the exchange rate:

$$E(RS_T) = S_0 \frac{P_f(0,T)}{P(0,T)}.$$

Perhaps even more damaging, credit risk makes it impossible to perfectly hedge the foreign exchange exposure of the carry trade with currency forward contracts.

[5] Let $R \equiv 1$.

It is not immediately clear whether the usual argument will suffice to obtain a statement of covered interest rate parity. A key contribution of my model is the recovery, in a general framework, of the statements of interest rate parity after introducing credit risk.

TABLE I. The carry trade

Event	Time 0	Time T
Short one US bond with maturity T	$+1$ [domestic bond oblig.] $+P(0,T)$ [USD]	
Exchange to pesos	$-P(0,T)$ [USD] $+\frac{P(0,T)}{S_0}$ [MXN]	
Buy MX bond with maturity T	$-\frac{P(0,T)}{S_0}$ [MXN] $+\frac{P(0,T)}{S_0 P_f(0,T)}$ [MX bonds]	
Collect MX bond payment		$-\frac{P(0,T)}{S_0 P_f(0,T)}$ [MX bonds] $\frac{P(0,T)}{S_0 P_f(0,T)} R$ [MXN]
Exchange to USD		$-\frac{P(0,T)}{S_0 P_f(0,T)} R$ [MXN] $+\frac{P(0,T)}{S_0 P_f(0,T)} R S_T$ [USD]
Pay US bond obligation		-1 [domestic bond oblig.] -1 [USD]
Net payoff (carry trade)		$R\frac{P(0,T)}{P_f(0,T)}\frac{S_T}{S_0} - 1$ [USD]
Net payoff (covered carry trade)		$(R-1)\frac{P(0,T)}{P_f(0,T)}\frac{S_T}{S_0} - 1$ $+\frac{F(0,T)P(0,T)}{P_f(0,T)S_0}$ [USD]

The unit for each transaction is stated in square brackets. For instance, the second event shows the result of exchanging $P(0,T)$ [USD] into the corresponding quantity of Mexican pesos at the spot rate S_0: A loss of $P(0,T)$ [USD] and a gain of $P(0,T)/S_0$ [MXN].

II. Literature Review

Explanations for the failure of interest rate parity fall into two distinct categories: The first identifies as the source behavioral peculiarities, irrationality, or market frictions. Yu (2013) introduces agents that fail to correctly estimate economic growth rates. Bacchetta and Van Wincoop (2010) points to transaction costs that force infrequent portfolio revision. Burnside, Han, Hirshleifer, and Wang (2011) proposes investor overconfidence. Du, Tepper, and Verdelhan (2018) suggests that limited leverage of banks restricts investors from taking advantage of deviations from covered interest rate parity.

The second category examines the ability of rare, crash/disaster risk to explain the deviations from uncovered interest rate parity: Since the carry trade exhibits large, negative skewness, observed carry trade profitability may simply be compensation for exposure to rare but large losses – the debate is ongoing. Burnside, Eichenbaum, Kleshchelski, and Rebelo (2010) finds that the peso problem[6] explains the positive payoffs of the carry trade; on the other hand, using a similar approach, Jurek (2014) finds that carry trades hedged with deep out of the money put options still have large, positive payoffs, and concludes the opposite. Lustig, Roussanov, and Verdelhan (2011) finds that a standard affine model can replicate carry trade returns, and suggests that differential exposure to global risk drives carry trade returns. Farhi and Gabaix (2015) finds that introducing disaster risk into a equilibrium model of foreign exchange produces the failure of uncovered interest rate parity, among numerous other stylized facts.

To my knowledge, my paper is the first to explain the failure of covered interest rate parity from the perspective of foreign sovereign credit risk. My paper is most similar to Tuckman and Porfirio (2003), which analyzes covered interest rate parity with credit risky interbank lending, under default-free foreign sovereign lending[7]; our analysis relaxes this latter assumption.

[6] "Peso problem" refers to "the effects on inference caused by low-probability events that do not occur in sample"; typically, it refers to the possibility of a large devaluation in one of the currencies (p.854).

[7] See their appendix 2.

My paper is also related to Farhi and Gabaix (2015), which models exchange rates with disaster risk in an equilibrium model; my no-arbitrage approach abstracts away much of the structure, and starts instead from a general class of assumed price evolutions. I draw a strikingly different conclusion: While Farhi and Gabaix (2015) assumes standard covered interest rate parity holds in the presence of disaster risk, I find that credit risk produces a new statement of covered interest rate parity, and that credit-risky CIP explain deviations from standard CIP. Hence, sovereign credit risk is an essential component of rare event risk that existing models overlook.

My model extends Amin and Jarrow (1991) to allow for credit-risky sovereign foreign debt. I model default via an extension of Jarrow and Turnbull (1995) to allow for time-varying default intensity and serial default. The model incorporates the observation in De Paoli, Hoggarth, and Saporta (2009) that sovereign default is associated with sharp currency depreciation.

My model does not incorporate counterparty risk in derivatives markets, on which literature is mixed: Levich (2012) identifies instances of counterparty default on forward contracts, and suggests that investors may be shifting from forward contracts to futures contracts to avoid counterparty risk; Arora, Gandhi, and Longstaff (2012) studies counterparty risk in CDS markets, and finds that it is vanishingly small. In any event, the existence of counterparty risk actually strengthens my result: Introducing counterparty risk into the derivatives market produces larger forward-implied default intensities.[8]

[8]In brief, this observation follows from credit-risky CIP relation (9). If there is counterparty risk, the counterparty-risk-free forward price is higher than the observed price of the forward contract with counterparty risk. Consequently, default intensities implied by counterparty-risky forward prices underestimate the true default intensities, which would result from using expression (9) with prices of forward contracts with no counterparty risk.

III. Model

Assume that, at the time of default, the defaultable foreign bond loses a fraction $1 - R$ of its face value, foreign interest rates jump up by θ, and the exchange rate jumps by multiple $1 + \eta > 0$. Note that devaluation on default is given by $\eta < 0$. I aim to study the consequences of these assumptions in a general setting.

To this end, consider an economy that trades in continuous time with no frictions over the interval $[0, \mathcal{T}]$. Describe the uncertainty in the economy by the probability space $(\Omega, \mathcal{F}, P)$, and a complete, right-continuous filtration $\{\mathcal{F}_t\}_{t\in[0,\mathcal{T}]}$. There are two $\mathcal{F}_t$-adapted[9], independent sources of randomness: An n-dimensional standard P- Brownian motion $W_t \equiv (W_i(t))_{i=1}^n$ and a nonexplosive point process[10] N_t with jump times τ_k. Jumps correspond to sovereign default. Assume that there exists an adapted, nonnegative process λ_t so that $M_t \equiv N_t - \int_0^t \lambda_s ds$ is a P-martingale.

A continuum of foreign (defaultable, default-free twin) and domestic (default-free) zero coupon bonds trade, as do the corresponding money market accounts. Let $P(t, T)$ denote the time t price of a domestic zero coupon bond with face value equal to one unit of domestic currency and maturity T. Let $P_0(t, T)$ and $P_f(t, T)$ denote the analogous default-free and defaultable foreign zero coupon bond prices.

Bond prices define the corresponding forward rates: $f(t, T) \equiv -\frac{\partial}{\partial T} P(t, T)$, $f_0(t, T) \equiv -\frac{\partial}{\partial T} P_0(t, T)$, and $f_1(t, T) \equiv -\frac{\partial}{\partial T} P_f(t, T)$. [11] Bond prices are then expressed in terms of forward rates by

$$P(t, T) = \exp\bigg(- \int_t^T f(t, y) dy \bigg),$$

[9]From now on, "adapted" means $\mathcal{F}_t$-adapted.
[10]As defined in Bremaud 1981, p.20
[11]To preclude arbitrage and ensure the existence of the forward rates, assume that $P(t, T) > 0$ for all $0 \le t \le T \le \mathcal{T}$, $P(t, t) = 1$ $P - a.e.$ for all $t \in [0, \mathcal{T}]$, and $\frac{\partial}{\partial T} \log P(t, T)$ exists for all $0 \le t \le T \le \mathcal{T}$; assume the same for the default-free forward rate f_0. For the defaultable forward rate, assume only nonnegativity and existence of the forward rate.

$$P_0(t,T) = \exp\Big(- \int_t^T f_0(t,y)dy \Big),$$

$$P_f(t,T) = \exp\Big(- \int_t^T f_1(t,y)dy \Big) e_1(t),$$

where $e_1(t)$ is the recovery process, representing the fraction of face value not yet lost to default by time t. Recall also that bond prices are defined in terms of yields y, y_f as follows:

$$P(t,T) = \exp(-(T-t)y(t,T)),$$

$$P_f(t,T) = \exp(-(T-t)y_f(t,T))$$

Forward rates additionally define corresponding short rates: $r_t \equiv f(t,t)$ is the domestic short rate, $r_0(t) \equiv f_0(t,t)$ is the foreign default-free short rate, and $r_1(t) \equiv f_1(t,t)$ is the foreign defaultable short rate. Each short rate defines a corresponding money market account (henceforth MMA), whose value at time t is the result is instantaneously accumulating interest at the short rate:

$$B_t \equiv \exp\Big(\int_0^t r_s ds \Big),$$

$$B_0(t) \equiv \exp\Big(\int_0^t r_0(s)ds \Big),$$

$$B_1(t) \equiv \exp\Big(\int_0^t r_1(s)ds \Big) e_1(t).$$

Denote the spot currency exchange rate at time t by S_t: At time t, one unit of foreign currency can be exchanged into S_t units of domestic currency.

Assume evolutions as follows:

A1: Domestic forward rate

$$(1) \qquad f(t,T) - f(0,T) = \int_0^t \alpha(s,T,\omega)ds + \int_0^t \sigma(s,T,\omega) \cdot dW_s,$$

where $\sigma(s,T,\omega)$ is an n-dimensional vector described by n functions σ_i, i.e. $(\sigma_1(s,T,\omega),...,\sigma_n(s,T,\omega))$, and $\int_0^t \sigma(s,T,\omega) \cdot dW_s \equiv \sum_{i=1}^n \int_0^t \sigma_i(s,T,\omega)dW_i(s)$. For notational convenience, I sometimes suppress explicit vector notation, so that $\int_0^t \sigma(s,T) \cdot dW_s \equiv \sum_{i=1}^n \int_0^t \sigma_i(s,T)dW_i(s)$.

The domestic forward rate evolves with continuous sample paths: It has no jumps, and is independent of the default process. Instantaneous sensitivity of the T-maturity forward at time t to the kth continuous source of randomness is given by $\sigma_k(t,T)$. The drift $\alpha(t,T)$ describes the average behavior at time t of the forward rate with maturity T over the next instant. The drift α and instantaneous volatilities σ are $\mathcal{F}_t$-adapted, i.e. the values $\alpha(t,T)$ and $\sigma(t,T)$ are known at time t for each $t \leq T$.[12]

A2: Foreign forward rates and default payout ratio

(2)

$$f_0(t,T) - f_0(0,T) = \int_0^t \alpha_0(s,T,\omega)ds + \int_0^t \sigma_f(s,T,\omega) \cdot dW_s + \int_0^t \theta(s,T)dN_s,$$

$$f_1(t,T) - f_1(0,T) = \int_0^t \alpha_1(s,T,\omega)ds + \int_0^t \sigma_f(s,T,\omega) \cdot dW_s + \int_0^t \theta(s,T)dN_s,$$

$$e_1(t) = R^k \text{ for } t \in [\tau_k, \tau_{k+1}).$$

The foreign forward rates are affected by the same continuous sources of randomness, but with sensitivities $\sigma_{f,k}(t,T)$. Additionally, the foreign forward jumps by

[12]A technical note: Some mild measurability and integrability conditions are required for regularity of the MMAs and bond prices; we assume all such conditions. See Jarrow and Madan (1995) Assumption 2.2 and the preceding discussion for details.

$\theta(\tau_j, T)$ at each time of default τ_j. Each default reduces the remaining face value of the defaultable foreign bond by a fraction $(1 - R)$; hence, the remaining face value after the kth default is R^k.

A3: Exchange rate

$$\text{(3)} \qquad \frac{dS_t}{S_{t-}} = \nu_t dt + \delta_t \cdot dW_t + \eta_t dN_t,$$

where $S_{t-} \equiv \lim_{u \nearrow t} S_u$, i.e. denotes the left limit. The exchange rate has drift $S_{t-}\nu_t$ and instantaneous volatilities $S_{t-}\delta_k(t)$; it jumps by fraction η_{τ_k} at the time of the kth default.

Assumptions A1-A3 imply[13] the following processes:

$$P(t,T) = P(0,T) \exp\bigg(\int_0^t (r_d(s) + b(s,T))ds + \int_0^t a(s,T) \cdot dW_s \bigg),$$

$$P_0(t,T) = P_0(0,T) \exp\bigg(\int_0^t r_0(s) + b_0(s,T)ds + \int_0^t a_f(s,T) \cdot dW_s \bigg),$$

$$\text{(4)} \qquad P_f(t,T) = P_f(0,T) \exp\bigg(\int_0^t r_1(s) + b_1(s,T)ds + \int_0^t \Theta(s,T) + \log R dN_s + \int_0^t a_f(s,T) \cdot dW_s \bigg),$$

$$S_t = S_0 \exp\bigg(\int_0^t (\nu_s - \frac{1}{2}\delta_s^2)ds + \int_0^t \delta_s \cdot dW_s + \int_0^t \log(1+\eta_s)dN_s \bigg),$$

where $b(s,T) \equiv -\int_s^T \alpha(s,y)dy$, $b_k(s,T) \equiv -\int_s^T \alpha_k(s,y)dy$, $a(s,T) \equiv -\int_s^T \sigma(s,y)dy$, $a_f(s,T) \equiv -\int_s^T \sigma_f(s,y)dy$, $\Theta(s,T) \equiv -\int_s^T \theta(s,y)dy$. For notational convenience, I suppress explicit vector notation, so that e.g. $\int a(s,T) \cdot dW_s$ means $\int \sum_{i=1}^n a_i(s,T)dW_i(s)$, $\int a^2(s,T) \cdot dW_s$ means $\int \sum_{i=1}^n a_i^2(s,T)dW_i(s)$.

[13]Derivation in appendix.

Consistency of the model requires that prices preclude arbitrage opportunities; conveniently, the process of excluding arbitrage opportunities produces the risk neutral dynamics of the market: If the market has no arbitrage opportunities, there exists an equivalent martingale measure[14], i.e. a probability measure equivalent to P, under which all relative asset prices are martingales, where relative asset prices are prices that are denominated in the domestic currency and discounted at the domestic MMA (e.g. the foreign defaultable bonds' relative asset prices are $P_f(t,\cdot)S_t/B_t$). As shown in the appendix, assumptions A1-A4[15] imply that there is exactly one such measure – the measure is identified in the appendix, and denoted by Q. It follows that assuming risk neutrality is equivalent to assuming Q-dynamics.[16] This identification facilitates analysis of a risk neutral market.

Denote the risk neutral drifts of the domestic forward rate, default-free foreign forward rate, defaultable foreign forward rate, and exchange rate by α_{RN}, $\alpha_{0,RN}$, $\alpha_{1,RN}$, and ν_{RN}, and define them by

$$
\begin{aligned}
df(t,T) &= \alpha_{RN}(t,T)dt + \sigma(t,T)\cdot d\tilde{W}_t, \\
df_0(t,T) &= \alpha_{0,RN}(t,T)dt + \sigma_f(t,T)\cdot d\tilde{W}_t + \theta(t,T)d\tilde{M}_t, \\
df_1(t,T) &= \alpha_{1,RN}(t,T)dt + \sigma_f(t,T)\cdot d\tilde{W}_t + \theta(t,T)d\tilde{M}_t, \\
\frac{dS_t}{S_{t-}} &= \nu_{RN}(t)dt + \delta_t \cdot d\tilde{W}_t + \eta_t d\tilde{M}_t,
\end{aligned}
\tag{5}
$$

[14]Harrison and Kreps (1979), Jarrow and Madan (1995).
[15]A4 is stated in the appendix.
[16]Technical note: This implies that, in our setting, assuming risk neutrality of the market is equivalent to assuming that P is in fact the martingale measure. To keep our results general, we distinguish between the equivalent martingale measure Q and the physical measure P. This notation lets us distinguish between results that hold without assuming risk neutrality (e.g. derivatives' prices) and those that do not (e.g. interpretation of futures price as market expectation).

where $\tilde{W}_i(t) \equiv W_i(t) - \int_0^t \gamma_i(s)ds$ and $\tilde{M}_t \equiv N_t - \int_0^t \mu_s \lambda_s ds$ are Q-martingales; Q, γ, and μ are defined in the appendix.

Proposition 1: The risk neutral drifts

(6)

$$\alpha_{RN}(t,T) = \sigma(t,T) \int_t^T \sigma(t,y)dy,$$

$$\alpha_{0,RN}(t,T) = \sigma_f(t,T)\bigg(\int_t^T \sigma_f(t,y)dy - \delta_t \bigg) + \mu\lambda\theta(t,T)(1 - e^{\Theta(t,T)}(1+\eta_t)),$$

$$\alpha_{1,RN}(t,T) = \sigma_f(t,T)\bigg(\int_t^T \sigma_f(t,y)dy - \delta_t \bigg) + \mu\lambda\theta(t,T)(1 - Re^{\Theta(t,T)}(1+\eta_t)),$$

$$\nu_{RN}(t) = r_d(t) - r_0(t) = r_d(t) - r_1(t) + \mu\lambda(1-R)(1+\eta).$$

IV. Covered Interest Parity

The analysis above produces new statements of covered interest rate parity, strictly from the requirement that the model is arbitrage-free – I have made no assumptions about preferences. First, note that the existence of default-free foreign bonds means the standard covered interest rate parity argument applies, using the default-free foreign bond and domestic bond. Following the trades in Table 1 produces the relation in terms of default-free foreign and domestic bonds:

$$F(0,T) = S_0 \frac{P_0(0,T)}{P(0,T)}.$$

Next, since defaultable foreign bonds are more easily observed than their default-free counterparts, I re-write the relation with defaultable bond prices in various settings.

Proposition 2: Covered interest parity, credit risk only.

Consider first the pure credit risk setting: Upon default, the foreign country's bonds lose a fraction $(1 - R)$ of face value, and there are no jumps in the foreign interest rate or exchange rate upon default, i.e. $\theta = \eta = 0$. In this case, covered interest rate parity becomes[17]

$$F(0,T) = S_0 \frac{P_f(0,T)}{P(0,T)} \exp\left((1-R)\int_0^T \mu_s \lambda_s ds\right). \tag{7}$$

Observe that this is almost exactly the standard statement of covered interest rate parity. The only modification is multiplication by an exponential term, which is the reciprocal of the expected recovery of face value of foreign bonds over the life of the forward contract. Since recovery is at most unity, the forward contract's value increases in credit risk, as measured by expected loss. This relation is intuitive: A forward currency contract with delivery date T is equivalent to a forward contract on a default-free foreign bond with maturity T, and the latter becomes more valuable relative to the defaultable foreign bond as credit risk increases.

Now consider the general setting: Upon default, the foreign country's bonds lose a fraction $(1 - R)$ of face value, interest rates jump by θ, and the exchange rate jumps down by a fraction η. Starting again with CIP for the default-free bonds and re-writing in terms of the defaultable foreign bond produces[18]

Proposition 3a: Covered interest rate parity, general setting.

$$F(0,T) = \left(S_0 \frac{P_f(0,T)}{P(0,T)} - \frac{Cov^Q(\frac{S_T}{B_T}, e_1(T))}{P(0,T)}\right) \exp\left((1-R)\int_0^T \mu_s \lambda_s ds\right), \tag{8}$$

[17]Derivation in appendix.

[18]Derivation in appendix.

where $Cov^Q(X,Y) \equiv E^Q(XY) - E^Q(X)E^Q(Y)$, and Q is the equivalent martingale measure.

I claim that the covariance term is positive; the positive correlation of S_T of $e_1(T)$ follows from two channels. Upon default, the exchange rate jumps down by fraction η, and e_1 falls by a multiple R. Additionally, the foreign interest rate jumps up, which reduces the exchange rate drift – see expression (5).

Consequently, introducing jumps in the exchange rate and foreign interest rate makes forward contracts less valuable, fixing the level of credit risk. This relation, too, is reasonable: Currency devaluation reduces the domestic price of replicating the forward contract. A jump in foreign interest rates does not change the payout of a default-free foreign bond, but reduces the domestic value of payout by reducing the exchange rate drift.

The expression further simplifies[19] to

Proposition 3b: Covered interest rate parity, general setting, simplified

$$(9) \qquad F(0,T) = S_0 \frac{P_f(0,T)}{P(0,T)} \exp\left((1-R) \int_0^T e^{\Theta(s,T)} (1+\eta_s)\mu_s \lambda_s ds \right).$$

Observe that foreign sovereign credit risk unambiguously makes forward contracts more valuable. From the preceding discussion,

$$F_{traditional\ CIP} < F_{general\ setting} < F_{credit\ risk\ only}.$$

Hence, this model is only capable of explaining standard CIP's underpricing of currency forward contracts, relative to market prices. Underpricing occurs in 82%

[19] Derivation in appendix.

of the US/Mexico sample; in this part of the sample, the model fully explains deviations from standard CIP. I describe the estimation of the model below.

V. Model Estimation

I test the new forward price formula (9) above, by estimating η. The formula requires spot and forward currency prices, and domestic and foreign zero coupon bond prices. I use government par bond yields to infer (instantaneous) forward interest rates (which in turn imply zero coupon bond prices), as described below. All data – government par bond yields, and spot and forward prices for currency exchange – are from Bloomberg,[20]

Forward curve estimation

First, note that the forward rate curve, in full generality, is described by a continuum of parameters, but yield curves are necessarily sampled at finitely many points. A simple but robust solution involves parametrizing the forward rate curve to be piecewise constant between the maturities observed on the yield curve; I impose this assumption for each country. Data is typically available for maturities of three months, six months, one year, two years, three years, five years, and ten years – details are in Table II. For convenience, denote these maturities by $\mathbb{T} \equiv \{0, 1/4, 1/2, 1, 2, 3, 5, 10\}$, measured in years. Enumerating $\mathbb{T} - \{10\}$ in order, the piecewise constant assumption means that for all $t_i \in \mathbb{T} - \{10\}$ and $t \in [t_i, t_{i+1})$, $f(t) = f(t_i)$ and $f_1(t) = f_1(t_i)$.

[20]Details in section "Data", preceding the appendix.

The assumed piecewise structure of the forward rates produces a recursive formula for zero coupon bond prices:

$$P(0) = 1,$$

$$P(t_i) = P(t)e^{-f(t)[t_i - t]} \tag{10}$$

for $t \leq t_i \in \mathbb{T} - \{10\}$. This expression will be used repeatedly below to write longer maturity zero coupon bond prices (many unknowns) in terms of shorter maturity zero coupon bond prices (known) and a single forward rate (unknown).

Expression (10) above, in conjunction with the appropriate par bond pricing equations, produces the forward rates. However, for a given maturity T, the par bond pricing equation for a T-maturity bond tends to differ between two different countries, due to hetereogeneity in coupon frequencies of bonds used to construct each country's yield curve – see Table II for details. For instance, the US and Canadian yield curves are both constructed using the corresponding government zero coupon bonds for maturities less than or equal to one year, and semi-annual coupon bonds for the remaining maturities; the Belgian yield curve uses zero coupon bonds for maturities of one year or less, but annual coupon bonds for the rest of the curve. Below, I detail for each country the procedure of mapping a daily observation of a yield curve to the implied forward rate curve.

US, Canada, Italy.

Par yield curve's associated coupon structure. Zero coupon bonds for maturities less than or equal to one year; semi-annual coupon bonds for remaining maturities.

Table II. Coupon Frequency for Bonds Used in Yield Curve Construction

Country	3M	6M	1Y	2Y	3Y	5Y	10Y
Belgium (I6)	0	0	0	A	A	A	A
Canada (I7)	0	0	0	S/A	S/A	S/A	S/A
France (I14)	0	0	0	0	0	0	0
Germany (I16)	0	0	0	0	0	0	0
Italy (I40)	0	0	0	S/A	S/A	S/A	S/A
Japan (I18)	0	0	S/A	S/A	S/A	S/A	S/A
Mexico (I251)	0	N/A	S/A	S/A	S/A	S/A	S/A
Netherlands (I20)	0	0	A	A	A	A	A
Sweden (I21)	0	0	A	A	A	A	A
Switzerland (I82)	0	0	A	A	A	A	A
UK (I22)	0	0	S/A	S/A	S/A	S/A	S/A
US (I25)	0	0	0	S/A	S/A	S/A	S/A
Australia (I1)	N/A	N/A	S/A	S/A	S/A	S/A	S/A

"A" denotes an annual coupon, and "S/A" denotes a semi-annual coupon. Bloomberg codes for the sovereign curves are in parentheses (e.g. The Bloomberg code for Belgium's par yield curve is I6.)

Forward rate calculations. The first three forward rates are obtained explicitly:

$$f(0) = y_{3/12},$$

$$f(3/12) = \frac{1}{1/2 - 1/4} \log \frac{P(3/12)}{P(6/12)},$$

$$f(6/12) = \frac{1}{1 - 1/2} \log \frac{P(6/12)}{P(1)}.$$

Proof. Expression (10) and the definition of yield to maturity produces $f(0) = y_{3/12}$, since $P(3/12) = e^{-y_{3/12}*3/12} = P(0)e^{-f(0)*3/12}$. Similarly, $P(6/12) = P(3/12)$ $e^{-f(3/12)*(1/2-1/4)}$, which simplifies to the stated expression above for $f(3/12)$. Similarly for $f(6/12)$. □

The remaining forward rates and computed numerically. Note that a par bond with maturity T years, face value 100, par yield y_T, and semi-annual coupon payments satisfies the equation[21]

$$100 = \sum_{j=1}^{2T} \frac{100 y_T}{2} P(j/2) + 100P(T), \tag{11}$$

which, in conjunction with equation (10), can be solved numerically for the remaining unknown forward rate with smallest maturity. To illustrate, the expressions for $T = 2$ become

$$100 = \sum_{j=1}^{4} \frac{100 y_T}{2} P(j/2) + 100P(2)$$

$$= \sum_{j=1}^{2} 50 y_2 P(j/2) + \sum_{j=3}^{4} 50 y_2 P(1) e^{-f(1)(j/2-1)} + 100P(1)e^{-f(1)(2-1)},$$

with second equality by expression (10). Since $P(1/2)$ and $P(1)$ are already known, $f(1)$ can be solved for numerically. Now that $f(1)$ is known, $P(1.5)$ and $P(2)$ are given by expression (1). Similarly, applying expressions (10) and (11) for $T = 3$ produces

$$100 = \sum_{j=1}^{4} 50 y_3 P(j/2) + \sum_{j=5}^{6} 50 y_3 P(2) e^{-f(2)(j/2-1)} + 100P(2)e^{-f(2)(2-1)},$$

which is solved numerically for $f(2)$. Now $P(2.5)$ and $P(3)$ are known. Apply the expressions again with $T = 5$ to obtain $f(3)$:

$$100 = \sum_{j=1}^{6} 50 y_5 P(j/2) + \sum_{j=7}^{10} 50 y_5 P(3) e^{-f(3)(j/2-3)} + 100P(3)e^{-f(3)*(5-3)}.$$

Now $P(3.5), P(4), P(4.5), P(5)$ are known. Apply the expressions with $T = 10$ to obtain $f(5)$:

$$100 = \sum_{j=1}^{10} 50 y_{10} P(j/2) + \sum_{j=11}^{20} 50 y_{10} P(5) e^{-f(5)(j/2-5)} + 100P(5)e^{-f(5)(10-5)}.$$

[21] For simplicity of notation, represent by $P(s)$ the quantity $P(t, t+s)$.

The complete forward rate curve has been derived from the yield curve observation; this process is applied for each daily yield curve observation, for each government.

Belgium.

Par yield curve's associated coupon structure. Zero coupon bonds for maturities less than or equal to one year; annual coupons for the remaining maturities.

Forward rate calcuations. The first three forward rates are obtained explicitly, exactly the same as with the US and Canada:

$$f(0) = y_{3/12},$$

$$f(3/12) = \frac{1}{1/2 - 1/4} \log \frac{P(3/12)}{P(6/12)},$$

$$f(6/12) = \frac{1}{1 - 1/2} \log \frac{P(6/12)}{P(1)}.$$

The par bond pricing equation for a T-maturity bond with yield y_T and an annual coupon is

$$1 = \sum_{j=1}^{T} y_T P(j) + P(T). \tag{12}$$

Note that annual coupon payments are convenient: For yields that are available on a yearly maturity grid (here, years 1, 2, and 3), explicit formulas are available for the corresponding forward rates. For Belgium, this is no longer the case starting with the five year yield, as there is no four year yield, so $f(3)$ and $f(5)$ must be computed numerically.

Use y_2 to get $f(1)$. Using expressions (10) and (12) with $T = 2$ produces

$$1 = y_2 P(1) + y_2 P(1)e^{-f(1)} + P(1)e^{-f(1)}$$

$$\Leftrightarrow f(1) = \log\left(\frac{(1+y_2)P(1)}{1-y_2P(1)}\right).$$

Use y_3 to get $f(2)$. Similarly, the expressions with $T = 3$ produces

$$1 = y_3(P(1) + P(2) + P(2)e^{-f(2)}) + P(2)e^{-f(2)}$$

$$\Leftrightarrow f(2) = \log\left(\frac{P(2)(1+y_3)}{1-y_3(P(1)+P(2))}\right)$$

Use y_5 to get $f(3)$. The expressions with $T = 5$:

$$1 = \sum_{j=1}^{5} y_5P(j) + P(5)$$

$$= y_5(P(1) + P(2) + P(3) + P(3)e^{-f(3)} + P(3)e^{-2f(3)}) + P(3)e^{-2f(3)}.$$

Use y_{10} to get $f(5)$. The expressions with $T = 10$:

$$1 = \sum_{j=1}^{10} y_{10}P(j) + P(10)$$

$$= y_{10}(P(1)+P(2)+P(3)+P(4)+P(5)(1+e^{-f(5)}+e^{-2f(5)}+e^{-3f(5)}+e^{-4f(5)}+e^{-5f(5)}))$$

$$+P(5)e^{-5f(5)}.$$

France and Germany.

Par yield curve's associated coupon structure. All zero coupon bonds.

Forward rate calculations. Zero coupon bonds satisfy $P(0,T) = e^{-y_T T}$ for all T. All forward rates are obtained explicitly using expression (10):

$$f(0) = y_{3/12},$$

$$f(3/12) = \frac{1}{1/2 - 1/4}\log\frac{P(3/12)}{P(6/12)},$$

$$f(6/12) = \frac{1}{1-1/2} \log \frac{P(6/12)}{P(1)},$$

$$f(1) = \log \frac{P(1)}{P(2)},$$

$$f(2) = \log \frac{P(2)}{P(3)},$$

$$f(3) = \frac{1}{2} \log \frac{P(3)}{P(5)},$$

$$f(5) = \frac{1}{5} \log \frac{P(5)}{P(10)}.$$

Japan and UK.

Par yield curve's associated coupon structure. Zero coupon for 3M and 6M, semi-annual coupons for the rest.

Forward rate calculations.

$$f(0) = y_{3/12},$$

$$f(3/12) = \frac{1}{1/2-1/4} \log \frac{P(3/12)}{P(6/12)}.$$

To get $f(6/12)$, use the par bond pricing equation for a one year bond:

$$100 = 50y_1(P(1/2)+P(1))+100P(1) = 50y_1(P(1/2)+P(1/2)e^{-f(6/12)/2})+P(1/2)e^{-f(6/12)/2}$$

$$\Leftrightarrow f(6/12) = 2\log\left(\frac{P(1/2)(100+50y_1)}{100-50y_1P(1/2)}\right).$$

To get $f(1), f(2), f(3)$, and $f(5)$, solve (in order)

$$100 = 50y_2(P(1/2) + P(1)(1+e^{-f(1)/2} + e^{-f(1)}) + 100P(1)e^{-f(1)},$$

$$100 = 50y_3(P(1/2) + P(1) + P(1.5) + P(2)(1+e^{-f(2)/2} + e^{-f(2)})) + 100P(2)e^{-f(2)},$$

$$100 = 50y_5(P(1/2)+\cdots+P(2.5)+P(3)(1+e^{-f(3)/2}+e^{-f(3)}+e^{-f(3)*3/2}+e^{-2f(3)}))+100P(3)e^{-2f(3)},$$

$$100 = 50y_{10}(P(1/2) + \cdots + P(5)(1 + e^{-f(5)/2} + \cdots + e^{-5f(5)})) + 100P(5)e^{-5f(5)}.$$

Mexico.

Par yield curve's associated coupon structure. Zero coupon for 3M, semi-annual coupons for the rest. The yield curve structure is the same as Japan and UK, but no six month yield is available.

Forward rate calculations.

$$f(0) = y_{3/12},$$

To obtain $f(3/12)$, use the par bond pricing equation for a one year bond, and all bond prices in terms of $P(3/12)$ and $f(3/12)$:

$$100 = 50y_1(P(6/12) + P(1)) + 100P(1)$$

$$= 50y_1(P(1/4)e^{-f(3/12)*3/12} + P(1/4)e^{-f(3/12)*9/12}) + 100P(3/12)e^{-f(3/12)*9/12}.$$

The procedure to obtain the remaining forward rates is identical to the corresponding procedure for Japan and UK.

Australia.

Par yield curve's associated coupon structure. Semi-annual coupon bonds for all maturities. The earliest maturity is one year. (i.e. Same as Mexico, but no three month yield is available.)

Forward rate calculations. To obtain $f(0)$, use the par bond pricing equation for a one year bond, and write all bond prices in terms of $f(0)$:

$$100 = -100P(1) + 50y_1(P(1/2) + P(1))$$

$$= -100e^{-f(0)} + 50y_1(e^{-f(0)/2} + e^{-f(0)}).$$

The other forward rate calculations are identical to those of the UK.

Netherlands, Sweden, and Switzerland.

Par yield curve's associated coupon structure. 3M and 6M zero coupon, remaining maturities are annual coupon bonds.

Forward rate calculations.

$$
\begin{aligned}
f(0) &= y_{3/12}, \\
f(3/12) &= \frac{1}{1/2 - 1/4} \log \frac{P(3/12)}{P(6/12)}, \\
f(6/12) &= 2 \log(P(1/2)(1 + y_1)), \\
f(1) &= \log\left(\frac{(1 + y_2)P(1)}{1 - y_2 P(1)}\right), \\
f(2) &= \log\left(\frac{P(2)(1 + y_3)}{1 - y_3(P(1) + P(2))}\right).
\end{aligned}
$$

Solve numerically for $f(3)$ and $f(5)$ below:

$$1 = y_5(P(1) + P(2) + P(3) + P(3)e^{-f(3)} + P(3)e^{-2f(3)}) + P(3)e^{-2f(3)},$$

$$1 = y_{10}(P(1)+\cdots+P(5)(1+e^{-f(5)}+e^{-2f(5)}+e^{-3f(5)}+e^{-4f(5)}+e^{-5f(5)}))+P(5)e^{-5f(5)}.$$

Applying covered interest parity with credit risk

Note that credit risk introduces four new parameters into the pricing equation for currency forward contracts. I estimate a one parameter version by making parameter assumptions $R = 0.6$, $\theta = 0$,[22] and constant η. The assumption of $R = 0.6$ is somewhat arbitrary, and simply scales the estimate of $1 + \eta$.[23] For

[22] This is the same as assuming θ constant; just scale the estimate of $1 + \eta$ in the discussion below.

[23] To illustrate the scaling, consider that Bharath and Shumway (2008) use $R = 0.75$: Using this value of R would produce estimates of $1 + \eta$ that are a multiple of 1.6 of the ones in my analysis.

Mexico, I assume $\mu\lambda = 0.04$, which is is the maximum likelihood estimate[24] of default intensity for Mexico: Since achieving independence in 1821, Mexico has defaulted on external debt eight times[25]. I drop from the sample any day in which the market forward price is lower than the price implied by standard CIP; this is 18% of the sample. For now, all currency pairs include USD (e.g. Results stated for Mexico refer to results from the joint analysis of Mexican and US yields and foreign exchange rates).

Given the data and parameter assumptions described above, I use covered interest rate parity relation (9) to procure weekly estimates of η, using the previous 14 days' data as follows: On date t, choose $\eta \in (-1, 0)$ to minimize

$$\sum_{j=-14}^{-1} \left(F_{model}(t+j, t+j+3mo) - F_{market}(t+j, t+j+3mo) \right)^2,$$

where $F_{model}(0, T)$ is given by CIP relation (9). On dates $t, t+1, ..., t+6$, price forward contracts using the minimizer η^* produced from the minimization problem above, i.e. for $j = 1, 2, 3, 4, 5, 6$,

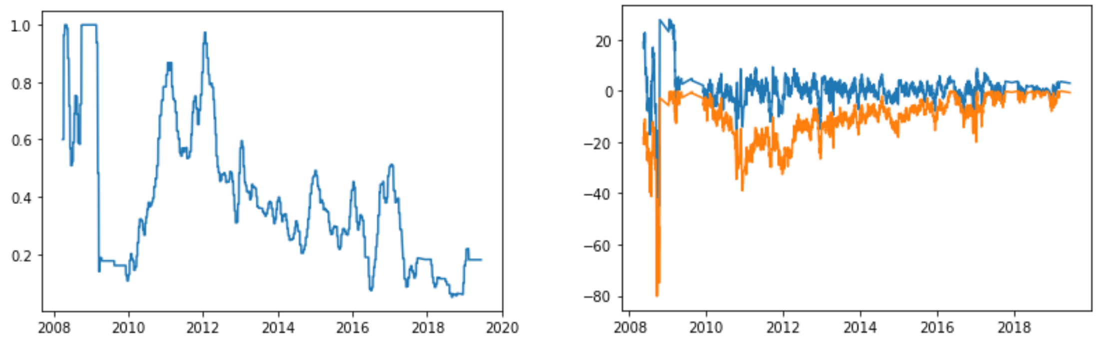
$$F_{model}(t+j, t+j+3mo) = S_{t+j} \frac{P_f(t+j, t+j+3mo)}{P(t+j, t+j+3mo)} \exp\left((1-R)(1+\eta^*)\frac{1}{4}\mu\lambda \right).$$

This procedure is repeated weekly for the sample, which produces the estimates of $1+\eta$ depicted in the plot below. For Mexico, the mean value is 0.402. If this were the true quantity η, a foreign bond close to maturity would lose 76% of its domestic value upon default, taking into account loss of face value.

[24] Classify each year by the number of times Mexico defaulted on its debt in that year; this provides a sample of iid $Poisson(\mu\lambda)$ random variables. The maximum likelihood estimate of $\mu\lambda$ in this setting is known to be the sample mean.

[25] Reinhart and Rogoff document defaults in 1827, 1833, 1844, 1866, 1898, 1914, 1928, 1982 in Tables 6.2, 6.4.

FIGURE 1. Weekly estimates of η and corresponding model forward pricing error for Mexico



On the left: Weekly estimates of η, using two-week-past data as described above. On the right: Pricing errors for standard CIP and credit-risky CIP. The orange curve depicts daily forward pricing error of standard CIP relative to market prices, i.e. $S_t \frac{P_f(t,t+3mo)}{P(t,t+3mo)} - F_{market}(t,t+3mo)$. The blue curve depicts daily forward pricing error of our calibrated model relative to market prices, i.e. $S_t \frac{P_f(t,t+3mo)}{P(t,t+3mo)} \exp((1-R)\int_t^{t+3mo}(1+\eta)\mu\lambda ds) - F_{market}(t,t+3mo)$. Errors are measured in increments of \$0.00001 per Mexican peso, as this is the minimum price fluctuation for Mexican peso futures contracts, and corresponds to a \$5 change in the contract's value; call these increments "points".

These estimates enable pricing of forward contracts according to CIP relation (9); the result is depicted in Figure 1. Traditional CIP persistently underprices forward contracts relative to the market; my model is able to fully correct this bias. The underpricing and correction are statistically significant: Traditional CIP has a mean pricing error of -15.57 points, and standard error of 0.233 points; the model has a mean pricing error -0.061 points and standard error of 0.149 points — Table III has details on all countries.

Implications for Creditworthy Countries

I argue for the relevance of this model for more creditworthy countries – say, G10 countries – as follows: Rare event probabilities are difficult to estimate; default is a rare event for creditworthy countries; hence, credit-risky CIP is the correct pricing equation, and forward prices no longer follow directly from easily observed quantities like yields and the spot currency exchange rate as under traditional CIP. Note that prices for CDS contracts, even if they are traded, may not provide reliable information about credit risk – see Siriwardane 2018.

The uncertainty regarding a country's credit risk parameters produces difficulty in pricing currency forward contracts. To illustrate, suppose that a country's default rate follows a Poisson process with default intensity λ, but λ is unknown. Given the observation of a one hundred year sample with no default, can we reasonably rule out a default probability of one in fifty years in favor of one in one hundred years?

The probability of observing such a sample given a true default probability of one in one hundred years is 0.368; given a true default probability of one in one fifty years, the probability of observing the sample drops to 0.135. In other words, neither default probability is an unreasonable candidate for the true default probability. But the two default probabilities produce meaningfully different values for forward prices: Assuming that standard CIP produces a forward price of 1, the first default intensity implies that the correct forward price is 10 basis points higher; the second default intensity implies that the correct forward price is 20 basis points higher.[26]

These price corrections seem small; such modest default rate uncertainty is in fact sufficient to explain a large fraction of daily mispricings relative to standard CIP in the sample for G10 countries. For instance, standard CIP underprices EUR forward contracts in 94.2% of the sample, using French interest rates, with a mean

[26]In computing these quantities, I use expression (7) for CIP, the setting with no jumps in the interest or exchange rate upon default.

mispricing of -6.82 basis points[27]; 82.2% of these mispricings can be explained by a default intensity of less than 1/100 under credit-risky CIP (see Table III).

Note from Figure II that standard CIP systematically underprices forward contracts, consistent with the existence of foreign sovereign credit risk. Further, the underpricing intensifies in times of crisis; observe the jump in mispricing across countries in 2008. As my model identifies larger CIP underpricing with higher foreign default intensity, this behavior of forward price deviations from standard CIP provides support for the recent notion that the US is a global provider of safe assets.

Following the same procedure I used for Mexico, I use the general CIP relation (9) to price forward contracts for the G10 countries, assuming a default intensity of $\mu\lambda = 0.02$. I find that most of the mispricings vanish – Table IV documents this finding.

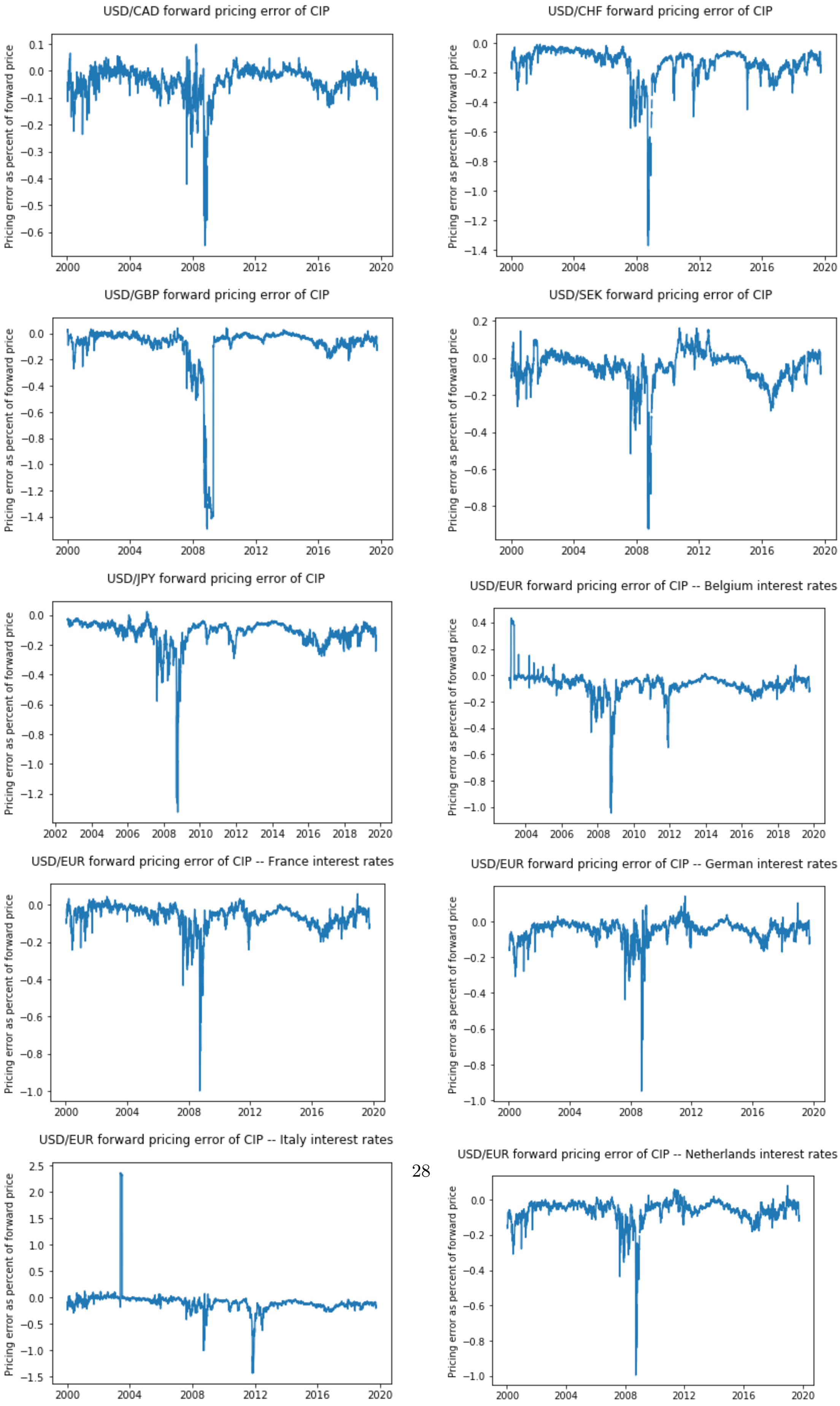
TABLE III. Forward pricing comparison: Standard and credit-risky CIP.

		Standard CIP error		Credit-risky CIP error	
Country	Fraction underpriced	Mean	Standard error	Mean	Standard error
Mexico	81.8	-15.57	0.233	-0.061	0.149
France	94.2	-6.82	0.105	-0.43	0.069
Belgium	96.2	-7.99	0.134	-0.71	0.091
Canada	82.3	-5.03	0.099	-0.34	0.064
Switzerland	99.9	-13.98	0.173	-2.22	0.131
UK	92.7	-10.31	0.329	-3.96	0.28
Sweden	74.7	-8.22	0.156	-0.878	0.104
Japan	99.8	-11.98	0.153	-1.27	0.11
Germany	89.9	-5.82	0.09	-0.25	0.061
Italy	90.3	-12.44	0.191	-1.86	0.149
Netherlands	96.4	-6.82	0.106	-0.46	0.07

Pricing errors are model forward price deviations from market forward prices, normalized by market forward prices, measured in basis points.

[27]Mispricing is measured as $\frac{Standard\ CIP\ price - F_{mkt}}{F_{mkt}}$.

FIGURE 2. G10 forward pricing error



VI. Decomposition of Carry Trade Profits

Note that for the preceding discussion, no discussion of preferences was required – CIP was derived purely from arbitrage considerations. Henceforth, I assume risk neutrality, as is common in the literature. This assumption facilitates analysis of market expectations, as the map from prices to expectations is then straightforward. In particular, currency futures prices are equal to expected future spot exchange rates, i.e. $G(0,T) = E^Q(S_T)$.[28]

The model produces a direct measure of carry trade profitability, based only on currency forward and futures prices. First, note that a currency futures contract has price[29]

Proposition 4: Currency futures price

$$G(t,T) = G(0,T)\exp\left(\int_0^t (a_f(s,T) - a(s,T) + \delta_s)\cdot d\tilde{W}_s\right)$$

$$\exp\left(-\frac{1}{2}\int_0^t (a_f(s,T) - a(s,T) + \delta_s)^2 ds\right) \tag{13}$$

$$\exp\left(\int_0^t (\Theta(s,T) + \log(1+\eta_s))dN_s\right)\exp\left(\int_0^t (1 - e^{\Theta(s,T)}(1+\eta_s)\mu_s\lambda_s ds\right)$$

where

$$G(0,T) = S_0\frac{P_f(0,T)}{P(0,T)}\exp\left(\int_0^T a(t,T)\cdot(a(t,T) - a_f(t,T) - \delta_t)dt\right)$$

$$\exp\left((1-R)\int_0^T e^{\Theta(t,T)}(1+\eta_t)\mu_t\lambda_t dt\right).$$

[28]See e.g. Jarrow 2015

[29]Follows from Lemma 1 in the appendix.

Recall from Table I that the uncovered carry trade produces a payoff at time T of $\frac{P(0,T)}{P_f(0,T)S_0} S_T e_1(T) - 1$. The expected value of this quantity is the expected payoff from the carry trade; this quantity is[30] the volatility exponential minus 1,

$$\exp\left(\int_0^T a(s,T) \cdot (a(s,T) - a_f(s,T) - \delta_s) ds \right) - 1. \tag{14}$$

In the setting with no jumps, this quantity is exactly the difference between the futures and forward price, normalized by the forward price. In the general setting, this quantity is proportional to the difference between futures and forward prices: Expressions (9) and (10) yield

(15)

$$G(0,T) - F(0,T) = C_T \left(\exp\left(\int_0^T a(t,T) \cdot (a(t,T) - a_f(t,T) - \delta_t) dt \right) - 1 \right),$$

where $C_T \equiv S_0 \frac{P_f(0,T)}{P(0,T)} \exp((1-R) \int_0^T e^{\Theta(t,T)}(1+\eta_t)\mu_t \lambda_t dt)$ is a deterministic quantity.

It follows from expression (12) that

Proposition 5: The signed difference of same-delivery futures and forward prices indicates the expected profitability of the uncovered carry trade.

If futures prices are higher than forward prices, currency speculation has a positive expected payoff; otherwise, the payoff is negative in expectation.[31] Interestingly, introducing credit risk and the possibility of a jump in exchange rates and foreign interest rates does not affect unconditional expected returns. Only the continuous

[30]Details in appendix.

[31]Note that this profitability indicator differs from Burnside Eichenbaum Rebelo 2011, who argue that the difference between forward and spot rates determines carry trade profitability. They assume that the spot exchange rate is a martingale, which is generally not the case in our setting.

component of the covariance between S_T and the inverse MMA determines the amount and sign of expected payoff from currency speculation.

However, empirically observed payoffs to the carry trade almost certainly do not equal the expected payoffs. There is no default in typical samples, as sovereign default is rare; consequently, currency speculation appears far more profitable than it is unconditionally. The expected payoff, conditional on no default, is

$$E^Q\left(\frac{P(0,T)}{P_f(0,T)S_0}S_T e_1(T) - 1 \middle| \text{no default in } [0,T]\right)$$

$$= \exp\left(\int_0^T a(s,T)\cdot(a(s,T) - a_f(s,T) - \delta_s)ds\right) \tag{16}$$

$$\exp\left(\int_0^T (1 - Re^{\Theta(s,T)}(1+\eta_s))\mu_s\lambda_s ds\right) - 1.$$

Note that, no matter how small the observed sample, if there has not been default, returns from currency speculation overstate expected returns. In the context of the carry trade, literature refers by a "peso problem" to the difference between the expected returns, conditional on no default, and the unconditional expected returns.

To estimate the conditional and unconditional expected payoffs of the carry trade, it remains to estimate the instantaneous volatilities of the forward rates and the spot exchange rate – these quantities are given by a principal components analysis of the daily changes in forward rates and percent change in exchange rate[32].

[32]Details in Jarrow (2002)

Conveniently, principal components analysis produces estimates of loadings on Brownian motion, separate from Poisson process estimation. Recall that the arbitrage pricing model posits that $n+1$ sources of randomness (n Brownian motions and one Poisson process) drive the evolution of interest rates and spot foreign exchange rates. To specify the model, we must estimate the number of sources of continuous randomness, the loadings of the interest rates and spot foreign exchange rate on the Brownian motions (i.e. $\sigma(s,T), \sigma_f(s,T), \delta_s$), and arrival rate and loadings on the Poisson process.

Jarrow (2002) demonstrates that a principal components analysis of the yield curve produces the loadings σ required to specify an HJM model with deterministic volatility functions: σ_i is the the ith eigenvector of the sample covariance matrix of the daily changes in the forward rates, scaled by the square root of the ith eigenvalue. Jolliffe (2002) describes numerous methods of choosing of n; I use the minimum number of components required to explain at least 90% of the variance. For our sample, this means a two factor model for each country pair – see Table V.

Observe that the natural extension of the procedure described in Jarrow (2002) will produce the interest rate and spot FX volatilities, even in the presence of the Poisson process and associated compensating drifts. The extension is as follows: Each daily observation consists of the changes from the previous day in the domestic and foreign forward rates, and the percent change in the spot foreign exchange rate; proceed as before to obtain loadings via PCA.

More precisely, start with m_d domestic forward rate maturities $T_1, ..., T_{m_d}$, m_f foreign forward rate maturities $T_1^f, ..., T_{m_f}^f$, and observations $\tilde{f}, \tilde{f}_f, \tilde{S}$, taken from a sample with no foreign sovereign default. Define the $m_d + m_f + 1$ dimensional

Table IV. Percent of total variation explained by principal components

Number of principal components	AUD/JPY	MXN/USD	MXN/JPY	MXN/SEK
1	97.0	87.1	94.3	86.1
2	98.6	90.6	96.2	89.6
3	98.9	92.9	97.1	92.6
4	99.2	94.8	97.9	94.1
5	99.5	96.2	98.6	95.5
6	99.7	96.9	99.1	96.7
7	99.7	97.6	99.4	97.7
8	99.8	98.2	99.6	98.5
9	99.9	98.7	99.8	99.0
10	99.9	98.9	99.9	99.4
11	100.0	99.3	99.9	99.7

The total variation refers to the daily changes in levels of the currency and corresponding countries' forward curves. For instance, 98.6% of the total variation in the JPY forward curve, AUD forward curve, and AUD/JPY are explained by a two factor model.

vector $x(t)$ by

$$x_i(t) = \begin{cases} \tilde{f}(t+\Delta, t+T_i) - \tilde{f}(t, t+T_i) & i \in [1, m_d] \\ \tilde{f}(t+\Delta, t+T^f_{i-m_d}) - \tilde{f}(t, t+T^f_{i-m_d}) & i \in [m_d+1, m_d+m_f] \\ \frac{\tilde{S}_{t+\Delta} - \tilde{S}_t}{\tilde{S}_t} & i = m_d+m_f+1 \end{cases}$$

To ensure time homogeneity of $x(t)$, assume that spot FX drift and volatilities (η_t and δ_t) are constants, and forward rate drifts and volatilities are deterministic functions of time to maturity (e.g. $\alpha(t,T) = \alpha(T-t)$). Now $x(t)$ is a time-homogeneous normal random variable with mean vector μ^* defined by

$$\mu^* = \Delta \begin{cases} \alpha(t, t+T_i) & i \in [1, m_d] \\ \alpha_f(t, t+T^f_{i-m_d}) & i \in [m_d+1, m_d+m_f] \\ \eta & i = m_d+m_f+1 \end{cases}$$

and covariance matrix Σ defined by $(\Sigma)_{ij} = \sum_{k=1}^{n} \tilde{\sigma}_k(i)\tilde{\sigma}_k(j)$, where

$$\tilde{\sigma}_k(i) = \begin{cases} \sigma_k(T_i) & i \in [1, m_d] \\ \sigma_{f,k}(T^f_{i-m_d}) & i \in [m_d+1, m_d+m_f] \\ \delta_k & i = m_d+m_f+1 \end{cases}$$

Letting a_i and l_i denote the ith eigenvector and eigenvalue of Σ, respectively, the volatility estimates are given by $\tilde{\sigma}_i = a_i\sqrt{l_i}$.

Observe that the estimation of Brownian loadings occurs without reference to the Poisson process, because the sample does not contain a sovereign credit event. Approximate changes in the observed rates then becomes

$$\begin{aligned} f_f(t+\Delta, T) - f_f(t,T) &\approx \alpha_f(t,T)\Delta + \sigma_f \cdot (W_{t+\Delta} - W_t) + \theta(t,T)(N_{t+\Delta} - N_t) \\ &= \alpha_f(t,T)\Delta + \sigma_f \cdot (W_{t+\Delta} - W_t), \end{aligned}$$

since $N_{t+\Delta} - N_t = 0$ in a sample with no default; the analogous result applies to the spot FX rate.

Having established the procedure to estimate the model, I now apply it descriptively and predictively. The descriptive analysis produces an estimate of the unconditional expected carry trade returns for the MXN/USD carry trade, and an estimate of the conditional quantity, given no default; the gap constitutes the peso problem inherent in the trade. I use the full sample of data to estimate the instantaneous volatilities, and then use expressions (14) and (16) to compute the desired quantities for a three month carry trade. I find that the three month volatility-exponential is 0.999868. In other words, \$1M USD invested in a carry trade is expected, unconditionally, to lose -\$27 USD. However, conditional on no default, the same investment is expected to produce \$8,025 USD, using as an estimate of

$1 + \eta$ the sample mean 0.402. I conclude that the carry trade's profitability is overwhelmingly driven by the peso problem, rather than positive expected payoffs from currency speculation.

The predictive analysis is as follows: Use the preceding three months' data to estimate the instantaneous volatilities, and use these volatilities to compute the risk neutral expected carry trade returns in the following three months. If the sign of this expected return is positive, go long the carry trade; otherwise, go short. I perform this analysis for AUD/JPY, USD/MXN, JPY/MXN, and SEK/MXN. The results are mixed: This "PCA carry trade" dominates the carry trade for one week, two week, one month, and three month carry trades for AUD/JPY, and fares well for JPY/MXN; the standard carry trade does better for USD/MXN and SEK/MXN. Unfortunately, almost none of these results have statistical significance at the 95% level – see Table V.

Figure 3. AUD/JPY carry trade is improved with PCA at all trade horizons

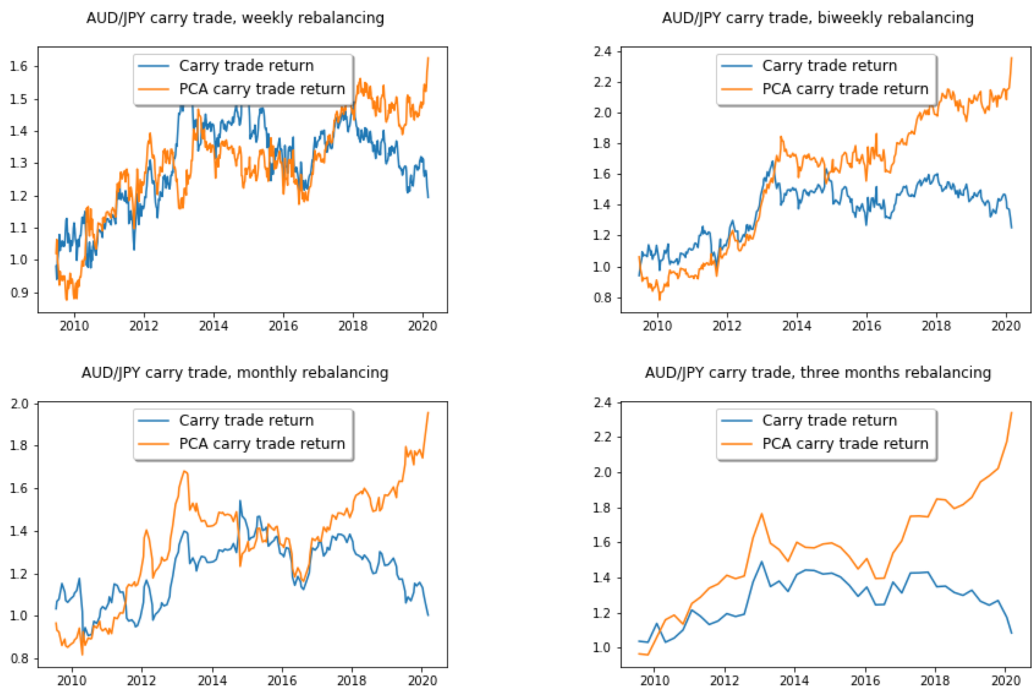


Figure 4. USD/MXN carry trade is not improved with PCA at most trade horizons

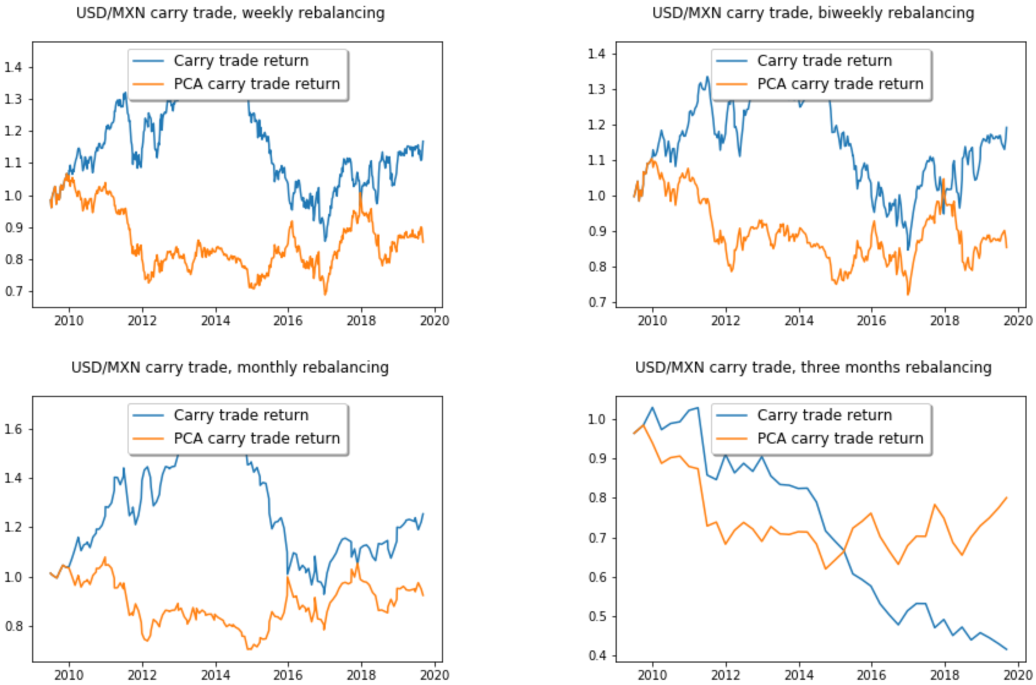


Figure 5. JPY/MXN carry trade is improved with PCA at most trade horizons

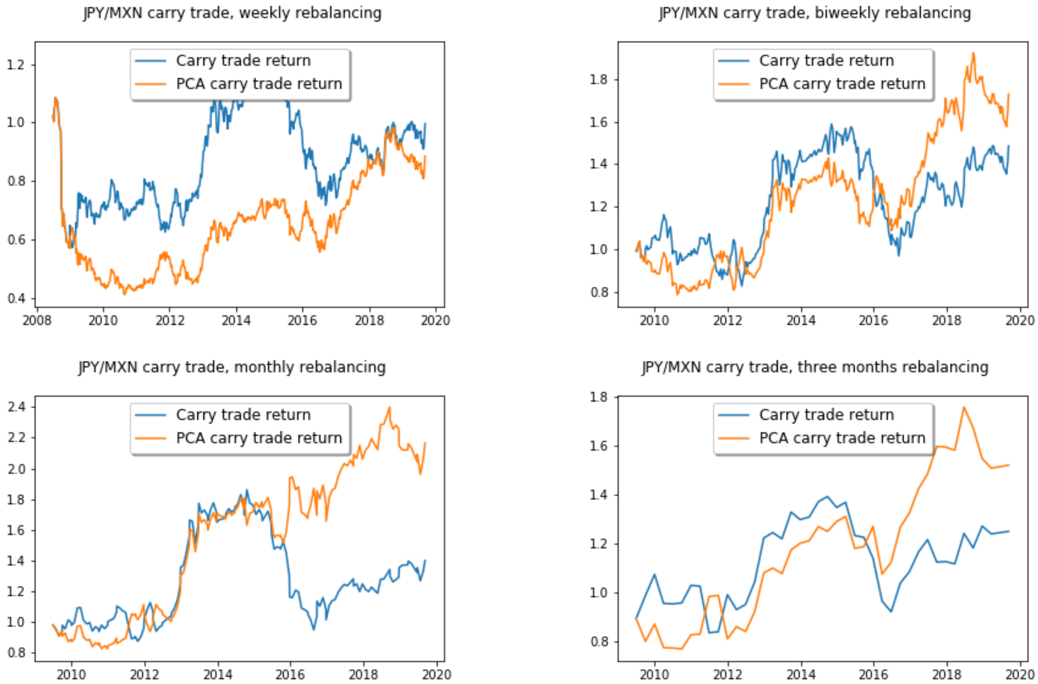


Figure 6. SEK/MXN carry trade is not improved with PCA at all trade horizons

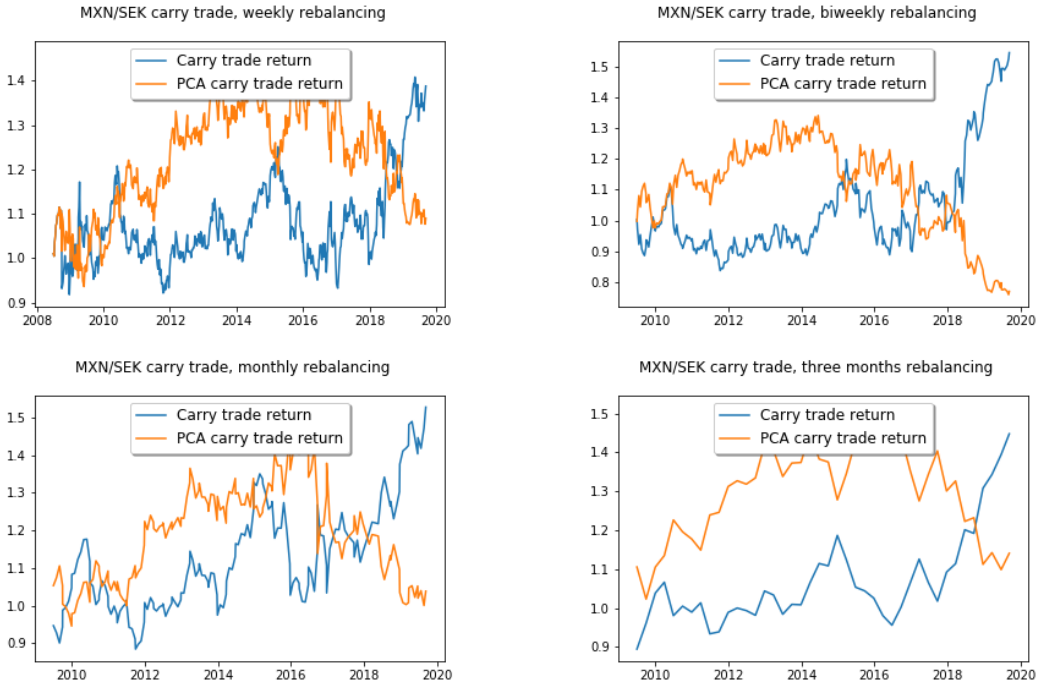


Table V. Carry trade returns are mostly not statistically different from zero at the 95% significance level

Trade horizon	One week	Two week	One month	Three months
AUD/JPY Carry	0.000463	0.00106	0.000655	0.00335
	(0.000751)	(0.00150)	(0.00270)	(0.00850)
AUD/JPY PCA	0.000976	0.00301*	0.00452	0.0208*
	(0.000750)	(0.00149)	(0.00268)	(0.00791)
MXN/USD Carry	0.0003918	0.000793	0.00195	0.00283
	(0.000648)	(0.00120)	(0.00255)	(0.00837)
MXN/USD PCA	-0.000305	-0.00115	-0.000373	-0.00613
	(0.000648)	(0.00120)	(0.00255)	(0.00833)
MXN/JPY Carry	0.000717	0.00170	0.00301	0.00847
	(0.000850)	(0.00163)	(0.00334)	(0.0123)
MXN/JPY PCA	0.00106	0.00219	0.00576	0.0131
	(0.000850)	(0.00163)	(0.00332)	(0.0122)
MXN/SEK Carry	0.000611	0.00172	0.00328	0.0101
	(0.000716)	(0.00140)	(0.00276)	(0.00775)
MXN/SEK PCA	0.000347	-0.000550	0.000848	0.00441
	(0.000717)	(0.00140)	(0.00277)	(0.00788)

Each entry contains the mean and standard error of returns; the standard error is reported below in parentheses (e.g. the standard AUD/JPY carry trade has a one week return of 0.000463 on average, with standard error of 0.000751). A star denotes a statistically significant positive return at a 95% significance level.

VII. Option Pricing

The model produces extensions of option pricing formulas presented in Amin Jarrow (1991).

Options on spot currency exchange

To simplify the analysis, let $\theta = 0$, $\eta_s \equiv \eta$.[33] Then a European call option on the spot exchange rate with strike K and maturity T has price[34]

$$C(0,T,K) = \sum_{j\geq 0}\Big(C_{1,j}\Phi(d_j) - C_2\Phi(d_j - \xi)\Big)\frac{\Lambda_T^j e^{-\Lambda_T}}{j!}, \tag{17}$$

where $\Phi(\cdot)$ is the standard normal cumulative density function,

$$C_{1,j} \equiv S_0 P_f(0,T)(1+\eta)^j \exp((1 - R(1+\eta))\Lambda_T),$$

$$C_2 \equiv KP(0,T),$$

$$d_j \equiv \frac{\log(C_{1,j}/C_2) + \xi^2/2}{\xi},$$

$$\xi \equiv \int_0^T (\sigma^*(s,T) + \delta_s - \sigma_f^*(s,T))^2 ds,$$

$$\Lambda_T \equiv \int_0^T \mu_s \lambda_s ds.$$

Sufficient precision[35] is obtained using a partial sum with only the first eight terms of the series. In general, to obtain a tail error of at most 10^{-k}, n must be at least[36]

$$\frac{\log(10^{-k}(1-\Lambda_T)e^{\Lambda_T}/M)}{\log(\Lambda_T)}, \tag{18}$$

where $M \equiv 2\max(S_0 P_f(0,T)e^{(1-R(1+\eta))\Lambda_T}, KP(0,T))$.

[33]These assumptions make the value of the call option dependent only on the number of defaults, not the timing. Further empirical analysis is needed to determine whether these assumptions are worth relaxing.
[34]Derivation in appendix.
[35]Price within 10^{-10}.
[36]Derivation in appendix.

An analogous argument produces the value of an analogous European put option on the spot exchange rate:

$$Put(0,T,K) = \sum_{j\geq 0}\Big(C_2\Phi(-d_j+\xi) - C_{1,j}\Phi(-d_j)\Big)\frac{\Lambda_T^j e^{-\Lambda_T}}{j!}. \tag{19}$$

Options on currency futures

Consider a European call option with strike K and maturity T on a futures contract with delivery $L > T$. Denoting its price by $C_G(0,T,L,K)$, we obtain

$$C_G(0,T,L,K) = \sum_{j\geq 0}\Big(C_{G,j}\Phi(d_{G,j}) - C_2\Phi(d_{G,j}-\xi_G)\Big)\frac{\Lambda_T^j e^{-\Lambda_T}}{j!}, \tag{20}$$

where

$$C_{G,j} \equiv F(0,L)P(0,T)(1+\eta)^j \exp(-\eta\Lambda_T)\exp((1-R)(1+\eta)\Lambda_L)$$

$$\exp\Big(\int_T^L \sigma^*(s,L)(\sigma^*(s,L) - \sigma_f^*(s,L) + \delta_s)ds\Big),$$

$$d_{G,j} \equiv \frac{\log(C_3/C_2) + \xi_G^2/2}{\xi_G},$$

$$\xi_G \equiv \int_0^T (\sigma^*(s,T) + \delta_s - \sigma_f^*(s,L))ds.$$

Proofs

European call option on spot exchange

Proof. The call price is given by the discounted expected value of the call payoff:

$$C(0,T) = E^Q\Big(\frac{S_T-K}{B_T}\Big)^+.$$

Note that, conditional on $N_T = j \in \{0, 1, ...\}$,

$$\frac{S_T}{B_T} = S_0 P_f(0,T)(1+\eta)^j \exp(-\eta\Lambda_T)\exp((1-R)(1+\eta)\Lambda_T)\mathcal{E}\Big((\delta - \sigma_f^*)d\tilde{W}\Big),$$

$$\frac{K}{B_T} = KP(0,T)\mathcal{E}\Big(-\sigma^* d\tilde{W}\Big).$$

For convenience, let $C_{1,j} \equiv S_0 P_f(0,T)(1+\eta)^j \exp((1 - R(1+\eta))\mu\lambda T)$, $C_2 \equiv KP(0,T)$. Then

$$E^Q\left(\left(\frac{S_T - K}{B_T}\right)^+ \middle| N_T = j\right) = E^Q(C_{1,j}\mathcal{E}((\delta - \sigma_f^*)d\tilde{W}) - C_2\mathcal{E}(-\sigma d\tilde{W}))$$

$$= C_{1,j}\Phi(d_j) - C_2\Phi(d_j - \xi),$$

where $\Phi(\cdot)$ is the standard normal cumulative density function,

$$d_j \equiv \frac{\log(C_{1,j}/C_2) + \xi^2/2}{\xi},$$

and $\xi \equiv \frac{1}{2}\int_0^T (\sigma^*(s,T) + \delta_s - \sigma_f^*(s,T))^2 ds$. Use $P(N_T = j)$ to obtain the call option's price:

$$C(0,T) = \sum_{j\geq 0} E^Q\left(\left(\frac{S_T - K}{B_T}\right)^+ \middle| N_T = j\right) P(N_T = j).$$

□

Tail error bound

Proof. The Cauchy-Schwarz inequality provides the bound on the tail error in the series:

$$\left|\sum_{j\geq n}(C_{1,j}\Phi(d_j) - KP(0,T)\Phi(d_j - \xi))\frac{(\mu\lambda T)^j}{j!}e^{-\mu\lambda T}\right|$$

$$\leq 2e^{-\mu\lambda T}\sum_{j\geq n}\max(C_{1,0}, KP(0,T))(\mu\lambda T)^j = \frac{M(\mu\lambda T)^n e^{-\mu\lambda T}}{1-\mu\lambda T}$$

with the first inequality by noting that $C_{1,j}$ is decreasing in j. □

Call option on currency futures

Proof. The call option's payoff at maturity is $(G(T,L)-K)^+$. Note that, conditional on $N_T = j \in \{0,1,...\}$,

$$\frac{G(T,L)}{B_T} = C_{G,j}\mathcal{E}\bigg(\int_0^T(\delta_s - \sigma_f^*(s,L)d\tilde{W}_s\bigg),$$

where

$$C_{G,j} \equiv F(0,L)P(0,T)(1+\eta)^j\exp(-\eta\Lambda_T)\exp((1-R)(1+\eta)\Lambda_L)$$

$$\exp\bigg(\int_T^L \sigma^*(s,L)(\sigma^*(s,L)-\sigma_f^*(s,L)+\delta_s)ds\bigg).$$

Note that

$$E^Q\bigg(C_{G,j}\mathcal{E}\bigg(\int_0^T(\delta_s-\sigma_f^*(s,L))d\tilde{W}_s\bigg) - C_2\mathcal{E}\bigg(\int_0^T\sigma^*(s,T)d\tilde{W}_s\bigg)\bigg)^+$$

$$= C_{G,j}\Phi(d_{G,j}) - C_2\Phi(d_{G,j}-\xi_G),$$

where

$$\exp\bigg(\int_T^L \sigma^*(s,L)(\sigma^*(s,L)-\sigma_f^*(s,L)+\delta_s)ds\bigg).$$

$$d_{G,j} \equiv \frac{\log(C_3/C_2)+\xi_G^2/2}{\xi_G},$$

$$\xi_G \equiv \int_0^T(\sigma^*(s,T)+\delta_s-\sigma_f^*(s,L))ds.$$

Apply the same conditioning argument as for the European option on the spot exchange rate to obtain the result. □

VIII. Conclusion

Sovereign credit risk and rapid exchange rate devaluations are capable of explaining persistent profits to the covered and uncovered carry trade. Credit-risky covered interest rate rate parity prices forward contracts in close agreement with market prices. The calibrated model indicates that carry trade returns are driven exclusively by a peso problem, and unconditional expected payoffs are minuscule. I demonstrate an application of the model to a long/short carry trade trading system, which in recent years dominates the standard carry trade for some popular carry trade currency pairs. I demonstrate that the credit risky model of foreign exchange accomodates option pricing formulas that are a natural extension of existing models.

I conclude that the richer[37] model of foreign exchange presented in this paper produces an explanation for puzzle in the exchange rate literature, and a potential improvement to an already-profitable trading strategy.

Data

The sample consists of daily observations for Mexico and the G10 countries, and spans the period between January 1st, 2000 to October 7th, 2019 for which three month, six month, one year, three year, five, and ten year yields are readily available on Bloomberg. For Mexico, this is from April 1st, 2008. For most G10 countries, this is the start of 2000; exceptions are Japan and Belgium, for which the samples start in 2002 and 2003, respectively.

[37]Primarily by incorporating credit risk and random interest rates.

Appendix

Derivation of price procceses

We derive the foreign bond prices; the domestic bond price derivation is analogous.

By definition, $P_0(t,T) = \exp(-\int_t^T f_0(t,y)dy)$ and $f_0(t,y) = f_0(0,y) + \int_0^t \alpha_0(s,y)ds + \int_0^t \sigma_f(s,y)dW_s + \int_0^t \theta(s,y)dN_s$, so $\log P_0(t,T) =$

$$-\int_t^T f_0(0,y)dy - \int_t^T \int_0^t \alpha_0(s,y)dsdy - \int_t^T \int_0^t \sigma_f(s,y)dW_s dy - \int_t^T \int_0^t \theta(s,y)dN_s dy.$$

By Fubini's theorem[38], this quantity is

$$-\int_t^T f_0(0,y)dy - \int_0^t \int_t^T \alpha_0(s,y)dyds - \int_0^t \int_t^T \sigma_f(s,y)dydW_s - \int_0^t \int_t^T \theta(s,y)dydN_s.$$

Noting that $-\int_0^t \int_t^T (\cdot)dyds = -\int_0^t \int_s^T (\cdot)dyds + \int_0^t (\cdot) \int_s^t dyds$, we rewrite as

$$-\int_t^T f_0(0,y)dy - \int_0^t \int_s^T \alpha_0(s,y)dyds - \int_0^t \int_s^T \sigma_f(s,y)dydW_s - \int_0^t \int_s^T \theta(s,y)dydN_s$$
$$+\int_0^t \int_s^t \alpha_0(s,y)dyds + \int_0^t \int_s^t \sigma_f(s,y)dydW_s + \int_0^t \int_s^t \theta(s,y)dydN_s.$$

Let $b_0(s,T) \equiv -\int_s^T \alpha_0(s,y)dy$, $a_f(s,T) \equiv -\int_s^T \sigma_f(s,y)dy$, and $\Theta(s,T) \equiv -\int_s^T \theta(s,y)dy$. Note that $-\int_t^T f_0(0,y)dy = -\int_0^T f_0(0,y)dy + \int_0^t f_0(0,y)dy$ and apply Fubini's theorem to the last line in the equation above; exponentiate to obtain the claimed expression for the foreign bond price.

Simplifying $e^{-\int_0^t f_1(0,y)dy}$ is identical; simply change the drift subscripts. Note that $P_f(t,T) = e^{-\int_0^t f_1(0,y)dy} e_1(t)$.

S_t follows from Ito's lemma.

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[38]See Jarrow Madan 1995 for the version with integrals with respect to point processes, and Heath Jarrow Morton 1992 for the version with integrals with respect to Brownian motions.

Derivation of risk neutral drifts

Standard arguments[39] equate absence of arbitrage with the existence of an equivalent martingale measure, i.e. a probability measure equivalent to P, under which all asset price processes – denominated in domestic currency and discounted at the domestic money market account – are martingales. Hence the relevant price processes are $Z(t,T) \equiv \frac{P(t,T)}{B_t}$, $Z_0(t,T) \equiv \frac{P_0(t,T)S_t}{B_t}$, $Z_f(t,T) \equiv \frac{P_f(t,T)S_t}{B_t}$, $Z_{r0}(t,T) \equiv \frac{B_0(t)S_t}{B_t}$, and $Z_{r1}(t,T) \equiv \frac{B_1(t)e_1(t)S_t}{B_t}$. The price processes above imply the following dynamics via Ito's lemma:

$$\frac{dZ(t,T)}{Z(t,T)} = [b(t,T) + \frac{1}{2}\sum_i a_i^2(t,T)]dt + \sum_i a_i(t,T)dW_i(t),$$

$$\frac{dZ_0(t,T)}{Z_0(t-,T)} = (\nu_t + r_0(t) - r_d(t) + b_0(t,T) + \frac{1}{2}\sum_i a_{fi}^2(t,T) + \sum_i a_{fi}\delta_i(t))dt$$
$$+ \sum_i (a_{fi}(t) + \delta_i(t))dW_t + (e^{\Theta(t,T)}(1+\eta_t) - 1)dN_t,$$
$$\frac{dZ_f(t,T)}{Z_f(t-,T)} = (\nu_t + r_1(t) - r_d(t) + b_1(t,T) + \frac{1}{2}\sum_i a_{fi}^2(t,T) + \sum_i a_{fi}\delta_i(t))dt$$
$$+ \sum_i (a_{fi}(t) + \delta_i(t))dW_t + (Re^{\Theta(t,T)}(1+\eta_t) - 1)dN_t,$$
$$\frac{dZ_{r0}(t)}{Z_{r0}(t)} = (r_0(t) - r_d(t) + \nu_t)dt + \sum_i \delta_i(t)dW_i(t) + \eta dN_t,$$
$$\frac{dZ_{r1}(t)}{Z_{r1}(t)} = (r_1(t) - r_d(t) + \nu_t)dt + \sum_i \delta_i(t)dW_i(t) + (R(1+\eta) - 1)dN_t,$$

Existence and uniqueness of the equivalent martingale measure. Note that there are five asset classes: The domestic bond, the foreign bonds, and the foreign MMAs, where the foreign assets have defaultable and non-defaultable versions. For convenience, denote each asset class' drift by d_k, where $k = 1$ denotes the domestic

[39] e.g. Harrison Kreps 1979, Jarrow Madan 1995.

bond, $k=2$ denotes the foreign default-free bond, $k=3$ denotes the foreign defaultable bond, $k=4$ denotes the foreign default-free foreign MMA, and $k=5$ denotes the foreign defaultable MMA. For instance, $d_1(t,T)=b(t,T)+\frac{1}{2}a^2(t,T)$, $d_2(t,T)=\nu_t+r_0(t)-r_d(t)+b_0(t,T)+\frac{1}{2}a_f^2(t,T)+a_f\cdot\delta_t$.

Pick n maturities $0\le S_1<\cdots<S_{n-1}$, and $m_1,m_2\ge 1$ with $m_1+m_2+m_3=n-1$. Given the dynamics above, no arbitrage is guaranteed by the invertibility of H_t, where

$$H_t=\begin{bmatrix} a_1(t,S_1) & \cdots & a_n(t,S_1) & 0 \\ & \vdots & & \\ a_1(t,S_{m_1}) & \cdots & a_n(t,S_{m_1}) & 0 \\ \tilde{a}_{f1}(t,S_{m_1+1}) & \cdots & \tilde{a}_{fn}(t,S_{m_1+1}) & e^{\Theta(t,S_{m_1})}(1+\eta_t)-1 \\ & \vdots & & \\ \tilde{a}_{f1}(t,S_{m_1+m_2}) & \cdots & \tilde{a}_{fn}(t,S_{m_1+m_2}) & e^{\Theta(t,S_{m_1+m_2})}(1+\eta_t)-1 \\ \tilde{a}_{f1}(t,S_{m_1+m_2+1}) & \cdots & \tilde{a}_{fn}(t,S_{m_1+m_2+1}) & Re^{\Theta(t,S_{m_1+m_2+1})}(1+\eta_t)-1 \\ & \vdots & & \\ \tilde{a}_{f1}(t,S_{n-1}) & \cdots & \tilde{a}_{fn}(t,S_{n-1}) & Re^{\Theta(t,S_{n-1})}(1+\eta_t)-1 \\ \delta_1(t) & \cdots & \delta_n(t) & \eta_t \\ \delta_1(t) & \cdots & \delta_n(t) & R(1+\eta_t)-1 \end{bmatrix},$$

where $\tilde{a}_{fi}(t,\cdot)\equiv a_{fi}(t,\cdot)+\delta_i(t)$.

Then the unique solutions μ and γ - assumed not to depend on the maturities chosen - define the market prices of risk:

$$H_t \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \\ \mu_t\lambda_t \end{bmatrix} = - \begin{bmatrix} d_1(t, S_1) \\ \vdots \\ d_3(t, S_{n-1}) \\ d_4(t) \\ d_5(t) \end{bmatrix}. \tag{21}$$

Hence, to guarantee a sensible model, we need assumption A4, stated below.

A4: Assume that H_t is invertible. Assume that the solutions γ and $\mu\lambda$ do not depend on the choice of maturities $S_1, ..., S_{n-1}$, and satisfy appropriate regularity conditions.[40]

Existence and uniqueness of equivalent martingale measure follows from A1-A4. Q defined by $\frac{dQ}{dP} = Z$ is the unique equivalent martingale measure, where

$$Z = \exp\bigg(\int_0^{S_1} \log \mu_s dN_s - \int_0^{S_1} (\mu_s - 1)\lambda_s ds + \sum \int_0^{S_1} \gamma_i(s) dW_i(s) - \frac{1}{2} \sum \int_0^{S_1} \gamma_i^2(s) ds \bigg), \tag{22}$$

where μ, γ are the solutions to the system of equations in A4.

Proof. By Jarrow Madan 1995, equivalent probability measures take the form given by Z for some adapted processes μ_s and γ so that μ is nonnegative, and the processes are suitably integrable. Further, under Q, $\tilde{W}_i(t) \equiv W_i(t) - \int_0^t \gamma_i(s)ds$ and $\tilde{M}_t \equiv N_t - \int_0^t \mu_s\lambda_s ds$ are martingales.

[40]See Jarrow Madan Assumption 3.1.

Note that, in defining Q, if we take μ and γ as the solution to the system of equations in A4, we obtain an equivalent probability measure, and this is the unique such measure makes the domestic-denominated relative asset prices martingales; i.e. this is the unique martingale measure. $\square$

Computation of risk neutral drifts.

Proof. Domestic bond drift. Start with the physical dynamics for the domestic bond, and write in terms of Q-dynamics:

$$\frac{dZ(t,T)}{Z(t,T)} = [b(t,T) + \frac{1}{2}\sum_i a_i^2(t,T)]dt + a(t,T)\cdot dW_t$$

$$= [b(t,T) + \frac{1}{2}\sum_i a_i^2(t,T) - a(t,T)\cdot\gamma_t]dt + a(t,T)\cdot d\tilde{W}_t.$$

Set the drift equal to zero and differentiate with respect to T to obtain

$$\alpha(t,T) = \sigma(t,T)\Big(\int_t^T \sigma(t,y)dy + \gamma_t\Big).$$

Plug this quantity into the physical dynamics of the domestic forward rate to obtain

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)\cdot dW_t$$

$$= \sigma(t,T)\int_t^T \sigma(t,y)dydt + \sigma(t,T)\cdot d\tilde{W}_t.$$

Default-free foreign bond drift. Analogously, start with the physical dynamics for the default-free foreign bond (relative price, domestic-denominated), and write in terms of Q-dynamics:

$$\frac{dZ_0(t,T)}{Z_0(t-,T)} = (\nu_t + r_0(t) - r_d(t) + b_0(t,T) + \frac{1}{2}\sum_i a_{fi}^2(t,T) + \sum_i a_{fi}\delta_i(t))dt$$

$$+\sum_i (a_{fi}(t) + \delta_i(t))dW_t + (e^{\Theta(t,T)}(1+\eta_t) - 1)dN_t,$$

$$= (\nu_t + r_0(t) - r_d(t) + b_0(t,T) + \frac{1}{2}\sum_i a_{fi}^2(t,T) + \sum_i a_{fi}\delta_i(t)$$

$$-(a_f(t,T) + \delta_t)\cdot\gamma_t + \mu\lambda(e^{\Theta(t,T)}(1+\eta) - 1)dt$$

$$+\sum_i (a_{fi}(t) + \delta_i(t))d\tilde{W}_t + (e^{\Theta(t,T)}(1+\eta_t) - 1)d\tilde{M}_t.$$

Set the drift equal to zero and differentiate with respect to T to obtain

$$\alpha_0(t,T) = \sigma_f(t,T)\bigg(\int_t^T \sigma_f(t,y)dy - \delta_t + \gamma_t\bigg) - \theta(t,T)e^{\Theta(t,T)}(1+\eta_t)\mu\lambda.$$

Plug this into

$$df_0(t,T) = \alpha_0(t,T)dt + a_f(t,T)\cdot dW_t + \theta(t,T)dN_t,$$

add and subtract $\theta(t,T)\mu\lambda$, and combine to get

$$df_0(t,T) = (\sigma_f(t,T)\bigg(\int_t^T \sigma_f(t,y)dy - \delta_t\bigg) + \mu\lambda\theta(t,T)(1 - e^{\Theta(t,T)}(1+\eta_t)))dt$$

$$+a_f(t,T)\cdot d\tilde{W}_t + \theta(t,T)d\tilde{M}_t.$$

Defaultable foreign bond drift. Follow the same procedure, but start with the dynamics for the defaultable foreign bond.

Exchange rate drift. For the first equality, start with the physical dynamics of the default-free foreign MMA (domestic-denominated, domestic-discounted), write in terms of Q-dynamics, and plug into the form given by A3. For the second equality, do the same but start with the physical dynamics of the defaultable foreign MMA. □

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First statement of covered interest parity

Proof. The result follows if $P_f(t,T) = P_0(t,T)\exp(\int_0^t \log R dN_s)\exp((R-1)\int_t^T \mu_s\lambda_s ds)$, which is proven below:

Note that $Z_f(t,T) = E^Q(Z_f(T,T)|\mathcal{F}_t) = E^Q(Z_0(T,T)e_1(T)|\mathcal{F}_t) = Z_0(t,T)E^Q(e_1(T)|\mathcal{F}_t)$, with the first equality since the relative price of the defaultable foreign bond is a martingale, the second equality since $P_0(T,T) = 1$, the third equality from independence of the recovery process $e_1(t)$ from $S_tP_0(t,T)/B_t$, and the martingality of the relative price of the default-free foreign bond. Multiply both sides by B_t/S_t to obtain

$$P_f(t,T) = P_0(t,T)\exp\bigg(\int_0^t \log R dN_s\bigg)E^Q\bigg(\exp\bigg(\int_t^T \log R dN_s\bigg)\bigg|\mathcal{F}_t\bigg);$$

it remains to show that the latter expectation evaluates to $\exp((R-1)\int_t^T \mu\lambda ds)$. The random variable in this expectation is equal to R^k with probability $\frac{(\int_t^T \mu\lambda ds)^k}{k!}e^{-\int_t^T \mu\lambda ds}$. Letting $\Lambda(t,T) \equiv \int_t^T \mu\lambda ds$, we obtain $E^Q(\exp(\int_t^T \log R dN_s)|\mathcal{F}_t) = \sum_{k\geq 0} R^k \frac{\Lambda^k(t,T)}{k!}e^{-\Lambda(t,T)}$ $= e^{\Lambda(t,T)(R-1)}\sum_{k\geq 0}\frac{(R\Lambda(t,T))^k}{k!}e^{-R\Lambda(t,T)} = e^{\Lambda(t,T)(R-1)}$. □

Second and third statement of covered interest parity

Proof. The derivation of the expression (9) starts, as before, with martingality of Z_f, but differs by a covariance term due to dependence:

$$Z_f(t,T) = E^Q(Z_0(T,T)e_1(T)|\mathcal{F}_t)$$

$$= Cov^Q(Z_0(T,T), e_1(T)|\mathcal{F}_t) + Z_0(t,T)E^Q(e_1(T)|\mathcal{F}_t),$$

so $P_f(t,T) = \frac{B_t}{S_t}Cov^Q(Z_0(T,T), e_1(T)|\mathcal{F}_t) + P_0(t,T)\exp((R-1)\int_t^T \mu_s\lambda_s)\exp(\int_0^t \log R dN_s)$. Substituting for $P_0(t,T)$ in the CIP expression with produces (9).

To obtain (10), recall that the forward price is defined to give the forward contract zero present value, i.e. $E^Q((S_T - F(0,T))\exp(-\int_0^T r_s ds)) = 0$. Recalling that

$$S_T = S_0 \frac{P_f(0,T)}{P(0,T)} \mathcal{E}\Big(\int_0^T (e^{\Theta(s,T)}(1+\eta_s) - 1) d\tilde{M}_s\Big) \mathcal{E}\Big(\int_0^T (\sigma^* - \sigma_f^* + \delta) d\tilde{W}_s\Big)$$

$$\exp\Big(\int_0^T \sigma^*(t,T)\cdot(\sigma^*(t,T) - \sigma_f^*(t,T) + \delta_t) dt\Big) \exp\Big(\int_0^T \mu_t \lambda_t e^{\Theta(t,T)}(1+\eta_t)(1-R) dt\Big), \text{ and}$$

$$\exp\Big(-\int_0^T r_s ds\Big) = P(0,T)\exp\Big(-\frac{1}{2}\int_0^T \sigma^*(t,T)^2 dt\Big)\exp\Big(-\int_0^T \sigma^*(t,T) d\tilde{W}_t\Big),$$

we simplify the volatility terms[41] to obtain

$$S_T \exp\Big(-\int_0^T r_s ds\Big) = S_0 P_f(0,T) \mathcal{E}\Big(\int_0^T (\delta - \sigma_f^*) d\tilde{W}\Big) \mathcal{E}\Big(\int_0^T (e^{\Theta}(1+\eta) - 1) d\tilde{M}\Big)$$

$$\exp\Big(\int_0^T \mu_t \lambda_t e^{\Theta(t,T)}(1+\eta_t)(1-R) dt\Big).$$

Take expected values to obtain the result:

$$F(0,T)P(0,T) = S_0 P_f(0,T) \exp\Big((1-R)\int_0^T e^{\Theta(s,T)}(1+\eta_s)\mu_s \lambda_s ds\Big).$$

□

Expected payoff to carry trade

Proof. Recall that

$$S_T = S_0 \frac{P_f(0,T)}{P(0,T)} \mathcal{E}\Big(\int_0^T (\sigma^*(t,T) - \sigma_f^*(t,T) + \delta_t) d\tilde{W}_t\Big) \mathcal{E}\Big(\int_0^T (e^{\Theta(t,T)}(1+\eta) - 1) d\tilde{M}_t\Big)$$

$$\exp\Big(\int_0^T \sigma^*(t,T)\cdot(\sigma^*(t,T) - \sigma_f^*(t,T) + \delta_t) dt\Big) \exp\Big(\int_0^T \mu_t \lambda_t e^{\Theta(t,T)}(1+\eta_t)(1-R) dt\Big).$$

[41] Collect ds terms in the exponential to obtain $\frac{1}{2}(\delta - \sigma_f)^2 - \frac{1}{2}(\sigma - \sigma_f + \delta)^2 + \sigma(\sigma - \sigma_f + \delta) - \frac{1}{2}\sigma^2$

Note that

$$\mathcal{E}\Big(\int_0^T (e^{\Theta(t,T)}(1+\eta)-1)d\tilde{M}_t\Big)\exp\Big(\int_0^T \log R dN_s\Big)$$

$$= \mathcal{E}\Big(\int_0^T (Re^{\Theta(t,T)}(1+\eta)-1)d\tilde{M}_t\Big)\exp\Big(\int_0^T (R-1)\mu\lambda e^{\Theta}(1+\eta)dt\Big),$$

and evaluate $E^Q(\frac{P(0,T)}{P_f(0,T)S_0}S_T\exp(\int_0^T \log R dN_s)-1)$ to obtain the result. □

Lemma 1: Expression for the exchange rate

$$S_T = S_0\frac{P_f(0,T)}{P(0,T)}\mathcal{E}\Big(\int_0^T (\sigma^*(t,T)-\sigma_f^*(t,T)+\delta_t)d\tilde{W}_t\Big)\mathcal{E}\Big(\int_0^T (e^{\Theta(t,T)}(1+\eta)-1)d\tilde{M}_t\Big)$$

$$\exp\Big(\int_0^T \sigma^*(t,T)\cdot(\sigma^*(t,T)-\sigma_f^*(t,T)+\delta_t)dt\Big)\exp\Big(\int_0^T \mu_t\lambda_t e^{\Theta(t,T)}(1+\eta_t)(1-R)dt\Big).$$

Proof. Use the risk neutral drift of the exchange rate (6) in the exchange rate dynamics (5) and solve to obtain

$$S_T = S_0\exp\Big(\int_0^T r_d(t)-r_1(t)+\mu_t\lambda_t(1-R)(1+\eta_t)dt\Big)$$

$$\mathcal{E}\Big(\delta_t d\tilde{W}_t\Big)\mathcal{E}\Big(\eta d\tilde{M}\Big).$$

Simplify the expression for the domestic short rate:

$$\exp\Big(\int_0^T r_d(t)dt\Big)$$

$$= \frac{1}{P(0,T)}\exp\Big(\int_0^T\int_0^t \sigma(s,t)\int_s^t \sigma(s,u)dudsdt\Big)\exp\Big(\int_0^T\int_0^t \sigma(s,t)d\tilde{W}_s dt\Big)$$

$$= \frac{1}{P(0,T)}\exp\Big(\frac{1}{2}\int_0^T \sigma^*(t,T)^2dt\Big)\exp\Big(\int_0^T \sigma^*(t,T)d\tilde{W}_t\Big),$$

where, in the last equality, the first exponential on the LHS is equal to the first exponential on the RHS due to Leibniz's rule, and the second follows from a version of Fubini's theorem for Brownian integrals.

By the same reasoning,

$$\exp\Big(-\int_0^T r_1(t)dt\Big) = \exp\Big(-\int_0^T f_1(0,t)dt\Big)$$
$$\exp\Big(-\frac{1}{2}\int_0^T \sigma_f^*(t,T)^2 dt + \int_0^T \sigma_f^*(t,T)\cdot\delta_t dt\Big)$$
$$\exp\Big(-\int_0^T\int_0^t \mu\lambda\theta(s,t)(1 - Re^{\Theta(s,t)}(1+\eta_s))dsdt\Big)$$
$$\exp\Big(-\int_0^T \sigma_f^*(t,T)d\tilde{W}_t + \int_0^T \Theta(t,T)d\tilde{M}_t\Big).$$

So we have

$$\exp\Big(\int_0^T r_d(t)dt\Big) = \frac{1}{P(0,T)}\exp\Big(\frac{1}{2}\int_0^T \sigma^*(t,T)^2 dt\Big)\exp\Big(\int_0^T \sigma^*(t,T)d\tilde{W}_t\Big),$$

$$\exp\Big(-\int_0^T r_1(t)dt\Big) = \exp\Big(-\int_0^T f_1(0,t)dt\Big)$$
$$\exp\Big(-\frac{1}{2}\int_0^T \sigma_f^*(t,T)^2 dt + \int_0^T \sigma_f^*(t,T)\cdot\delta_t dt\Big).$$
$$\exp\Big(\int_0^T \mu_t\lambda_t\Theta(t,T)dt + \int_0^T \mu_t\lambda_t(1+\eta_t)R(1-e^{\Theta(t,T)})dt\Big)$$
$$\exp\Big(-\int_0^T \sigma_f^*(t,T)d\tilde{W}_t + \int_0^T \Theta(t,T)d\tilde{M}_t\Big)$$

Collecting coefficients in S_T, we obtain

$$dt:\ f(0,t) + \frac{1}{2}\sigma^*(t,T)^2 - f_1(0,t) - \frac{1}{2}\sigma_f^*(t,T)^2 + \sigma_f^*(t,T)\cdot\delta_t + \mu\lambda\Theta(t,T)$$
$$+\mu\lambda(1+\eta)R(1-e^{\Theta}) - \mu\lambda\Theta + \mu\lambda(1-R)(1+\eta) - \frac{1}{2}\delta_t^2 - \eta\mu\lambda,$$

$$d\tilde{W}_t : \delta_t + \sigma^*(t,T) - \sigma_f^*(t,T),$$

$$dN_t : \log(1+\eta) + \Theta(t,T).$$

The drift coefficient simplifies to

$$dt: \; f(0,t)+\frac{1}{2}\sigma^*(t,T)^2-f_1(0,t)-\frac{1}{2}\sigma_f^*(t,T)^2+\sigma_f^*(t,T)\cdot\delta_t+\mu\lambda(1+\eta)(1-Re^{\Theta})-\frac{1}{2}\delta_t^2-\eta\mu\lambda$$

$$= f(0,t)+\frac{1}{2}\sigma^*(t,T)^2-f_1(0,t)-\frac{1}{2}\sigma_f^*(t,T)^2+\sigma_f^*(t,T)\cdot\delta_t+\mu\lambda(1-Re^{\Theta}(1+\eta))-\frac{1}{2}\delta_t^2$$

Noting that $\exp(\int_0^T f(0,t)-f_1(0,t)dt)=\frac{P_1(0,T)}{P(0,T)}=\frac{P_f(0,T)}{P(0,T)}$, S_T becomes

$$S_T = S_0\frac{P_f(0,T)}{P(0,T)}\exp\bigg(\int_0^T \frac{1}{2}\sigma^*(t,T)^2-\frac{1}{2}\sigma_f^*(t,T)^2+\sigma_f^*(t,T)\cdot\delta_t+\mu\lambda(1-Re^{\Theta}(1+\eta))-\frac{1}{2}\delta_t^2\bigg)$$

$$\exp\bigg(\int_0^T \delta_t+\sigma^*(t,T)-\sigma_f^*(t,T)d\tilde{W}_t\bigg)\exp\bigg(\int_0^T \log(1+\eta)+\Theta(t,T)dN_t\bigg)$$

$$= S_0\frac{P_f(0,T)}{P(0,T)}\exp\bigg(\int_0^T \frac{1}{2}\sigma^*(t,T)^2-\frac{1}{2}\sigma_f^*(t,T)^2+\sigma_f^*(t,T)\cdot\delta_t+\mu\lambda(1-Re^{\Theta}(1+\eta))-\frac{1}{2}\delta_t^2\bigg)$$

$$\mathcal{E}\bigg(\int_0^T \sigma^*(t,T)-\sigma_f^*(t,T)+\delta_t d\tilde{W}_t\bigg)\exp\bigg(\frac{1}{2}\int_0^T(\sigma^*(t,T)-\sigma_f^*(t,T)+\delta_t)^2dt\bigg)$$

$$\mathcal{E}\bigg(\int_0^T (e^{\Theta(t,T)}(1+\eta)-1)d\tilde{M}_t\bigg)\exp\bigg(\int_0^T (e^{\Theta(t,T)}(1+\eta)-1)\mu\lambda dt\bigg).$$

Note that $\frac{1}{2}(\sigma^*(t,T)-\sigma_f^*(t,T)-\delta_t)^2+\frac{1}{2}\sigma^*(t,T)^2-\frac{1}{2}\sigma_f^*(t,T)^2+\sigma_f^*(t,T)\cdot\delta_t-\frac{1}{2}\delta_t^2 = \sigma^*(t,T)\cdot(\sigma^*(t,T)-\sigma_f^*(t,T)+\delta_t)$, and $\mu\lambda(1-Re^{\Theta}(1+\eta))+(e^{\Theta}(1+\eta)-1)\mu\lambda = \mu\lambda e^{\Theta}(1+\eta)(1-R)$, so

$$S_T = S_0\frac{P_f(0,T)}{P(0,T)}\mathcal{E}\bigg(\int_0^T (\sigma^*(t,T)-\sigma_f^*(t,T)+\delta_t)d\tilde{W}_t\bigg)\mathcal{E}\bigg(\int_0^T (e^{\Theta(t,T)}(1+\eta)-1)d\tilde{M}_t\bigg)$$

$$\exp\bigg(\int_0^T a_d(t,T)\cdot(a_d(t,T)-a_f(t,T)-\delta_t)dt\bigg)\exp\bigg(\int_0^T \mu_t\lambda_t e^{\Theta(t,T)}(1+\eta_t)(1-R)dt\bigg).$$

□

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Equation (4),

$$P_0(t,T) = P_0(0,T)\exp\bigg(\int_0^T (r_0(s) + b_0(s,T))ds + \int_0^t a_f(s,T)\cdot dW_s + \int_0^T \Theta(s,T)dN_s\bigg).$$