

# Adverbs of Quantification

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## 1 Introduction

Adverbs of quantification in David Lewis's sense are a group of words that semantically have a quantifying role, and morpho-syntactically are adverbs that can occur in the pre-verbal auxiliary position of English sentences (Lewis, 1975). (1) is a list of common adverbs of quantification, and (2) are some sentences where they are used. (3) lists some complex items. Lewis's paper called attention to the interest and complexity of establishing a compositional structure and semantics for adverbs of quantification, and made a specific proposal about it.

- (1) always      invariably      universally  
usually      mostly      generally  
often      frequently      commonly  
rarely      infrequently      seldom  
sometimes      occasionally  
never

- (2) a. The fog usually lifts before noon here.  
b. If a dart hits a unit square, it almost never hits it on the diagonal.  
c. People who commit mass murders generally suffer from an extraordinary lack of impulse control.

- (3) almost always      nearly always  
almost never      nearly never

Calling the adverbs quantificational is justified by the fact that sentences with adverbs of quantification (henceforth AQS) sometimes have paraphrases or near-

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paraphrases involving quantificational determiners.<sup>1</sup> For instance sentence (4a) is paraphrased by (4b) with a quantificational determiner *most* and nominal generalized quantifier *most times*. In sentence (5b), the paraphrase has a nominal quantifier *every quadratic equation* in subject position.<sup>2</sup>

- (4) a. This clock is usually off by at least half an hour.
- b. At most times, the time showing on this clock differs from the actual time by at least half an hour.
  
- (5) a. Quadratic equations always have two real solutions.
- b. Every quadratic equation has two real solutions.

In (4a), *usually* presumably quantifies times, and AQs are sometimes morphologically related to the root *time* (for instance *some-times*, or Swedish *al-tid*, morphologically ‘all time’). However in (5) time is irrelevant, and on the face of it equations are being quantified, just as in (5b) with a nominal quantifier.<sup>3</sup> Similarly, (6a) with the AQ *sometimes* is talking about adverbs of quantification in English and other languages, not about how the nature of adverbs of quantification changes over time.

- (6) a. Adverbs of quantification are sometimes morphologically related to the root *time*.
- b. Some adverbs of quantification are morphologically related to the root *time*.

An important group of examples are ones where the AQ occurs together with an adverbial *if*-clause, see (7).<sup>4</sup> A feature of these examples is that what is quantified is introduced by some phrases or combination of phrases in the *if*-clause. In (7c) one can reasonably say that headlines are quantified, because of the near paraphrase (8). The phrase *a headline* is part of the *if*-clause in (7c).

- (7) a. If you drop an unbreakable object, it always lands on something more valuable.
- b. If two cars are driving in opposite directions on a long road with a one-way bridge, they always meet at the bridge.
- c. If a headline ends in a question mark, the answer is usually “no”.
  
- (8) Most headlines that end with a question mark are answered negatively.

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<sup>1</sup>As pointed out by Zoltan Szabo, similar paraphrases are found in Russell (1905), where they go in the other direction. Russell says that  $C(\text{everything})$  means “ $C(x)$  is always true”.

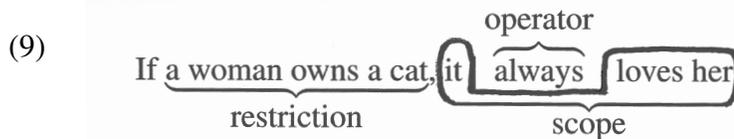
<sup>2</sup>This leaves open why the determiners in the paraphrases are to be considered quantificational, and Lewis does not try to give an account of this. As worked out in Sections 3 and 4, the terminology is justified in Lewis’s theory, where the semantics of AQs is isomorphic to the semantics of determiners, as analyzed in generalized quantifier theory.

<sup>3</sup>This point is made by Lewis. I’m not going to say this every time I could.

<sup>4</sup>(7a) and (7b) are cases of Murphy’s law.

## 2 Logical forms for AQs

A *logical form* or *compositional structure* in natural language semantics is a structure that encodes in its syntax how the semantics of parts of a sentence are composed to give a semantics for the whole, so that each structure has a unique semantic interpretation. Usually, a logical form (or LF) is a complex of functions or operators, and the arguments of those functions or operators. Accordingly, an important issue in establishing the logical form for a sentence is identifying the main function or operator, and establishing the number and identity of the arguments of that operator. For sentences with an AQ and an *if*-clause, Lewis made the hypothesis that the adverb of quantification is the main operator, and that it has *two* arguments. The *if*-clause supplies one argument (called the *restriction* of the AQ), and the main clause of the sentence (minus the AQ itself) supplies the other argument, termed the *scope*. This is shown graphically in (9).



Beyond articulating sentences with an AQ into an operator, a restriction, and scope, Lewis hypothesized that any number of variables can be quantified by the AQ. This is the analysis of AQs as *unselective* quantifiers. The variables that are to be quantified are those corresponding to indefinite descriptions. For expository purposes, here we begin by writing the quantified indices as numerical subscripts on the AQ.<sup>5</sup> In a notation with the operator on the left, the logical form of a sentence with an AQ is as exemplified in (10), where indices appear on the operator [always], and also on the phrases [a woman]<sub>1</sub>, [a cat]<sub>2</sub>, [it]<sub>2</sub>, and [her]<sub>1</sub> in the restriction and scope. Two arguments follow the AQ.<sup>6</sup>

(10) [[always]<sub>1,2</sub> [a woman<sub>1</sub> owns a cat<sub>2</sub>] [it<sub>2</sub> loves her<sub>1</sub>]]

Lewis proposed that the semantics of the structure is quantification over *cases*, which are tuples of individuals. With two quantifying indices, the quantified cases have two slots, and the semantics of (10) is as stated in (11).

(11) Semantics of (10)  
 For every case  $\langle x_1, x_2 \rangle$  such that  $x_1$  is a woman,  $x_2$  is a cat, and  $x_1$  owns  $x_2$ :  $x_2$  loves  $x_1$ .

<sup>5</sup>Then in Section 4, the subscripts are eliminated, by encoding the variables that are quantified in the semantics.

<sup>6</sup>Lewis encodes the same information by introducing variables such as  $x$  and  $y$ , with the understanding that all such variables in the restriction are quantified. This is (10) in the notation used by Lewis: always if  $x$  is a woman and  $y$  is a cat and  $x$  owns  $y$ ,  $y$  loves  $x$ .

How can the structure (10) be derived systematically, and what is the source of the clauses such as “ $x_1$  is a woman” seen in (11)? The second chapter of Irene Heim’s dissertation addressed this with a sequence of syntactic operations (Heim, 1982). We start with a syntactic structure (12a), that already has indices on indefinite descriptions and pronouns. First, indefinite descriptions are adjoined to a dominating clause, introducing an index on the moved constituent, and leaving behind an indexed empty category that functions as a variable.<sup>7</sup> See (12b), where the two indefinite descriptions have been moved to the level of the restrictive clause, introducing the empty categories  $e_1$  and  $e_2$ . Second, the indices of indefinite descriptions are copied to the minimal c-commanding quantificational adverb, which in this case is *always*.<sup>8</sup> Finally, an indexed indefinite description  $[a \alpha]_i$  is re-written as  $[e_i \alpha]$ . See (12c), where the indices 1 and 2 have been copied as subscripts onto  $[\text{always}]_{1,2}$ , and where  $[a \text{ cat}]_2$  has been rewritten as  $[e_2 \text{ cat}]$ .<sup>9</sup> In our example, these steps transform the restriction into a sequence of atomic formulas, consisting of relation symbols with indexed arguments (traces or pronouns) that have the status of bound variables. Indices coming from indefinite descriptions appear as subscripts on the AQ.

- (12)a.  $[[\text{always}] [a \text{ woman}_1 \text{ owns a cat}_2] [\text{it}_2 \text{ loves her}_1]]$   
 b.  $[[\text{always}][[a \text{ woman}]_1 [[a \text{ cat}]_2 [e_1 \text{ owns } e_2]]] [\text{it}_2 \text{ loves her}_1]]$   
 c.  $[[\text{always}]_{1,2}[[e_1 \text{ woman}] [[e_2 \text{ cat}] [e_1 \text{ owns } e_2]]] [\text{it}_2 \text{ loves her}_1]]$

Heim pointed out that additional treatment is required for sentences such as (13a) where there is an indefinite description in the scope. Somehow, *a shed* should be interpreted existentially in the scope, rather than being quantified by *usually*. For this, Heim proposed that there is an existential operator adjoined to the scope, and that the indefinite  $[a \text{ shed}]_3$  is indexed with it by the procedure that copies the index of an indefinite description to the minimal c-commanding adverbial operator. The result in this case is (13b), where the index originally on  $[a \text{ shed}]_3$  ends up as a subscript on the existential adverb of quantification  $\exists_3$ , and as a variable in the atomic formula  $[e_3 \text{ shed}]$ .

- (13)a. If a woman<sub>1</sub> owns a cat<sub>2</sub>, she<sub>1</sub> usually keeps it<sub>2</sub> in a shed<sub>3</sub>.  
 b.  $[\text{usually}_{1,2} [[a \text{ woman}]_1 [[a \text{ cat}]_2 [e_1 \text{ owns } e_2]]] [\exists_3 [[e_3 \text{ shed}] [she_1 \text{ keeps } it_2 \text{ in } e_3]]]]$

Revising the syntactic procedure stated earlier, first an existential operator  $\exists$  is adjoined to the scope of each adverb of quantification, and then we proceed as

<sup>7</sup>Adjunction is the tree transformation that maps  $[_A \dots X \dots ]$  to  $[_A X_i [_A \dots e_i \dots ]]$ .

<sup>8</sup>A node A c-commands a node B if and only if the node dominating A dominates B, and A does not dominate B.

<sup>9</sup>This last step, which is not found in Heim, is included here in order to produce something that has the syntactic shape of an atomic formula.

before, with  $\exists$  together with AQs treated as a target of indexing.<sup>10</sup>

Copying indices from indefinite descriptions and nothing else has another good consequence. When as in (14) there is an indexed pronoun in the *if*-clause, the index does not get copied, because it is an index on a pronoun rather than an indefinite description. This explains why the pronoun  $[it_3]$  is interpreted in the context (perhaps it refers to a certain famous beautiful and friendly cat), rather than being quantified by the AQ.

(14) If a woman<sub>1</sub> owns [a cat]<sub>2</sub> that resembles it<sub>3</sub>, it<sub>2</sub> always loves her<sub>1</sub>.

### 3 Semantics of the LF

The case-quantifying semantics for *always* needs to be generalized to other adverbs of quantification. A neat way of achieving this is to assume that AQs *always*, *usually*, *never*, (and so forth) have the same root meanings as nominal determiners *every*, *most*, *no*, (and so forth), except that the AQs quantify cases rather than individuals. (15) gives the semantics for some quantificational determiners, as they are defined in generalized quantifier theory (Barwise and Cooper, 1981).  $A$  and  $B$  are sets of individuals.

(15) Let  $A$  and  $B$  be sets of individuals. Two-place relations corresponding to determiners have the following truth conditions.

<b>every</b> ( $A, B$ )	true iff $A \subseteq B$
<b>most</b> ( $A, B$ )	true iff the cardinality of $A \cap B$ is greater than the cardinality of $A - B$
<b>no</b> ( $A, B$ )	true iff $A \cap B = \emptyset$

(16) uses the same definitions for the root meanings of some adverbs of quantification. The switch from (15) is that  $A$  and  $B$  are sets of cases, rather than sets of individuals.

(16) Let  $A$  and  $B$  be sets of cases. Two-place relations corresponding to adverbs of quantification have the following truth conditions.

<b>always</b> ( $A, B$ )	true iff $A \subseteq B$
<b>usually</b> ( $A, B$ )	true iff the cardinality of $A \cap B$ is greater than the cardinality of $A - B$
<b>never</b> ( $A, B$ )	true iff $A \cap B = \emptyset$

The relation between AQs and quantificational determiners is supported by the fact that the phrases of the form *in D case* or *in D cases*, where  $D$  is a determiner, can function as AQs. See the examples in (17).<sup>11</sup>

<sup>10</sup> $\exists$  is not an adverb of quantification, because it has one argument rather than two.

<sup>11</sup>While Lewis theorized about quantifying cases, the noun *case* hardly figured in his database

- (17)a. If a woman owns a cat,  $\left\{ \begin{array}{l} \text{in every case} \\ \text{in most cases} \\ \text{in some cases} \end{array} \right\}$  it loves her.
- b. If a woman owns a cat, it  $\left\{ \begin{array}{l} \text{in all cases} \\ \text{in most cases} \\ \text{in some cases} \end{array} \right\}$  loves her.

To link up the root meanings with the LF, we use the indices on the AQ to extract a set of cases from the restriction, and another set of cases from the scope. The root meaning of the AQ takes those two sets of cases as arguments. In (18), the lambda binder  $\lambda n_1, \dots, n_k$  is understood to be of an abstractor over cases, so that  $\lambda n_1, \dots, n_k \phi$  and  $\lambda n_1, \dots, n_k \psi$  are sets of cases of length  $k$ . Here  $n_1, \dots, n_k$  are the numerical indices seen in the LF.

- (18) Case quantification interpretation for the logical forms of AQs  
 A clause of the form  $[\text{AQ}_{n_1, \dots, n_k} \phi \psi]$  is semantically equivalent to  $\text{AQ}(\lambda n_1, \dots, n_k \phi, \lambda n_1, \dots, n_k \psi)$ .

Let us work through an example that exercises everything in the analysis. Sentence (19a) has indefinite descriptions in both the restriction and scope, and also has pronouns in both the restriction and scope. The sentence is transformed into an LF that is interpretable by the semantics by a sequence of syntactic transformations, as follows. In (19b), an existential operator is added to the scope of *always*. In (19c), indefinite descriptions are adjoined to dominating clauses, either the *if*-clause or the main clause, leaving behind indexed traces. In (19d), the indices of indefinite descriptions are copied to the minimal c-commanding adverbial operator, which is *always* in the case of  $[\text{a woman}]_1$  and  $[\text{a cat}]_2$  in the restriction, and  $\exists$  in the case of  $[\text{a shed}]_4$  in the scope.

- (19)a. If a woman<sub>1</sub> owns a cat<sub>2</sub> that resembles it<sub>3</sub>, she<sub>1</sub> always keeps it<sub>2</sub> in [a shed]<sub>4</sub>.
- b.  $\exists$  *adjunction*  
 [always  
 [a woman<sub>1</sub> owns [a cat]<sub>2</sub> that resembles it<sub>3</sub>]  
 [ $\exists$ [she<sub>1</sub> always keeps it<sub>2</sub> in [a shed]<sub>4</sub>]]]
- c. *Movement of indefinite descriptions*  
 [always  
 [[a woman]<sub>1</sub>[[a cat]<sub>2</sub> that [e<sub>2</sub> resembles it<sub>3</sub>]][e<sub>1</sub> owns e<sub>2</sub>]]  
 [ $\exists$ [[a shed]<sub>4</sub>[e<sub>1</sub> keep e<sub>2</sub> in e<sub>4</sub>]]]]]
- d. *Indexing of indefinite descriptions to AQ and  $\exists$*   
 [always<sub>1,2</sub>  
 [[a woman]<sub>1</sub>[[a cat]<sub>2</sub> that [e<sub>2</sub> resembles it<sub>3</sub>]][e<sub>1</sub> owns e<sub>2</sub>]]  
 [ $\exists$ <sub>4</sub>[[a shed]<sub>4</sub>[e<sub>1</sub> keep e<sub>2</sub> in e<sub>4</sub>]]]]]

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of examples. Moltmann (2017) is a study of uses and interpretations of this noun.

The semantics stated in (18) maps the restriction to a set of cases, by binding the indices on the AQ. In this case, there is a free pronoun  $[it_3]$ , which intuitively has to take its value from the context. Writing the value for this pronoun as  $g(3)$ , where  $g$  is an assignment function used to give values to free variables, we obtain the set of cases defined in (20).<sup>12</sup>

- (20) *Set of cases derived from the restriction (= A)*  
 $[[\lambda 1 2][[a\ woman]_1[[a\ cat]_2\ that\ [e_2\ resembles\ it_3]][e_1\ owns\ e_2]]]$

The set of all pairs  $\langle x_1, x_2 \rangle$  such that  
 $x_1$  is a woman and  
 $x_2$  is a cat and  
 $x_2$  resembles  $g(3)$  and  
 $x_1$  owns  $x_2$ .

Similarly, a set of cases is derived from the scope, by binding the same indices 1 and 2. In this case, there is an existential quantifier on the scope, which quantifies the index 4. The set of cases described in (21) is the result.

- (21) *Set of cases derived from the scope (= B)*  
 $[[\lambda 1 2][\exists_4[[a\ shed]_4[e_1\ keep\ e_2\ in\ e_4]]]]]$

The set of all pairs  $\langle x_1, x_2 \rangle$  such that there is an  $x_4$  such that  
 $x_4$  is a shed and  
 $x_1$  keeps  $x_2$  in  $x_4$ .

According to (18), these two sets of cases become arguments of the root meaning for the AQ, which in this case is the subset relation on sets of cases that is defined in (16). The resulting truth condition is worked out in (22).

- (22) Let **always** be the relation between sets of cases defined in (16). Let  $A$  and  $B$  be the sets of cases defined in (20) and (21). The structure

$[always_{1,2}$   
 $[[a\ woman]_1[[a\ cat]_2\ that\ [e_2\ resembles\ it_3]][e_1\ owns\ e_2]]$   
 $[\exists_4[[a\ shed]_4[e_1\ keep\ e_2\ in\ e_4]]]]]$

is true with respect to an assignment  $g$  if and only if **always**( $A, B$ ). That is, it is true iff the set of cases

$$\left\{ \left\langle x_1, x_2 \right\rangle \left| \begin{array}{l} x_1 \text{ is a woman and} \\ x_2 \text{ is a cat and} \\ x_2 \text{ resembles } g(3) \text{ and} \\ x_1 \text{ owns } x_2 \end{array} \right. \right\}$$

is a subset of the set of cases

<sup>12</sup>While write “ $\lambda 1 2$ ” to indicate that the subscript indices on the AQ are used to abstract cases, this notation not part of the LF.

$$\left\{ \langle x_1, x_2 \rangle \left| \exists x_4 \left[ \begin{array}{l} x_4 \text{ is a shed and} \\ x_1 \text{ keeps } x_2 \text{ in } x_4 \end{array} \right] \right. \right\}.$$

The truth condition in this case is equivalent to the intuitively correct condition that is stated in (23) using universal and existential quantifiers over individuals.

- (23) The structure is true with respect to assignment  $g$  iff for every  $x_1$  and every  $x_2$  such that  $x_1$  is a woman and  $x_2$  is a cat and  $x_2$  resembles  $g(3)$  and  $x_1$  owns  $x_2$ , there is an  $x_4$  such that  $x_4$  is a shed and  $x_1$  keeps  $x_2$  in  $x_4$ .

## 4 Dynamic case quantification

In the analysis from Sections 2 and 3, variables in the *if*-clause get quantified by an adverb of quantification, as expressed by indexing in the notation with subscripts on the AQ. The movement and indexing steps ensure that the indices that are quantified are exactly the maximal-scope indefinite descriptions in the restriction of the adverb. This account of the meaning of AQs and the interaction between AQs and indefinite descriptions is mediated by the syntax of LFs, and by the syntactic transformations that construct the LFs. While the theory is workable, the syntactic transformations are not motivated by any observable aspects of English syntax. A response is to try to treat the interaction between AQs and indefinite descriptions semantically, instead of syntactically. In particular, we would like for it to be a *semantic* property of the restriction that certain variables get quantified by an AQ.<sup>13</sup>

Some terminology from Karttunen (1969) hints at a way of conceptualizing the problem. The sentence (24) introduces two *discourse referents* (one for a woman, and one for a cat). The semantics for this clause should encode possible values for these discourse referents that witness the truth of the sentence, in the sense that they are values for the discourse referents that make the sentence true. We will look at an account that develops this idea, while staying as close as possible to Lewis’s formalization in terms of cases.

- (24) A woman owns a cat.

A significant fact is that indefinite descriptions allow for *discourse anaphora*, where one sentence contains an indefinite description, and a following sentence contains a pronoun that refers back to the indefinite description. This is illustrated in (25) for various types of indefinite descriptions. Examples (26a,b) indicate that

<sup>13</sup>This follows the plot of Heim (1982), where first an analysis using indexing and existential closure is stated, and then the analysis is re-formulated in dynamic terms. Rather than following Heim’s file change semantics, the second part as stated here is close to Paul Dekker’s formulation of dynamic semantics, see Dekker (2012).

nominals with cardinal determiners, and nominals with the plural determiner *some* can also be used in the AQ construction.

- (25)a. A woman owns a crocodile. She keeps it in her swimming pool.  
 b. Two people in the department specialize in historical linguistics. They have different interests, though.  
 c. Some people in the department are interested in deep learning. They have organized a reading group.
- (26)a. If two people have the same last name, they are usually related.  
 b. If some people have the same last name, they always know each other.  
 c. If some people are jointly interested in an arcane subject, they usually organize a reading group on it.

A clue about a way to proceed technically comes from the interpretation rule (18) in the last section, where a set of cases  $\lambda n_1, \dots, n_k \phi$  is obtained from the restriction, and used as the first argument of the AQ. If the semantics could be modified so that this set of cases is directly the semantic value of the restriction, this could simply be fed in as the first argument of the AQ. Notice by the way that an element of  $\lambda n_1, \dots, n_k \phi$  can be viewed as a case that records choices for the discourse referents corresponding to the top-level indefinite descriptions in  $\phi$ . Anyway, to summarize the basic idea, the semantic value of sentence (27) should be the set of cases (27b), or equivalently (27c).

- (27) Illustration of case semantics for clauses (Version 1)  
 a. A woman owns a cat.  
 b. The set of cases  $\langle x, y \rangle$  such that  $x$  is a woman,  $y$  is a cat, and  $x$  owns  $y$ .  
 c.  $\lambda xy. x$  is a woman and  $y$  is a cat and  $x$  owns  $y$

This design does not take into account the fact that a clause can contain pronouns that refer back to something earlier in the sentence or discourse, as we saw in Section 2. What set of cases is contributed by clause (28a), which has a pronoun  $[it_i]$  that intuitively has to pick up its referent from the context? An answer is to say that it denotes a set of cases *relative to a case*. In (28b),  $c$  is an arbitrary case, and the semantics takes the value of the indexed pronoun  $[it_i]$  to be  $c[1]$ , which is the first element of  $c$ .<sup>14</sup> An additional idea is that *cases are cumulative*. In (28), relative to an arbitrary case  $c$ , a case in the semantics of (28a) as described in (28b) is formed by incrementing  $c$  with a witness  $y$  for the cat, and a witness  $x$  for the woman.

- (28) Illustration of case semantics for clauses (Version 2)  
 a. A woman owns a cat that resembles  $it_i$ . (Sentence A)

<sup>14</sup>The choice of  $i$  in  $[it_i]$  depends details of the sub-clausal semantics. (34) switches to  $[it_3]$ , on the assumption that 1 and 2 are discourse referents for the woman and the cat.

b. Relative to a case  $c$ , sentence A denotes the set of cases of the form  $xyz$ , where  $x$  is a woman,  $y$  is a cat,  $x$  owns  $y$ , and  $y$  resembles  $c[1]$ .

To explain the increment notation, where  $c$  is a tuple  $\langle z_1, \dots, z_k \rangle$ ,  $xc$  designates the tuple  $\langle x, z_1, \dots, z_k \rangle$ . Where  $d = xc$ , the new value  $x$  is added at the beginning in the tuple, and can be accessed as  $d[1]$ . Pronouns index into cases. Relative to case  $c$ , the value of the pronoun  $[it_i]$  is  $c[i]$ .

(29) introduces some terminology that will be used in defining the semantics of AQs. Where A is a clause and  $c$  is a case,  $cA$  is the set of cases corresponding to clause A, relative to case  $c$ . The semantics will be phrased as a recursive definition of  $cA$ , for arbitrary  $c$  and various choices of clause A. The *domain* of A, written  $Do(A)$ , is the set of all cases  $c$  such that  $cA$  is non-empty.<sup>15</sup> This projects away witnesses, and is conceived of as the set of cases relative to which A is true.

(29) Let  $c$  and  $d$  be cases, and  $A$  be a clause.

$cA$         Semantics of A as a set of cases, relative to case  $c$ .  
 $Do(A)$     The set of all  $c$  such that  $cA$  is non-empty.

For a clause headed by *always*, we assume the logical form [always A B], where the restriction A and the scope B are clauses that may contain indefinite descriptions and pronouns. Unlike in Sections 2 and 3, there is no indexing on the AQ, and no existential AQ adjoined to the scope. Given the format (29), we need to define the set of cases  $c[\text{always A B}]$  for arbitrary  $c$ , A, and B. In the term  $c[\text{always A B}]$ , the part following  $c$  is a clause, just like A in the term  $cA$ . Before looking at a definition, let us check empirically whether a clause of the form  $c[\text{always A B}]$  sets up any discourse referents. (30) checks whether an indefinite descriptions in A or B sets up a discourse referents that can be referred to with a pronoun in later discourse. In (30a), [a woman] in the restriction can not be picked up with the pronoun *she* in the second sentence. In (30b), [a shed] in the scope can not be picked up with the pronoun *it* in the second sentence. This is partial evidence that clauses headed by AQs do not set up discourse referents.

(30)a. If a woman owns a cat, she always keeps it in a shed. #She is cruel.  
 b. If a woman owns a cat, she always keeps it in a shed. #It is resentful.

Clauses that do not set up discourse referents are known as *tests*. A test A has the property that  $cA$  is either the unit set of  $c$ , or the empty set. The value  $\{c\}$  corresponds to A being true relative to  $c$ , and the value  $\emptyset$  corresponds to A being false relative to  $c$ . This is illustrated in (31) for a transitive clause with two pronouns.

(31)  $c[\text{he}_1 \text{ admires her}_2]$  =  $\{c\}$  if  $c[1]$  admires  $c[2]$   
   =  $\emptyset$  otherwise.

<sup>15</sup>To explain the terminology,  $Do(A)$  is the domain of the relation that holds between cases  $d$  and  $e$  iff  $e$  is an element of  $dA$ .

(32) is a definition of the semantics of *always*. It has the same superstructure as (31), with a description of conditions for the value being the unit set  $\{c\}$ , corresponding to truth relative to  $c$ . The condition requires that B be true relative to each case in  $cA$ . Since cases in  $cA$  record witnesses for indefinite descriptions in A, this allows indefinite descriptions in A to set up antecedents for pronouns in B. This is a core result of dynamic semantics for adverbs of quantification.

(32) Semantics for *always*

$$\begin{aligned} c[\text{always } A \text{ B}] &= \{c\} && \text{if } cA \subseteq Do(B) \\ &= \emptyset && \text{otherwise.} \end{aligned}$$

To illustrate, we assume the semantic values described in (34) and (35) for the restriction A and the scope B of the sentence that is repeated in (33). The semantics (32) refers to  $cA$  and  $Do(B)$ .  $cA$  is already described in (34), and it can be rewritten as in (36). Given the definition in (29),  $Do(B)$  is (37). The semantics (32) performs a subset check between the two sets of cases, and we obtain the semantics (38). The value  $\{c\}$  corresponds to truth, and truth conditionally, (38) is equivalent to the truth condition for the same sentence at the end of Section 3. An interesting change in the mechanics is that while before cases of length two were quantified, now longer cases beginning with all of  $c$  are quantified. This does not matter, because since  $c$  is constant, the short cases are in one-to-one correspondence with the long cases.

(33) If a woman owns a cat that resembles it, she always keeps it in a shed.

(34) Where A is [a woman owns a cat that resembles it<sub>3</sub>] and  $c$  is a case,  $cA$  is the set of all cases  $x_1x_2c$  such that

- $x_1$  is a woman and
- $x_2$  is a cat and
- $x_2$  resembles  $c[1]$  and
- $x_1$  owns  $x_2$ .

(35) Where B is [she<sub>1</sub> keeps it<sub>2</sub> in a shed] and  $d$  is a case,  $dB$  is the set of all cases  $yd$  such that

- $d[1]$  keeps  $d[2]$  in  $y$  and
- $y$  is a shed.

$$(36) \left\{ d \left| \left[ \begin{array}{l} d[1] \text{ is a woman and} \\ d[2] \text{ is a cat and} \\ d[1] \text{ resembles } d[3] \text{ and} \\ d[1] \text{ owns } d[2] \end{array} \right] \right. \right\}$$

$$(37) \left\{ d \mid \exists y \left[ \begin{array}{l} d[1] \text{ keeps } d[2] \text{ in } y \text{ and} \\ y \text{ is a shed} \end{array} \right] \right\}$$

(38)  $c[\text{always [a woman owns a cat that resembles it]} [\text{she keeps it in a shed}]]$

$$\begin{aligned}
&= \{c\} \text{ if} \\
&\left\{ d \left[ \begin{array}{l} d[1] \text{ is a woman and} \\ d[2] \text{ is a cat and} \\ d[1] \text{ resembles } d[3] \text{ and} \\ d[1] \text{ owns } d[2] \end{array} \right] \right\} \subseteq \left\{ d | \exists y \left[ \begin{array}{l} d[1] \text{ keeps } d[2] \text{ in } y \text{ and} \\ y \text{ is a shed} \end{array} \right] \right\} \\
&= \emptyset \text{ otherwise.}
\end{aligned}$$

In (39), the analysis is generalized to arbitrary adverbs of quantification that have root meanings that are relations between sets of cases.

(39) Dynamic case-quantifying semantics for adverbs of quantification

$$\begin{aligned}
c[Q A B] &= \{c\} && \text{if } Q(cA, Do(B)) \\
&= \emptyset && \text{otherwise.}
\end{aligned}$$

There is work to do in completing the sub-sentential parts of the semantic analysis, which is only illustrated by example here. (40) gives the primitive semantics of an indefinite description, and of some atomic formulas with indexed arguments. (41) defines a dynamic conjunction. With these one can work out the semantics of the discourse anaphora in (42), assuming that dynamic conjunction is used for combining [a woman] with [ $e_1$  rests], and combining [[a woman][ $e_1$  rests]] with [ $she_1$  is tired].

$$\begin{aligned}
(40) \quad c[\text{a woman}] & \quad \{xc | x \text{ is a woman}\} \\
c[e_1 \text{ rests}] & \quad \{d | d = c \wedge c[1] \text{ rests}\} \\
c[she_1 \text{ is tired}] & \quad \{d | d = c \wedge c[1] \text{ is tired}\}
\end{aligned}$$

(41) Dynamic conjunction

$$b[A B] \quad \{d | \exists c [c \epsilon [bA] \wedge d \epsilon [cB]]\}$$

$$\begin{aligned}
(42) \quad c[[\text{a woman}][e_1 \text{ rests}]] [she_1 \text{ is tired}] \\
&= \{yc | y \text{ is a woman and } y \text{ rests and } y \text{ is tired}\}
\end{aligned}$$

Summing up the results of this section, by switching to a dynamic semantics where clauses directly contribute sets of cases (relative to a case), it is possible to eliminate indexing from the formulation in Sections 2 and 3, and to build Lewis's hypothesis of unselective case quantification quite directly into the compositional semantics of adverbs of quantification.

## 5 Syntactic splitting

In the simple picture from Section 2, the main clause supplies the scope of an AQ, and the *if*-clause supplies the restriction. But components of the main clause can also contribute to the restriction. This is illustrated by the sentences in (43), where the paraphrases in (44) indicate that the subject is contributing to the restriction.

- (43)a. A man who owns a dog usually resembles it.
- b. A dog is usually happy if it has a happy owner.

- (44)a. Most men who own a dog resemble it.
- b. Most dogs that have a happy owner are happy.

Focusing on bare plural subjects, Diesing (1992) pointed out that with certain predicates, the subject can ambiguously contribute to the restriction or to the scope. This is shown by the competing paraphrases in (45b,c) for sentence (45a). In the reading (45b), the subject *ghosts* has contributed to the restriction, and is quantified by *usually*, or by *most* in the paraphrase. In the reading (45c), the subject has contributed to the scope, taking on an existential reading, as indicated by the determiner *some* in the paraphrase, or by the existential *there*-construction in (45d).

- (45)a. Ghosts are usually visible.
- b. Most ghosts are visible.
- c. On most occasions, some ghosts are visible.
- d. On most occasions, there are ghosts visible.

With other predicates, a bare plural subject contributes unambiguously to the restriction. Sentence (46a) has the reading (46b), and can not have the reading (46c).

- (46)a. Ghosts are usually honest.
- b. Most ghosts are honest.
- c. #On most occasions, there are some honest ghosts.

Diesing proposed that these data follow from a syntactic splitting procedure that maps the verb phrase to the scope of an AQ, and the rest of the sentence to the restriction. Example (46a) without the adverb has the structure (47). The subject is outside the VP, and so is mapped to the restriction. Example (45) has a more complicated syntactic structure, where the subject has been raised out of a base position inside the VP, as in (48.) Semantically, the subject *ghosts* can be either in its raised position, resulting in splitting into the restriction of the AQ, or in its base position, resulting in splitting into the restriction. This accounts for the two readings of (45a).

(47) [<sub>S</sub> ghosts [<sub>VP</sub> are honest]]

(48) [<sub>S</sub> ghosts<sub>i</sub> [<sub>VP</sub> e<sub>i</sub> are visible]]

The predicates where the subject is raised out of VP on Diesing's analysis are the *stage level predicates* in the sense of Carlson (1977). Predicates such as 'being honest' are his individual-level predicates. Carlson had proposed an ontological distinction between stages or time-slices of individuals, and individuals in the or-

dinary sense. Some predicates are primitively predicates of stages, and others are primitively predicates of individuals. In Diesing’s development, the distinction is argument-structural, with only stage-level predicates having VP-internal subjects.

Support for the VP-splitting analysis comes from German and Dutch, where the possibilities for restriction and scope readings correlate with surface word order. (49) is a German word-final embedded clause, where the bare plural object *books about wombats* follows the adverb of quantification *immer* ‘always’. This sentence has a scope reading for the bare plural, as gloss at the bottom in (49). In (50), the object is to the left of the AQ, and gets a restriction reading. These data (which are from Diesing 1992) follow from the hypothesis that the AQ is at the left edge of the verb phrase, and that arguments in the the VP can be optionally moved leftward out of the VP and past the adverb, by the operation of scrambling. When the bare plural object is not scrambled, it is in the VP and is mapped to the scope. When it is scrambled, it is outside VP and is mapped to the restriction. Thus we see evidence in German surface phrase order for the hypothesis that material inside the VP is mapped to the scope of an adverb of quantification.

- (49) . . . daß Otto immer Bücher über Wombats liest.  
 . . . that Otto always books about wombats reads  
 that Otto is always reading some books about Wombats.  
 that Otto, whenever he reads, reads books about Wombats

- (50) . . . daß Otto Bücher über Wombats immer liest.  
 . . . that Otto books about wombats always reads  
 that whenever there are books about wombats, Otto reads them.

The issue of splitting is largely independent of the approach to semantic interpretation. Diesing (1992) assumed the indexing analysis from Sections 2 and 3. We can say that a clause headed by an AQ is first split by mapping the VP to the scope and the rest of the sentence to the restriction. Then indexing proceeds as before, with adjunction of an existential adverb to the scope, and indexing of each indefinite to the minimal c-commanding AQ. Or we can say that splitting proceeds syntactically, with the interpretation of the resulting configuration by the dynamic procedure in Section 4.

## 6 More on restrictive *if*-clauses

A dramatic aspect of Lewis’s analysis of adverbs of quantification used in combination with *if*-clauses is that *if* has no meaning of its own. It is simply a syntactic marker for the role of being a restriction of an operator. This comes up in some other places. Kratzer (1978) proposed that in the examples (51) and (52), the modals are the main operators, while the *if*-clauses supply arguments of the

modals.

- (51)a. If Jack eats the panna cotta, he will become sick.
- b. If Jack eats the panna cotta, he might become sick.

- (52)a. If Jack had eaten the panna cotta, he would have become sick.
- b. If Jack had eaten the panna cotta, he might have become sick.

Lewis (1976) looked at contexts with a probabilistic pragmatics, and at the phenomenon of *if*-clauses expressing conditional probability in such contexts. (53) exemplifies some syntactic constructions where such interpretations arise. (53a) can be understood to describe the probabilistic judgment that the probabilistic measure of the set of options where John ate the panna cotta and is sick is significantly greater than the probabilistic measure of the set of options where John ate the panna cotta and is not sick. Or equivalently, the probability that Jack is sick, conditioned on his eating the panna cotta, is significantly greater than 0.5. Similarly, suppose we stipulate that belief states are characterized by probability measures, so that (53b) is true if and only if Keisha's epistemic probability measure assigns high probabilistic measure to the proposition that Jack ate the panna cotta. With this theoretical and empirical mindset, (53c) can be understood to describe a constraint on Keisha's epistemic probability measure that is stated in terms of conditional probability—it seems to be true if and only if Keisha's epistemic probability measure is such that the measure of the set of options where John ate the panna cotta and is sick, divided by the measure of the set of options where Jack ate the panna cotta is high (close to 1.0). Or equivalently, Keisha's epistemic probability measure is such that the probability of Jack being sick, conditioned on his eating the panna cotta, is high.<sup>16</sup>

- (53)a. If Jack ate the panna cotta, he is probably sick.
- b. Keisha believes that Jack ate the panna cotta.
- c. Keisha believes that if Jack ate the panna cotta, he is sick.

Lewis (1976) proved a technical result that there is no semantics that defines a proposition contributed by *if p then q* in terms of propositions *p* and *q*, and also allows the conditional-probability readings seen in (52) to be derived compositionally. In other words, the logical form of (53) could not be as in (54), with [if Jack ate the panna cotta, he is sick] contributing a proposition that is the argument of *probably*. Instead (going beyond what Lewis said explicitly) the *if*-clause supplies an argument of a two-place conditional probability operator.

- (54) [probably [if Jack ate the panna cotta, he is sick]]

Kratzer (1986) generalized the cases of adverbs of quantification, modals, and

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<sup>16</sup>Khoo (2016) is a recent discussion of this empirical picture and counterarguments to it.

probabilistic readings to the hypothesis that *if* is never meaningful, and summarized the situation like this: “The history of the conditional is the story of a syntactic mistake. There is no two-place *if* . . . *then* connective in the logical forms for natural languages. *If*-clauses are devices for restricting the domains of various operators.”

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