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A Research Note on Strategy in Coalition Gaming*

The debate between William Riker, Robert Butterworth, and others on the validity of the size principle in coalition gaming has uncovered some interesting arguments. However, none of the authors have considered the importance of long-term strategy in the side payment bargaining process. The purpose of this paper is to bring coalition gaming closer to real-world situations by introducing two strategies which result in winning coalitions of greater than and smaller than minimum size. In the process two conclusions are reached: 1) Butterworth's argument that the size of the winning coalition is indeterminate when side payments are permitted is correct; 2) Riker's revised definition of a winning coalition, the set of positive gainers, is not sustained in that a coalition of one can result from a possible maximization strategy in contradiction to the rule which states that a coalition of one will have a payoff less than zero.

Before analyzing long-term strategy the groundwork of the game must be established. We are dealing here with the five-person game devised by Riker.[1] In it the players in the winning coalition are paid five points each by the

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members of the losing coalition. Table 1 depicts the payoff schedule in this zero-sum game.

Table 1 Payoffs in a Five-Person Game

Number of Players in Winning Coalition	Payoffs to Players: A	B	C	D	E
5	0	0	0	0	0
4	5	5	5	5	-20
3	10	10	10	-15	-15

As the object of the game is to maximize gain, the coalition will continue to evict members as long as it is profitable. In a game without side payments the minimum size of the winning coalition will be three and the payoff schedule for that outcome is found in line 3 of Table 1. However, real-world coalitions are seldom as egalitarian as the one depicted here. Payoffs to the members of the winning coalition are rarely equated. Thus, in order to bring more realism into coalition gaming, side payments must be introduced into the model.

The effect of side payments on coalition theory was brilliantly argued by Butterworth.[2] He argued that the size of the winning coalition is indeterminate when bribes are allowed. Table 2 is an example of what can occur.

Table 2 A Rational Bribe

Situation	Payoffs to Players: A	B	C	D	E
1) 4 Member Coalition (ABCD)	5	5	5	5	-20
2) A,B,C, Oust D	10	10	10	-15	-15
3) D Bribes to Stay In					
a) Bribe	6	6	6	-18	0
b) Result	11	11	11	-13	-20

In an effort to maximize their gain the members of the winning coalition in this example decide to oust player D. The resulting payoff schedule is represented in line 2 of Table 2. However, player D has the option of bribing players A,B, and C by offering them six points each in order to remain a member of the winning coalition. By doing so he reduces his net loss from twenty to eighteen points. Players A,B, and C would obviously accept the bribe because it increases their winnings more than an eviction of player D would. The result of the bribe is the four person winning coalition with the payoff depicted in line 3b of Table 2.

In a rebuttal to this critique of his theory, Riker amended his definition of a winning coalition.[3] The set of positive gainers now constituted such a group. In the example presented in Table 2, a winning coalition of minimum size would again exist, comprised of players A,B, and C. However, by examining two hypothetical yet rational strategies which can be used by individual players in this game we can conclude that Butterworth is correct when he argues that coalition size is indeterminate.

STRATEGY AS THE ANSWER TO THE LONE LOSER'S PROBLEM

The predicament of the lone loser in coalition gaming presents an interesting problem. The first player to be ousted is at a serious disadvantage because he does not have the alternative of bribing the others in order to remain a member of their group. To do so would require him to pay the other four players six points, for a total of twenty-four. It is rational for him to admit defeat in order to reduce his losses to twenty points. It is possible, however, for E to get back into the winning coalition in a short period of time by using side payments and developing a minimization of loss strategy.

Analysis of E's position begins at the point where a four man winning coalition has been established. At this point players A,B, and C decide to oust D. Table 3 depicts the policy options available to D in lines 2,3, and 4. Briefly stated, D can accept defeat, bribe A,B, and C for continued membership in the coalition, or accept a bribe from E to join him. In

Table 3 Minimization of Loss Strategy

Situation	Payoffs to Players: A	B	C	D	E
1) Four Man Coalition	5	5	5	5	-20
2) ABC Force D Out, D Does Nothing					
Spoils of Victory	5	5	5	-20	5
Result	10	10	10	-15	-15
3) ABC Force D Out, D Bribes ABC					
Bribe by D	6	6	6	-18	0
Result	11	11	11	-13	-20
4) ABC Force D Out, E Bribes D					
Spoils of Victory	5	5	5	-20	5
Bribe by E to D	0	0	0	3	-3
Result	10	10	10	-12	-18
5) E Offers Conditional Bribe					
Bribe by E to ABC	1	1	1	0	-3
Result	11	11	11	-12	-21
6) ABC Force C Out, C Bribes					
Bribe by C	6	6	-18	0	6
Result	17	17	-7	-12	-15

order to minimize his losses D will naturally choose the last option. Player E offers the bribe because D would otherwise have made an offer to A,B, and C, which would have left E's payoff at a dismal -20. The payoff which results from the bribe by E to D (10, 10, 10, -12, -18) is beneficial to both members of the losing coalition.

Player E's next move is to approach two members of the winning coalition, say A and B, and offer the coalition a bribe under two conditions: 1) that they allow him to enter the winning coalition; 2) on the next move the coalition, now composed of players A,B,C, and E, oust C. These two moves are shown in lines 5 and 6 of Table 3. In line 5, E's position suffers to the extent that he is worse off than when he started. However, due to the second condition stipulated in his bribe to A and B, it quickly improves to the point where it is better than at any other stage of the game. This highlights the importance of strategy in coalition gaming. In line 6 I assume that C bribes his way back into the winning coalition. If he were to accept a bribe from D it would decrease the payoff to A,B, and E by one point. This, however, would have no bearing on my conclusion because the position of E would still be better than it was previously.

We are able to solve the problem of the lone loser only by formulating a long-term strategy. To re-enter the winning coalition E first has to draw one man out, enter the coalition by bribery, and then force a second player to either bribe or leave the coalition. The last step is necessary in order to make the strategy a rational alternative. Analysis of strategy enables us to better understand the process of minimization of losses.

It is also possible to hypothesize a strategy of maximization of gain.

A MAXIMIZATION OF GAIN STRATEGY

It is possible for one member of a winning coalition to become the sole positive gainer by implementing a strategy of playing the remaining members off against each other. A graphic representation of this strategy appears in Table 4.

Table 4 A Maximization Strategy

Situation	Payoffs to Players: A	B	C	D	E
1) Four Man Coalition	5	5	5	5	-20
2) ABC Force D to Bribe					
Bribe by D	6	6	6	-18	0
Result	11	11	11	-13	-20
3) ABD Force C to Bribe					
Bribe by C	6	6	-18	6	0
Result	17	17	-7	-7	-20
4) ACD Force B to Bribe					
Bribe by B	6	-18	6	6	0
Result	23	-1	-1	-1	-20

Let us assume that player A devises this strategy when he is a member of a four man winning coalition. His first move is to join with players B and C to force D out. In analyzing his position D realizes that he would lose eighteen points if he bribed the coalition as opposed to the twenty points he would otherwise be forced to forfeit. If A can convince D that the latter's losses will be further minimized in subsequent moves of the game (by informing him that this tactic will be used against B and C), D will be influenced to offer the bribe and reject any offers from E. It is rational, therefore, for D to offer A,B, and C six points to remain a member of the winning coalition.

Player A's next moves follow the same pattern. He unites with B and D to force a bribe from C and then turns the tables on B. In each case a bribe is the rational alternative for the endangered player.

The result of this strategy is a four man winning coalition (A,B,C,D) with payoffs (23, -1, -1, -1, -20). Player A has attained a commanding position and becomes the sole positive gainer.[4]

CONCLUSIONS

Coalition behavior as modeled by Riker is rarely witnessed in real-world politics. Instead, one is more likely to see individual members attempt to maximize their personal gain or minimize their losses. In order to accomplish these tasks the players would be wise to devise long-term strategies such as the ones I have presented here.

As a result of the introduction of strategy into the coalition gaming process the conclusion is reached that the size of the winning coalition is indeterminate, a conclusion originally reached by Butterworth. In the examples presented above the size of the winning coalition is either three or four, the deciding factor being the course of action which the player to be evicted pursues. It is important to note that in neither example presented above is a stable situation reached. Coalition behavior is a dynamic process and the size of the winning coalition will expand and contract as each player attempts to improve his position.

More important, perhaps, is the existence of just one positive gainer in the final payoff schedule of the maximization of gain strategy. This refutes Riker's revised definition of a winning coalition, the set of positive gainers, which was offered in reply to Butterworth's critique. If we were to accept this definition then player A would represent the winning coalition, thus giving us a coalition of one. However, this is incorrect because a coalition of one, also according to Riker, must have a payoff less than zero. In other words, if we accept this definition the result is a coalition of smaller than minimum size. If we reject this definition on the grounds of the above stated rule on payoffs to coalitions of one, the result is a coalition of larger than minimum size. This coalition is made up of players A,B,C, and D with the payoff schedule (23, -1, -1, -1, -20). As a consequence of this paradox the validity of the size principle remains in doubt.

Footnotes

1. Riker, William H., The Theory of Political Coalitions, Yale University Press, New Haven, 1962.

2. Butterworth, Robert Lyle, "A Research Note on the Size of Winning Coalitions," American Political Science Review, 65 (September 1971), pp. 741-745.

3. Riker, William H., "Comment on Butterworth: A Research Note on the Size of Winning Coalitions," American Political Science Review, 65 (September 1971).

4. It should be noted at this point that I am not proposing a stable solution, but, rather, one possible situation. The logic of A's strategy may indeed be used against him in subsequent moves of the game.

Guernica (1937) by Pablo Picasso
Extended Loan from the artist to The Museum of Modern Art, New York City, N. Y.