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Risk Preferences Necessary to Choose Life Insurance Funding of Buy-Sell Arrangements

by

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Abstract

Pratt-Arrow risk aversion coefficients are derived such that term life insurance funding of buy-sell arrangements is preferred by decision makers with risk preferences greater than those breakeven coefficients. Given previous estimates of farmers' risk preferences, anything greater than a 25 percent loading of actuarially fair premiums would discourage life insurance funding.

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Introduction

Buy-sell arrangements are typically suggested for closely held farm businesses. Upon the occurrence of an event, such as the death of an owner, the mechanism for an orderly transfer of ownership can then occur, since contract negotiations had previously occurred when the arrangement was drawn. One detail of formulating buy-sell arrangements is financing the property transfer. If the buy-sell is written for the death of an owner, then life insurance becomes a financing option.

The purpose of this article is to determine the risk aversion under which term life insurance would be used to fund buy-sell arrangements. Since the decision involves selecting a financing option to transfer property, a simple criterion might be to minimize the cost of financing. However, since death is a probabilistic event, the business may not be transferred in any given year, and thus financing costs become stochastic. So in this article the disutility of financing a fixed value of property is minimized using stochastic dominance procedures.

Optimal life insurance purchases by individuals for the income needs of surviving dependents have been extensively analyzed. Although the solution procedures have often differed, the basic technique used by most researchers has been to find the conditions necessary to maximize the combined utility from lifetime consumption and bequests at death. For instance, Hakansson incorporated life insurance and the bequest motive into a model of investment and consumption strategies under risk and an uncertain life. The portfolio-composition decision, the financing decision, the consumption decision, and the insurance decision were all analyzed in one model using discrete-time dynamic programming. The model objective was to maximize expected utility from consumption and the bequest left at death. General solutions to the optimal purchase of life insurance were obtained only under highly specialized conditions.

Likewise, Fisher used a discrete-time model in which the length of life is uncertain to examine life-cycle patterns of consumption, savings, and insurance purchases. Specific utility functions were assigned to period consumption and to possible bequests. The utility functions were additive with various weightings. The results indicated that an individual who receives labor income is more likely to purchase insurance than an individual who lives off the proceeds of his wealth, who is unlikely ever to purchase life insurance. An individual may buy insurance which is heavily loaded against him if the bequest utility function is sufficiently strong. He may reject fair or even favorable insurance if the bequest function is weak. Other research on optimal life insurance includes articles by Borch, and Richard, who used differential equations, and Fortune, who used a mean-variance approach.

The use of life insurance to fund buy-sell arrangements is typically discussed in most life insurance textbooks. Yet the economics of funding buy-sell arrangements have not been thoroughly researched. Tauer analyzed the decision with general stochastic dominance using the probability distribution of present costs of life insurance versus installment payments. He found that the decision depends upon the age, discount rate, and risk aversion of the decision-maker, but that partial funding with life insurance was optimal in many cases. A limitation of that study was the use of present costs when the financing duration is often 40 years or more, with drastically different cash flow timings between the financing options.

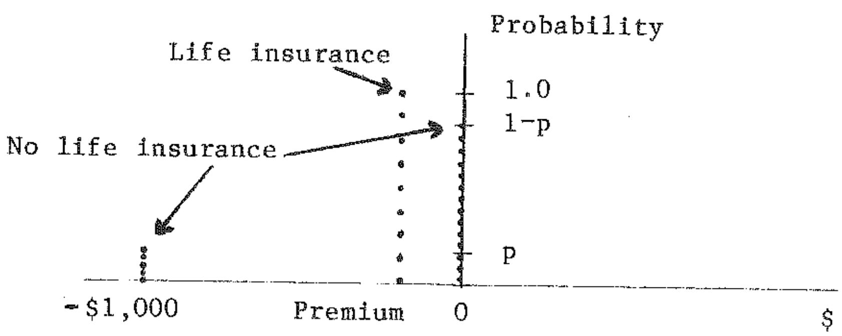
The Decision

Assume that to finance the purchase of the business upon the death of a co-owner, a decision-maker has the option to purchase life insurance on the co-owner's life, or to use equity and debt financing. Although individuals would use various combinations of equity and debt financing, the present value of any financing arrangement should be approximately equal to the cash price. The decision period is yearly but sequential so that at the start of any year the decision-maker can elect whether or not to purchase a one-year term life insurance policy on his co-owner. It is assumed that the co-owner will remain insurable if insurance is not purchased in any given year. Yearly term insurance is invariable guaranteed renewable and the cost of that adjunct is reflected in the yearly premium, but is not analyzed here. Two outcomes are possible during the year: the co-owner may live or die. The cost of life insurance for either outcome is the premium cost. If the co-owner dies, the insurance proceeds will be used to purchase the business property. The cost of no life insurance is zero if the co-owner lives but is the value of the business property if the co-owner dies. Table 1 illustrates these costs for a $1,000 business interest. The probability distributions are depicted in Figure 1.

Table 1: Costs of Transferring $1,000 of Business Property at the Death of a Co-owner

Outcome	Probability	Cost of Life Insurance	Cost With No Life Insurance
Live	1 - p	Premium	$ 0
Die	p	Premium	1,000

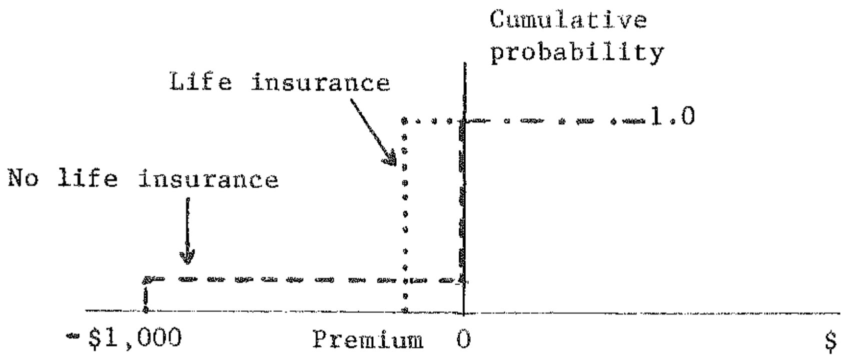
FIGURE 1. Probability Distribution of Life Insurance and No Life Insurance



Both income and estate taxes can be ignored in these transactions. Insurance premiums for buy-sell arrangements are not tax deductible even when the business pays the premiums. Although interest on debt payments would be deductible, the after-tax present value of those payments would be zero if the after-tax discount rate is the same as the after-tax interest rate.

By computing the cumulative distributions (Figure 2), it is clear that neither financing option dominates the other by first degree stochastic dominance since the cumulative distributions cross (Kroll and Levy). In fact, since the life insurance cumulative distribution reaches the value of 1 (it is a vertical line) before the no life insurance cumulative distribution does, and since the no life insurance option has the lowest outcome of the two options, neither of the two options dominates the other by second degree stochastic dominance. However, since the cumulative functions cross only once, and no life insurance is a riskier option since it has the lowest outcome, then for a constant risk aversion individual (negative exponential utility function), there will exist a breakeven aversion to risk coefficient (Arrow-Pratt's absolute risk aversion coefficient) such that life insurance

FIGURE 2. Cumulative Distribution of Life Insurance and No Life Insurance



would be preferred at higher coefficient values, and no life insurance would be preferred at lower value coefficients (Hammond).1/ Since neither option dominates the other by second degree stochastic dominance, that breakeven coefficient must be greater than zero. Otherwise, life insurance would dominate no life insurance by second degree stochastic dominance.

The task then is to compute the risk aversion coefficient separating preference for life insurance and no life insurance. Knowledge of this coefficient is useful regardless of the functional form of a decision-maker's utility function since Hammond shows that the breakeven aversion to risk coefficient derived from the constant risk aversion utility function can be used as the boundary risk aversion coefficient for any non-constant risk aversion utility function to determine preference between any two distributions. This is especially useful for decreasing (or increasing) aversion to risk utility functions where the largest aversion to risk coefficient is less than the boundary value. If a nonconstant aversion to risk utility function displays a risk aversion coefficient throughout its relevant range below the boundary

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1/From the negative exponential utility function, $u = -e^{-cx}$, where $c>0$, the computation of Arrow-Pratt's absolute risk aversion coefficient $r = -u''/u'$, results in $r = c$, a constant risk aversion, regardless of income level x.

value computed from the constant risk aversion utility function (negative exponential), then no life insurance would be preferred. Life insurance would be preferred if the nonconstant aversion to risk utility function displays risk aversion coefficients always greater than the boundary risk coefficient. In that case, the lowest risk aversion value of a decreasing aversion to risk utility function must be greater than the boundary risk aversion coefficient before life insurance would be preferred. Hammond's theorem allows drawing conclusions for most utility functions without computing utility values for those utility functions. Only utility values for the negative exponential utility function must be computed. Identical results can be obtained by the stochastic dominance with respect to a function (general stochastic dominance) procedure derived by Meyer, except that the boundary risk coefficient need not be constant with Meyer's procedure, and can be a function of the outcome variable.

Analysis

To determine the breakeven aversion to risk coefficient for any age, it is necessary to know the premium cost and probability of death for that age. Conditional probability of death (alive at beginning of the year) for ages 1 through 100 were taken from the New Male Basic Table, which was used to derive the 1980 CSO Table. Premium costs for yearly term insurance for each age were computed as actuarially fair insurance by multiplying the probability of death by the amount of insurance ($1,000). To allow for administrative costs, yearly premiums were marked up by 10, 25, 50, and 100 percent. 2/

2/Margins built into the 1980 CSO Table are computed as a quadratic function of age and reciprocal of the custate expectation of life. In addition to these margins a company would need to recover the costs of doing business in its premiums.

The expected utility of each of the two options was computed and compared at various risk aversion coefficients by the exponential utility formulas: E (utility of life insurance) = $-e^{-cx}$, where x is the insurance premium, c is the risk coefficient, and e is the natural exponential; and E (utility of no life insurance) = $p\ (-e^{-c(-1000)}) + (1-p)\ (-1)$, where p is the probability of death. The risk coefficient, c, was increased from 0 to .02 in increments of .0001, and utilities were computed and compared at each increment.

Results at various ages and premium markup percentages are given in Table 2. With actuarially fair insurance (zero loading), the breakeven risk coefficient is either .0001, .0002, or .0003. Insurance is preferred above these coefficients; no insurance is preferred below them. Although the constant aversion to risk utility function was used, the results are applicable to any utility function that exhibits a nonconstant aversion to risk coefficient that remains above or below a breakeven coefficient. At the higher ages the probability of death increases and so does the actuarially fair premium. Thus, the probability distribution of life insurance, which is a vertical line, shifts to the left by the amount of the premium increase. The right point of the no life insurance probability distribution will shift down but will not change vertically, while the left point will shift up by that same amount, with no vertical change. The cumulative distribution function of life insurance will shift vertically, while the cumulative function for no life insurance will shift up horizontally. The cumulative functions will cross at a lower outcome value but higher probability.

At any age the expected cost of insurance and no insurance will be identical with actuarially fair premiums. As soon as the first risk aversion coefficient is reached, .0001, life insurance is preferred since it has the

Table 2: Breakeven Risk Aversion Coefficients for Life Insurance Preference ($1000 Face Value)*

Premium Markup	Age								
	10	20	30	40	50	60	70	80	90
(percent)									
0	.0003	.0001	.0002	.0002	.0001	.0001	.0001	.0001	.0001
10	.0009	.0010	.0010	.0010	.0010	.0010	.0010	.0011	.0012
25	.0022	.0022	.0022	.0022	.0022	.0022	.0023	.0024	.0028
50	.0039	.0039	.0039	.0039	.0039	.0039	.0040	.0043	.0051
100	.0063	.0063	.0063	.0064	.0064	.0065	.0067	.0073	.0091

*Yearly term insurance is preferred for greater risk aversion coefficients. No life insurance is preferred for lower risk aversion coefficients. To adjust the coefficients for any multiple of $1,000, divide the coefficients by that multiple.

same expected cost as no insurance, but is a less-risky option. (The values of .0002 and .0003, instead of .0001, occur at some ages because of rounding errors.)

With any premium loading at any age, the risk aversion coefficient must increase before life insurance is preferred. The reason is that life insurance becomes more costly. It may also be considered more risky in comparison to no life insurance, whose probability distribution would not be altered.

At any percentage markup the breakeven risk aversion coefficient increases with age. Because the actuarially fair premium increases with age, loading as a percentage of that premium means that costs increase in absolute value. This makes insurance more costly, requiring larger values of risk aversion before it is preferred. Tauer using discounted costs found breakeven risk coefficients between .00125 and .0025, which compares to these results with premium markups from 10 to 25 percent.

These results have been obtained based upon a $1,000 business purchase. To alter the breakeven risk aversion coefficients to be applicable to any multiple of $1,000, it is necessary to divide the computed risk aversion coefficient by that same multiple (Tauer). This is because multiplying x and dividing c by the same value in the negative exponential utility function, $u = -e^{-cx}$, results in the same utility value for comparison.

Thus far the analysis has considered only two funding options, either no life insurance or complete life insurance funding. The possibility exists, however, to use any percentage of life insurance funding. Six options were analyzed: 0, 20, 40, 60, 80, or 100-percent funding with life insurance. The negative exponential utility function was again used to determine which of the six funding options provided the highest expected utility at various risk aversion coefficients. The coefficient was again varied from .0 to .02 in increments of .0001. The results for ages 40 and 60 are reported in Tables 3 and 4 respectively. Results for other ages are similar.

With zero loading of the premium, the preferred option is no life insurance until the risk coefficient of .0001, the first increment of the coefficient. With a zero loading the expected cost of no life insurance and complete life insurance is equal, as is any combination of insurance or no insurance funding. As soon as the first risk aversion coefficient is reached (i.e., .0001), the least risk averse option is selected, which is 100-percent life insurance.

Table 3. Risk Aversion Coefficients for Preference for Various Percentages of Life Insurance Funding, $1,000 Business Purchase, Age 40

Percentage of Life Insurance	Markup of Premium in Percent				
	0	10	25	50	100
0	.0	.0	.0	.0	.0
20	NP	NP	.0003	.0005	.0008
40	NP	NP	.0004	.0006	.0010
60	NP	.0002	.0005	.0009	.0014
80	.0001	.0004	.0008	.0014	.0023
100	.0002	.0010	.0022	.0039	.0064

NP means that percentage of life insurance is never preferred.

Table 4. Risk Aversion Coefficients for Preference for Various Percentages of Life Insurance Funding, $1,000 Business Purchase, Age 60

Percentage of Life Insurance	Markup of Premium in Percent				
	0	10	25	50	100
0	.0	.0	.0	.0	.0
20	NP	NP	.0003	.0005	.0008
40	NP	NP	.0004	.0006	.0011
60	NP	.0002	.0005	.0009	.0015
80	NP	.0004	.0008	.0014	.0024
100	.0001	.0010	.0022	.0039	.0065

NP means that percentage of life insurance is never preferred.

With a 10-percent loading, 60-percent insurance funding is typically preferred at a risk level of .0002. It requires a risk coefficient of five times that magnitude (.0010) before 100-percent life insurance is preferred. With markups of 25, 50, and 100 percent, it requires risk coefficients of .0003, .0005, and .0008 respectively before 20-percent insurance funding is preferred. It requires slightly higher risk aversion coefficients before 40 and 60-percent insurance funding is preferred, much higher coefficients before 80-percent insurance, and substantially higher coefficients before 100-percent insurance is preferred.

Concluding Remarks

Empirical evidence of risk preferences suggests that the majority of farmers have utility functions such that Pratt's risk coefficients fall within a range of -.0002 to .0003 at their average income level (Young et al.). Purchasing a co-owner's share of the business would normally be outside a farmer's average income, but the evidence supporting decreasing or increasing risk aversion as wealth increases is mixed (Wilson and Eidman). If one can assume that most decision-makers have risk coefficients in the interval -.0002 to .0003 over all wealth levels, then any percentage of life insurance funding for any size business purchase at any age is a rational decision except when the premium markup is 50 or 100 percent. In those cases, a decision-maker would probably not fund more than 60 percent of the purchase with life insurance.

If decision-makers are only slightly risk averse, a 10-percent markup of actuarially fair premiums would encourage 40 or 60-percent funding of the buy-sell arrangement with life insurance. With zero loading, 100-percent life insurance funding would be used. However, with anything greater than a 25-percent loading, little or no life insurance would be used at any age. Twenty five percent loading would be premiums of $1.56, $1.18, $2.15, $5.73, and $15.06 for a 20, 30, 40, 50, and 60-year-old person respectively. One-year renewable term insurance is available at these and lower premium rates (Gaines and Cosson).

Yet, many risk averse farmers do not utilize life insurance in buy-sell arrangements. Several explanations are possible. Many farmers are myopic and do not plan ahead. Other farmers are averse to the thought of planning for a death, even a partner's death. Even if a buy-sell arrangement exists, an owner may place a lower subjective probability on the death of a partner. This would increase the utility value of no life insurance. Some farmers may be averse to a co-owner or the business purchasing and benefitting from life insurance on their lives. Finally, the owners may have insufficient funds to purchase life insurance, although it may be the optimal decision without the cash-flow constraint. Survival or growth of the farm business requires reinvesting all earnings. The fact that insufficient cash or cash flow exists to purchase the business if necessary is not relevant since that problem is not imminent.

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