OPTIMAL MOSO BAMBOO FOREST MANAGEMENT: A DYNAMIC MODEL

A Thesis

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by

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ABSTRACT

Moso bamboo forest management involves making decisions about the timing and quantity of bamboo stem harvests and bamboo shoot harvests. In my Masters thesis, I solve for the optimal bamboo stem harvest and bamboo shoot harvest policy using a numerical dynamic model that nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem. I use a Chapman-Richards growth function as my model for bamboo biomass accumulation. I compare the optimal bamboo stem harvest and bamboo shoot thinning policy with actual data on bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China. I find that while the actual bamboo stem and bamboo shoot harvests come close to approximating the optimal harvesting policy, there are some differences between actual harvests and optimal bamboo harvests. My results have important implications for bamboo forest management and, to the extent that some of the differences between actual harvests and optimal bamboo harvests reflect possible sub-optimal behavior on the part of Moso bamboo forest managers, for ways to improve Moso bamboo forest management and policy.

BIOGRAPHICAL SKETCH

Tong Wu comes from Taiyuan, a city in Shanxi Province in China. Prior to graduate school, Tong attended the 2+2 joint program between China Agricultural University and the University of Maryland, receiving a B.S. in Agricultural Economics and Management from China Agricultural University, and a B.S. in Agricultural and Resource Economics from the University of Maryland. During college, Tong became passionate about natural resource economics when taking "AREC 453: Natural Resources and Public Policy" taught by Professor Erik Lichtenberg. Her enthusiasm for applying quantitative methods to real life natural resource management problems encouraged her to continue in the academia as a graduate student. She decided to pursue a Masters in the field of resource economics and to immerse herself in research. She chose to pursue her Masters in Applied Economics and Management at Cornell University, where she got trained in economic theory and math skills. She got interested in dynamic programming when taking Professor C.-Y. Cynthia Lin Lawell's Ph.D. class "AEM 7500: Resource Economics" during her first year as a Masters student at Cornell, and decided to apply this method to forest management in China for her Masters thesis research.

Tong was raised in a single-parent family by her mom and grandparents. Her love of nature and curiosity was developed by listening to book readings from her grandpa before she could even read. Tong enjoyed playing with snails, birds, and insects when she was young, and her love of nature continues to this day. In her spare time, Tong is dedicated to nature. She was a member of the Community Learning Garden at the

University of Maryland and has enjoyed hiking while a student at Cornell. Tong loves to read and watch anime. She absolutely loves cats and dogs (and all other fluffy animals), and plans to have her own cat this year. She has always found her peace internally and enjoys quiet time alone as well as time together with friends.

Tong's Masters thesis research was selected for presentation at the Forests & Livelihoods: Assessment, Research, and Engagement (FLARE) Annual Meeting at the University of Michigan in August 2019; as well as at the Faustmann Symposium in October 2019 in Darmstadt, Germany. She is very honored to be selected to present her Masters thesis research at these prestigious conferences.

Tong will be joining the Ph.D. program in Applied Economics and Management at Cornell University this Fall Semester 2019, and is looking forward to continuing her research on Moso bamboo forest management and natural resource economics for her Ph.D. dissertation.

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1. Introduction

Moso bamboo (*Phyllostachys edulis*) is the single most important bamboo species in China, accounting for 74% of China's bamboo forest area ("China Forestry and Grassland Administration", 2018), and the third most important source of timber in China. Both bamboo shoots and bamboo stems are harvested as valuable products: bamboo shoots are a traditional food source, and bamboo stem are used as timber for paper making, flooring, and construction (Fu, 2001).

Optimal Moso bamboo management is a complex dynamic problem. Moso bamboo forest management involves making decisions about the timing and quantity of bamboo stem harvests and bamboo shoot harvests. Both bamboo stems and bamboo shoots are products that are sold on the market. Bamboo shoots grow annually from a bamboo plant's underground rhizomes. Bamboo stem continue to grow each year until age 4-5 years, while bamboo shoots only grow within a year. The harvesting of bamboo stems entails cutting down the bamboo plant, while the harvesting of bamboo shoots does not.

In this paper, we solve for the optimal bamboo stem harvest and bamboo shoot harvest policy using a numerical dynamic model that nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem. The inner finite-horizon within-year daily dynamic programming problem captures daily bamboo shoot growth within a year. The outer finite-horizon between-year annual dynamic programming problem captures annual bamboo stem growth from year to year. We use a Chapman-Richards growth function as our model for bamboo biomass accumulation. We compare the optimal bamboo stem harvest and bamboo shoot harvest policy with actual data on bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China.

The results of our numerical dynamic model suggest that since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year and on whether the previous year was a high-precipitation year, and since bamboo stem continue to grow each year until age 4-5 years, while bamboo shoots only grow within a year, it is generally optimal not to harvest any bamboo stem until the first day of the last year, and to harvest all the bamboo shoots at the end of each year.

When we allow for uncertainty in rain, which increases the number of bamboo shoots in the following year, we find that when the probability of having a high-precipitation year is high enough, then it will be optimal to harvest a little bamboo stem on the first day of an earlier year, and harvest all the remaining bamboo stem on the 1st day of the last year. When the probability of having a high-precipitation year is high enough and relative price of bamboo stem is high enough, then it will be optimal to harvest a little bamboo stem on the first day of an earlier year, and harvest all the remaining bamboo stem on the first day of a subsequent year.

We find that the actual bamboo stem and bamboo shoot harvests come close to approximating the optimal harvesting strategy, but have some features that differ from what our model suggests to be optimal. Our results have important implications for bamboo forest management and, to the extent that some of the differences between actual harvests and optimal bamboo harvests reflect possible sub-optimal behavior on the part of Moso bamboo forest managers, for ways to improve Moso bamboo forest management and policy.

The balance of our paper proceeds as follows. Section 2 provides background information on forestry and bamboo forests in China. We discuss the previous literature in

Section 3. We describe our numerical dynamic model of bamboo forest management in Section 4. We describe our chosen parameter values in Section 5. Section 6 presents the results of our numerical dynamic model. In Section 7, we compare the dynamically optimal harvesting strategies derived from our model with data on actual bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China. We discuss and conclude in Section 8.

2. Background on Forestry and Bamboo Forests in China

China has 208 million hectares of forest area, covering 21.63% of total area of the country and constituting a total stock volume of 15,173 million cubic meters. China ranks among the top countries in terms of total forest resources, although it ranks lower in terms of forest resources per capita. China accounts for 25% of the world demand for forest products, but only 5% of the world's forest area. The distribution of forest resources in China is unevenly centered in the Northeast and the Southwest (China Forestry and Grassland Administration, 2018).

China's forest deficiency results from a combination of various factors. Historically, China was a feudal society that relied on the brutal extraction of natural resources. Land cultivation in China started thousands years ago, and since then there has been a continued conflict between forest and crop cultivation. Firewood collection, charcoal making, land reclamation, brick making, and house construction contributed to deforestation in the preindustrial periods (Fang and Xie, 1994). It was not until industrialization, however, when massive deforestation took place. The Great Leap forward (1958-1962) campaign destroyed forests by popularizing the usage of homemade furnaces. Even after the campaign, deforestation intensified, providing cheap logs for industrialization (Wang, Kooten, and Wilson, 2004). During the transition from a central planning system to a market economy, local households had insecure ownership rights over tress, leading to forest clearing and deforestation (Démurger, Hou, and Yang 2009).

The forest sector acted as a base for Chinese industrial development in the first several decades after 1949. Although there are experts who appealed to ecosystem protection (Wang, Kooten, and Wilson, 2004), there was little focus on forest stock preservation. Prior to 1982, forest policies in China defined the priority of forest management to be timber production

(Démurger, Hou, and Yang, 2009). Although the China Forestry Administration was in place, there was no formal legislation. It was not until 1979 that the first forest law got passed and became effective in 1984 (Wang, Kooten, and Wilson, 2004). In the l990s, China started its transition to preservation and restoration-oriented forest management policies. Six major forestry conservation, restoration, expanding, and commercially developing programs were launched in order to recover and better manage forest resources in China (Démurger, Hou, and Yang, 2009).

Land ownership in China is unique and worth mentioning because it affects decisions making of forest operations on the land. Land ownership appears to affect forest management through stumpage price and management objectives. Lands in China are state owned or collectively owned. In contrast with private land ownership in the US, only usage of land can be owned by private agents rather than the land itself. Forest lands owned by the state are managed by state owned forest farm, where both land and forests belong to the state. It is not usually the case for collectively owned forest lands, where lands belong to collectives, but forests belong to rural households or private entities. (Démurger, Hou, and Yang, 2009). This is how the term "private forest" arises and always causes confusion.

In this paper, we focus on bamboo forests. Bamboo grows faster compared to other forest types, which is consistent with goal of forest conservation and developing programs discussed above. China has abundant bamboo species and total stock ("China Forestry and Grassland Administration", 2018). Of the bamboo forest resources in China, 6.6% are in state forests, 51.4% are in collective forests, and 42.0% are in private forests (Démurger, Hou, and Yang, 2009). China has established pilot futures market in Fujian province, where bamboo change can be traced through market price (Wang et al. 2007).

Bamboo (*Bambusoideae*) is categorized as woody grass rather than a tree in plant science. Bamboo is distributed mostly in tropical areas, subtropical areas, and temperate zones in Asia. They survive even at 4000 meters elevation form sea level (Scurlock, Dayton, and Hames, 1999). There are 107 genera and 1300 species of bamboo worldwide (Zhu, 2001). China has the world's most copious bamboo forest resources, with more than 500 bamboo species in 39 genera. China has 6.01 million hectares of bamboo forest, which accounts for 3% of the country's total forest area. Eighty-nine percent of China's bamboo forests are located in eight provinces: Fujian, Jiangxi, Zhejiang, Hunan, Sichuan, Guangdong, Guangxi and Anhui. Moso bamboo (*Phyllostachys pubescens*) occurs most extensively and is the single most important bamboo species in China since it accounts for 74% of China's bamboo forest area ("China Forestry and Grassland Administration", 2018).

Moso bamboo distributes mostly in subtropical provinces include Fujian, Hunan, Zhejiang, and Jiangxi. The mean annual temperature where Moso bamboo grows well varies from 15 to 21°C (59 to 69.8°F), and the mean temperature of the coldest month is 1 to 12°C (33.8 to 53.6°F). Annual precipitation higher than 800mm (31.5 inches) and soil fertile loam deeper than 60cm (23.5 inches) with pH of 4.5 to 7.0 are ideal for Moso bamboo growth. Extreme temperature, amount of precipitation, and soil condition influence shoot growth for different areas (Fu, 2001). Moso bamboos reproduce by bamboo shoots grown from runners, which are called rhizomes. Rhizomes are mature underground stems while shoots are buds of new bamboo. The reproduction of Moso bamboo is no different from other grass, while its usage suggests bamboo to be included as tree species in China. Generally, Moso bamboos reach mature age after five years.

Moso bamboo emerges from its rhizomes, which is an underground stem system that expands out. Rhizomes are usually beneath the ground, whereas when its expansion encounter rocks they might jump out of the ground and round the hard hindrance from its top. Moso bamboo start to shoot above the ground from late March randomly. Shoots are initially covered by brown sheath with hairs. Half of the shoots rust naturally in the ground before growing into bamboo, leaving a shallow pit on the ground while another half continue to grow higher into bamboo stems. Bamboo diameter depends solely on the diameter of shoots that will not change in later growth periods (personal communication, bamboo specialist at Zhejiang Provincial Key Laboratory of Bamboo of Zhejiang Provincial Academy of Forestry, August 2018). The harvesting of bamboo stems and bamboo shoots are forms of thinning; there is little need to dig out all rhizomes and rotate the whole stand.

Bamboo shoots grow annually from a bamboo plant's underground rhizomes. As long as the rhizome has not been destroyed, bamboo shoots can still emerge from rhizomes. A bamboo plant may have rhizomes that extends massively and thus can have lots of nodes for shoots growth. The number of newly grown bamboo is the number of bamboo shoots minus number of shoots harvested. This is to say that when all the shoots emerge, they either degenerate, are harvested, or are left in the ground and grow into a newly grown bamboo stem (personal communication, bamboo specialist at Zhejiang Provincial Key Laboratory of Bamboo of Zhejiang Provincial Academy of Forestry, August 2018).

Moso bamboo growth is in a cycle of two years in China that is called "Du", with one Du equals to two calendar years. This cycle is determined due to biological features of a bamboo forest. In an "on" year, shoots merge rapidly and numerously. On the contrary, there are little shoots emergence during an "off" year. Due to this particular growth pattern, bamboo forest becomes an uneven stand with individuals from different Dus. First Du refers to 0-1 year bamboo. Send Du refers to 2-3 years bamboo. Third Du refers to 4-5 years bamboo. And forth Du refers to 6-7 years bamboo. Each Du bamboo has its characteristics for specialties to distinguish them. The first Du ones are in light green with white fuzz on stems and decade sheath at stem bottom. The second Du ones are in darker colors but without fuzz. Their node thorns are harder. The third Du ones are dark green, while the forth Du ones are in whitish green. Even with these descriptions, distinguishing bamboo ages is a difficult task for non-specialists (personal communication, bamboo specialist at Zhejiang Key Laboratory of Bamboo of Zhejiang Academy of Forestry, August 2018).

In China, bamboo forest harvest decisions and shoots harvest decisions are made according to on and off years. Shoots harvest, especially winter shoots harvest take place during on years. In addition, stem harvest take place mainly during on years. This is because every time a stem harvest decision is made, all forth Du bamboo are clear cut for the whole forest in Zhejiang province. Clear cut of forth Du bamboo usually takes place for sixth-year-old bamboo rather than waiting until they grow into seven-year-old ones. Due to this clear cut pattern, massive shoots emergence and clear cut of forth Du bamboo take place simultaneously in the on year. When bamboo stem are harvested, which means it is going to be an off year next year, number of shoots next year is going to be less next year since bamboo stand developing more for leaves and rhizomes during an off year. For the off year, relatively less shoots harvest activities are performed and little stem cutting is operated (personal communication, bamboo specialist at Zhejiang Provincial Key Laboratory of Bamboo of Zhejiang Provincial Academy of Forestry, August 2018).

The harvesting of bamboo stems and bamboo shoots will both change the age structure of the bamboo stand, but in different patterns. Harvesting bamboo stems will reduce the amount of bamboo plants in their age class when harvested. This increases the proportion of bamboo plants in young age class.

The harvesting of bamboo shoots is a natural process of thinning since without human intervention, more than half of the shoots will degenerate and die naturally before they grow into bamboo plants. Shoots harvesting is thus a thinning activity that takes these weak shoots out before their death (Jiang, 2007). The harvesting activities will be directly influencing density of the bamboo stand and decreasing proportion of young bamboo plants compared to un-thinned bamboo stands. Harvesting one shoot eliminates one future bamboo plant from the beginning. However, harvesting shoots does not necessarily reduce total bamboo biomass in the future since thinning creates more space for other bamboo plants left in the ground to grow.

In forest management in the United States, forest thinning (silviculture) generally produces low quality logs that incur a cost due to their low market value. A unique feature of bamboo shoot thinning is that by harvesting bamboo shoots, bamboo farmers are also able to sell shoots as a by-product with a high market price.

Bamboo farmers generally follow a pattern of intensive harvest of shoots at the beginning of shoots emergence and preserve remaining shoots for later bamboo growth.

Bamboo shoots grow into bamboo plant after around 55 days, not including the dormancy period of shoots underground during the winter (Su, 2012). Generally, since Moso bamboo stems reach their maximum biomass at age 4-5 years (Zhang et al., 2014; Zhuang et al., 2015) and mature at age 5-6 years, it would be very economically inefficient to harvest bamboo stem after 10 years.

3. Previous Literature

Forest management is a dynamic problem because trees (and bamboo) take time to grow. We build on the seminal model of the optimal rotation time for a forest developed by Faustmann (1849) and elaborated upon by Samuelson (1976). Since then, the Faustmann model has been extended in many ways (Newman, 1988), including to even-aged forest management (Jackson, 1980; Chang, 1983), uneven-aged forest management (Chang, 1981; Hall, 1983), externalities (Berck, 1981; Bowes, 1983; Calish et al., 1978; Hartman, 1976; Nguyen, 1979; Strang, 1983), taxation (Chang, 1982; Klemperer, 1979; Pearse, 1967; Rideout, 1982; Ollikainen, 1991), evolving stumpage price (Bare and Waggener, 1980; Gregersen, 1975; McConnell et al., 1983; Hardie et al., 1984; Newman et al., 1985), a one-time change of unchanged factors (Nautiyal and Williams, 1990), uncertainty (Chang, 1998), the intertemporal allocation of consumption (Deegen et al., 2011), rotation and thinning (Arimizu, 1958), optimal density (Amidon and Akin, 1968), net present discounted value of future payoff (Kilkki and Väisänen, 1969), forest production control (Hool, 1965), production control with markov process (Hool 1966), and thinning decisions (Amidon and Akin, 1968).

Although simple rotation decisions have been addressed by dynamic optimization, more applications of this methodology appear in making complicated thinning decisions or combined decisions in following literature. Amidon and Akin (1968) do an assessment of optimal thinning with dynamic programming on solving deterministic, one-dimensional present worth optimization by choosing a trajectory of forest density. Brodie et al. (1978) show consistent result using forward recursion in thinning decision. More complicated thinning decisions on thinning type, intensity and timing are brought out in 1979 while adding diameter into the forest growth model (Brodie and Kao, 1979). Chen et al. (1980) demonstrate optimal thinning plan between rotations as well as an optimum rotation age considering diameter and basal area in their growth model. In addition to value of timber, forage value has first been considered simultaneously in optimal rotation and thinning (Ritters et al., 1982).

Dynamic programming has been found to be sensitive to parameter changes (Amidon and Akin, 1968), and is more flexible and more efficient (Brodie et al., 1978) compared to previous marginal analysis in forest economics. Multiple descriptors and objectives can be incorporated into the model (Ritters et al., 1982). Its advantages further extents capability of dynamic programming on complicated dynamic forest management.

In most of the forest economics literature, growth simulation models or yield models characterize objective timber yield for tree species of interest in dynamic programming. Both growth simulation model and yield model describe productivity of a tree standing as a function of multiple variables such as age, temperature, soil, rainfall, slope, and rooting depth (Tyler, Macmillan, and Dutch, 1996). If the objective of the forest owners is instead profit maximization, then the market price of the timber also becomes significant factor in the payoff function.

Previous studies have some typical focuses that are worth mentioning. Sophisticated studies on forestry management utilizing dynamic optimization have been done (Riitters, 1982; Haight, 1985; Yousefpour, 2009) with specific focus on developed countries. It is not surprising that dynamic optimization for forest management first suffices in more affluence areas since these areas are high in demand for sustainable development and cost effectiveness. Fewer studies are carried out in developing countries and poor areas.

In addition to amount of studies been done, tree species is another aspect where further studies can be carried out. Pine and fir are two major types of tree species researchers interested in, due to their popularity in the western world and well developed productivity simulation models. Management of these tree species can be cost ineffective for the fact that they require intensive thinning machinery and labor.

Fast economic development in developing countries and a higher demand of sustainability generate incentive for managing forest with dynamic optimization in these regions. Different political structures, forestry contexts, objectives, and previous sivilcultural practices demonstrate various research opportunities on forest management in developing countries. The literature review done so far demonstrates possibility of applying dynamic optimization in Chinese forestry.

The major dilemma for dynamic optimization of bamboo forests is that bamboo has always been ignored in the US due to stronger interest in planting timber rich trees, low demand for bamboo shoots, and very limited history cultivating bamboo forests. Only very few literatures about bamboo have been done focusing on bamboo management in the US, comparing to great amount of timber forest species such as fir and oak discussed in the former sections. Introduced as a potential bio energy crop but proved to be less productive, bamboo is still been considered as a woody grass with very constrained usage in the US, since there are more other out productive trees to be cultivated (Scurlock, Dayton, and Hames, 1999). To date, no previous study used dynamic optimization in bamboo forest management as it has been done for Douglas fir or oak, which are more popular and traditional tree species for US.

4. Dynamic Model of Moso Bamboo Management

We solve for the optimal bamboo stem harvest and bamboo shoot harvest policy using a numerical dynamic model that nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem. The inner finite-horizon within-year daily dynamic programming problem captures daily bamboo shoot growth within a year. The outer finite-horizon between-year annual dynamic programming problem captures annual bamboo stem growth from year to year.

There are two periods in a year when shoots are harvested. During the winter, shoots remain dormant underground, and can be sold for higher price due to its texture. During the spring, shoots emerge from ground but have a lower price compared to shoots in the winter. Bamboo shoots grow into bamboo plants after around 55 days, not including the dormancy period of shoots underground during the winter (Su, 2012).

Our dynamic programming model will be a finite sequence of finite-horizon problems. In particular, we nest an inner dynamic optimization problem within an outer dynamic optimization problem. The inner dynamic optimization problem is a finite-horizon dynamic programming problem that runs from 0 to 55 days representing the bamboo shoot season in one year. Each finite horizon runs from 0-55 days, representing the bamboo shoot season in one year. Since the values for bamboo shoot age are 0-55 days, the values for day in year are 0-55 days, followed by the final bin for “Day in year” representing “56 or more days, including the winter season dormant period”, for a total of 57 bins.

Generally, Moso bamboo stems reach their maximum biomass at age 4-5 years (Zhang et al., 2014; Zhuang et al., 2015) and mature at age 5-6 years. In our numerical dynamic model, we allow bamboo managers the possibility of letting bamboo stem grow to age 10 years, well past their age of maximum biomass, if it is optimal for them to do so. Since it would be very economically inefficient to harvest bamboo stem after 10 years, however, we model bamboo stem growth with a finite horizon of 10 years. We therefore have a finite sequence of 10 one-year finite horizon problems. Thus, the outer dynamic optimization problem is a finite-horizon dynamic programming problem that runs from 0 to 10 years.

Because we have two time variables, year *y* and day-in-year *d*, in a finite horizon, nested backward iteration is applicable. We can think of the management of different age classes of bamboo that co-exist at the same time as a (possibly infinite) set of these finite-horizon dynamic programming problems (each representing a finite sequence of finite-horizon problems), one for each set of bamboo emerged as shoots at the same time, but possibly with spillovers across these problems.

We start by modelling the harvesting of bamboo that emerged from rhizomes at the same time (and therefore of the same age class). The control (action) variables are the bamboo shoots harvest decision a_s and the bamboo stem harvest decision a_b. The state variables are the number of bamboo stem per hectare n_b and the number of bamboo shoots per hectare n_s. The bamboo biomass for an individual bamboo plant is given by the bamboo biomass Y_b. The bamboo shoot biomass for an individual bamboo shoot is given by the bamboo shoot biomass Y_s. The time variables are year $y = 0,...,Y$ and day-in-year $d = 0,...,D$, where we set $Y = 10$ and $D = 56$. The final bin for day-in-year, $D = 56$, represents “56 or more days, including the winter season dormant period”, for a total of 57 bins.

Since during the dormant period nothing will be growing, we do not need to model any growth of bamboo shoots during the dormant period, and therefore do not need to model the specific day during the winter dormant period that any winter bamboo shoot harvesting takes place. So we will lump the entire winter dormant period into one value (bin) for day-in-year, which is the final bin of day-in-year.

The value function, which is the present discounted value (PDV) of the entire stream of per-period payoffs when the bamboo shoot harvest and bamboo stem harvest decisions are chosen optimally, is given by the following Bellman equation:

$$V(s,d,y) = \max_{a_b, a_s} \pi(a_s, a_b, s) + \beta E[V(s', d', y') \mid s, a_s, a_b, d, y].$$

Since we nest a finite-horizon within-year dynamic problem within an outer annual dynamic problem, we use two different discount factors: a daily discount factor β_d and an annual discount factor β_y. We set the annual discount factor to be $\beta_y = 0.9$. We set the daily discount factor to be $\beta_d = \exp(\ln(\beta_y)/(D+1)) = 0.9981$, will yield an annual discount rate of 0.9 over 56 days.

Since we are assuming that the same bamboo plant can be harvested for shoots and/or later for bamboo, it suffices to declare one constant to be the initial number of bamboo plants (stems) per hectare. In general, there will be around 1,750 to 3,000 bamboo stem per hectare in Zhejiang province. To make our dynamic programming problem easier to solve, we discretize action variables as proportions of the initial number of bamboo plants (stems) per hectare. To make our dynamic programming problem easier to solve, a_s is expressed as a percentage of n_{b0}, the initial number of bamboo plants (stems) per hectare, for simplicity using dynamic optimization. The bamboo shoots harvest decision a_s thus ranges from harvesting 0% to 100% of the initial number n_{b0} of bamboo stem per hectare, in units of 10%. To make our dynamic programming problem easier to solve, a_b is expressed as a percentage of n_{b0}, the initial number of bamboo plants (stems) per hectare, for the same reason. Since the harvest of bamboo stem is unlikely to exceed more than half of the total number of bamboo plants, the bamboo stem harvest decision a_b ranges from harvesting 0% to 50% of the initial number of bamboo plants per hectare, in units of 10%.

Since the number of bamboo stem can only change via the bamboo stem harvest decision a_b, and since a_s is in units of 10% of n_{b0}, then the number of bamboo plants per hectare n_b ranges from 0 to n_{b0}, in units of (n_{b0}/ 10). Within any given year the number of shoots changes via shoots harvest decision a_s, and a_s is in units of 10% of n_{b0}, then the number of bamboo shoots per hectare n_s ranges from 0 to n_{b0}, in units of (n_{b0} / 10).

For the transition density for number of bamboo plants, the number of bamboo can only change via the bamboo stem harvest decision a_b , which can decrease the number of bamboo plants by a percentage of the initial number of bamboo plants. The number of bamboo can only change via the bamboo stem harvest decision a_s, which can decrease the number of bamboo plants by a percentage of the initial number of bamboo plants n_{b0}. Given a_b, we obtain the next period's number of bamboo plants by subtracting a_b from this period's number of bamboo plants. Since both number of bamboo plants and a_b are in the same units (i.e., 10% of initial number of bamboo plants), next period's number of bamboo plants is given by:

$$
\begin{aligned}
n_{b,y,d+1} &= \max\{n_{b,y,d} - a_{b,y,d}, 0\} \\
n_{b,y+1,0} &= \max\{n_{b,y,D} - a_{b,y,D}, 0\}.
\end{aligned}
$$

For the transition density for number of bamboo shoots within a year: during each year, the number of bamboo shoots will change via the bamboo shoots harvest decision a_s, which can decrease the number of bamboo plants by a percentage of the initial number of bamboo plants n_{b0}. Since both number of bamboo shoots and a_s are in the same units (i.e., 10% of initial number of bamboo plants) each bin of a_s decreases the number of bamboo shoots by a bin. Thus, the number of bamboo plants the next day this year is given by:

$$n_{s,y,d+1} = \max\{n_{s,y,d} - a_{s,y,d}, 0\}.$$

The transition density for number of bamboo shoots between years is more complicated. Li et al. (2016) find that Moso bamboo shoots emergence is positively related to bamboo stem density: with a higher bamboo stem density, there are more rhizomes underground, and thus a larger possibility for bamboo shoots emergence. According to Zhang and Ding (1997), new bamboo shoots emergence is positively correlated with Leaf Area Index (LAI) and positively correlated with precipitation in July, August, September over the previous year, which is the period of bamboo shoots formation. LAI is defined as leaf area/ground area, and thus is an index that characterize the bamboo stem density of a Moso bamboo stand. The number of bamboo shoots at the beginning of the year depends on the number of remaining bamboo plants at the beginning of the year (remaining after bamboo stem are harvested the previous year), since remaining bamboo plants affect density of the bamboo stand. When bamboo stem are harvested, which means it is going to be an off year next year, number of shoots next year is going to be less next year since bamboo stand developing more for leaves and rhizomes during an off year.

We incorporate the positive correlation between number of shoots and bamboo plants by subtracting this year's bamboo harvest (which is equal to 0 and does not affect number of bamboo shoots if it is within the year and bamboo harvest has not taken place). We implement the positive correlation between number of bamboo shoots at the beginning of the year and on the number of remaining bamboo plants at the beginning of the year (remaining after bamboo stem are harvested the previous year) as follows:

$$n_{s,y+1,0} = \max\{n_{b,y+1,0}, 0\} \ .$$

Later, in the stochastic model, we add the positive correlation with the precipitation during the months of July, August, and September from the previous year.

There are multiple available models to measure the growth and productivity of a Moso bamboo plant. Allometric equations and logistic functions have been used for characterizing bamboo growth. An allometric model predicts biomass using diameter at breast height. Biological studies suggest using the Chapman-Richards model (Richards, 1959), which is flexible growth model for plants, and has been used to predict Moso bamboo height (Yen, 2016). The Chapman-Richards model for bamboo stem growth for an individual bamboo plant is given by:

$$Y_b = A_b * \left(1 - Q_b * \exp\left(-\alpha_b * t_b\right)\right)^{1/(1-v_b)},$$

where Y_b is the total above ground biomass for bamboo, A_b is the maximum possible bamboo biomass for an individual bamboo plant, α_b is related to growth rate for bamboo, t_b is the age of bamboo in years, and v_b determines curve shape and the location of inflection point of bamboo stem growth. Since Q_b is the intercept of the growth model, Q_b is set to one in order to satisfy a biomass of zero when age is zero. A_b and t_b can be obtained from data. The Chapman-Richards model for bamboo stem growth yields the following equation of motion for bamboo stem biomass:

$$\frac{dY_b(t_b)}{dt_b} = \frac{\alpha_b}{1-v_b} A_b Q_b \left(1 - Q_b \exp(-\alpha_b t_b)\right)^{\frac{1}{1-v_b}-1} \exp(-\alpha_b t_b),$$

which can be written in discrete time as:

$$Y_b(t_b+1) - Y_b(t_b) = \frac{\alpha_b}{1-v_b} A_b Q_b \left(1 - Q_b \exp(-\alpha_b t_b)\right)^{\frac{1}{1-v_b}-1} \exp(-\alpha_b t_b)\,.$$

In addition to a model for bamboo stem growth, we also need a model for bamboo shoot growth. Bamboo shoot biomass accumulation has been described using logistic curve (Zhou, 1998). As the literature constructing a growth model for bamboo shoots is sparse, as Chapman-Richards model is a generalized logistic curve, and since bamboo shoots are young bamboo plants, we adopt and separately parameterize a Chapman-Richards model for bamboo shoot growth as well. The Chapman-Richards model for bamboo shoot growth for an individual bamboo plant is given by:

$$Y_s = A_s * \left(1 - Q_s * \exp\left(-\alpha_s * t_s\right)\right)^{1/(1-v_s)},$$

where Y_s is the total biomass for shoots, A_s is the maximum possible bamboo shoot biomass for an individual bamboo plant, α_s is related to growth rate for bamboo shoots, t_s is the age of bamboo shoots in days, and v_s determines curve shape and the location of inflection point of bamboo shoots growth (Zhang and Li, 2003). Since Q_s is the intercept of the growth model, Q_s is set to one in order to satisfy a biomass of zero when age is zero. The Chapman-Richards model for bamboo shoot growth yields the following equation of motion for bamboo shoot biomass:

$$\frac{dY_s(t_s)}{dt_s} = \frac{\alpha_s}{1-v_s} A_s Q_s \left(1 - Q_s \exp(-\alpha_s t_s)\right)^{\frac{1}{1-v_s}-1} \exp(-\alpha_s t_s)\,,$$

which can be written in discrete time as:

$$Y_s(t_s+1) - Y_s(t_s) = \frac{\alpha_s}{1-v_s} A_s Q_s \left(1 - Q_s \exp(-\alpha_s t_s)\right)^{\frac{1}{1-v_s}-1} \exp(-\alpha_s t_s)\,.$$

The equation for bamboo biomass is the equation for bamboo biomass for an individual bamboo plant that has not yet been harvested for bamboo stem. It therefore should not be a function of any of the action variables. It also does not depend on the current bamboo biomass, but only on time. We do not need to discretize or round Y_b, since it is not a state variable. Bamboo stem biomass Y_b ranges from 0 to 22 kg.

Similarly, the equation for bamboo shoots biomass is the equation for bamboo shoots biomass for an individual bamboo plant that has not yet been harvested for bamboo shoots. It therefore should not be a function of any of the action variables. It also does not depend on the current bamboo shoots biomass, but only on time. We do not need to discretize or round Y_s, since it is not a state variable. Bamboo shoot biomass Y_s ranges from 0 to 1 kg, in units of 0.1 kg.

The per-period payoff function is a function of state variables, action values, day in year, and year. In our numerical model, we assume a constant price for bamboo stem and a constate price for bamboo stem, assumptions which we will relax in our dynamic structural econometric model. We assume that the daily fixed cost is c_0. This includes the cost of maintenance.

Since this is a finite horizon problem, the value functions and policy functions are functions of both measures of time. There are 2 nested dynamic programming problems: The inner backwards iteration is that for each year, there is a finite-horizon dynamic programming problem whose last time period is the final bin for "day in year" representing "56 or more days, including the winter season dormant period", after which no more bamboo shoot or bamboo stem harvesting decisions can be made in that year. The outer backwards iteration is that there are 10 possible years during which one can make bamboo stem and/or bamboo shoot harvesting decisions. The outer backwards iteration gives the value function by year for the first day (day 0) in each year.

The terminal condition for the outer backwards iteraction is that the value function for the first day (day 0) of year "10 + 1" is 0. In other words, there is no continuation value after year 10. The terminal condition for the inner day-in-year backwards iteration is that, except in the last year, when there is no continuation value after the last day of the last year, the continuation in the last day of the year is the expected value of the value function on the first day (day 0) of the next year. We can think of the management of different age classes of bamboo that co-exist at the same time as a (possibly infinite) set of these finite-horizon dynamic programming problems (each representing a finite sequence of finite-horizon problems), one for each set of bamboo planted at the same time, but possibly with spillovers across these problems. But for now, we start by modeling the harvesting of bamboo that was all planted at the same time (and therefore of the same age class). From Xu et al. (2017), there are on average 400 stems of bamboo in the same one-year age class per hectare.

Our per-period payoff function is:

$$\pi(s,a,d,y) = R_b(s,a,d,y) - C_b(s,a,d,y) + R_s(s,a,d,y) - C_s(s,a,d,y) - c_0,$$

where $R_b(s,a,d,y)$ is bamboo stem harvest revenue, $C_b(s,a,d,y)$ is bamboo stem harvest cost, $R_s(s,a,d,y)$ is bamboo shoot harvest revenue, $C_s(s,a,d,y)$ is bamboo shoot harvest revenue, and c_0 is a daily fixed cost.

Bamboo stem harvest revenue $R_b(s,a,d,y)$ is given by:

$$R_b(s,a,d,y) = p_b \tau a_b n_{b0} Y_b,$$

where p_b is the bamboo stem price and τ is a conversion coefficient to convert bamboo stem price and bamboo stem quantity $a_b n_{b0} Y_b$ to comparable units, as explained in more detail below.

Bamboo stem harvest cost $C_b(s,a,d,y)$ is given by:

$$C_b(s,a,d,y) = c_b \tau a_b n_{b0} Y_b \,,$$

where c_b is the unit cost of bamboo stem harvest and τ is a conversion coefficient to convert the unit cost of bamboo stem harvest and bamboo stem quantity $a_b n_{b0} Y_b$ to comparable units, as explained in more detail in below.

Bamboo shoot harvest revenue $R_s(s,a,d,y)$ is given by:

$$R_s(s,a,d,y) = p_s \tau a_s n_{b0} Y_s \,,$$

where p_s is the bamboo shoots price and τ is a conversion coefficient to convert bamboo shoots price and bamboo shoots quantity $a_s n_{b0} Y_s$ to comparable units, as explained in more detail below.

Bamboo shoot harvest cost $C_b(s,a,d,y)$ is given by:

$$C_s(s,a,d,y) = c_s \tau a_s n_{b0} Y_s \,,$$

where c_s is the unit cost of bamboo shoot harvest and τ is a conversion coefficient to convert the unit cost of bamboo shoot harvest and bamboo shoots quantity $a_s n_{b0} Y_s$ to comparable units, as explained in more detail in below.

The bamboo forest manager chooses the bamboo stem harvest strategy and the bamboo shoot harvest strategy to maximize the present discounted value of the entire strem of per-period payoffs.

In addition to a deterministic specification of our dynamic programming model, we also run our model allowing rain to be stochastic. In our stochastic rain specification: there are four state variables: the number n_b of bamboo stem per hectare, the number n_s of bamboo shoots per hectare, precipitation *precip*, and lagged precipitation *precip_lagged* . We discretize n_b into 11 units and n_s into 11 units, in units of 40. We discretize *precip* and *precip_lagged* into 2 values each: 0 (low) and 1 (high). The probability that precipitation is high, *rain_high_prob*, is a parameter in this model whose value we vary.

We implement the positive correlation of the number of bamboo shoots per hectare with precipitation during the months of July, August, and September of the previous year as a bin increment of 1 if *precip_lagged* is equal to 1 (high):

$$n_{s,y+1,0} = \min\{\max\{n_{b,y+1,0} + precip_{y-1,D}, 0\}, N_b\} \quad .$$

5. Parameter Values

5.1. Bamboo shoot price

According to data from the National Agricultural Products Business Information Public Service Platform operated by China's Ministry of Commerce, the bamboo shoots price on May 27, 2018 was ¥4.60/kg ($0.61/kg) in Jiaxing, Zhejiang province; and ¥16/kg ($2.50/kg) in Anshan, Liaoning province. Bamboo shoot prices varies vary for spring bamboo shoots and winter bamboo shoots. Winter bamboo shoots are younger bamboos than spring bamboo shoots, where the former were hidden in the ground and the latter are older shoots that grow above ground. Due to difficulties of locating and harvesting underground winter bamboo shoots, as well as popular preference over more tender taste, winter bamboo shoots has higher market price than spring bamboo shoots. According to data from National Agricultural Products Business Information Public Service Platform operated by China's Ministry of Commerce, bamboo shoots prices remain stable in Liaoning province, which is not a major source of bamboo, but volatile in Zhejiang province, one of the eight major provinces with bamboo forests. The bamboo shoot price in a representative market in Jiaxing in Zhejiang province varied from ¥3.06/kg to ¥24.75/kg in 2017. Bamboo shoot prices in Zhejiang province followed a similar pattern from 2014 to 2018, with the highest prices in the winter. Bamboo shoot prices are in the range of ¥2/kg to ¥40/kg, with highest price appearing in November and lowest price appearing in May in general. In our numerical model, we set the bamboo shoot price p_s to 20 ¥/kg.

5.2. Bamboo stem price

Unlike for bamboo shoots, daily data on bamboo stem price is not available from government operated databases. According to Wu and Cao (2016), the 2012 Moso bamboo stem price is ¥1.39/kg in Zhejiang province. Meng, Liu, and Wu (2014) find average bamboo stem price to be ¥0.79/kg.

The biomass we predicted using Chapman-Richards model cannot been directly used to calculate total revenue in per period payoff because shoots and stem price are recorded in yuan per kg, which is the weight that contains both biomass and water. We need coefficients that transfer biomass into kilograms for both bamboo forest products. The Chapman-Richard's model predicts biomass Y_b and Y_s in units of kilograms. The price we have are for weights rather than weights of biomass, which is dry weight. Since weight include water and biomass, we use a conversion coefficient τ to transfer biomass into its actual weight.

In our numerical model, we vary the bamboo stem price p_b from 1.4 ¥/kg to 40 ¥/kg.

5.3. Bamboo forest maintenance cost, harvesting costs, and planting costs

Bamboo management and harvest costs vary for different provinces. According to a recent study using randomized data from representative sampling in three provinces, the major costs of bamboo forest are labor costs, along with a minor cost of fertilizer (Wu and Cao, 2016). The average labor cost in Zhejiang province is ¥125 per worker per day. One hectare Moso bamboo is estimated to cost ¥4100 annually on labor. Other important information from their study is conclusions on input-output relationship in bamboo forest management. Increasing fertilizer usage significantly boosts bamboo shoots yields, while has little effect on stems output. This is not true for increase in labor input, which leads to higher stems and shoots quantity at the same time (Wu and Cao, 2016). Fertilizer costs are pure maintenance costs, while labor costs are consists of maintenance costs and harvesting costs.

Land area is not the only factor affecting harvesting costs. Land quality such as slope of land affects cost of bamboo stems harvest (Wu and Cao, 2016; Dong et al., 2015). Both costs depend on area of land quality of land, and thus are functions of land area and land quality, where land quality can be characterized using slope and soil quality.

Meng, Liu and Wu (2014) specifiy detailed labor hours for bamboo forest maintenance, stems harvest, shoots harvest, and plantation. Labor hours differ for timber forest and timber-shoots forest, where the former aims at bamboo stem production and the latter has two major products. All estimates are based on 395 bamboo plants per hectare in below.

Maintenance costs vary in the range of 5.4 to 14.4 work-day per hectare annually, with average labor cost of ¥125/work-day in Zhejiang province. Average maintenance cost per hectare in Zhejiang province is estimated to be ¥675/ha to ¥1,800/ha annually. The difference in costs is due to age of bamboo forests. In their study, "forest formation year" is used to name the ninth year when maintenance costs decreases sharply. Intensive labor is ideal before bamboo forest formation year, and decreases sharply after forest formation year. Average shoots harvesting costs are estimated to be 1.9 work-day per hectare annually, which is ¥237.5/ha per year. Bamboo stem harvesting cost is 14 work-day per hectare annually, which is ¥1,750/ha per year. Bamboo plantation cost includes land finishing costs, planting labor costs, and bamboo mother plants costs, accordingly account for 39.4, 30.9 work-day per hector, and ¥15 per mother plant. These add up to a ¥14,712.5/ha one-time cost. (Meng, Liu and Wu, 2014)

All bamboo forests included in our data are naturally grown from long ago rather than planted, and thus planting cost is zero

According to Mr. Jianping Pan, who is the manager of Fumin Bamboo Shoot Specialized Cooperative, bamboo harvest can be fast, one worker can harvest 1 mu (about 667 square meters) of bamboo per day. For bamboo stem, workers get paid daily with a rate of 300 yuan per day and harvest 1,250 to 2,000 kg of bamboo stem. For spring shoots, workers got paid daily, with a rate of 150 to 180 yuan per day. Winter shoots are more expensive and harder to find than spring shoots, and thus workers get paid for 300 yuan per day and can harvest about 15 to 20 kg per day. Bamboo forest has its major cost of fertilizer, which cost 1000 yuan per year, with an additional cost of 2,000 yuan on labor to spread fertilizer. Bamboo stem density ranges from 1,750 to 3,000 per ha for different areas.

For the estimates for harvesting costs in our numerical model, we specify the unit cost c_s of bamboo shoot harvest for one hectare of shoots as 1.9 labor-day per year, and the unit cost c_b of bamboo stem harvest for one hectare of stems as 14 labor-day per year, with a flat rate of 125 ¥/work-day.

5.4. Parameters in Chapman-Richards model of bamboo stem growth

For the maximum possible bamboo stem biomass A_b for a single bamboo plant in the specific area, the maximum possible bamboo biomass for a single bamboo plant depends on land quality such as slope, precipitation, soil type, and temperature of the bamboo field we are interested in. Different bamboo fields have different carrying capacity. In Yen (2016), the maximum bamboo height is revealed for different diameter at breast height (DBH) Moso bamboo groups. As we are interested in not only height growth but also diameter increases, however, we use biomass to more comprehensively capture growth in bamboo stems. Yen (2016) calculated maximize stem biomass for Moso bamboo in central Taiwan to be 15.88 kg per plant with standard deviation of 2.51 kg, taking Moso bamboos under its 5th year growth into account. Zhang et al. (2014) find that the maximum stem biomass for an eight-year-old Moso bamboo has average biomass of 15.06 kg, with a standard deviation of 6.58 kg. Moso bamboo with longer age will have higher maximum stem biomass, while stem biomass accumulation slow down in the mature age for Moso bamboo, which is generally 5-6 years after plantation age.

For the growth rate α_b for Moso bamboo, the growth rate for Moso bamboo differs with studies as well. In Yen (2016), the growth rate of Moso bamboo height is revealed to be in the rage of 0.0980 to 0.1006 per day in the first five years of growth. However, we need the range for biomass accumulation in our model. According to Xu et al. (2011), the major biomass accumulation occurred along with the fast elongation of bamboo stem in the early stage of bamboo growth. In the stage where first shoot shell detached and branch emergence, bamboo biomass tripled. To estimate the biomass accumulation rate for Moso bamboo, we compare bamboo stem biomass in different age groups. The growth rate is in the range of 0.060 to 0.196 per stage (2 years) in Zhang et al. (2014), and a further estimate of 0.03 to 0.098 per year on average.

For the age t_b of bamboo forest in years, since Moso bamboo generally matures after 5-6 years, it would be very economically inefficient to harvest it after 10 years. We will assume age of Moso bamboo to be in the range of 0 to 10 years for now.

For v_b, which determines when the bamboo stem reaches its maximum biomass, from Zhang et al. (2014), the maximum biomass accumulation occurs at 4-5 years during Moso bamboo growth period. Zhuang et al. (2015) found consistent result that the maximum biomass accumulation occurs after 4.62 year of growth for Moso bamboo in Fujian province. The value for it is calculated as 1-4.62/11=0.58.

5.5. Parameters in Chapman-Richards model of bamboo shoot growth

For A_s, the maximum possible bamboo shoots biomass for a single bamboo plant in the specific area: Shoots biomass is basically the dried weight of shoots. This is estimated to be around 0.98kg in Su et al. (2013) in Anhui province and around 0.8kg in Zhejiang province (Xu et al. 2011).

For the growth rate α_s for bamboo shoots, the growth rate for bamboo shoots is more rapid than that for bamboo stem (Song et al., 2016).

For the age t_s of bamboo shoots in days, bamboo shoots grow into bamboo plant after around 55 days (Shi et al., 2013). A more recent study reveals this date to be 35-40 days after bamboo shoots emerging from soil (Song et al., 2016). Due to its relatively short period of growth, age of bamboo shoots is measured in days rather than years.

For v_s, which determines when the bamboo shoot reaches its maximum biomass, according to Song et al. (2016), maximum growth occurs at around 24 to 30 days.

6. Results of Numerical Model

We solve the deterministic model for each of the following sets of parameter values for bamboo stem price p_b and bamboo shoot price p_s:

Specification 1: $p_b = 1.4$ and $p_s = 20$

Specification 2: $p_b = 5$ and $p_s = 20$

Specification 3: $p_b = 10$ and $p_s = 20$

Specification 4: $p_b = 15$ and $p_s = 20$

Specification 5: $p_b = 20$ and $p_s = 20$

Specification 6: $p_b = 25$ and $p_s = 20$

Specification 7: $p_b = 21$ and $p_s = 20$

Specification 8: $p_b = 30$ and $p_s = 20$

Specification 9: $p_b = 40$ and $p_s = 20$

For each of the nine Specifications for bamboo stem price and bamboo shoot price above, we solve for the value function, the bamboo stem harvest policy function, and the bamboo shoot harvest policy function, each as a function of the number of bamboo stem per hectare and the number of bamboo shoots per hectare. Since our dynamic model nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem, there is a separate value function and policy function for each day of each year.

For each of the nine Specifications for bamboo stem price and bamboo shoot price above, we simulate optimal trajectories for each action and state variable over 11 years starting form the following initial values for the state variables:

Simulation 1 (initial state 3): $n_b = n_{bo} = 400$ and $n_s = n_{bo} = 400$.

Simulation 2 (initial state 6): $n_b = n_{bo} / 2 = 200$ and $n_s = n_{bo} / 2 = 200$.

Simulation 3 (initial state 11): $n_b = n_{bo} / 5 = 80$ and $n_s = n_{bo} / 5 = 80$.

Figure 1 presents the results of our deterministic model for Specification 1: $p_b = 1.4$ and $p_s = 20$. Figure 2 presents the results of our deterministic model for Specification 7: $p_b = 21$ and $p_s = 20$. Figure 3 presents the results of our deterministic model for Specification 6: $p_b = 25$ and $p_s = 20$. Figure 4 presents the results of our deterministic model for Specification 8: $p_b = 30$ and $p_s = 20$. Figure 5 presents the results of our deterministic model for Specification 9: $p_b = 40$ and $p_s = 20$.

For the value functions, each graph represents a different year, and within each graph we plot the value function for each day of the year in a different color ranging from red for the first day in that year to blue for the last day in that year. Similarly, for the policy functions, each graph represents a different year, and within each graph we plot the policy function for each day of the year in a different color ranging from red for the first day in that year to blue for the last day in that year.

According to our results, the value function increases dramatically with number of bamboo stem per hectare, but the slope declines with year. The value function increases with number of bamboo shoots per hectare, but shifts down with year. The value of value function increase with day of year, except in last days of each year (when is also when policy function has actions (harvests) being taken).

In earlier years, there are trade-offs between number of bamboo per hectare and individual maximum possible biomass since they will compete for nutrition (Feng, Gong, and

Wu, 2012), The trade offs will be between bamboo density and total maximum possible biomass, not individual biomass because number of shoots can also play a role in here.

In later years, the value function increases with number of bamboo per hectare because the most rapid stem biomass accumulation takes place in forth or firth year of Moso bamboo growth and slows down since then (Su, 2012). Holding number of shoots per hectare fixed, maximize bamboo density is a good way to increase value of value function ip to some density (Yu, 2011).

According to the results for the bamboo stem harvest policy function for the deterministic model, in all but the last year, optimal bamboo harvest is 0 until the later days of each year. When relative price of bamboo stem is high enough (and higher than bamboo shoot price; e.g., Specification 7: $p_b = 21$ and $p_s = 20$; Specification 6: $p_b = 25$ and $p_s = 20$), then the optimal bamboo harvest may be non-zero in an earlier year (e.g., second to last year).

According to the results for the bamboo shoot harvest policy function for the deterministic model, the bamboo shoots harvest does not depend on number of bamboo stem. The bamboo shoot harvest occurs towards the end of each year, and is an increasing function of the number of bamboo shoots.

According to the optimal trajectories figures for the deterministic model, the optimal trajectories of bamboo stem harvest is to harvest bamboo stem on the first day (day 0) of the last year. When the relative price of bamboo stem is high enough (and higher than bamboo shoot price; e.g., Specification 7: $p_b = 21$ in Figure 2 and $p_s = 20$; Specification 6: $p_b = 25$ and $p_s = 20$ in Figure 3), then the optimal bamboo stem harvest may be non-zero in an earlier year (e.g., harvest on the first day (day 0) of second to last year).

The optimal trajectories of bamboo shoot harvest is to harvest all the bamboo shoots at the end of each year. When the relative price of bamboo stem is high enough (and higher than bamboo shoot price; e.g., Specification 7: $p_b = 21$ and $p_s = 20$ in Figure 2; Specification 6: $p_b = 25$ and $p_s = 20$ in Figure 3) so that the optimal bamboo stem harvest may be non-zero in an earlier year (e.g., harvest on the first day (day 0) of second to last year), then if there is no bamboo stem remaining at the beginning of the years after harvest (e.g., the last year), then no bamboo shoots can be harvested then either.

These optimal trajectories suggest that since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year, and since bamboo stem continue to grow each year until age 4-5 years, while bamboo shoots only grow within a year, it is optimal not to harvest any bamboo stem until the first day of the last year, and it is optimal to harvest all the bamboo shoots at the end of each year. When the relative price of bamboo stem is high enough (and higher than bamboo shoot price; e.g., Specification 7: $p_b = 21$ and $p_s = 20$ in Figure 2; Specification 6: $p_b = 25$ and $p_s = 20$ in Figure 3), then the optimal bamboo stem harvest may be non-zero in an earlier year (e.g., harvest on the first day (day 0) of second to last year), in which case there is no bamboo harvest or bamboo shoots harvest in the last year, so there are no bamboo and no bamboo shoots in the last year to harvest.

The optimal trajectories suggest that since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year, and since bamboo stem continue to grow each year until age 4-5 years, while bamboo shoots only grow within a year, it is optimal not to harvest any bamboo stem until the first day of the last year, and it is optimal to harvest all the bamboo shoots at the end of each year.

When relative price of bamboo stem is high enough (and higher than bamboo shoot price; e.g., Specification 7: $p_b = 21$ and $p_s = 20$ in Figure 2; Specification 6: $p_b = 25$ and $p_s = 20$ in Figure 3), then the optimal bamboo stem harvest may be non-zero in an earlier year (e.g., harvest on the first day (day 0) of second to last year), in which case there is no bamboo stem harvest or bamboo shoots harvest in the last year, so there are no bamboo stem and no bamboo shoots in the last year to harvest.

Reasons to wait many years to harvest bamboo stem are as follows. First, bamboo stem harvest is irreversible; once harvested bamboo stem do not grow back unless new bamboo stem are planted. Second, bamboo shoots do not contribute to future bamboo stem, so once you harvest a bamboo stem, the number of bamboo stem per hectare will never go back up. Third, the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year, and bamboo shoots are profitable, so the more years we keep bamboo stem, the more years we will be able to harvest bamboo shoots. Fourth, the bamboo stem continue to grow each year until age 4-5 years, so for the first 4 years or so, the longer you wait, the more bamboo stem biomass you have per bamboo stem.

It is still optimal to wait to the last year to harvest bamboo when the price of bamboo stem is the same as the price of bamboo shoots (e.g., Specification 5: $p_b = 20$ and $p_s = 20$). It is not enough for bamboo stem to be same price as bamboo shoot, since harvesting bamboo stem early means sacrificing multiple years of future bamboo shoots.

When the relative price of bamboo stem is high enough (and higher than bamboo shoot price; e.g., Specification 7: $p_b = 21$ and $p_s = 20$ in Figure 2; Specification 6: $p_b = 25$ and $p_s = 20$ in Figure 3), then the optimal bamboo stem harvest may be non-zero in an earlier year, since the net benefit from harvesting now and getting the higher profits from the high bamboo stem price ends up outweighing the net benefits from waiting to harvest in order to get more years of bamboo shoot harvest, especially since bamboo stem biomass reaches its maximum and stops increasing after 4-5 years.

Reasons to harvest bamboo stem on the first day of the (last) year are as follows. The number of bamboo shoots that year only depend on the number of bamboo stem on the first day of the year, so keeping bamboo stems any longer will not affect profits from bamboo shoots. Likewise, if bamboo stem biomass growth within a year is negligible relative to bamboo stem biomass growth from year to year, especially after bamboo stem biomass growth has reached its maximum at age 4-5 years, and if bamboo stem prices do not vary within a year, then delaying bamboo stem harvests to a later day that year would not increase bamboo stem revenue but would incur a time cost of delay.

Reasons to harvest all the bamboo shoots on the last day of each year are as follows. First, there is no benefit of keeping bamboo shoots past 1 year, since they will no longer be bamboo shoots next year, and since they do not affect the number of bamboo stem next year. Second, bamboo shoots only grow within a year. Third, bamboo shoots grow within a year, so if you wait until the last day of the year, you get the most bamboo shoot biomass possible. Fourth, bamboo shoots are a renewable resource, since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year, thus it is OK to deplete all the bamboo shoots since you will have more next year as long as you have bamboo stem next year.

This optimal solution might also characterize other products that grow on trees that are renewable and can be harvested at more frequent intervals than the trees themselves. Examples include fruits, nuts, sap, and maple syrup.

For the stochastic model, we run the following sets of Specifications 1 and 4-9:

Version A: *rain_high_prob* = 0. This version is equivalent to our deterministic model.
Version B: *rain_high_prob* = 0.2

Version C: *rain_high_prob* = 0.5

Version D: *rain_high_prob* = 0.8

Version E: *rain_high_prob* = 1

We make 4 sets of all value function and policy function plots for Versions B-E:

Set 1: *precip* = 0, *precip_lagged* = 0

Set 2: *precip* = 0, *precip_lagged* = 1

Set 3: *precip* = 1, *precip_lagged* = 0

Set 4: *precip* = 1, *precip_lagged* = 1

Figure 6 presents the results of Specification 1, Version C, Set 1 of our stochastic model. Figure 7 presents the results of Specification 1, Version E, Set 1 of our stochastic model. Figure 8 presents the results of Specification 5, Version C, Set 1 of our stochastic model. Figure 9 presents the results of Specification 5, Version C, Set 2 of our stochastic model. Figure 10 presents the results of Specification 5, Version E, Set 1 of our stochastic model. Figure 11 presents the results of Specification 7, Version C, Set 1 of our stochastic model. Figure 12 presents the results of Specification 7, Version D, Set 1 of our stochastic model. Figure 13 presents the results of Specification 7, Version E, Set 1 of our stochastic model. Figure 14 presents the results of Specification 8, Version C, Set 1 of our stochastic model. Figure 15 presents the results of Specification 8, Version D, Set 1 of our stochastic model. Figure 16 presents the results of Specification 8, Version E, Set 1 of our stochastic model. Figure 17 presents the results of Specification 9, Version C, Set 1 of our stochastic model. Figure 18 presents the results of Specification 9, Version D, Set 1 of our stochastic model. Figure 19 presents the results of Specification 9, Version E, Set 1 of our stochastic model.

The optimal trajectories for the stochastic model suggest that since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year, and since bamboo stem continue to grow each year until age 4-5 years, while bamboo shoots only grow within a year, it is generally optimal not to harvest any bamboo stem until the first day of the last year, and to harvest all the bamboo shoots at the end of each year.

When relative price of bamboo stem is high enough (and higher than bamboo shoot price; e.g., Specification 7: $p_b = 21$ and $p_s = 20$; Specification 6: $p_b = 25$ and $p_s = 20$), then the optimal bamboo stem harvest may be non-zero in an earlier year (e.g., harvest on the first day (day 0) of second to last year), in which case there is no bamboo stem harvest or bamboo shoots harvest in the last year, so there are no bamboo and no bamboo shoots in the last year to harvest.

When rain_high_prob is high enough (e.g., Version C: *rain_high_prob* = 0.5; Version D: *rain_high_prob* = 0.8) then it is optimal to harvest a little bamboo stem on the first day of the second-to-last year, and harvest all the remaining bamboo stem on the first day of the last year.

When *rain_high_prob* is extremely high (e.g. Version E: *rain_high_prob* = 1) then it is optimal to harvest a little bamboo stem on the first day of an earlier year (e.g., 5th year), and harvest all the remaining bamboo stem on the first day of the last year

When *rain_high_prob* is high enough and relative price of bamboo stem is high enough, then will harvest a little bamboo stem on the first day of an earlier year, and harvest all the remaining bamboo stem on the first day of a subsequent year

Reasons to wait many years to harvest bamboo stem are as follows. First, bamboo stem harvest is irreversible; once harvested bamboo stem do not grow back unless new bamboo stem are planted. Second, bamboo shoots do not contribute to future bamboo stem, so once you harvest a bamboo stem, the number of bamboo stem per hectare will never go back up. Third, the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year, and bamboo shoots are profitable, so the more years we keep bamboo stem, the more years we will be able to harvest bamboo shoots. Fourth, bamboo stem continue to grow each year until age 4-5 years, so for the first 4 years or so, the longer you wait, the more bamboo stem biomass you have per bamboo stem.

It is still optimal to wait to the last year to harvest bamboo when the price of bamboo is the same as the price of bamboo shoots (e.g., Specification 5: $p_b = 20$ and $p_s = 20$). It is not enough for bamboo to be same price as bamboo shoot, since harvesting bamboo stem early means sacrificing multiple years of future bamboo shoots.

When the relative price of bamboo stem is high enough (and higher than bamboo shoot price; e.g., Specification 7: $p_b = 21$ and $p_s = 20$; Specification 6: $p_b = 25$ and $p_s = 20$), then the optimal bamboo stem harvest may be non-zero in an earlier year, since the net benefit from harvesting now and getting the higher profits from the high bamboo stem price ends up outweighing the net benefits from waiting to harvest in order to get more years of bamboo shoot harvest and more bamboo stem biomass harvest earlier, especially since bamboo stem biomass reaches its maximum and stops increasing after 4-5 years.

The reason to harvest bamboo stem on the first day of the (last) year is that the number of bamboo shoots that year only depends on the number of bamboo stem on the first day of the year, so keeping bamboo any longer will not affect profits from bamboo shoots. Likewise, if bamboo stem biomass growth within a year is negligible relative to bamboo stem biomass growth from year to year, especially after bamboo stem biomass growth has reached its maximum at age 4-5 years, and if bamboo stem prices do not vary within a year, then delaying bamboo stem harvests to a later day that year would not increase bamboo stem revenue but would incur a time cost of delay.

Reasons to harvest all the bamboo shoots on the last day of each year are as follows. First, there is no benefit of keeping bamboo shoots past the first year, since they will no longer be bamboo shoots next year, and since they do not affect the number of bamboo stem next year. Second, bamboo shoots only grow within a year. Third, bamboo shoots grow within a year, so if you wait until the last day of the year, you get the most bamboo shoot biomass possible. Fourth, bamboo shoots are a renewable resource, since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year, thus it is OK to deplete all the bamboo shoots since you will have more next year as long as you have bamboo stem next year.

The reason to harvest a little bamboo stem in an earlier year when *rain_prob_high* is enough is as follows. When there is a high probability of rain, then possible to still have many bamboo shoots in next and subsequent years even if some bamboo stem have been harvested. Thus, bamboo managers may benefit from earlier profit from some bamboo harvest without forgoing too much bamboo shoot harvest in subsequent years.

This optimal solution might also characterize other products that grow on trees that are renewable and can be harvested at more frequent intervals than the trees themselves. Examples include fruits, nuts, sap, and maple syrup.

7. Comparing Optimal Bamboo Management with Actual Harvest Decisions

We compare the optimal bamboo stem harvest and bamboo shoot thinning policy as given by our numerical dynamic model with actual data on bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China.

7.1. Data on actual bamboo shoot and bamboo stem harvest

We collect data on actual bamboo shoot harvest and bamboo stem harvest decisions on 20m by 20m plots in Shanchuan Township and Sian Township in Zhejiang province in China. Our data set includes 20 plots in Sian Township over the years 2017 and 2018; 15 plots in Shanchuan Township in 2017, and one plot in Shanchuan Township in 2018.

Our dataset includes the bamboo stem density for different sample plots in multiple years. However, number of shoots per hectare is seldomly reported unless under careful experimental design, whereas weights of shoots harvested is a commonly recorded variable. Our estimation of the number of shoots per hectare relies on converting the total weight of bamboo shoots into number of bamboo shoots per hectare and shoots harvest decisions, which we define to be proportion of shoots harvested of maximum possible shoots weights. The dilemma here is that we observe weight of shoots harvested, but do not observe either total amount of shoots or number of harvested shoots.

Ideally, we would like to convert the bamboo shoots harvested data and any estimated weight of bamboo shoots into the proportion of bamboo shoots harvested. Even though we can estimate the total possible weight of bamboo shoots, the actual weight of bamboo shoots would be different if some bamboo shoots were previously harvested that season. In addition, we cannot simply subtract the weight of bamboo shoots harvested earlier in the season from our estimate the total possible weight of bamboo shoots as a function of bamboo stems (culm), since those bamboo shoots that were harvested earlier in the season would have grown or changed in weight if they had not been harvested. So it would be ideal if we made the harvesting decision in terms of proportion of bamboo shoots harvested, so that we can model the weight and change in weight of the remaining bamboo shoots.

We estimate the unobserved bamboo shoot state and control variables as follows. First, for each plot and each day, we convert the weight of bamboo shoots harvest into the number of bamboo shoots harvested per hectare by dividing the weight of bamboo shoots harvest by the bamboo shoot biomass per bamboo shoot that day of the year from Chapman-Richard's model for bamboo shoot growth, assuming that bamboo shoots start growing from the beginning of winter shooting; and then multiplying by 25 to convert from the 0.04 hectare plots to 1 hectare.

We then impute the maximum number of bamboo shoots in the ground in the absence of bamboo shoot harvest for each sample plot in each bamboo growth year. To do so, we apply the following model from Zheng (1998) to estimate the weight of bamboo shoots in the ground that remain after all the bamboo shoots have been harvested that season:

$$w_b = 0.0018 * d_b^{\ 2.8637} \ ,$$

where w_b is weight of an individual bamboo shoot and d_b is its maximum diameter. As we do not have data on the maximum diameter of bamboo shoots, we use data on the diameter at breast height (DBH) of each newly grown bamboo stem that year to represent the diameter at breast height of bamboo shoots if they were to grow until the end of that season. For each sample plot and each year in our data set, we use data on the diameter at breast height (DBH) of newly grown bamboo stem, representing the diameter at breast height of bamboo shoots if they were to grow until the end of that season, to estimate the weight of a bamboo shoot if were to grow until the end of the season. Then, for each sample plot and each year, to calculate the weight of bamboo shoots per hectare on this sample plot that are not harvested, we take the sum over all the newly grown bamboo stems of the respective weights of a bamboo shoot if were to grow until the end of the season for that sample plot in that year. We convert the weight of bamboo shoots that are not harvested by the end of the season into the number of bamboo shoots that are not harvested per hectare by dividing the weight of bamboo shoots not harvested by the bamboo shoot biomass per bamboo shoot from Chapman-Richard's model for bamboo shoot growth, assuming that the unharvested bamboo shoots must have grown from the beginning of winter shooting until the last day of spring shooting, and then multiplying by 25 to convert from the 0.04 hectare plots to 1 hectare.

For each plot, to calculate the number n_s of bamboo shoots per hectare at the beginning of the season, in the absence of any bamboo shoots harvest, we add the total number of bamboo shoots harvested over the season to the total number of bamboo shoots that remain unharvested at the end of the season.

For each day on each sample plot, we calculate the bamboo shoots harvest action variable a_s as the number of shoots harvested per acre that day on that sample plot by the number n_s of bamboo shoots per hectare on that sample plot at the beginning of the season, in the absence of any bamboo shoots harvest.

We then calculate the number n_s of bamboo shoots per acre for each day on each sample plot as the number n_s of bamboo shoots per acre on that sample plot the previous day that season minus the number of bamboo shoots per acre harvested on that sample plot on the previous day that season.

7.2. Comparing actual with optimal

Figure 20 presents the actual data on the number of bamboo stem harvested on each sample plot in Zhejiang province in our data set. Time series plots of the number of bamboo stem harvested on each sample plot in Sian Township are in blue. Time series plots of the number of bamboo stem harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) and April 30 (last day of spring shooting) of each year.

As seen in Figure 20, most of the bamboo stem harvesting in Shanchuan Township (in green) takes place in the first few months of winter shooting and at the start of spring shooting of the second (and last) bamboo growth year of our data set. The harvesting of bamboo stem at the start of shooting is consistent with our dynamically optimal solution to harvest any bamboo stem until the first day of the last year, since the number of bamboo shoots that bamboo growth year only depends on the number of bamboo stem on the first day of the bamboo growth year, so keeping bamboo any longer will not affect profits from bamboo shoots.

In Sian Township, however, the bamboo stem harvesting (in blue) takes place at end of the last year of the data set. Whether the bamboo stem is harvested at the beginning or end of the last year would not affect the number or profits from bamboo shoots. Delaying the bamboo stem harvest in the last year from the beginning to the end of the year would only by dynamically optimal if the growth in bamboo stem biomass and any increase bamboo stem price over the course of the last year outweighed the time cost of delaying the profits from harvesting bamboo stem at the beginning of the last year.

Figure 21 presents time series plots of the cumulative fraction of bamboo stem harvested by age of bamboo on each sample plot. No bamboo stems aged 0-2 are harvested, which is consistent with the dynamically optimally policy of waiting until the last bamboo growth years to harvest bamboo stem. All the bamboo stems of age 6 are harvested in sample plots in Sian Township during the last bamboo growth year of our data set, which is consistent with the dynamically optimally policy of waiting until the last bamboo growth years to harvest bamboo stem, but on some plots the bamboo stem harvest takes place after spring shooting, rather than at the beginning of shooting. On sample plots in Shanchuan Township, the majority of the bamboo stem of age 5 are harvested in the last bamboo growth year of our data set, either during the first third of winter shooting or at the beginning of spring shooting, which is consistent with the dynamically optimally policy of waiting until the first day of one of the last bamboo growth years to harvest bamboo stem.

Figure 22 presents the actual data on weight of bamboo shoots harvested on each sample plot in Zhejiang province in our data set. Time series plots of the weight of bamboo shoots harvested on each sample plot in Sian Township are in blue. Time series plots of the weight of bamboo shoots harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) and April 30 (last day of spring shooting) of each year.

As seen in Figure 22, bamboo shoots are harvested about a third of the way into the winter shooting season, and then during the latter two-thirds of the spring shooting season. In contrast, our dynamic model suggests that it is optimal to harvest all the bamboo shoots on the last day of the shooting season, rather than harvest bamboo shoots before the last day of the shooting season. It is possible that the bamboo managers are weighing the benefits of an increased bamboo shoot biomass from waiting to harvest with any foregone profits from a particularly high bamboo shoot price in middle of the season. It is also possible that bamboo managers worry that flooding the bamboo shoot market at the end of the season may lead to a lower bamboo price than the price they would receive from spreading their harvest over a longer interval of time.

Figure 23 presents time series plots of the cumulative fraction of bamboo shoots harvested, as imputed above, on each sample plot. On all the sample plots in Shanchuan Township (in green), at least 80 percent of the bamboo shoots are harvested by the end of the spring shooting season in the first bamboo growth year of our data set. This is nearly consistent with the dynamically optimal policy of harvesting all the bamboo shoots by the end of the season, although our model suggests that they should harvest all of their bamboo shoots. During the second bamboo growth year of our data set, however, no bamboo shoot harvesting is done on the sample plots in Shanchuan Township, which is contrary to the dynamically optimal policy of harvesting all the bamboo shoots by the end of the season.

On all the sample plots in Sian Township (in blue), at least 90 percent of the bamboo shoots are harvested by the end of the spring shooting season in the first bamboo growth year of our data set. This is nearly consistent with the dynamically optimal policy of harvesting all the bamboo shoots by the end of the season, although our model suggests that they should harvest all of their bamboo shoots. During the second bamboo growth year, some sample plots in Sian Township similarly harvest at least 87 percent of the bamboo shoots by the end of the spring shooting season, which again is nearly consistent with the dynamically optimal policy of harvesting all the bamboo shoots by the end of the season. On several other sample plots in Sian Township, however, bamboo shoot harvests during the second bamboo growth year only take place during the first half of the winter shooting season, and at least 50 percent of the bamboo shoots remain unharvested by the end of the spring shooting season. Harvesting bamboo shoots during the first half of the winter shooting season only is contrary to the dynamically optimal policy unless the price at the during the first half of the winter shooting season is extremely high, and high enough to outweigh any bamboo shoot growth during that second bamboo growth year. In addition, leaving bamboo shoots remaining at the end of the spring shooting season may be sub-optimal. We hope to improve our modeling of winter shooting in future work.

Thus, results of our comparison between the optimal bamboo stem harvest and bamboo shoot harvest given by our dynamic model with the data on actual bamboo stem harvests and bamboo shoos harvest is that actual bamboo stem and bamboo shoot harvests come close to approximating the optimal harvesting strategy, but have some features that differ from what our model suggests to be optimal.

8. Discussion and Conclusion

Moso bamboo forest management involves making decisions about the timing and quantity of bamboo stem harvests and bamboo shoot harvests. In this paper, we solve for the optimal bamboo stem harvest and bamboo shoot harvest policy using a numerical dynamic model that nests an inner finite-horizon within-year daily dynamic programming problem within an outer finite-horizon between-year annual dynamic programming problem. We use a Chapman-Richards growth function as our model for bamboo biomass accumulation.

The results of our numerical dynamic model suggest that since the number of bamboo shoots at the beginning of each year depend on the number of bamboo stem remaining at the beginning of each year and on whether the previous year was a high-precipitation year, and since bamboo stem continue to grow each year until age 4-5 years, while bamboo shoots only grow within a year, it is generally optimal not to harvest any bamboo stem until the first day of the last year, and to harvest all the bamboo shoots at the end of each year.

When we allow for uncertainty in rain, which increases the number of bamboo shoots in the following year, we find that when the probability of having a high-precipitation year is high enough, then it will be optimal to harvest a little bamboo stem on the first day of an earlier year, and harvest all the remaining bamboo stem on the 1st day of the last year. When the probability of having a high-precipitation year is high enough and relative price of bamboo stem is high enough, then it will be optimal to harvest a little bamboo stem on the first day of an earlier year, and harvest all the remaining bamboo stem on the first day of a subsequent year.

We compare the optimal bamboo stem harvest and bamboo shoot thinning policy with actual data on bamboo shoot and bamboo stem harvests on multiple bamboo plots in multiple townships in Zhejiang province in China. We find that the actual bamboo stem and bamboo shoot harvests come close to approximating the optimal harvesting strategy, but have some features that differ from what our model suggests to be optimal. Our results have important implications for bamboo forest management and, to the extent that some of the differences between actual harvests and optimal bamboo harvests reflect possible sub-optimal behavior on the part of Moso bamboo forest managers, for ways to improve Moso bamboo forest management and policy.

The remaining differences between actual harvests and optimal bamboo harvests may reflect features that we do not capture in our model, including variation in bamboo shoot price and/or bamboo stem price over time, particularly within a season; concerns that harvesting all the bamboo shoots at the same time may flood the market and lower the bamboo shoot price they receive; concerns that harvesting all the bamboo stem at the same time may flood the market and lower the bamboo stem price they receive; capacity and/or labor constraints on the number of bamboo stems that are feasible to harvest in one day; capacity and/or labor constraints on the number of bamboo shoots that are feasible to harvest in one day; liquidity constraints during the season that may lead bamboo managers to harvest some bamboo shoots or bamboo stem early; winter shooting; variation in age of bamboo stem in a bamboo forest; the possibility that unharvested bamboo shoots may become newly grown bamboo stem; carbon sequestration motives; alternative crops or uses of the land; and/or actual parameter values that differ from the ones we use in the model. In future work, we plan account for these considerations by estimating a dynamic structural econometric model using data on the actual bamboo shoot harvest, bamboo stem harvest, bamboo shoot price, and bamboo stem harvest, which will enable us to estimate the parameters econometrically.

If some of the differences between actual harvests and optimal harvests arise because of economic constraints such as liquidity constraints and/or labor constraints, it is possible that some of these constraints can be ameliorated by well-designed instituitions or policies.

Our results have important implications for Moso bamboo forest management in particular, and forest management more generally. Our optimal solution might also characterize other products that grow on trees that are renewable and can be harvested at more frequent intervals than the trees themselves. Examples include fruits, nuts, sap, and maple syrup.

References

Amidon, E. L., & Akin, G. S. (1968). Dynamic programming to determine optimum levels of growing stock. Forest Science, 14(3), 287-291.

Arimizu, T. (1958). Regulation of the cut by dynamic programming. Journal of the Operations Research Society of Japan, 1(4), 175-182.

Bare, B. B., & Waggener, T. R. (1980). Forest land values and return on investment. Forest Science, 26(1), 91-96.

Barnett, H. J., & Morse, C. (2013). Scarcity and growth: The economics of natural resource availability. RFF Press, 3(3), 550-552.

Bellman, R. (1954). The theory of dynamic programming. Bulletin of the American Mathematical Society, 60(6), 503-515.

Ben-Zhi, Z., Mao-Yi, F., Jin-Zhong, X., Xiao-Sheng, Y., & Zheng-Cai, L. (2005). Ecological functions of bamboo forest: research and application. Journal of Forestry Research, 16(2), 143-147.

Berck, P. (1981). Optimal management of renewable resources with growing demand and stock externalities. Journal of Environmental Economics and Management, 8(2), 105-117.

Blackwell, D. (1965). Discounted dynamic programming. The Annals of Mathematical Statistics, 36(1), 226-235.

Bowes, M. D. (1983). Economic foundations of public forestland management. Resources for the Future.

Brodie, J. D., & Kao, C. (1979). Optimizing thinning in Douglas-fir with three-descriptor dynamic programming to account for accelerated diameter growth. Forest Science, 25(4): 665-672

Brodie, J. D., Adams, D. M., & Kao, C. (1978). Analysis of economic impacts on thinning and rotation for Douglas-fir, using dynamic programming. Forest Science, 24(4), 513-522.

Brouwer, D. R., & Jansen, J. D. (2002, January). Dynamic optimization of water flooding with smart wells using optimal control theory. In European Petroleum Conference. Society of Petroleum Engineers.

Bystriakova, N., Kapos, V., Lysenko, I., & Stapleton, C. M. A. (2003). Distribution and conservation status of forest bamboo biodiversity in the Asia-Pacific Region. Biodiversity & Conservation, 12(9), 1833-1841.

Calish, S., Fight, R. D., & Teeguarden, D. E. (1978). How do nontimber values affect Douglas-fir rotations?. Journal of Forestry, 76(4), 217-221.

Chang, S., & Yeh, W. W. G. (1973). Optimal Allocation of Artificial Aeration Along A Polluted Stream Using Dynamic Programming. Jawra Journal of The American Water Resources Association, 9(5), 985-997.

Chang, S. (1982). An Economic Analysis of Forest Taxation's Impact on Optimal Rotation Age. Land Economics, 58(3), 310-323.

Chang, S. J. (1981). Determination of the optimal growing stock and cutting cycle for an uneven-aged stand. Forest science, 27(4), 739-744.

Chang, S. J. (1983). Rotation age, management intensity, and the economic factors of timber production: Do changes in stumpage price, interest rate, regeneration cost, and forest taxation matter?. Forest science, 29(2), 267-277.

Chaudhuri, K. (1988). Dynamic optimization of combined harvesting of a two-species fishery. Ecological Modelling, 41(1-2), 17-25.

Chen, C. M., Rose, D. W., & Leary, R. A. (1980). Derivation of optimal stand density over time—a discrete stage, continuous state dynamic programming solution. Forest science, 26(2), 217-227.

China Forestry and Grassland Administration. (2018). “Forest Resources in China.”

Clark, W. G. (1976). The lessons of the Peruvian anchoveta fishery. California Cooperative Oceanic Fisheries Investigations Reports, 19, 57-63.

Deegen, P., Hostettler, M., & Navarro, G. A. (2011). The Faustmann model as a model for a forestry of prices. European journal of forest research, 130(3), 353-368.

Démurger, S., Hou, Y., & Yang, W.. (2009). Forest management policies and resource balance in China: an assessment of the current situation. The Journal of Environment & Development, 18(1), 17-41.

Dixit, A. K., & Pindyck, R. S. (1994). Investment under uncertainty. Princeton University Press.

Dong, D., Tong, Y., Yi., X., Zhang., C., & Gu, M.. (2015) Harvesting Cost and Economics Efficiency Analysis of Bamboo Forests in Various Slope Hill Lands. Journal of Bamboo Research, 34(4), 23-27. [in Chinese]

Du, H., Zhou, G., Fan, W., Ge, H., Xu, X., Shi, Y., & Fan, W. (2010). Spatial heterogeneity and carbon contribution of aboveground biomass of moso bamboo by using geostatistical theory. Plant Ecology, 207(1), 131-139.

Evans, G.C. (1930), Mathematical Introduction to Economics. Mcgraw Hill Book And Co..

Fang, J. Q., & Xie, Z. (1994). Deforestation in preindustrial China: The Loess Plateau region as an example. Chemosphere, 29(5), 983-999.

Faustmann, M. (1849). Calculation of the Value which Forest Land and Immature Stands Possess for Forestry. General Journal of Forestry and Hunting. [in German]

Feng, J., Gong, Q., & Wu, H.. (2012). Research on the standing bamboo density in Phyllostachys pubescens forest.” Hunan Forestry Science and Technology, 39(4):37-40. [in Chinese]

Fogel, M. M., Duckstein, L., & Kisiel, C. C. (1976). Optimum control of irrigation water application. Journal of Hydrology, 28(2), 343-358

Fu, J. (2001). Chinese moso bamboo: its importance. Bamboo, 22(5), 5-7.

Fu, S., Xu, X., Wu, W.. (2015). Analysis on the determinants of Phyllostachys edulis’s input-output efficiency. Journal of Zhejiang A&F University, 32(4): 596-602. [in Chinese]

Gregersen, H. M. (1975). Effect of inflation on evaluation of forestry investments. Journal of Forestry, 73(9), 570-572.

Hall, D. O. (1983). Financial maturity for even-aged and all-aged stands. Forest science, 29(4), 833-836.

Hall, W. A., & Buras, N. (1961). The dynamic programming approach to water‐resources development. Journal of Geophysical Research, 66(2), 517-520.

Hall, W. A., Butcher, W. S., & Esogbue, A. (1968). Optimization of the operation of a multiple‐purpose reservoir by dynamic programming. Water Resources Research, 4(3), 471-477.

Hardin, G. (1968). The tragedy of the commons. science, 162(3859), 1243-1248.

Hardie, I. W., Daberkow, J. N., & McConnell, K. E. (1984). A timber harvesting model with variable rotation lengths. Forest Science, 30(2), 511-523.

Hartman, R. (1976). The harvesting decision whena standing forest has valuea. Economic inquiry, 14(1), 52-58.

Hotelling, Harold. (1931). The economics of exhaustible resources. Journal of Political Economy, 39 (2), 137-175.

Jackson, D. H. (1980). The microeconomics of the timber industry. Westview Press.

Jiang, Z. (2007). Bamboo and rattan in the world. Beijing: China Forestry Pub. House.

Katsigris, E., Bull, G. Q., White, A., Barr, C., Barney, K., Bun, Y., . . . Weyerhaeuser, H. (2004). The china forest products trade: Overview of asia-pacific supplying countries, impacts and implications. The International Forestry Review, 6(3/4), 237-253.

Kamien, M. I., & Schwartz, N. L. (2012). Dynamic optimization: the calculus of variations and optimal control in economics and management. Courier Corporation.

Kelman, J., Stedinger, J. R., Cooper, L. A., Hsu, E., & Yuan, S. Q. (1990). Sampling stochastic dynamic programming applied to reservoir operation. Water Resources Research, 26(3), 447-454.

Kilkki P., & Väisänen U. (1969). Determination of the optimum cutting policy for the forest stand by means of dynamic programming. Acta Forestalia Fennica, 102.

Klemperer, W. D. (1979). Inflation and present value of timber income after taxes. journal of Forestry, 77(2), 94-96.

Kirk, D. E. (2012). Optimal control theory: an introduction. Courier Corporation,.

Labadie, J. W., Grigg, N. S., & Trotta, P. D. (1974). Minimization of combined sewer overflows by large-scale mathematical programming. Computers & Operations Research, 1(3-4), 421-435.

Li, C, Zhou, G. M., Shi, Y. J., Zhou, Y. F., Xu, X.J., Zhang, Y. P., Fan, Y. Q., Shen, Z. M.. (2016). Effects of old bamboo forests and relevant management measures on growth of new bamboo forests. Acta Ecologica Sinica, 36(8): 2243-2254. [in Chinese]

Liebman, J. C., & Lynn, W. R. (1966). The optimal allocation of stream dissolved oxygen. Water Resources Research, 2(3), 581-591.

McConnell, K. E., Daberkow, J. N., & Hardie, I. W. (1983). Planning timber production with evolving prices and costs. Land Economics, 59(3), 292-299.

Meng, H., Liu, Q., and Wu, W.. (2014). Study on the cost-benefit and carbon sequestration capacities of Phyllostachys edulis plantations under different types of management. Journal of Zhejiang A&F University, 31(6): 959-964. [in Chinese]

Mesterton - Gibbons, M. (1988). On the optimal policy for combining harvesting of predator and prey. Natural Resource Modeling, 3(1), 63-90.

Nath, A. J., Das, G., & Das, A. K. (2009). Above ground standing biomass and carbon storage in village bamboos in North East India. Biomass and Bioenergy, 33(9), 1188-1196.

National Agricultural Products Business Information Public Service Platform, People's Republic of China Ministry of Commerce. (2018, May 28). Retrieved from http://nc.mofcom.gov.cn/ [in Chinese].

Nautiyal, J. C., & Williams, J. S. (1990). Response of optimal stand rotation and management intensity to one-time changes in stumpage price, management cost, and discount rate. Forest Science, 36(2), 212-223.

Newman, D. H., Gilbert, C. B., & Hyde, W. F. (1985). The optimal forest rotation with evolving prices. Land Economics, 61(4), 347-353.

Newman, D. H. (1988). The optimal forest rotation: a discussion and annotated bibliography. Gen. Tech. Rep. SE-48. Asheville, NC: US Department of Agriculture, Forest Service, Southeastern Forest Experiment Station.

Nguyen, D.. (1979). Environmental services and the optimum rotation problem in forest management." Journal of Environmental Management, 8(2): 127-136

Ollikainen, M. (1991). The Effect of Nontimber Taxes on the Harvest Timing—The Case of Private Nonindustrial Forest Owners: A Note. Forest science, 37(1), 356-363.

Pearse, P. H. (1967). The optimum forest rotation. The Forestry Chronicle, 43(2), 178-195.

Ramsey, F. P. (1928). A mathematical theory of saving. The economic journal, 38(152), 543-559.

Ramirez, W. F. (1987). Application of optimal control theory to enhanced oil recovery (Vol. 21). Elsevier.

Ren, G., Young, S. S., Wang, L., Wang, W., Long, Y., Wu, R., ... & Yu, D. W. (2015). Effectiveness of China's national forest protection program and nature reserves. Conservation Biology, 29(5), 1368-1377.

Richards, F. J. (1959). A flexible growth function for empirical use. Journal of experimental Botany, 10(2), 290-301.

Rideout, D. B. (1982). Micro-Economic Properties of Four Local Forest Tax Methods. (doctoral dissertation). University of Washington, Seattle, United States.

Riordan, C. (1971). General multistage marginal cost dynamic programing model for the optimization of a class of investment - pricing decisions. Water Resources Research, 7(2), 245-253.

Roos, C. F. (1925). A mathematical theory of competition. American Journal of Mathematics, 47(3), 163-175.

Samuelson, P. A. (1976). Economics of forestry in an evolving society. Economic inquiry, 14(4), 466-492.

Scurlock, J. M. O., Dayton, D. C., & Hames, B. (2000). Bamboo: an overlooked biomass resource?. Biomass and bioenergy, 19(4), 229-244.

SFA (2014b) Report of forest resources in China (2009–2013). China Forestry Press, Beijing, p 86.

Slade, M. E. (1982). Trends in natural-resource commodity prices: an analysis of the time domain. Journal of Environmental Economics and Management, 9(2), 122-137.

Sniedovich, M. (1979). Reliability - constrained reservoir control problems: 1. Methodological issues. Water Resources Research, 15(6), 1574-1582.

Song, X., Zhou, G., Jiang, H., Yu, S., Fu, J., Li, W., ... & Peng, C. (2011). Carbon sequestration by Chinese bamboo forests and their ecological benefits: assessment of potential, problems, and future challenges. Environmental Reviews, 19(NA), 418-428.

Song, X., Gu, H., Wang, M., Zhou, G., & Li, Q. (2016). Management practices regulate the response of Moso bamboo foliar stoichiometry to nitrogen deposition. Scientific reports, 6, 24107.

Stedinger, J. R., Sule, B. F., & Loucks, D. P. (1984). Stochastic dynamic programming models for reservoir operation optimization. Water resources research, 20(11), 1499-1505.

Strang, W. J. (1983). On the optimal forest harvesting decision. Economic Inquiry, 21(4), 576-583.

Su, W.. (2012) "Fertilization Theory and Practice for Phyllostachys edulis Stand Based on Growth and Nutrient Accumulation Rules.". (doctoral dissertation). Chinese Academy of Forestry, Beijing, China.

Tyler, A. L., MacMillan, D. C., & Dutch, J. (1996). Models to predict the General Yield Class of Douglas fir, Japanese larch and Scots pine on better quality land in Scotland. Forestry: An International Journal of Forest Research, 69(1), 13-24.

Wang, G., Innes, J. L., Dai, S., & He, G. (2008). Achieving sustainable rural development in Southern China: the contribution of bamboo forestry. The International Journal of Sustainable Development & World Ecology, 15(5), 484-495.

Wang, G., Innes, J. L., Lei, J., Dai, S., & Wu, S. W. (2007). China's forestry reforms. Science, 318(5856), 1556-1557.

Wang, S., Van Kooten, G. C., & Wilson, B. (2004). Mosaic of reform: forest policy in post-1978 China. Forest Policy and Economics, 6(1), 71-83.

Wei, Q., Jiao, C., Guo, L., Ding, Y., Cao, J., Feng, J., ... & Yang, G. (2017). Exploring key cellular processes and candidate genes regulating the primary thickening growth of Moso underground shoots. New Phytologist, 214(1), 81-96.

Wu, W., and Cao, X.. (2016) Study on the relationship between input, output and economic benefit of Moso bamboo plantation. Journal of Nanjing Forestry University (Natural Sciences Edition), 40(3), 108-114. [in Chinese]

Xi, W., Wang, F., Shi, P., Dai, E., Anoruo, A. O., Bi, H., ... & Li, W. (2014). Challenges to sustainable development in China: A review of six large-scale forest restoration and land conservation programs. Journal of sustainable forestry, 33(5), 435-453.

Xiao, X., Luo Y., & Ni, C.. (1997). Study on the Bambo+o Shoot Yield of Phyllostachys pubescens. Journal of Fujian Forestry and Technology, 24(3), 24-27. [in Chinese]

Xu, L., Shi, Y., Zhou, G., Xu, X., Liu, E., Zhou, Y., ... & Deng, X. (2017). Temporal change in aboveground culms carbon stocks in the Moso bamboo forests and its driving factors in Zhejiang Province, China. Forests, 8(10), 371.

Yen, T. M. (2015). Comparing aboveground structure and aboveground carbon storage of an age series of moso bamboo forests subjected to different management strategies. Journal of forest research, 20(1), 1-8.

Yen, T. M. (2016). Culm height development, biomass accumulation and carbon storage in an initial growth stage for a fast-growing moso bamboo (Phyllostachy pubescens). Botanical studies, 57(1), 10.

Yu, Lin. (2011) Study on Density Effect of Phyllostachys edulis Stands in Southern Anhui Province. (doctoral dissertation). Chinese Academy of Forestry, Beijing, China..

Zakirov, I., Aanonsen, S. I., Zakirov, E. S., & Palatnik, B. M. (1996, September). Optimizing reservoir performance by automatic allocation of well rates. In ECMOR V-5th European Conference on the Mathematics of Oil Recovery.

Zhang, C., & Ding, X. (1997) Studies on yield of Phyllostachys Heterocycla var. Pubescens stands. Journal of Bamboo Research, 16(3), 31-36. [in Chinese]

Zhao-gang, L., & Feng-ri, L. (2003). The generalized Chapman-Richards function and applications to tree and stand growth. Journal of Forestry Research, 14(1), 19-26.

Zheng, Y., Hong, W., & Zhang, W. (1998). Study on Prediction of Bamboo Shoot Weight and Yield. Scientia Silvae Sinicae, 34(1), 69-72. [in Chinese]

Zhou, Fangchun. (1998). Mathematical Model for Bamboo Growth. Bamboo Research, 19(1), 74-78. [In Chinese]

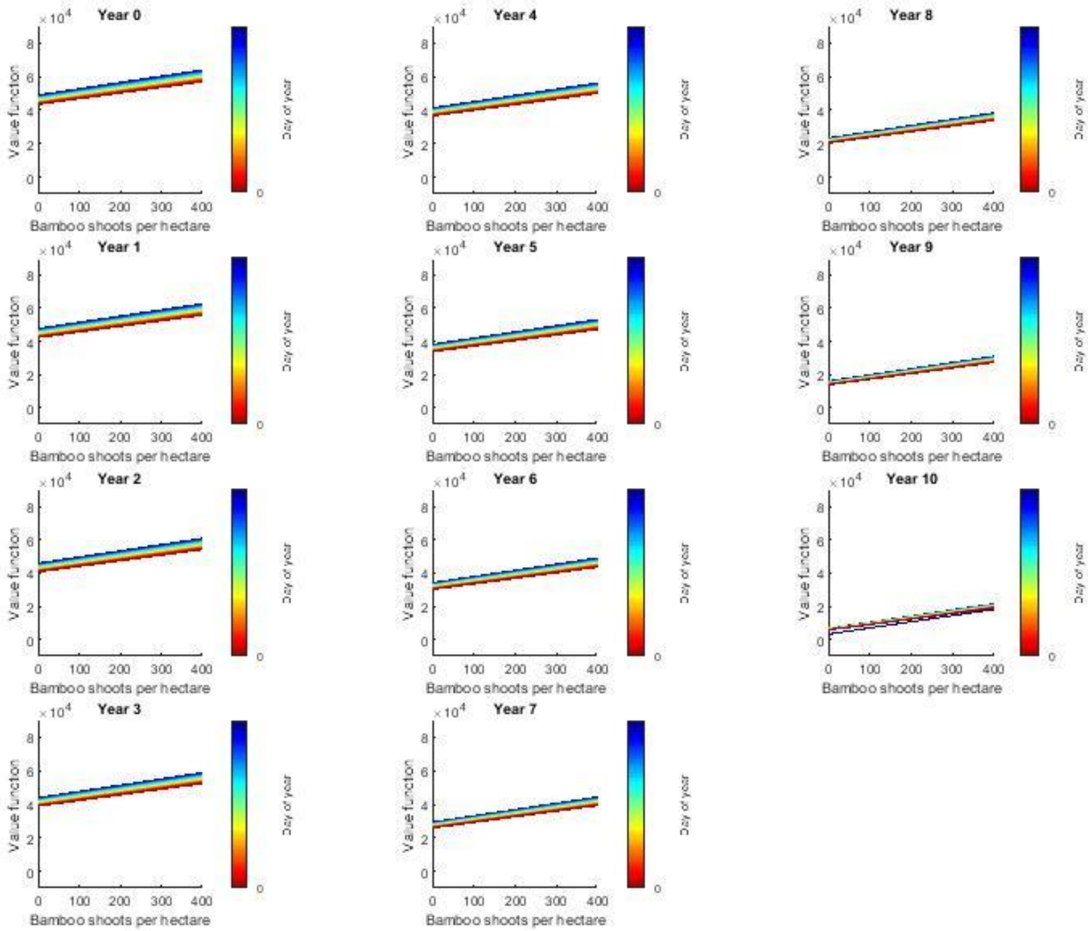
Zhou, G. M. (2006). Carbon Storage, Fixation, and Distribution in Moso Bamboo Forest Ecological System. (doctoral dissertation). Zhejiang University, Hangzhou, China.

Zhu, Zhaohua. (2001). Sustainable development of the bamboo and rattan sectors in tropical China. China Forestry Publishing House.

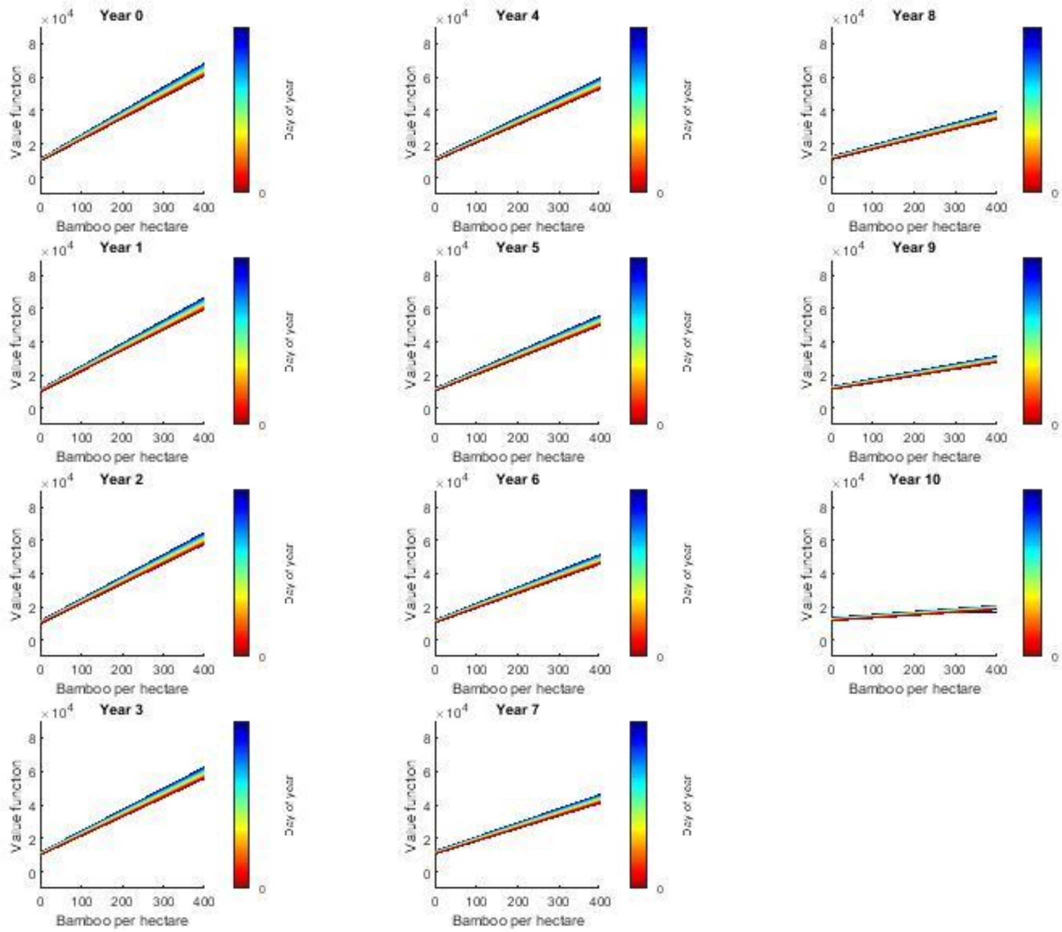
Zong, Y., & Chen, X. (2000). The 1998 flood on the Yangtze, China. Natural Hazards, 22(2), 165-184.

Figure 1. Deterministic Model, Specification 1: $p_b = 1.4$ and $p_s = 20$

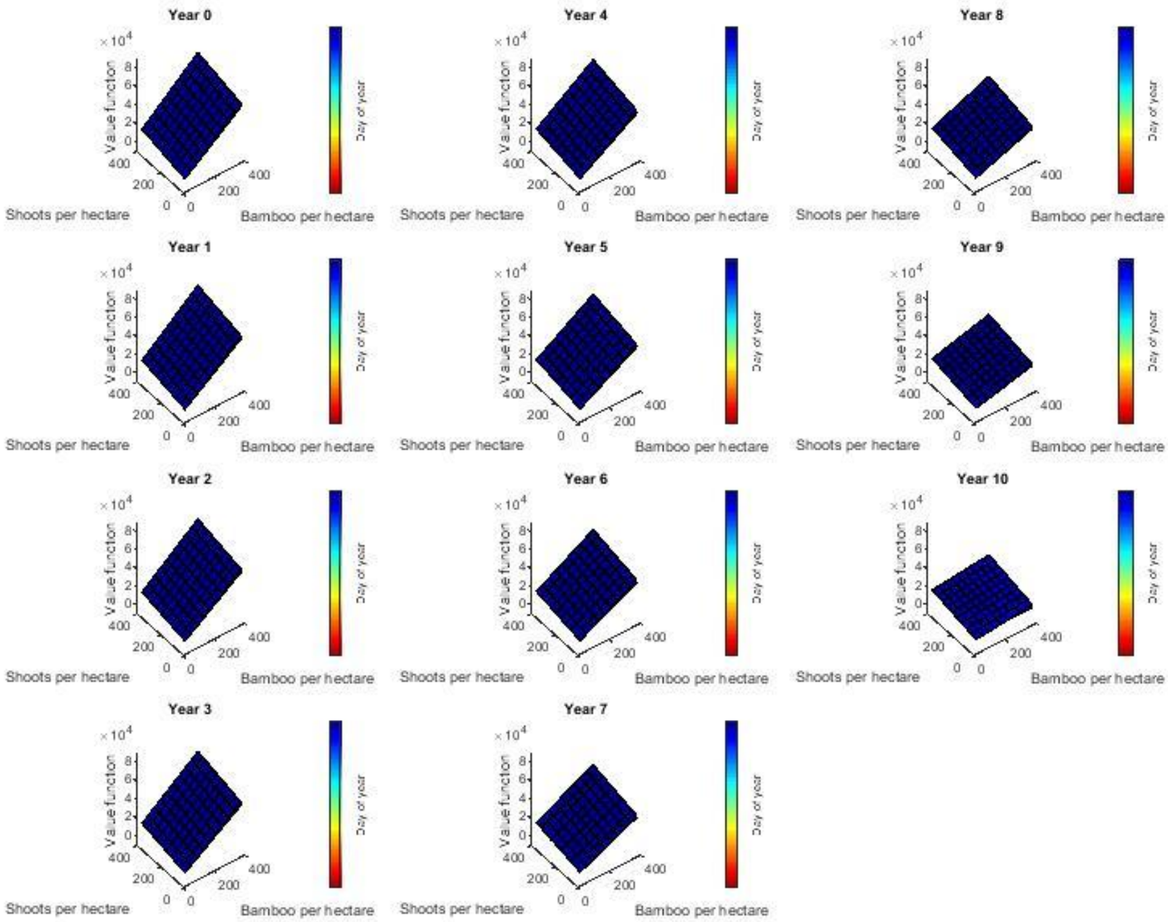
a) Value function as function of bamboo shoots per hectare



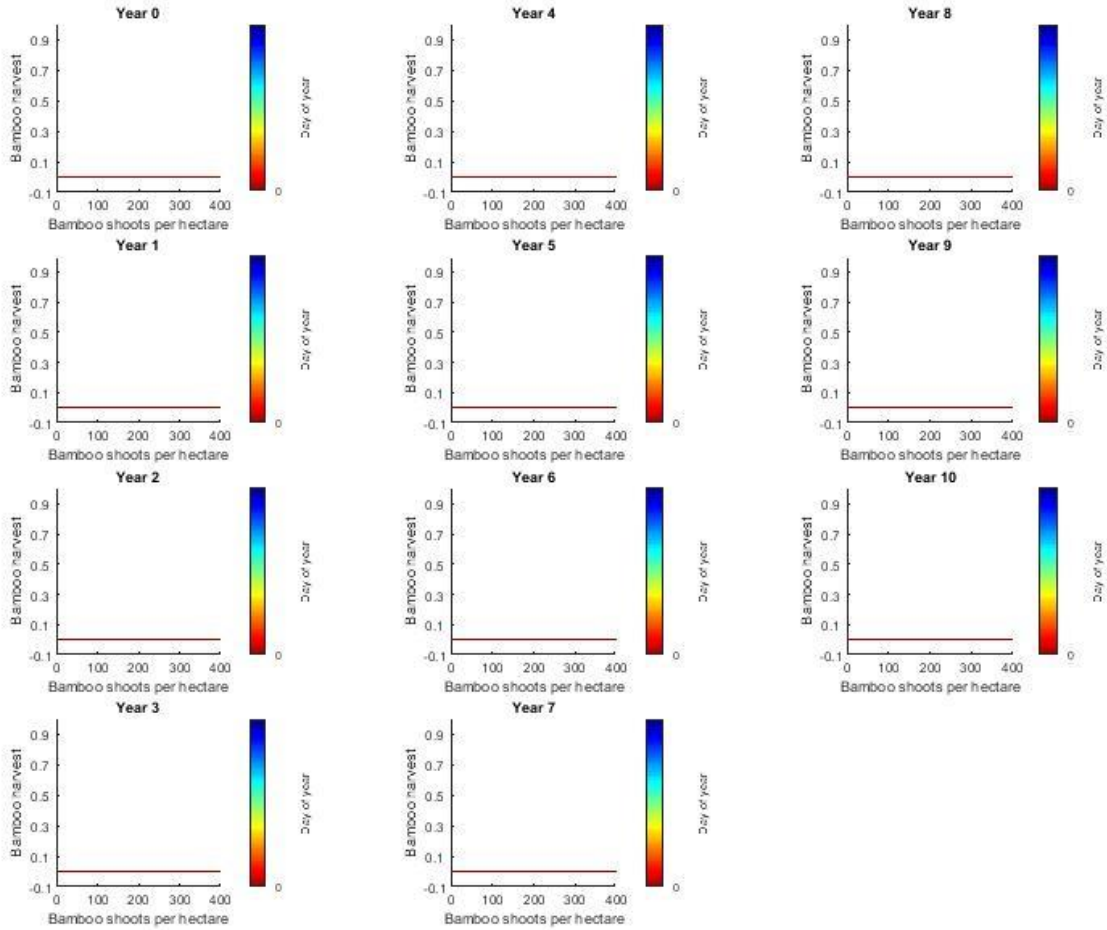
b) Value function as function of bamboo stem per hectare



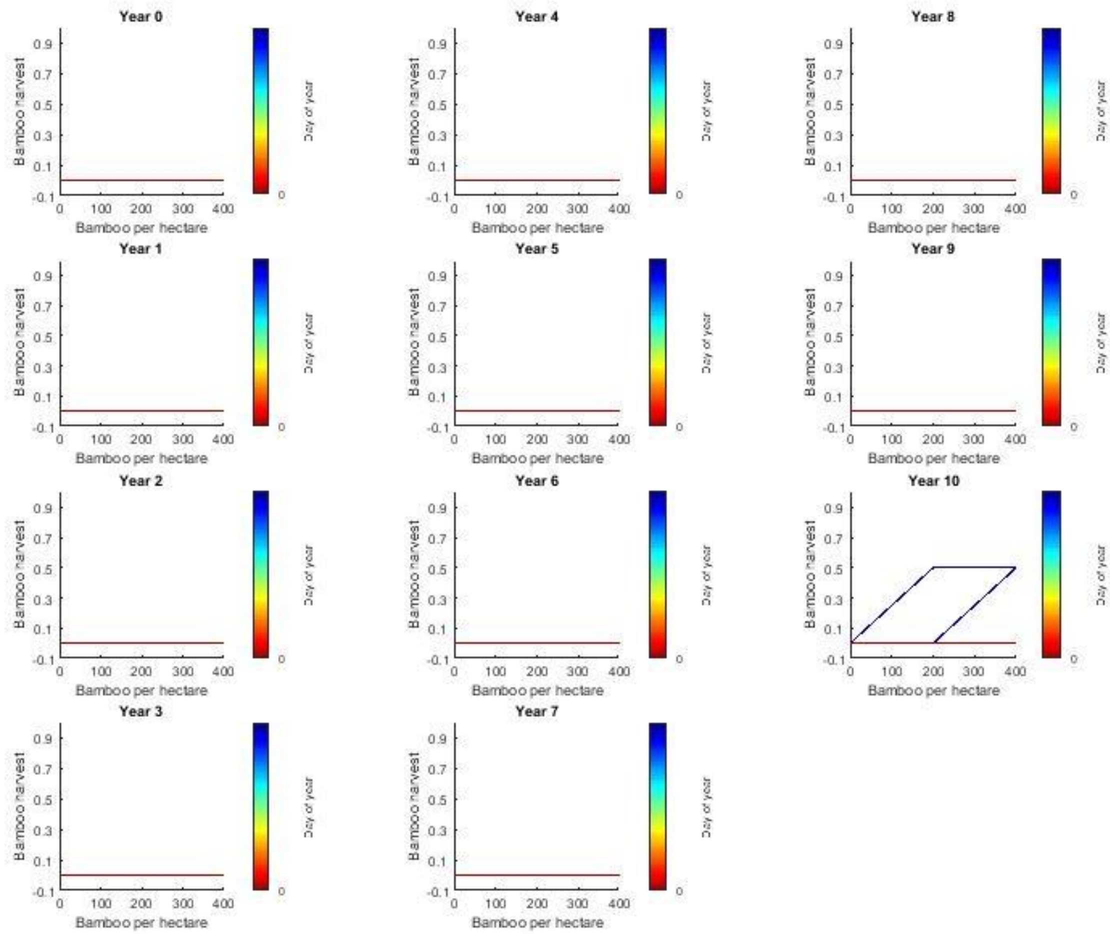
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



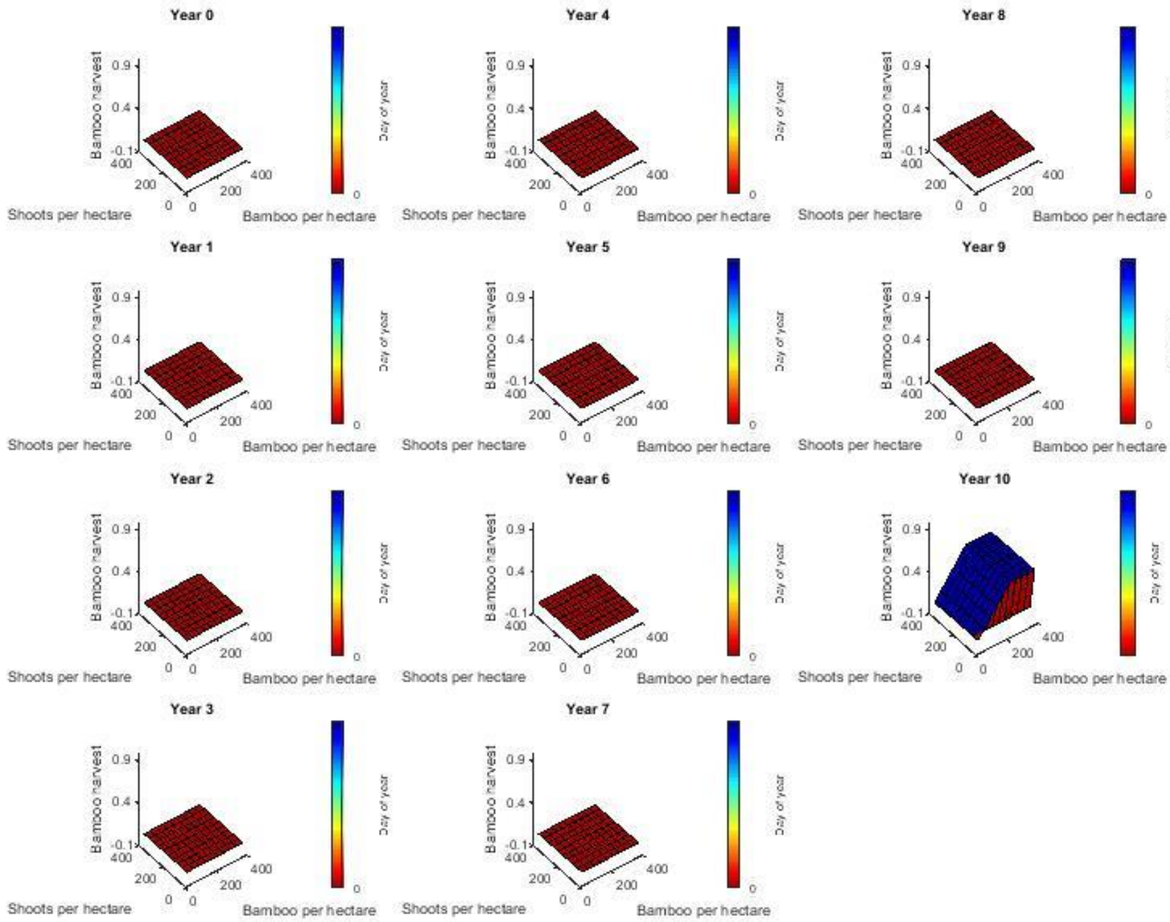
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



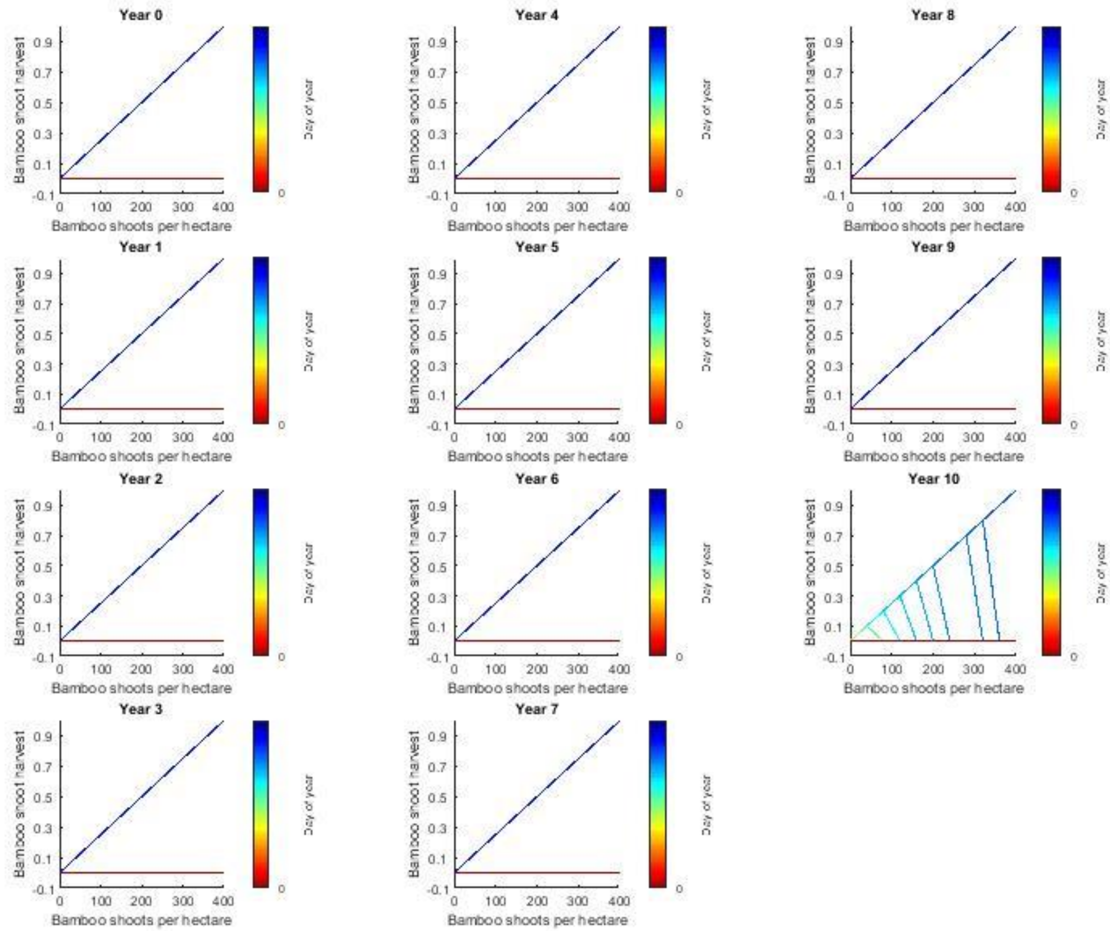
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



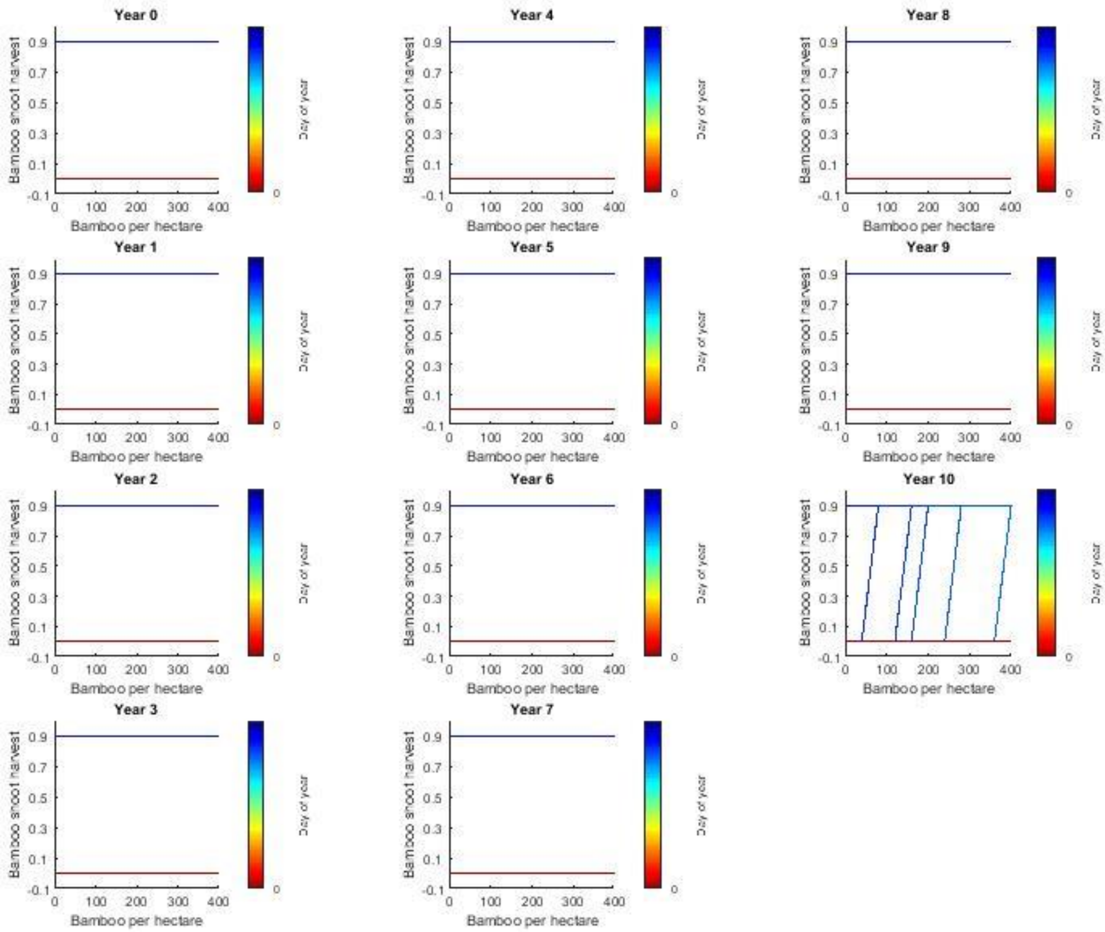
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



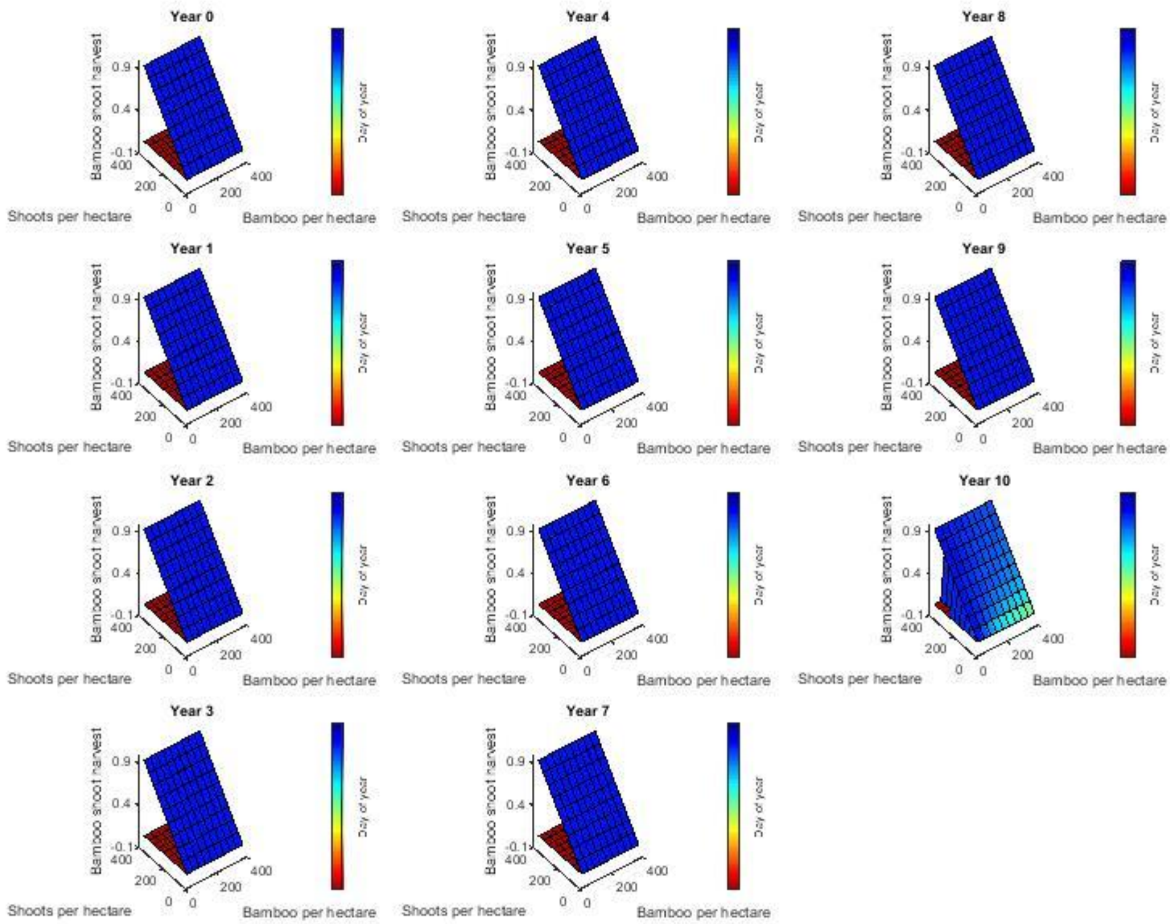
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



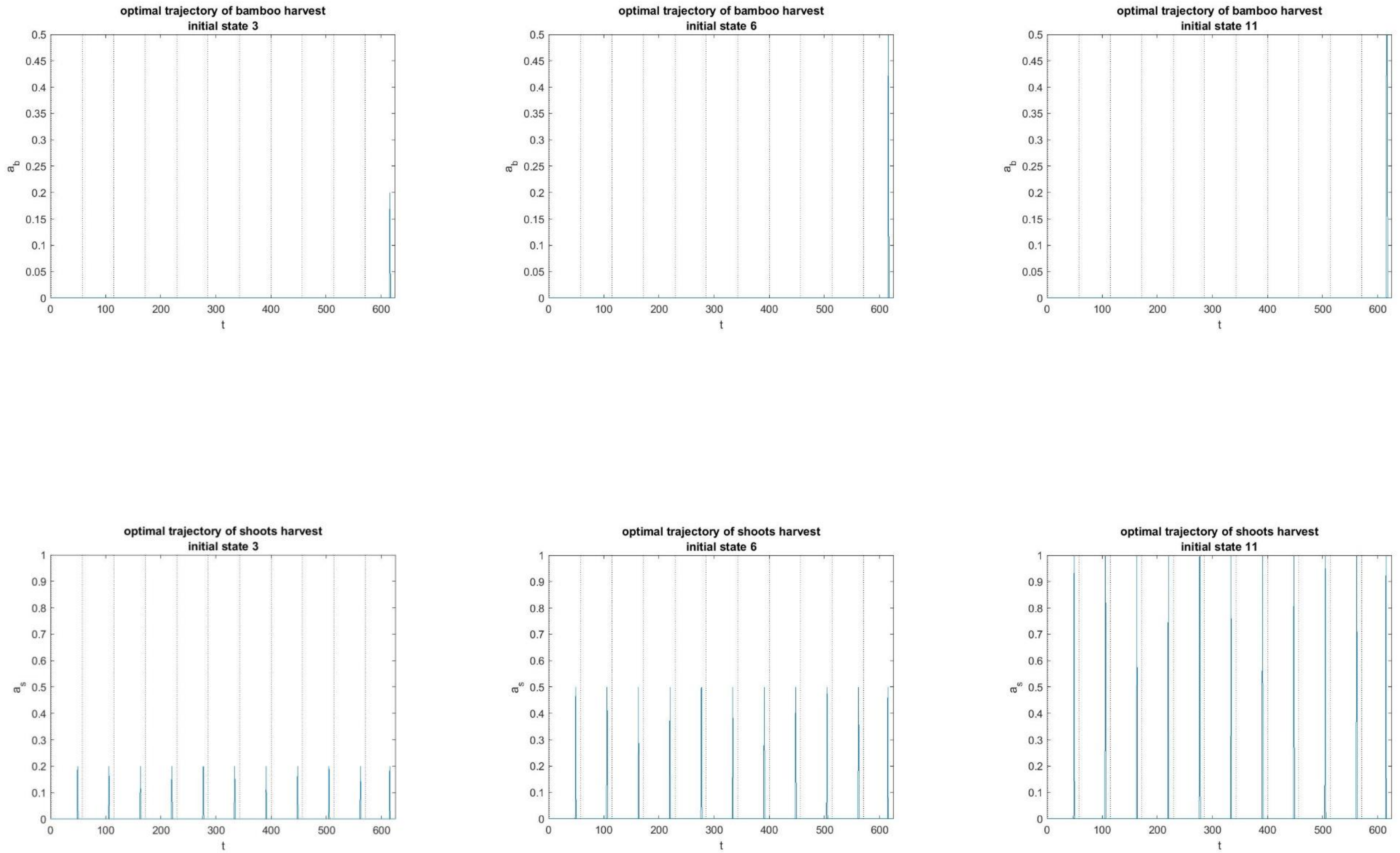
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

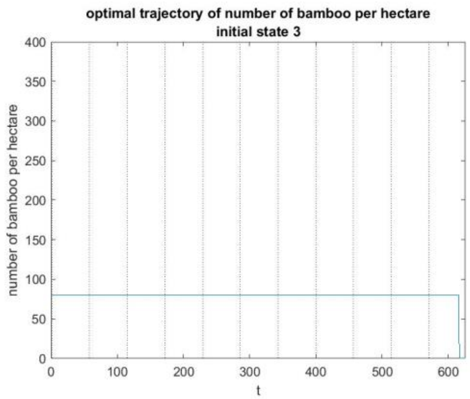


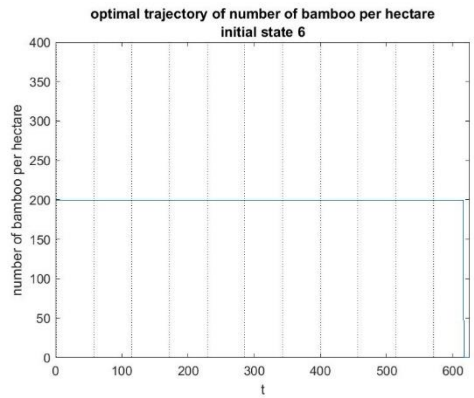
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

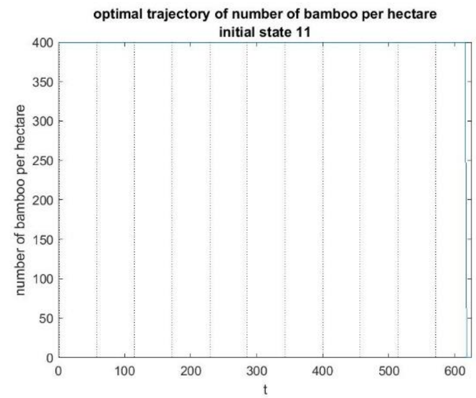


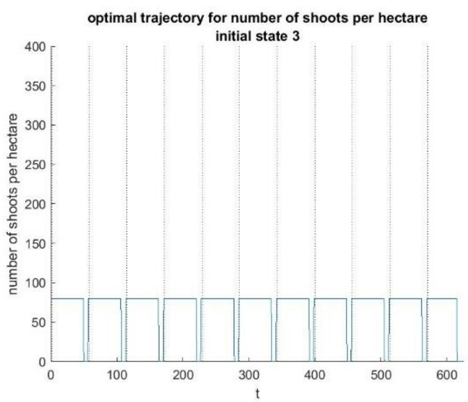
j) Optimal trajectories for each action and state variable over 11 years

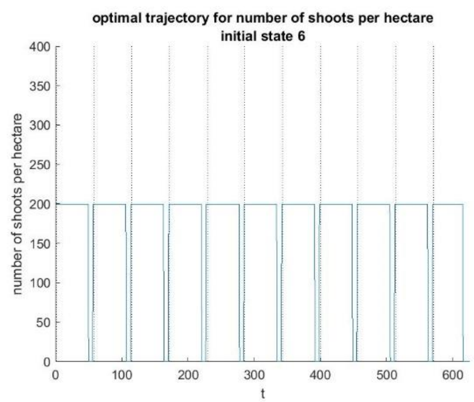



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

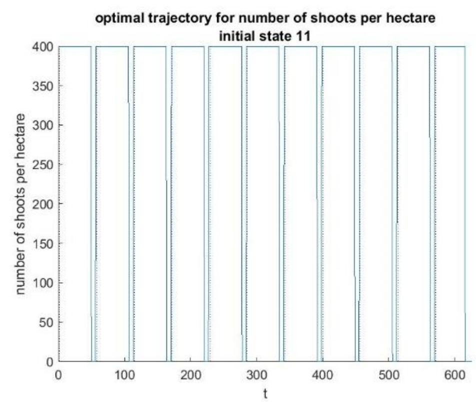
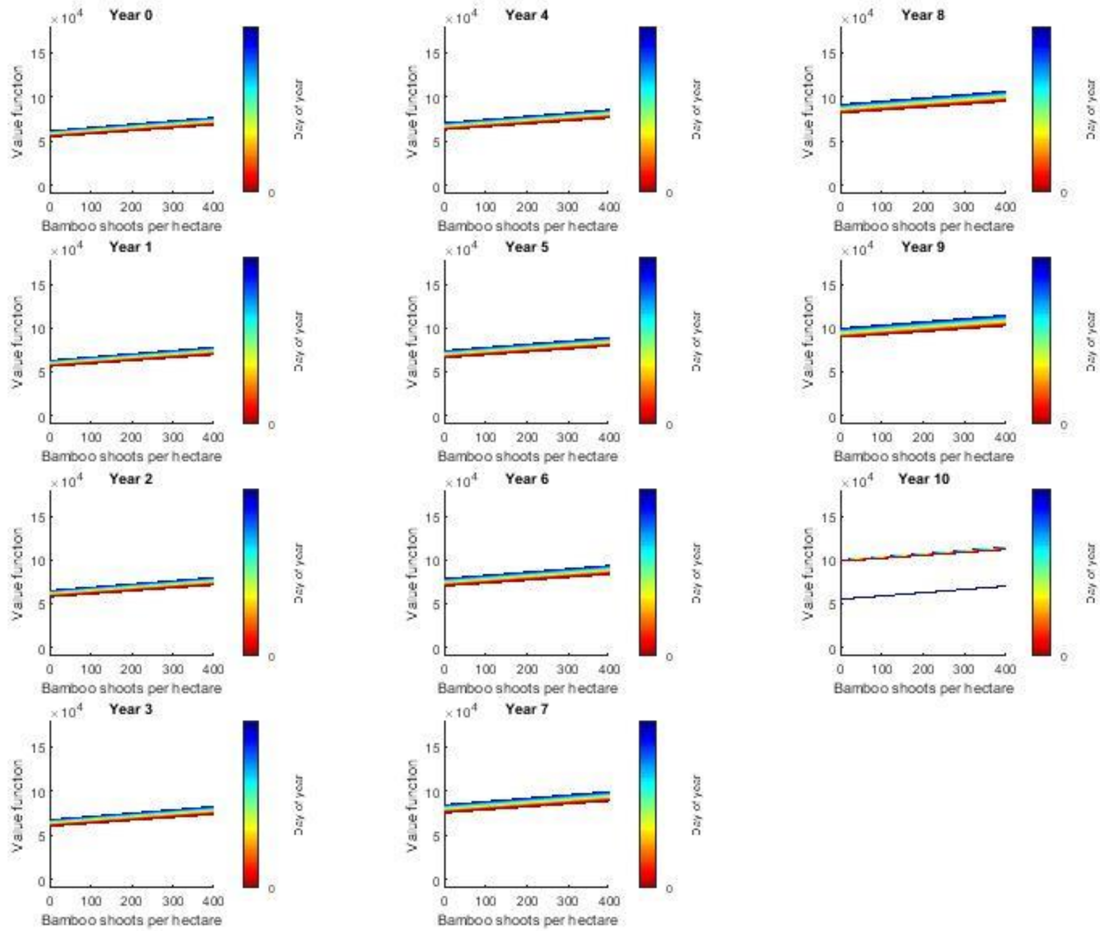
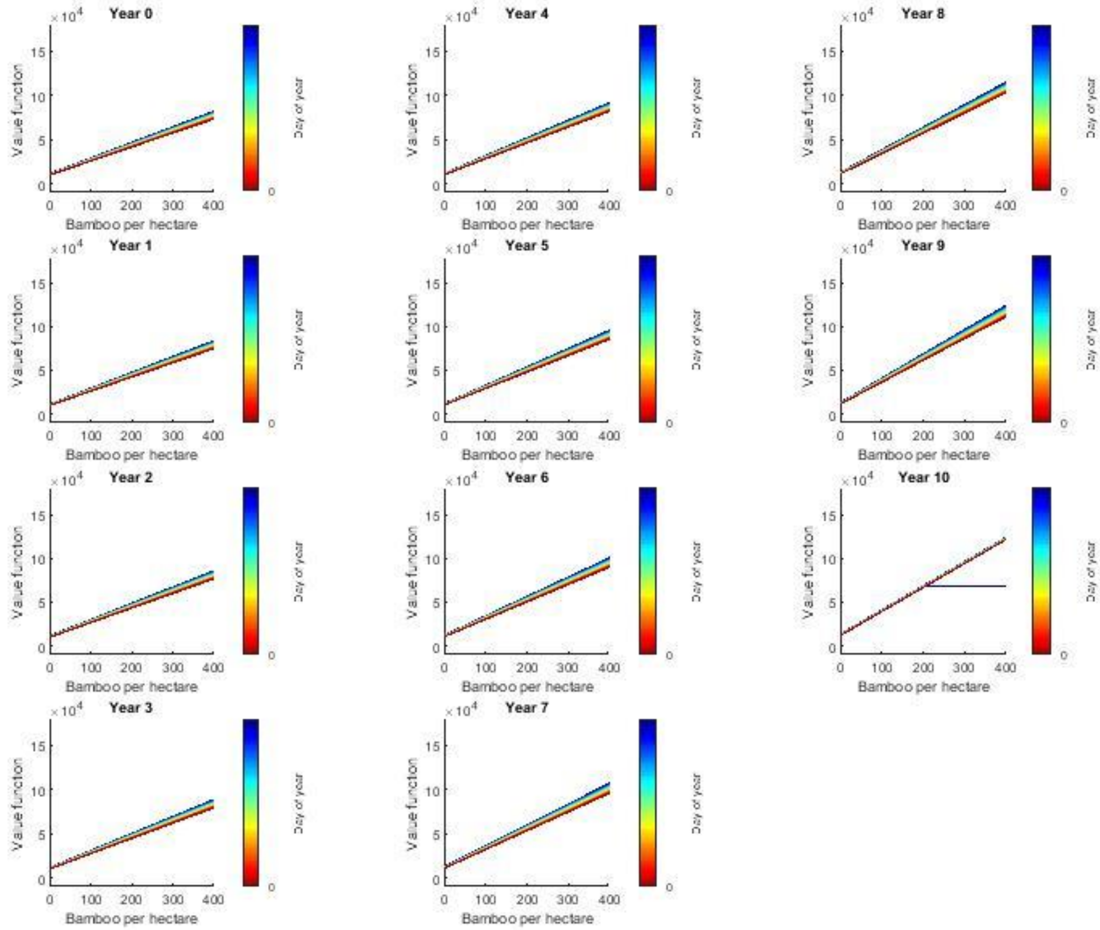

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

Figure 2. Deterministic Model, Specification 7: $p_b = 21$ and $p_s = 20$

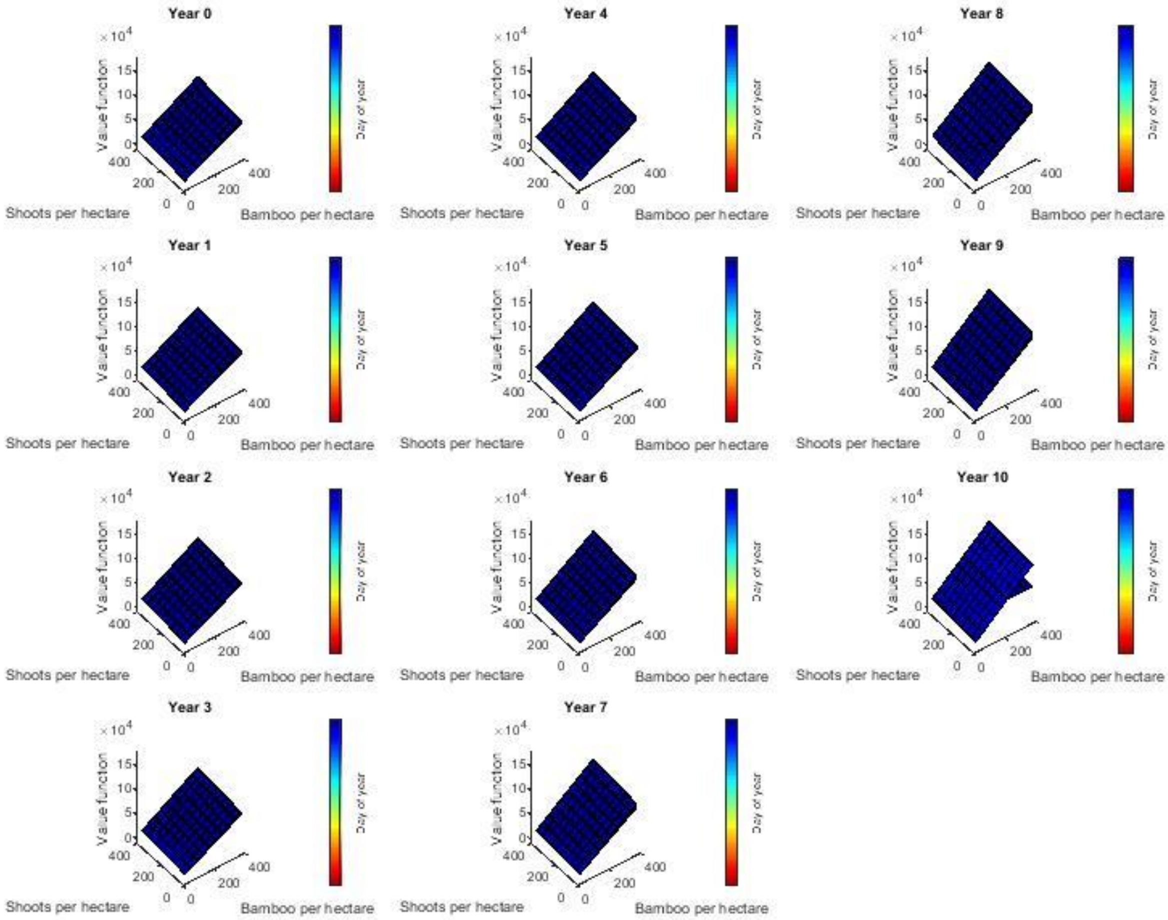
a) Value function as function of bamboo shoots per hectare



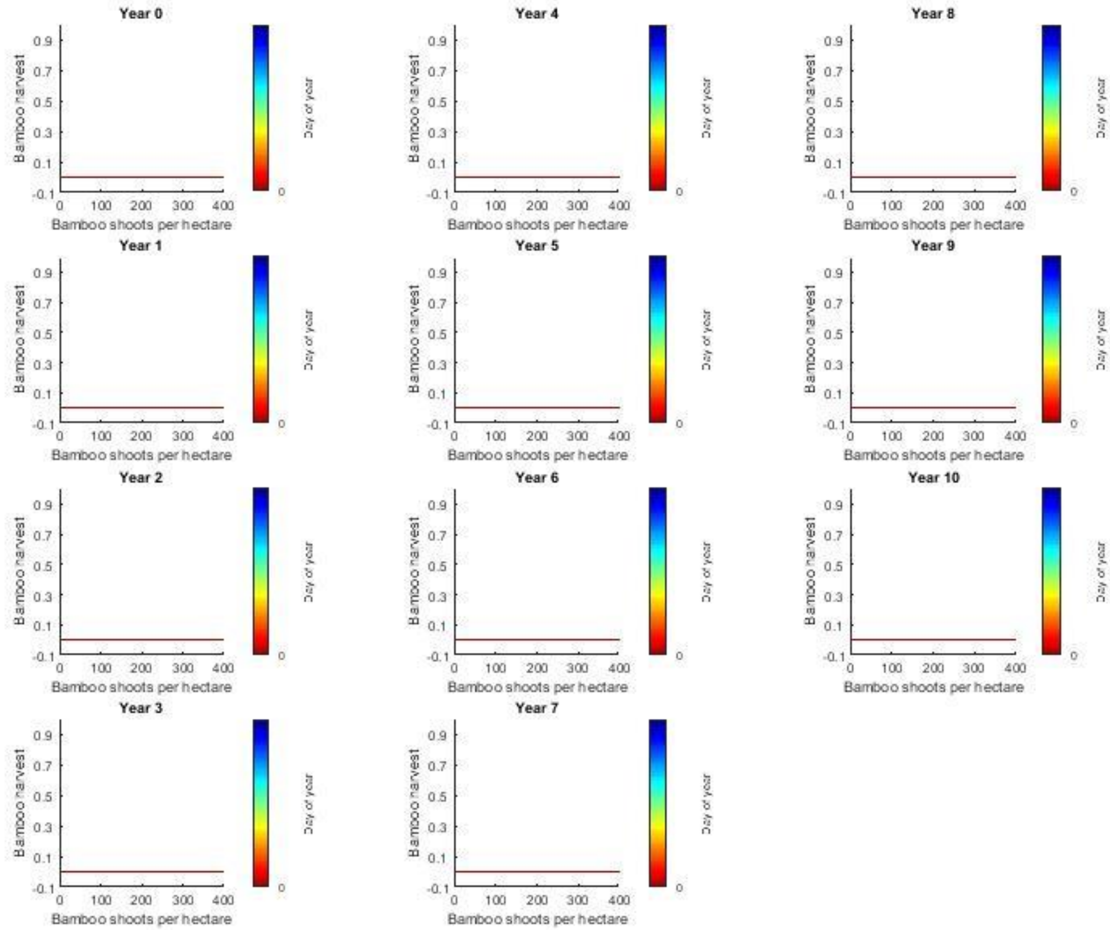
b) Value function as function of bamboo stem per hectare



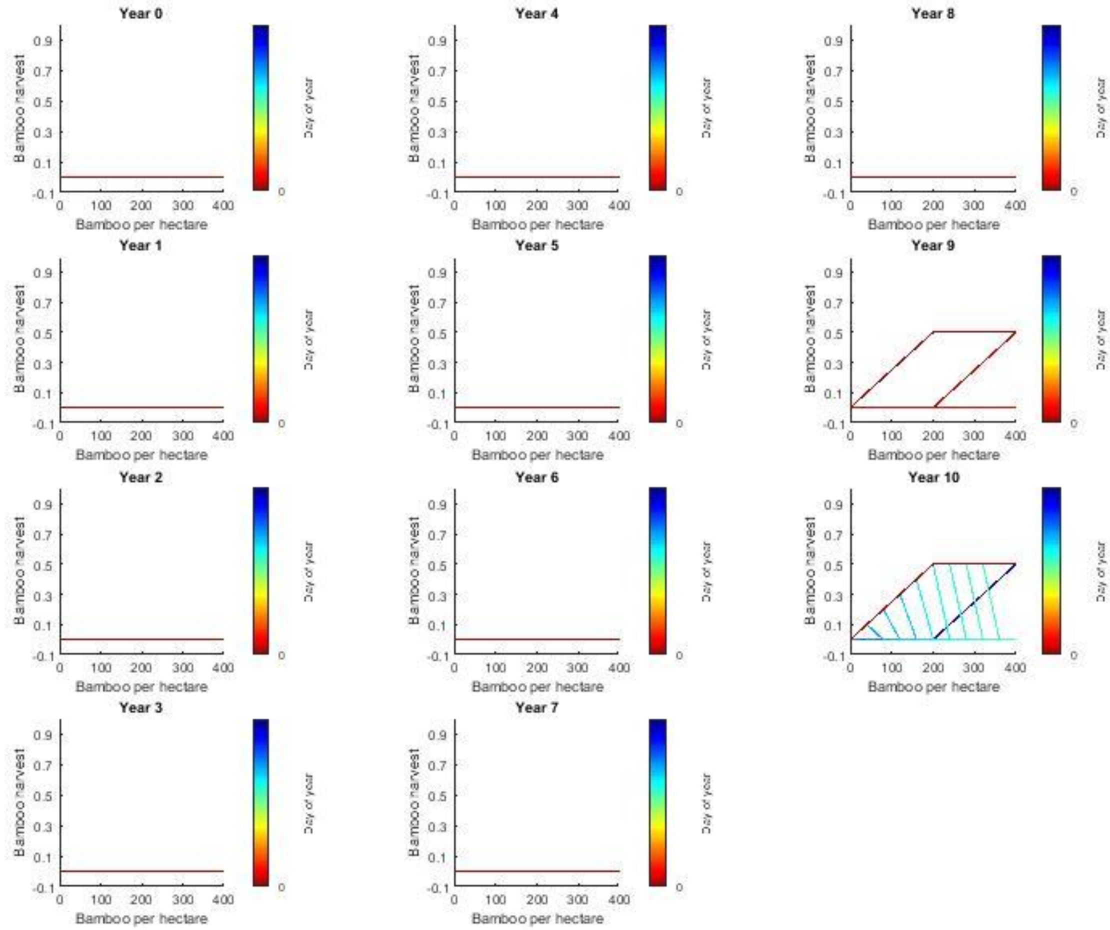
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



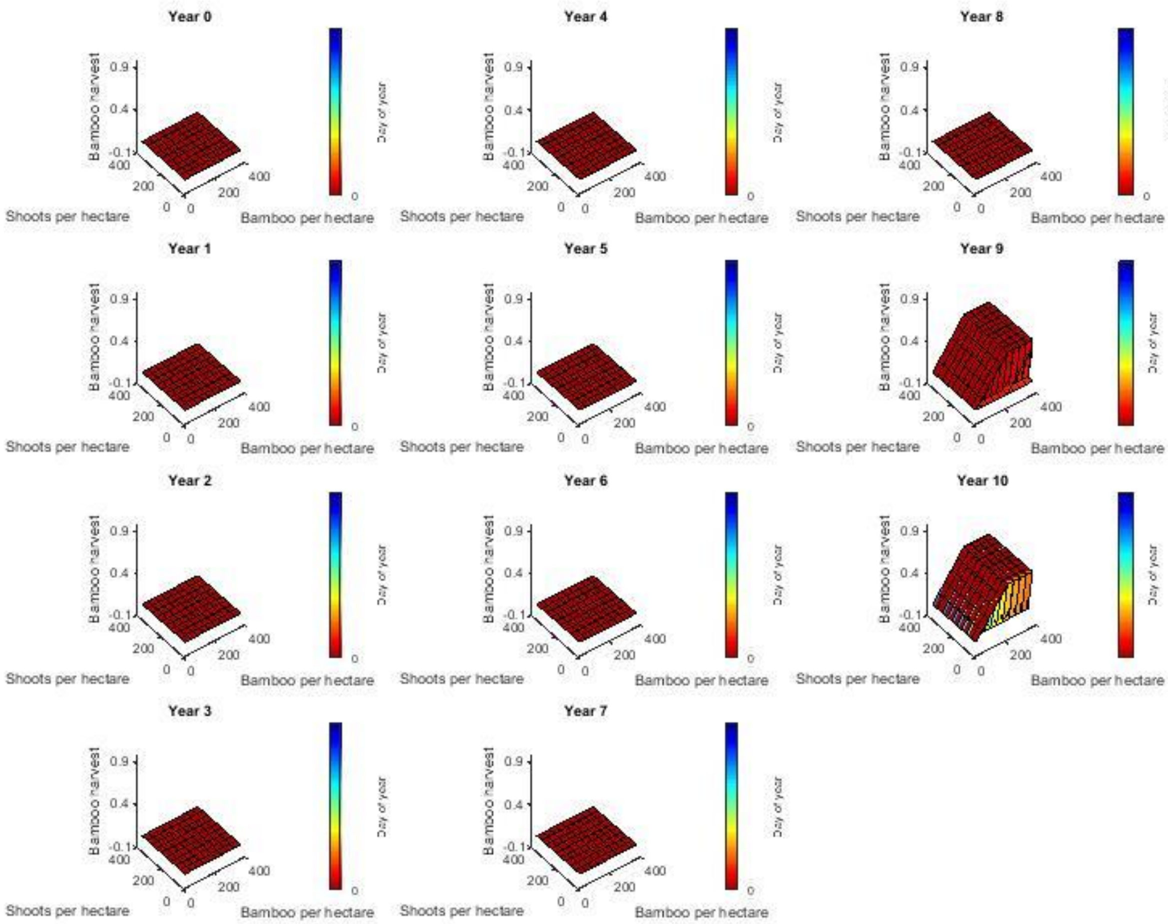
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



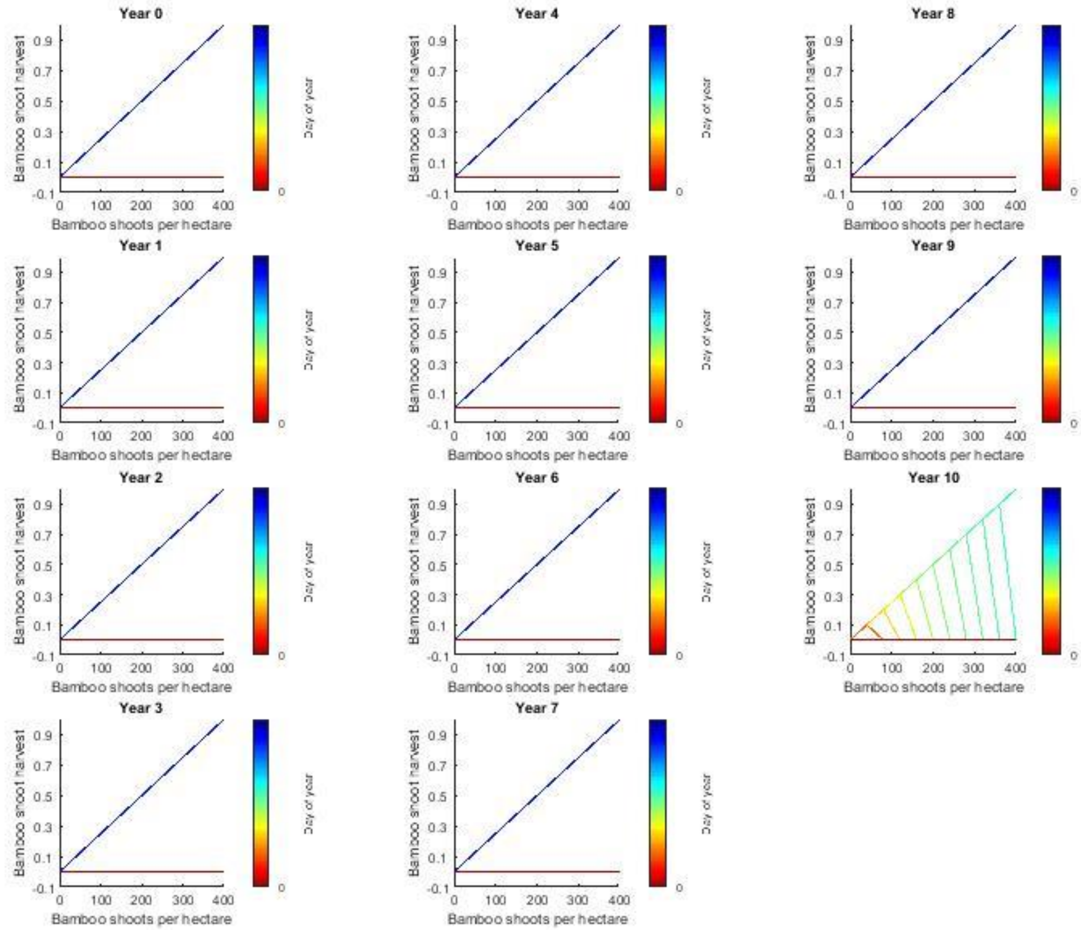
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



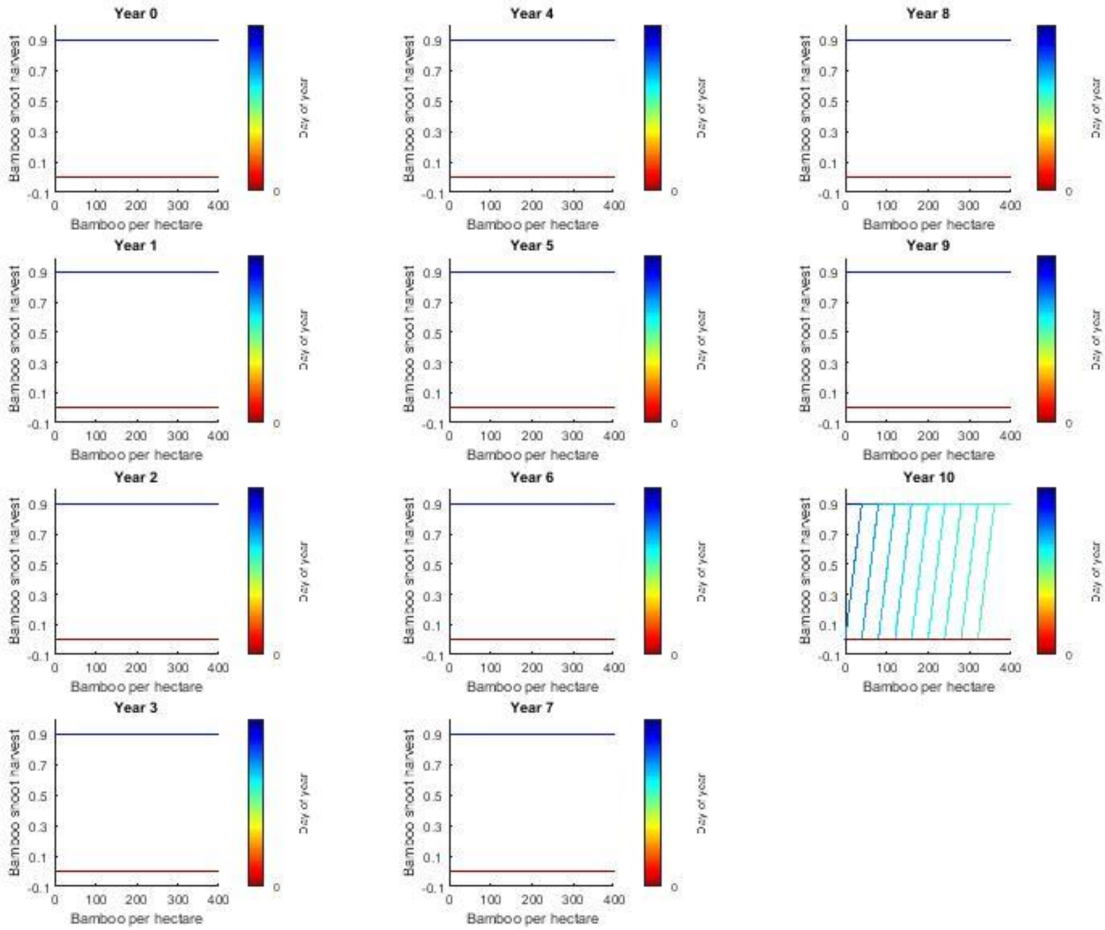
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



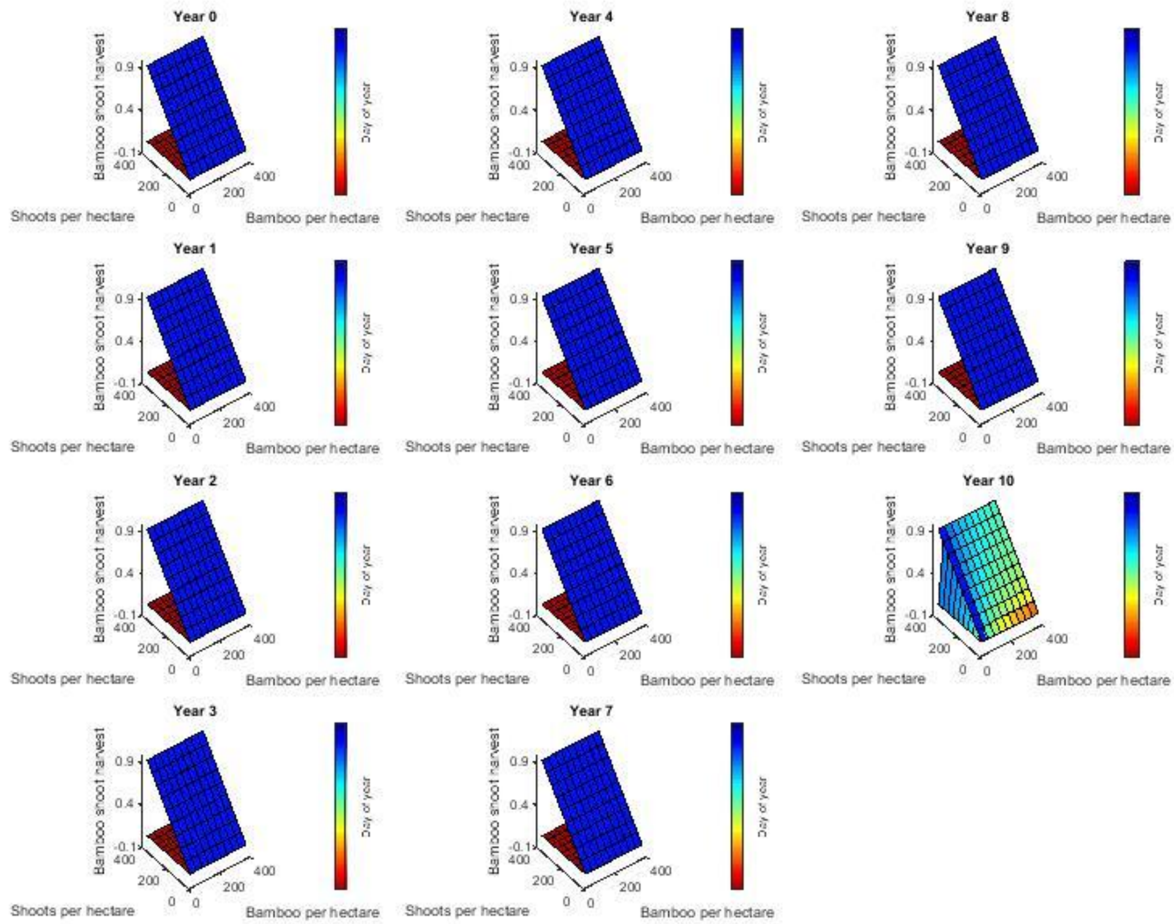
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



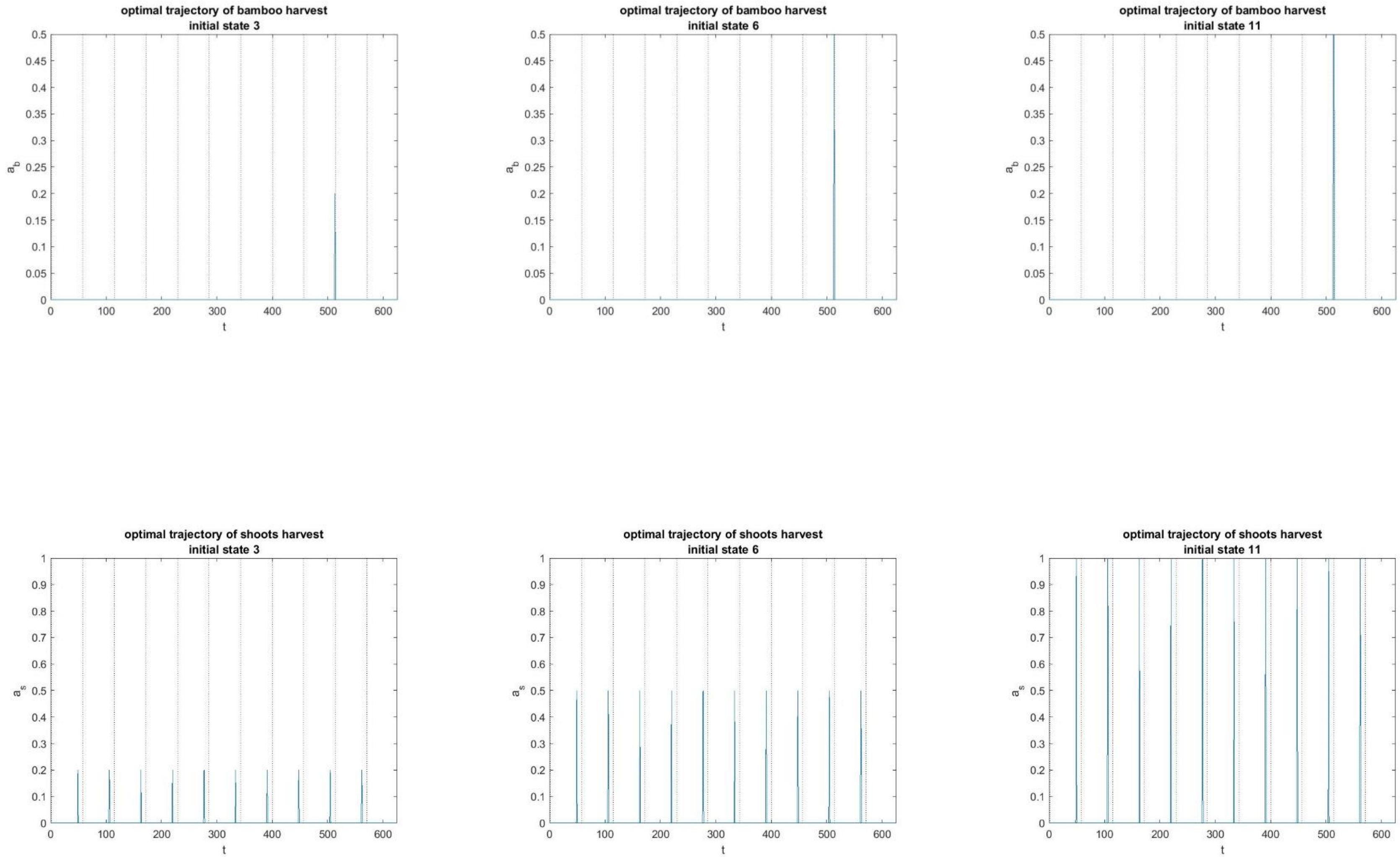
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

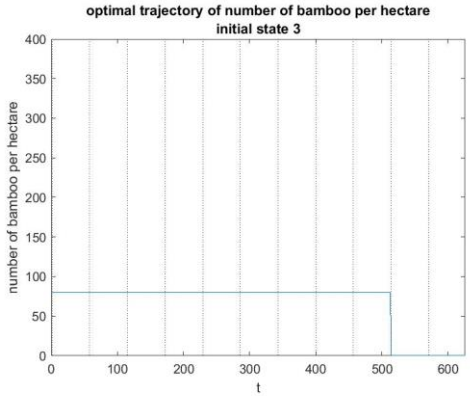


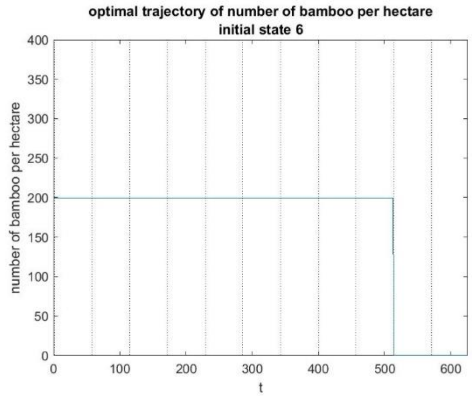
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

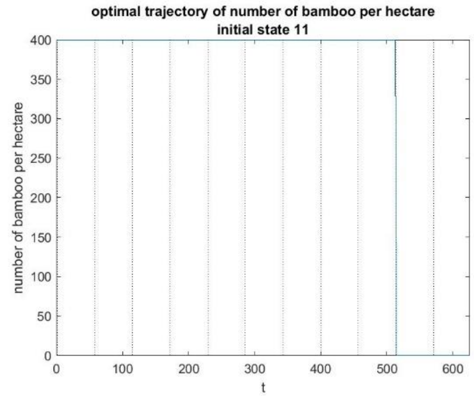


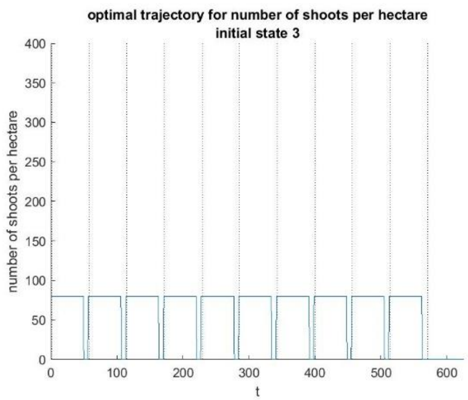
j) Optimal trajectories for each action and state variable over 11 years

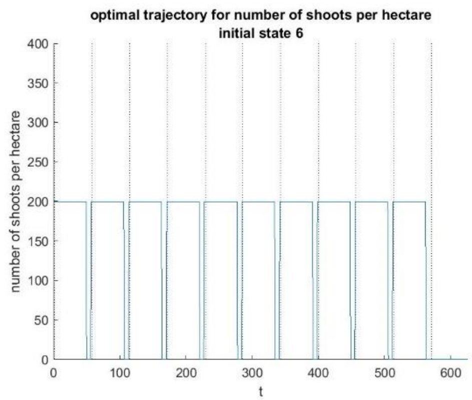



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

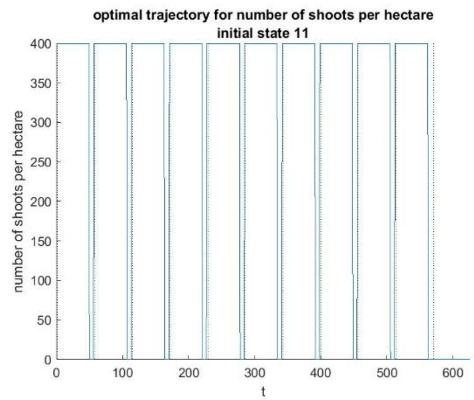
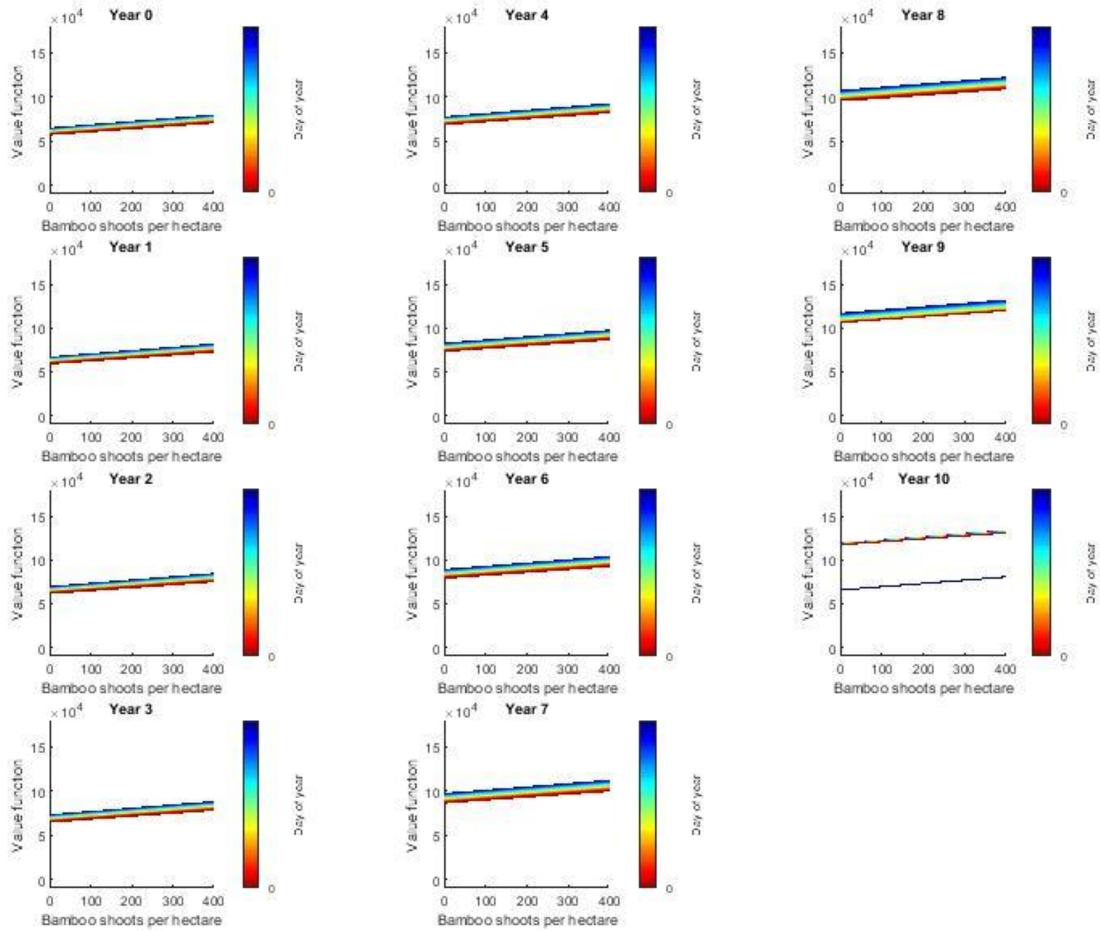
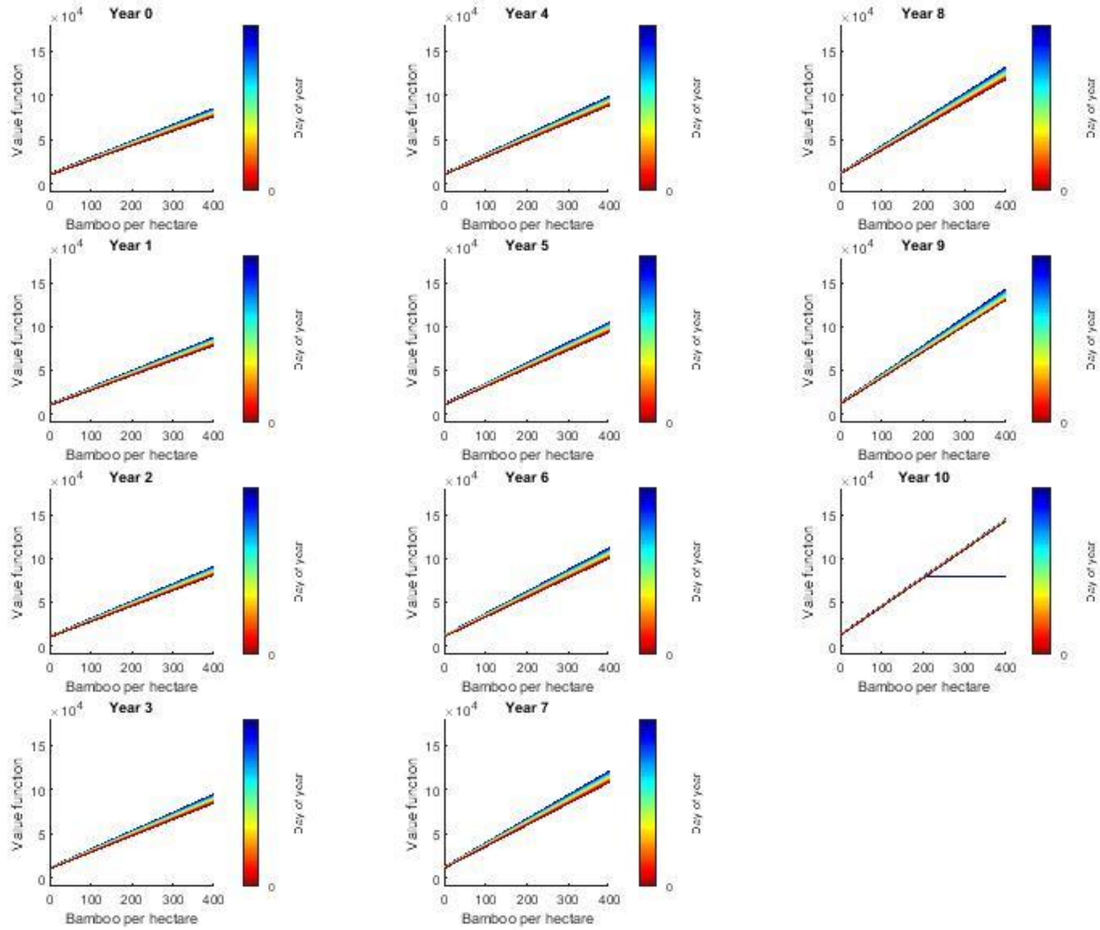

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

Figure 3: Deterministic Model, Specification 6: $p_b = 25$ and $p_s = 20$

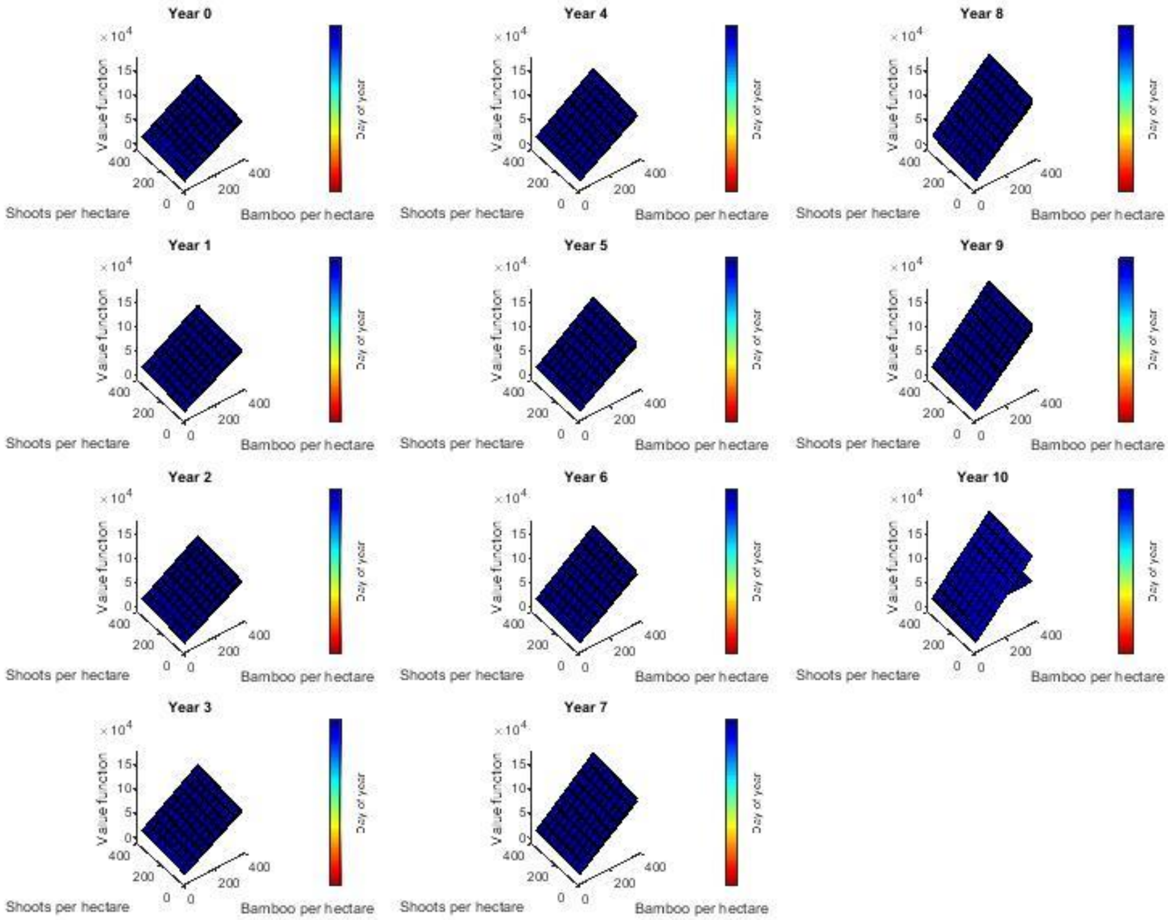
a) Value function as function of bamboo shoots per hectare



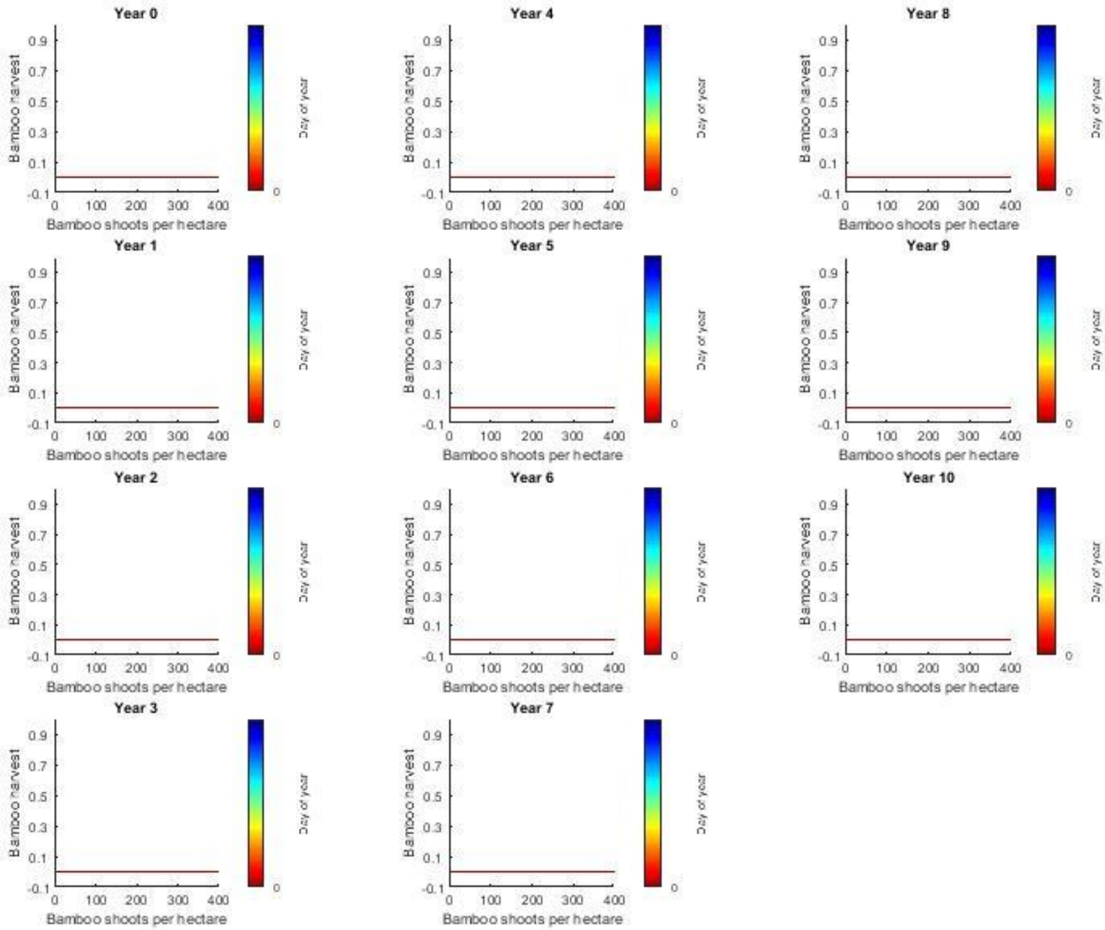
b) Value function as function of bamboo stem per hectare



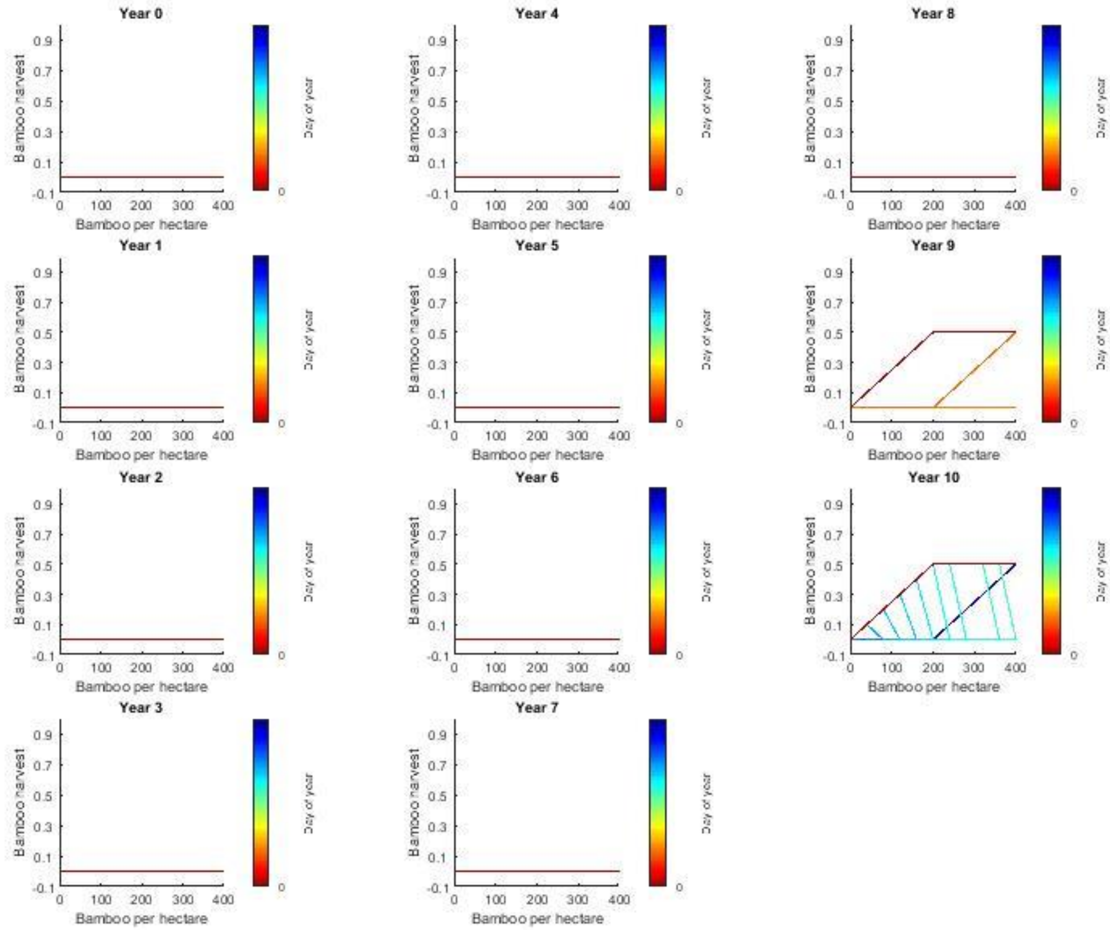
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



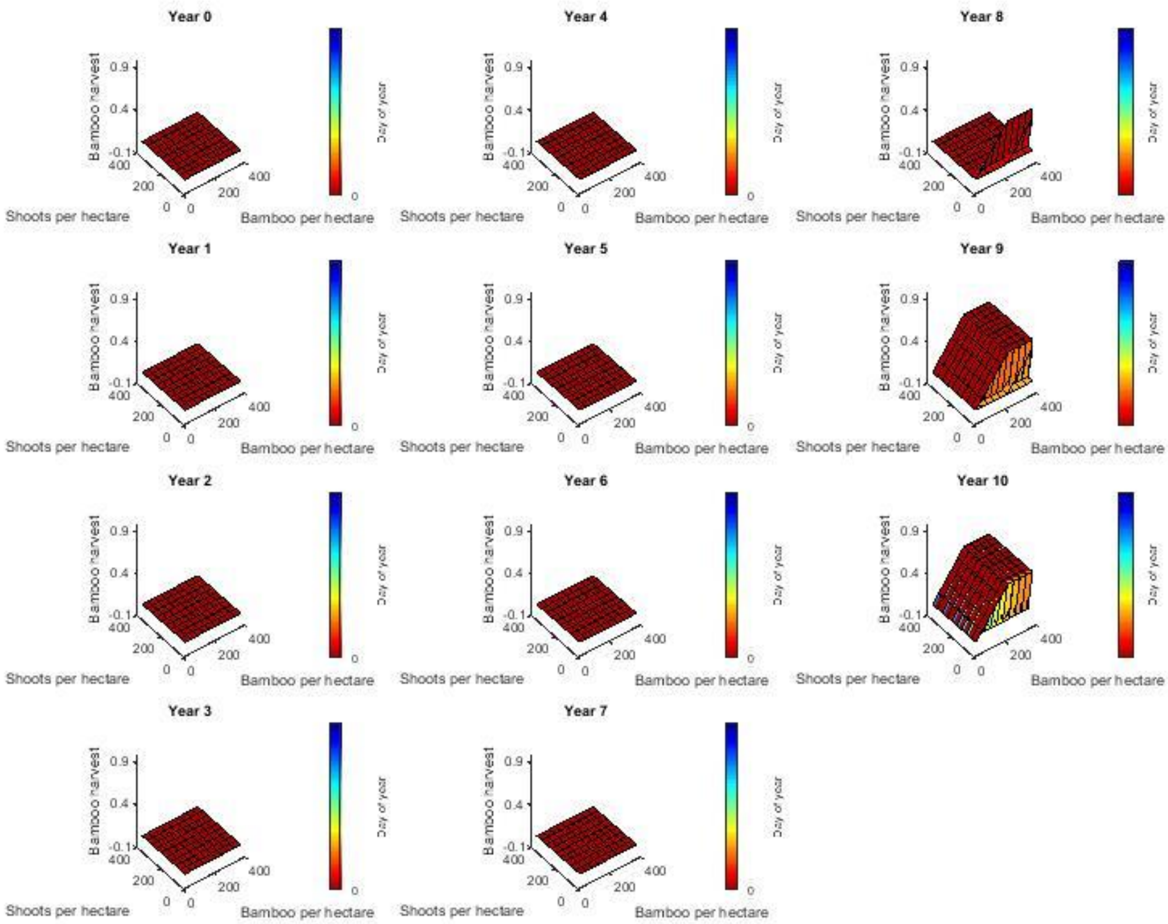
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



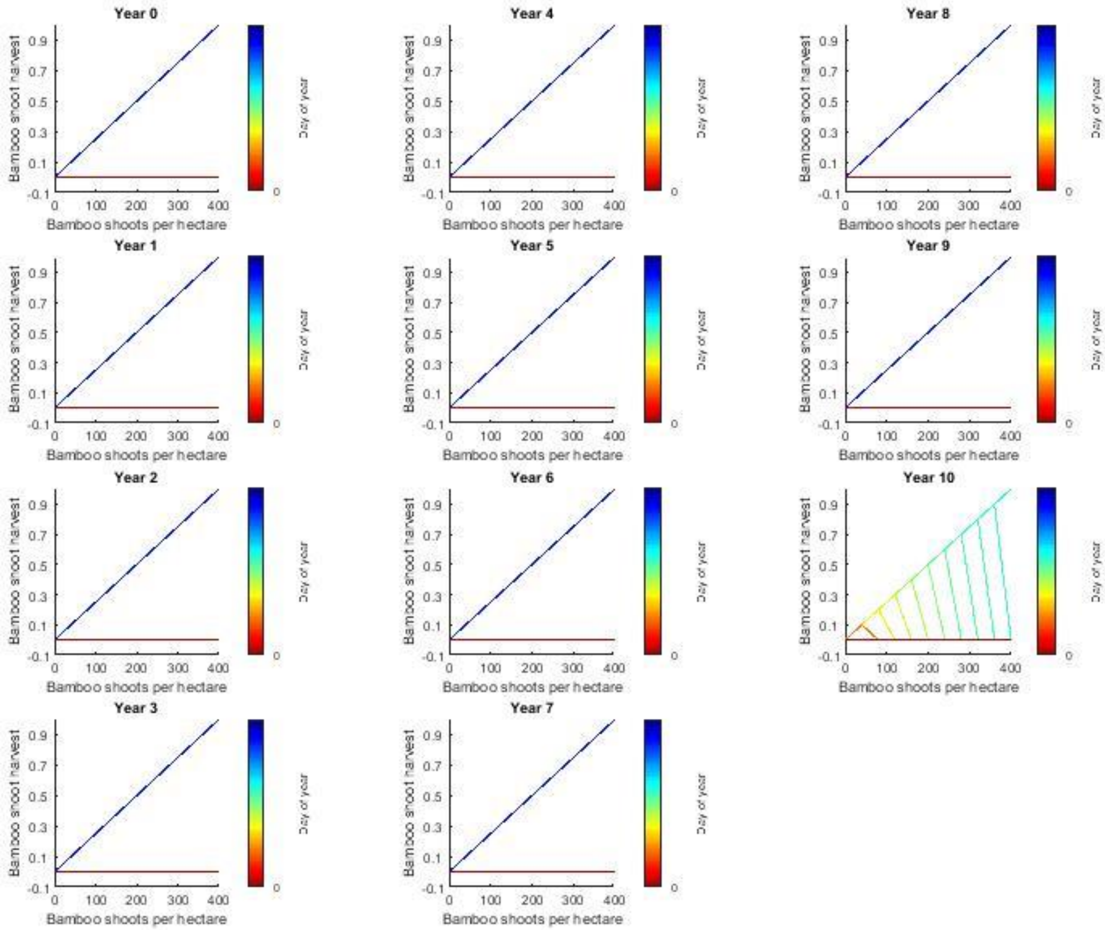
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



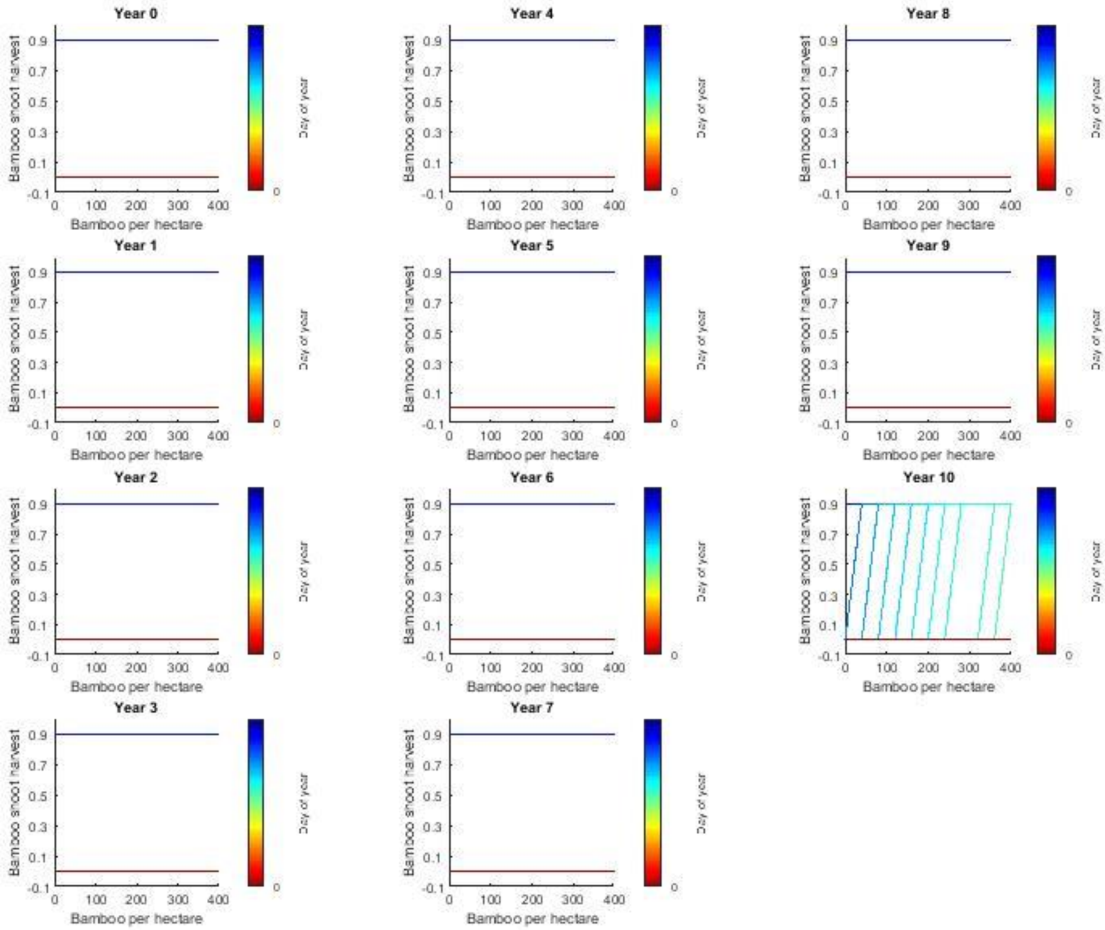
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



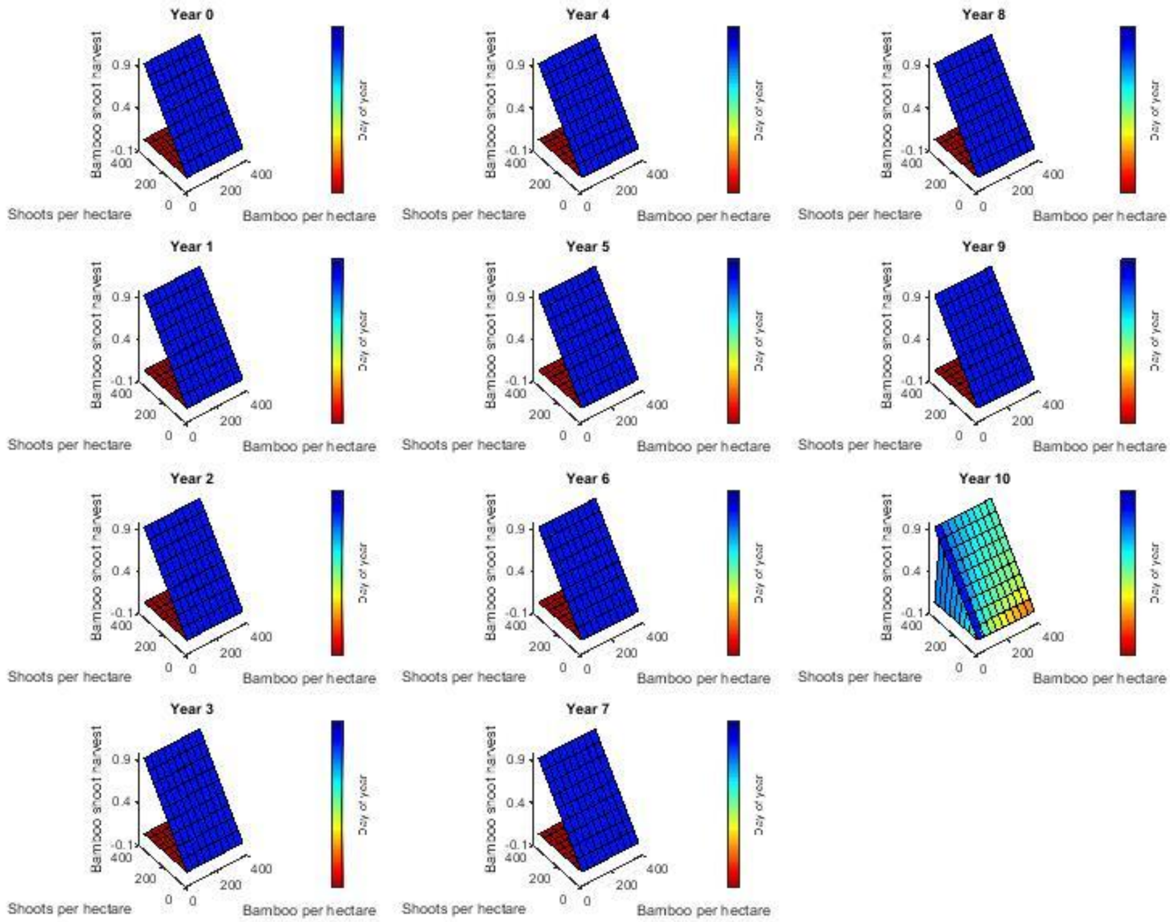
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



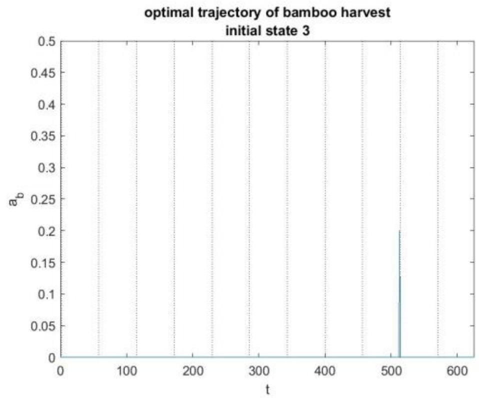
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

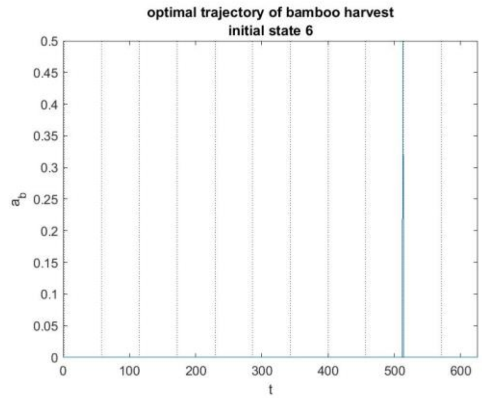


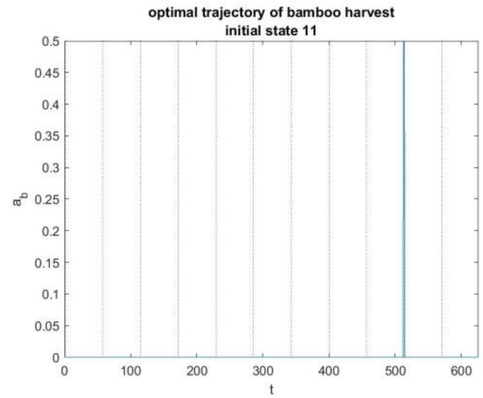
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

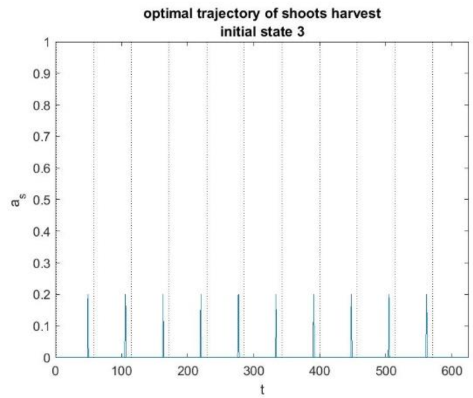


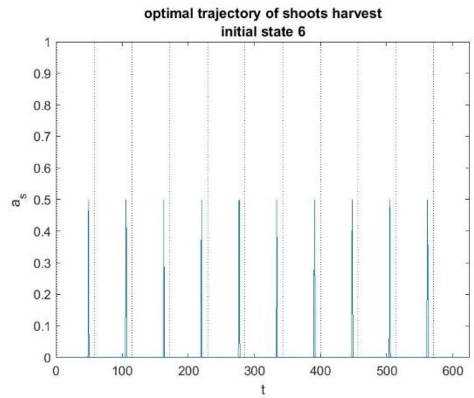
j) Optimal trajectories for each action and state variable over 11 years

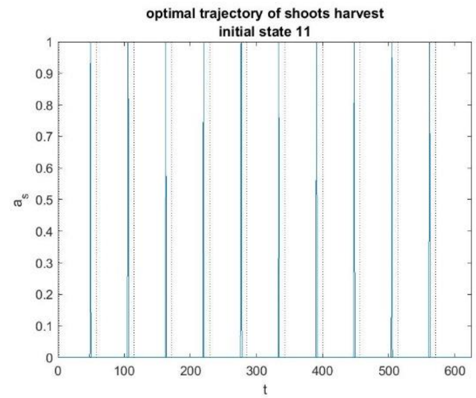


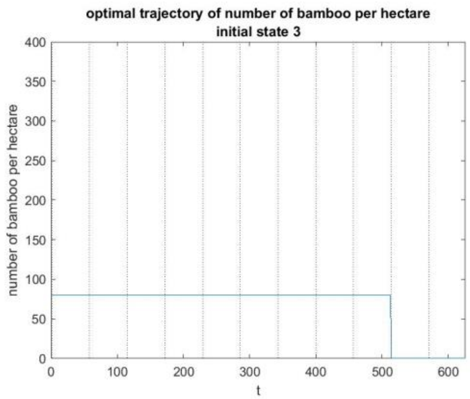
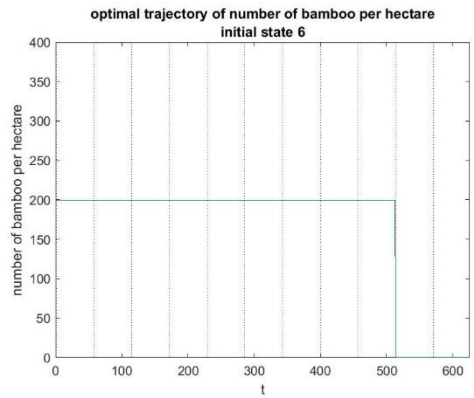
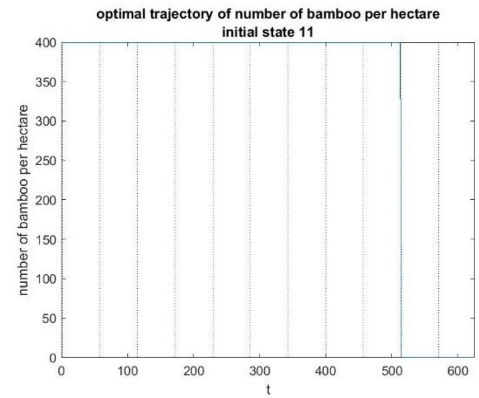











optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t

optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t

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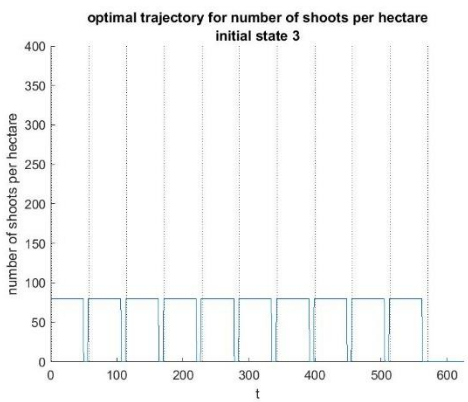
initial state 11
number of bamboo per hectare
t

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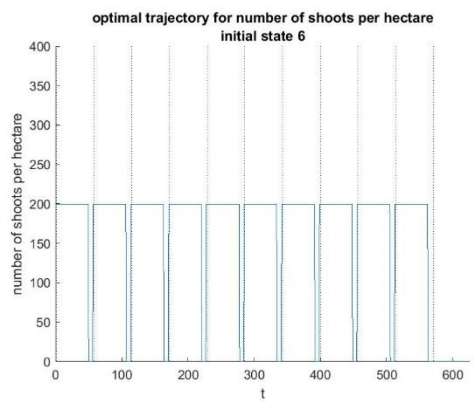
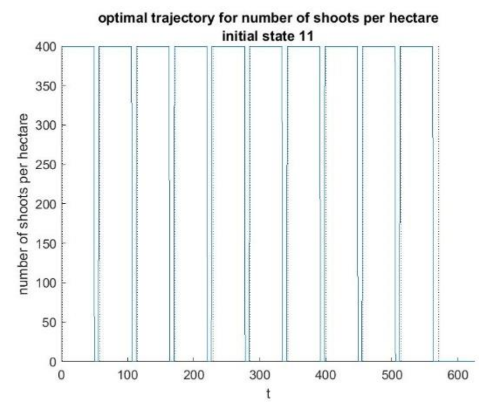
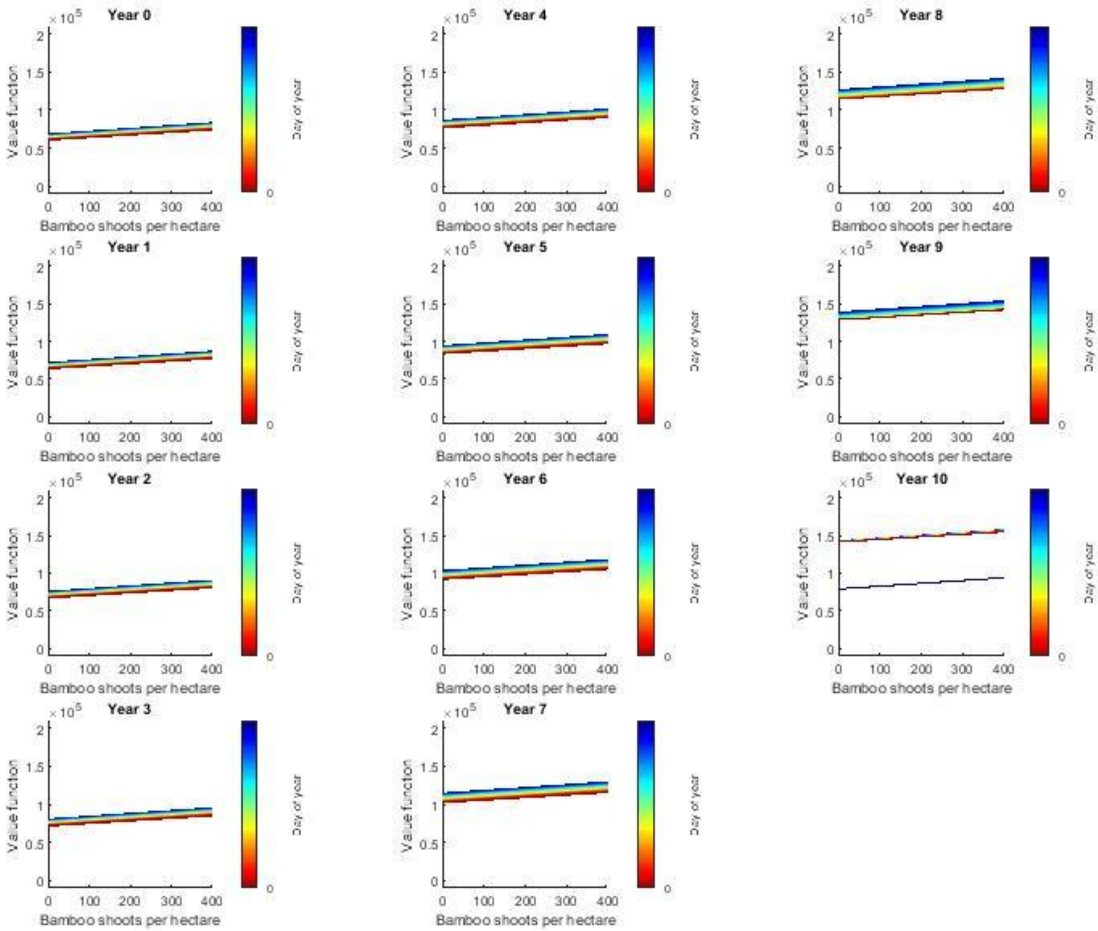
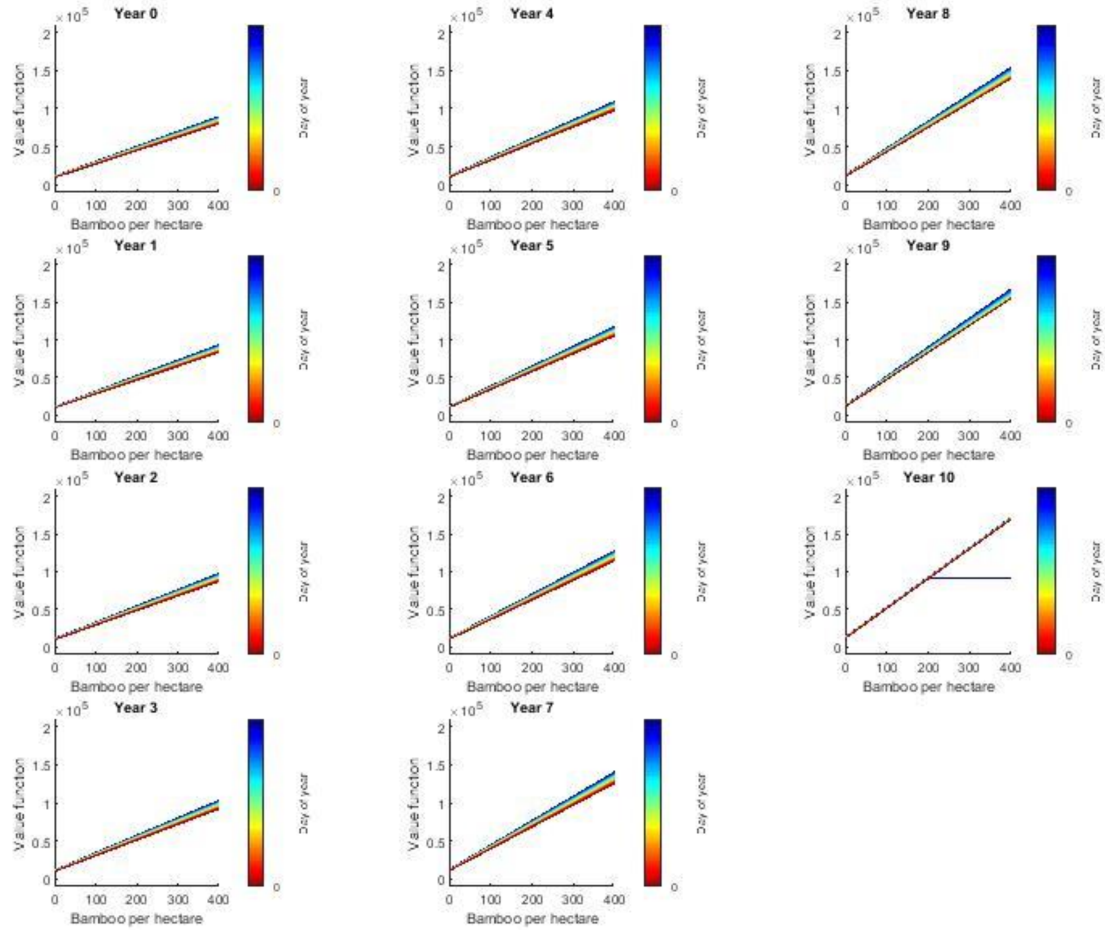
initial state 3
number of shoots per hectare
t

optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

Figure 4: Deterministic Model, Specification 8: $p_b = 30$ and $p_s = 20$

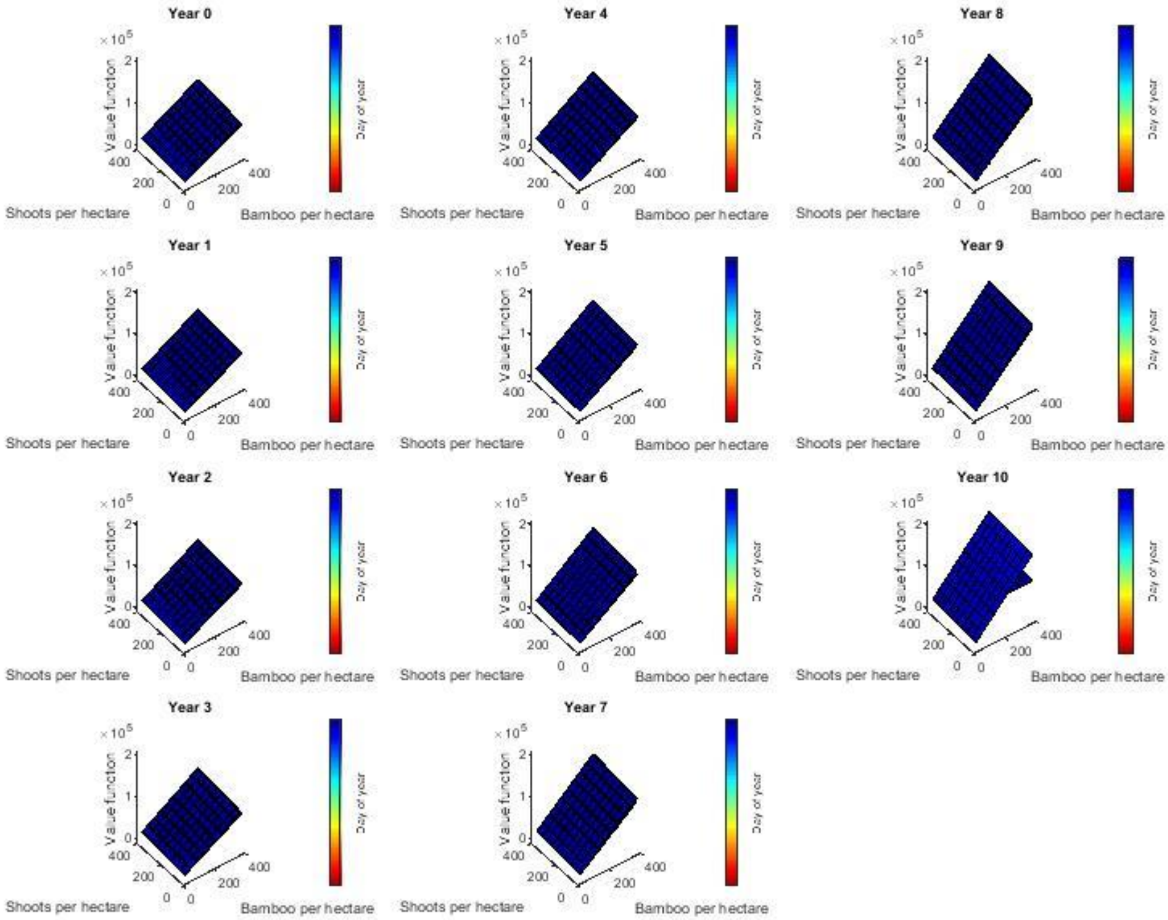
a) Value function as function of bamboo shoots per hectare



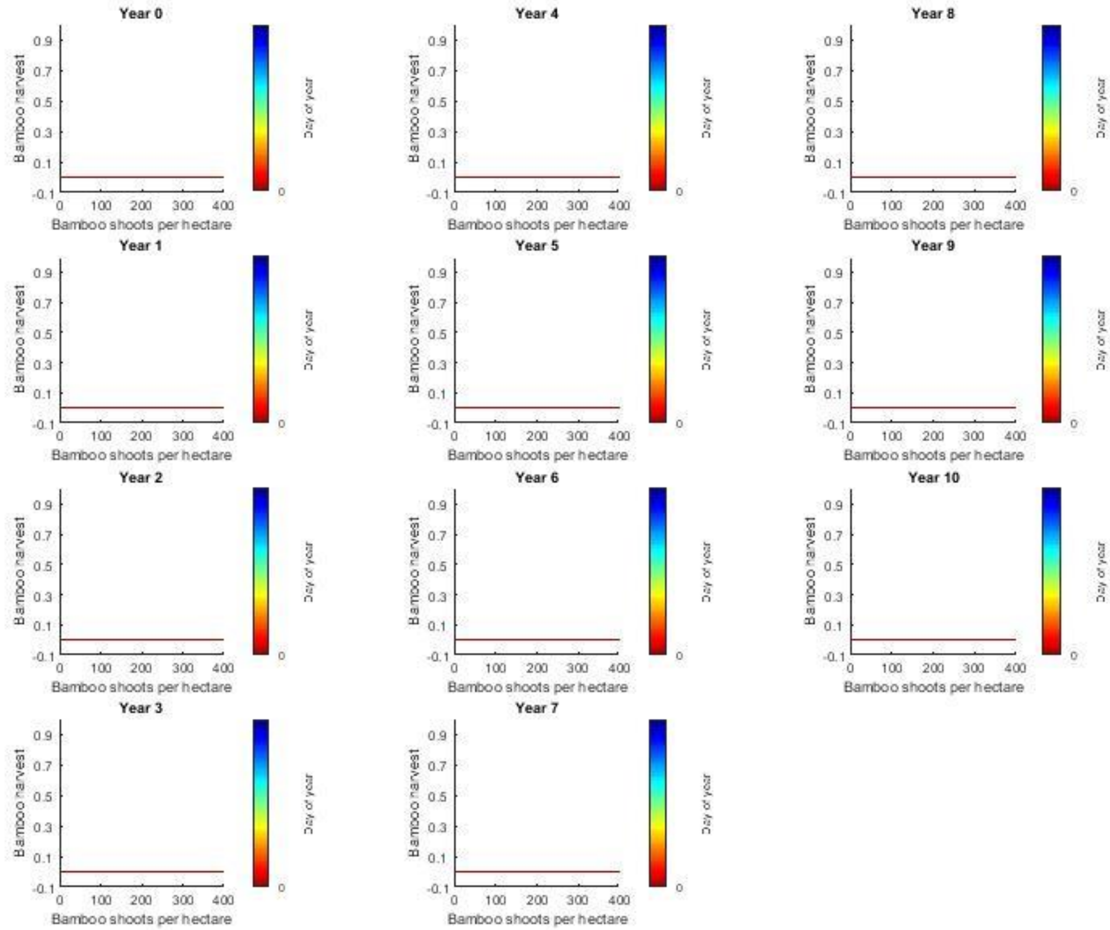
b) Value function as function of bamboo stem per hectare



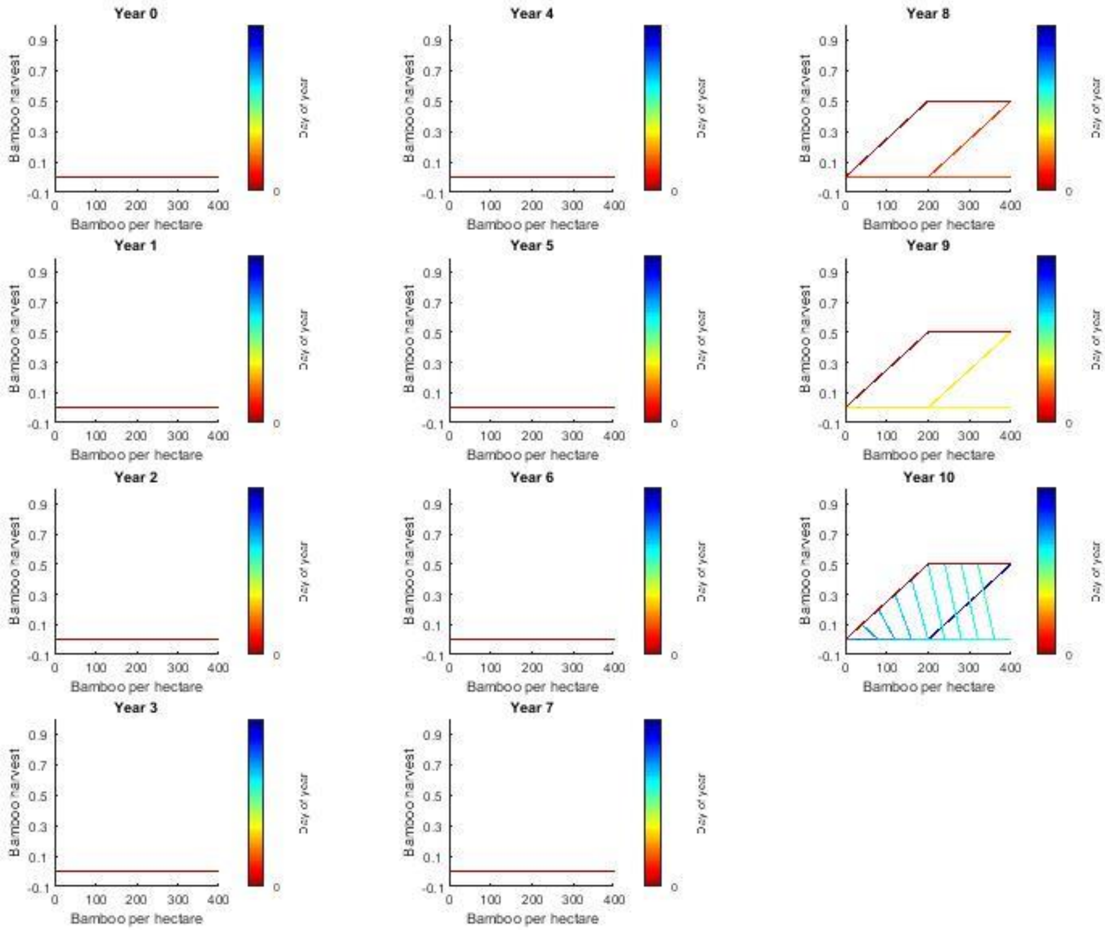
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



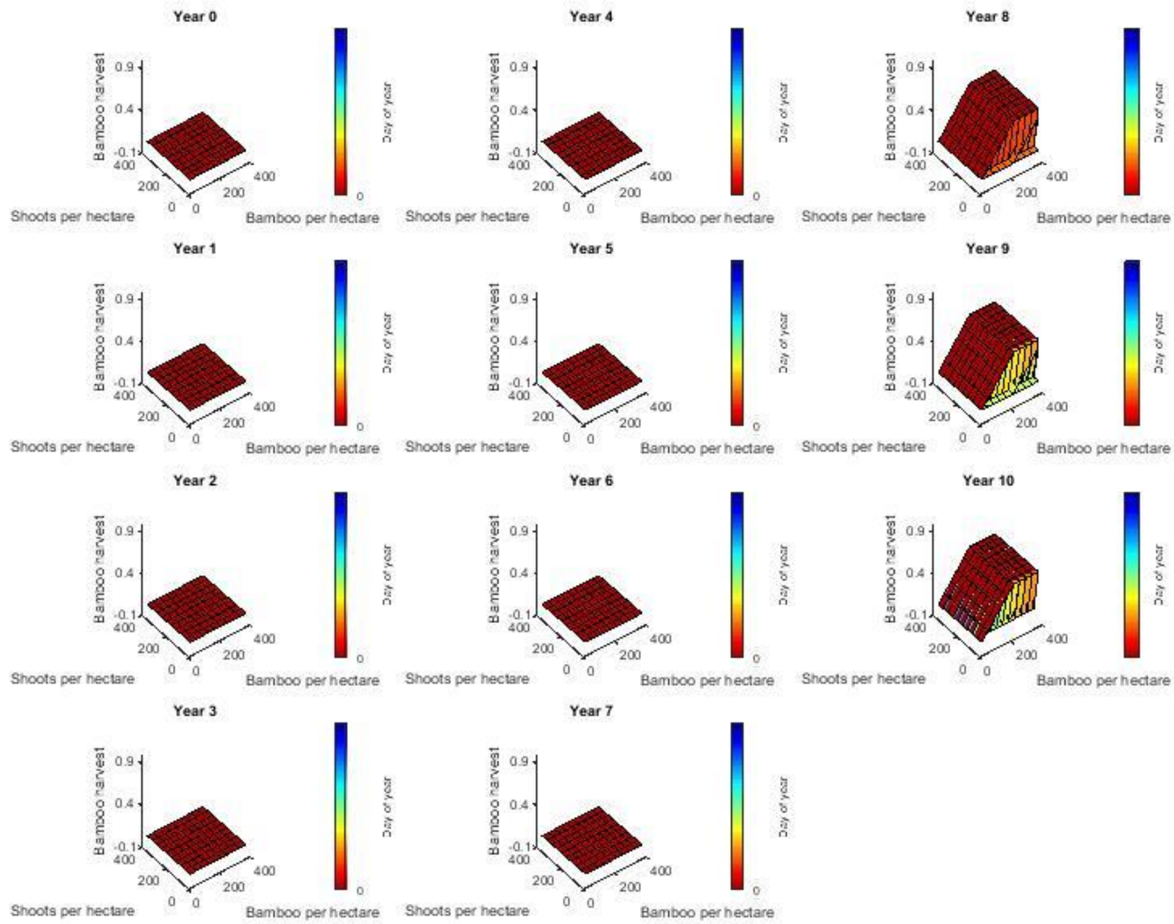
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



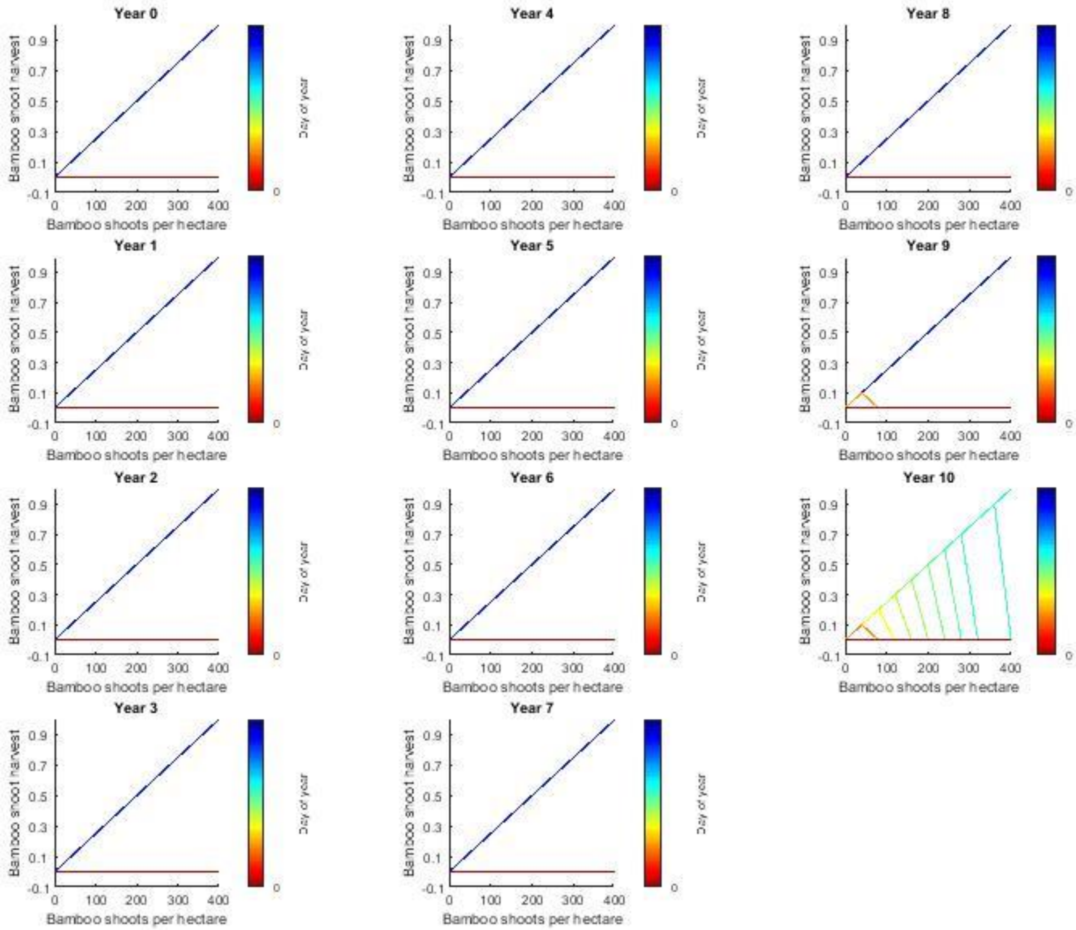
e) **Bamboo stem harvest policy function as function of bamboo stem per hectare**



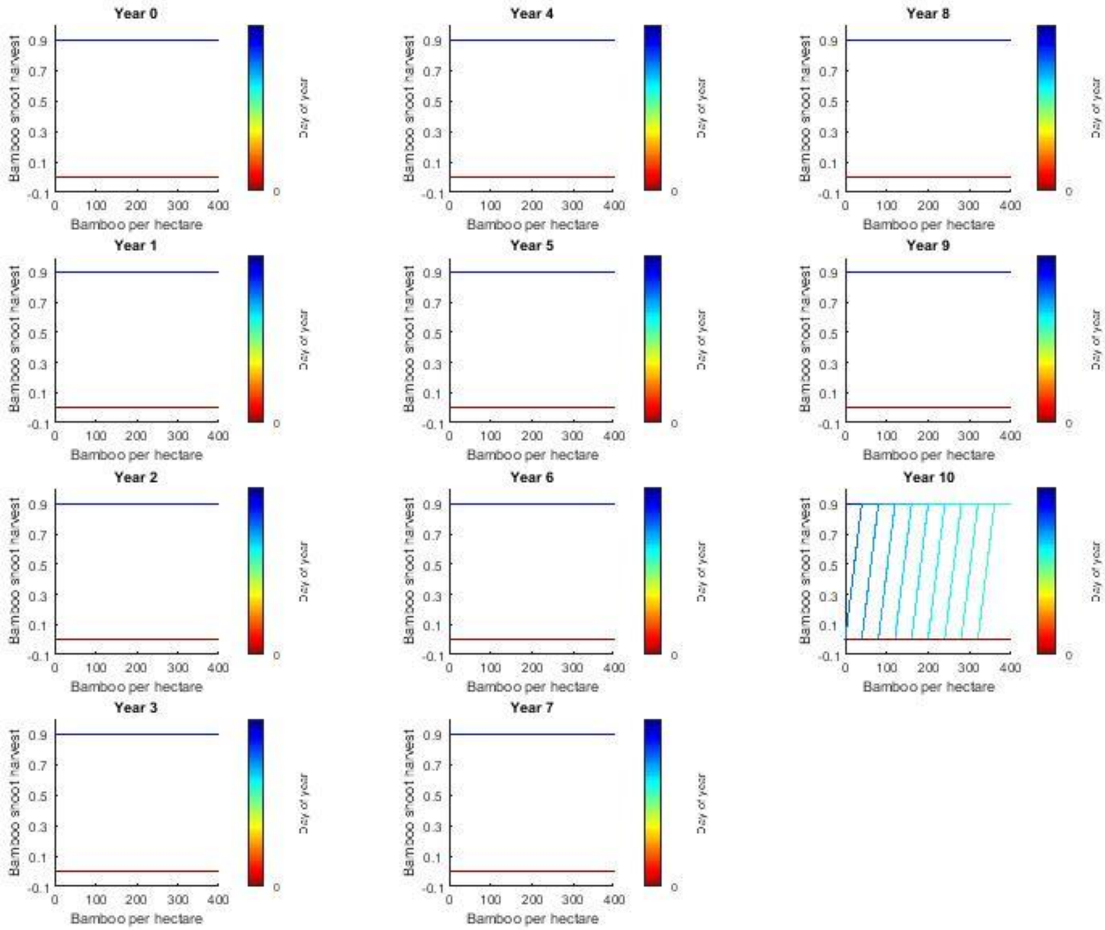
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



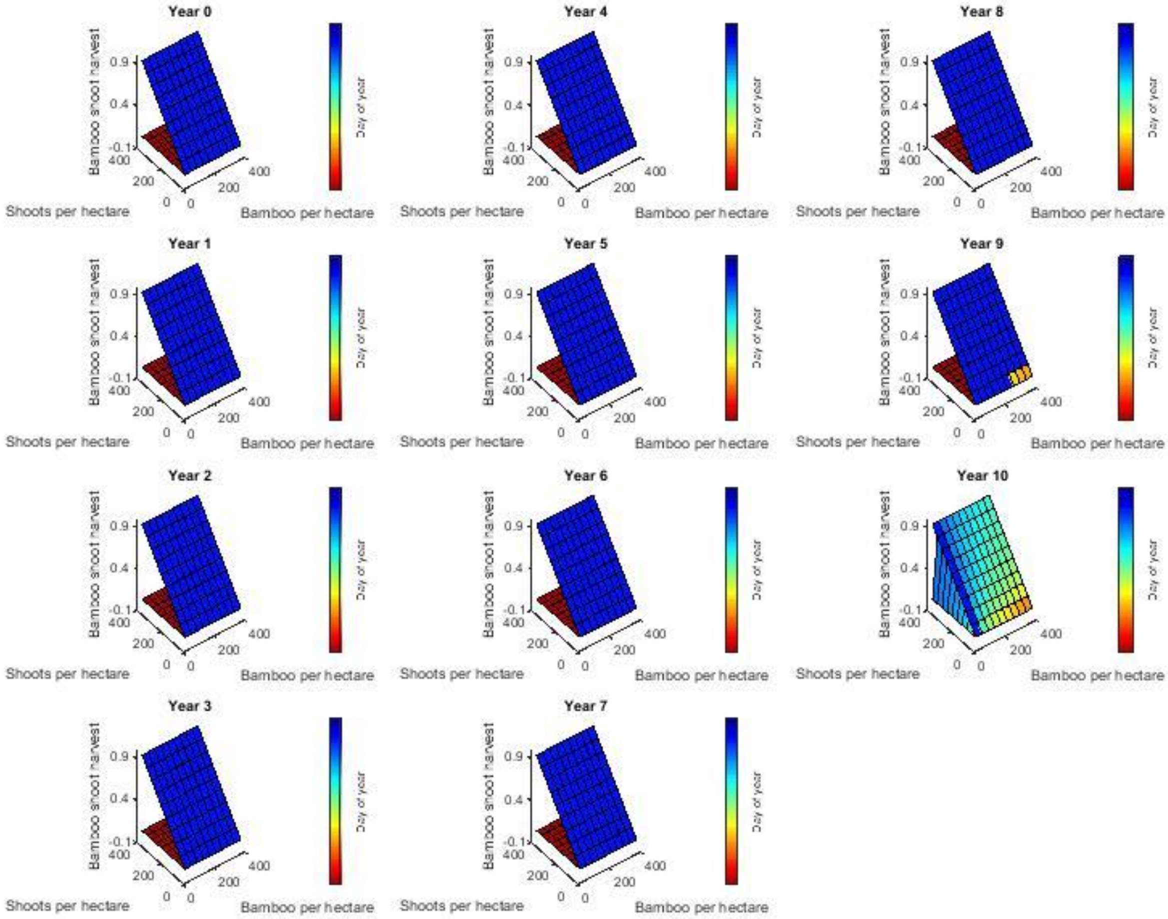
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



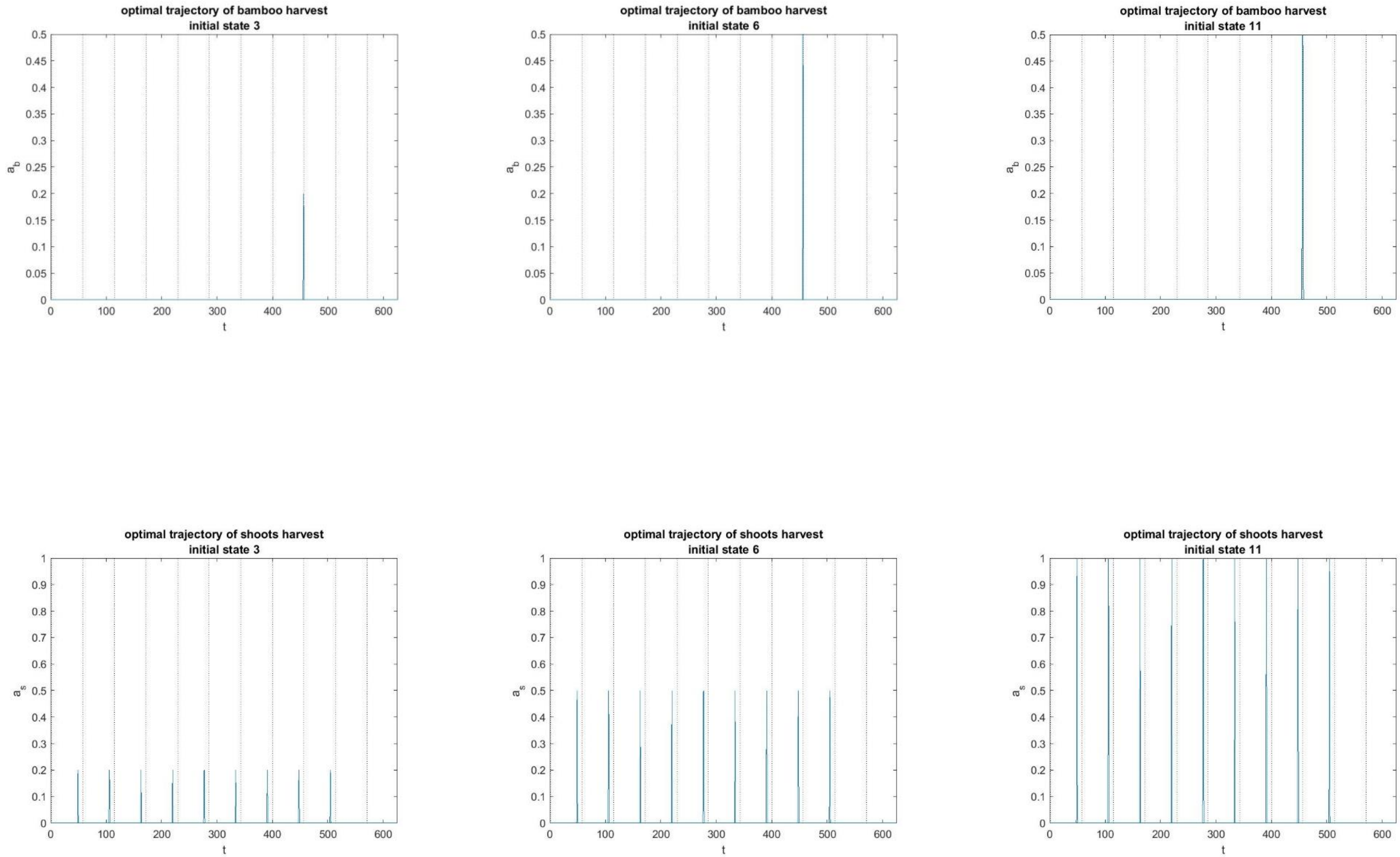
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

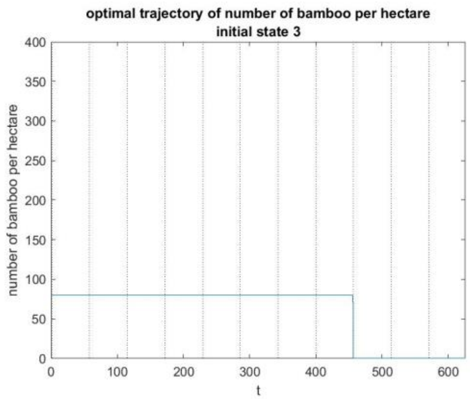


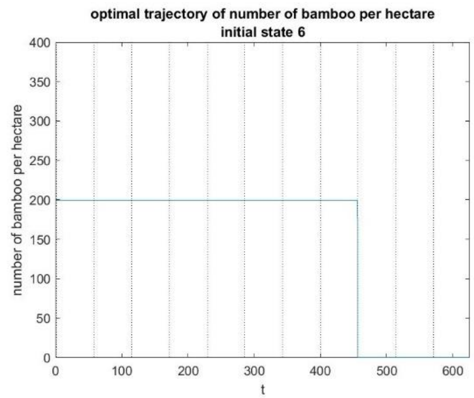
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

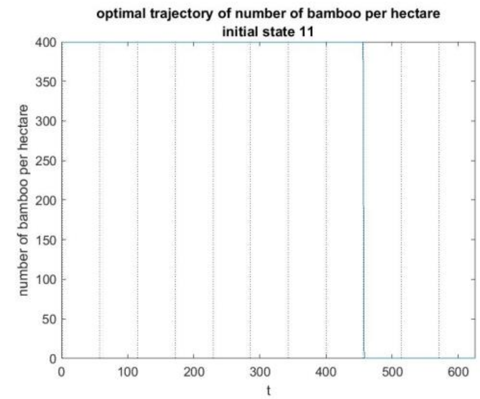


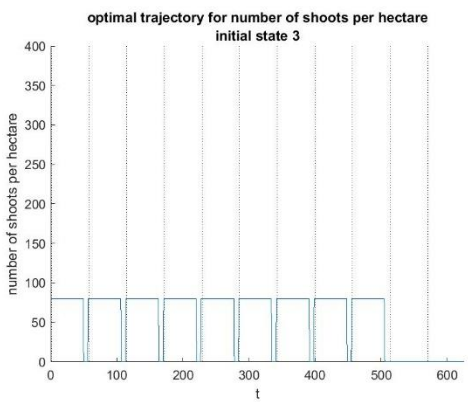
j) Optimal trajectories for each action and state variable over 11 years

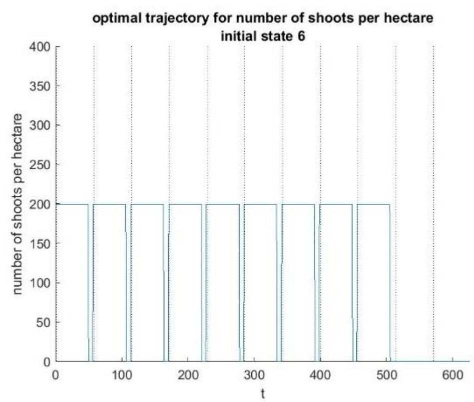



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

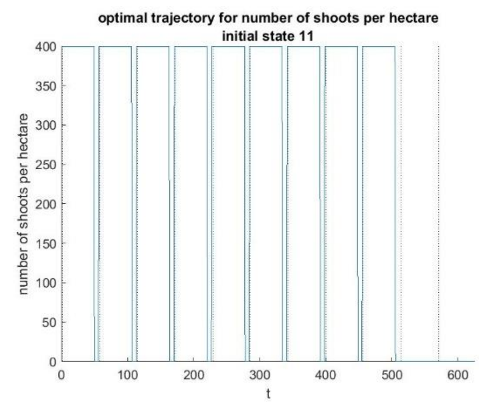
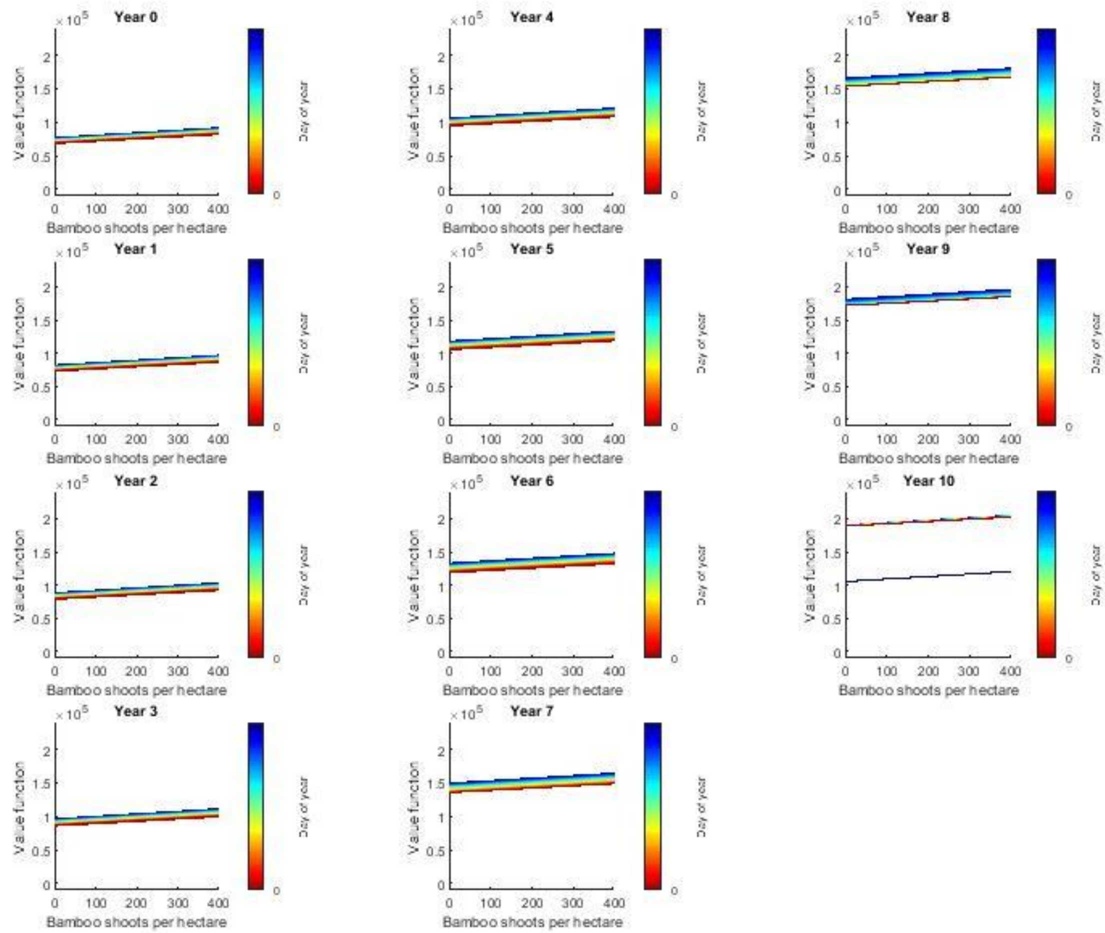
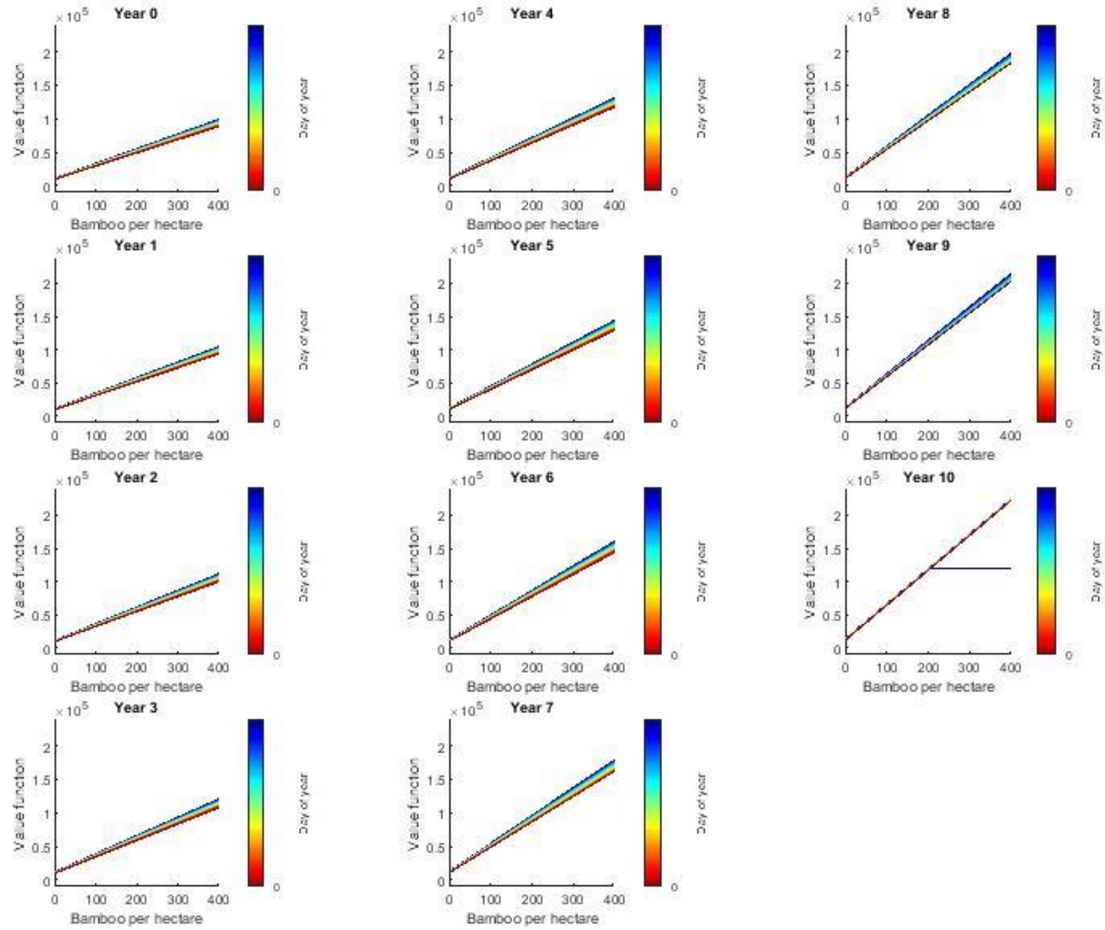

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

Figure 5: Deterministic Model, Specification 9: $p_b = 40$ and $p_s = 20$

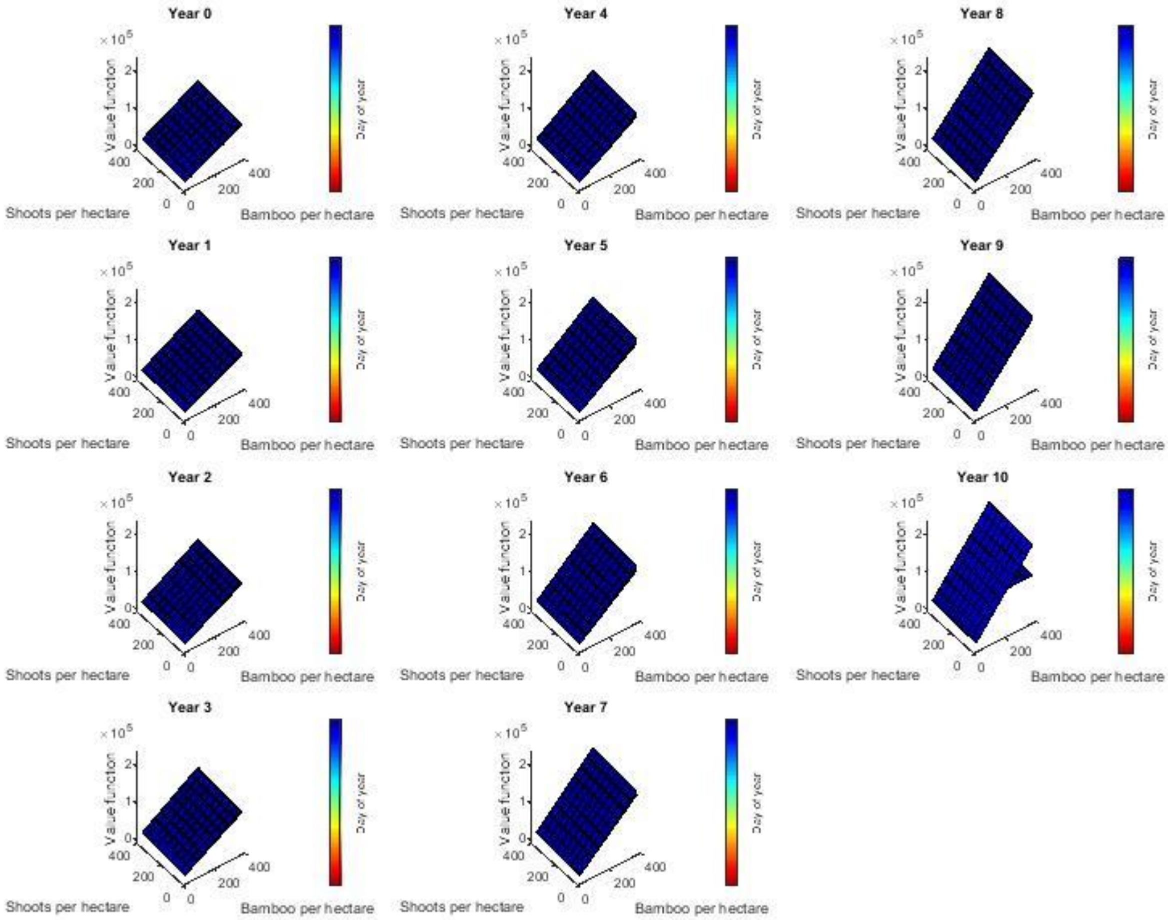
a) Value function as function of bamboo shoots per hectare



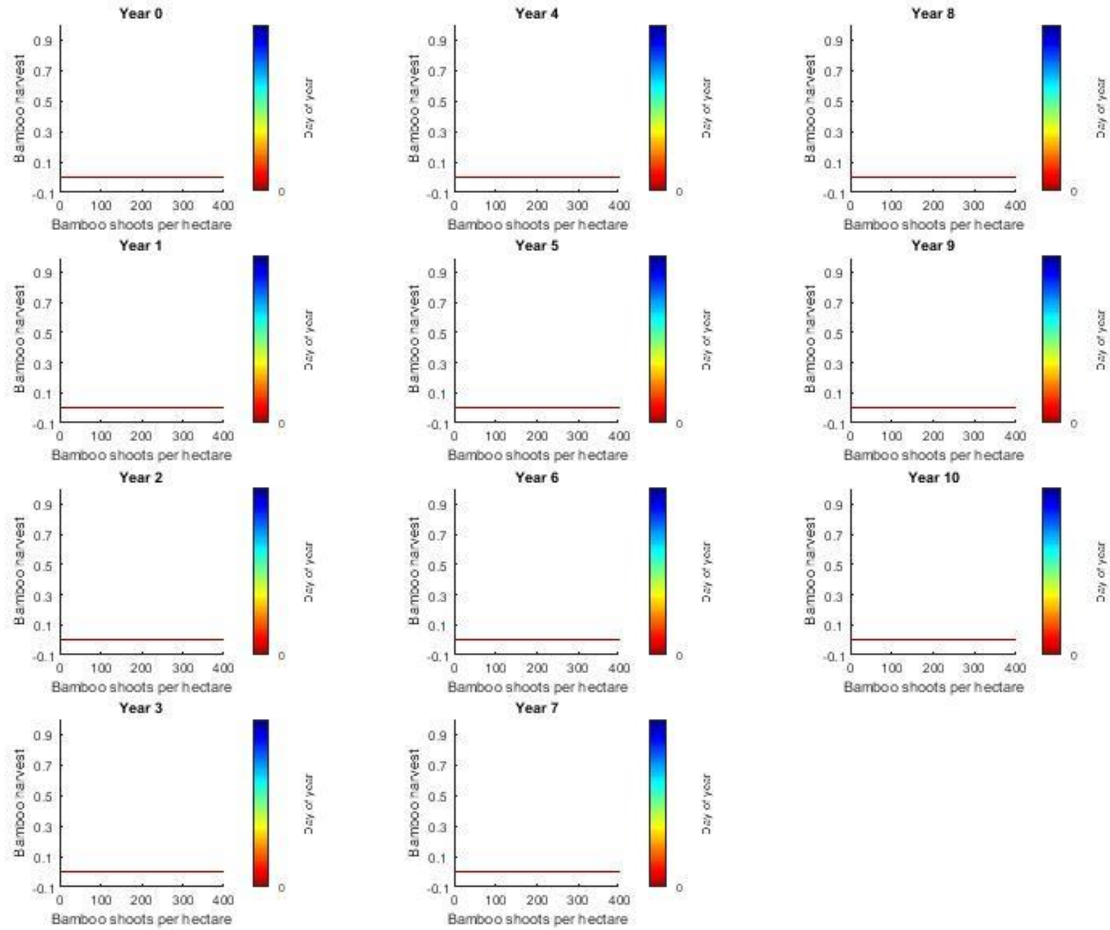
b) Value function as function of bamboo stem per hectare



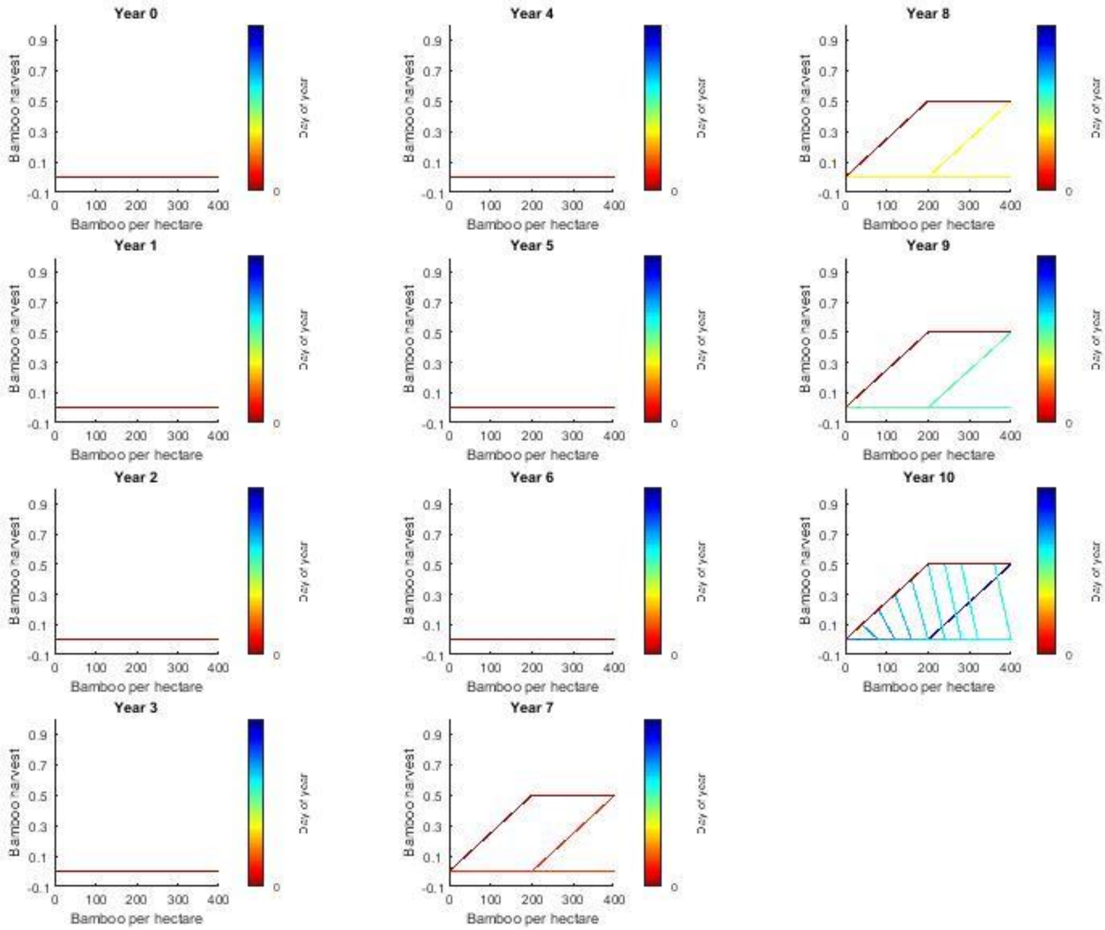
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



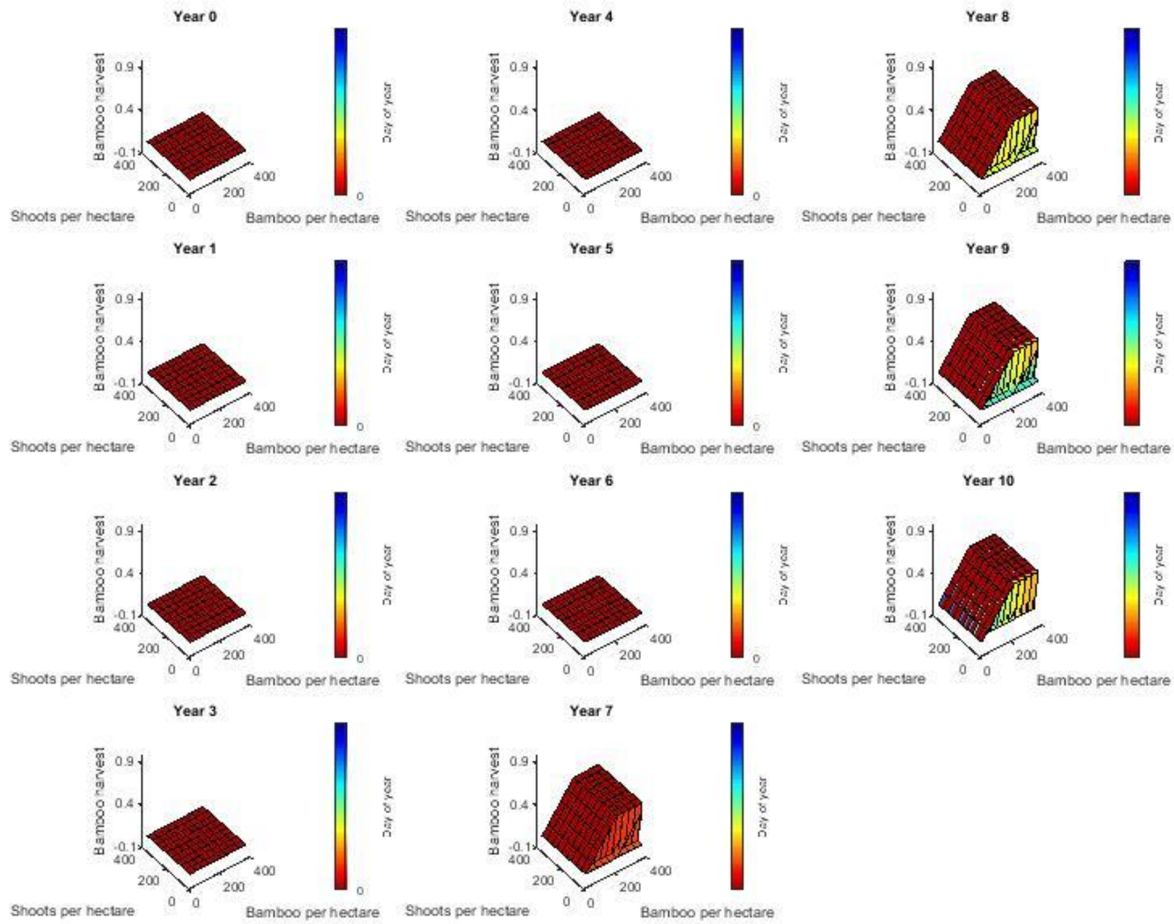
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



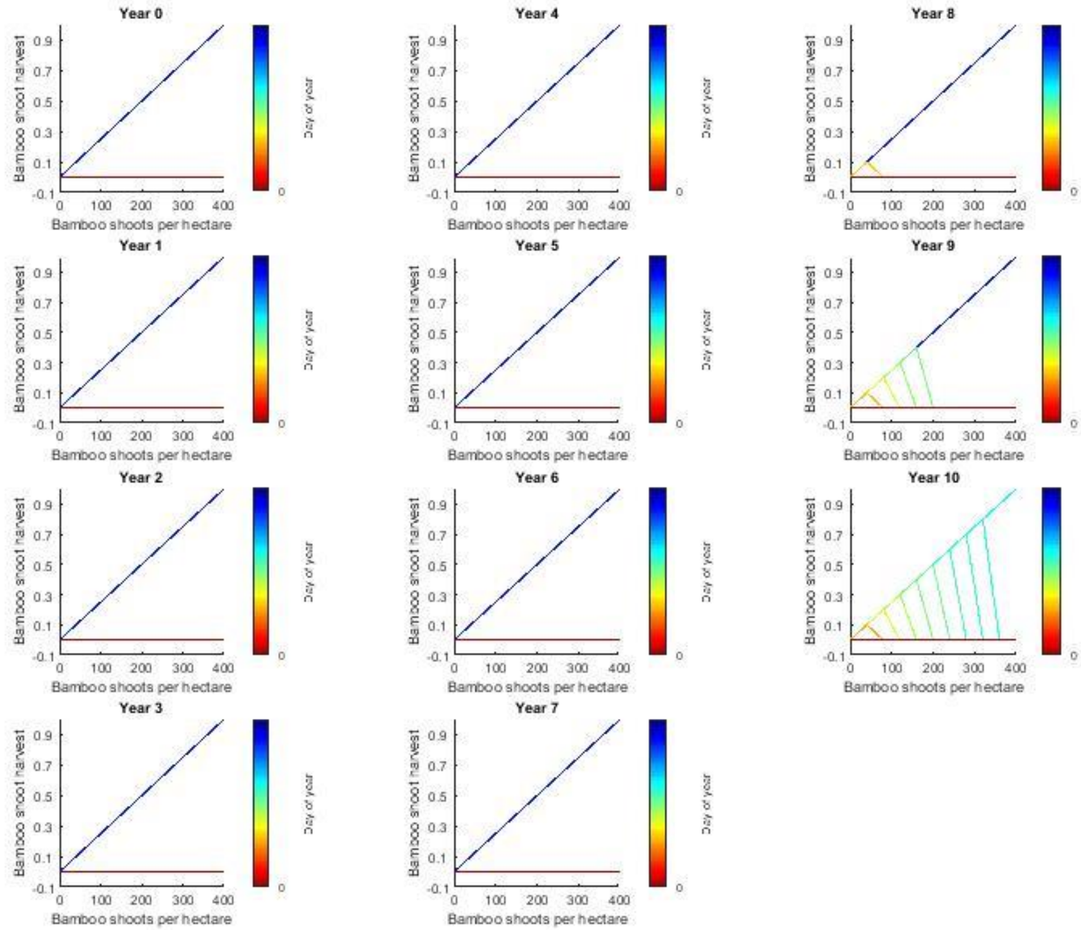
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



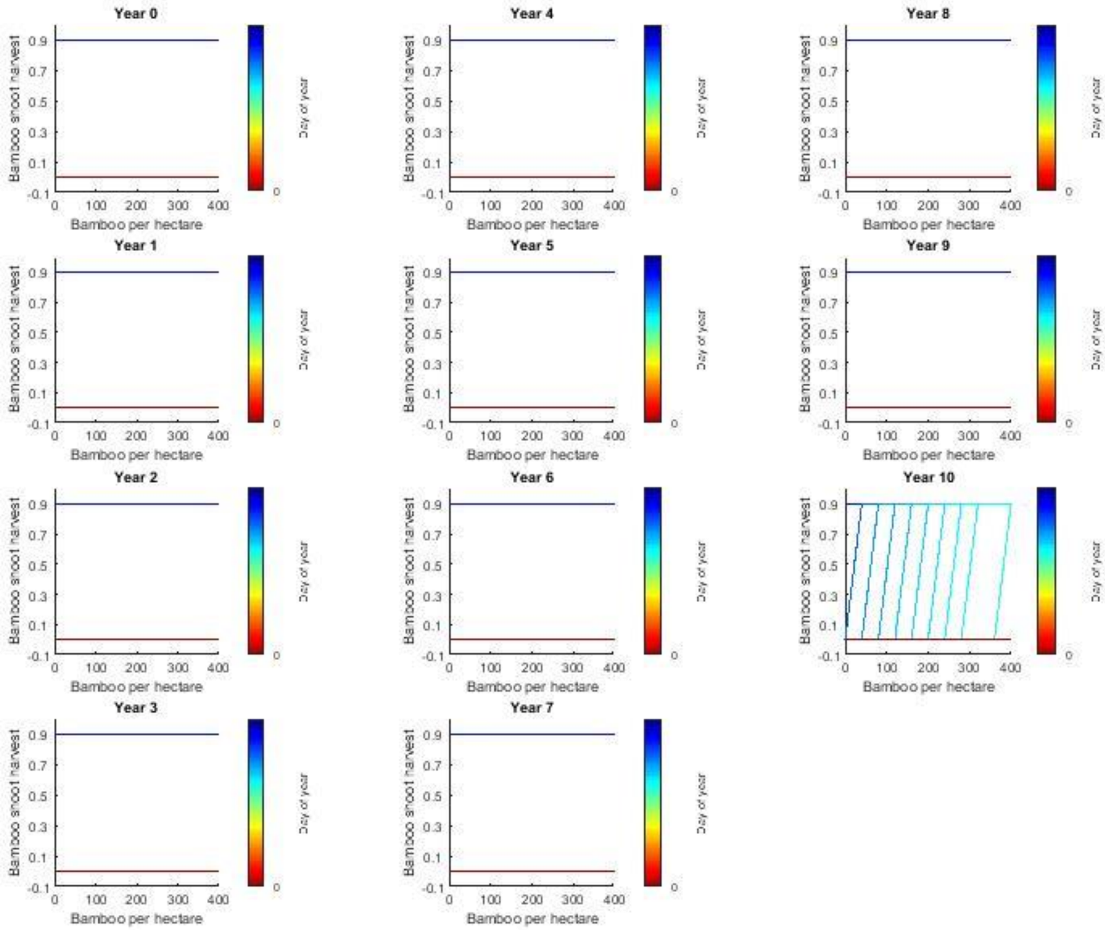
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



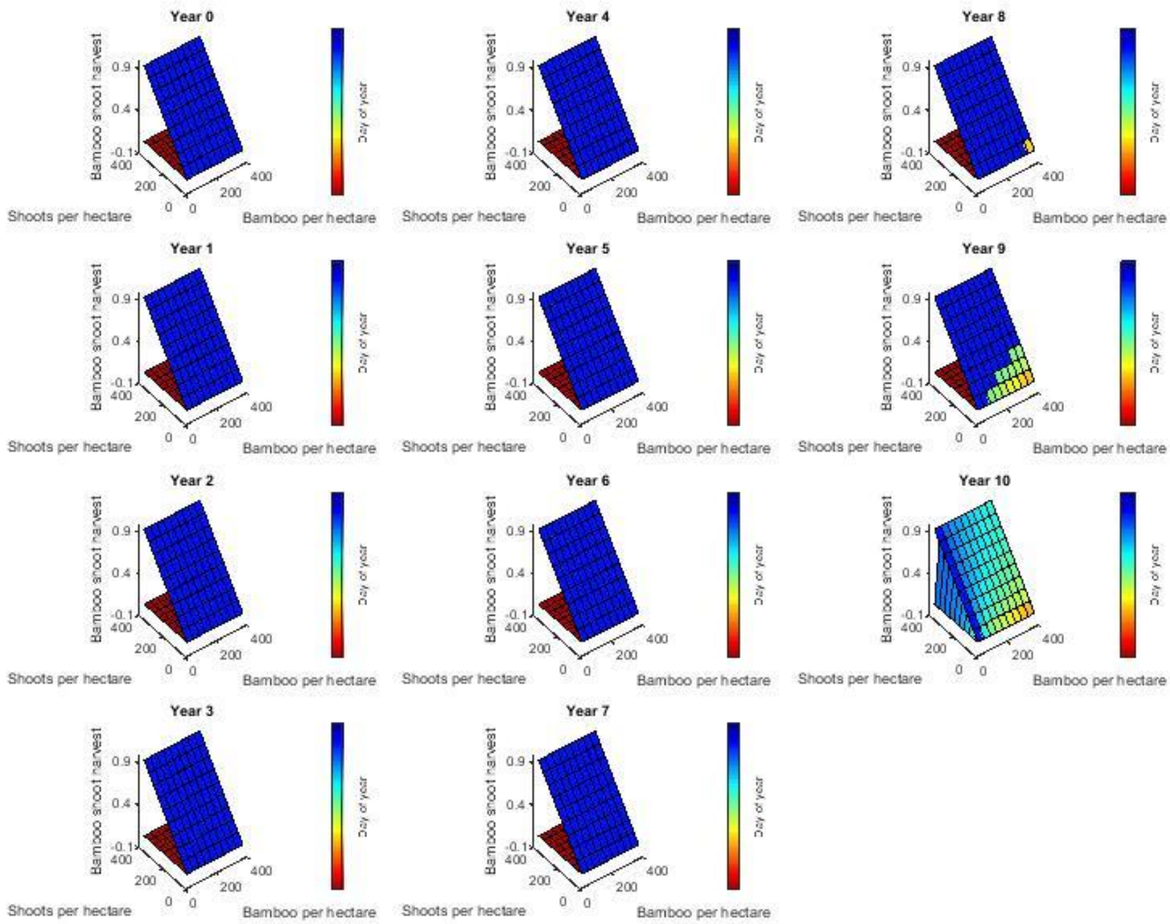
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



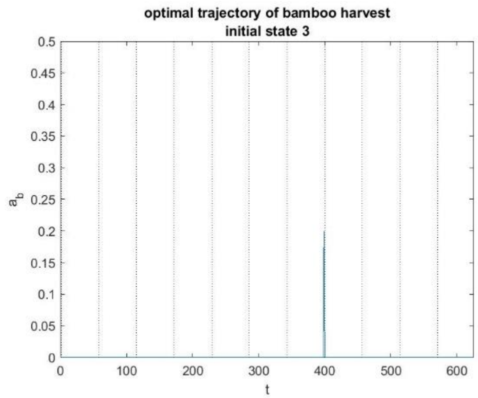
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

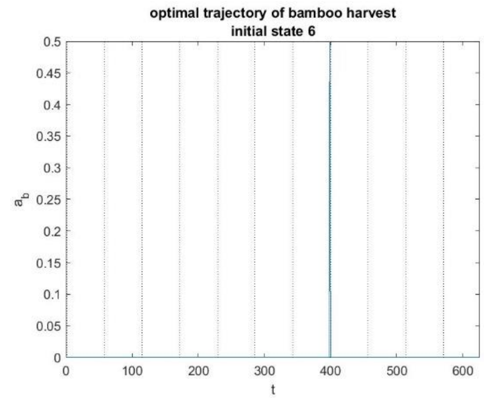


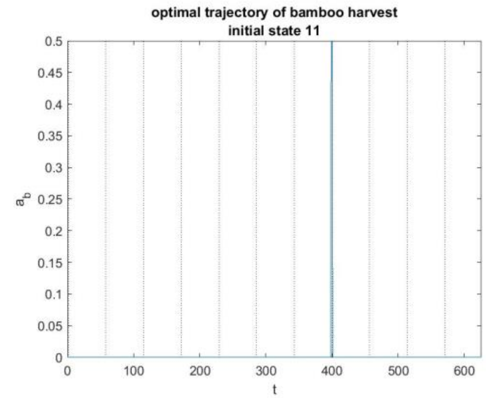
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

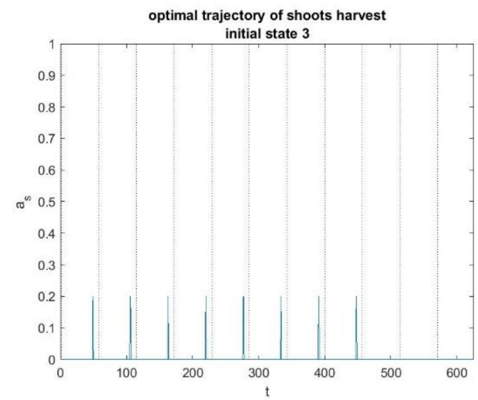


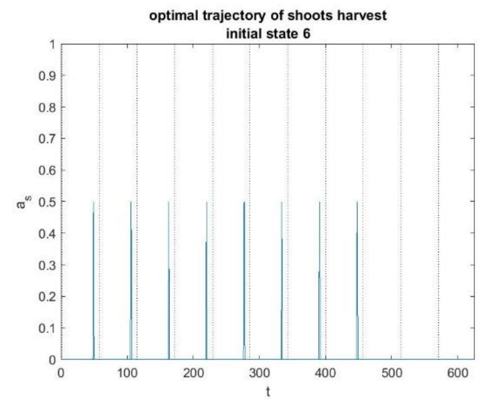
j) Optimal trajectories for each action and state variable over 11 years

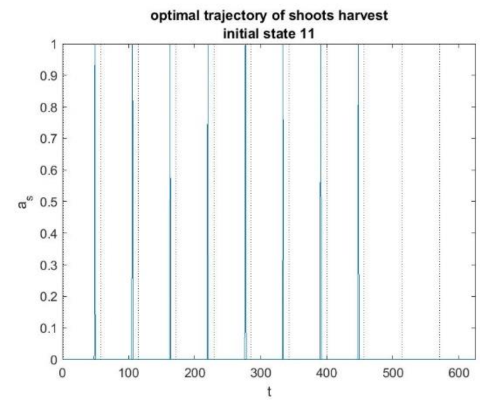


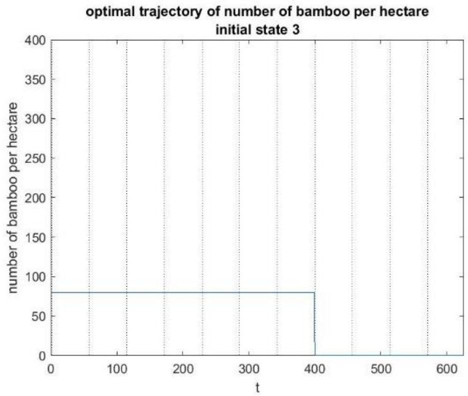


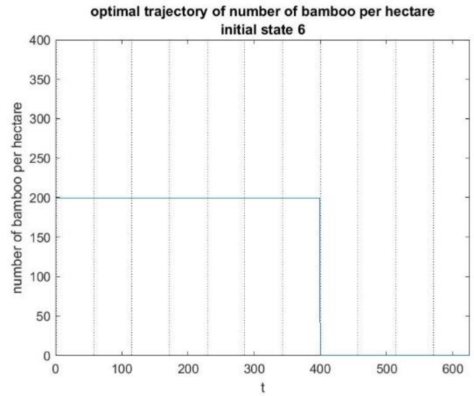


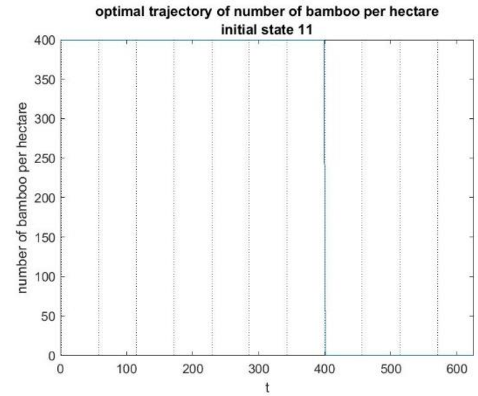


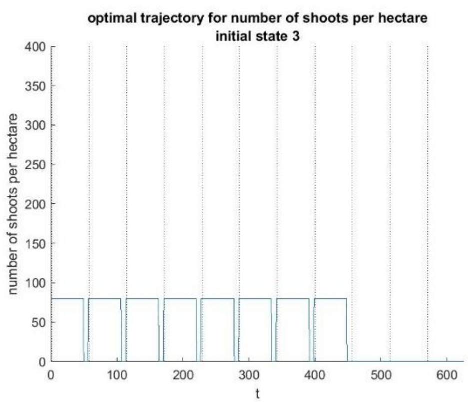


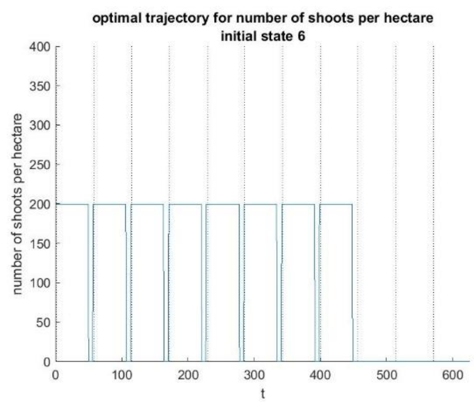



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

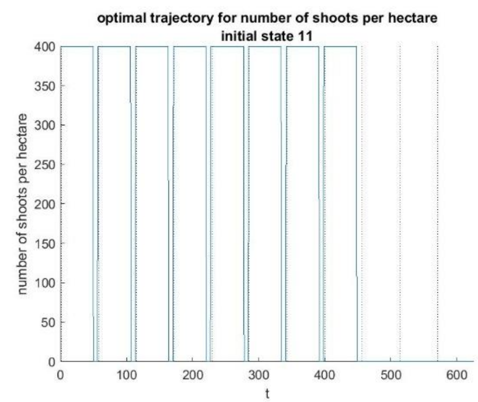
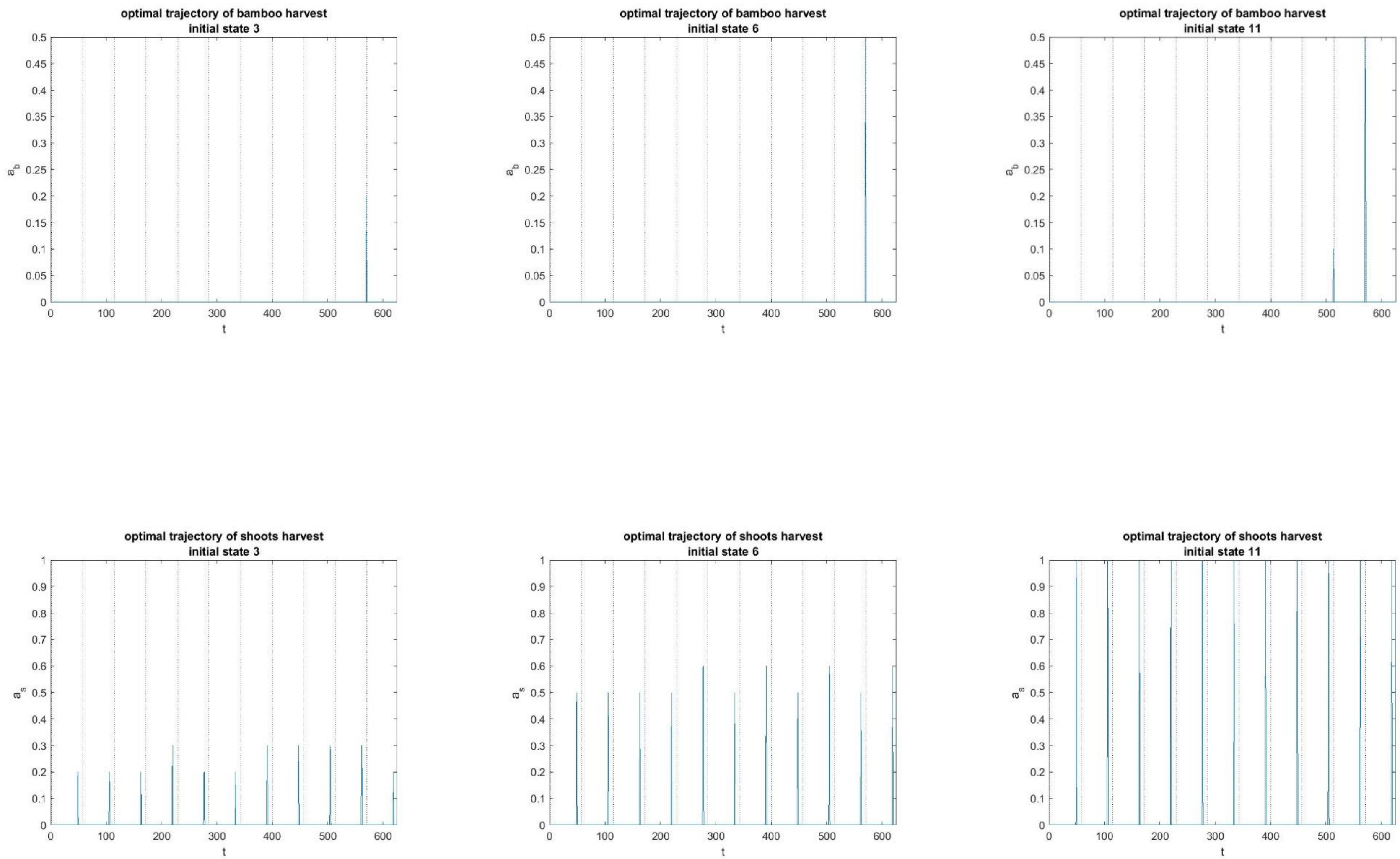
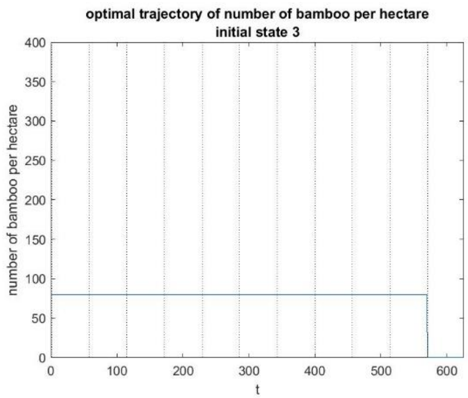
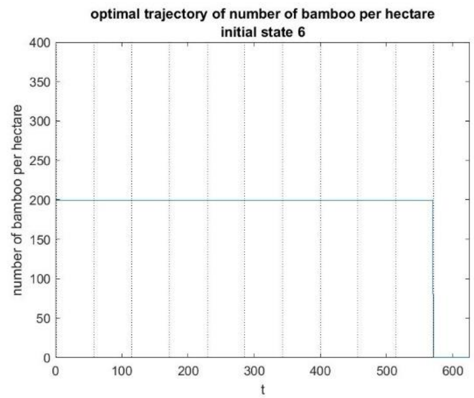
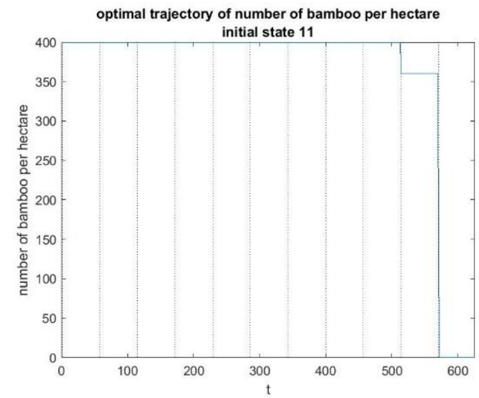

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

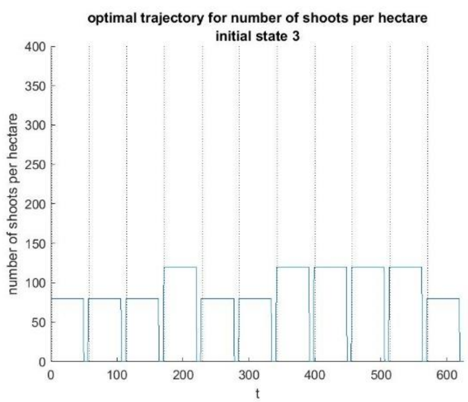
Figure 6: Stochastic Model, Specification 1, Version C, Set 1
a) Optimal trajectories for each action and state variable over 11 years

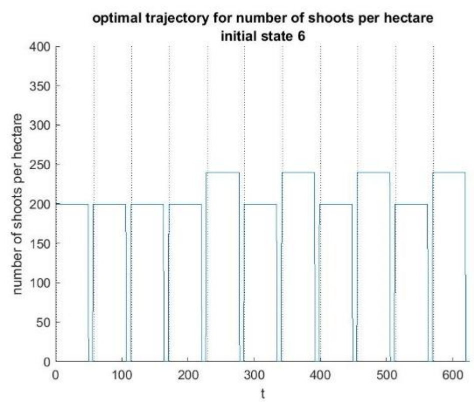



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

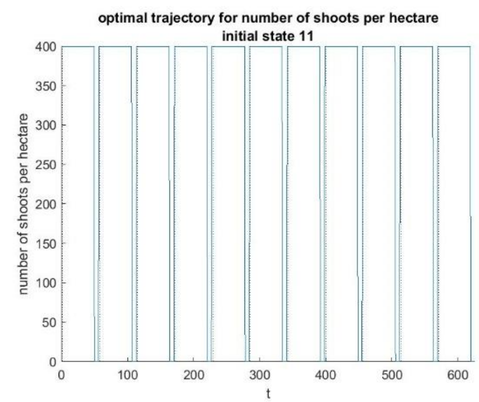
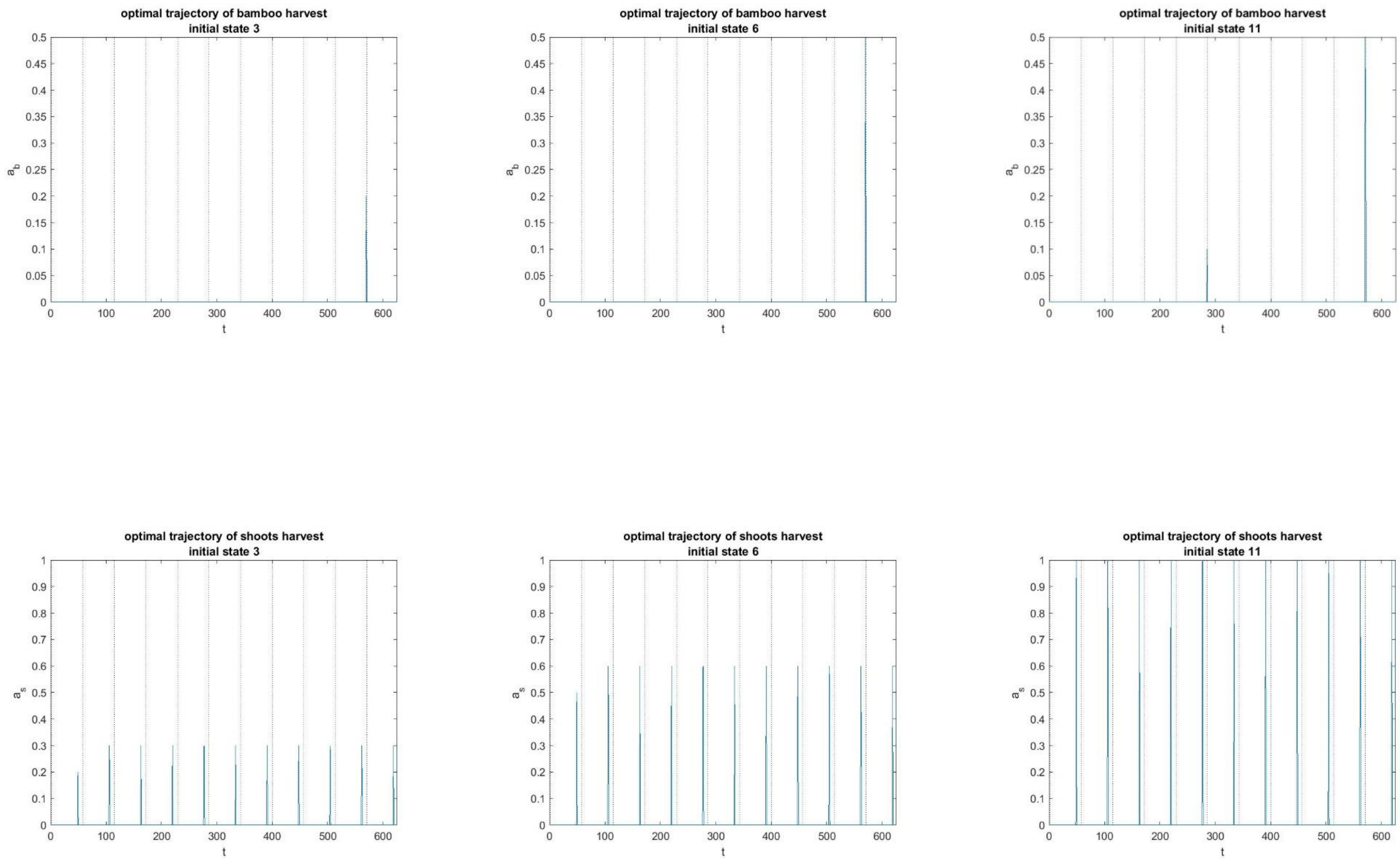
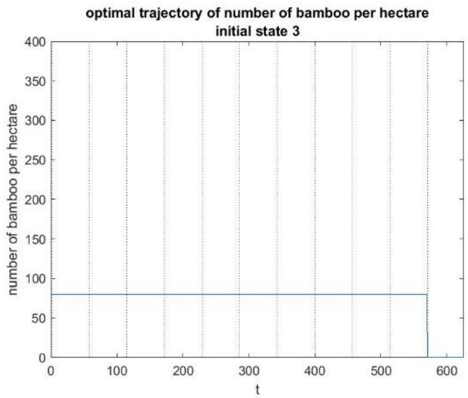
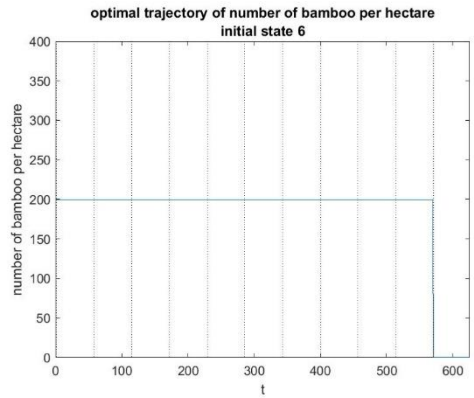
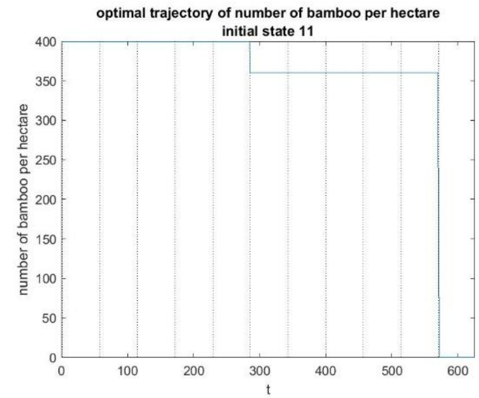

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

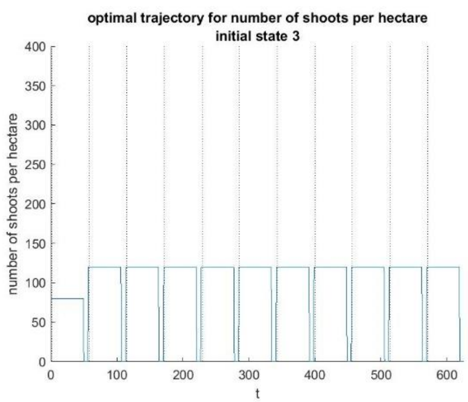
Figure 7: Stochastic Model, Specification 1, Version E, Set 1
a) Optimal trajectories for each action and state variable over 11 years

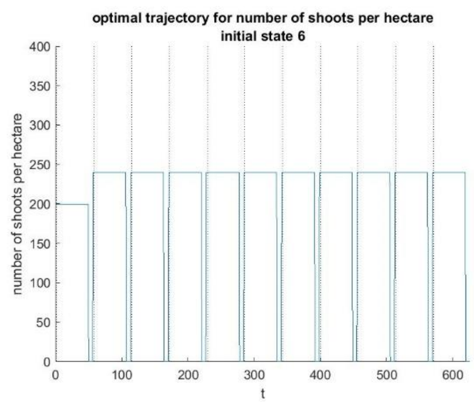



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

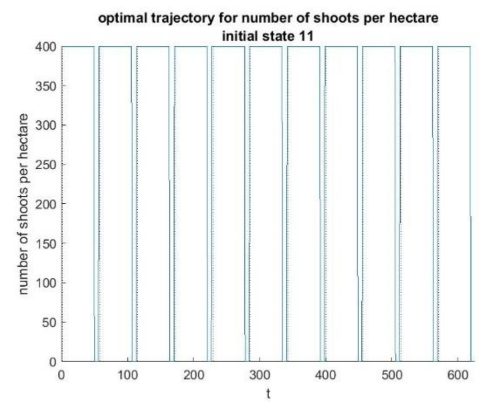
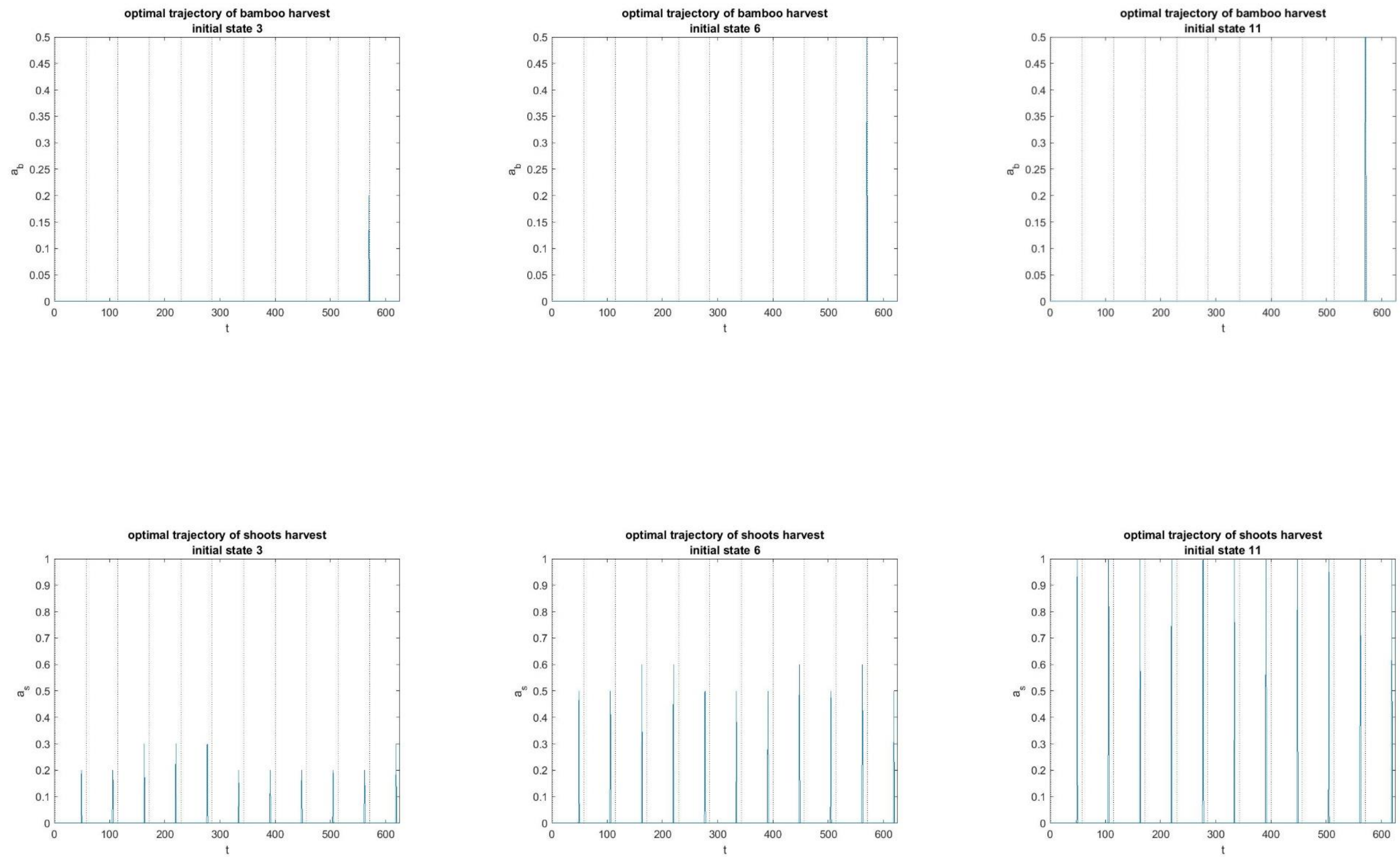
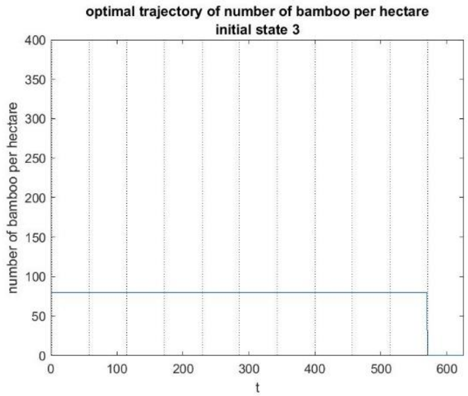
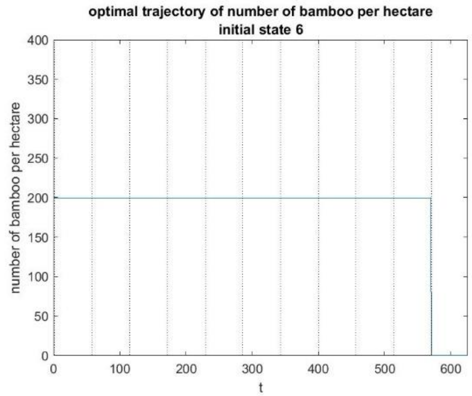
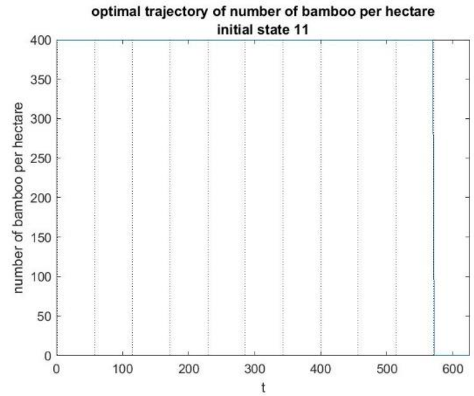

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

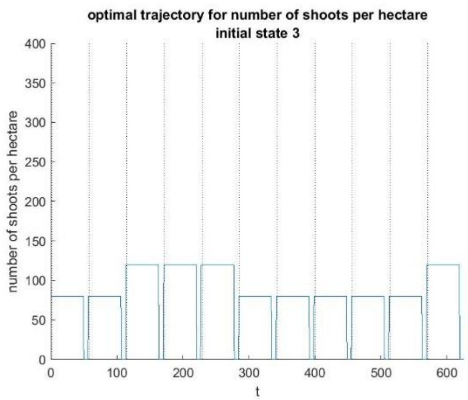
Figure 8: Stochastic Model, Specification 5, Version C, Set 1
a) Optimal trajectories for each action and state variable over 11 years

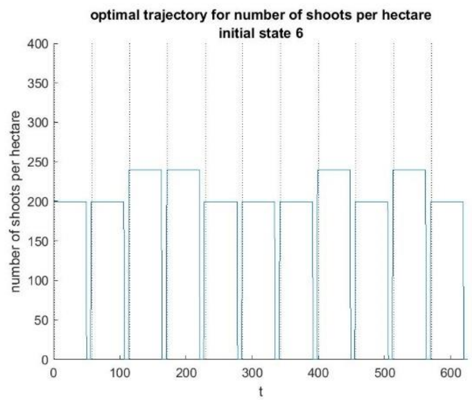



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

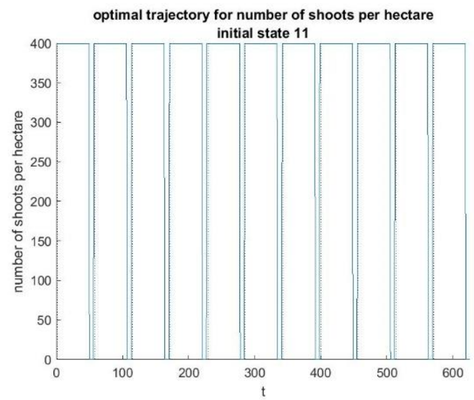
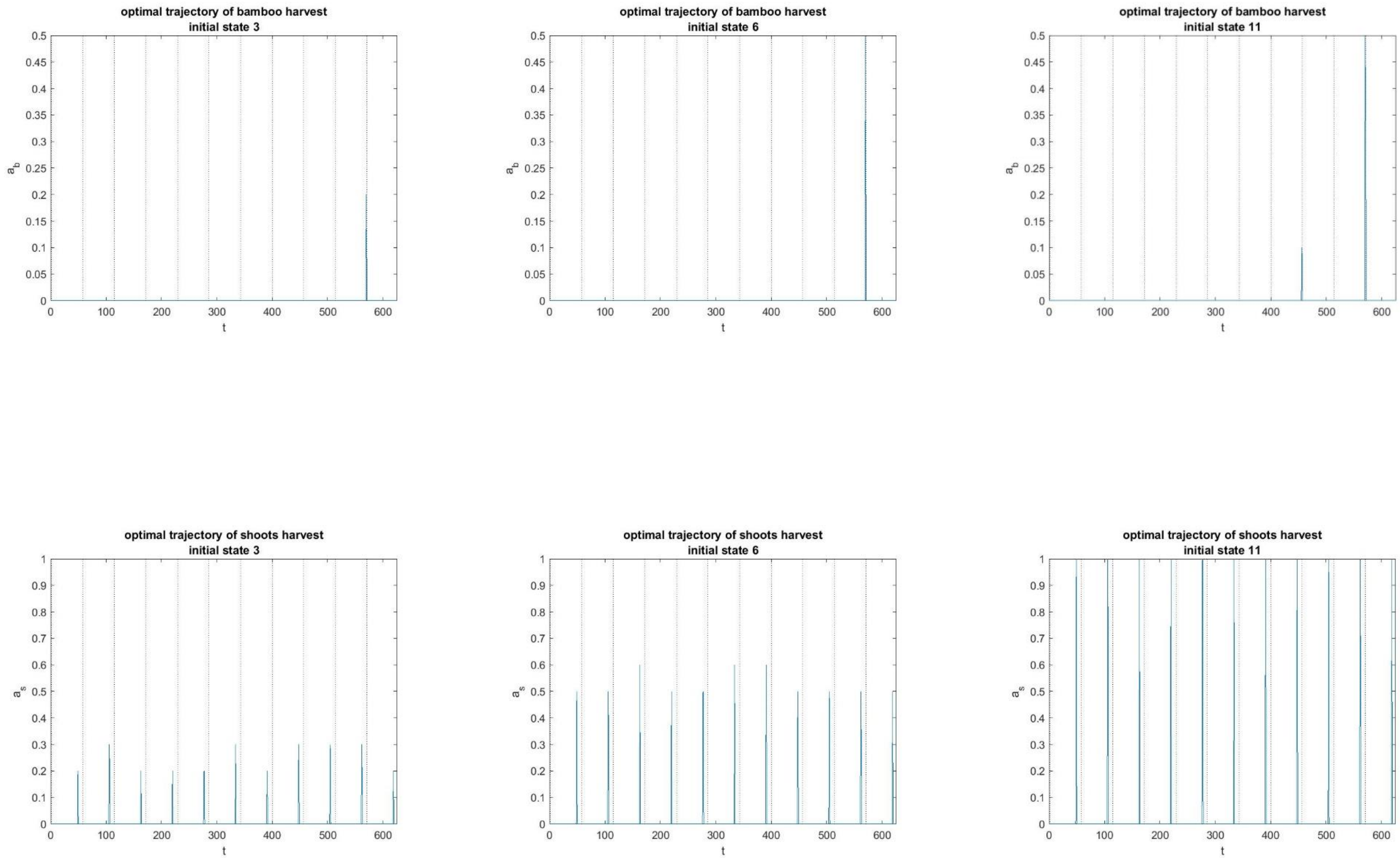
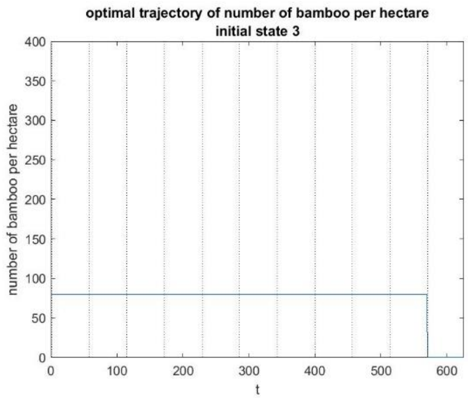
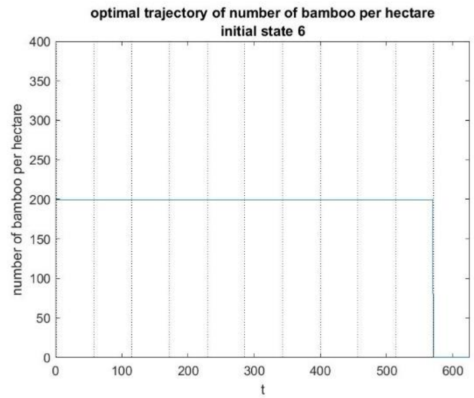
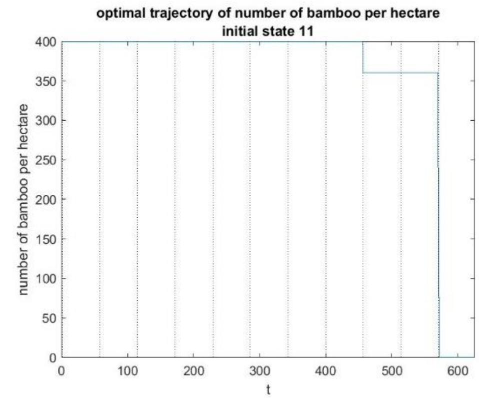

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

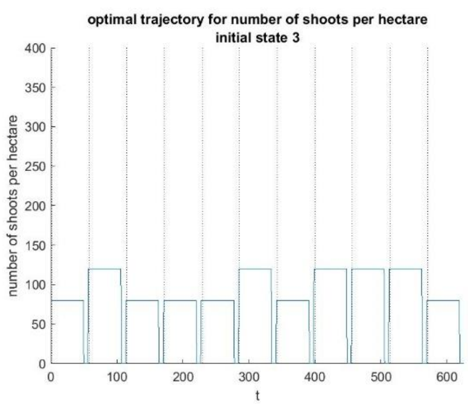
Figure 9: Stochastic Model, Specification 5, Version C, Set 2
a) Optimal trajectories for each action and state variable over 11 years

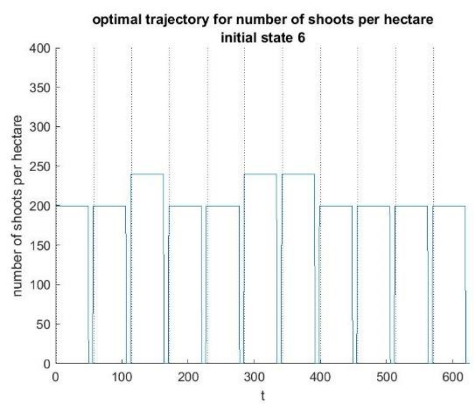



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

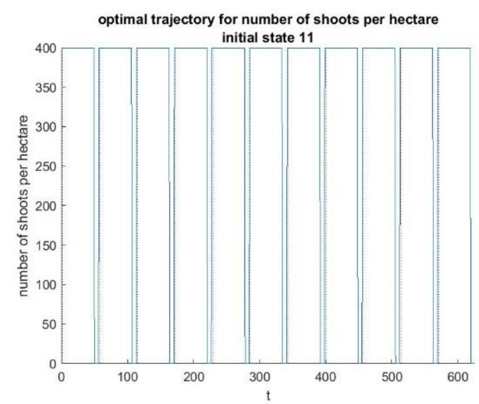
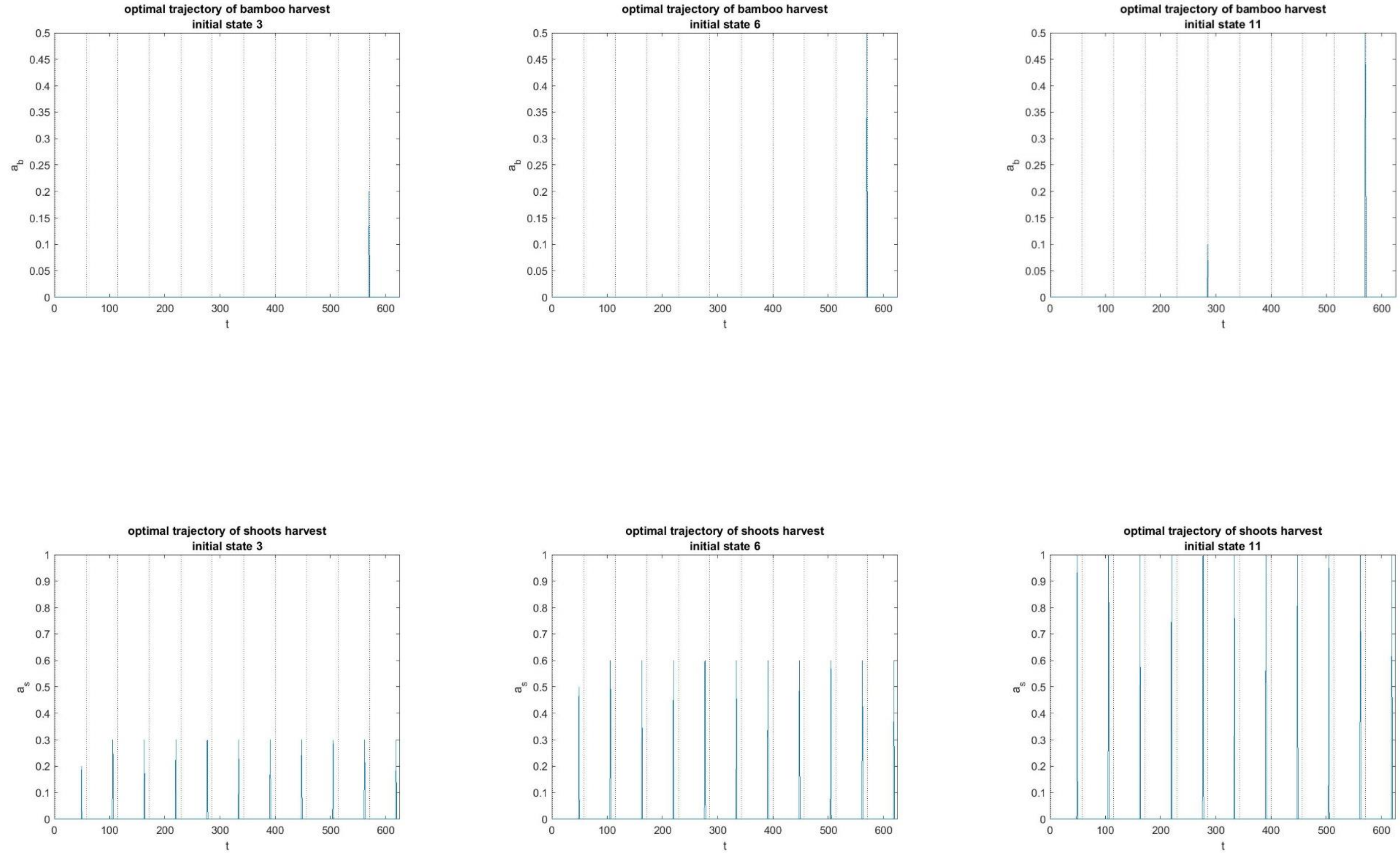
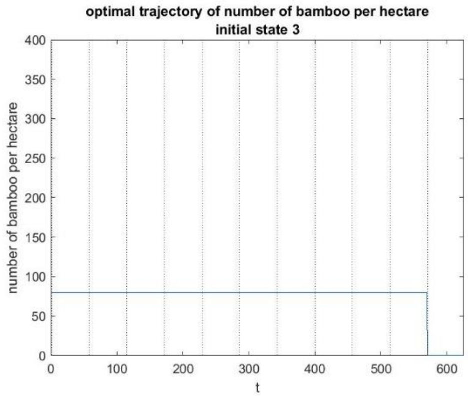
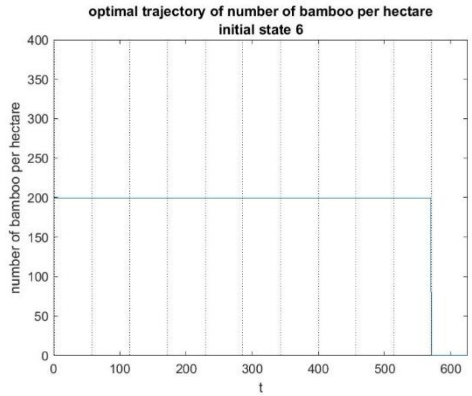
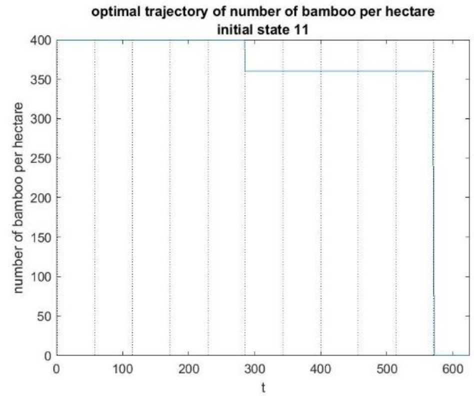

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

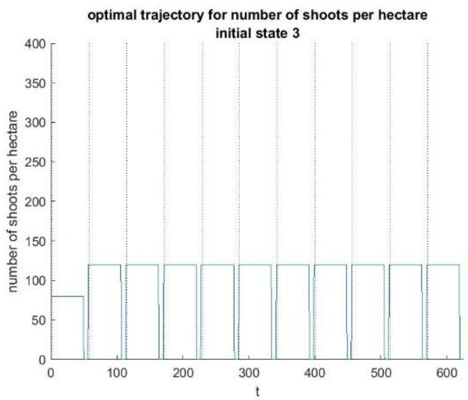
Figure 10: Stochastic Model, Specification 5, Version E, Set 1
a) Optimal trajectories for each action and state variable over 11 years

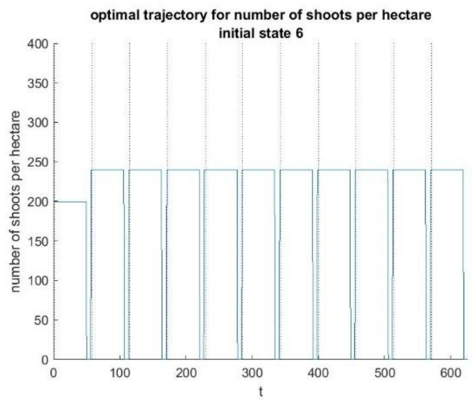



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

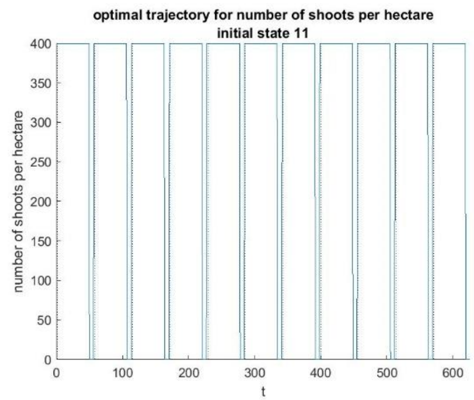
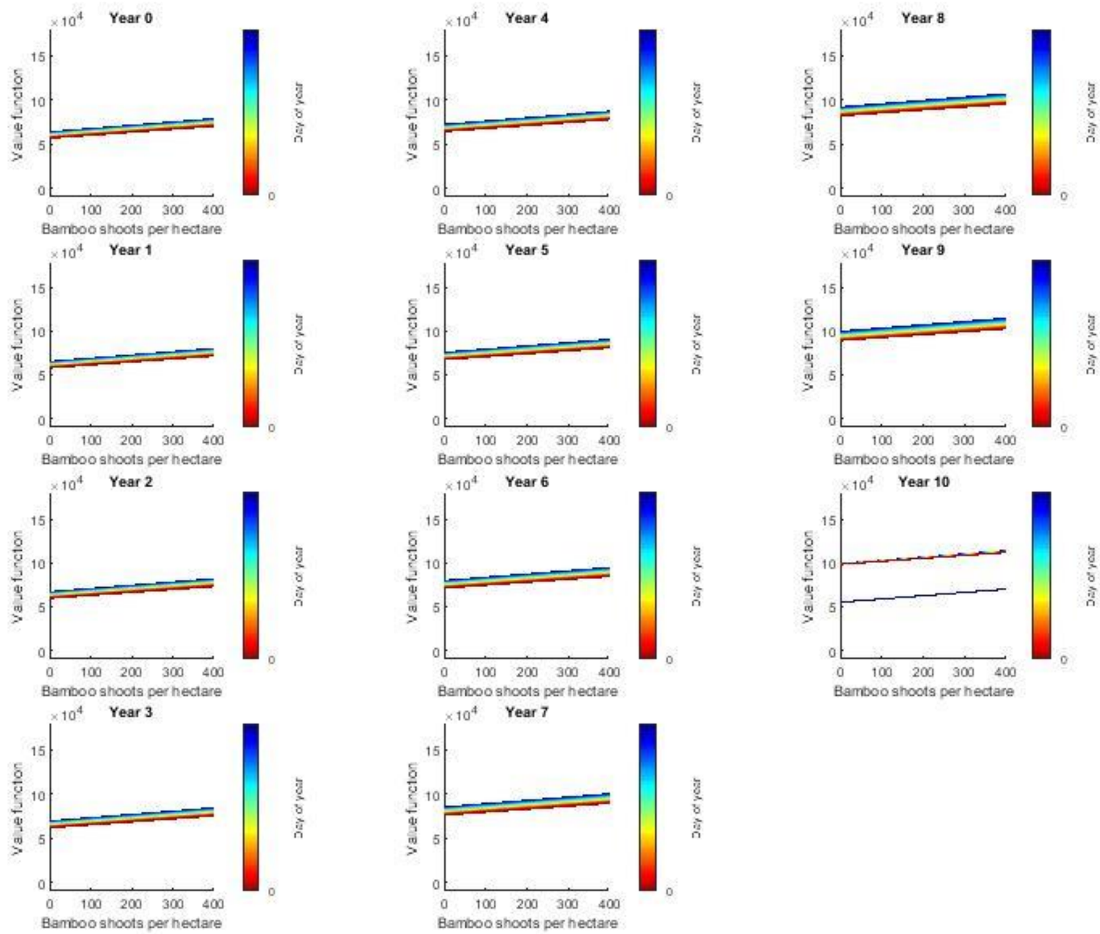
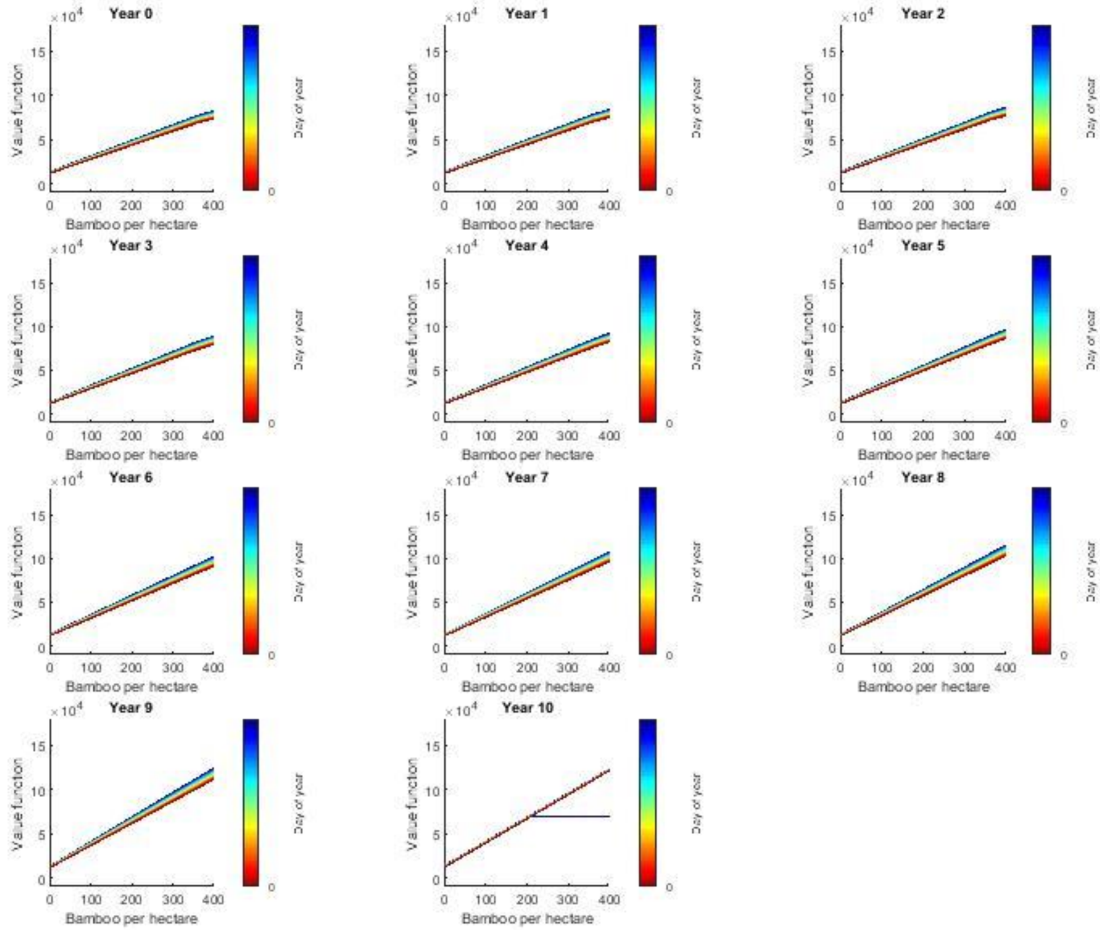

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

Figure 11: Stochastic Model, Specification 7, Version C, Set 1

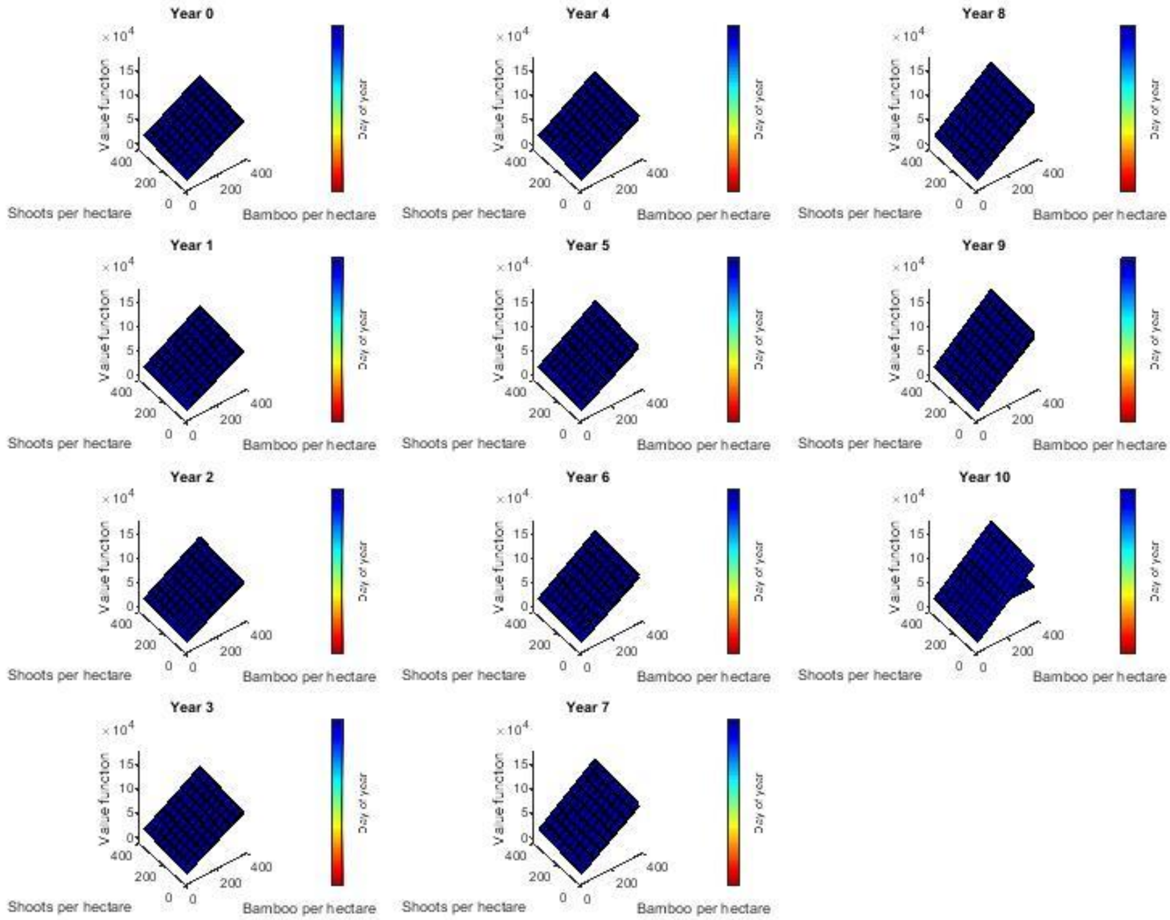
a) Value function as function of bamboo shoots per hectare



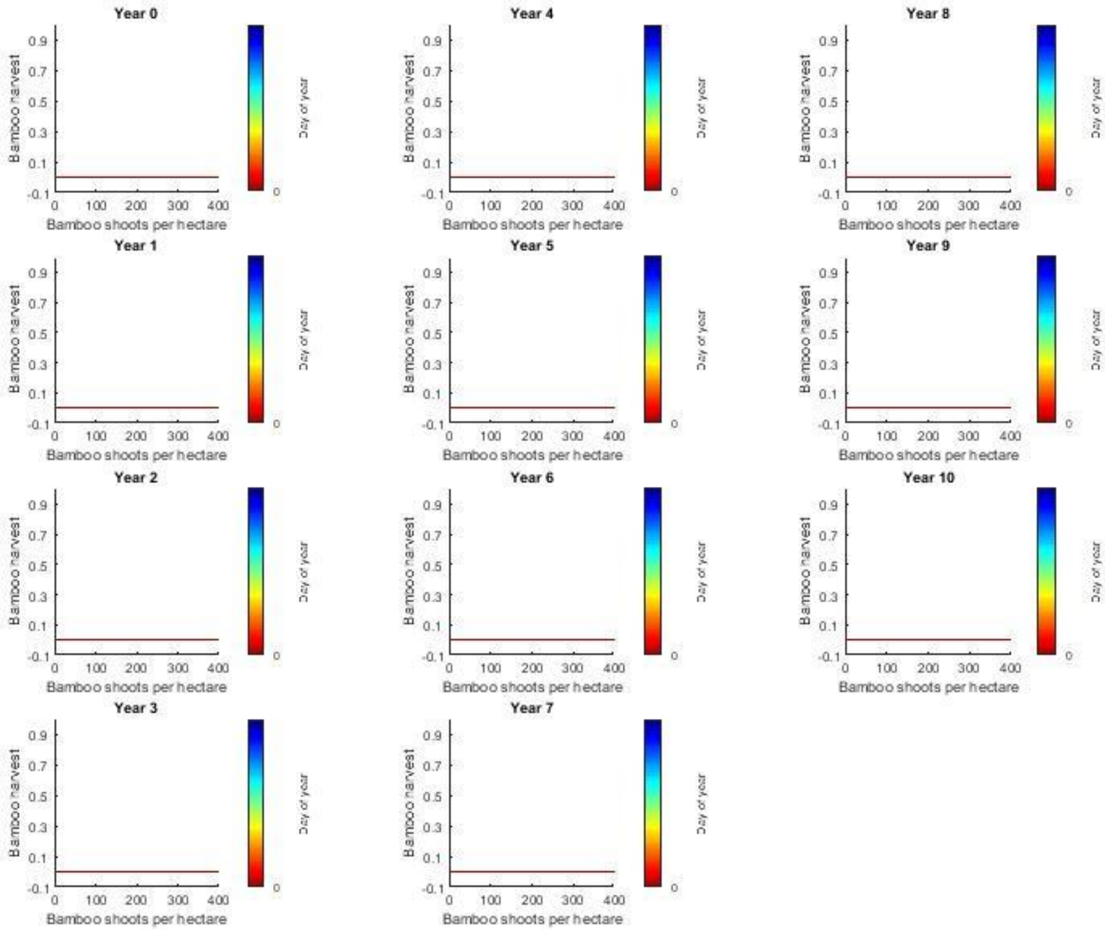
b) Value function as function of bamboo stem per hectare



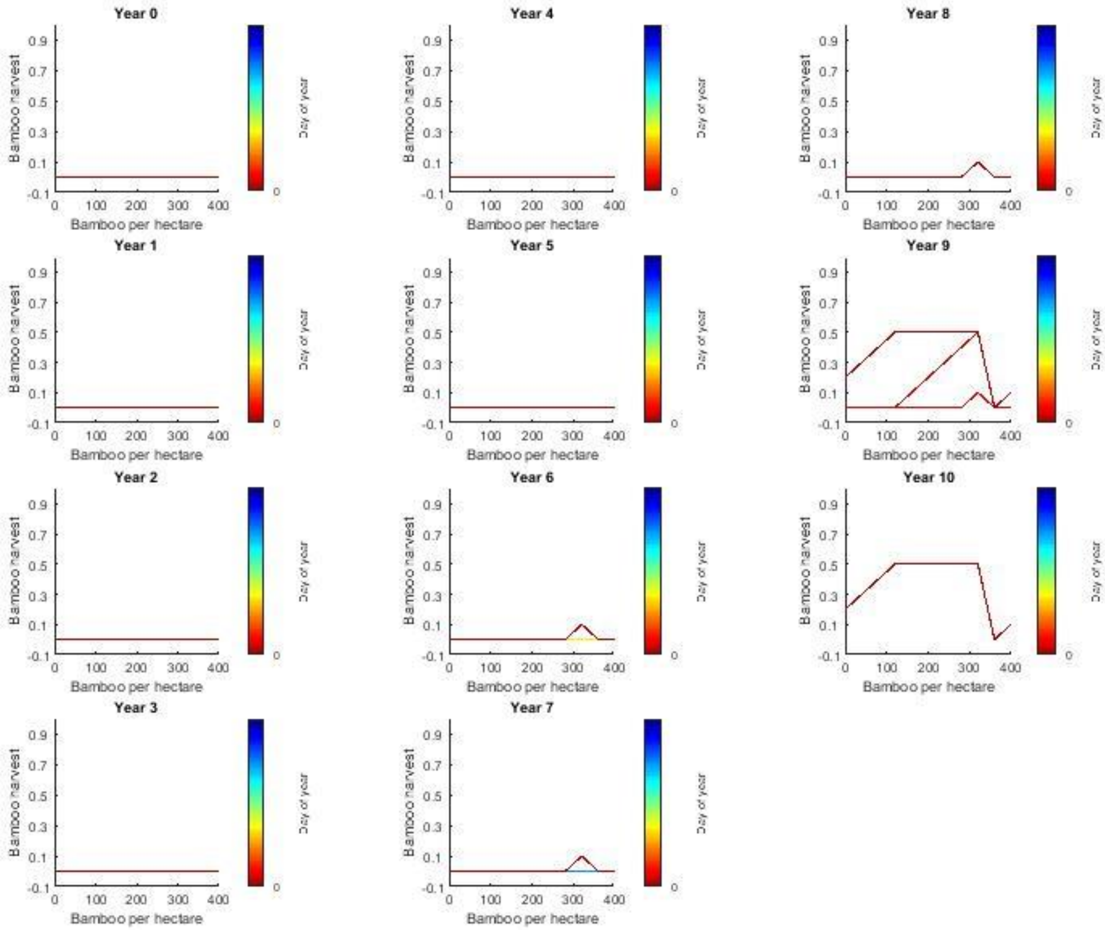
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



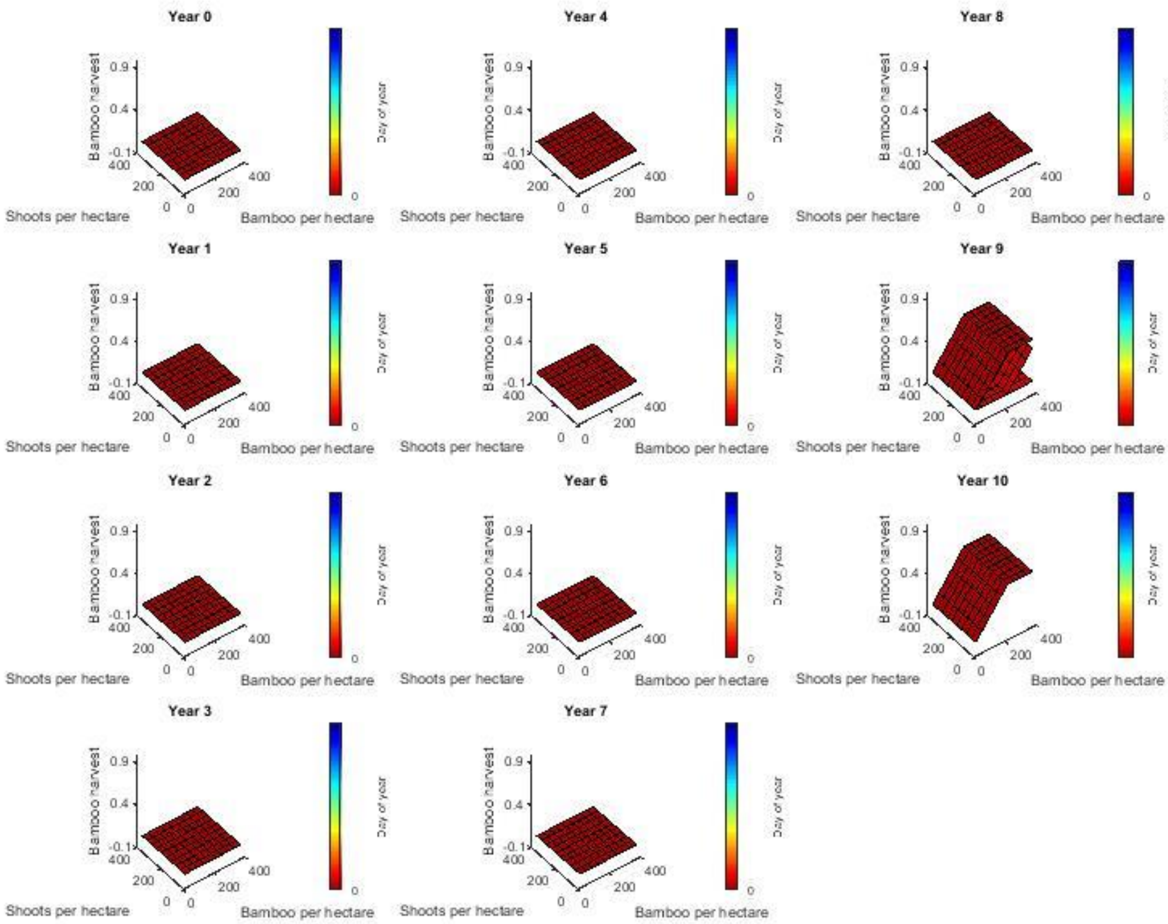
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



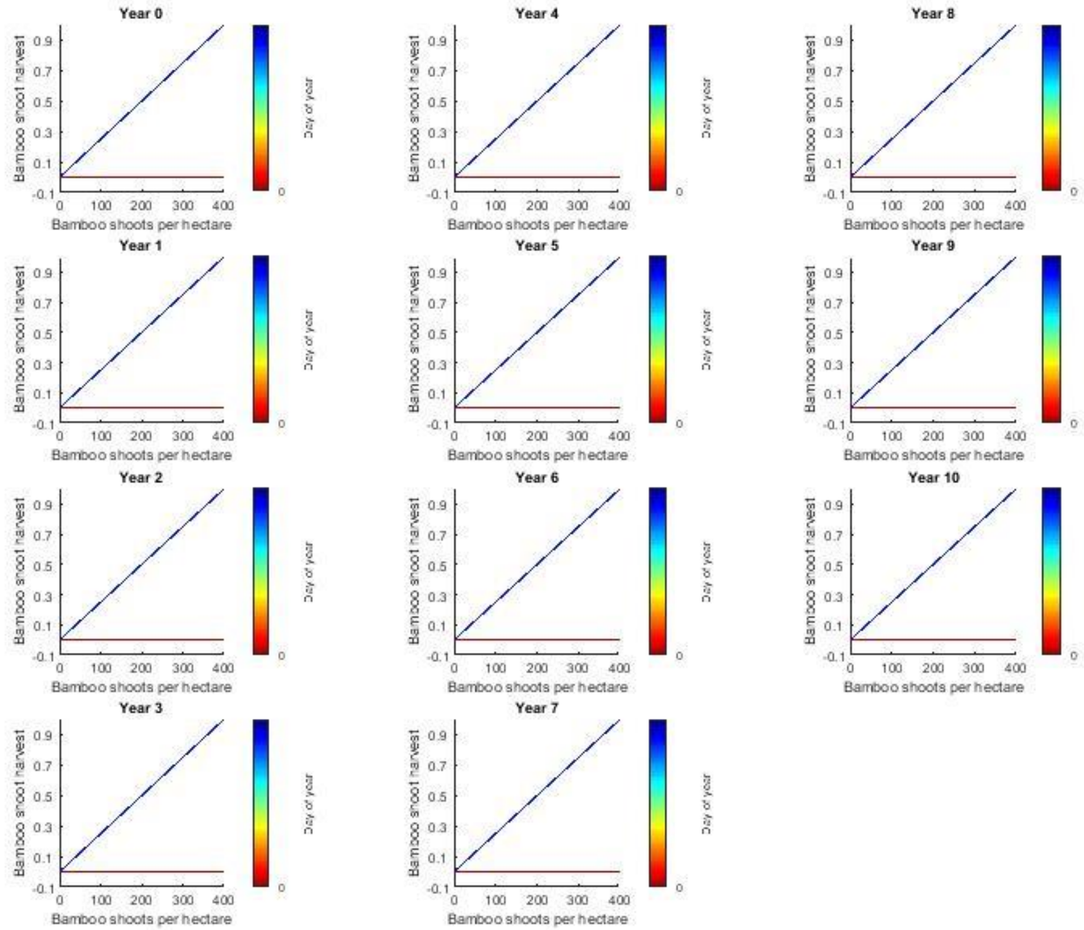
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



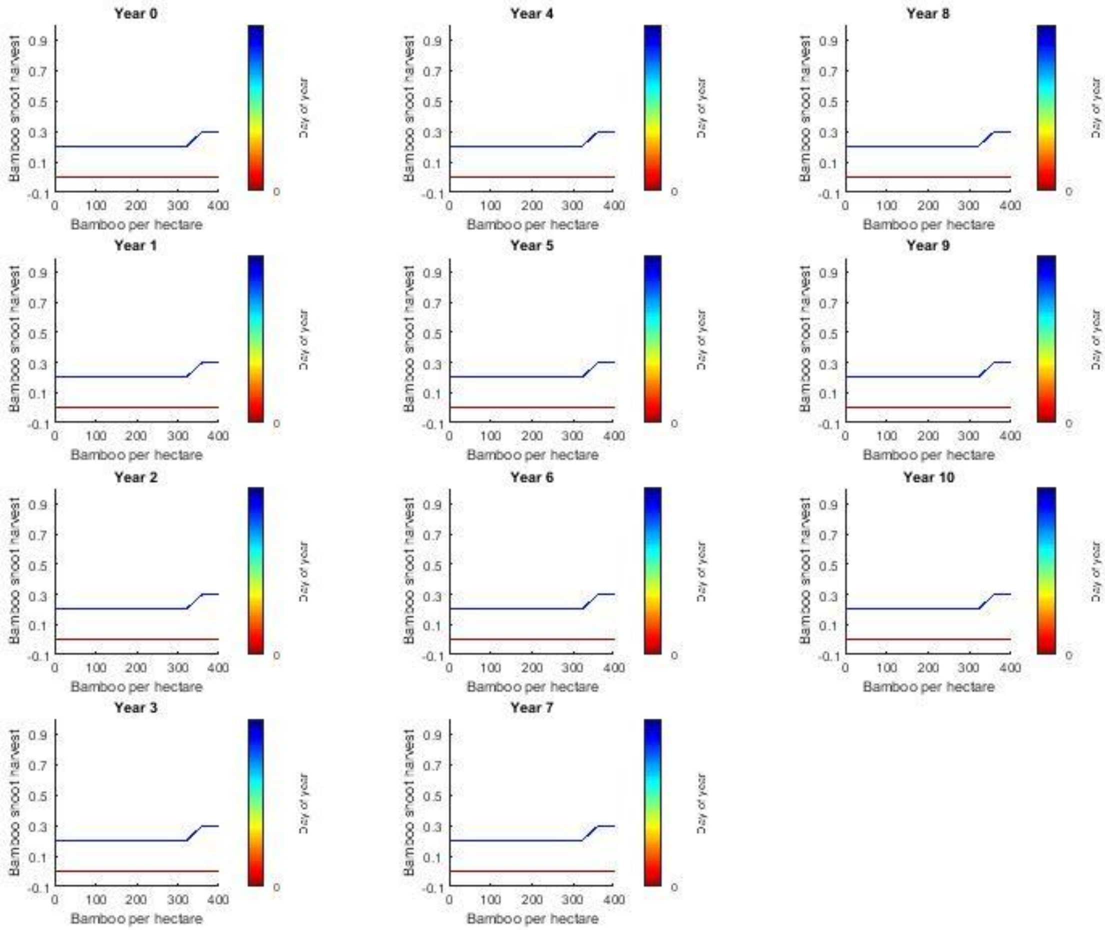
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



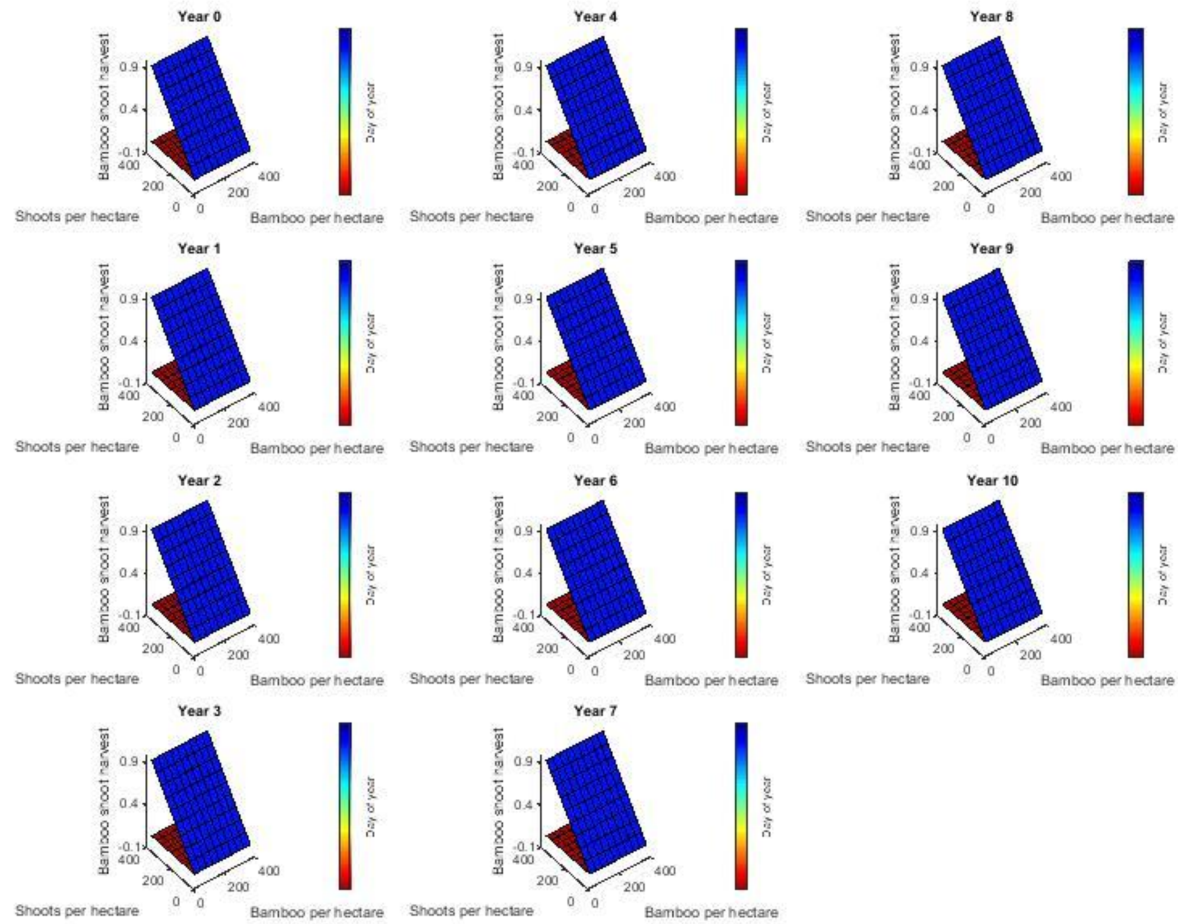
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



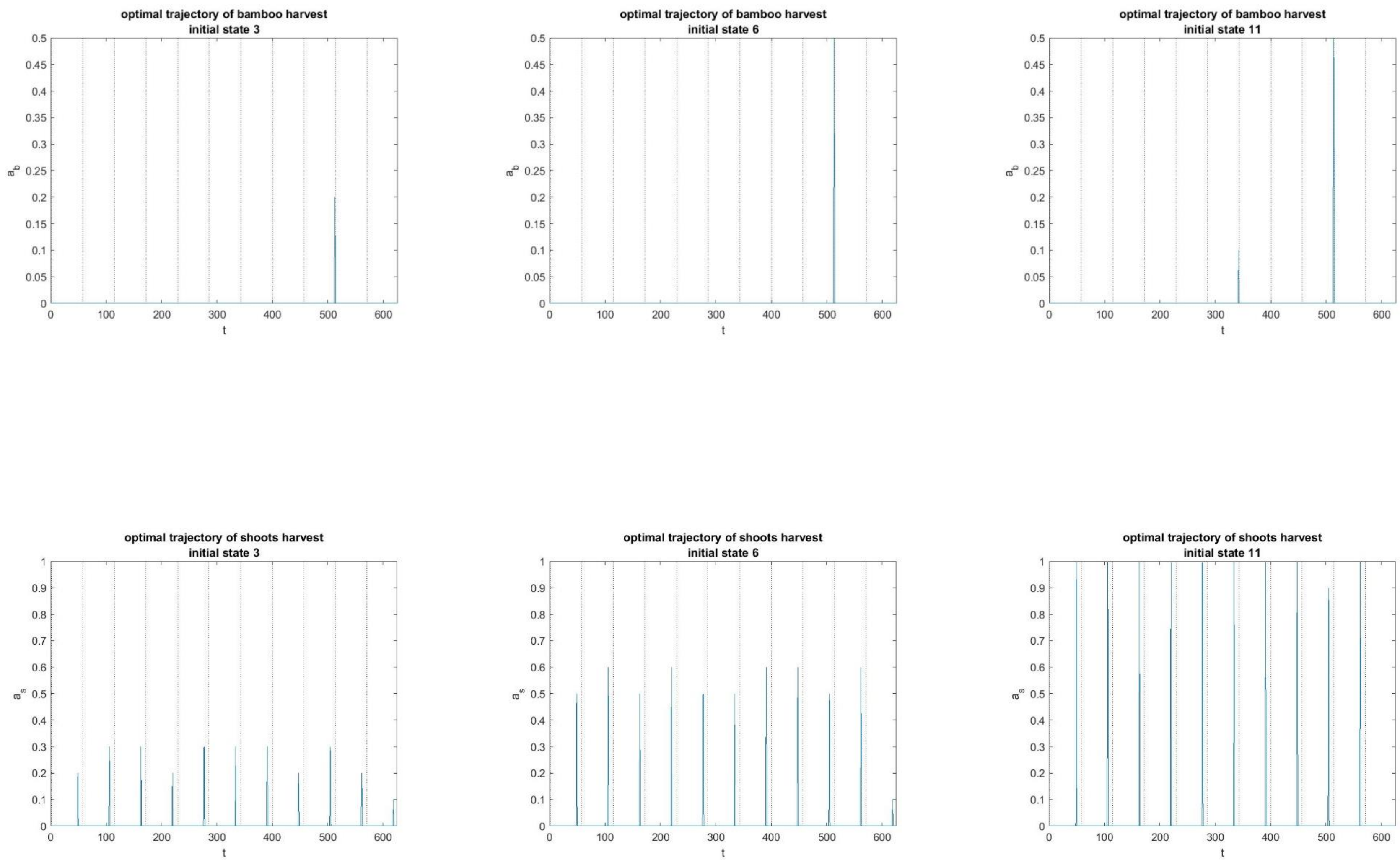
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare



i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



j) Optimal trajectories for each action and state variable over 11 years



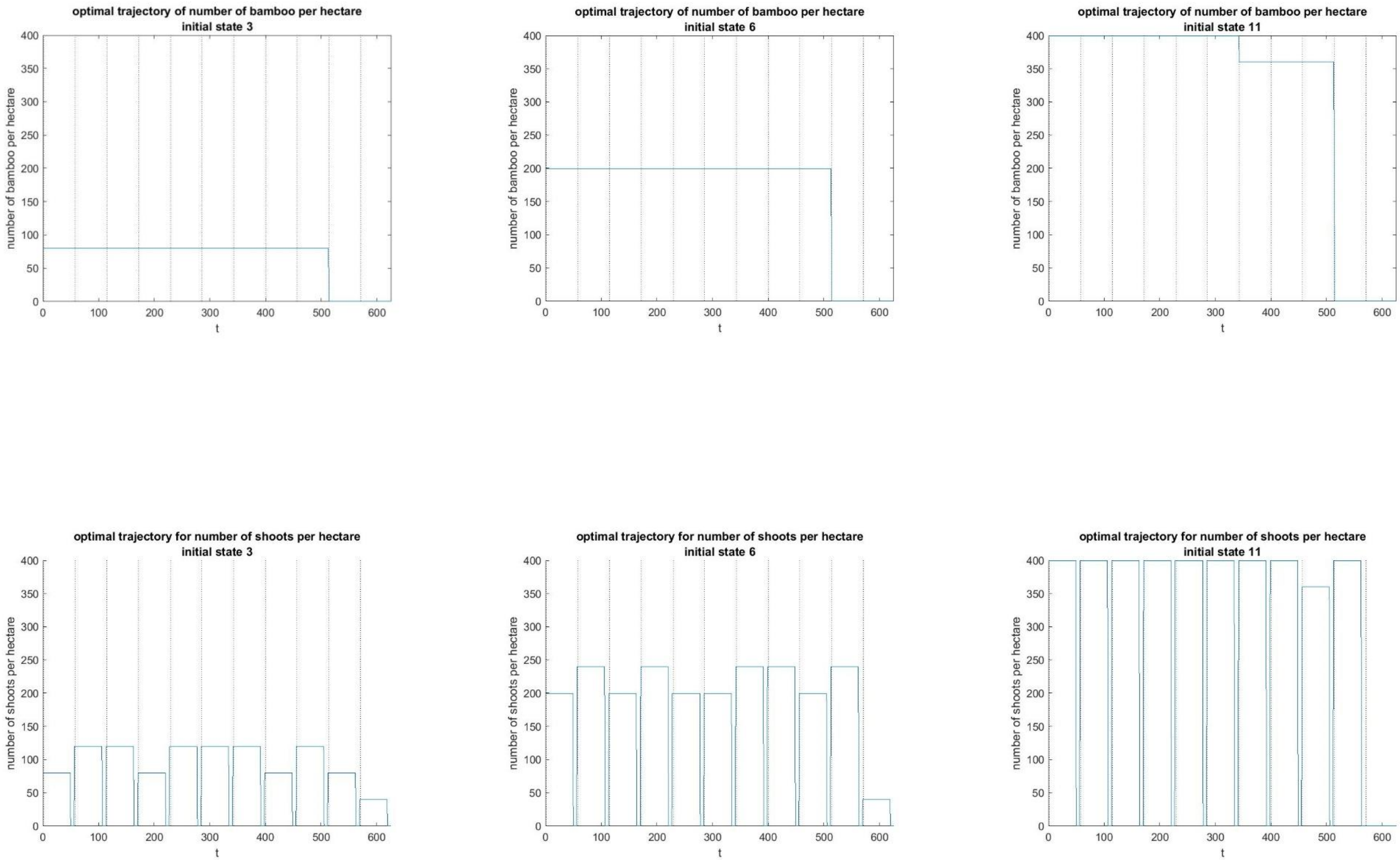
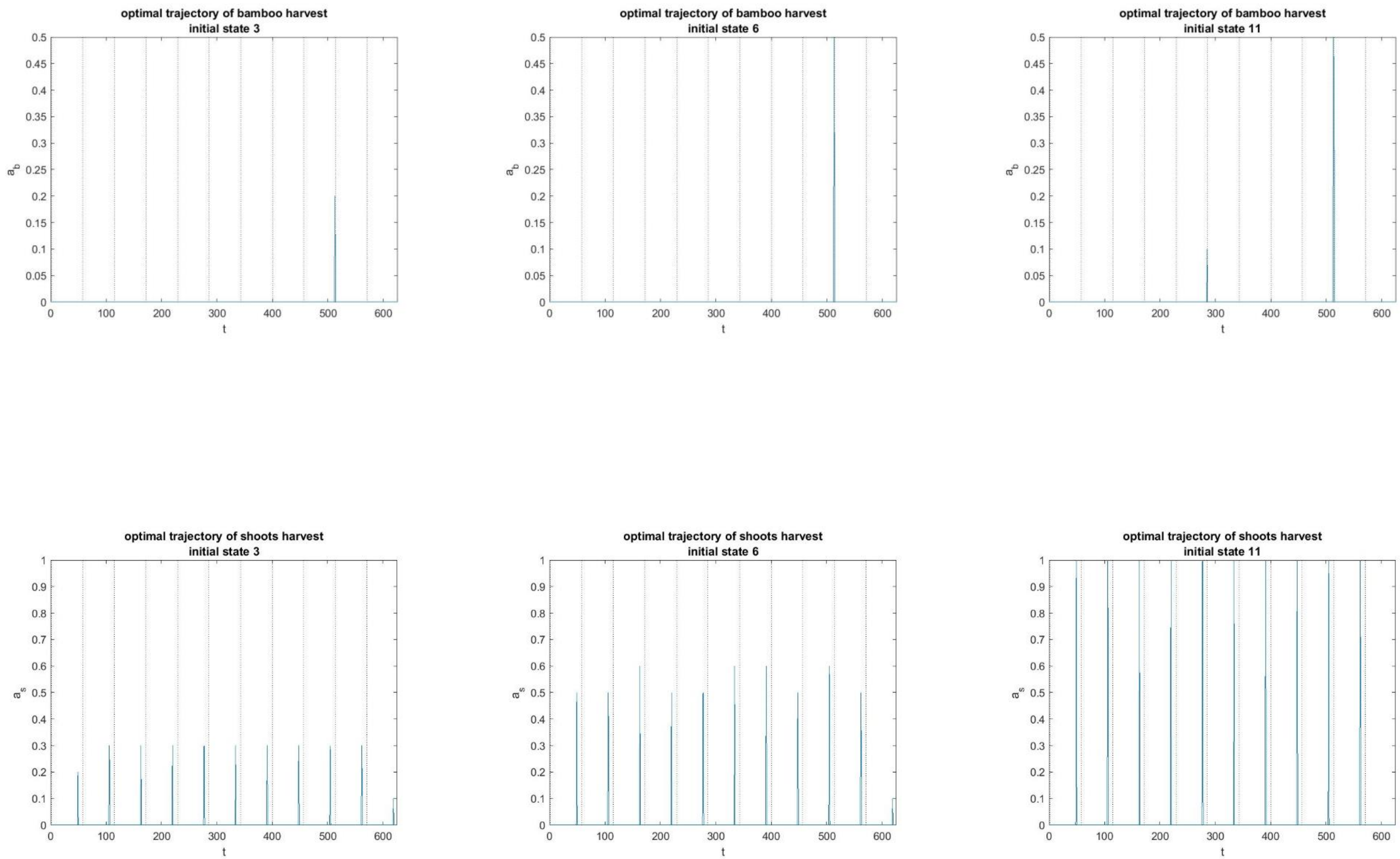
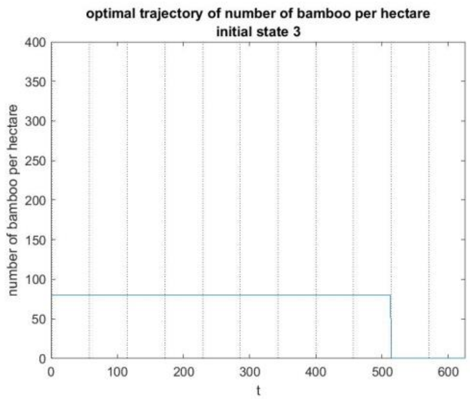
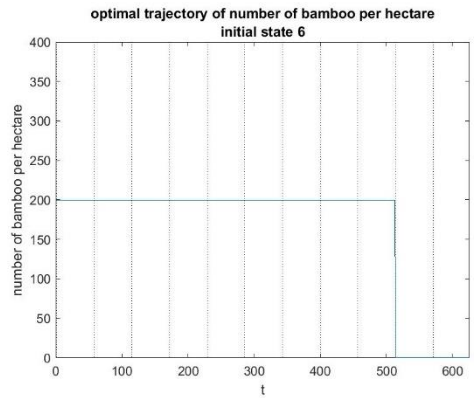
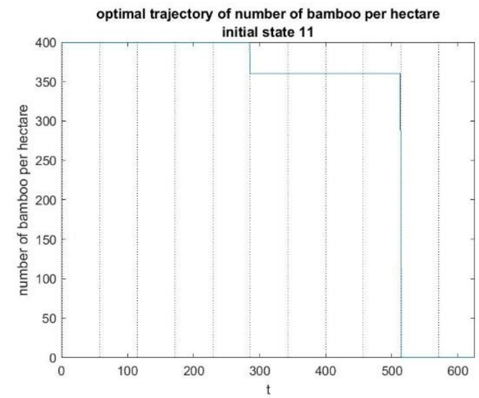

optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t
optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t
optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t
optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t
optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t
optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

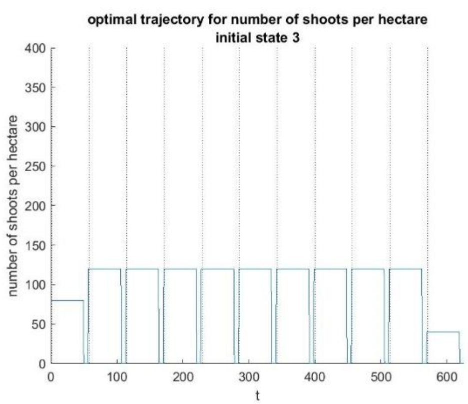
Figure 12: Stochastic Model, Specification 7, Version D, Set 1
a) Optimal trajectories for each action and state variable over 11 years

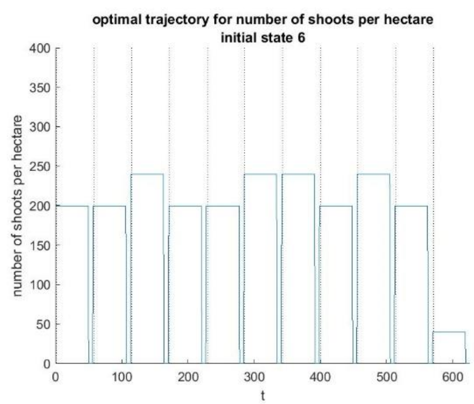



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

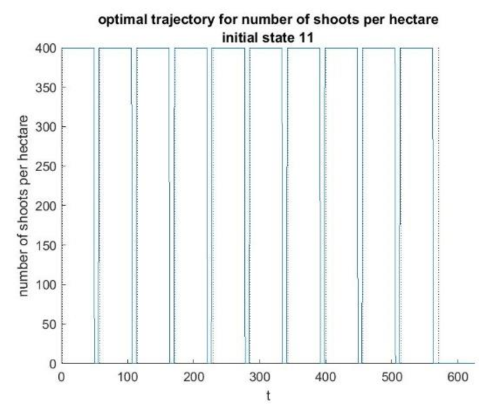
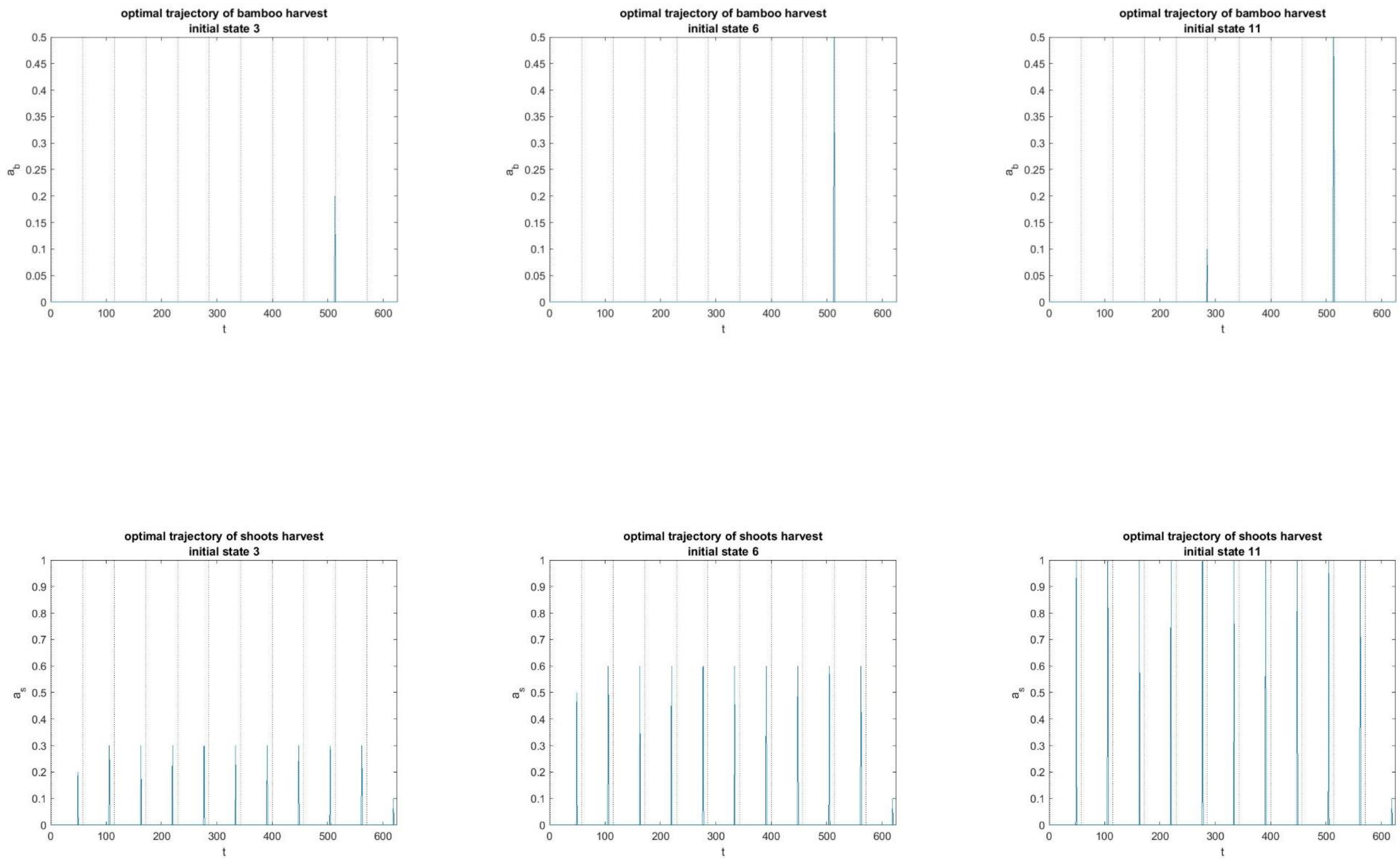
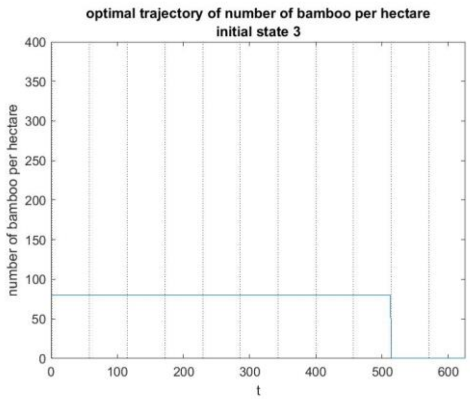
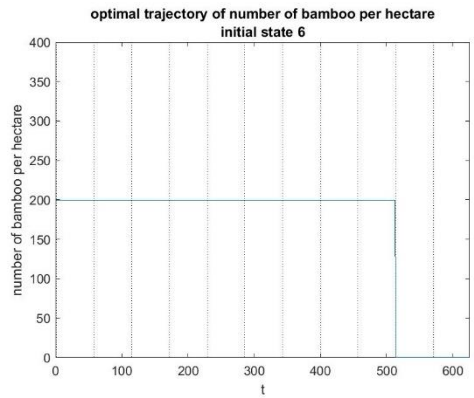
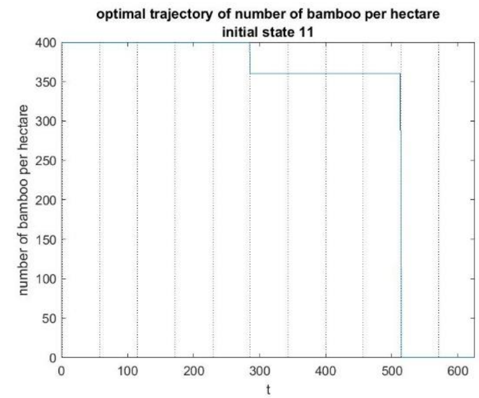

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

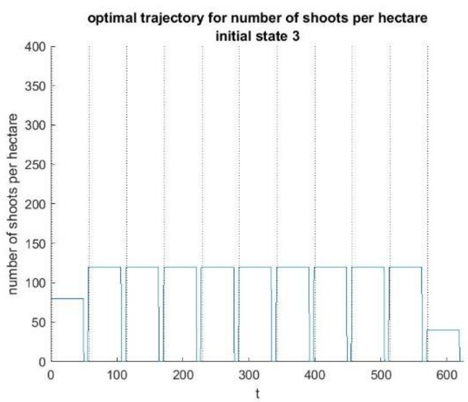
Figure 13: Stochastic Model, Specification 7, Version E, Set 1
a) Optimal trajectories for each action and state variable over 11 years

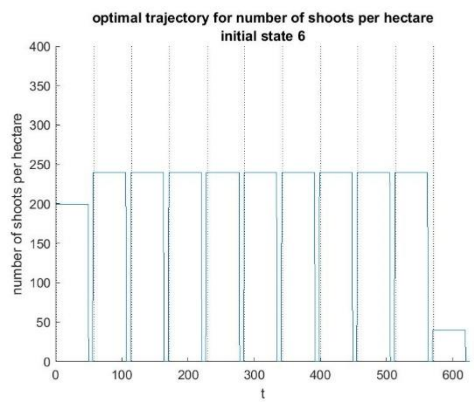



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600

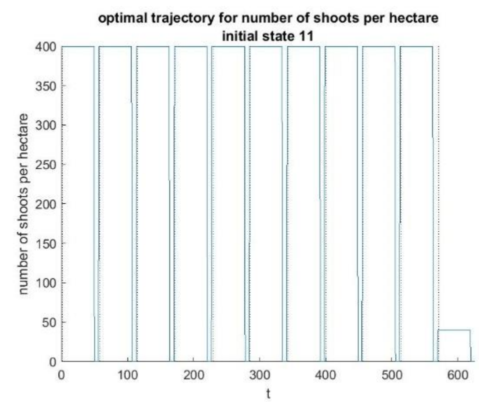
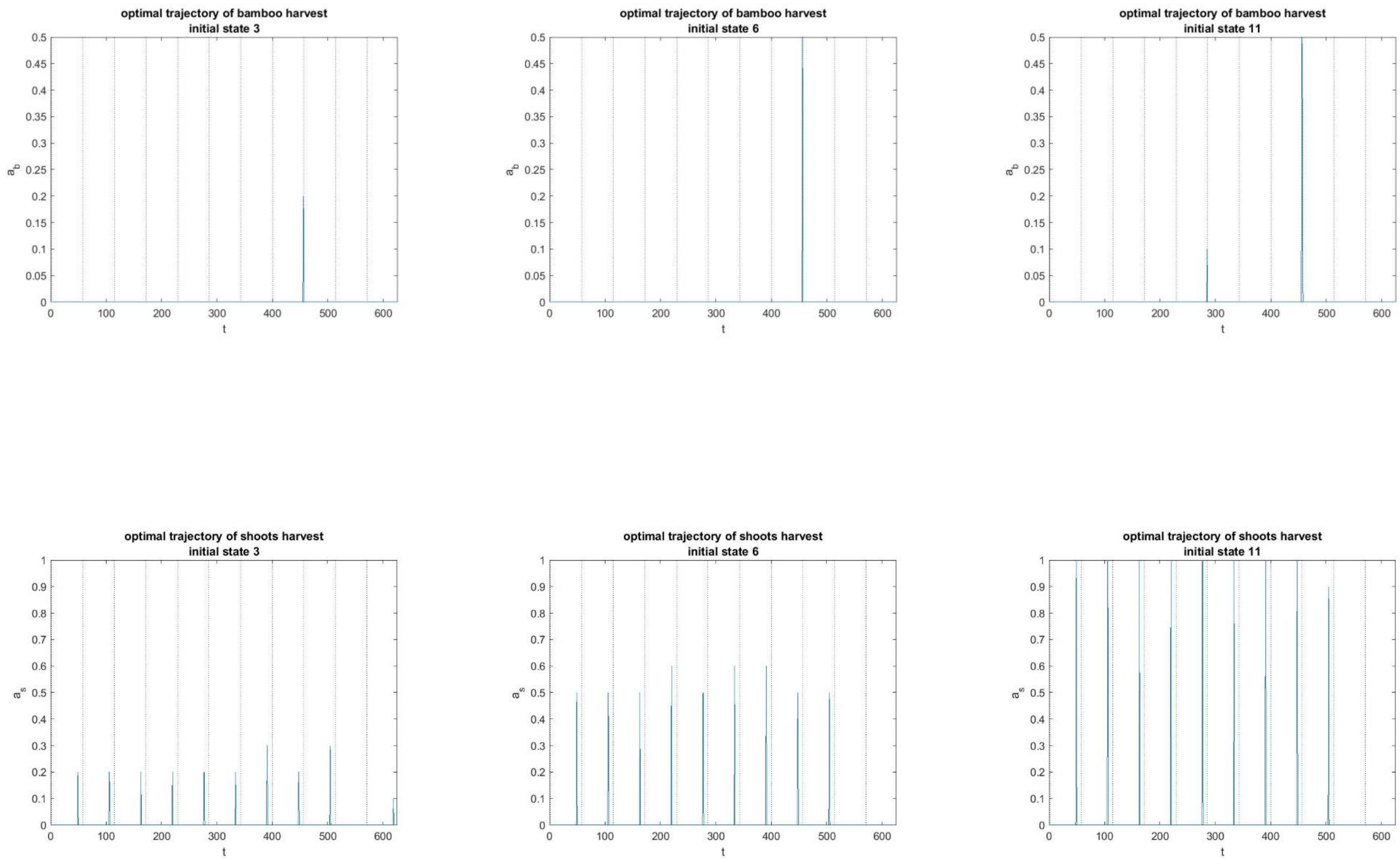
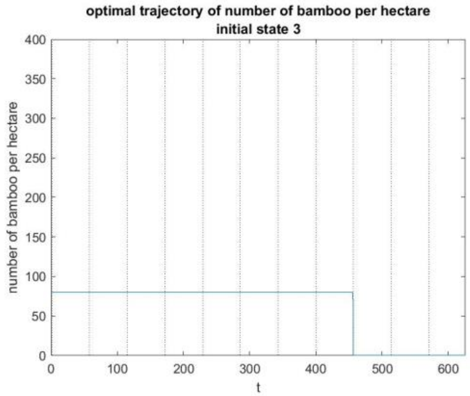
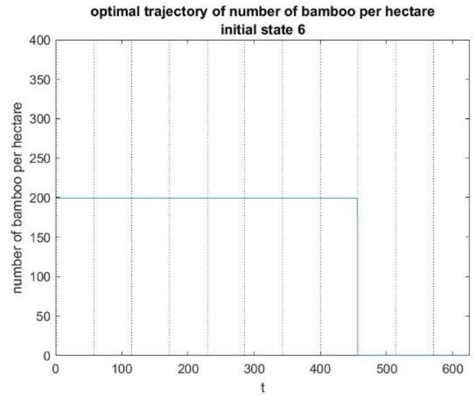
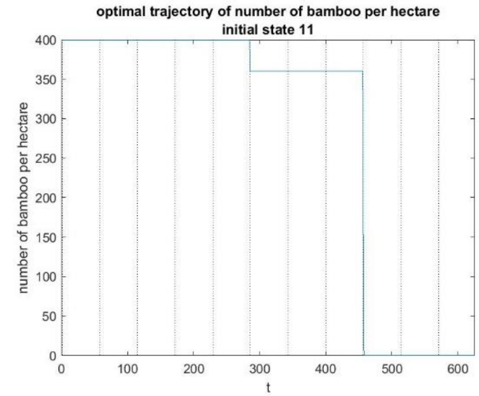

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600

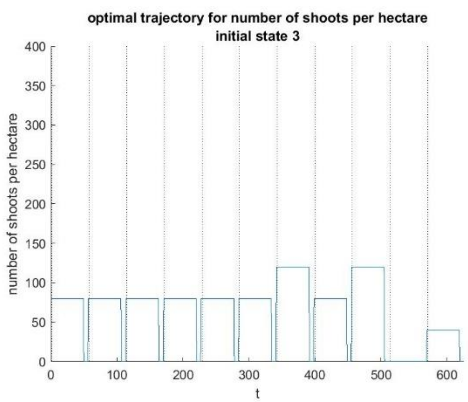
Figure 14: Stochastic Model, Specification 8, Version C, Set 1
a) Optimal trajectories for each action and state variable over 11 years

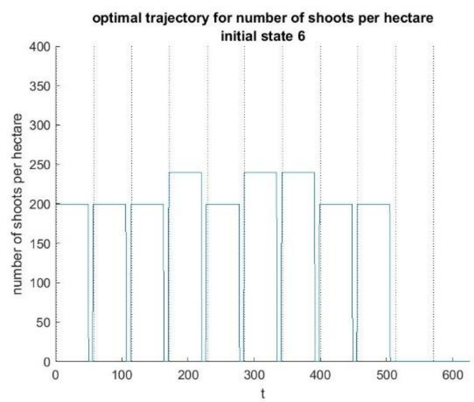



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600

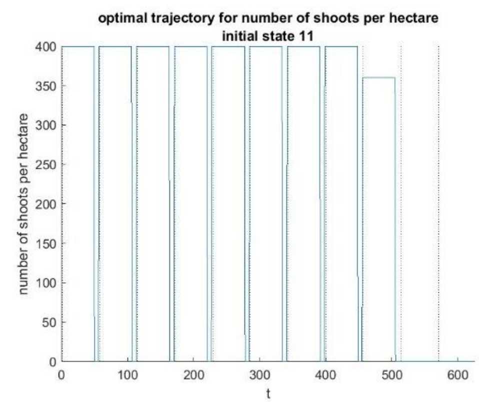
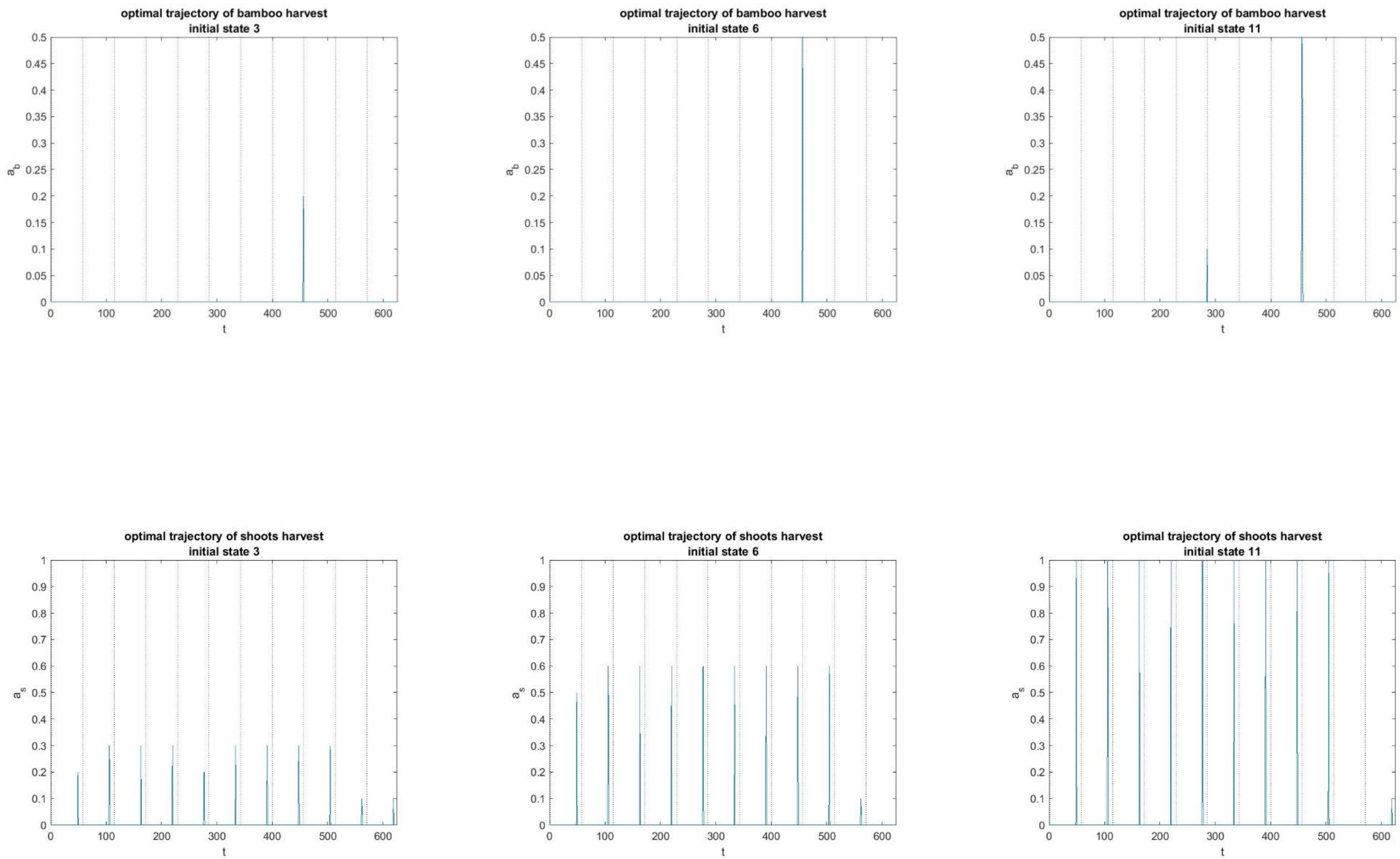
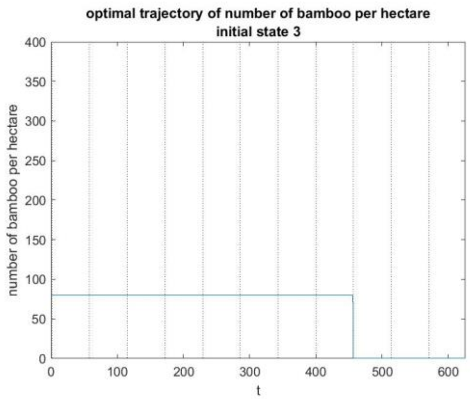
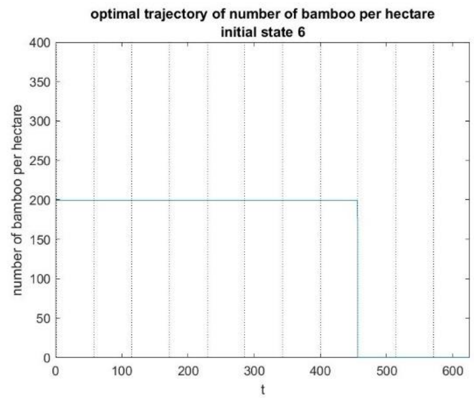
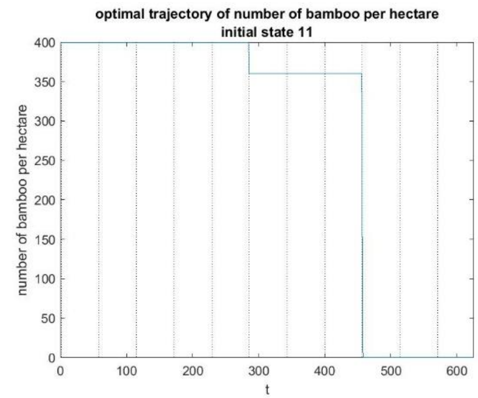

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600

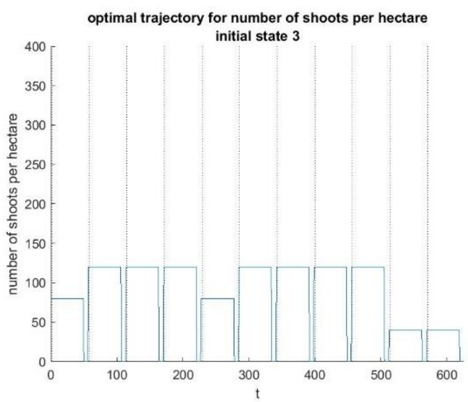
Figure 15: Stochastic Model, Specification 8, Version D, Set 1
a) Optimal trajectories for each action and state variable over 11 years

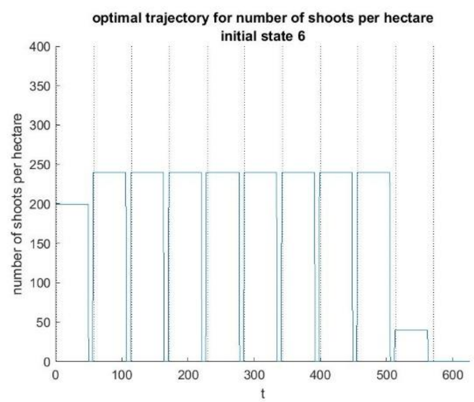



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600

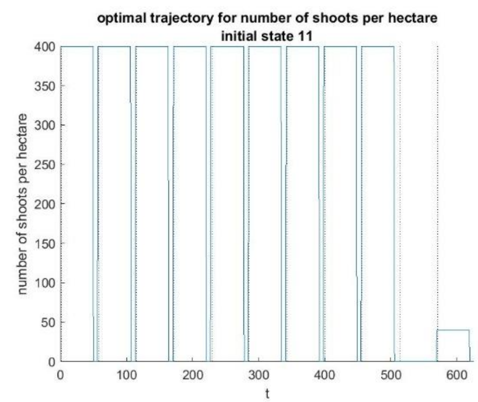
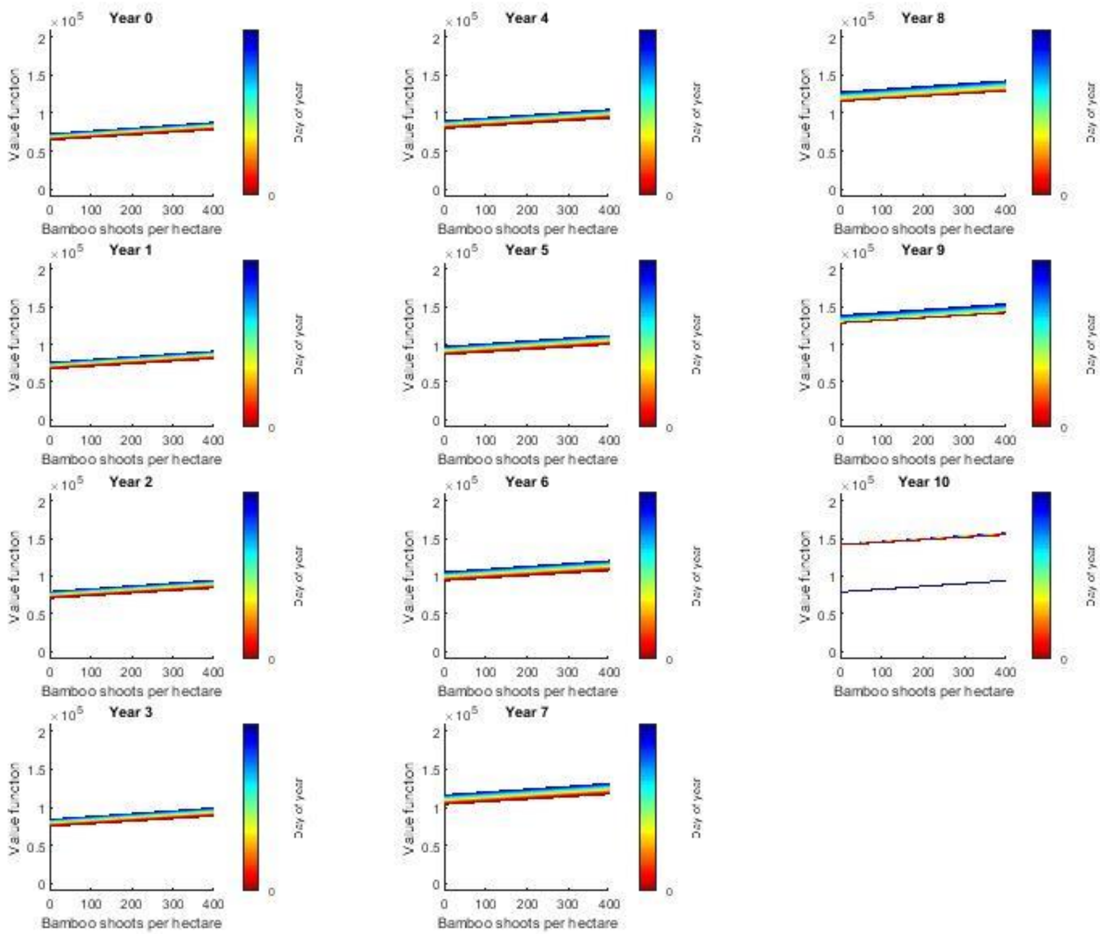
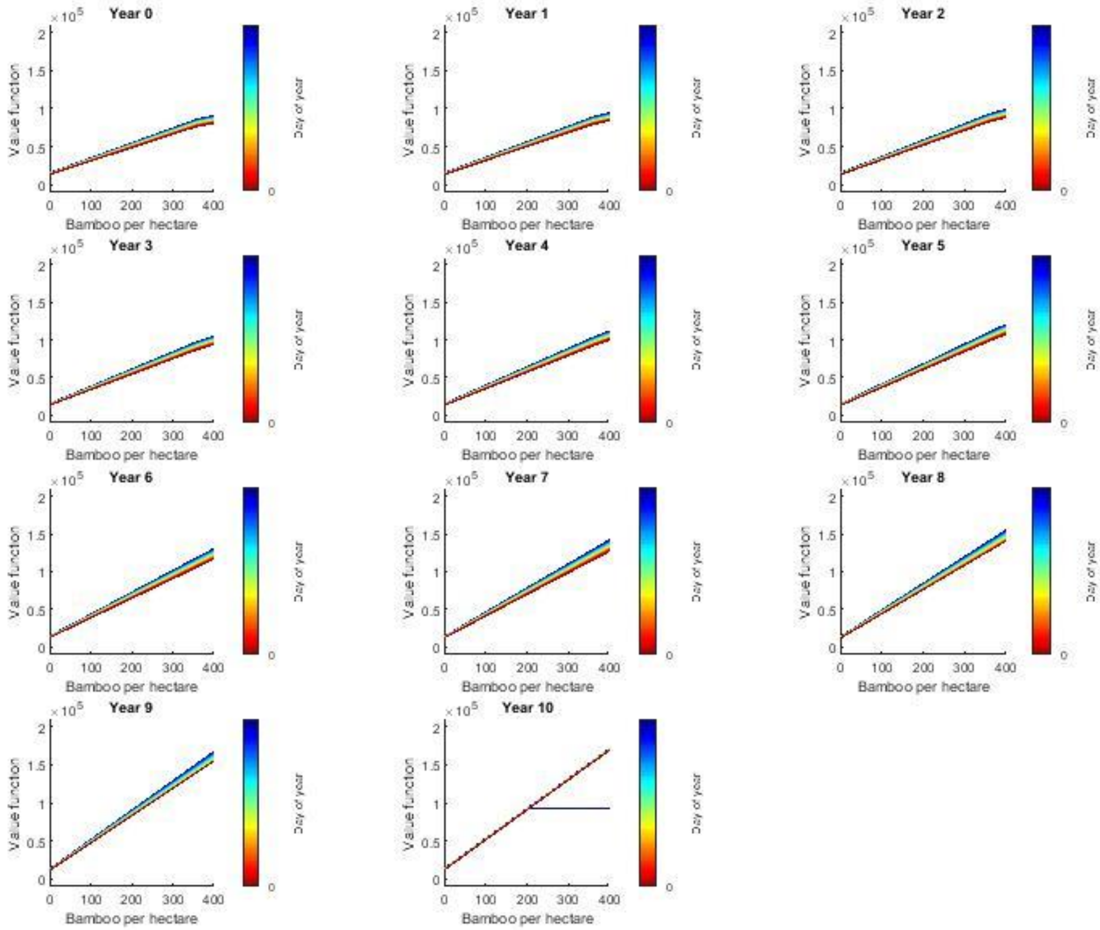

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t
0 50 100 150 200 250 300 350 400
0 100 200 300 400 500 600

Figure 16: Stochastic Model, Specification 8, Version E, Set 1

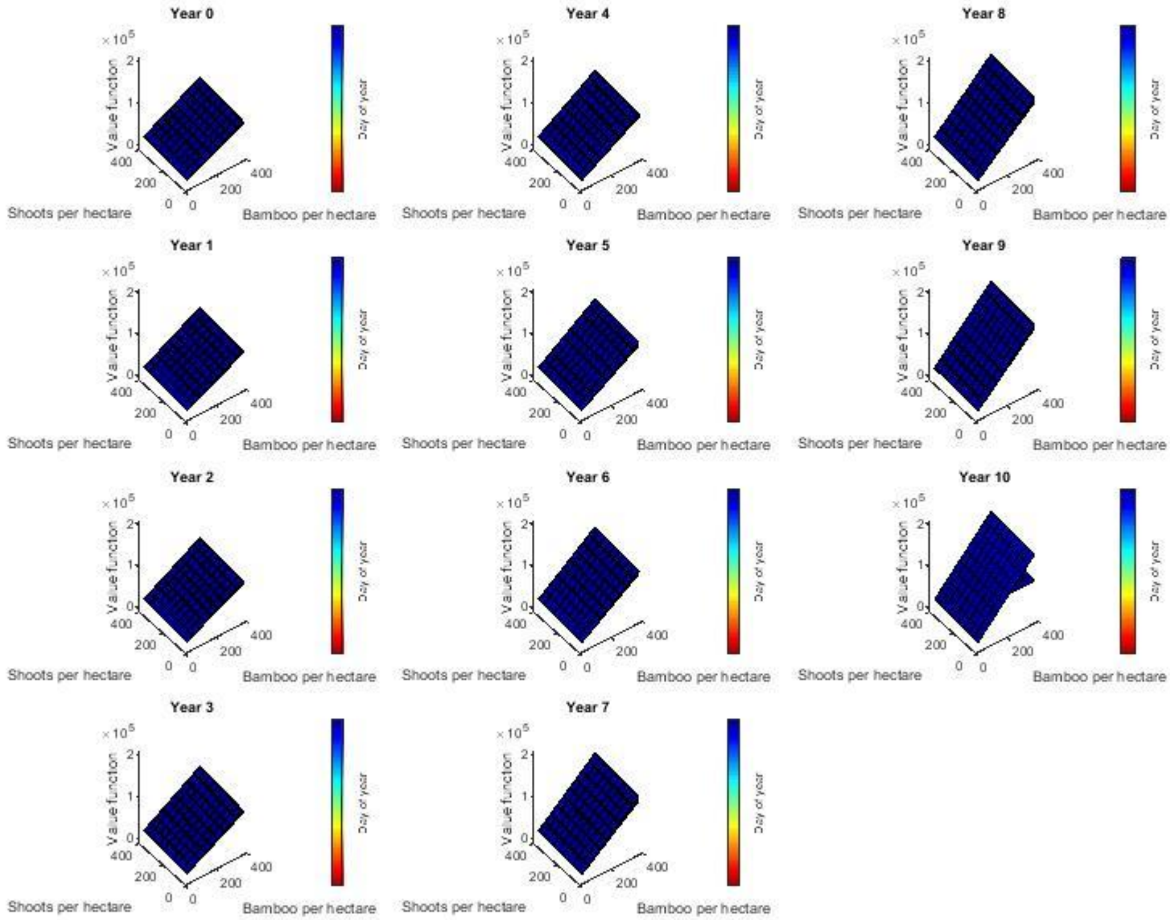
a) Value function as function of bamboo shoots per hectare



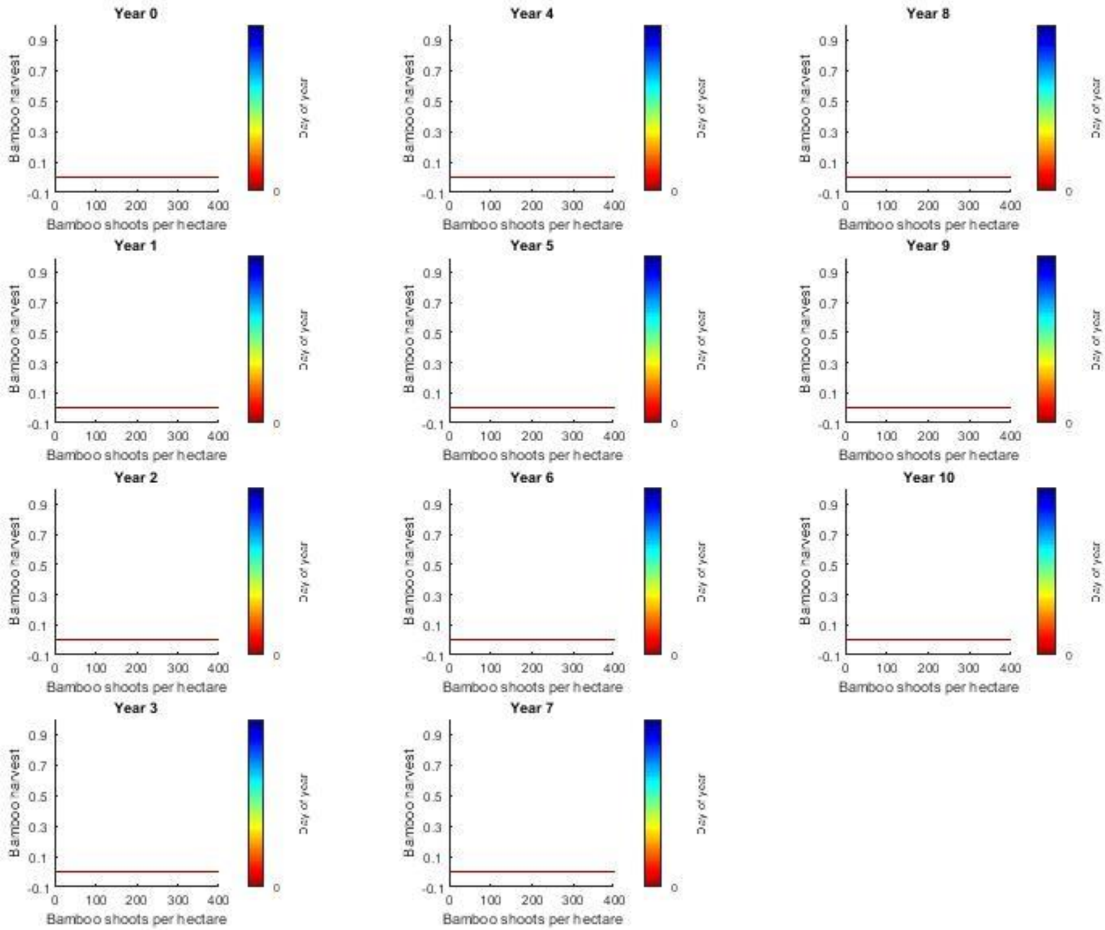
b) Value function as function of bamboo stem per hectare



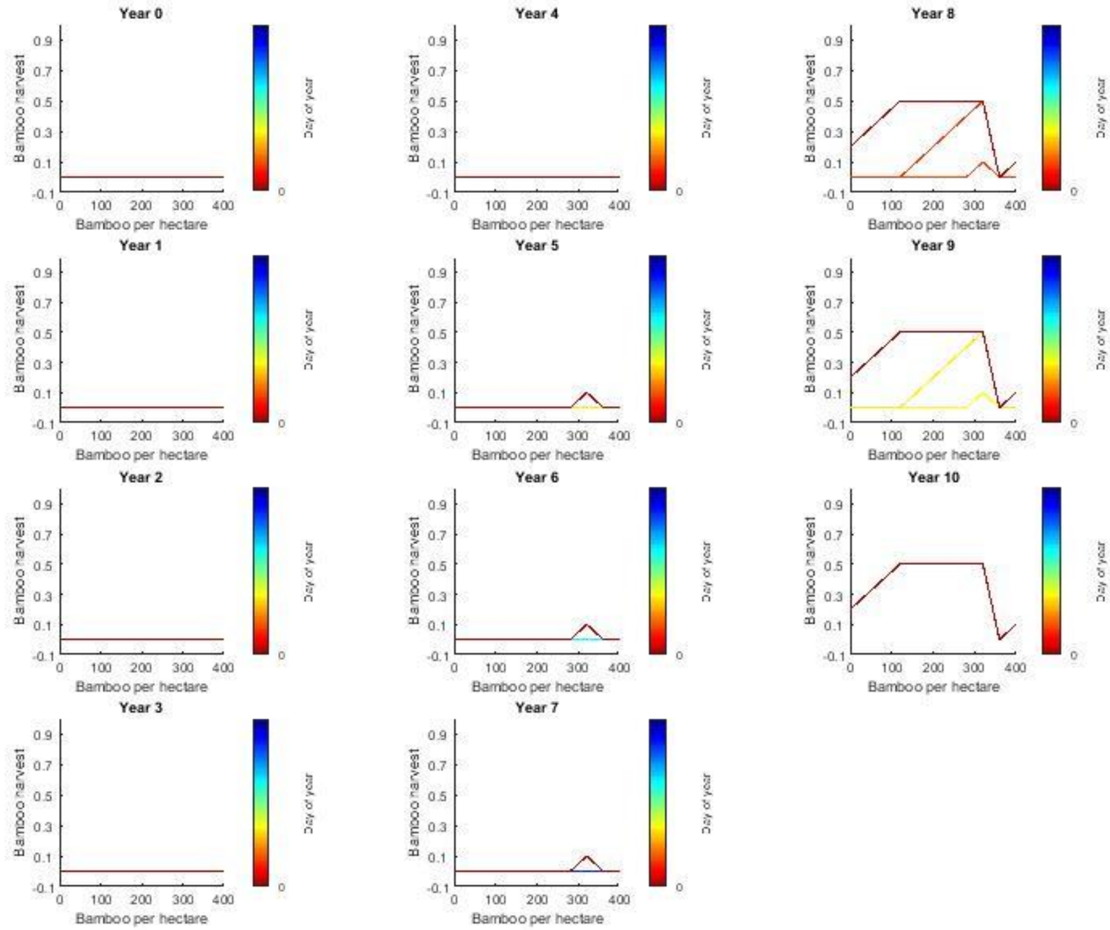
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



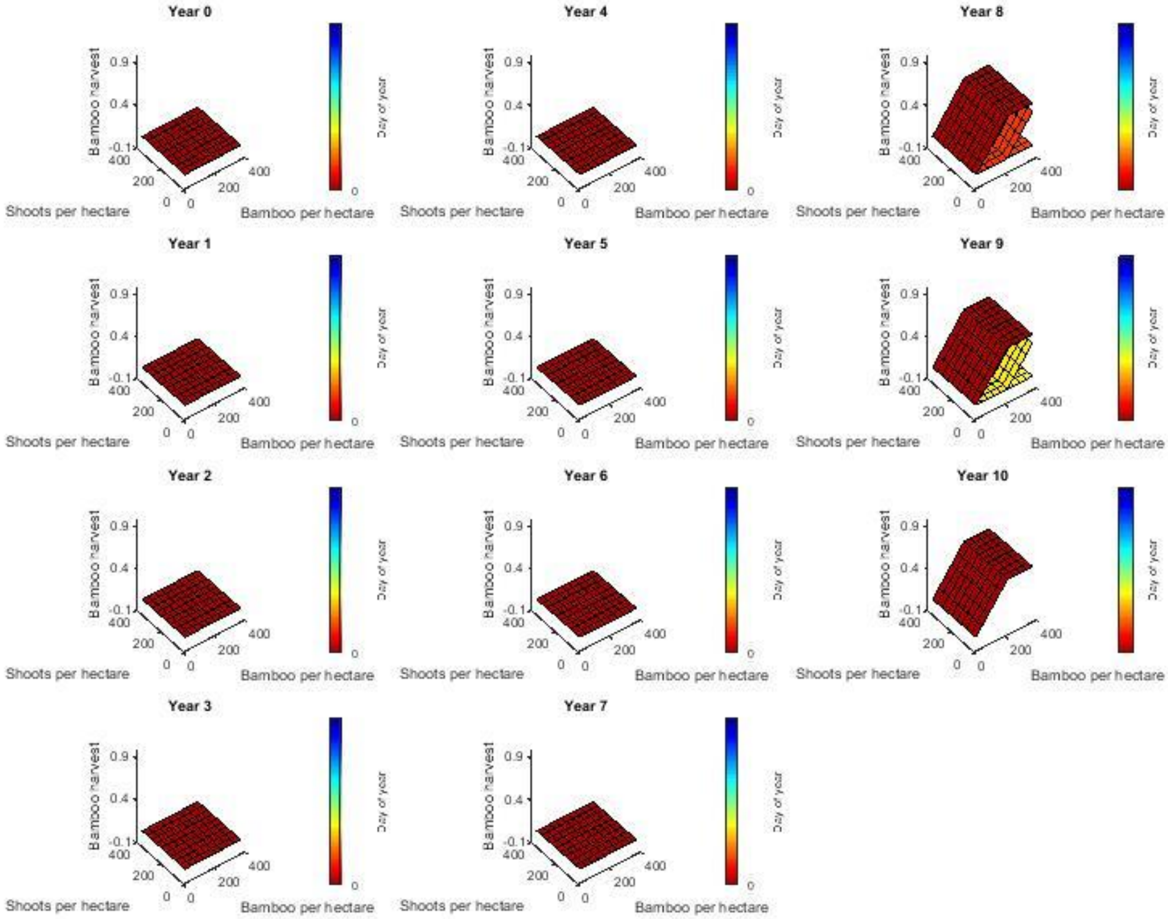
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



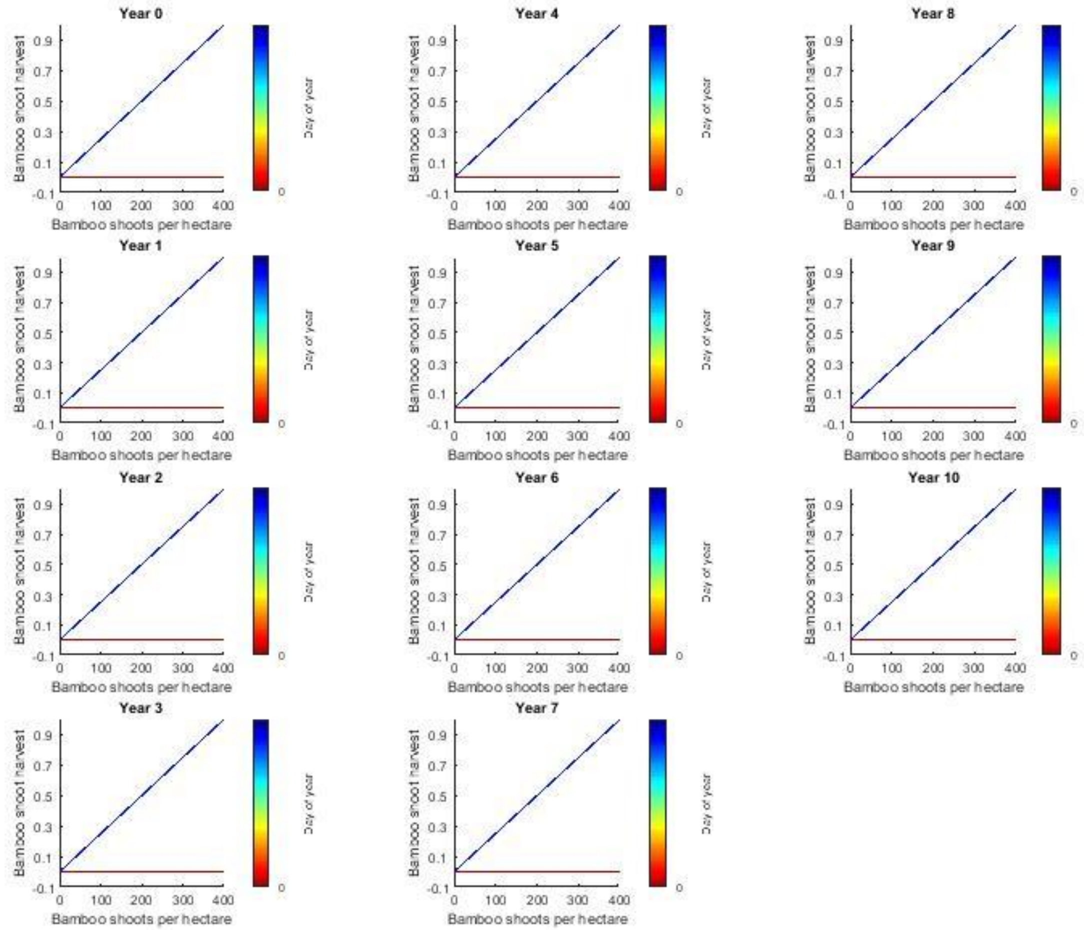
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



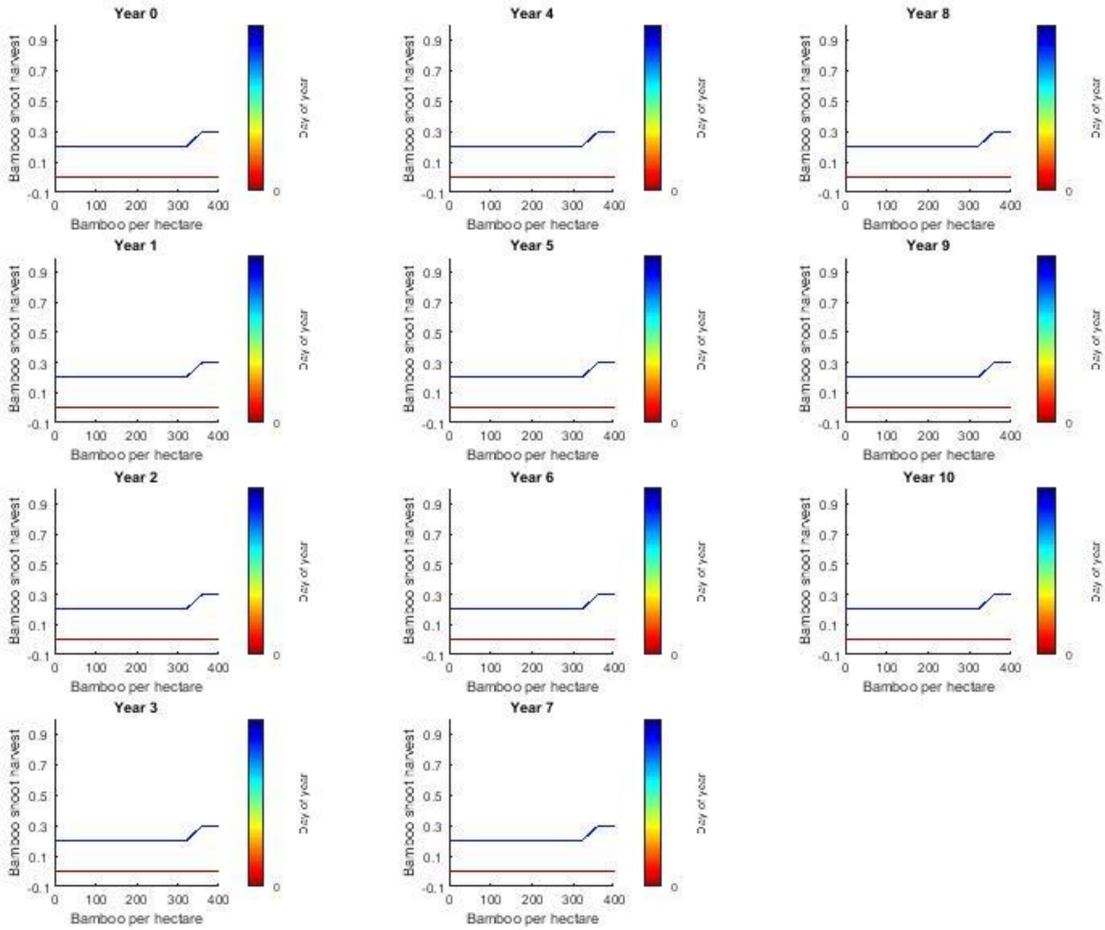
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



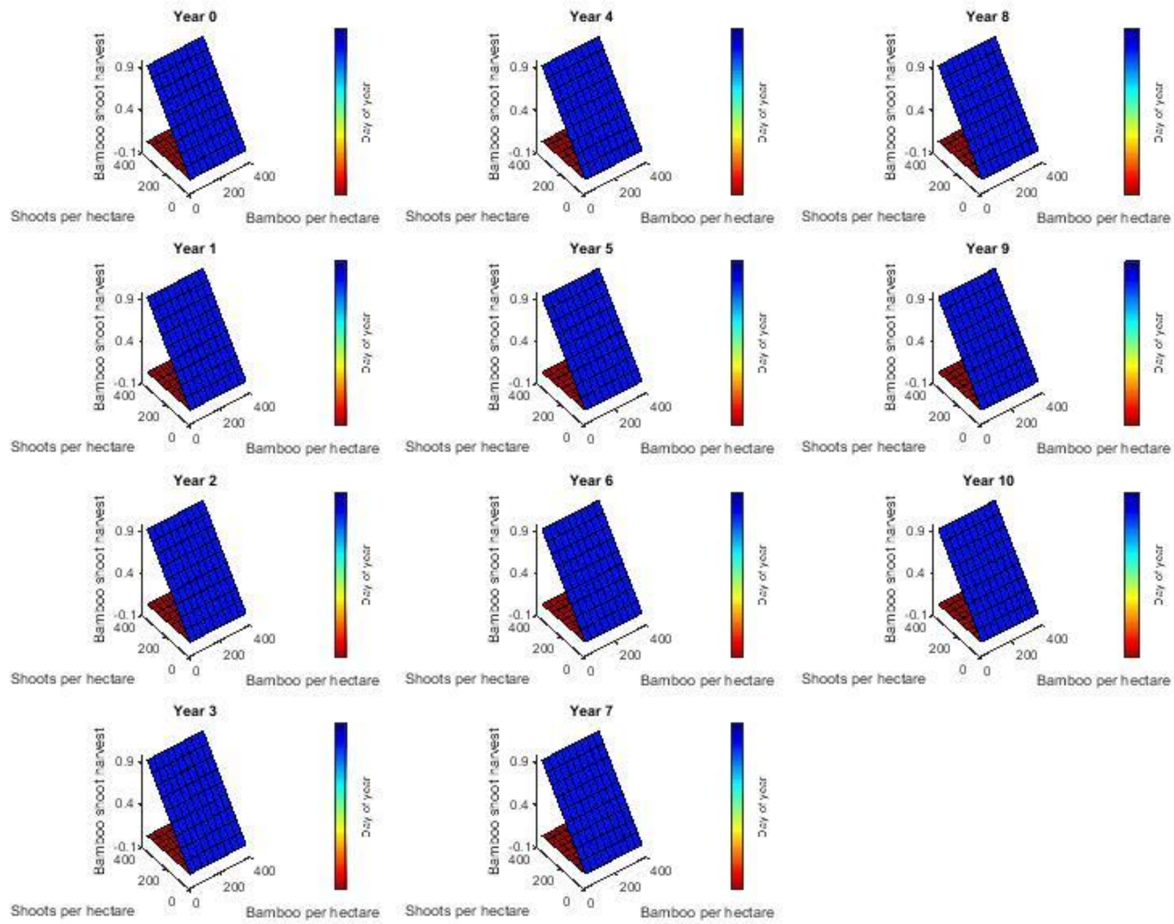
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



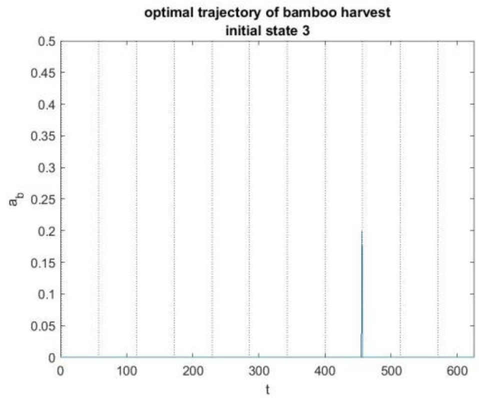
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

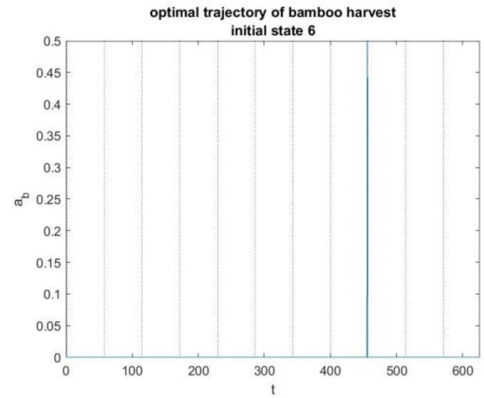


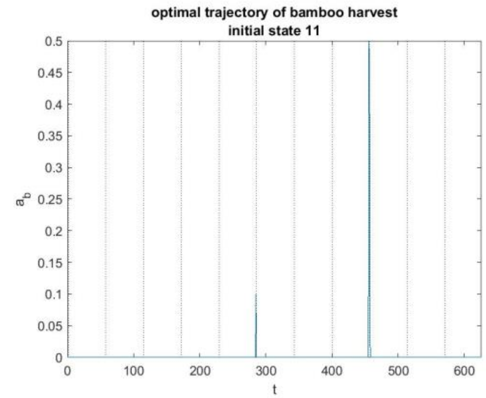
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

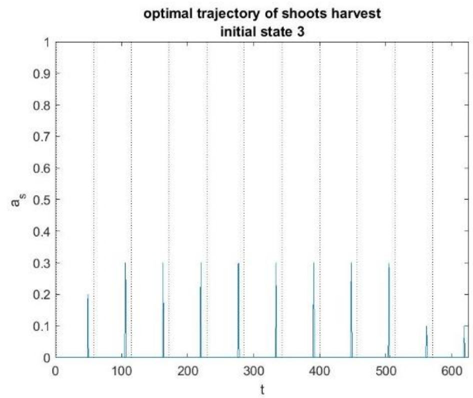


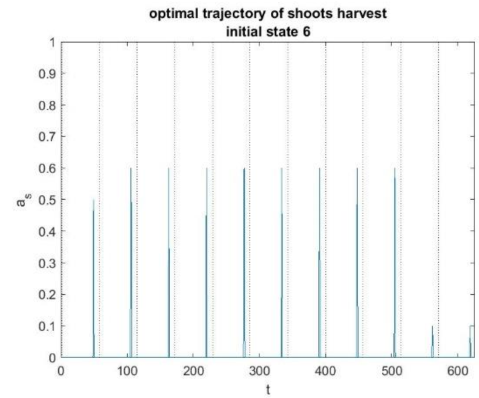
j) Optimal trajectories for each action and state variable over 11 years

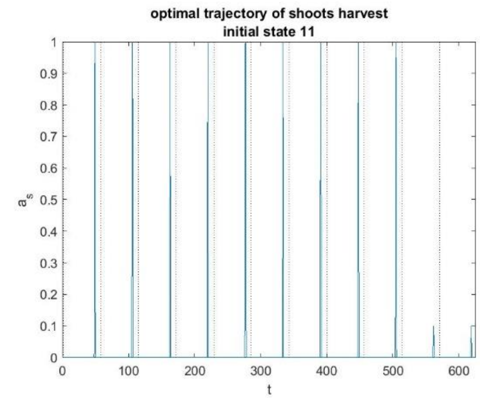


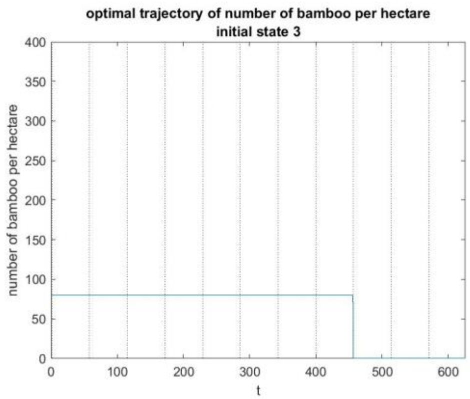


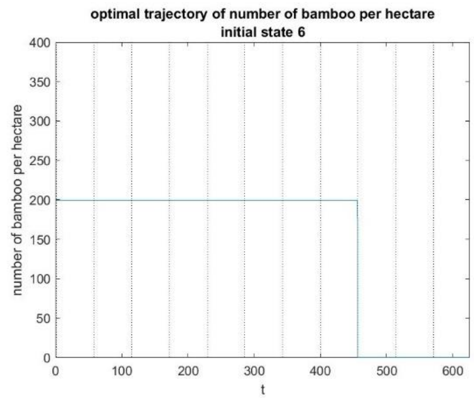


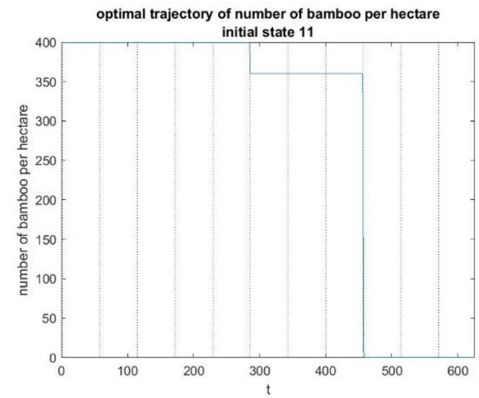


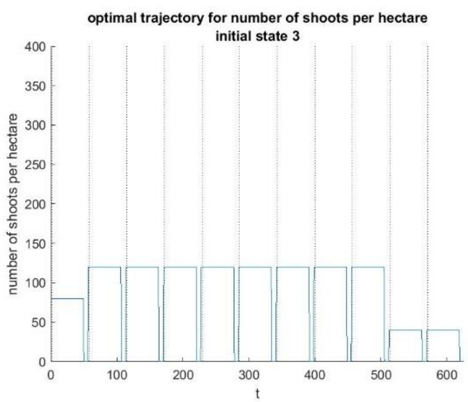


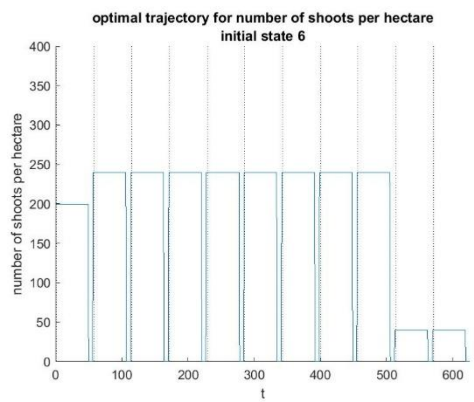



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

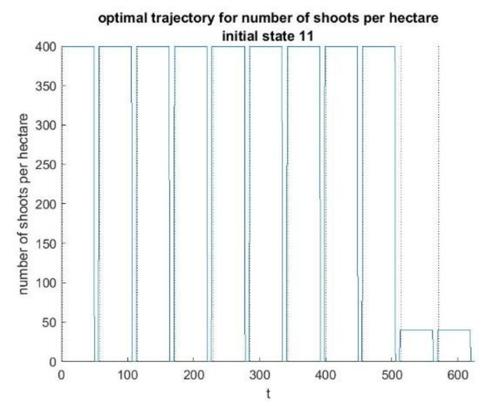
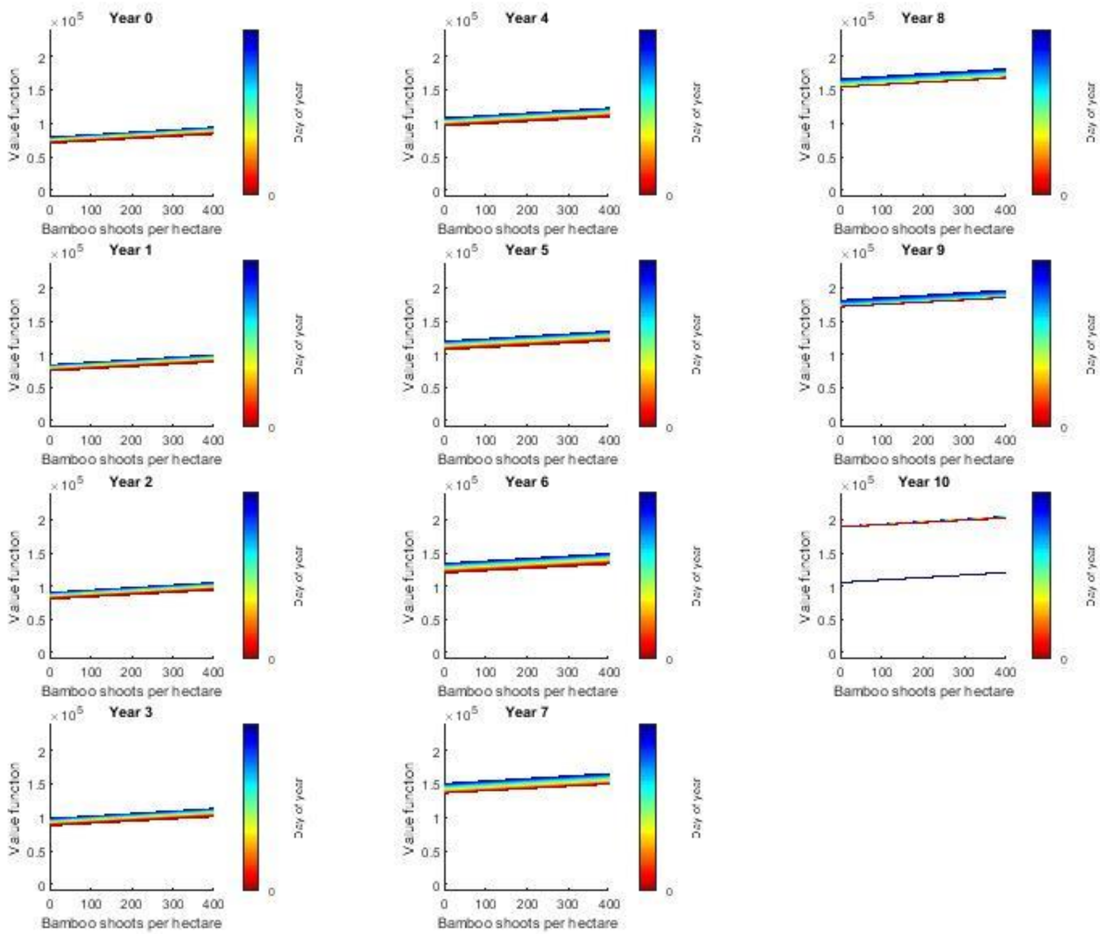
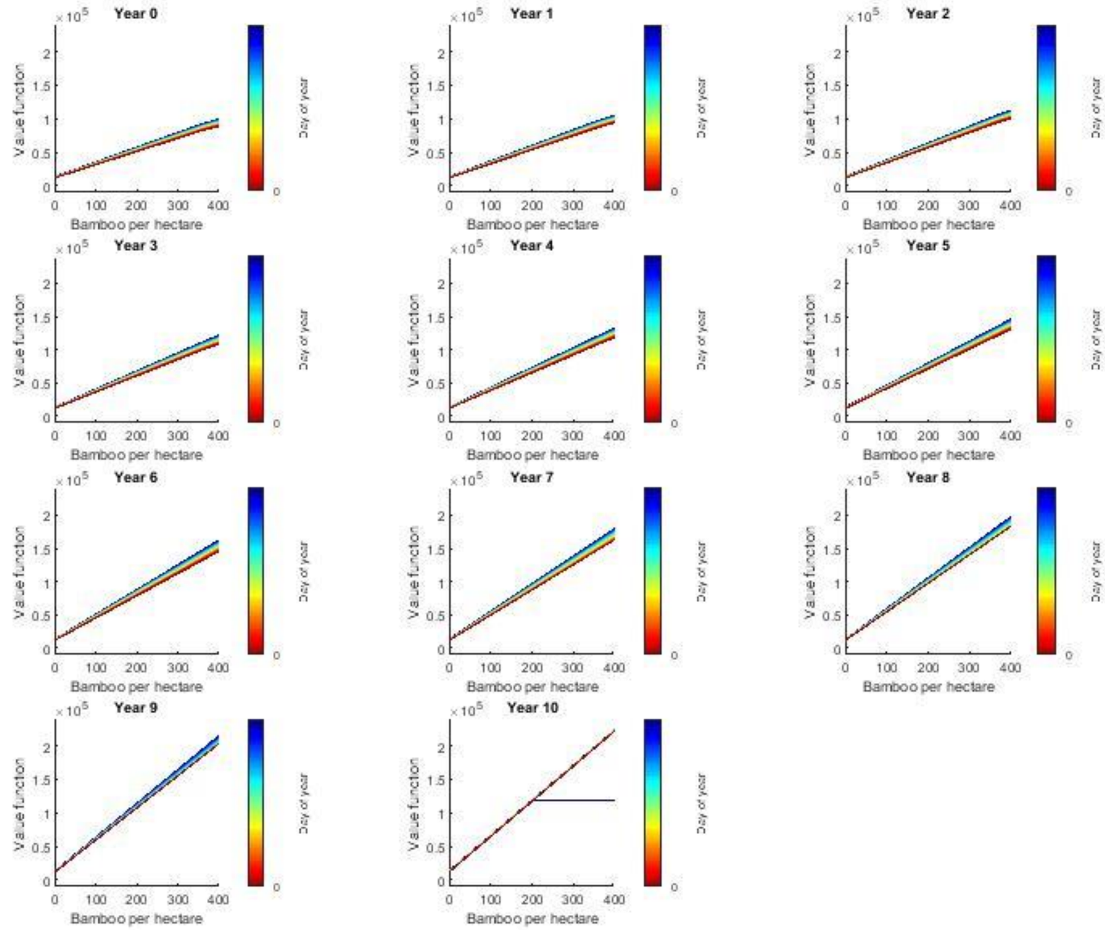

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

Figure 17: Stochastic Model, Specification 9, Version C, Set 1

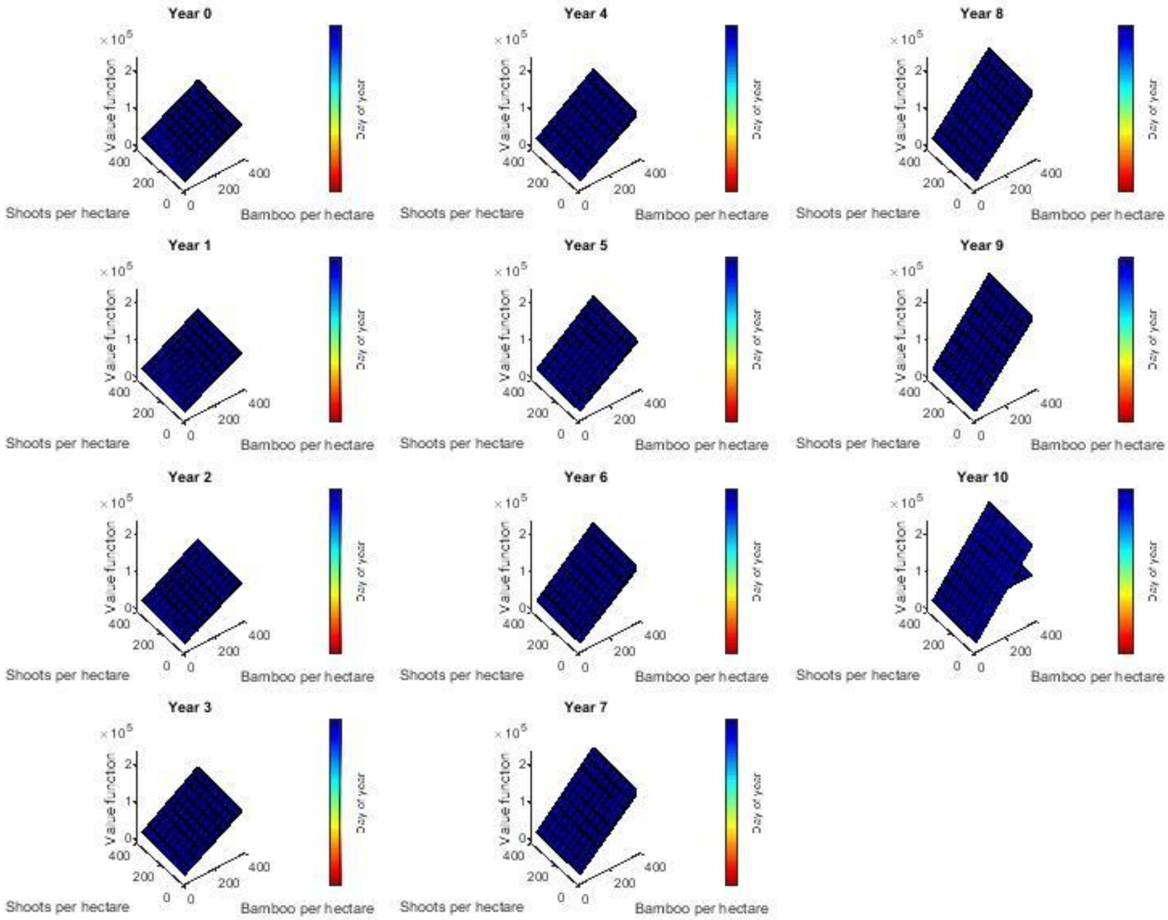
a) Value function as function of bamboo shoots per hectare



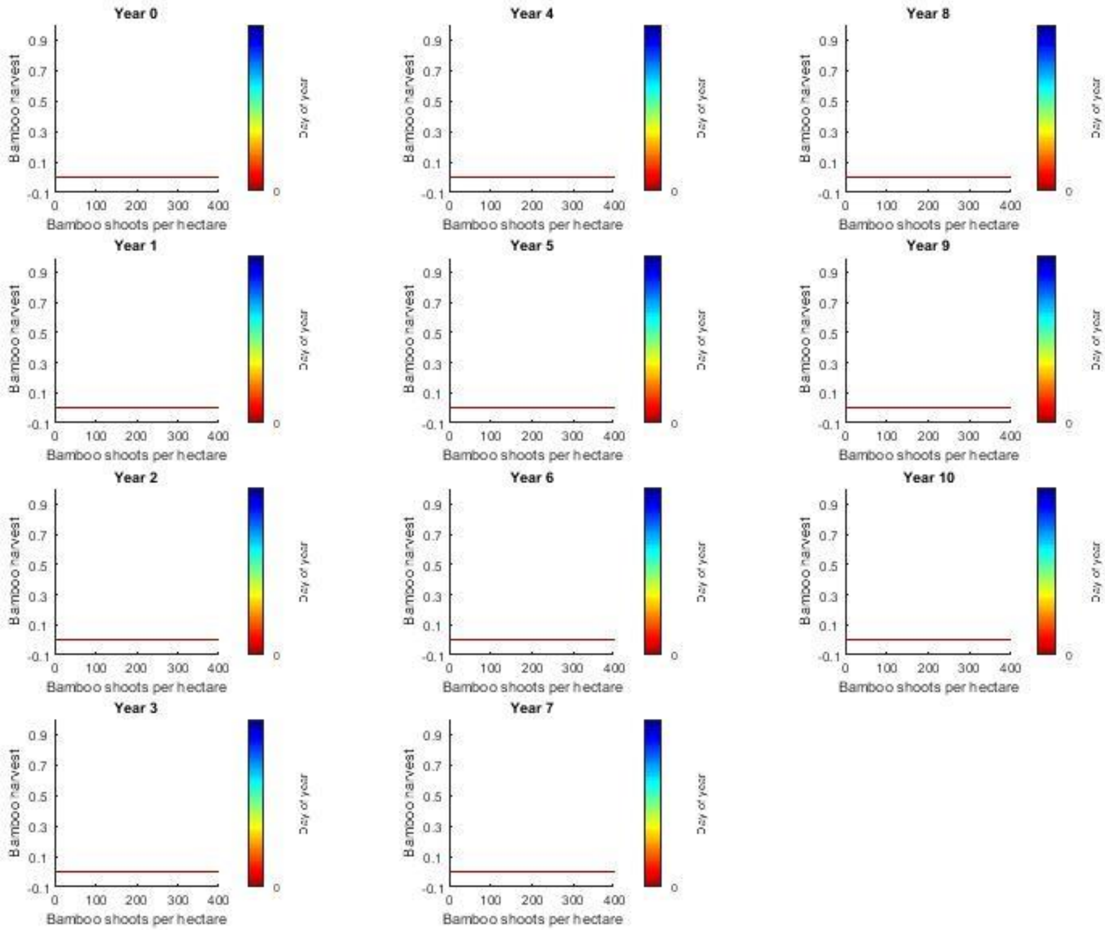
b) Value function as function of bamboo stem per hectare



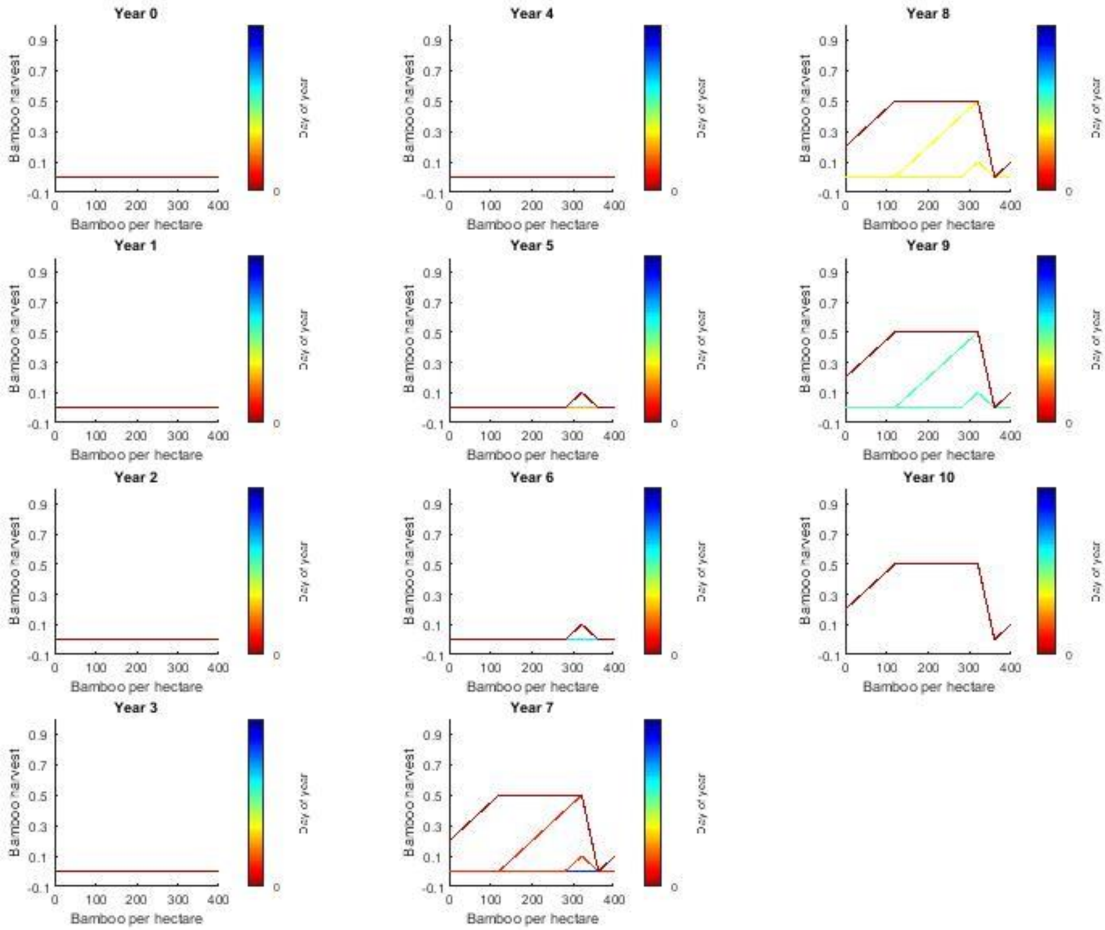
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



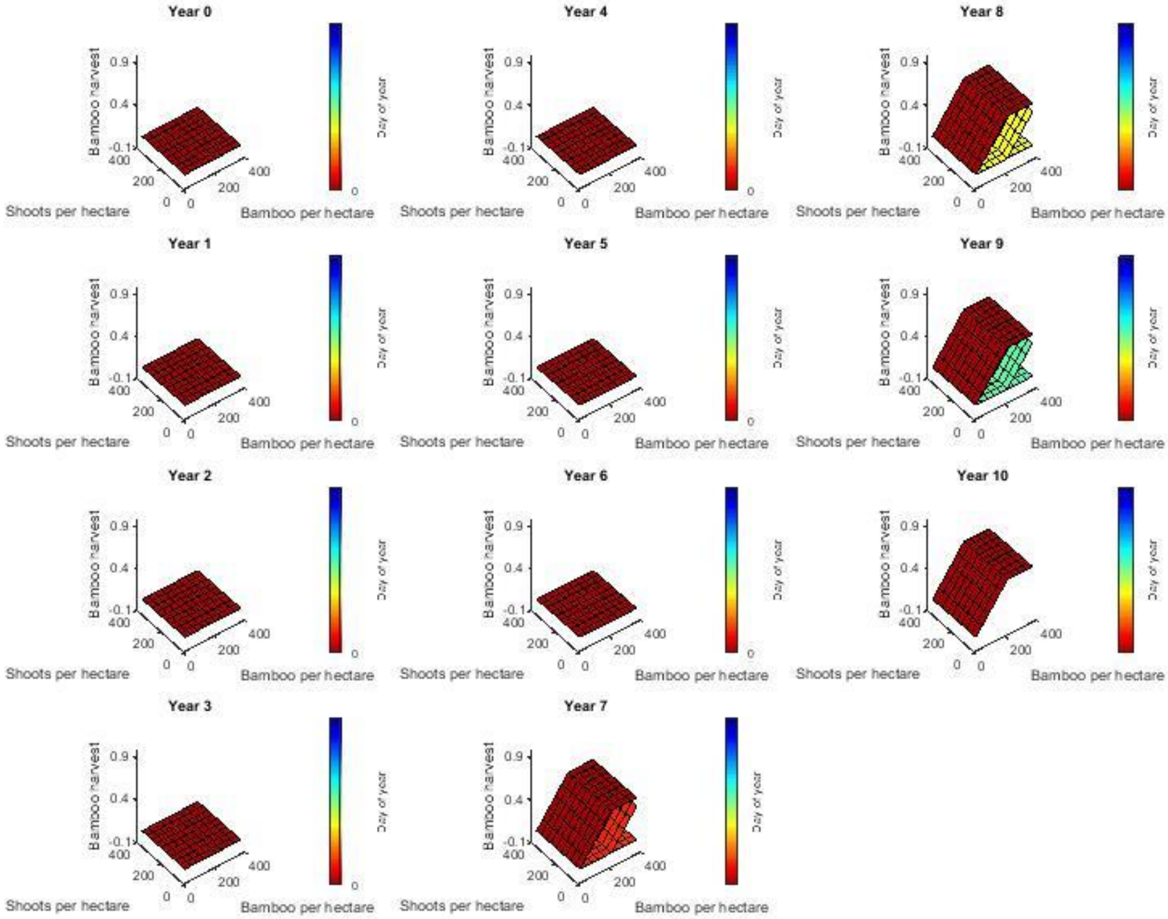
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



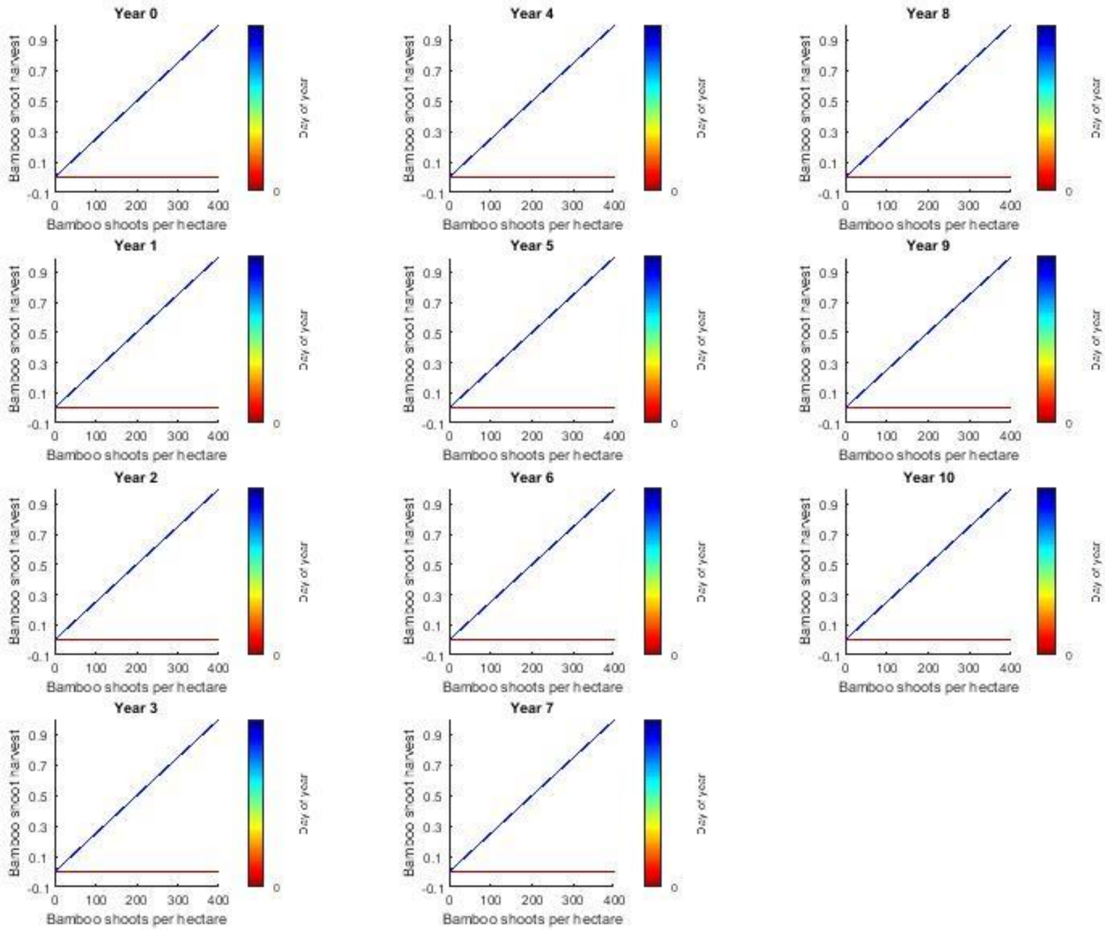
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



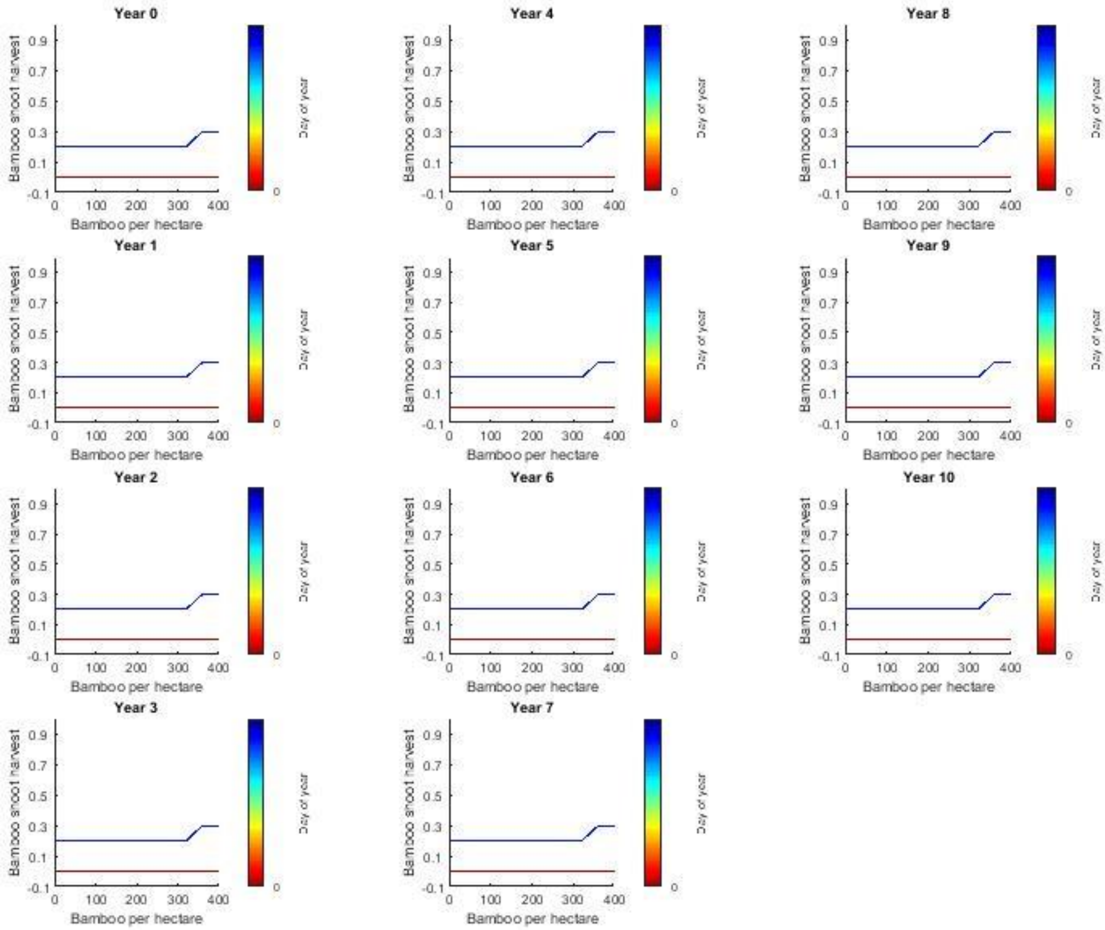
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



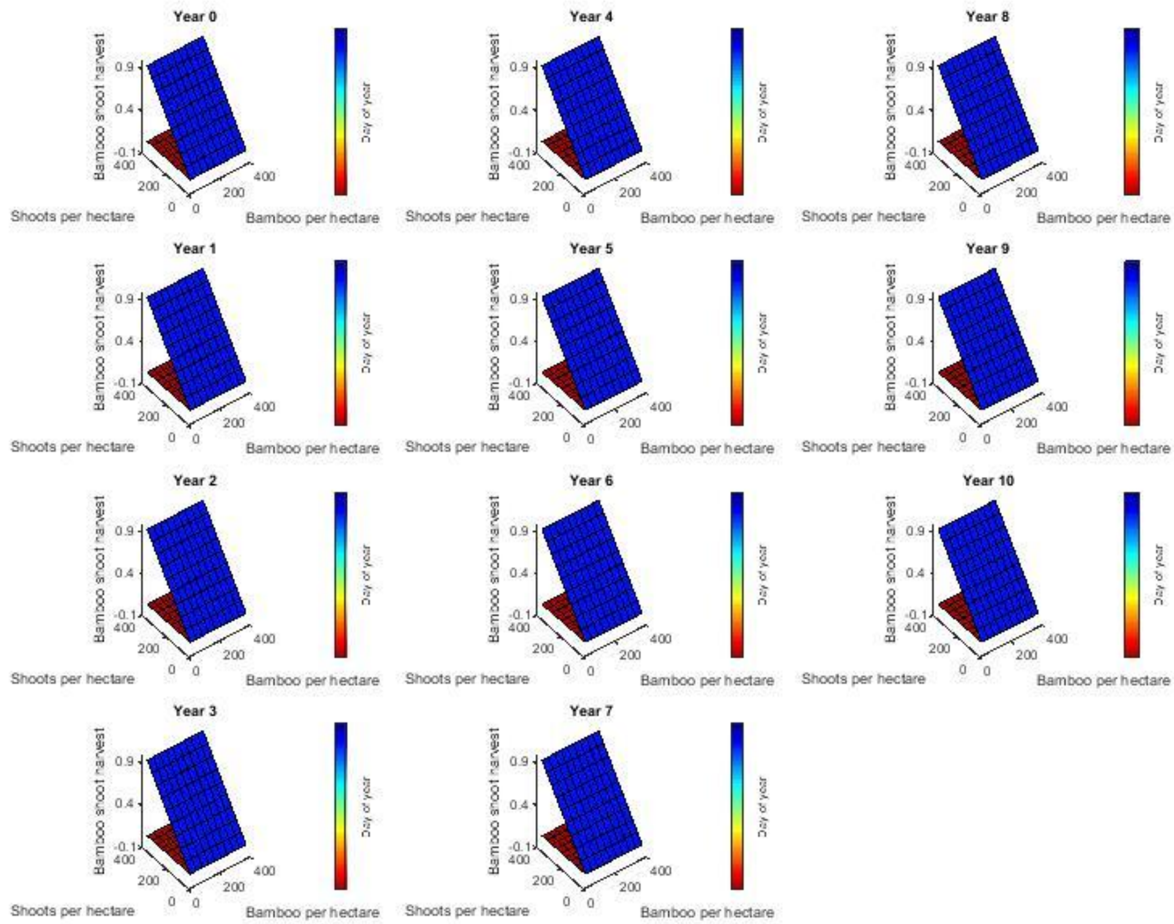
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



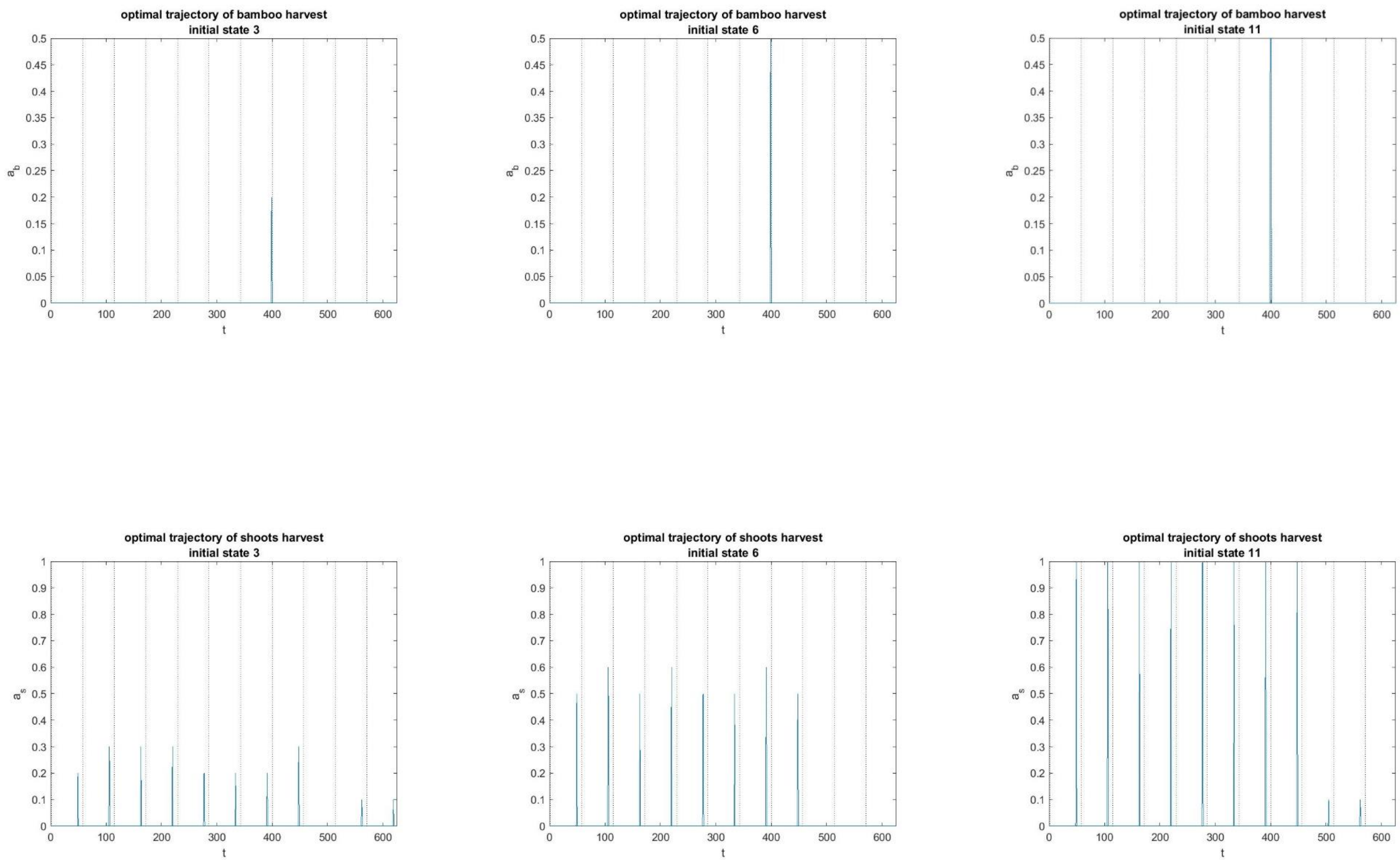
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

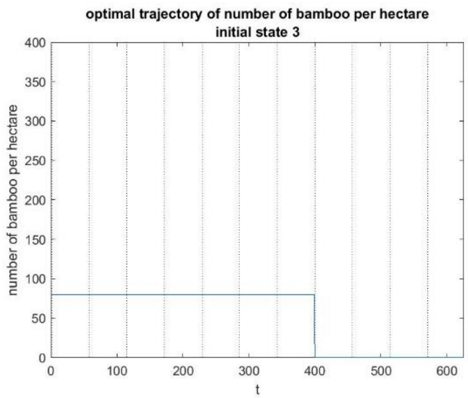


i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



j) Optimal trajectories for each action and state variable over 11 years




optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t
0
50
100
150
200
250
300
350
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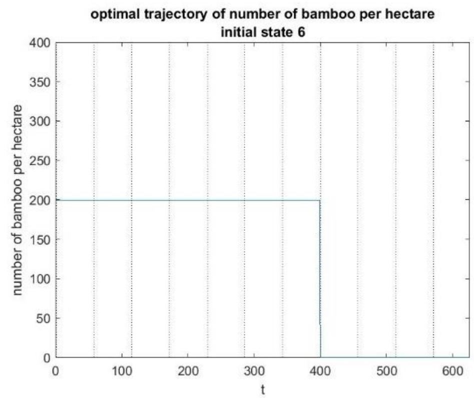
500
600


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t
0
50
100
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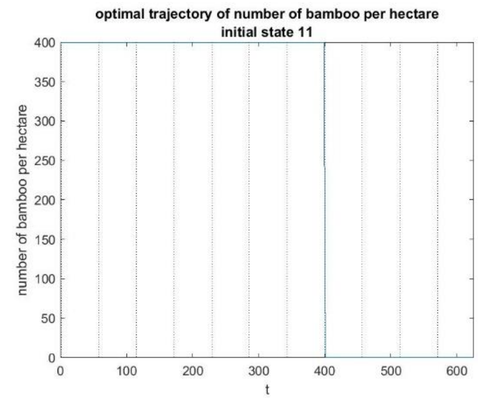
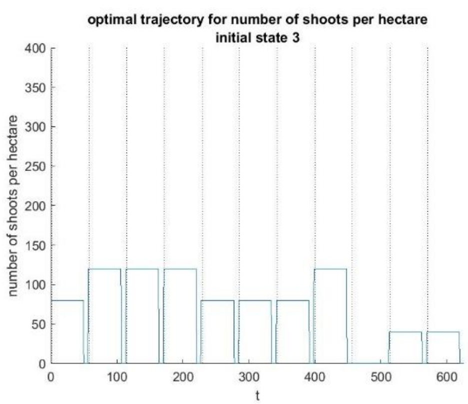
200
250
300
350
400
500
600


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t
0
50
100
150
200
250
300
350
400
500
600


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t
0
50
100
150
200
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400
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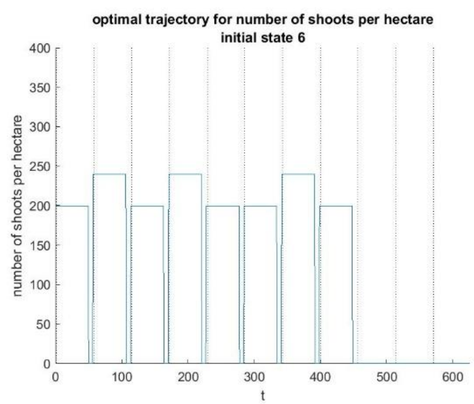
600


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t
0
50
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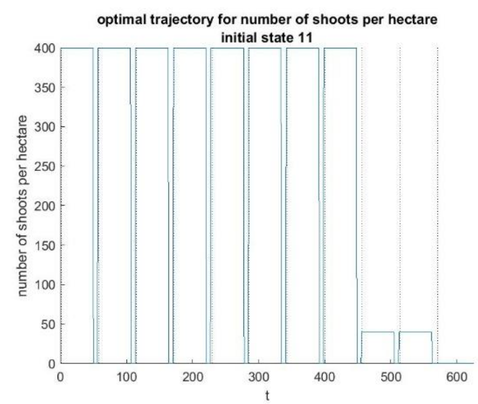
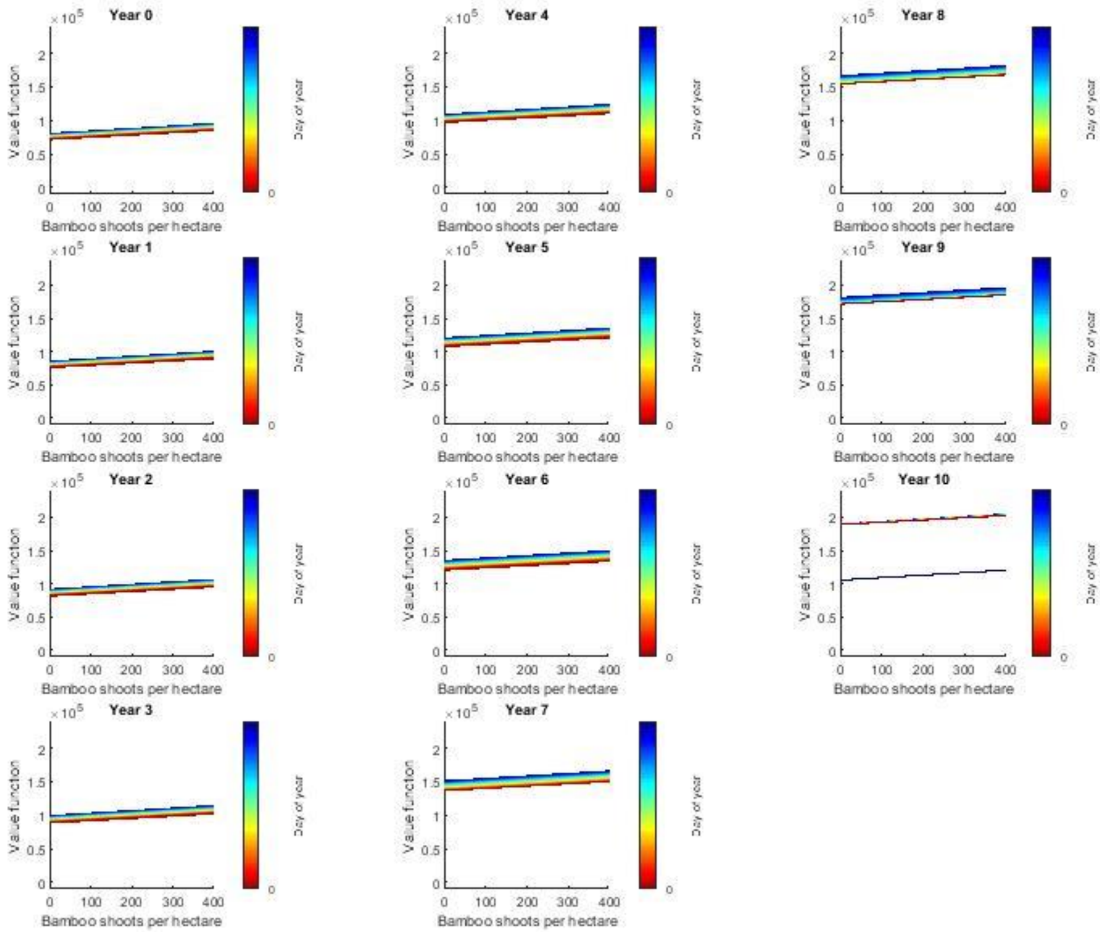
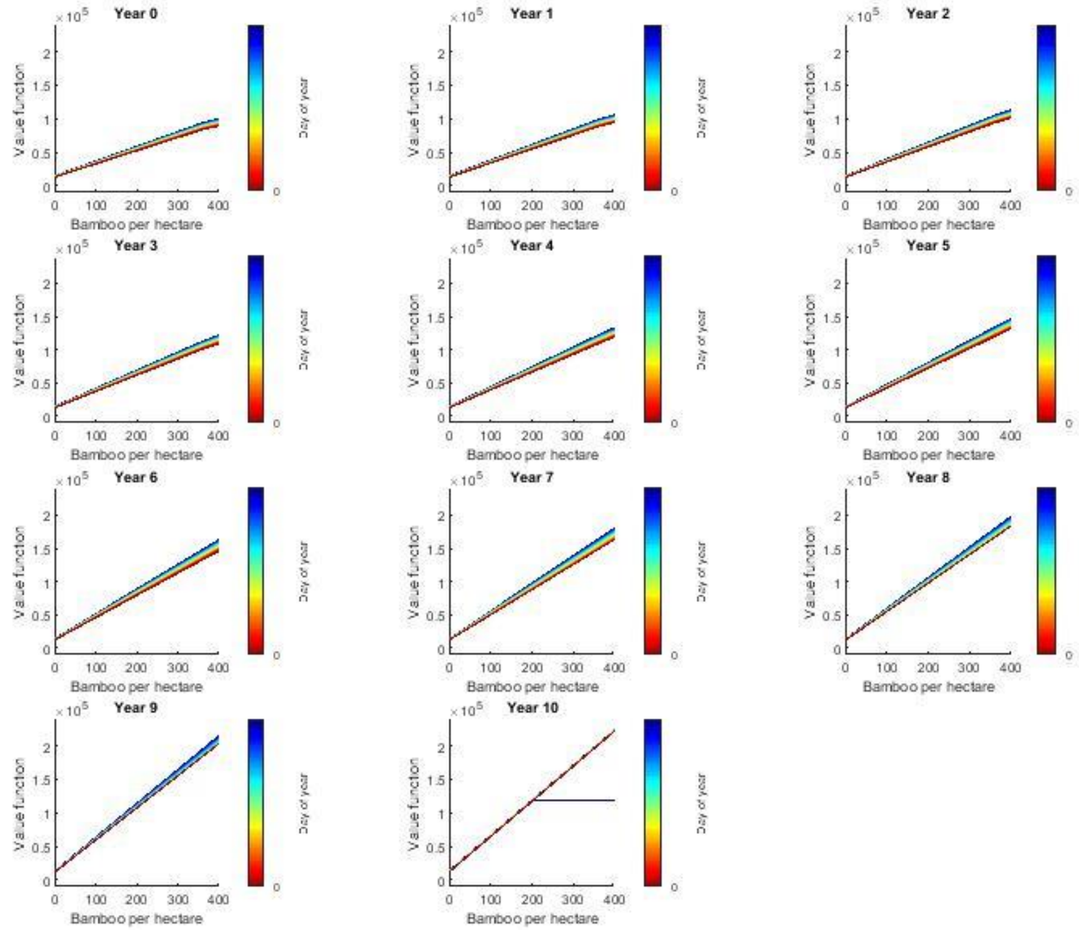

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t
0
50
100
150
200
250
300
350
400
500
600

Figure 18: Stochastic Model, Specification 9, Version D, Set 1

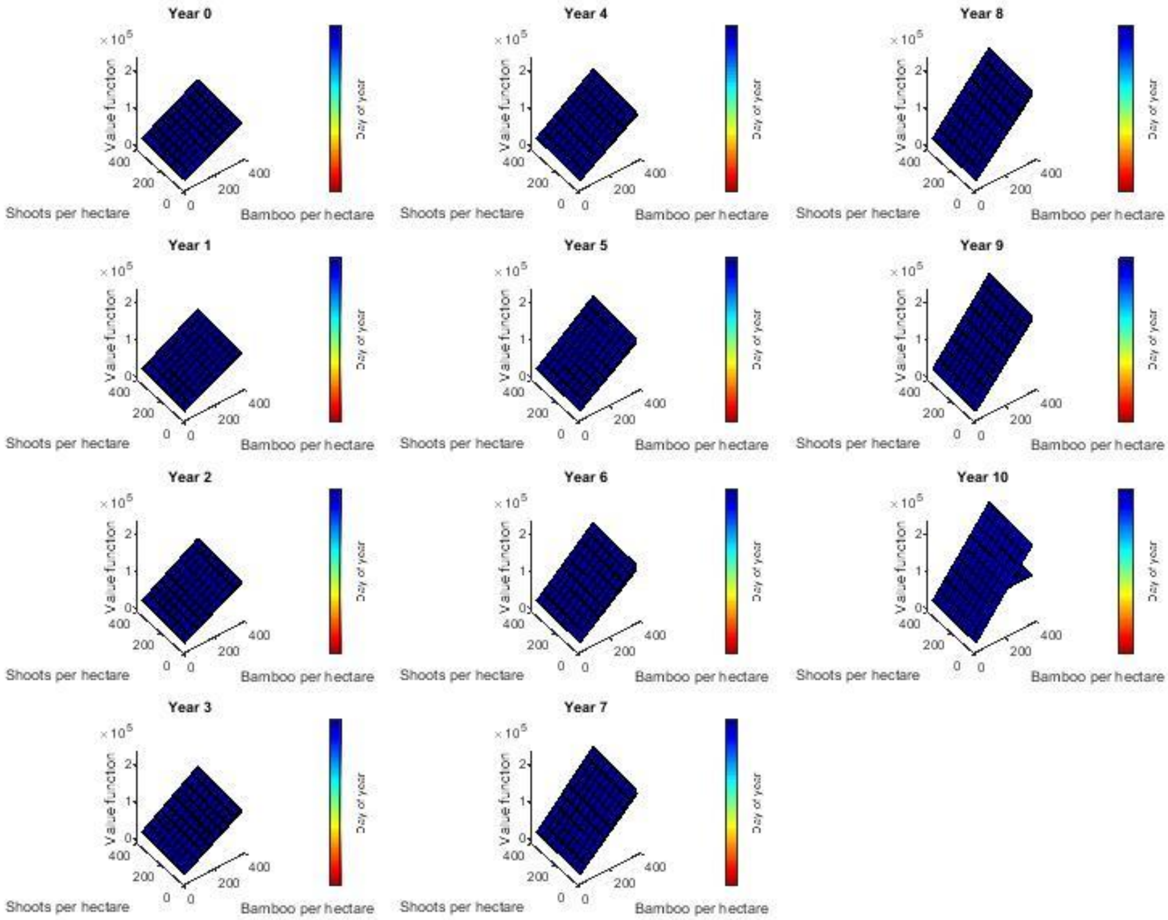
a) Value function as function of bamboo shoots per hectare



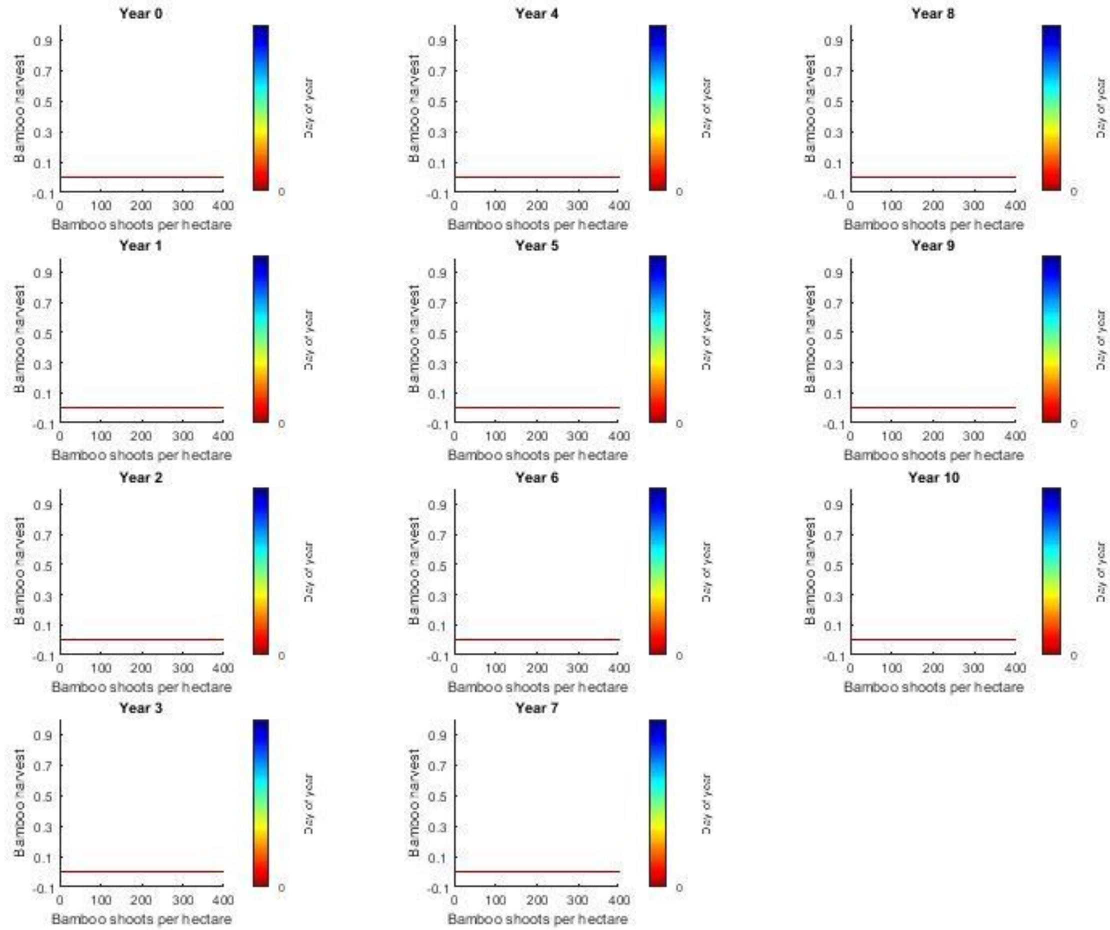
b) Value function as function of bamboo stem per hectare



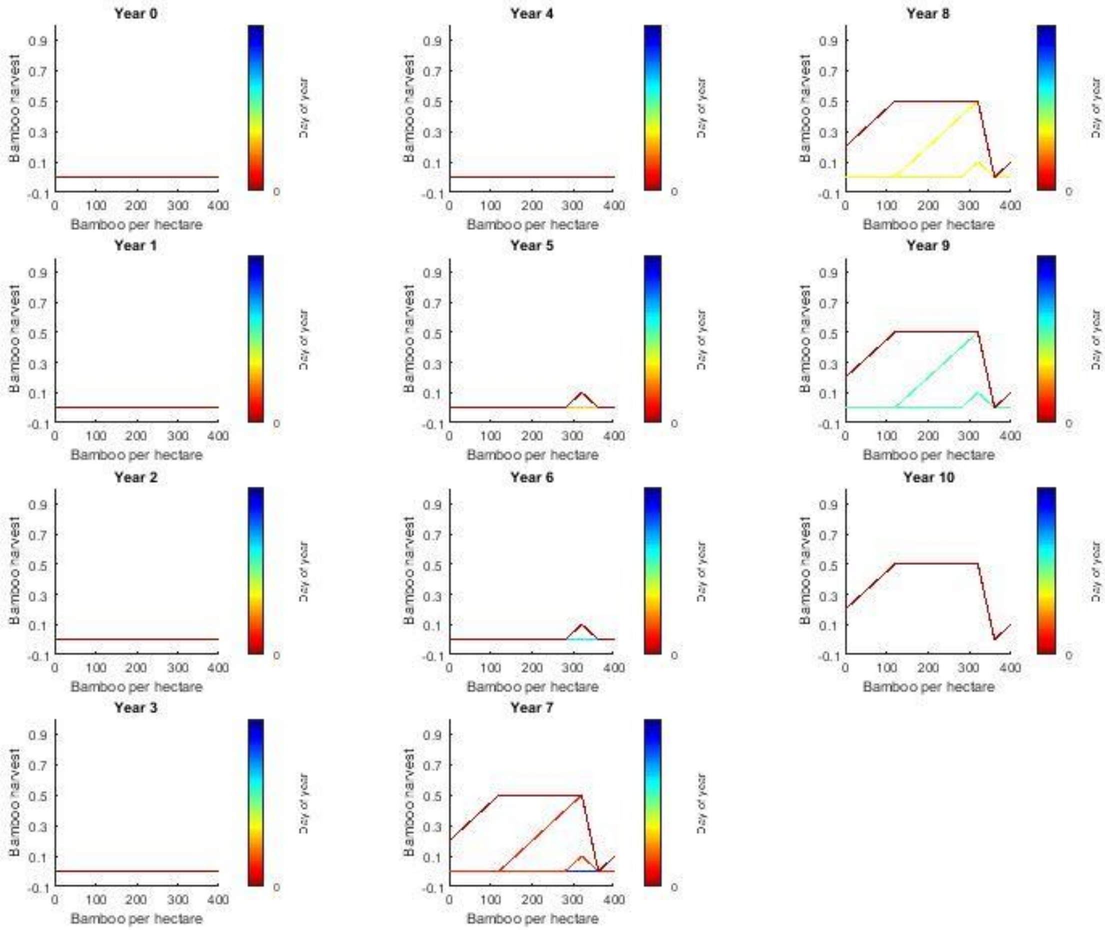
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



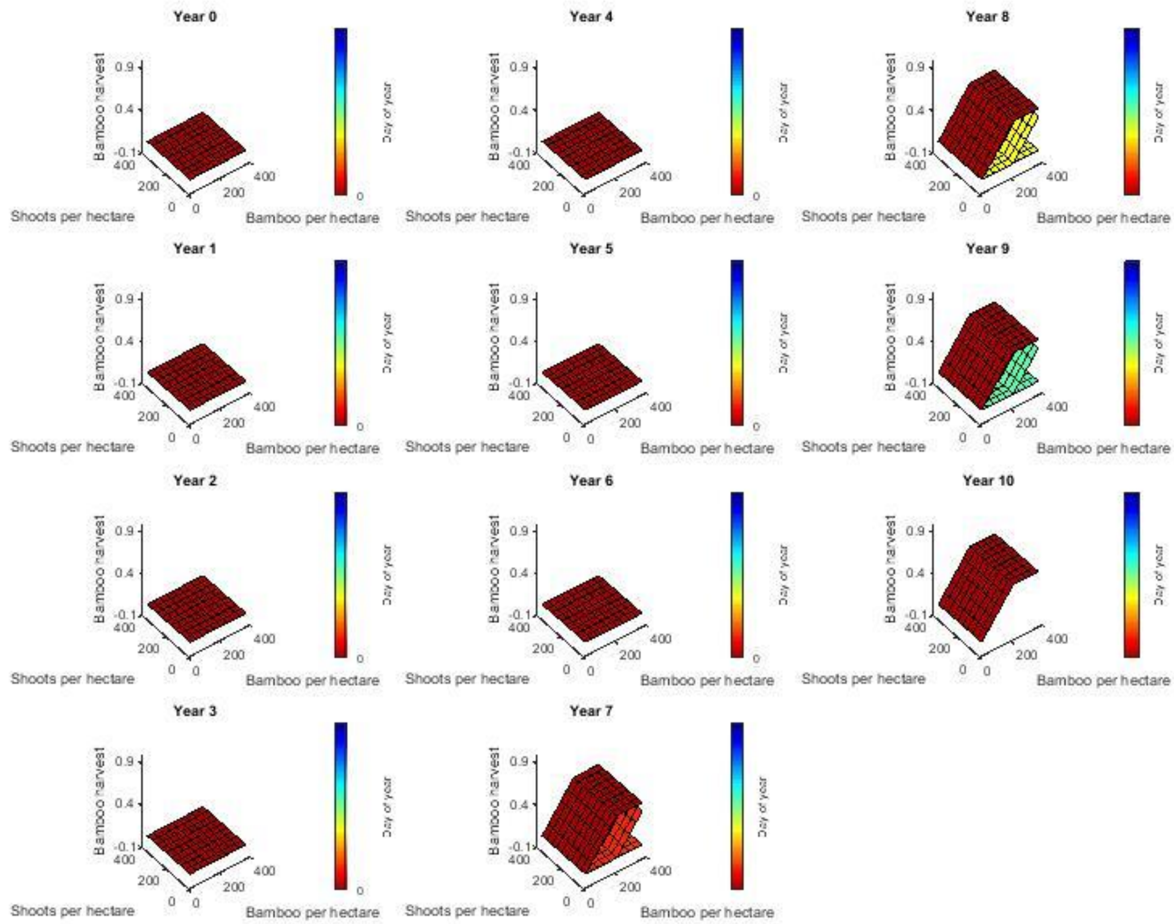
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



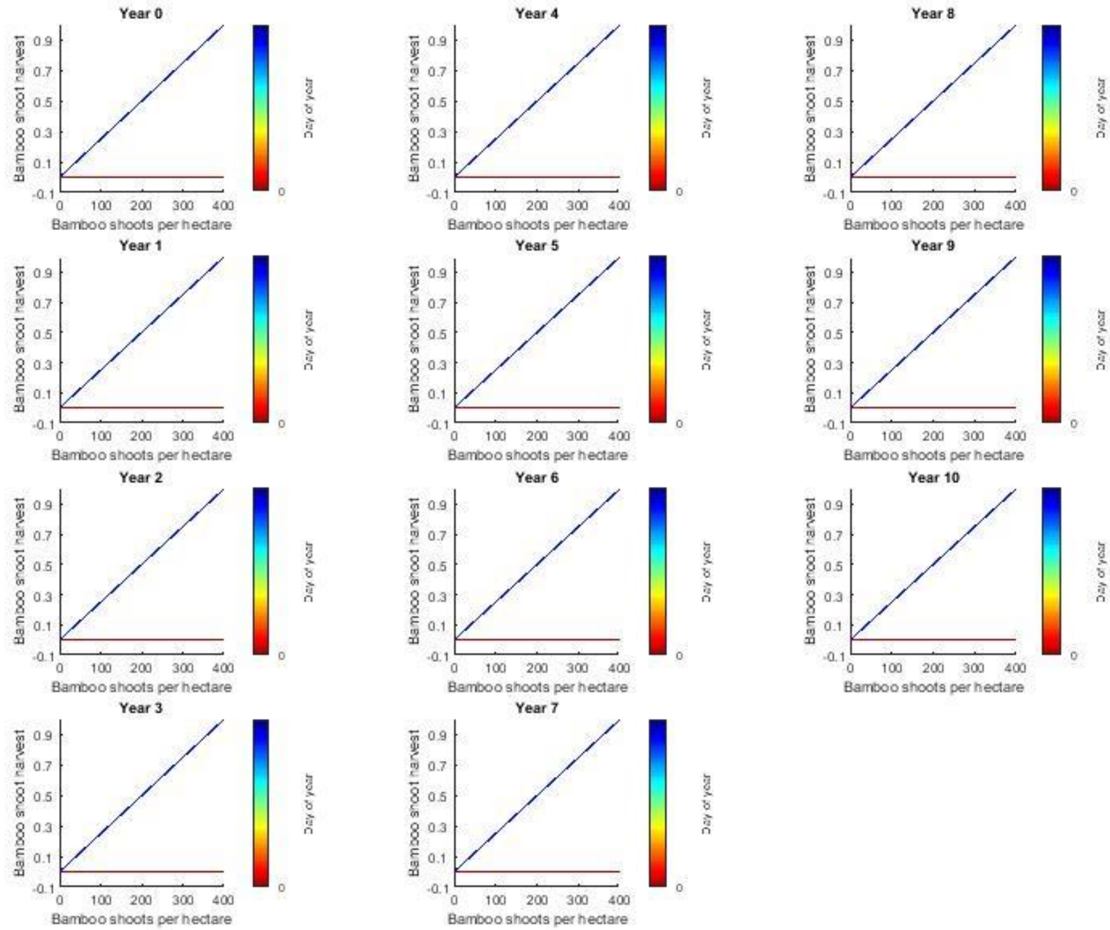
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



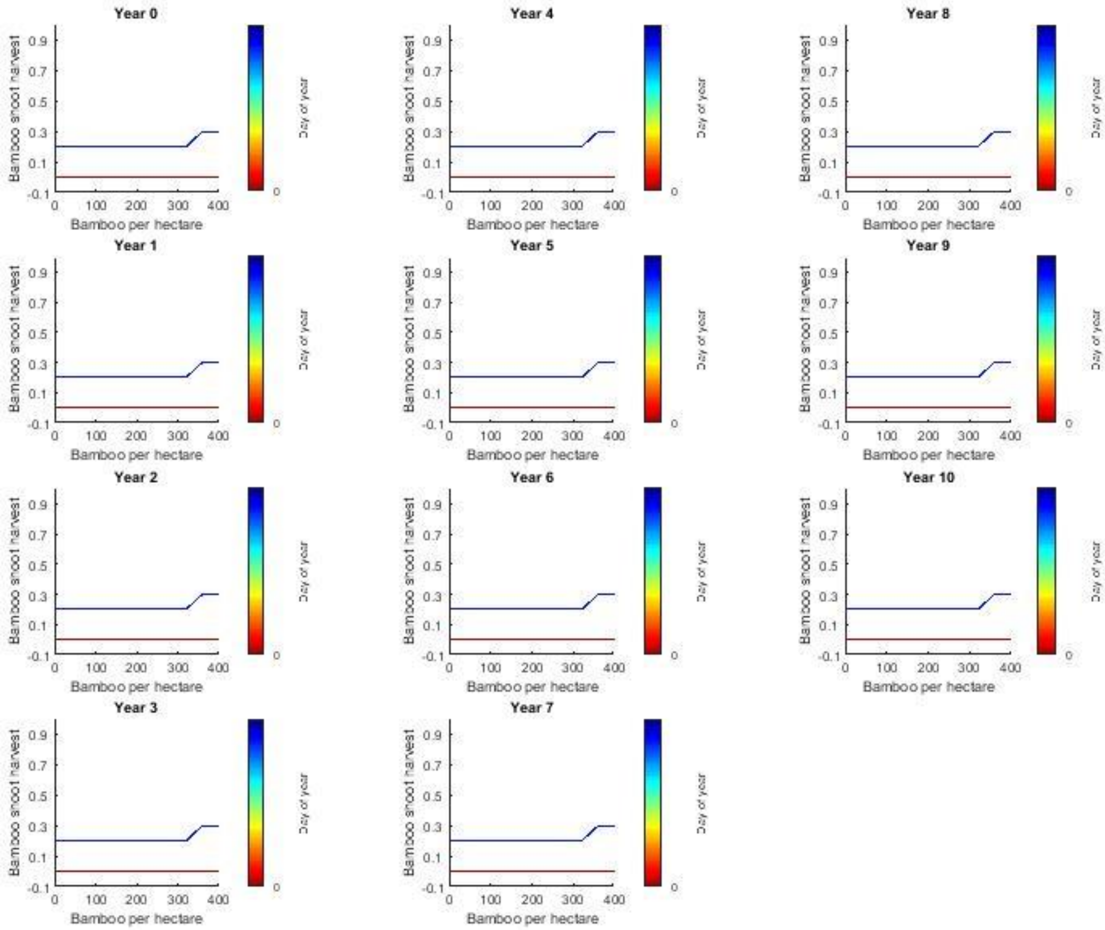
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



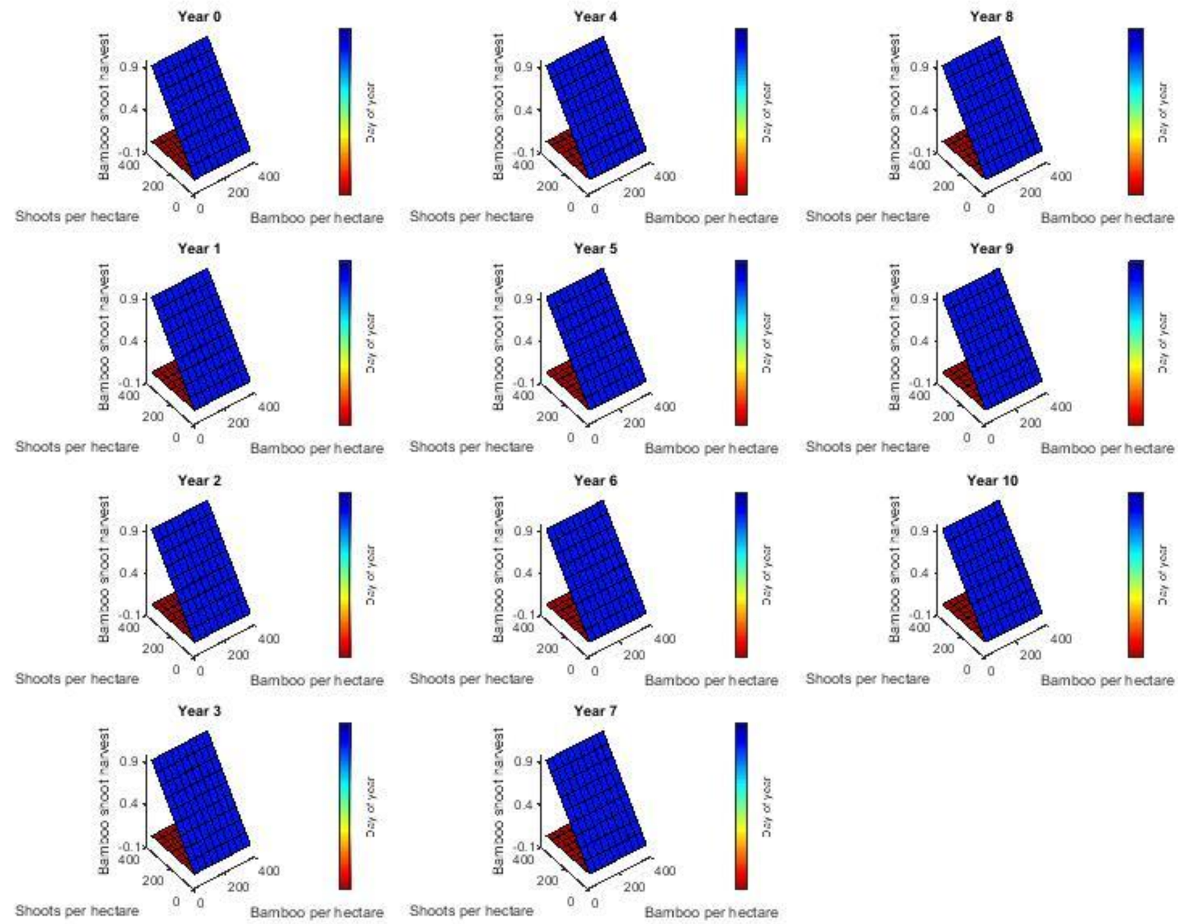
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



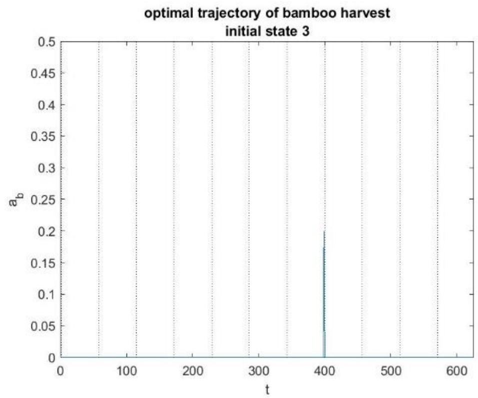
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

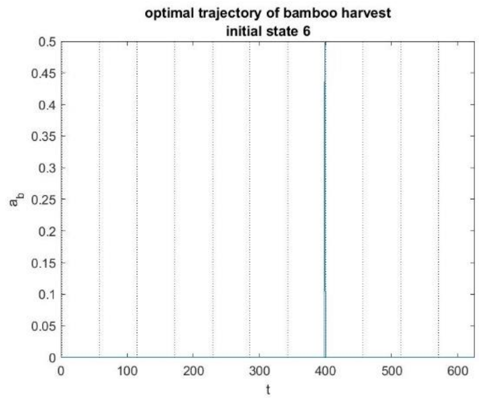


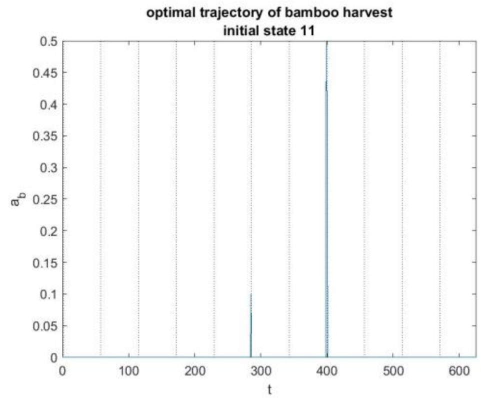
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

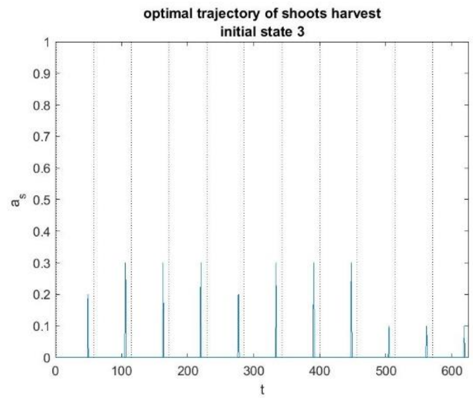


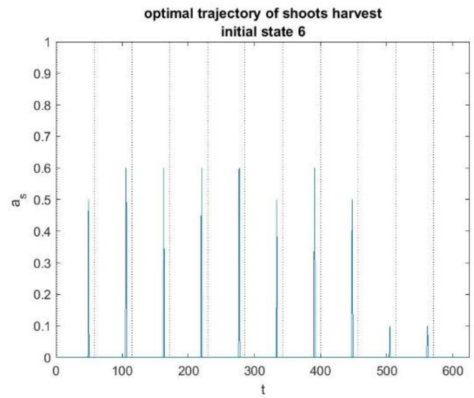
j) Optimal trajectories for each action and state variable over 11 years

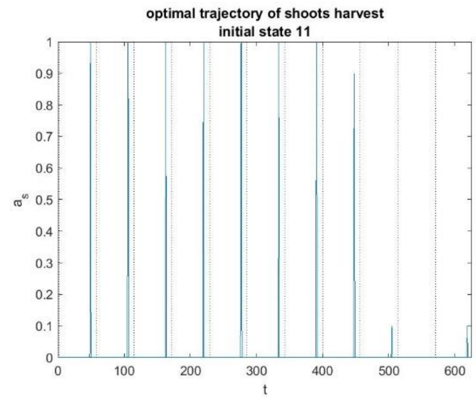


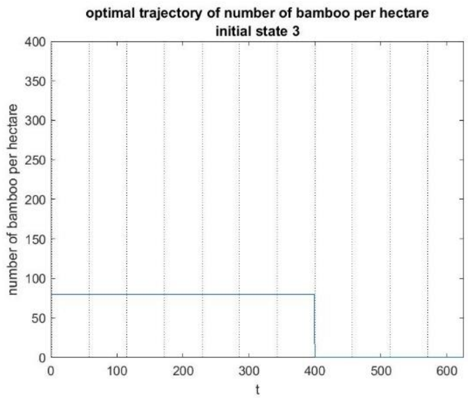


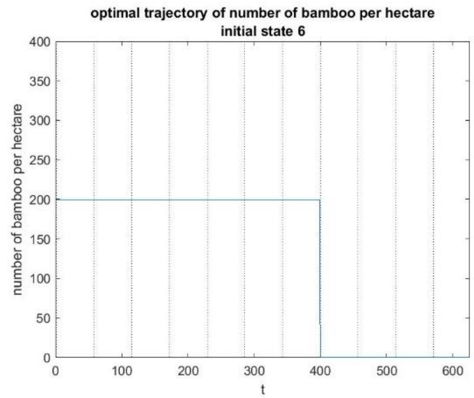


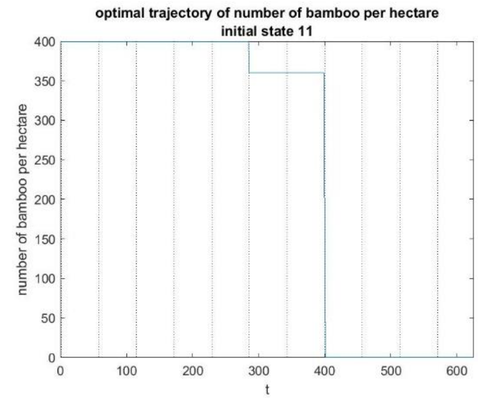


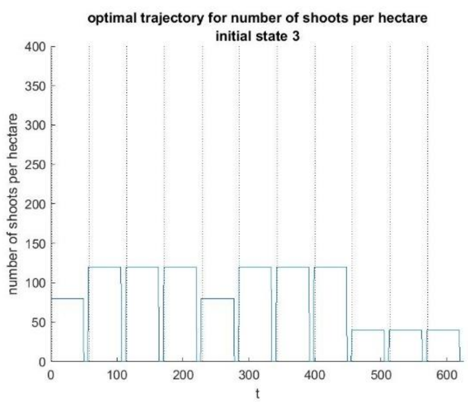


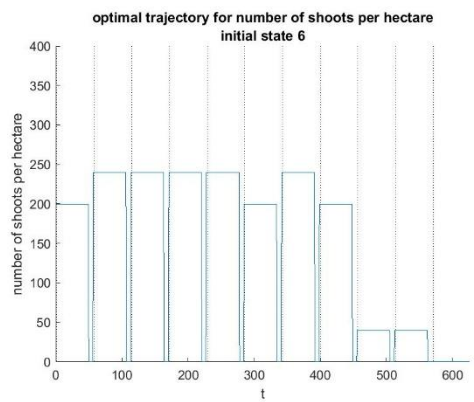



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

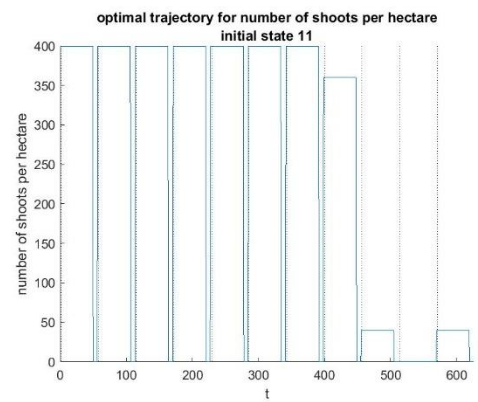
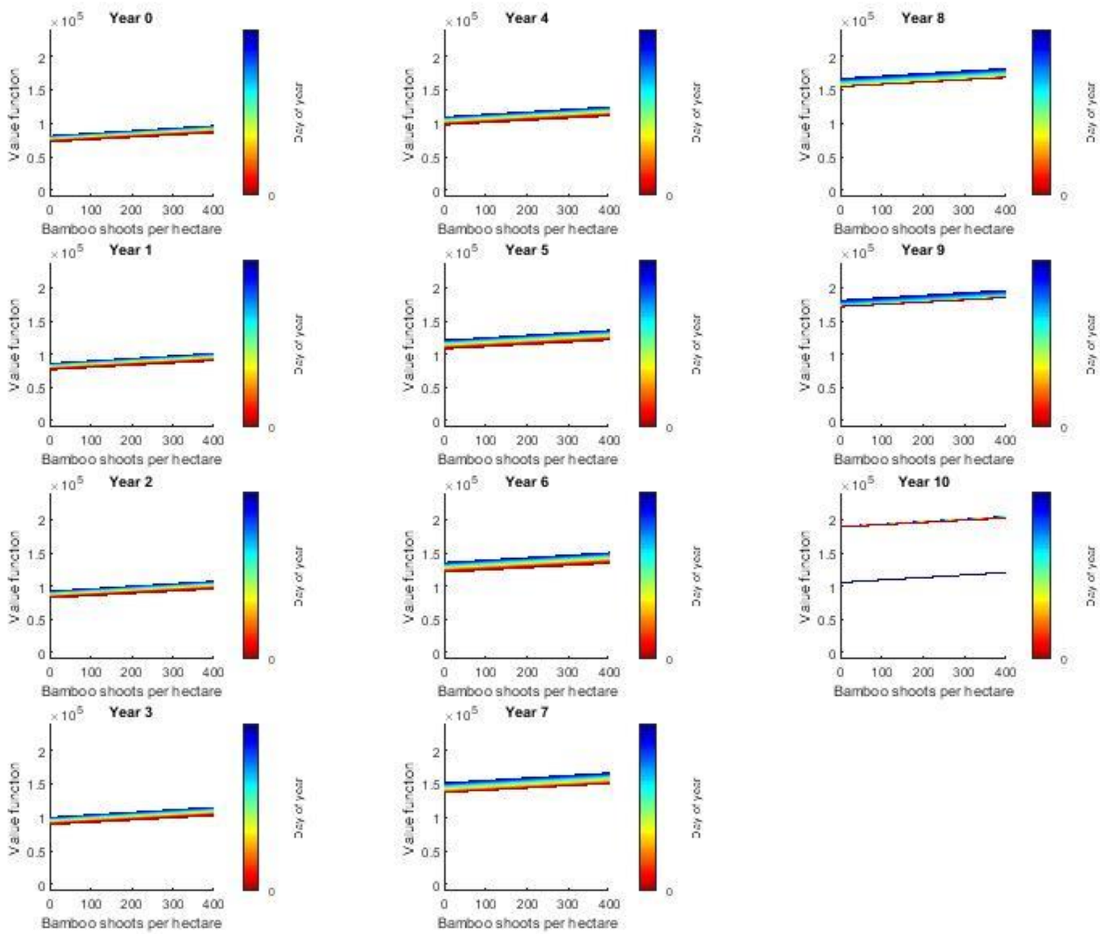
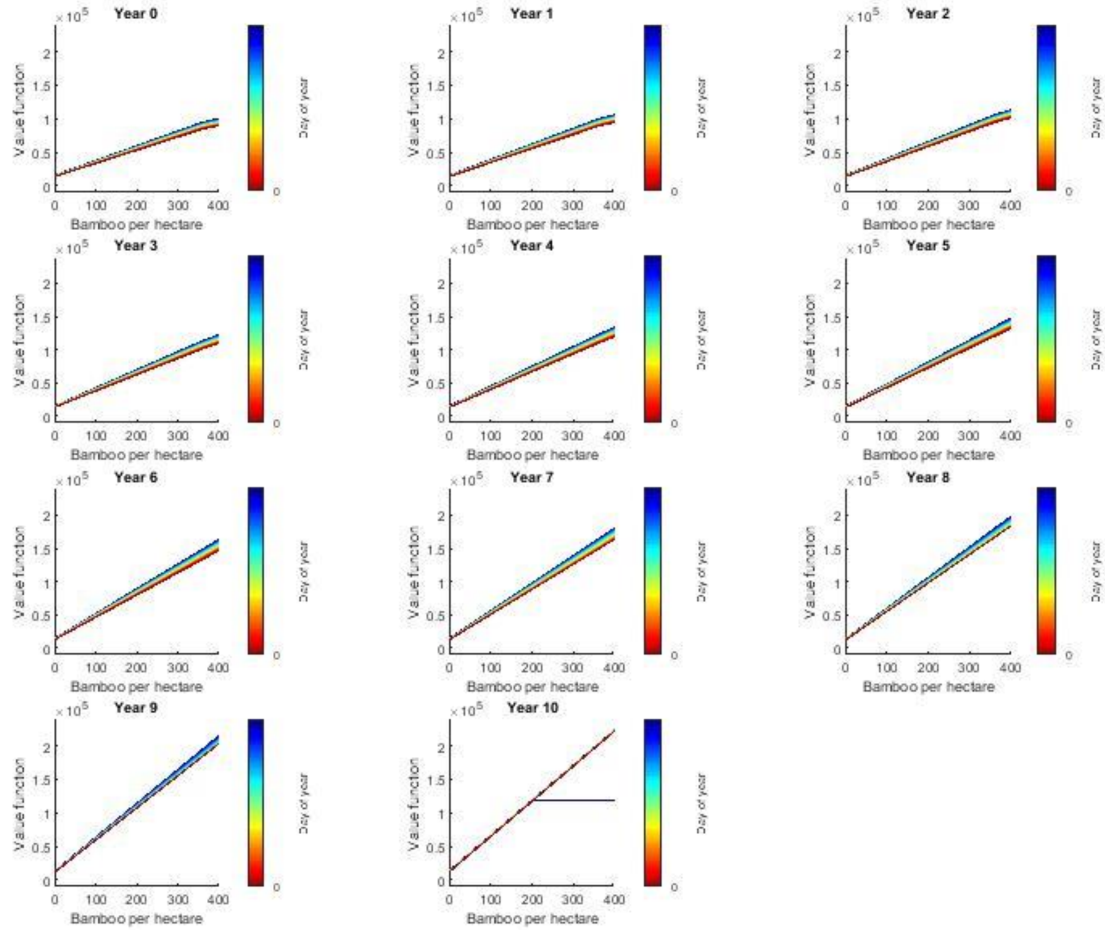

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

Figure 19: Stochastic Model, Specification 9, Version E, Set 1

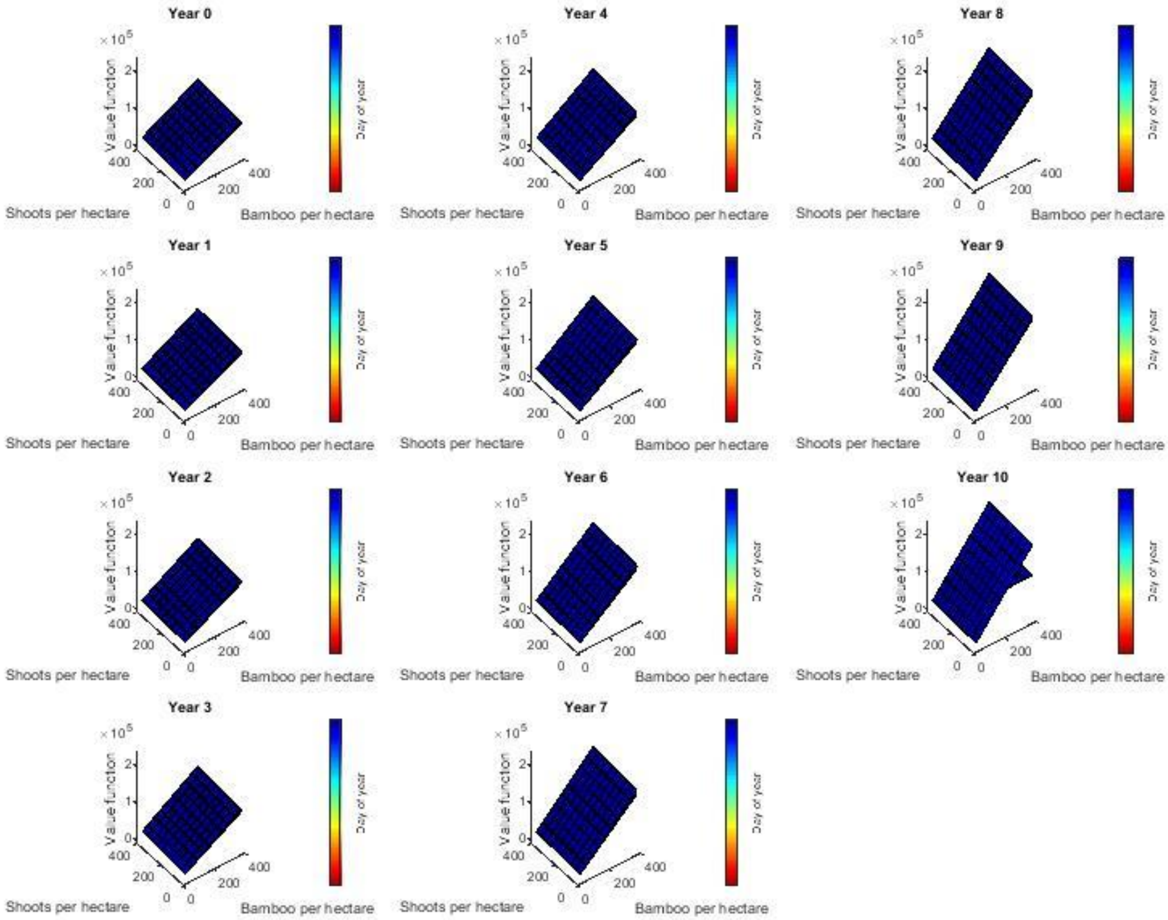
a) Value function as function of bamboo shoots per hectare



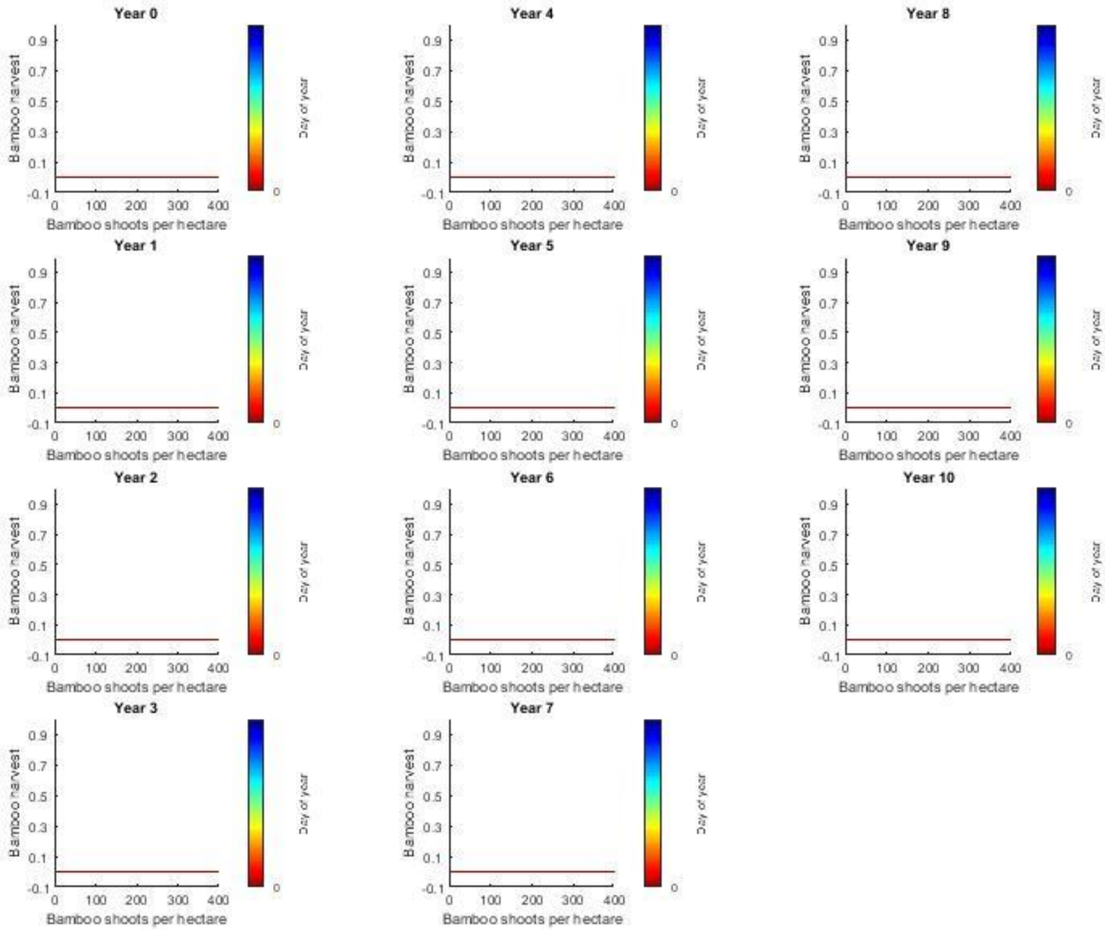
b) Value function as function of bamboo stem per hectare



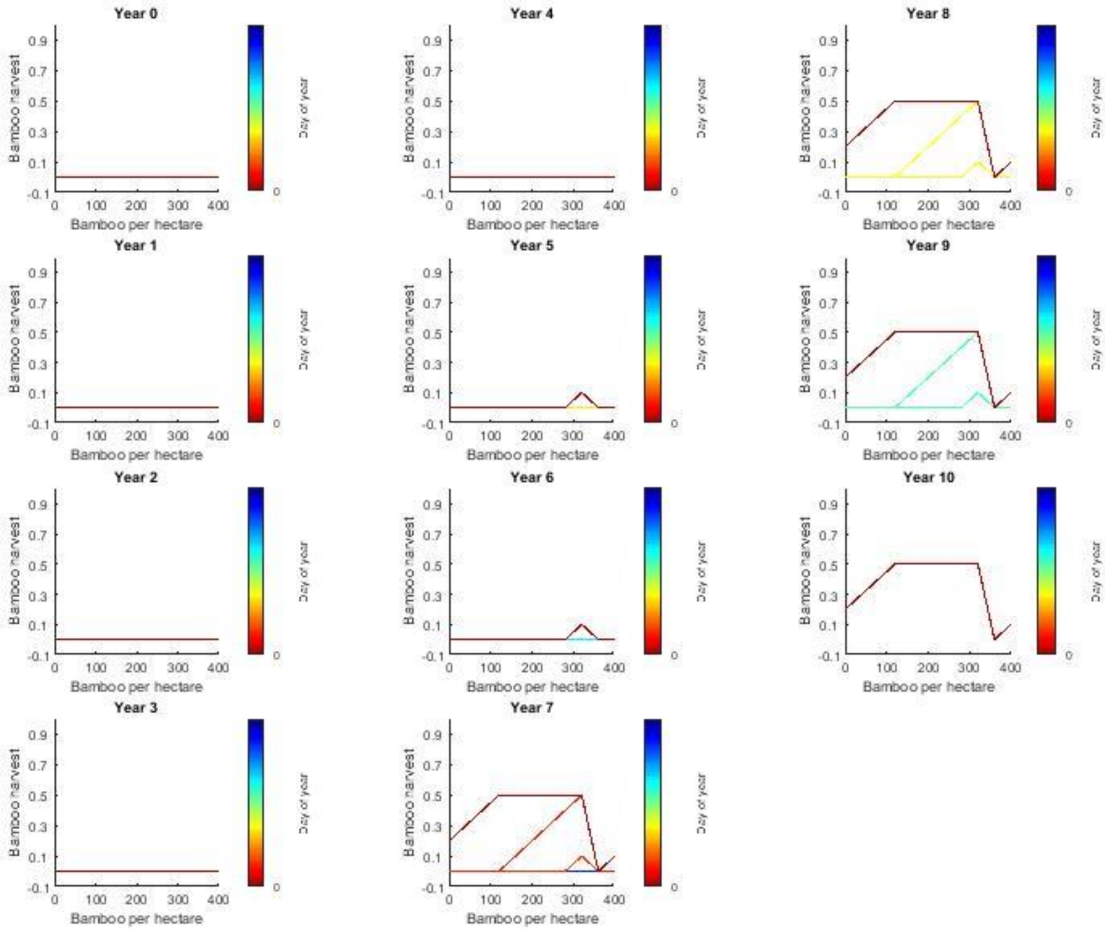
c) Value function as function of bamboo shoots per hectare and bamboo stem per hectare



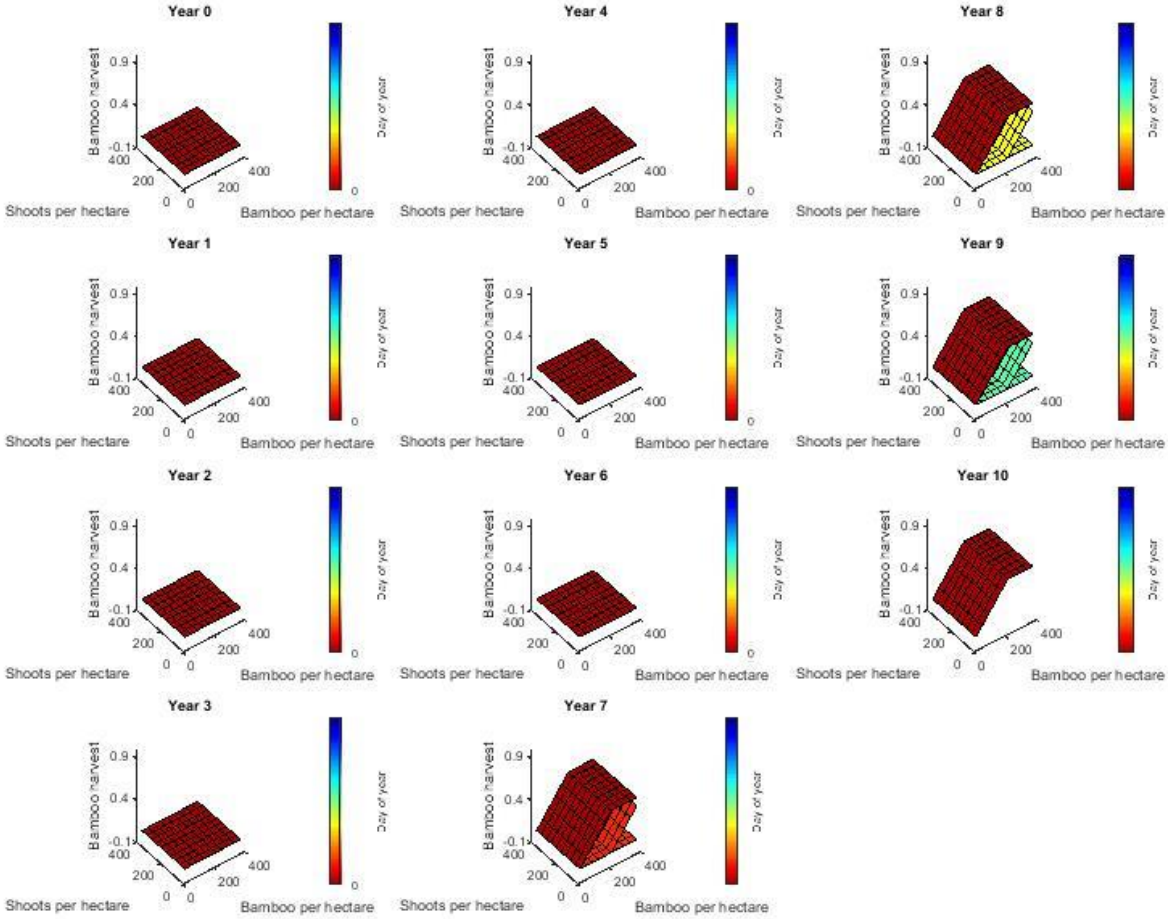
d) Bamboo stem harvest policy function as function of bamboo shoots per hectare



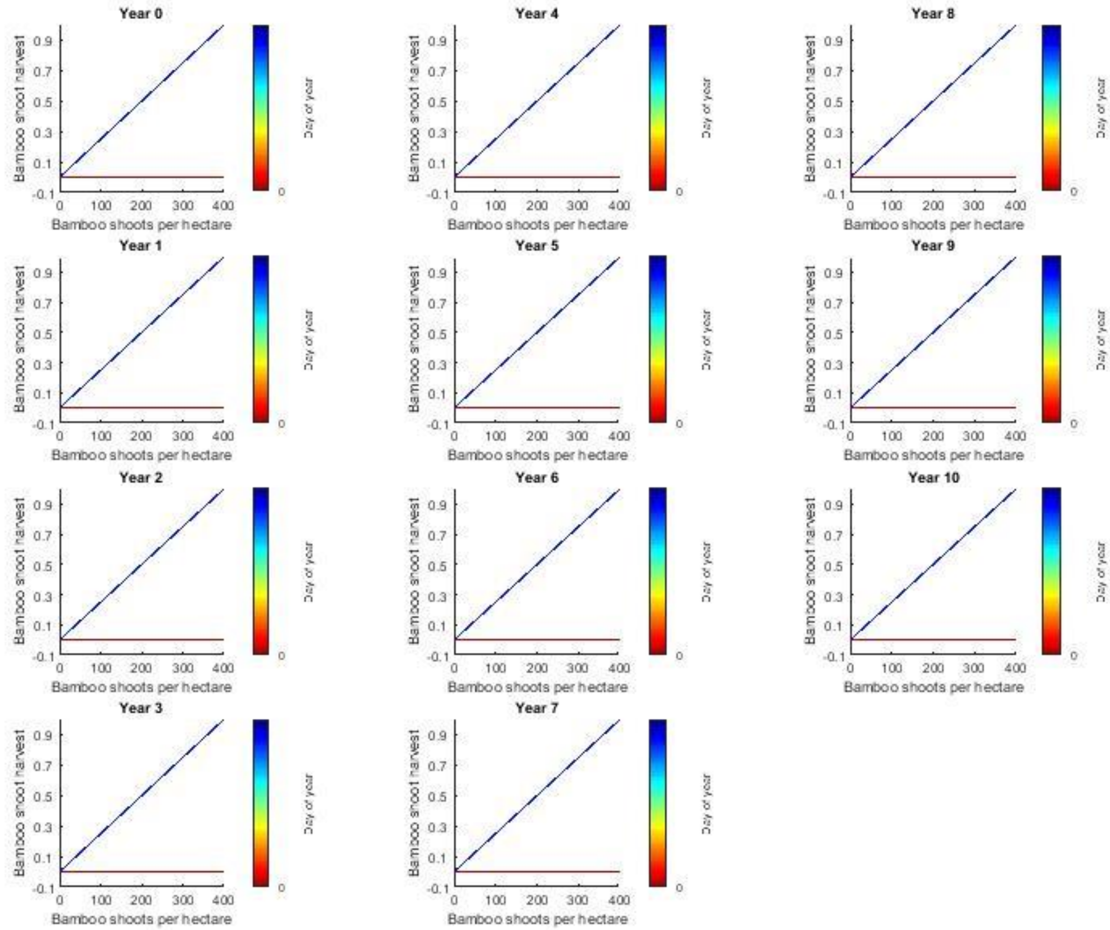
e) Bamboo stem harvest policy function as function of bamboo stem per hectare



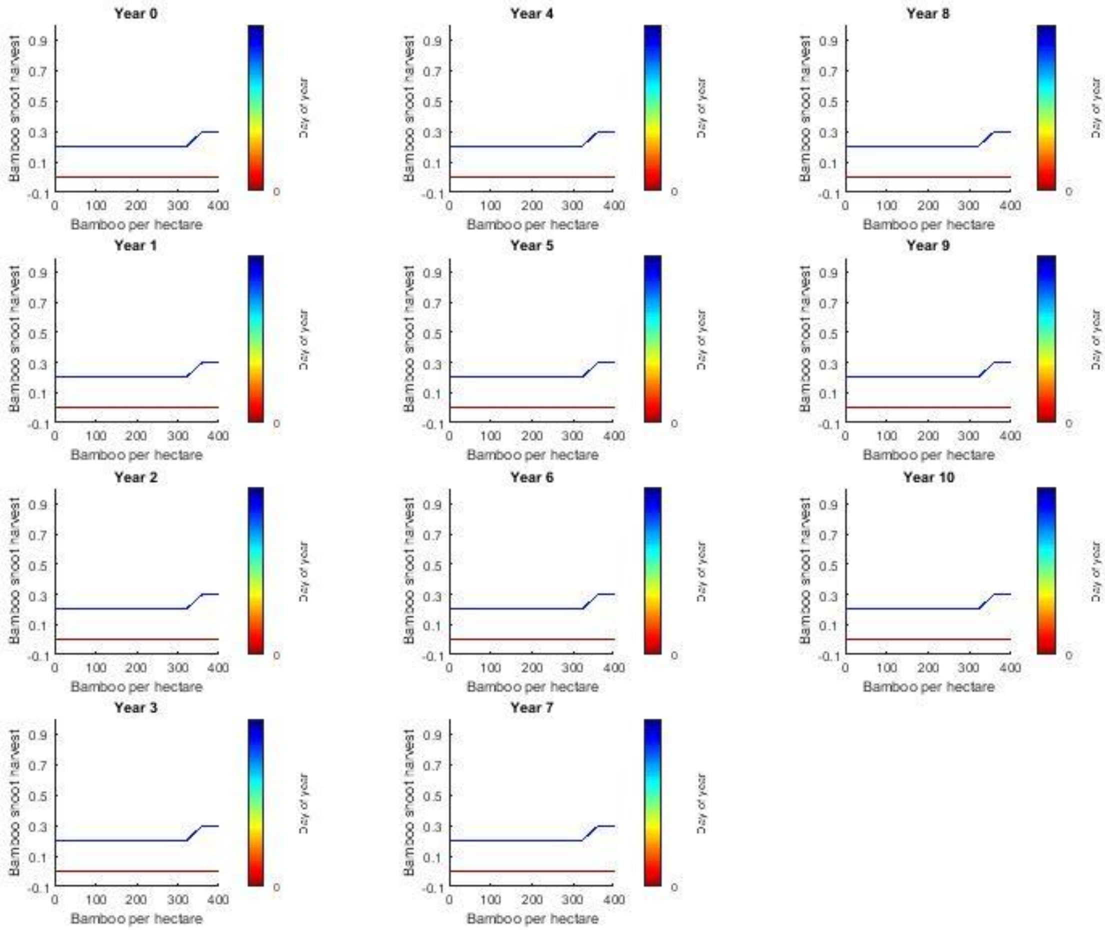
f) Bamboo stem harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare



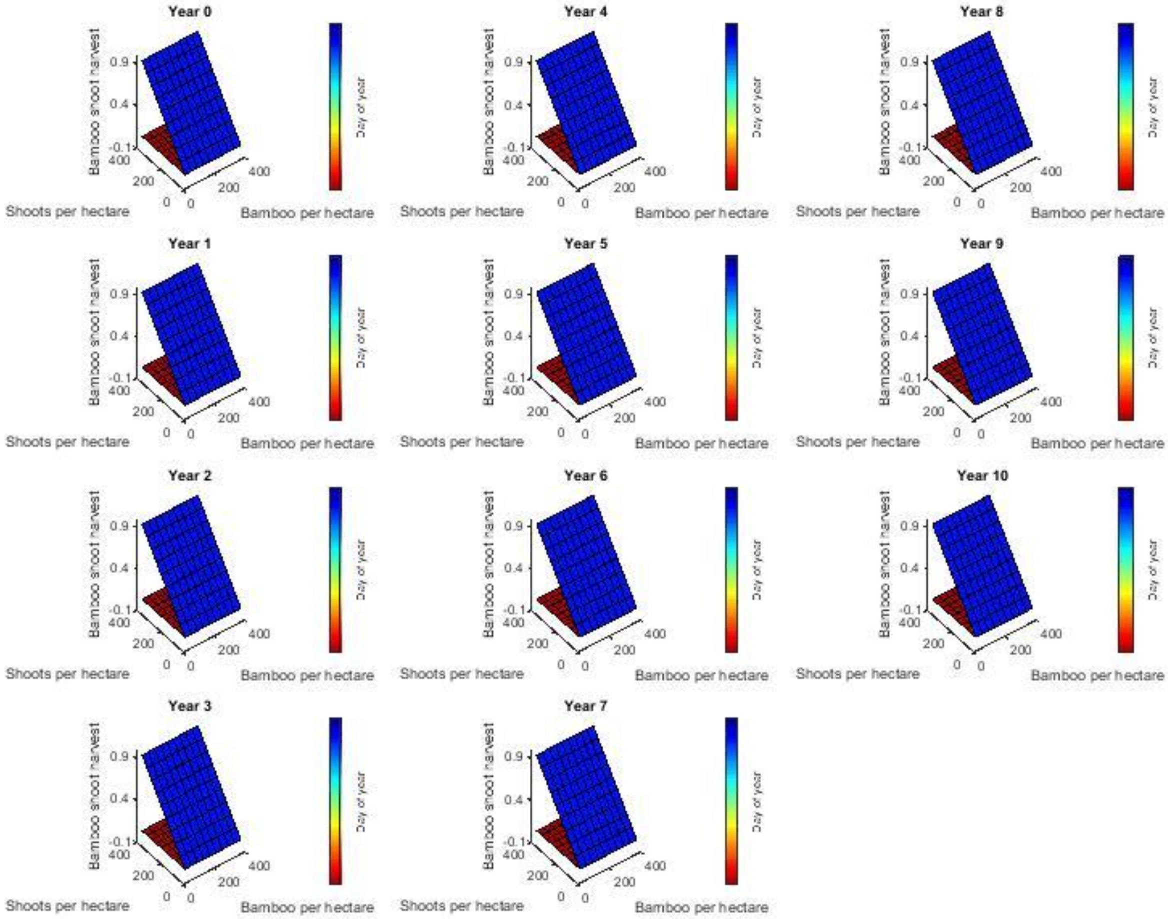
g) Bamboo shoot harvest policy function as function of bamboo shoots per hectare



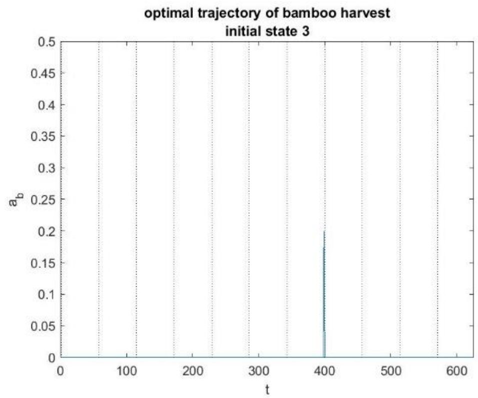
h) Bamboo shoot harvest policy function as function of bamboo stem per hectare

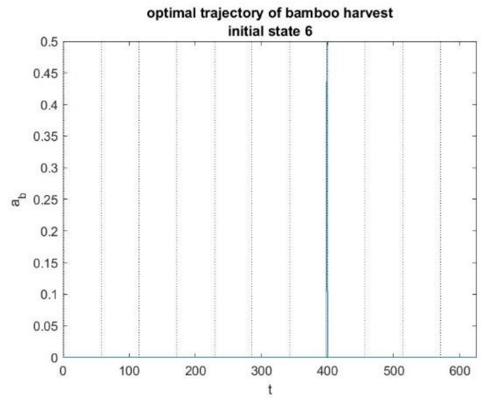


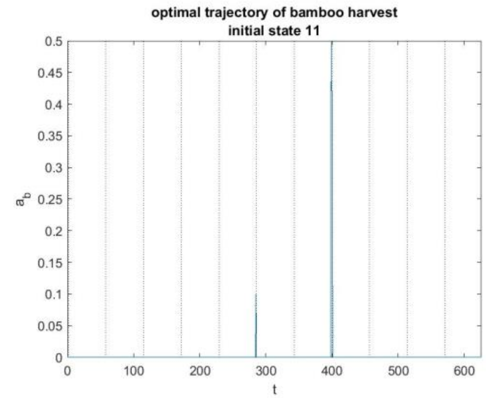
i) Bamboo shoot harvest policy function as function of bamboo shoots per hectare and bamboo stem per hectare

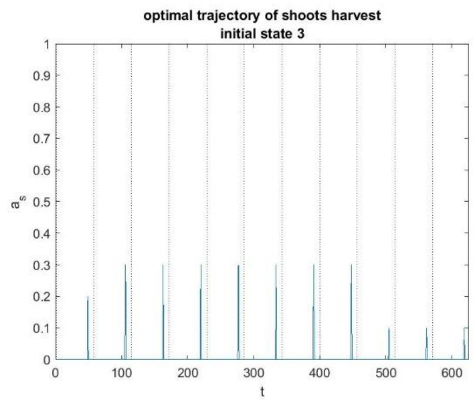


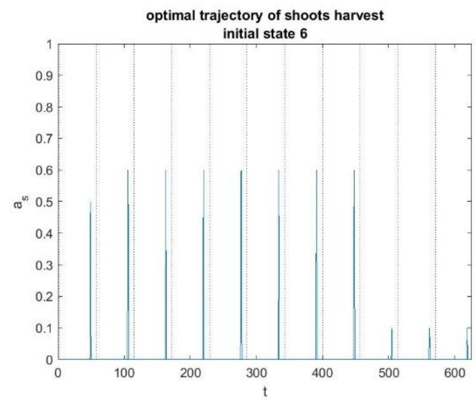
j) Optimal trajectories for each action and state variable over 11 years

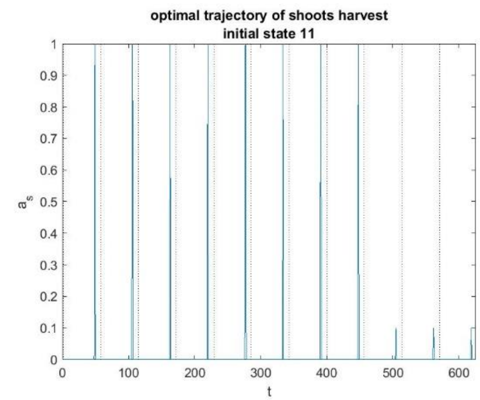


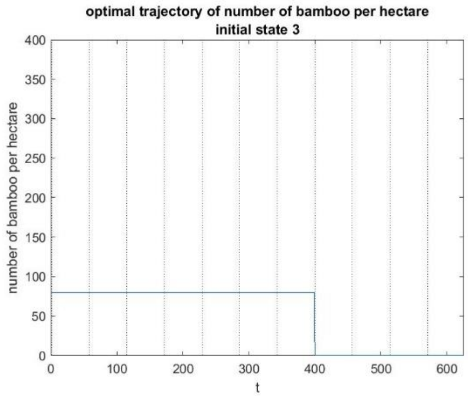


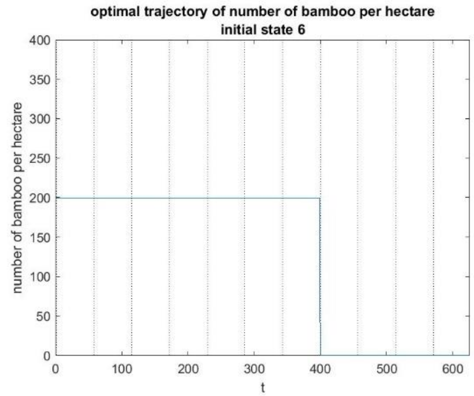


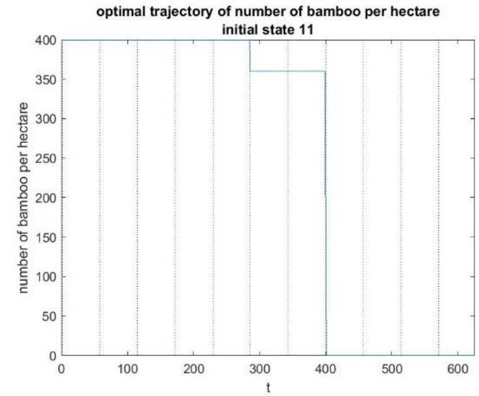


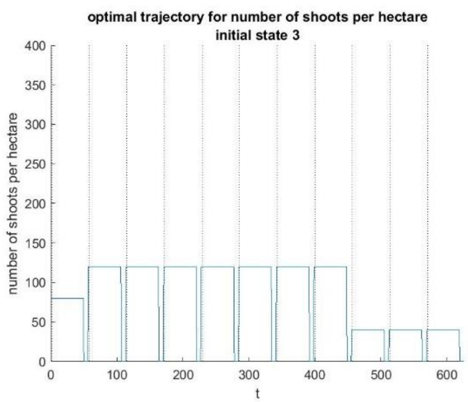


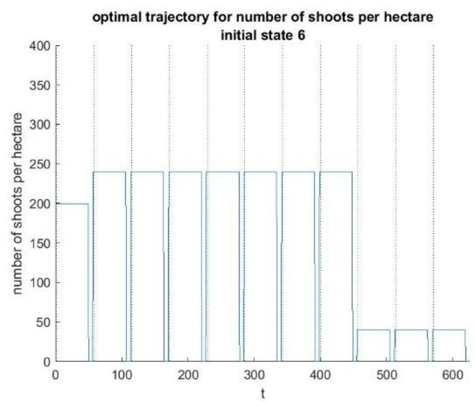



optimal trajectory of number of bamboo per hectare
initial state 3
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 6
number of bamboo per hectare
t


optimal trajectory of number of bamboo per hectare
initial state 11
number of bamboo per hectare
t


optimal trajectory for number of shoots per hectare
initial state 3
number of shoots per hectare
t


optimal trajectory for number of shoots per hectare
initial state 6
number of shoots per hectare
t

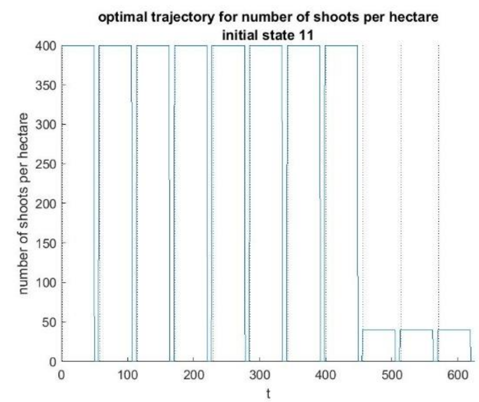
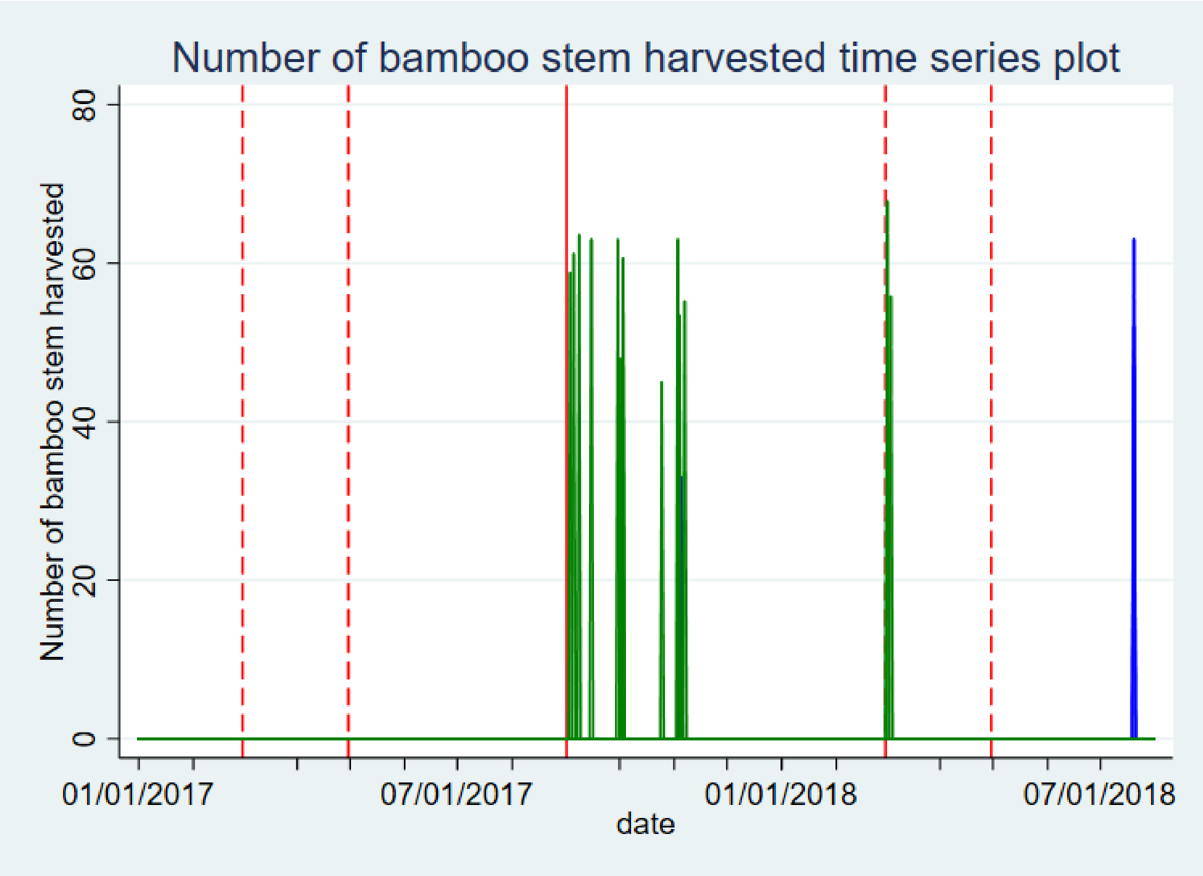
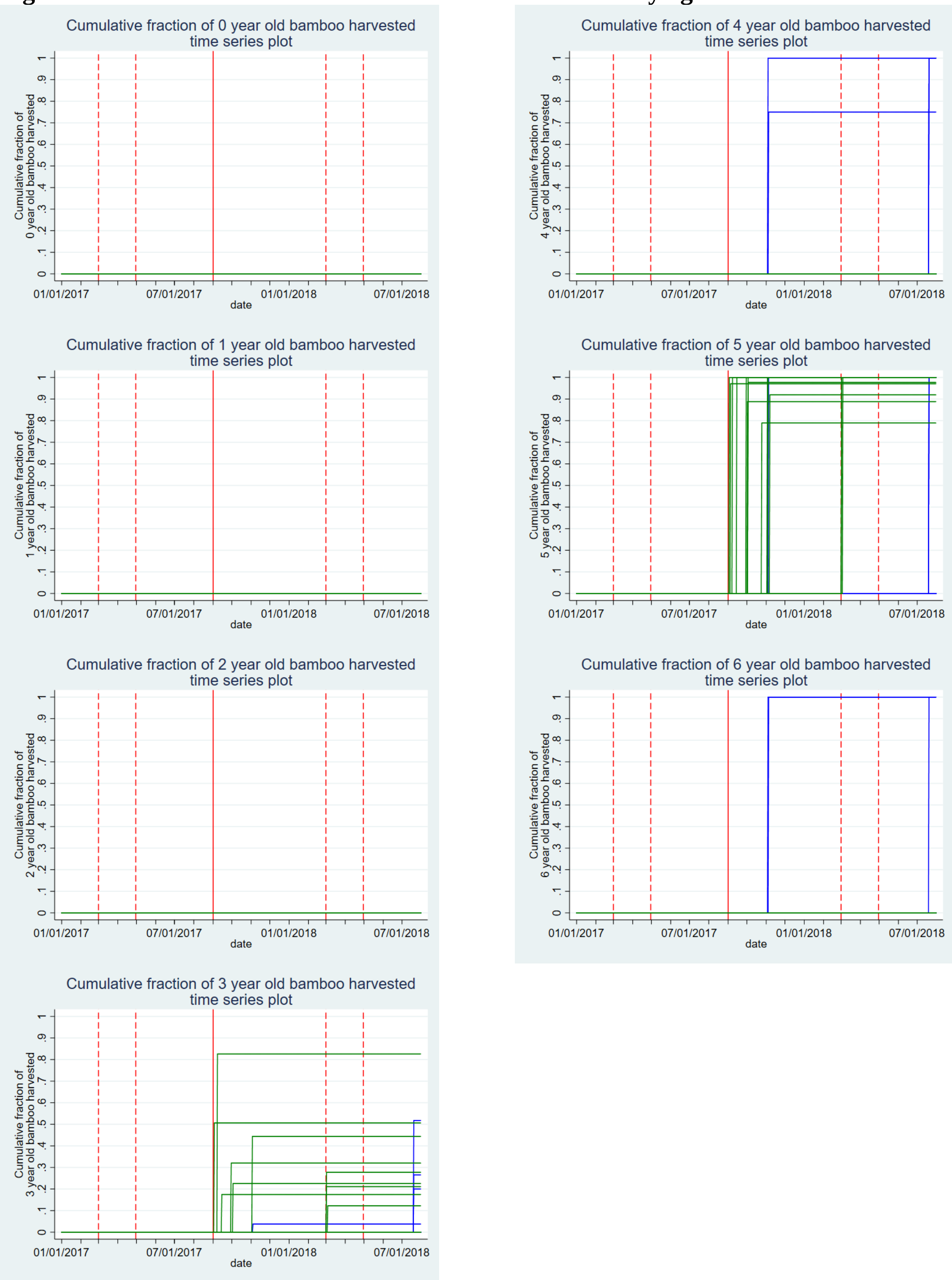

optimal trajectory for number of shoots per hectare
initial state 11
number of shoots per hectare
t

Figure 20: Number of Bamboo Stem Harvested



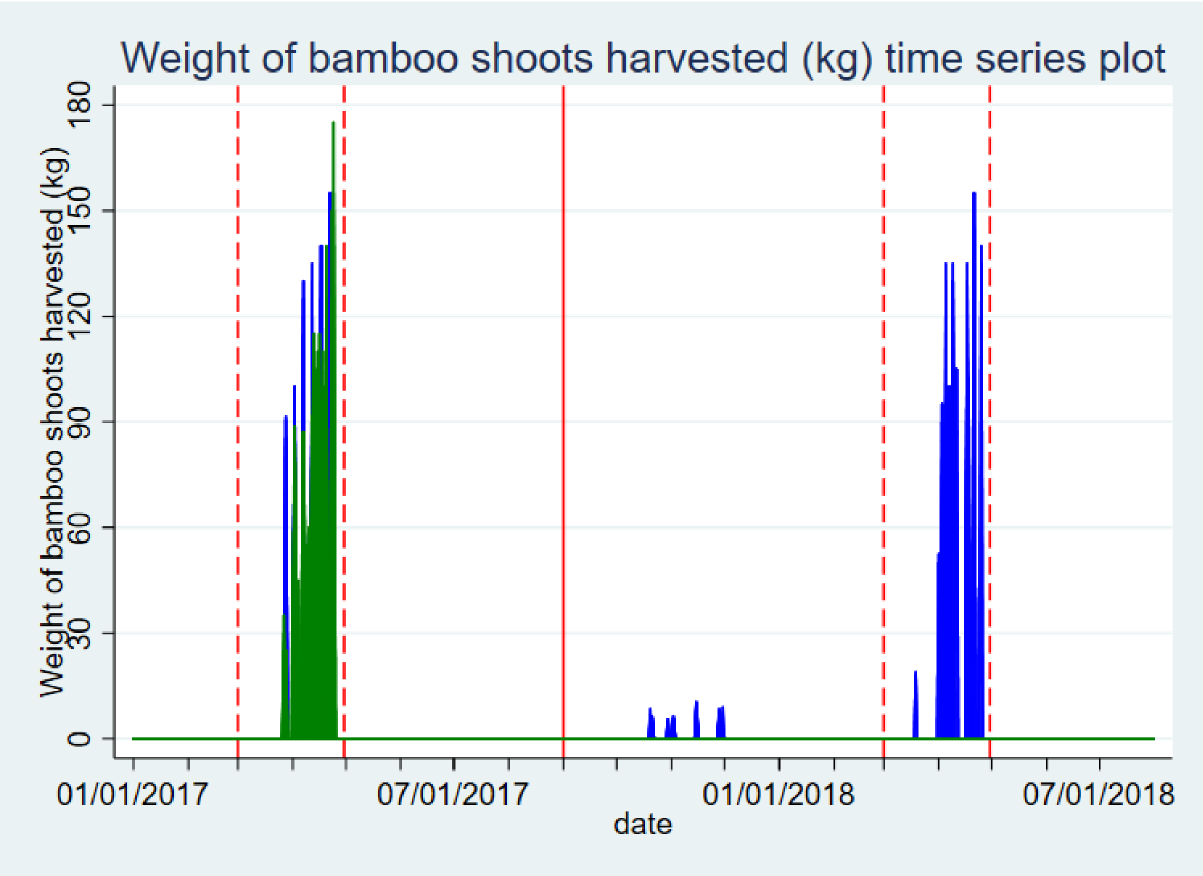
Notes: Time series plots of the number of bamboo stem harvested on each sample plot in Sian Township are in blue. Time series plots of the number of bamboo stem harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) and April 30 (last day of spring shooting) of each year.

Figure 21: Cumulative fraction of bamboo stem harvested by age of bamboo stem



Notes: Time series plots of the cumulative fraction of bamboo stem harvested by age of bamboo on each sample plot in Sian Township are in blue. Time series plots of the cumulative fraction of bamboo stem harvested by age of bamboo on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) and April 30 (last day of spring shooting) of each year.

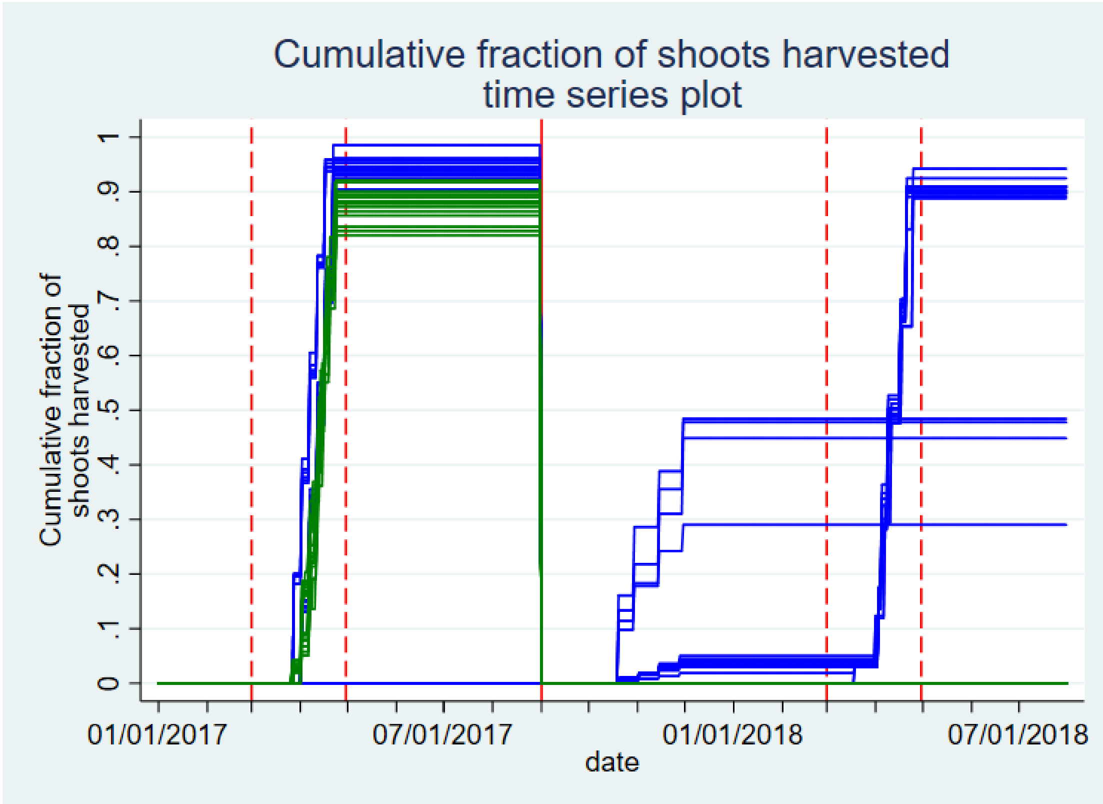
Figure 22: Actual Weight of Bamboo Shoots Harvested



Notes: Time series plots of the weight of bamboo shoots harvested on each sample plot in Sian Township are in blue. Time series plots of the weight of bamboo shoots harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) and April 30 (last day of spring shooting) of each year.

s

Figure 23: Cumulative fraction of bamboo shoots harvested



Notes: Time series plots of the cumulative fraction of bamboo shoots harvested on each sample plot in Sian Township are in blue. Time series plots of the cumulative fraction of bamboo shoots harvested on each sample plot in Shanchuan Township are in green. Vertical lines in red that go from the top to the bottom of the graph denote September 1 (first day of winter shooting) of each year. Dashed vertical lines in red that go from the top to the bottom of the graph denote March 1 (first day of spring shooting) and April 30 (last day of spring shooting) of each year.