ON BANK BAILOUTS

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Chapter 1

I develop a theoretical model for investigating the cost of failure to commit in the provision of bailouts to financial institutions. When a financial institution fails, the fiscal authority often deviates from its *ex-ante* no-bailout commitment: the *ex-post* best response is to bailout. The fiscal authority's time inconsistency creates moral hazard. I calculate the welfare loss from the failure to commit. In the model, as long as the fiscal authority is able to commit to a pre-determined bailout policy, the outcome is typically constrained efficient. Furthermore, a higher probability of bank run is not always welfare reducing. Increased run probability can be beneficial by making financial institutions more cautious, thus decreasing the moral hazard loss. Regulations on short-term interest rates offered by financial institutions can deter moral hazard, particularly when the run probability is small.

Chapter 2

This chapter analyzes the optimal bailout policy in an interconnected banking system. Banks are allowed to deposit in each other to hedge against idiosyncratic liquidity shocks. When a fraction of banks in the economy are hit by a liquidity shock and become insolvent, there are potential spillovers to solvent banks. In this case, the optimal bailout policy is not always either a full bailout or zero bailout. It is sometimes optimal for the fiscal authority to provide partial

bailouts that are just sufficient to prevent spillovers. The decision of the fiscal authority depends on how much pressure from taxpayers and banks. If the urgency to save the banking sector outweighs the utility from public goods, a full bailout is optimal, and vice versa. When the two effects are comparable, the optimal decision of the fiscal authority is partial bailout.

BIOGRAPHICAL SKETCH

I am a sixth year graduate student in the Department of Economics at Cornell University. Prior to the Ph.D program, I received a Bachelor of Science in Economics, Actuarial Mathematics, and Statistics from the University of Michigan – Ann Arbor. My research focus is on Macroeconomics, Banking, and Monetary Economics. My current research is on the inability of the government to commit to a bailout policy, and the social cost arising thereof. I am also working on understanding the motivation of the government in providing partial bailouts in an interconnected banking system.

This document is dedicated to all Cornell graduate students.

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CHAPTER 1

BANK BAILOUTS: THE COST OF INABILITY TO COMMIT

1.1 Introduction

There is an important debate on whether the fiscal authority should bail out insolvent financial institutions. On one hand, bailouts create moral hazard because they incentivizes financial institutions to hold riskier portfolios or to offer higher short-term payoffs to investors. On the other hand, bailouts could potentially save financial institutions from a crisis caused by forces outside the banking sector.

Over the past decades, various regulations have been employed with the intention of reducing moral hazard and failures of financial institutions. The Federal Deposit Insurance Corporation (FDIC), the Glass-Steagall Act, and Regulation Q were created in accordance with the 1933 Banking Act to battle bank failures. The DoddFrank Wall Street Reform and Consumer Protection Act was signed into the U.S. federal law in 2010, which introduced extensive oversight on financial institutions.

Recent bank failures, however, have shown that financial institutions are fragile despite substantial efforts and regulations by policymakers. In particular, the S&L crisis of the 1980s and the failure of Washington Mutual in 2008 proved that depository institutions are vulnerable to runs even with deposit insurance protection from the FDIC.

Even when restricting bailouts is optimal, it is often difficult for the fiscal authority to commit to such policy. The bailout episode during the financial crisis of 2008, in which the U.S. Treasury allocated over $700 billion to purchase distressed assets from banks, is a leading example of the commitment failure by the fiscal authority. To restore the confidence in the credit market, the bailout package was approved over substantial opposition. Financial institutions are aware of the commitment issue and often exploit it for their own benefits.

Since financial institutions are susceptible to failures, and bailouts are inevitable, it remains crucial to understand the extent of moral hazard in financial institutions arising from bailouts. Existing literature has focused heavily on the comparison between the flexible bailout regime and the zero-bailout regime.[1] Even when bailouts create moral hazard, the zero-bailout policy is not always superior (Keister (2016)). The flexible bailout regime is preferred when effective financial regulatory tools are available (Keister and Narasiman (2016)).

This paper takes a different direction and focuses on the isolation of the social cost of bailouts that arises solely from the commitment failure of the fiscal authority. It differs from the existing literature in that the comparison is made between the flexible bailout regime and the pre-committed bailout regime.[2] The main result of this paper is that moral hazard arises predominantly from the failure of the fiscal authority to commit to a pre-determined bailout policy.

Diamond and Dybvig (1983) introduced an environment that allows for panic-based runs on a bank. A panic-based run is generated solely by the belief of the depositors that all the other depositors are running, which is unrelated to the fundamentals of the economy. However, as long as the depositors believe that all the other depositors are not running, the panic-based run will be

[1]The flexible bailout regime is when the fiscal authority decides on the bailout policy after financial institutions have failed.

[2]The pre-committed bailout regime is when the fiscal authority commits to the ex-ante optimal bailout policy before financial institutions fail.

avoided. By merely changing the belief of the depositors, a catastrophic equilibrium outcome can be averted.

Peck and Shell (2003) refined the equilibria in Diamond and Dybvig (1983) by introducing the pre-deposit game. Peck and Shell (2003) runs are panic-based, driven by sunspots. See also Shell and Zhang (2017).[3] Additionally, by having a finite number of depositors, there is aggregate uncertainty on the demand for liquidity in the economy.[4] This allows for a clear distinction in the causes of a bank run. A fundamental-driven run is caused by a high demand for liquidity; a panic-based run is caused by the belief of the depositors that the other depositors are running the bank. In my terminology in this paper (following Keister), a run can be either panic-based *or* fundamental-driven.

Keister (2016) introduced the fiscal authority into the Diamond and Dybvig (1983) framework. The fiscal authority collects taxes to fund the public good and bailouts. When there is a bank run, the fiscal authority is able to bail out banks but doing so is costly to the public as bailouts reduce the resources available for public good provision.

In this paper, I adapt the environment of Shell and Zhang (2017) with the fiscal authority from Keister (2016). By having a finite number of depositors in each bank, the magnitude of the demand for liquidity is stochastic, adding an additional layer of uncertainty for the fiscal authority when providing bailouts; the fiscal authority does not have the information on whether a run is fundamental-driven or panic-based.

[3]The sunspot equilibrium concept was introduced by Shell (1977). See also Cass and Shell (1983).

[4]There is no aggregate uncertainty in Diamond and Dybvig (1983) and Keister (2016) because there is a measure of depositors.

The source of a bank run is crucial. A high demand for liquidity corresponds to urgency in consumptions. This could due to an emergency or a strong desire for immediate consumption, and is referred to as the "impulse demand" in Shell and Zhang (2017). Therefore, the fiscal authority should respond with a higher level of bailout if the bank run is fundamental-driven. On the other hand, if the bank run is panic-based, the fiscal authority should hold back and cut down on bailouts since the urgency of the early withdrawals is lower.

Two economies are analyzed in this paper: (1) the economy under the fiscal authority with commitment (the pre-committed bailout regime) and (2) the economy under the fiscal authority without commitment (the flexible bailout regime). The fiscal authority with commitment is assumed to have the ability to pre-commit to a bailout policy prior to the occurence of a bank run. The fiscal authority without commitment decides on the bailout policy after a bank run is observed. The difference in the welfares achieved in the two economies is the welfare loss from the inability of the fiscal authority to commit. The amount of endowment required to compensate the depositors in each of the two economies to bring them to the constrained efficient welfare level is also computed. This amount (in units of endowment) indicates the "real" loss from moral hazard.

The main result of this paper is that when the fiscal authority is able to commit to a bailout policy, the economy is typically constrained efficient. However, if the fiscal authority is unable to commit, the economy is never constrained efficient. The second result is that a higher probability of bank run does not always hurt welfare. It can strategically induce banks to be more conservative and hence, reduce moral hazard and increase welfare. The third result is that allowing the fiscal authority to regulate short-term interest rates offered by banks can increase the efficiency of the economy.[5] When the probability of a bank run is low, the ability of the fiscal authority to regulate short-term interest rates makes the economy constrained efficient even when the fiscal authority is unable to commit.

The paper is organized in the following way: Section 2 introduces the framework. Section 3 analyzes the constrained efficient allocation. Section 4 analyzes the economy under the fiscal authority with commitment. Section 5 analyzes the economy under the fiscal authority without commitment. Section 6 investigates a potential policy tool to reduce the welfare loss from the inability of the fiscal authority to commit. Section 7 concludes the paper. The proofs of all Lemmas and Propositions are in the Appendix.

1.2 The Banking Environment

There is a fiscal authority, a measure of banks, and two groups of depositors, D_A and D_B. Each bank serves exactly two depositors, one from each group. There are three periods $t = 0, 1, 2$. In period 0, each depositor is endowed with y units of consumption good that is subject to a fractional tax τ. There is an infinitely divisible, constant returns to scale technology that returns either 1 unit of consumption good in period 1 or $R > 1$ units in period 2 for each unit of consumption good invested in period 0. There is also a costless storage technology between periods. Both the banks and the depositors have access to both technologies.

The depositors are identical in period 0. In period 1, each group of depositors

[5]Regulation Q is an example of regulations on short-term interest rates.

turns impatient with probability p and patient with probability $1-p$ independently.[6] The sunspot states α and β are realized in period 1 with probabilities $1-s$ and s respectively.[7]

The depositors observe their own types (patient or impatient) and the sunspot state. Types are private information. Each depositor maximizes her own utility by choosing to withdraw in period 1 (early) or period 2 (late) contingent on her type and sunspots. The early withdrawals are bounded by the sequential service constraint. If both depositors in a bank decide to withdraw in early, each depositor has 1/2 probability to be the first in the line. The depositors make their withdrawal decisions before they observe their positions in the line.

Define c_1 and c_2 as the withdrawals in period 1 and 2 respectively and x as the private consumption of a depositor. Let $u(x)$ be the utility for an impatient depositor and $v(x)$ be the utility for a patient depositor where

$$u(x) = A\frac{x^{1-\gamma}}{1-\gamma},$$

$$v(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

$\gamma > 1$ is the measure of relative risk aversion and $A \geq 1$ is the impulse demand of the impatient depositors.[8] An impatient depositor only derives utility from period-1 consumption ($x = c_1$). A patient depositor only derives utility from period-2 consumption but can costlessly store period-1 withdrawal c_1 for period-2 consumption ($x = c_1 + c_2$).

[6]This implies that liquidity shocks are perfectly correlated among the banks. This deters the banks from pooling their resources to hedge against the shocks.

[7]Sunspots are used for equilibrium coordination, which will be discussed in later sections.

[8]It is natural to assume that $A \geq 1$ since an impatient depositor typically has a more urgent need for consumption compared to a patient depositor.

Each bank writes a deposit contract that maximizes the ex-ante expected utilities of its own depositors from private consumptions (withdrawals). Since there are only two depositors in each bank and the early withdrawals are bounded by the sequential service constraint,[9] the deposit contract can be summarized as a scalar $c \in [0, 2(1-\tau)y)$, where c is the payment to the first in line in period 1. If both depositors withdraw in period 1, the first in line receives c and the second in line receives the leftover consumption good in the bank $(2(1-\tau)y - c)$. If only one depositor decides to withdraw in period 1, she receives c. The leftover consumption good remains in the technology so the other depositor receives $(2(1-\tau)y - c)R$ in period 2. If both depositors withdraw in period 2, they split the resources in the bank evenly. Each depositor receives $(1-\tau)yR$.

The fiscal authority chooses the bailout policy that maximizes welfare, which is the sum of the expected utilities of all the depositors from both private consumption and public good consumption.[10] The utility of each depositor from public good provision is given by

$$\Gamma(g) = G\frac{g^{1-\gamma}}{1-\gamma},$$

where g is the level of public good funded by the fiscal authority, γ is the measure of relative risk aversion and $G > 0$ is the measure of the relative value of public good.[11]

The bailout policy chosen by the fiscal authority is the scalar $B \in [0, 2\tau y)$, where B is the amount of bailout paid to each bank that experiences a bank run (both depositors withdrawing early). A bank run is fundamental-driven

[9] See Wallace (1988)

[10] The inclusion of the fiscal authority in the bank run model was first done by Keister (2016).

[11] Larger G implies a higher cost of bailouts.

if all the depositors who withdraw early are impatient. A bank run is panic-based if at least one of the depositors who withdraw early is patient.[12] The bailout payments are used by the banks to fund their second-in-line depositors' withdrawals.[13]

If one or no depositor withdraws early, all tax revenue goes to public good provision, $g = 2\tau y$. If there is a bank run, bailout payments are funded from tax revenues. In general, the level of the public good provision is $g = 2\tau y - B$.[14] The reduction of public good provision is the cost of bailouts in this model.

1.2.1 Timeline

From Peck and Shell (2003), events taking place in period 0 are referred to as the pre-deposit game. Events that take place in periods 1 and 2 are referred to as the post-deposit game. In the pre-deposit game, each depositor receives an endowment of y units of consumption good and pays a tax τy to the fiscal authority. Then, each bank writes a deposit contract c. Each depositor observes the deposit contract and decides whether to deposit her disposable income $(1 - \tau)y$ into the bank. This marks the end of the pre-deposit game. In the post-deposit game, each depositor observes her own type and sunspots before deciding to withdraw early or late.

If the fiscal authority is able to commit, it chooses a bailout policy B right

[12]The theoretical distinction between panic-based and fundamental-driven runs was introduced by Keister and Narasiman (2016).

[13]By the time the fiscal authority realizes that there is a bank run, the first-in-line depositor has already left the bank. Therefore, the bailout payment made to a bank can only be added to the withdrawal of the second-in-line depositor.

[14]Since the types within each group of depositors are perfectly correlated, the ex-post outcome in all the banks are identical. This is a slight abuse of notation but eventually either all banks receive a bailout B, and $g = 2\tau y - B$, or all banks receive no bailout, and $g = 2\tau y$.

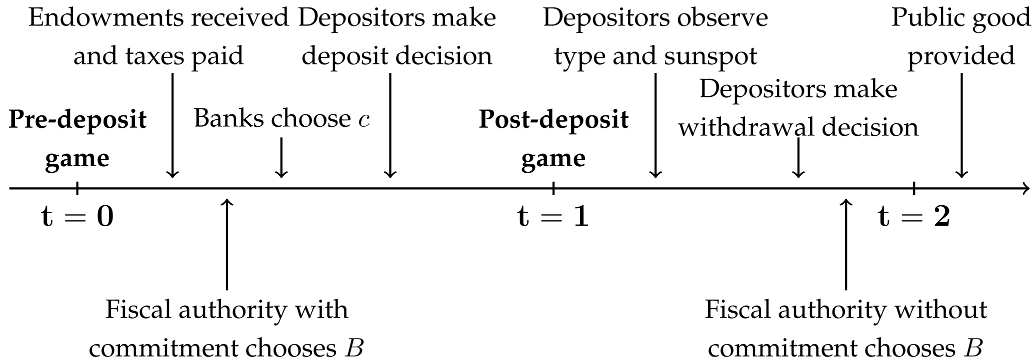

Figure 1.1: Timeline

after taxes are collected and before the banks write a deposit contract. Since the banks are identical ex-ante, the same bailout policy applies to every bank in the economy. The bailout policy is observed immediately by the banks and the depositors.

If the fiscal authority is unable to commit, it chooses a bailout policy B only when after it observes that both depositors in a bank withdraw early. Since banks are also identical ex-post (due to perfectly correlated types within the groups of depositors D_A and D_B), the optimal bailout responses of the fiscal authority for all the banks in the economy are identical.

Figure 1.1 summarizes the series of events in the pre-deposit game and the post-deposit game under the fiscal authority with commitment and the fiscal authority without commitment.

1.3 The Constrained Efficient Allocation

This section solves for the constrained efficient allocation that is used as a benchmark for later comparisons. I suppose that there is a welfare maximizing planner that makes decisions for both the fiscal authority and the banks. The planner, however, is unable to observe the types or dictate the decisions of the depositors.[15] The deposit contract c and the bailout policy B chosen by the planner determines the constrained efficient allocation. The welfare achieved by the constrained efficient allocation gives a good benchmark for identifying the degree of welfare loss the economy incurs by moving from a centralized institutional setting to a decentralized one.

1.3.1 The Post-deposit Game

Taking the deposit contract c and the bailout policy B from the pre-deposit game as given, the depositors have to simultaneously decide when to withdraw after observing their own types and sunspots. This is a two-player static game with incomplete information. The expected utility of a depositor from withdrawing early or late depends on the decision and the type of the other depositor. A depositor always withdraws early if she is impatient, so the analysis focuses on the patient depositors.

Definition 1. *The panic equilibrium in the post-deposit game is an equilibrium in which both depositors withdraw early regardless of their types.*

[15]The unconstrained first-best, which will be introduced later, is when the planner can observe the types of the depositors and dictate their decisions. This important distinction was introduced by Ennis and Keister (2016).

Definition 2. *The non-panic equilibrium in the post-deposit game is an equilibrium in which both depositors withdraw early (late) if and only if they are impatient (patient).*

Since c is the amount promised to the first early withdrawal, a higher c increases the expected utility of a patient depositor from withdrawing early. A higher c also decreases the expected amount of resources available at period 2 and thus lowers the expected utility of a patient depositor from withdrawing late.

Lemma 1. *If $\gamma < 1 + \ln 2/\ln R$, there exists a threshold $\bar{c}^{early}(B)$ such that a panic equilibrium in the post-deposit game exists if and only if $c > \bar{c}^{early}(B)$.*

Lemma 1 is an extension of Shell and Zhang (2018). It shows that for any given B, if c is sufficiently small (i.e. $c \leq \bar{c}^{early}(B)$), then a patient depositor withdraws late regardless of her belief about the action of the other depositor. Then, there is no panic equilibrium.

Lemma 2. *If $\gamma < 1+\ln 2/\ln R$, there exists a threshold $\bar{c}^{wait}(B)$ such that a non-panic equilibrium in the post-deposit game exists if and only if $c \leq \bar{c}^{wait}(B)$.*

Lemma 2 implies that for any given B, if c is sufficiently high (i.e. $c > \bar{c}^{wait}(B)$) a patient depositor withdraws early regardless of her belief about the action of the other depositor. Therefore, there is no non-panic equilibrium.

Lemma 3. *$\bar{c}^{early}(B) < \bar{c}^{wait}(B)$ for $B \in [0, 2\tau y]$ if*

$$\gamma < \min\{2, 1 + \ln 2/\ln R\}. \tag{1.1}$$

If $\bar{c}^{early}(B) < c \leq \bar{c}^{wait}(B)$, both the panic and non-panic equilibria exist in the post-deposit game. The deposit contract c is large enough so that a patient

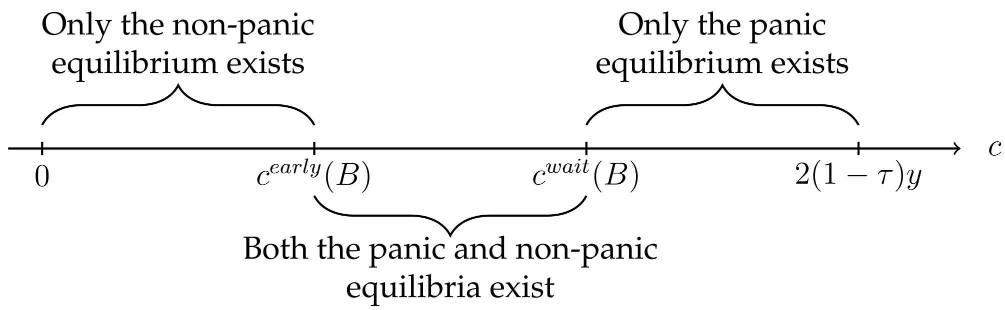

Figure 1.2: Equilibria of the Post-deposit Game

depositor withdraws early if she believes that the other depositor – if patient – also withdraws early. However, c is low enough such that a patient depositor withdraws late if she believes that the other depositor - if patient - also withdraws late. In this case, the actions of the depositors exhibit strategic complementarity. The depositors coordinate on the equilibria via sunspots. Without loss of generality, the depositors play the non-panic (panic) equilibrium in state α (β),

For the remainder of the paper, the analyses only focus on the range of γ that satisfies 1.1. This excludes the possibility of strategic substitutability in the actions of the two depositors.[16] Figure 1.2 summarizes the equilibria of the post-deposit game for $c \in [0, 2(1-\tau)y]$, given B.[17]

The thresholds $c^{early}(B)$ and $c^{wait}(B)$ are strictly decreasing functions of B. Higher B increases the expected utility of withdrawing early since bailout is added to the second early withdrawal. Therefore, higher B has to be compensated with a lower c in order to keep the propensity to run and the actions of

[16] Strategic substitutability is when a patient depositor withdraws early (late) if she believes that the other depositor - if patient - withdraws late (early). See Shell and Zhang (2018)

[17] The detailed derivations of the post-deposit game can be found in Section A.1 of the Appendix. Figure 1.2 is adapted from Shell and Zhang (2018).

the patient depositors the same.

The constraint $c \leq \bar{c}^{wait}(B)$ is the Incentive Compatibility Constraint (ICC). It has to be satisfied in order for the depositors to be willing to deposit during the pre-deposit game. If the ICC is violated, the ex-ante expected utility of a depositor from depositing is lower than autarky. The depositors would not have deposited their endowments into the banks and there would not be a post-deposit game.

Define the following sets:

$$S^{early} = \{(c, B) \in \mathbb{R}^2_+ | c \leq \bar{c}^{early}(B), B \leq 2\tau y\}$$
$$S^{wait} = \{(c, B) \in \mathbb{R}^2_+ | c \leq \bar{c}^{wait}(B), B \leq 2\tau y\}.$$

According to Lemma 3, S^{early} is a strict subset of S^{wait} as long as (1.1) holds.

Definition 3. *An allocation (c, B) is Dominant Strategy Incentive Compatible (DSIC) if and only if $(c, B) \in S^{early}$.*

The term "dominant strategy" comes from the fact that withdrawing late is the dominant strategy for patient depositors when $(c, B) \in S^{early}$. The expected utility from depositing is higher than autarky regardless of sunspots.

Definition 4. *An allocation (c, B) is Bayesian Incentive Compatible (BIC) if and only if $(c, B) \in S^{wait}$.*

If an allocation (c, B) is BIC, the ex-post expected utility from depositing might be higher or lower than autarky, depending on sunspots. However, depositors are still better off depositing ex-ante. Notice that DSIC is a strictly stronger condition than BIC as long as (1.1) holds.[18]

[18] The terms DSIC and BIC were introduced into the bank run literature by Shell and Zhang

1.3.2 The Pre-deposit Game

In the pre-deposit game, the planner decides (c, B) that maximizes the welfare of the economy. The welfare functions are different in the non-panic equilibrium and the panic equilibrium. Let $\widehat{W}(c, B)$ be the ex-ante expected welfare in the non-panic equilibrium, and is given by

$$\begin{aligned}\widehat{W}(c, B) = p^2 \big[u(c) + u(2(1-\tau)y - c + B) + 2\Gamma(2\tau y - B)\big] \\ + 2p(1-p)\big[u(c) + v((2(1-\tau)y - c)R) + 2\Gamma(2\tau y)\big] \\ + (1-p)^2\big[2v((1-\tau)yR) + 2\Gamma(2\tau y)\big]. \end{aligned} \tag{1.2}$$

In the non-panic equilibrium, only impatient depositors withdraw early. With probability p^2, both depositors are impatient and withdraw early in every bank. Their utilities are given by u. The first depositor receives c and the second receives the leftover resources plus the bailout $(2(1-\tau)y - c + B)$. The utility of each depositor from public good provision is given by Γ. Since every bank receives the same bailout B, the aggregate bailout in the economy is B. The level of the public good provision is $g = 2\tau y - B$.

With probability $2p(1-p)$, exactly one depositor is impatient and withdraws early (in each bank). The impatient depositor with utility function u receives c in period 1 and the patient depositor with utility function v receives the returns from the leftover consumption good that remains in the technology $(2(1-\tau)y - c)R$ in period 2. Since there is exactly one withdrawal in period 1 in every bank, there is no bailout. All tax revenue goes towards the public good provision, $g = 2\tau y$.

With probability $(1-p)^2$, both depositors are patient and withdraw late in

(2018).

every bank. The two depositors split the returns from the investment in period 2. Each depositor receives $(1-\tau)yR$. Let $W^{run}(c, B)$ be ex-ante expected welfare in the panic equilibrium, and is given by

$$
\begin{aligned}
W^{run}(c, B) = p^2 \big[u(c) + u(2(1-\tau)y - c + B)\big] \\
&+ p(1-p)\big[u(c) + v(2(1-\tau)y - c + B) + v(c) \\
&\qquad + u(2(1-\tau)y - c + B)\big] \\
&+ (1-p)^2\big[v(c) + v(2(1-\tau)y - c + B)\big] + 2\Gamma(2\tau y - B).
\end{aligned}
$$

In the panic equilibrium, all depositors withdraw early regardless of their types. The first depositor receives c and the second depositor $2(1-\tau)y - c + B$. With probability p^2, both depositors are impatient and their utilities are given by u.

With probability $2p(1-p)$, exactly one depositor is impatient in each bank. If the impatient depositor withdraws first, the sum of the utilities of the two depositors is $u(c) + v(2(1-\tau)y - c + B)$. If the impatient depositor withdraws second, the sum is $v(c) + u(2(1-\tau)y - c + B)$. Each of these events happens with probability 1/2.

With probability $(1-p)^2$, both depositors are patient and their utilities are given by v. Since there are always two withdrawals in period 1, bailout payment B is made to all banks with certainty. The level of public good provision is always $2\tau y - B$ in the panic equilibrium.

If the planner chooses a DSIC allocation such that $(c, B) \in S^{early}$, the ex-ante expected welfare is $\widehat{W}(c, B)$ since only the non-panic equilibrium exists in the post-deposit game. If the allocation is not BIC such that $(c, B) \notin S^{wait}$, the ex-ante expected welfare is $W^{run}(c, B)$ since only the panic equilibrium exists in the post-deposit game. If the allocation is BIC but not DSIC such that $(c, B) \in S^{wait} \backslash S^{early}$, sunspots matter. With probability $1-s$ (s), the sunspot state α (β) realizes and the depositors play the non-panic (panic) equilibrium in the post-deposit game, which gives expected welfare $\widehat{W}(c, B)$ ($W^{run}(c, B)$). Let $\widetilde{W}(c, B; s)$ be the ex-ante expected welfare when $(c, B) \in S^{wait} \backslash S^{early}$. Then, we have

$$\widetilde{W}(c, B; s) = (1-s)\widehat{W}(c, B) + sW^{run}(c, B). \tag{1.3}$$

The planner's problem is

$$\max_{(c,B) \in S^{wait}} \quad W(c, B; s) = \begin{cases} \widehat{W}(c, B) & \text{if } (c, B) \in S^{early} \\ \widetilde{W}(c, B; s) & \text{if } (c, B) \notin S^{early} \end{cases}. \tag{1.4}$$

The planner's choices are constrained by $(c, B) \in S^{wait}$ to ensure that the ICC is satisfied so that the depositors accept the deposit contract. Let (c^*, B^*) be the solution to (1.4). By definition, (c^*, B^*) is the constrained efficient allocation.

1.3.3 Characterization of the Constrained Efficient Allocation

This section provides a full characterization of the constrained efficient allocation. Let $(\widehat{c}, \widehat{B})$ be the unconstrained efficient allocation, where

$$(\widehat{c}, \widehat{B}) \equiv \underset{(c,B) \in \mathbb{R}^2_+}{\arg\max} \, \widehat{W}(c, B).$$

The welfare that the unconstrained efficient allocation maximizes is exactly the welfare of the non-panic equilibrium $\widehat{W}(c, B)$. The unconstrained efficient allocation is assumed to be chosen by an unconstrained planner with the ability to observe the types of the depositors and dictate their actions. Thus, the unconstrained efficient economy is equivalent to the constrained efficient economy with truth-telling depositors such that the panic equilibrium never occurs.

Therefore, the unconstrained planner maximizes the welfare in the non-panic equilibrium and is not subject to the ICC constraint or the DSIC constraint.

The unconstained efficient allocation $(\widehat{c}, \widehat{B})$ is implementable in the constrained efficient economy if and only if it is DSIC.[19] When the unconstrained efficient allocation is not implementable, further analyses are required to solve for the constrained efficient allocation. The constrained efficient allocation can be characterized into the following 3 cases based on the implementability of the unconstrained efficient allocation $(\widehat{c}, \widehat{B})$:

1) $(\widehat{c}, \widehat{B})$ is DSIC,

2) $(\widehat{c}, \widehat{B})$ is BIC but not DSIC, or

3) $(\widehat{c}, \widehat{B})$ is neither BIC nor DSIC.

Lemma 4. *There exist $A^{early}, A^{wait} \in \mathbb{R}$, where $A^{wait} > A^{early}$, such that the unconstrained efficient allocation is DSIC when $A \leq A^{early}$ (Case 1), BIC but not DSIC when $A^{early} < A \leq A^{wait}$ (Case 2) and neither BIC nor DSIC when $A > A^{wait}$ (Case 3) .*

Lemma 4 suggests that the value of the impulse demand parameter A dictates which of the three cases the unconstrained efficient allocation falls in. The deposit contract c and bailout policy B set by the planner are positively correlated with the intensity of the impulse demand of the impatient depositors. Holding everything else equal, higher impulse demand incentivizes the planner to divert resources from late withdrawals to early withdrawals, hence higher c. In addition, it also incentivizes the planner to redistribute resources from public consumption to private consumption, hence higher B.

[19]The parameter values are restricted such that the unconstrained efficient allocation always dominates any implementable panic-based run permitting allocations. The detail of the parameter restrictions can be found in Section A.2 of the Appendix.

Higher c and B increase the propensity of the patient depositors to run since more resources are diverted towards the early withdrawals. When the impulse demand is sufficiently low ($A \leq A^{early}$), the propensity to run is sufficiently low such that the unconstrained efficient allocation $(\widehat{c}, \widehat{B})$ is DSIC. As the impulse demand gets higher into the mid-range ($A^{early} < A \leq A^{wait}$), the propensity to run is high enough that the $(\widehat{c}, \widehat{B})$ is no longer DSIC but still low enough such that $(\widehat{c}, \widehat{B})$ is still BIC. When the impulse demand gets sufficiently high ($A > A^{wait}$), the propensity to run is so high that $(\widehat{c}, \widehat{B})$ is neither DSIC nor BIC.

Case 1: The Unconstrained Efficient Allocation is DSIC

If $(\widehat{c}, \widehat{B})$ is DSIC, then $(\widehat{c}, \widehat{B}) \in S^{early}$ is implementable in the constrained efficient economy. The unconstrained efficient allocation is also the solution to (1.4), and is given by

$$(c^*, B^*) = (\widehat{c}, \widehat{B}).$$

The constrained efficient allocation is panic-based run-proof for $s \in [0, 1]$ and independent of s.

Case 2: The Unconstrained Efficient Allocation is BIC but not DSIC

Since $(\widehat{c}, \widehat{B}) \notin S^{early}$, the unconstrained efficient allocation is not implementable in the constrained efficient economy except for the special case of $s = 0$.[20] The extent of the deviation of the constrained efficient allocation (c^*, B^*) from the unconstrained efficient allocation $(\widehat{c}, \widehat{B})$ depends on the panic-based run probability s.

[20]The unconstrained and constrained efficient economies are identical when $s = 0$ because $\widehat{W}(c, B) = \widetilde{W}(c, B; 0)$

Define $(\widetilde{c}, \widetilde{B})$ as the best panic-based run-permitting allocation and (c^{early}, B^{early}) as the best panic-based run-proof allocation when $(\widehat{c}, \widehat{B})$ is non-implementable. They are given by

$$\begin{aligned}(\widetilde{c}, \widetilde{B}) &:= \underset{(c,B)\in\mathbb{R}_+^2}{\arg\max}\, \widetilde{W}(c, B; s) \\ B^{early} &:= \underset{B\in[0,2\tau y]}{\arg\max}\, \widehat{W}(\overline{c}^{early}(B), B) \\ c^{early} &:= \overline{c}^{early}(B^{early}).\end{aligned} \tag{1.5}$$

The constrained efficient allocation is either the best panic-based run-permitting allocation $(\widetilde{c}, \widetilde{B})$ or the best panic-based run-proof allocation (c^{early}, B^{early}), whichever gives the higher welfare.

Proposition 5. *In Case 2, there exists $s_0 \in (0, 1]$ such that for any $s \geq s_0$, the constrained efficient allocation is panic-based run-proof, and for $s < s_0$, the constrained efficient allocation panic-based run-permitting.*

$$(c^*, B^*) = \begin{cases} (\widetilde{c}, \widetilde{B}) & \text{if } s < s_0 \\ (c^{early}, B^{early}) & \text{if } s \geq s_0 \end{cases}$$

The benefit of tolerating panic-based runs is that the planner can offer larger deposit contracts c and larger bailout policies B to efficiently reallocate resources from the withdrawals of patient depositors and the public good provision to the withdrawals of impatient depositors. On the other hand, tolerating panic-based runs increases the expected amount of investment liquidated early and allows for the possibility of the patient depositors consuming the resources allocated for the impatient depositors.

It can be seen from (1.5) that the best panic-based run-proof allocation (c^{early}, B^{early}) and the corresponding welfare $\widehat{W}(c^{early}, B^{early})$ are independent of

s. On the other hand, the best panic-based run-permitting welfare $\widetilde{W}(\widetilde{c}, \widetilde{B}; s)$ is decreasing in s. Higher s exacerbates the inefficiency that arises from tolerating panic-based runs. Proposition 5 suggests that the best panic-based run-proof welfare is higher when $s \geq s_0$, while the panic-based run-permitting welfare is higher otherwise.

When the panic-based run probability is sufficiently small ($s < s_0$), the benefits of tolerating runs outweigh the inefficiencies that arise from it. The constrained efficient allocation is panic-based run-permitting and dependent on s. As s gets large ($s \geq s_0$), the inefficiencies arising from tolerating panic-based runs overshadow the benefits. The constrained efficient allocation jumps from panic-based run-permitting to panic-based run-proof. The constrained efficient allocation (c^*, B^*) is discontinuous at s_0 but the corresponding welfare $\widehat{W}(c^*, B^*)$ is continuous as it is comprised of the envelope of two welfare functions: the best panic-based run-proof welfare $\widehat{W}(c^{early}, B^{early})$ and the best panic-based run-permitting welfare $\widetilde{W}(\widetilde{c}, \widetilde{B}; s)$.

Case 3: The Unconstrained Efficient Allocation is neither BIC nor DSIC

Similar to Case 2, the unconstrained efficient allocation is not implementable in the constrained efficient economy. Moreover, the unconstrained efficient allocation is not BIC. When s is sufficiently close to 0, $(\widetilde{c}, \widetilde{B})$ is also not BIC. That makes $(\widetilde{c}, \widetilde{B})$ non-implementable in the constrained efficient economy since the ICC is violated.

Let (c^{wait}, B^{wait}) be the best run-permitting allocation when $(\widetilde{c}, \widetilde{B})$ is non-implementable, where

$$B^{wait} := \max_{B\in[0,2\tau y]} \widetilde{W}(\bar{c}^{wait}(B), B; s)$$

$$c^{wait} := \bar{c}^{wait}(B^{wait}).$$

Proposition 6. *In Case 3, there exist $s_1 \in (0,1]$ and $s_2 \in (0, s_1]$ such that for $s \geq s_1$, the constrained efficient allocation is panic-based run-proof; for $s_2 \leq s < s_1$, the constrained efficient allocation tolerates panic-based runs and the ICC is non-binding; and for $s < s_2$, the constrained efficient allocation still tolerates panic-based runs but the ICC is binding.*

$$(c^*, B^*) = \begin{cases} (c^{wait}, B^{wait}) & \textit{if } s < s_2 \\ (\widetilde{c}, \widetilde{B}) & \textit{if } s_2 \leq s < s_1 \\ (c^{early}, B^{early}) & \textit{if } s \geq s_1 \end{cases}$$

The equilibrium characterization in Case 3 suggested by Proposition 6 is similar to the equilibrium characterization in Case 2 suggested by Proposition 5 except that the ICC is binding for a positive measure of the panic-based run probability, $s \in [0, s_2)$. The intuition behind the jump of the constrained efficient allocation from panic-based run-permitting to panic-based run-proof at s_1 is similar to Case 2.

In Case 3, the high impulse demand ($A > A^{wait}$) incentivizes the planner to direct more resources toward the impatient depositors by choosing a higher deposit contract and a more generous bailout policy. This also makes early withdrawals more attractive for the patient depositors. For a sufficiently small panic-based run probability ($s < s_2$), the allocation chosen by the planner makes early withdrawals so attractive that the patient depositors choose to withdraw early regardless of their beliefs about the actions of the other depositors. The ICC is violated. Therefore, the best panic-based run-permitting allocation is given by (c^{wait}, B^{wait}) and the ICC is binding. The corresponding welfare $\widetilde{W}(c^{wait}, B^{wait}; s)$ is strictly decreasing in s.

As s increases, the probability of a patient depositor receiving an early withdrawal increases. The incentive for the planner to choose a high deposit contract and a generous bailout policy decreases. As s gets sufficiently large ($s \geq s_2$), the allocation chosen by the planner no longer violates the ICC. The best panic-based run-permitting allocation is $(\widetilde{c}, \widetilde{B})$ and the ICC is not binding. The corresponding welfare $\widetilde{W}(\widetilde{c}, \widetilde{B}; s)$ is still strictly decreasing in s.

Numerical examples and detailed explanations for the three cases are in Section A.3 of the Appendix.

1.4 The Fiscal Authority with Commitment

This section analyzes the economy in which the fiscal authority is able to commit to a bailout policy.

This problem can be solved by backward induction, starting from the post-deposit game, followed by the banks' decisions and the fiscal authority's. Given any allocation (c, B), the outcome in the post-deposit game is identical to the one in the constrained efficient economy.

The banking industry is assumed to be competitive. The free entry condition ensures that each bank in the economy maximizes the ex-ante expected utilities of its depositors from withdrawals. Since all banks are identical, it is sufficient to analyze the decision of the representative bank. The representative bank solves the following problem:

$$\max_{c} \quad W_{pri}(c,B;s) = \begin{cases} \widehat{W}_{pri}(c,B) & \text{if } (c,B) \in S^{early} \\ \widetilde{W}_{pri}(c,B;s) & \text{if } (c,B) \notin S^{early} \end{cases}$$
$$\text{subject to} \quad (c,B) \in S^{wait}. \tag{1.6}$$

The objective function of the bank is $W_{pri}(c,B;s)$, which differs from the ex-ante expected welfare $W(c,B;s)$ in that it excludes the utilities from public good provision. This is the key to the moral hazard that arises because banks do not internalize the cost of bailouts, which cost is the distortion between private consumption and public consumption.

Let $\bar{c}_1^*(B)$ denote the solution to problem (1.6). The fiscal authority, when choosing the bailout policy B, foresees the response of the bank, $\bar{c}_1^*(B)$. The problem of the fiscal authority is given by

$$\max_{B} \quad W(c_1^*(B),B;s) = \begin{cases} \widehat{W}(\bar{c}_1^*(B),B) & \text{if } (\bar{c}_1^*(B),B) \in S^{early} \\ \widetilde{W}(\bar{c}_1^*(B),B;s) & \text{if } (\bar{c}_1^*(B),B) \notin S^{early} \end{cases}$$
$$\text{subject to} \quad (\bar{c}_1^*(B),B) \in S^{wait}, \tag{1.7}$$

where $\widehat{W}$ and $\widetilde{W}$ are as defined in (1.2) and (1.3). Let B_1^* denote the solution to problem (1.7) and $c_1^* \equiv \bar{c}_1^*(B_1^*)$. The equilibrium allocation under the fiscal authority with commitment is (c_1^*, B_1^*).

Proposition 7. *The economy in which the fiscal authority can commit is constrained efficient, except for a measure of run probability $s \in (s_0, s_0 + k_0)$ in Case 2 and $s \in (s_1, s_1 + k_1)$ in Case 3, where $k_0 \geq 0$ and $k_1 \geq 0$.*

As long as the fiscal authority is able make a commitment on bailouts, the economy is typically constrained efficient. The potential moral hazard that arises from the failure of the bank to internalize the cost of bailouts can typically be eliminated.

The intuition is that when the bailout policy is pre-committed by the fiscal authority, the bank takes the bailout policy as given and realizes that the deposit contract it sets does not affect the amount of bailout it receives in the event of a bank run. Another way to see this is that maximizing $\widehat{W}_{pri}$ and $\widehat{W}$ ($\widetilde{W}_{pri}$ and $\widetilde{W}$) are equivalent since the difference between the two functions $\widehat{W} - \widehat{W}_{pri}$ ($\widetilde{W} - \widetilde{W}_{pri}$) only depends on B, which the bank takes as given. Therefore, there is usually no moral hazard under the fiscal authority with commitment.

This economy is not constrained efficient only when s is at the lower end of the range in which the constrained efficient allocation is panic-based run-proof. The constrained efficient allocation $(c^*, B^*) = (c^{early}, B^{early})$ cannot be supported as an equilibrium because the deposit contract set by the bank in response to bailout policy B^{early} committed by the fiscal authority is not c^{early}, or $\bar{c}_1^*(B^{early}) \neq c^{early} = c^*$. Since the bank does not internalize the cost of bailouts, it is willing to tolerate a higher panic-based run probability s than the fiscal authority. The discrepancy in the levels of toleration towards panic-based runs between the banks and the fiscal authority causes the wedge between the welfare achieved in this economy and the constrained efficient welfare.

1.4.1 Numerical Example

This section includes a numerical example to show the allocation and the welfare of the economy under the fiscal authority with commitment. The numerical example uses the following parameter values:

$$R = 1.5 \qquad \gamma = 1.01 \qquad D = 1 \qquad y = 3 \qquad p = 0.5 \qquad \tau = 0.5. \tag{1.8}$$

Figure 1.3 shows the equilibrium allocation and the welfare in the economy under the fiscal authority with commitment and the constrained efficient economy. $A = 1.65$ is such that the constrained efficient allocation falls in Case 2.

The wedge between the blue and red plots in Figure 1.3 shows the range $s \in (s_0, s_0+k_0)$ in which the economy under the fiscal auhority is not constrained efficient. Within this range, the fiscal authority realizes that a panic-based run-proof contract is optimal. However, since the bank does not internalize the cost of bailouts, it still prefers to offer a panic-based run-permitting contract. When this happens, the fiscal authority can either allow the bank to offer a panic-based run-permitting contract or lower the bailout B to incentivize the bank to offer a panic-based run-proof contract. Incentivizing the bank is costly. The fiscal authority has to lower bailout way below the optimal level B^* in order for the bank to prefer a panic-based run-proof contract over a run-permitting one (as seen from the Bailout plot in Figure 1.3).

When s is sufficiently close to s_0, the fiscal authority still allows the banks to offer a panic-based run-permitting contract. The cost of incentivizing the bank to offer a panic-based run-proof contract outweighs the cost of tolerating a panic-based run-permitting contract. However, when s gets higher, the cost of tolerating a panic-based run-permitting contract increases. At the same time,

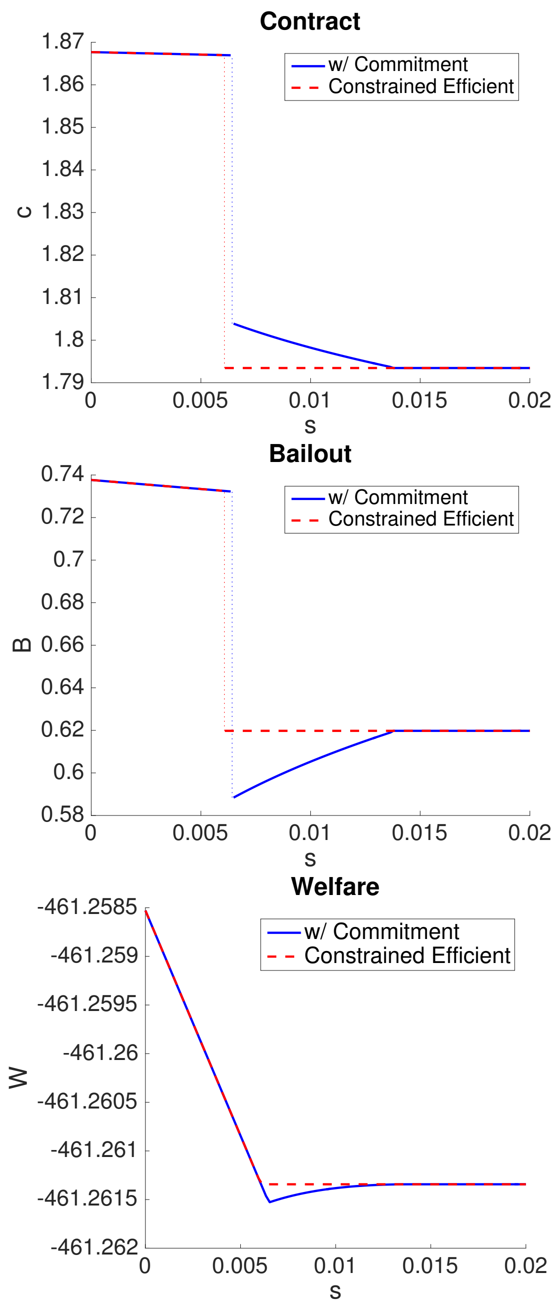

Figure 1.3: Equilibrium under Fiscal Authority with Commitment (Case 2)

the cost of incentivizing the bank to offer a panic-based run-proof contract decreases. Higher s forces the bank to be more conservative. The fiscal authority does not have to lower the bailout amount as far below the optimal level B^* to incentivize the bank to offer a panic-based run-proof contract. Therefore, as s gets sufficiently high above s_0, the fiscal authority starts lowering the bailout to make the bank offer a panic-based run-proof contract. This switch is captured by the discontinuity in the allocation of the economy under the fiscal authority with commitment as seen in Figure 1.3.

Notice that once the fiscal authority decides to lower the bailout to incentivize the bank to offer a panic-based run-proof contract, the ex-ante expected welfare is increasing in s. A higher panic-based run probability does not always hurt welfare. Higher s induces banks to behave more conservatively, and consequently decreases moral hazard and increases welfare.

1.5 The Fiscal Authority without Commitment

This section analyzes the economy in which the fiscal authority is unable to pre-commit to a bailout policy. The fiscal authority chooses the bailout policy B after they observe that both depositors withdraw in period 1. By backward induction, the post-deposit game is first analyzed followed by the pre-deposit game.

1.5.1 The Post-deposit Game

Taking the deposit contract c as given, the post-deposit game can be solved as an extensive-form game with imperfect information. The fiscal authority observes the withdrawal decisions of the depositors but not sunspots and the types of the depositors. The fiscal authority has to infer sunspots and the types of the depositors from the withdrawal decisions of the depositors.

Each depositor - if patient - has to choose her withdrawal plan for each of the two sunspot states. There are three pure strategies for each depositor: (1) to withdraw late in both states α and β, (2) to withdraw late in state α and early in state β, or (3) to withdraw early in both states.[21]

Definition 5. *The separating equilibrium in the post-deposit game under the fiscal authority without commitment is defined as the symmetric separating Perfect Bayesian Equilibrium (PBE) in which both depositors choose action (2).*

Definition 6. *The "good" ("bad") pooling equilibrium in the post-deposit game under the fiscal authority without commitment is defined as the symmetric pooling Perfect Bayesian Equilibrium (PBE) in which both depositors choose action (1) ((3)).*

The Separating Equilibrium

Suppose both depositors - if patient - choose action (2). When the fiscal authority observes that both depositors withdraw early, there is a positive probability that the second in line is patient. Let $\widetilde{p}$ be the conditional probability of the second in line being impatient given that both depositors withdraw early, and is

[21] Without loss of generality, the option for which a patient depositor withdraws early in state α and late in state β is omitted.

given by

$$\widetilde{p} = \frac{(1-s)p^2 + sp}{(1-s)p^2 + s}.$$

With probability $1-s$, state α realizes and all the patient depositors withdraw late. The probability of both depositors withdrawing early in this state is p^2. With probability s, state β realizes and all depositors withdraw early regardless of their types. The probability of both depositors withdrawing early in this state is 1. In general, the probability of the fiscal authority observing that both depositors withdraw early is $(1-s)p^2 + s$. The probability of the two depositors withdrawing early and the second in line being impatient can be derived in a similar way and is given by $(1-s)p^2 + sp$.

The bailout decision of the fiscal authority is essentially a trade off between the withdrawal of the second in line and the public good provision. The fiscal authority's problem is given by

$$\max_{B \in [0, 2\tau y]} \quad \widetilde{p}u(2(1-\tau)y - c + B) + (1-\widetilde{p})v(2(1-\tau)y - c + B) + 2\Gamma(2\tau y - B). \tag{1.9}$$

Notice that the utility from the first in line is not in the objective function because the bailout payment is only used to fund the withdrawal of the second in line. The utility from the public good provision appears in the objective function because the resources for bailouts are taken away from the public good provision.

Let B^*_{sep} be the solution to (1.9). The bailout response of the fiscal authority under the assumption that both depositors choose action (2) is given by:

$$B^*_{sep}(c) = \max\{2\tau y - K_{sep}(2y - c), 0\}, \tag{1.10}$$

where K_{sep} is a constant that is positively correlated with the relative weights on the utility of the public good provision and the utility of the withdrawal of the second in line ($\frac{2D}{\widetilde{p}A+(1-\widetilde{p})}$). Notice that the relative weight is a function of s. K_{sep} is given by

$$K_{sep} = \frac{\left[\frac{2D}{\widetilde{p}A+(1-\widetilde{p})}\right]^{1/\gamma}}{1+\left[\frac{2D}{\widetilde{p}A+(1-\widetilde{p})}\right]^{1/\gamma}}.$$

Holding the deposit contract c constant, the bailout response of the fiscal authority is negatively correlated with s. As s gets larger, the fiscal authority gets decreasingly certain that the second-in-line depositor is impatient. Therefore, the amount of resources that the fiscal authority intends to reallocate from public goods to the withdrawal of the second in line decreases.

When the value of the public good provision is sufficiently high (D is large), it could be welfare improving for the fiscal authority to have a negative bailout. This allows the fiscal authority to impose an additional tax on the withdrawal of the second in line to fund the public good. In this case, the non-negativity constraint for the bailout is binding and $B^*_{sep}(c) = 0$. For the remainder of the paper, the focus is on the parameter values such that the non-negativity constraint for $B^*_{sep}(c)$ is non-binding. This ensures that $B^*_{sep}(c) > 0$.

Lemma 8. *If* $\gamma < \min\{2, 1+\ln 2/\ln R\}$, *there exists an interval* $(\underline{c}_{sep}, \bar{c}_{sep}] \subset (0, 2(1-\tau)y)$ *such that the separating equilibrium exists in the post-deposit game under the fiscal authority without commitment if and only if* $c \in (\underline{c}_{sep}, \bar{c}_{sep}]$.

In order for the fiscal authority's bailout response $B^*_{sep}(c)$ to be such that it is incentive compatible for the depositors to choose action (2), the propensity of the patient depositors to run has to be sufficiently high. This is ensured by having a high enough deposit contract c such that a patient depositor is better off withdrawing early if she believes that the other depositor - if patient - also withdraws early. The propensity to run also has to be low enough such that a patient depositor withdraws late if she believes that the other depositor - if patient - also withdraws late. Lemma 8 suggests that there is a measure of deposit contract $(\underline{c}_{sep}, \bar{c}_{sep}]$ that ensures that both conditions are satisfied and the separating equilibrium exists in the post-deposit game.

The "Good" and "Bad" Pooling Equilibrium

The "good" pooling equilibrium is when both depositors choose action (1), which is to withdraw late in both states α and β if they are patient. When both depositors withdraw early, the fiscal authority knows with certainty that the second in line is impatient ($\widetilde{p} = 1$). The fiscal authority's problem is given by

$$\max_{B \in [0, 2\tau y]} \quad u(2(1-\tau)y - c + B) + 2\Gamma(2\tau y - B). \tag{1.11}$$

Let B^*_{good} be the solution to (1.11). The bailout response of the fiscal authority under the assumption that both depositors choose action (1) is given by

$$B^*_{good}(c) = \max\{2\tau y - K_{good}(2y - c), 0\}, \tag{1.12}$$

where K_{good} is a constant that is positively correlated with the relative weights on the utility from public good and the utility from the withdrawal of the second-in-line ($\frac{2D}{A}$). K_{good} is given by

$$K_{good} = \frac{\left(\frac{2D}{A}\right)^{1/\gamma}}{1 + \left(\frac{2D}{A}\right)^{1/\gamma}}.$$

The bailout response of the fiscal authority is independent of s since the depositors' actions are independent of the sunspot state. The fiscal authority always knows that the probability of the second in line being impatient is 1.

The analysis of the "bad" pooling equilibrium is similar to the "good" pooling equilibrium. When both depositors choose action (3), they withdraw early regardless of their types. Therefore, observing two early withdrawals does not provide the fiscal authority with additional information about the type of the second in line. The conditional probability of the second in line being impatient given two early withdrawals is $\widetilde{p} = p$, which is equal to the probability of any given depositor being impatient.

Lemma 9. *If $\gamma < \min\{2, 1 + \ln 2/\ln R\}$, the "good" ("bad") pooling equilibrium exists in the post-deposit game under the fiscal authority without commitment as long as $c \leq \underline{c}_{sep}$ ($c > \bar{c}_{sep}$).*

To ensure the existence of the "good" pooling equilibrium in the post-deposit game, the best response of the fiscal authority on the bailout $B^*_{good}(c)$ has to be such that it is incentive compatible for the depositors to choose action (1). This happens when the deposit contract c is low ($c \leq \underline{c}_{sep}$) so that the propensity to run of the patient depositors is low enough such that a patient depositor withdraws late if she believes that the other depositor is also withdrawing late. In order for it to be incentive compatible for the depositors to play action (3), c has to be high enough such that a patient depositor withdraws early if she believes that the other depositor - if patient - also withdraws early. Figure 1.4 provides a summary of the type of equilibrium that exists for a given deposit contract $c \in [0, 2(1 - \tau)y]$.

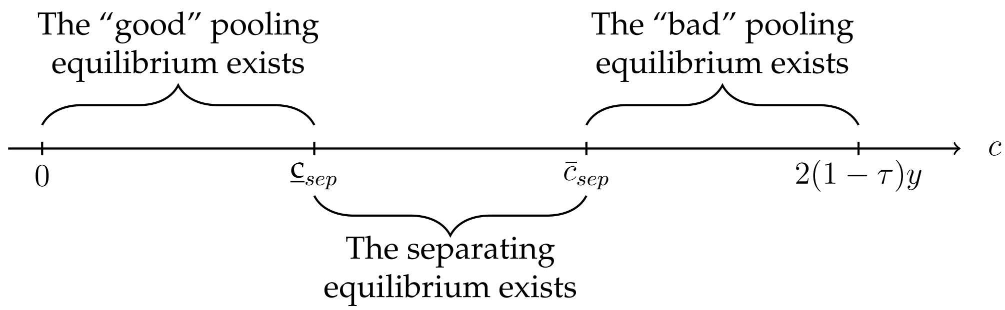

Figure 1.4: PBEs of the Post-deposit Game

1.5.2 The Pre-deposit Game

Since the bailout policy is decided by the fiscal authority later on when both depositors in a bank withdraw in period 1, the bank knows that the deposit contract it offers affects the decision of the fiscal authority on the amount of bailout. It is clear from the response functions of the fiscal authority in (1.10) and (1.12) that the bailout response is an increasing function of c. The bank does not internalize the cost of bailouts but is able to indirectly increase the bailout amount by increasing c. This is the driving force of moral hazard that arises due to the lack of commitment of the fiscal authority in this model.

The bank knows that the choice of its deposit contract dictates the type of equilibrium in the post-deposit game based on Lemma 8 and 9. The representative bank's problem is given by

$$\max_{c \in [0, \bar{c}_{sep}]} \begin{cases} \widehat{W}_{pri}(c, B^*_{good}(c)) & \text{if } c \leq \underline{c}_{sep} \\ \widetilde{W}_{pri}(c, B^*_{sep}(c); s) & \text{if } c > \underline{c}_{sep} \end{cases}. \tag{1.13}$$

The bank only maximizes the ex-ante expected utilities of its own depositors from withdrawals W_{pri} due to the competitive nature of the banking industry.

$\widehat{W}_{pri}(c, B^*_{good}(c))$ is the sum of the ex-ante expected utilities from withdrawals of the two depositors when there is the "good" pooling equilibrium in the post-deposit game. $\widetilde{W}_{pri}(c, B^*_{sep}(c); s)$ is the sum of the ex-ante expected utilities from withdrawals of the two depositors when there is the separating equilibrium in the post-deposit game. Let c_2^* denote the solution to (1.13).[22] The full equilibrium characterization can be found in Section A.5 of the Appendix.

Proposition 10. *The economy in which the fiscal authority is unable to commit is never constrained efficient.*

The main takeaway from Proposition 10 is that the inability of the fiscal authority to commit to a bailout policy is the main driving force of the moral hazard in the banking industry. If the fiscal authority is able to commit, the outcome is typically constrained efficient (Proposition 7). The disparity between the commitment case and the non-commitment case is mainly due to the failure of the bank to internalize the cost of bailouts and the ability of the bank to indirectly affect the bailout it receives by altering the deposit contract it offers.

1.5.3 Numerical Example

The same parameter values from (1.8) are used. Figure 1.5 shows the equilibrium outcome in the non-commitment case together with the equilibrium

[22]There exists a range $c \in (c_{good}, \underline{c}_{sep}]$ such that it is incentive compatible for the depositors to both choose action (1) or both choose action (3). Choosing action (1) in both sunspot states can still be supported as an equilibrium. However, it does not give a fair comparison between the commitment and non-commitment case. This is because in the commitment case, the depositors are assumed to always play the non-panic equilibrium in α state and panic equilibrium in β state whenever both equilibria exist. The focus of the parameter values for the remainder of the paper is such that $c_2^* \notin (c_{good}, \underline{c}_{sep}]$. The detail on the parameter restrictions can be found in Section A.6 of the Appendix.

outcomes in the commitment case and the constrained efficient economy when $A = 1.65$. The plots of the contract, bailout, and welfare for the non-commitment case and the constrained efficient economy are identical to Figure 1.3. For the entire measure of s shown in Figure 1.5, the separating equilibrium exists in the post-deposit game under the fiscal authority without commitment.

Notice that the deposit contract c is higher under the fiscal authority without commitment than the one under the fiscal authority with commitment and the one in the constrained efficient economy. This is due to the bank's expectation that the fiscal authority will increase bailout B if it offers a higher deposit contract c. In equilibrium, bailout B under the fiscal authority without commitment is also higher than the ones in the other two economies. The ex-ante expected welfare under the fiscal authority without commitment is strictly lower than the constrained efficient welfare and the welfare under the fiscal authority with commitment.

The Cost of Non-commitment

In order to better illustrate the cost of moral hazard under the fiscal authority with and without commitment, the amounts of additional endowments on top of y that are required to bring the equilibrium welfare under both cases to the constrained efficient welfare are computed. Define Δ_y^C and Δ_y^{NC} respectively as the corresponding amounts of additional endowments required for each depositor in the commitment and non-commitment case. The difference $\Delta_y^{NC} - \Delta_y^C$ is the cost of the inability of the fiscal authority to commit.

Figure 1.6 shows the plot of Δ_y^C and Δ_y^{NC} using the same parameter values

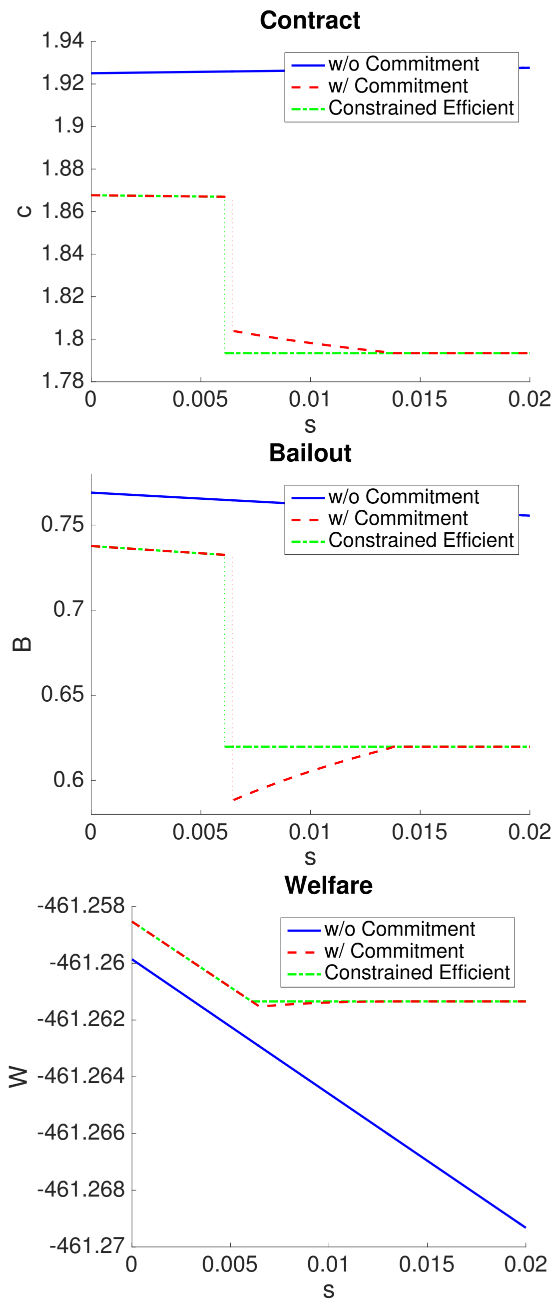

Figure 1.5: Equilibrium under Fiscal Authority without Commitment

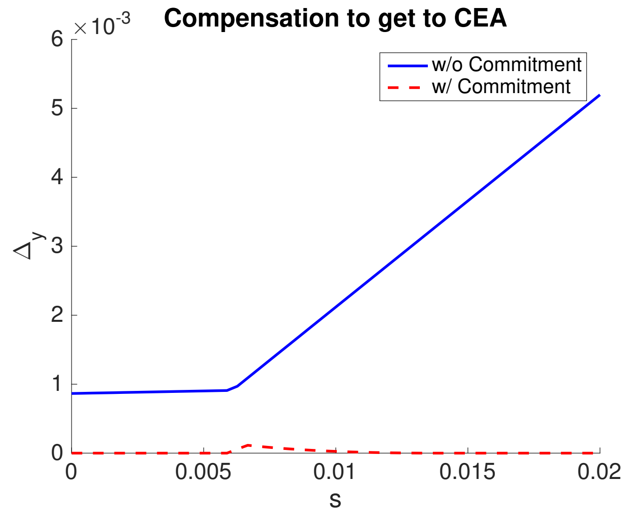

Figure 1.6:

from (1.8) and $A = 1.65$. Notice that Δ_y^C is relatively close to or equal to zero for the entire measure of s. When the fiscal authority is able to commit, the moral hazard cost is minimal. Also, Δ_y^{NC} is strictly increasing in s. As the panic-based run probability gets larger, the moral hazard cost of the inability of the fiscal authority to commit increases. For small s, Δ_y^C is less responsive to an increase in s. This is because constrained efficient contract is panic-based run-permitting while the non-commitment case has the separating equilibrium in the post-deposit game. In equilibrium, the depositors play the same strategies in the post-deposit game in both economies. As s gets larger, the constrained efficient contract switches to panic-based run-proof but the non-commitment case still has the separating equilibrium in the post-deposit game. The depositors play a different strategy in each economy. The constrained efficient welfare is constant in s but the welfare in the non-commitment case is strictly decreasing in s. Therefore, Δ_y^{NC} is more responsive to an increase in s.

1.6 Controlling Short-term Interest Rates

This section evaluates the efficacy of an additional policy tool for the fiscal authority in eliminating the moral hazard from bailouts.

Since banks tend to offer a deposit contract c higher than the efficient level, it can be welfare improving if the fiscal authority is able to place an upper bound on the contract offered by banks.[23] The fiscal authority chooses an upper bound on the deposit contract $\bar{c}$ right after taxes are collected.

Proposition 11. *Suppose the fiscal authority is able to place an upper bound on contracts. When the fiscal authority is able to commit, the economy is always constrained efficient. When the fiscal authority is unable to commit, the economy is constrained efficient if $s \in [0, \bar{s})$, where $\bar{s} = 1$ if the constrained efficient allocation is in Case 1, $\bar{s} \in (0, s_0]$ in Case 2, and $\bar{s} = 0$ in Case 3.*

When the fiscal authority is able to commit, the optimal choices for bailout and the upper bound for contracts are trivial. The fiscal authority chooses $B = B^*$ and $\bar{c} = c^*$.[24] The upper bound for contracts is binding and the bank chooses c^*. In equilibrium, the allocation is identical to the constrained efficient allocation. The economy is constrained efficient for $s \in [0, 1]$.

When the fiscal authority is unable to commit, the optimal choice for $\bar{c}$ is less trivial. Assuming that the fiscal authority chooses the deposit contract on behalf

[23]Contract is essentially the short-term interest rate offered by the bank. The short-term rate can be calculated by $\frac{c}{(1-\tau)y}$.

[24]Recall that (c^*, B^*) is the constrained efficient allocation.

of the bank, the fiscal authority's problem is given by

$$c^*_{optimal} \equiv \underset{c\in[0,\bar{c}_{sep}]}{\arg\max} \begin{cases} \widehat{W}(c, B^*_{good}(c)) & \text{if } c \leq \underline{c}_{sep} \\ \widetilde{W}(c, B^*_{sep}(c); s) & \text{if } c > \underline{c}_{sep} \end{cases}. \tag{1.14}$$

(1.14) differs from (1.13) and in that the ex-ante expected welfare is maximized in (1.14) but only the ex-ante expected utilities from withdrawals are maximized in (1.13). The contract $c^*_{optimal}$ chosen by the fiscal authority gives the highest possible welfare in the non-commitment case. Naturally, the fiscal authority's optimal choice for the upper bound is $\bar{c} = c^*_{optimal}$.[25]

When $A \leq A^{early}$ (Case 1), the constrained efficient allocation is $(\widehat{c}, \widehat{B})$ and is panic-based run-proof for $s \in [0, 1]$. The solution to (1.14) is also $\widehat{c}$. The fiscal authority's optimal upper bound for contracts is $\bar{c} = \widehat{c}$. Since the bank typically chooses a deposit contract that is higher than the constrained efficient level, the upper bound is strictly binding. The bank's optimal deposit contract is $\widehat{c}$, which gives the "good" equilibrium in the post-deposit game. The fiscal authority's bailout response in the post-deposit game is $B^*_{good}(\widehat{c}) = \widehat{B}$. The equilibrium allocation turns out to be $(\widehat{c}, \widehat{B})$, which is identical to the constrained efficient allocation. Therefore, the economy is constrained efficient for $s \in [0, 1]$.

When $A^{early} < A \leq A^{wait}$ (Case 2), the constrained efficient allocation is $(\widetilde{c}, \widetilde{B})$ and is panic-based run-permitting for $s \in [0, s_0)$. It can be shown that the solution to (1.14) is also $\widetilde{c}$ for $s \in [0, \bar{s})$. The fiscal authority then chooses $\widetilde{c}$ as the upper bound and the bank responds with $\widetilde{c}$ since the upper bound is strictly binding, as shown in Figure 1.5. The equilibrium allocation is $(\widetilde{c}, \widetilde{B})$, which is identical to the constrained efficient allocation. Therefore, the economy is con-

[25]It remains to check that when $\bar{c} = c^*_{optimal}$, the bank responds by choosing $c^*_{optimal}$. That ensures that the optimal upper bound of the fiscal authority is $\bar{c} = c^*_{optimal}$

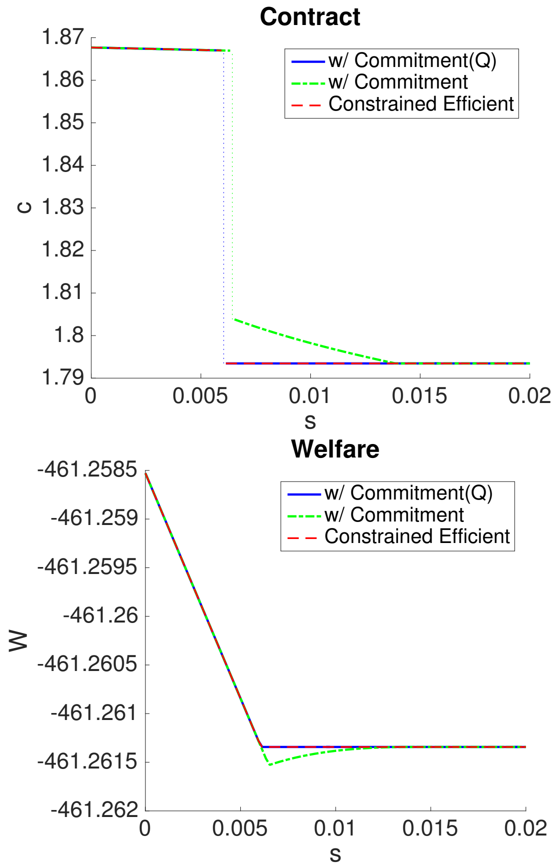

Figure 1.7: “Regulation Q” under Fiscal Authority with Commitment

strained efficient for $s \in [0, \bar{s})$. When $s \in [\bar{s}, 1]$, the constrained efficient allocation is (c^{early}, B^{early}). However, c^{early} is not necessarily the solution to (1.14). Therefore, the economy is not necessarily constrained efficient.

1.6.1 Numerical Example

The numerical example in this section uses the parameter values as in (1.8) and $A = 1.65$. The constrained efficient allocation is in Case 2. Figure 1.7 shows the equilibrium contract and welfare under the fiscal authority with commitment. Notice that if the fiscal authority is able to impose an upper bound on contracts (shown in Figure 1.7 as “w/ Commitment (Q)”), the economy is always constrained efficient. The measure of s in which the economy is initially not constrained efficient is generated by the bank setting higher than socially optimal c. Once the fiscal authority sets $\bar{c} = c^*$, the bank is forced to lower its contract to the constrained efficient level c^*.

Figure 1.8 shows the equilibrium contract, bailout and welfare under the fiscal authority without commitment. Notice that when the fiscal authority is able to impose an upper bound on contracts, the economy is constrained efficient for small panic-based run probability $s \in [0, \bar{s})$. In this case, $\bar{s} = s_0$. The economy is constrained efficient for the entire measure of s such that the constrained efficient allocation is panic-based run-permitting. When $s \in [s_0, 1]$, the economy is not necessarily constrained efficient. In this numerical example, it happens that the economy is not constrained efficient. However, the welfare is still strictly higher than the case in which the fiscal authority is not able to impose an upper bound on contracts.

1.7 Concluding Remarks

Bailouts do create moral hazard. The model in this paper suggests that moral hazard can mostly be eliminated if the fiscal authority is able to make a credible commitment on the bailout policy since most of the welfare loss from moral hazard comes from the inability of the fiscal authority to commit.

In practice, it can be difficult for the fiscal authority to credibly commit. Therefore, additional policy tools are necessary to reduce the moral hazard arising from bailouts. This model suggests that controlling short-term interest rates

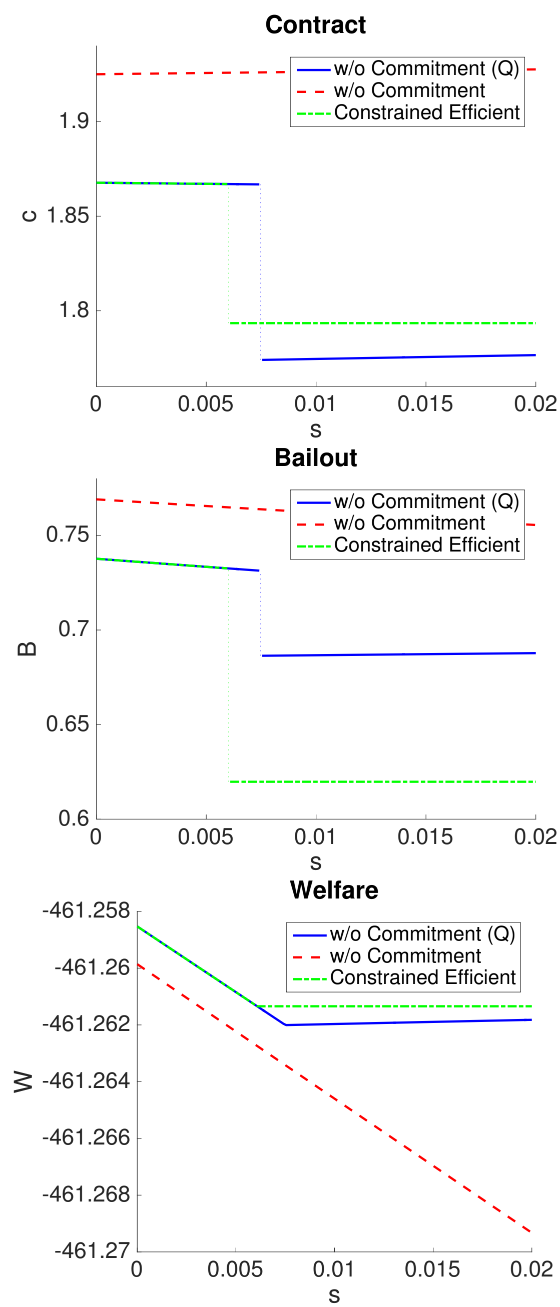

Figure 1.8: "Regulation Q" under Fiscal Authority without Commitment

set by the banks can reduce the welfare loss from moral hazard. It is particularly effective when the probability of a bank run is low, which is generally the case in economies with developed financial system, such as the OECD countries.

Keister (2016) compared the bailout regime (similar to the fiscal authority without commitment) to the no-bailout regime (similar to the fiscal authority with commitment but instead of committing to an ex-ante optimal level of bailout, the fiscal authority commits to zero bailout). He found that even though bailouts create moral hazard, a commitment to zero-bailout policy is not always superior. This paper goes in a different direction and focuses on the welfare loss that comes solely from the inability of the fiscal authority to commit to a bailout policy.

This model also suggests that the cost of the inability of the fiscal authority to commit to a bailout policy is increasing in the probability of bank runs. Economies with an unstable financial system, which is particularly common in underdeveloped countries, suffer a greater cost from moral hazard. Therefore, restrictive regulations on financial institutions are essential more so in underdeveloped countries than developed countries.

This model assumes that taxes are exogenous. The model can be generalized to the case in which the tax rate is chosen endogenously by the fiscal authority. This could be a potential future research. However, preliminary numerical exercises have shown that the qualitative results of this model preserve after this generalization.

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CHAPTER 2

THE OPTIMAL BAILOUT POLICY IN AN INTERBANK NETWORK

2.1 Introduction

The concern and skepticism regarding the bailouts of financial institutions at the expense of tax payers have been rising especially since the financial crisis of 2008. Arguments against bailouts based predominantly on the huge cost of these bailout packages and the future incentive of financial institutions to exploit such action from the government. On the other hand, the arguments for bailouts stand on the fact that the insolvencies of financial institutions are often "temporary" and that bailouts are essential to back these financial institutions over a brief rough patch. Additionally, by maintaining solvencies in failing financial institutions, bailouts prevent negative spillovers to other solvent financial institutions that could potentially induce a larger financial crisis. In the financial crisis of 2008, the arguments agaist the refusal to bailout Lehman Brothers were established on the fact that the failure of Lehman had created spillovers that caused multiple other financial institutions to fail. It also created major distrusts in the financial sector, which eventually led to the financial crisis.

The model in this paper aims to highlight three motivations behind bailouts of financial institutions: redistribution of resources between private and public consumption, prevention of costly bankruptcies, and prevention of negative spillovers. This model focuses on the decision making of and the trade-off faced by the fiscal authority regarding the level of bailouts made to financial institutions during a financial crisis. The results of this paper agree with the arguments supporting bailouts, particularly on the benefits they generate from preventing negative spillovers from insolvent to solvent financial institutions.

This paper, however, does not take into consideration the moral hazard that arise from the expectation of the financial institutions to receive bailouts during a financial crisis. It also does not consider the advantages of building a tough reputation of the fiscal authority by restricting bailouts to break the common "too big to fail" belief among financial institutions. The unwillingness of the government to provide a bailout to Lehman Brothers in 2008 provided a huge lesson to the financial sector that big financial institutions do not always receive bailouts from the government when they fail, though this lesson had lost its essence due to the enormous bailout package that came later in the crisis.

The main result of this paper is that partial bailouts are often optimal due to the advantages of bailouts from bankruptcy and spillover preventions. A key parameter in the model that dictates the optimal level of bailout is the cost of bailout. A high cost of bailout can be interpreted as a high valuation of public good consumption such that taking resources away from public goods to fund bailouts creates a huge welfare loss. It is intuitive that when the cost of bailout is sufficiently low, it is optimal for the fiscal authority to provide a full bailout, which covers all the losses incurred by the failing financial institutions. On the other hand, with a sufficiently high cost of bailout, the zero bailout policy is optimal. There exists an intermediate range of the cost of bailout such that it is optimal for the fiscal authority to cover a fraction of the losses incurred by the failing financial institutions. Within this range, there exists a positive measure of the cost of bailout such that the optimal level of bailout is exactly sufficient to prevent a spillover such that the failure of one financial institution does not cause another financial institution to fail. This highlights the significance of recognizing the role of bailouts in preventing financial spillovers, which strengthens the arguments against the failure to bailout Lehman Brothers during the financial crisis.[1]

This paper is organized as follows: Section 2 describes the interbank network environment, Section 3 introduces the financial crisis into the model, Section 4 analyzes the optimal bailout of the fiscal authority, Section 5 concludes the paper. The proofs of all Lemmas are in the Appendix.

2.2 The Banking Environment

There are two banks denoted {A,B}. There is an investment technology that gives 1 at $t = 1$ or $R > 1$ at $t = 2$ for each unit of consumption good invested at $t = 0$. There are two states of the world $\{S_1, S_2\}$ that realizes at $t = 1$. We can assume that each state realizes with equal positive probability but this could be generalized to any positive probabilities. At each state, exactly one of the two banks incurs a loss where a fraction $\bar{\sigma} \in [0, 1]$ of its investment becomes worthless. We can denote σ_A and σ_B as the proportion of investment that has become worthless at $t = 1$ for banks A and B respectively. The states are summarized in Table 2.1.

Each bank has a measure of depositors each of whom has probability π of turning impatient at $t = 1$ and probability $1 - \pi$ of turning patient at $t = 1$. Denote x_1 as the withdrawal at $t = 1$ and x_2 as the withdrawal at $t = 2$. The

[1]There also exists a positive measure of the cost of bailouts such that the optimal level of bailout is exactly sufficient to prevent the failing financial institution to become insolvent. This is to emphasize the role of bailouts in preventing costly bankruptcies.

State	σ_A	σ_B
S_1	$\bar{\sigma}$	0
S_2	0	$\bar{\sigma}$

Table 2.1: States

utility of an impatient depositor is $u(x_1)$ and of a patient depositor is $u(x_1 + x_2)$ where u is a twice continuously differentiable function satisfying $u' > 0$ and $u'' < 0$.[2]

2.2.1 Social Planner

Since there is no aggregate uncertainty, the economy as a whole always has $1 - \frac{\bar{\sigma}}{2}$ portion of investment that has not become worthless at $t = 1$. The social planner's problem is

$$\max_{c_1,c_2} \quad \pi u(c_1) + (1-\pi)u(c_2)$$
$$\text{subject to } (1-\pi)c_2 \leq \Big(1 - \frac{\bar{\sigma}}{2} - \pi c_1\Big)R, \tag{2.1}$$

which can be rewritten as

$$\max_{c_1} \quad \pi u(c_1) + (1-\pi)u\left(\frac{\big(1 - \frac{\bar{\sigma}}{2} - \pi c_1\big)R}{1-\pi}\right). \tag{2.2}$$

The first order condition is

$$u'(c_1) = Ru'\left(\frac{\big(1 - \frac{\bar{\sigma}}{2} - \pi c_1\big)R}{1-\pi}\right) = Ru'(c_1). \tag{2.3}$$

Since $R > 1$ and $u'' < 0$, it can be inferred that $c_2 > c_1$, which means that the incentive compatibility constraint is satisfied by the social planner's allocation.[3]

[2]The distinction between a patient and an impatient depositor is on the timing of the demand for consumption. An impatient depositor only derives utility from consumption in $t = 1$, while a patient depositor is indifferent between consumption in $t = 1$ or $t = 2$.

[3]The incentive compatibility constraint is said to be satisfied if the bank pays its patient depositors in $t = 2$ at least as much as it pays its impatient depositors in $t = 1$.

Let c_1 and c_2 denote the social planner's allocation for the remainder of the paper.

2.2.2 Decentralized Economy and Risk Sharing

There is a potential benefit from risk sharing between bank A and bank B in the decentralized economy because the risks they face are perfectly negatively correlated. To perfectly risk share, the two banks could enter a contract in which the bank with $\sigma_k = \bar{\sigma}$ can claim either $\frac{\bar{\sigma}}{2}$ at $t = 1$, $\frac{\bar{\sigma}}{2}R$ at $t = 2$ or any linear combination of the two from the other bank with $\sigma_k = 0$. This way, the decentralized economy can achieve the welfare of the social planner. For both banks, each impatient depositor receives c_1 and each patient depositor receives c_2. Without loss of generality, the following analysis focuses on state S_1, in which $\sigma_A = \bar{\sigma}$ and $\sigma_B = 0$.

Case 1: $\bar{\sigma} \leq 1 - \pi c_1$

In this case, $\bar{\sigma}$ is sufficiently small such that bank A still has enough resources to pay all the impatient depositors c_1 at $t = 1$. Both banks liquidate πc_1 of their total investment to pay their respective impatient depositors. At $t = 2$, bank A has $[(1 - \bar{\sigma}) - \pi c_1]R$ available from its own investment and, under the contract with bank B, can claim $\frac{\bar{\sigma}}{2}R$ from bank B. The total amount of resources available for patient depositors in bank A is $[1 - \frac{\bar{\sigma}}{2} - \pi c_1]R$. On the other hand, bank B has $(1 - \pi c_1)R$ available from its own investment but, under the contract with bank A, it has to pay bank A $\frac{\bar{\sigma}}{2}R$. The total amount of resources available for patient depositors in bank B is also $[1 - \frac{\bar{\sigma}}{2} - \pi c_1]R$. This means that all patient depositors, regardless of which bank they deposited in, receive $c_2 = \frac{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}{1-\pi}$

at $t = 2$. The social planner's allocation is identical to the equilibrium allocation in the decentralized economy.

Case 2: $1 - \pi c_1 < \bar{\sigma} \leq 2(1 - \pi c_1)$

Since $\bar{\sigma}$ is large enough such that bank A cannot afford to pay all its mpatient depositors c_1 at $t = 1$, it has to withdraw the remaining amount $\pi c_1 - (1 - \bar{\sigma})$ from bank B. That gives bank A just enough resources at $t = 1$ to meet its liabilities πc_1. On the other hand, bank B has to liquidate πc_1 of its investment to meet the demand of the impatient depositors and another $\pi c_1 - (1 - \bar{\sigma})$ for bank A. The total liquidation for bank B is $2\pi c_1 - (1 - \bar{\sigma})$.[4] At $t = 2$, bank A has 0 left from its own investment and is entitled to claim $\left[\frac{\bar{\sigma}}{2} - [\pi c_1 - (1 - \bar{\sigma})]\right] R$ from bank B. The total amount of resources available for patient depositors in bank A is then $[1 - \frac{\bar{\sigma}}{2} - \pi c_1]R$. Bank B has $[1 - 2\pi c_1 + (1 - \bar{\sigma})]R$ available from its own investment at $t = 2$. After paying $[1 - \frac{\bar{\sigma}}{2} - \pi c_1]R$ to bank A, it has $[1 - \frac{\bar{\sigma}}{2} - \pi c_1]R$ left for its own patient depositors. Therefore, all patient depositors in the economy receives $c_2 = \frac{\left(1 - \frac{\bar{\sigma}}{2} - \pi c_1\right)R}{1 - \pi}$ at $t = 2$. Again, the incentive compatibility constraint is satisfied and the social planner's allocation can be supported as the equilibrium allocation in the decentralized economy.

2.2.3 Bank Runs and Bankruptcies

To simplify the model, it is assumed that a bank run only happens when patient depositors anticipate that they will receive less than c_1 even if they believe that all other patient depositors will withdraw at $t = 2$. In other words, there is no

[4]Since c_1 is chosen by the social planner, it has to satisfy $2\pi c_1 - (1 - \bar{\sigma}) \leq 1$. Otherwise, there is not enough resources in the entire economy to even satisfy only the need of the patient depositors. This means that $\bar{\sigma} > 2(1 - \pi c_1)$ will never happen.

run on the bank as long as the incentive compatibility constraint is satisfied. In the language of sunspot, the sunspot probability in this model is zero, which eliminates the possibility of multiple equilibria and panic-based bank runs.[5]

There are two ways for a bank to go into bankruptcy. The first way is that there is not enough resources available to the bank at $t = 1$ to satisfy all the impatient depositors even if it liquidates all its investments and withdraws all its claims the other bank. This happens when the amount of resources in the bank is less than πc_1 at $t = 1$. The second way is that the bank anticipates that its remaining resources after paying impatient depositors in $t = 1$, together with any possible claims that it has on other banks at $t = 2$, are not enough to pay each patient depositor at least c_1 at $t = 2$. The second way is basically saying that the bank will also go into bankruptcy if it fails to satisfy the incentive compatibility constraint.

When a bank goes into bankruptcy, it liquidates all its investments and claims on the bank at $t = 1$, and evenly distributes the available resources to all its depositors. If the other bank has a claim on the bank that goes into bankruptcy, the remaining resources have to be shared proportionately between the depositors and the other bank.

2.3 Financial Crisis

A financial crisis in this model is defined as an unanticipated shock $\varepsilon > 0$ on the investment of one of the two banks, on top of the anticipated shock σ discussed earlier. Since this shock is unanticipated, all agents in the economy, including

[5]Similar assumption has been made in Allen and Gale (2000)

State	σ_A	σ_B
S_1	$\bar{\sigma}$	0
S_2	0	$\bar{\sigma}$
S_3	$\bar{\sigma}+\varepsilon$	0
S_4	$\bar{\sigma}$	ε
S_5	ε	$\bar{\sigma}$
S_6	0	$\bar{\sigma}+\varepsilon$

Table 2.2: States including Financial Crisis

the social planner, perceive that the ex-ante probability of this unanticipated shock occuring is zero. The financial crisis can be formally written as four additional states, as shown in Table 2.2.

States S_1 and S_2 are both anticipated to occur with positive probabililities while states S_3, S_4, S_5, and S_6 are anticipated to occur with zero probability. Since states S_3 and S_4 are symmetrical to states S_5 and S_6, without loss of generality, the remainder of this paper focuses only on states S_3 and S_4.

2.3.1 State S_3

Define c_1^k and c_2^k as the amount paid by bank k to its impatient and patient depositors respectively, where $k \in \{\text{A, B}\}$.

Case 1: $\varepsilon \leq 1 - \bar{\sigma} - \pi c_1$

Since bank A has enough resources from liquidating its investments to satisfy the payments to impatient depositors, bank A does not make any withdrawals from bank B. Both banks liquidate πc_1 of their own investments at $t = 1$ to pay their respective impatient depositors. At $t = 2$, bank A has $(1 - \bar{\sigma} - \varepsilon - \pi c_1)R$ and withdraws $\frac{\bar{\sigma}}{2}R$ from bank B. Each patient depositor in bank A receives $\frac{(1-\frac{\bar{\sigma}}{2}-\varepsilon-\pi c_1)R}{1-\pi}$. To satisfy the incentive compatibility constraint of the patient depositors, it has to be the case that patient depositors receive at least as much as c_1, which is equivalent to when

$$\varepsilon \leq 1 - \frac{\bar{\sigma}}{2} - (\frac{1-\pi}{R} + \pi)c_1. \tag{2.4}$$

If condition (2.4) is violated, bank A has to go into bankruptcy and liquidate all its investments and claims on bank B in $t = 1$. The total amount available for bank A after liquidation is $1 - \frac{\bar{\sigma}}{2} - \varepsilon$, which is also the amount each impatient or patient depositor receives. Regardless of whether bank A goes into bankruptcy, bank B is still able to pay c_1 to each of its impatient depositors and c_2 to each of its patient depositors.

Case 2: $1 - \bar{\sigma} - \pi c_1 < \varepsilon \leq 1 - \frac{\bar{\sigma}}{2} - \pi c_1$

In this case, bank A does not have enough investment to liquidate to pay πc_1 to its impatient depositors unless it withdraws some of its claims on bank B. The amount bank A withdraws from bank B is $\pi c_1 - (1-\bar{\sigma}-\varepsilon)$. At $t = 2$, bank A's only available resources is its remaining claim on bank B, which is $(1 - \frac{\bar{\sigma}}{2} - \varepsilon - \pi c_1)R$. Each patient depositor in bank A receives $\frac{(1-\frac{\bar{\sigma}}{2}-\varepsilon-\pi c_1)R}{1-\pi}$, which is the same as in Case 1. Similar to Case 1, bank B is also able to pay c_1 and c_2 to its impatient and patient depositors respectively.

Case 3: $\varepsilon > 1 - \frac{\bar{\sigma}}{2} - \pi c_1$

In this case, bank A cannot afford to pay each of its impatient depositors c_1 even if it withdraws all its claims on bank B. Bank A goes into bankruptcy and each depositor receives $1 - \frac{\bar{\sigma}}{2} - \varepsilon$. Bank B remains completely solvent just like in the previous two cases.

The outcome in State S_3 can be summarized as follows:

$$
\begin{aligned}
c_1^A(\varepsilon) &= \begin{cases} c_1 & \varepsilon \leq 1 - \frac{\bar{\sigma}}{2} - (\frac{1-\pi}{R} + \pi)c_1 \\ 1 - \frac{\bar{\sigma}}{2} - \varepsilon & \varepsilon > 1 - \frac{\bar{\sigma}}{2} - (\frac{1-\pi}{R} + \pi)c_1 \end{cases}, \\
c_2^A(\varepsilon) &= \begin{cases} \frac{(1-\frac{\bar{\sigma}}{2}-\varepsilon-\pi c_1)R}{1-\pi} & \varepsilon \leq 1 - \frac{\bar{\sigma}}{2} - (\frac{1-\pi}{R} + \pi)c_1 \\ 1 - \frac{\bar{\sigma}}{2} - \varepsilon & \varepsilon > 1 - \frac{\bar{\sigma}}{2} - (\frac{1-\pi}{R} + \pi)c_1 \end{cases}, \\
c_1^B(\varepsilon) &= c_1, \\
c_2^B(\varepsilon) &= c_2. \qquad (2.5)
\end{aligned}
$$

Notice that for any $\varepsilon \in [0, 1 - \bar{\sigma}]$, bank B remains unaffected by the financial crisis. There is no contagion in state S_3.

2.3.2 State S_4

The analysis in this section is restricted to the case where $\bar{\sigma} \leq 1 - \pi c_1$ such that Bank A does not have to depend on withdrawals from Bank B to fulfil its early liabilities to its depositors at $t = 1$.

Case 1: $\varepsilon > 1 - \pi c_1$

When ε is within this range, the unanticipated shock is so large that bank B does not have enough resources even if it liquidates all its remaining investment. Bank B then goes into bankruptcy. Each depositor in bank B receives $\frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}}$ and bank A recieves $\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}$. It is easy to show that for ε this large, it is always true that $\pi c_1 > \frac{(1-\epsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}$ such that Bank A depletes its entire withdrawal from Bank B from fulfilling its liabilities to its impatient depositors at $t = 1$. Bank A is able to pay each of its patient depositor $\frac{1}{1-\pi}\left[1 - \bar{\sigma} + \frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}} - \pi c_1\right]R$. The incentive compatibility constraint is satisfied for Bank A if its patient depositors' withdrawals in $t = 2$ is at least as much as its impatient depositors' withdrawals in $t = 1$. This happens if and only if

$$\varepsilon \leq 1 - \frac{2c_1 + \bar{\sigma}}{\bar{\sigma}}\left[\left(\frac{1-\pi}{R} + \pi\right)c_1 - (1 - \bar{\sigma})\right]. \tag{2.6}$$

Otherwise, $c_1^A > c_2^A$ and bank A has to go into bankruptcy.

Case 2: $\varepsilon \leq 1 - \pi c_1$

Since both ε and $\bar{\sigma}$ are sufficiently small, both banks are able to use their own investment liquidation to fulfill their liabilities to their respective depositors at $t = 1$. At $t = 2$, bank B has to pay a total of $(1 - \frac{\bar{\sigma}}{2} - \pi c_1)R$ to its patient depositors and $\frac{\bar{\sigma}}{2}R$ to bank A. The total amount of liabilities is $(1 - \pi c_1)R$, which is larger than the amount of remaining resources bank B has $(1 - \pi c_1 - \varepsilon)R$. The reamining resources has to be divided proportionately to the patient depositors and bank A. Each patient depositor receives $\frac{1}{1-\pi}\frac{(1-\frac{\bar{\sigma}}{2}-\pi c_1)(1-\varepsilon-\pi c_1)R}{1-\pi c_1}$ while bank A receives $\frac{1-\varepsilon-\pi c_1}{1-\pi c_1}\frac{\bar{\sigma}}{2}R$. Bank B satisfies the incentive compatibility constraint if and only if

$$\varepsilon \leq (1 - \pi c_1)\left[1 - \frac{(1-\pi)c_1}{(1 - \frac{\bar{\sigma}}{2} - \pi c_1)R}\right]. \tag{2.7}$$

Suppose (2.7) holds such that Bank B is able to satisfy the incentive compatibility constraint. Bank A has $(1-\bar{\sigma}-\pi c_1)R$ remaining from its own investment at $t=2$. Together with $\frac{1-\varepsilon-\pi c_1}{1-\pi c_1}\frac{\bar{\sigma}}{2}R$ it receives from bank B, it has a total of $(1-\bar{\sigma}-\pi c_1)R+\frac{1-\varepsilon-\pi c_1}{1-\pi c_1}\frac{\bar{\sigma}}{2}R$. Each patient depositor in bank A receives $\frac{1}{1-\pi}\big[(1-\bar{\sigma}-\pi c_1)R+\frac{1-\varepsilon-\pi c_1}{1-\pi c_1}\frac{\bar{\sigma}}{2}R\big]$. Bank A can then satisfy the incentive compatibility constraint if and only if

$$\varepsilon \leq (1-\pi c_1)\left[\frac{2}{\bar{\sigma}}\left(1-\Big(\frac{1-\pi}{R}+\pi\Big)c_1\right)-1\right]. \tag{2.8}$$

Lemma 12. *If $\bar{\sigma} \leq 1-\pi c_1$, Bank A does not go into bankruptcy as long as Bank B does not go into bankruptcy.*[6]

Now suppose that (2.7) is violated such that Bank B does go into bankcruptcy. Bank B's investments are divided proportionately among its depositors and Bank A. Each depositor in bank B receives $\frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}}$ and bank A receives $\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}$.

Notice that the amount received by bank A is no longer in the investment technology. If bank A does not use up this amount at $t=1$, the amount will not be compounded by R at $t=2$. If $\pi c_1 \geq \frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}$ or equivalently $\varepsilon \geq 1-\pi c_1\frac{2c_1+\bar{\sigma}}{\bar{\sigma}}$, bank A depletes the resources it receives from bank B when making payments to its impatient depositors at $t=1$.

Lemma 13. *When Bank B goes into bankruptcy, Bank A always depletes the resources it receives from Bank B from paying its patient depositors if and only if*

$$\bar{\sigma} \leq (1-\pi c_1)\frac{2\pi c_1 R}{(1-\pi c_1)(1-\pi)+\pi c_1}.^{7} \tag{2.9}$$

[6] Lemma 12 is mathematically equivalent to $(1-\pi c_1)\big[\frac{2}{\bar{\sigma}}\big(1-\big(\frac{1-\pi}{R}+\pi\big)c_1\big)-1\big] \geq (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big]$ as long as $\bar{\sigma} \leq 1-\pi c_1$

[7] Lemma 13 is mathematically equivalent to $(1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \geq 1-\pi c_1\frac{2c_1+\bar{\sigma}}{\bar{\sigma}}$ if and only if $\bar{\sigma} \leq (1-\pi c_1)\frac{2\pi c_1 R}{(1-\pi c_1)(1-\pi)+\pi c_1}$

If (2.9) holds, Bank A has to liquidate $\pi c_1 - \frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}$ of its investment to fulfill the liabilities for its impatient depositors at $t = 1$. Bank A is then able to pay $\frac{1}{1-\pi}\big(1 - \bar{\sigma} - \pi c_1 + \frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}\big)R$ to each of its patient depositors at $t = 2$. Bank A satisfies the incentive compatibility constraint if and only if

$$\varepsilon \leq 1 - \frac{2c_1 + \bar{\sigma}}{\bar{\sigma}}\left[\left(\frac{1-\pi}{R} + \pi\right)c_1 - (1-\bar{\sigma})\right]. \tag{2.10}$$

The analysis in thie section focuses on the more interesting case in which there exists a range of unanticipated shock ε such that Bank B goes into bankruptcy but Bank A does not. The parameter combinations that satisfy

$$(1-\pi c_1)\left[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\right] < 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\left[\left(\frac{1-\pi}{R}+\pi\right)c_1 - (1-\bar{\sigma})\right] \tag{2.11}$$

ensure the existence of such range of ε.

Assume that $\bar{\sigma} \leq 1 - \pi c_1$, (2.9), and (2.11) hold for the remainder of the paper.[8] This ensures that Bank A does not go into bankruptcy unless Bank B does. It also ensures that when Bank B goes into bankruptcy and Bank A does not, Bank A's total payment to its patient depositors is always larger than the amount Bank A receives from the bankruptcy liquidation of Bank B.

[8]Section B.1 in the Appendix analyzes the outcome when (2.9) or (2.11) does not hold.

The outcome in State S_4 can be summarized as follows:

$$c_1^A(\varepsilon)=\begin{cases} c_1 & \text{if } \varepsilon \le (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] \\ c_1 & \text{if } \begin{array}{l}(1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] \\ \quad < \varepsilon \le 1-\frac{2c_1+\bar\sigma}{\bar\sigma}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1-(1-\bar\sigma)\big]\end{array} \\ 1-\bar\sigma+\frac{(1-\varepsilon)\bar\sigma}{2c_1+\bar\sigma} & \text{if } \varepsilon > 1-\frac{2c_1+\bar\sigma}{\bar\sigma}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1-(1-\bar\sigma)\big] \end{cases}$$

$$c_2^A(\varepsilon)=\begin{cases} c_2 & \text{if } \varepsilon \le (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] \\ \frac{1}{1-\pi}\big(1-\bar\sigma-\pi c_1+\frac{(1-\varepsilon)\bar\sigma}{2c_1+\bar\sigma}\big)R & \text{if } \begin{array}{l}(1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] \\ \quad < \varepsilon \le 1-\frac{2c_1+\bar\sigma}{\bar\sigma}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1-(1-\bar\sigma)\big]\end{array} \\ 1-\bar\sigma+\frac{(1-\varepsilon)\bar\sigma}{2c_1+\bar\sigma} & \text{if } \varepsilon > 1-\frac{2c_1+\bar\sigma}{\bar\sigma}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1-(1-\bar\sigma)\big] \end{cases}$$

$$c_1^B(\varepsilon)=\begin{cases} c_1 & \text{if } \varepsilon \le (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] \\ \frac{2(1-\varepsilon)c_1}{2c_1+\bar\sigma} & \text{if } \varepsilon > (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] \end{cases}$$

$$c_2^B(\varepsilon)=\begin{cases} \frac{1}{1-\pi}\frac{(1-\frac{\bar\sigma}{2}-\pi c_1)(1-\varepsilon-\pi c_1)R}{1-\pi c_1} & \text{if } \varepsilon \le (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] \\ \frac{2(1-\varepsilon)c_1}{2c_1+\bar\sigma} & \text{if } \varepsilon > (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] \end{cases}. \tag{2.12}$$

When $\varepsilon \le (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big]$, the unanticipated shock is sufficiently small such that Bank B can still satisfy the incentive compatibility constraint and thus does not go into bankruptcy. Each impatient depositor in Bank B receives c_1 at $t=1$ and each patient depositor in Bank B receives the return of the remaining investment $\frac{1}{1-\pi}\frac{(1-\frac{\bar\sigma}{2}-\pi c_1)(1-\varepsilon-\pi c_1)R}{1-\pi c_1}$ at $t=2$. According to Lemma 12, Bank A also does not go into bankruptcy. Each impatient depositor receives c_1 at $t=1$. Since Bank A receives the full $\frac{\bar\sigma}{2}R$ from Bank B at $t=2$, it is able to pay each of its patient depositor c_2.

When $(1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar\sigma}{2}-\pi c_1)R}\big] < \varepsilon \le 1-\frac{2c_1+\bar\sigma}{\bar\sigma}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1-(1-\bar\sigma)\big]$, Bank B fails to satisfy the incentive compatibility constraint and goes into bankruptcy. After liquidation at $t=1$, each depositor in Bank B receives $\frac{2(1-\varepsilon)c_1}{2c_1+\bar\sigma}$ and Bank

A receives $\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}$ from Bank B. According to Lemma 13, Bank A's total payment to its impatient depositors πc_1 is larger than the amount it receives from Bank B. Therefore, Bank A has to liquidate some of its own investment at $t = 1$ to be able to pay each of its impatient depositor c_1. Each patient depositor in Bank A ends up receiving $\frac{1}{1-\pi}\big(1 - \bar{\sigma} - \pi c_1 + \frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}\big)R$.

When $\varepsilon > (1 - \pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big]$, Bank B fails to satisfy the incentive compatibility constraint and goes into bankruptcy. In this case, the unanticipated shock is large enough such that Bank B's bankruptcy causes Bank A to fail to satisfy the incentive compatibility constraint and has to go into bankruptcy as well. This is the contagion effect that causes Bank A, which is not hit by the unanticipated shock, to fail merely because of its connection to Bank B. After liquidation, each depositor in Bank B receives $\frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}}$ and Bank A receives $\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}$ from Bank B. Since Bank A also goes into bankruptcy, each depositor in bank A receives $1 - \bar{\sigma} + \frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}$ after Bank A's liquidation.

The analyses of the parameter combinations that violate (2.9), or (2.11) can be found in section B.1 of the Appendix.

2.4 Fiscal Authority

This section adds the role of fiscal authority into the economy. For simplicity, assume that each depositor is endowed with y units of consumption good and the utility function u satisfies $u'(\lambda c_1) = Ru'(\lambda c_2)$ for any $\lambda > 0$.[9]

The fiscal authority collects an exogenous lump-sum tax $\tau \in [0, y]$ from the

[9]The CRRA utility function satisfies this property.

endowments of the depositors. Each depositor ends up with after-tax income of $y_d = y - \tau$. The tax revenue is used by the fiscal authority to invest in the technology to provide public good g at $t = 2$. Each depositor derives utility from public good. The utility is described by $Dv(g)$, where $D > 0$ is the measure of the value of the public good provision to the depositors. Also, assume that $v' > 0$ and $v'' < 0$.

2.4.1 Financial Crisis and Optimal Bailout

During a financial crisis, an unexpected shock hits one of the two banks in the economy. The fiscal authority does not have additional channels to raise more taxes after a financial crisis hits. However, it could potentially be welfare improving for the fiscal authority to divert some of the tax revenue to fund the bailout for the bank hit by the unanticipated shock. The fiscal authority chooses the fraction of distressed assets to bailout $b \in [0, \varepsilon]$. The fiscal authority then transfers $y_d b$ of its investment to the bank hit by the unexpected shock.

Notice that ε is the fraction of assets that becomes worthless after the unanticipated shock hits a bank and b is the fraction of assets being transferred by the fiscal authority from tax revenue to the bank hit by the shock. The function $c_t^i(\varepsilon)$ for $t = 1, 2$ and $i = A, B$ from (2.12) shows the withdrawals of the patient and impatient depositors in Banks A and B respectively for each unit of consumption good deposited in $t = 0$. As long as $u'(\lambda c_1) = Ru'(\lambda c_2)$ holds, the withdrawals for the depositors can simply be scaled by $y_d c_t^i(\varepsilon)$ if the depositors deposit y_d units of consumption good at $t = 0$.

Once the fiscal authority agrees to compensate b fraction of assets to the bank hit by the unanticipated shock, the withdrawals of the depositors is simply $y_d c_t^i(\varepsilon - b)$. To save b fraction of distress assets, the fiscal authority has to transfer $y_d b$ units of its investment in the technology to the bank hit by the shock. That leaves $(2\tau - y_d b)R$ units of consumption good going into public goods at $t = 2$.

The fiscal authority chooses the fraction of distressed assets b to compensate the bank hit by the unanticipated shock to maximize the overall welfare of the economy. The overall welfare is the total utility of the depositors in Banks A and B from withdrawals and public good consumption. The fiscal authority's problem is the following:

$$\max_{b\in[0,\varepsilon]} \quad \pi\big[u(y_d c_1^A(\varepsilon - b)) + u(y_d c_1^B(\varepsilon - b)\big] \\ + (1-\pi)\big[u(y_d c_2^A(\varepsilon - b)) + u(y_d c_2^B(\varepsilon - b))\big] + 2Dv((2\tau - y_d b)R). \tag{2.13}$$

The welfare function in (2.13) shows the trade-off faced by the fiscal authority between subsidizing private withdrawals and providing public goods. The withdrawals of the depositors for any given bailout level $b \in [0, \varepsilon]$ in State S_3 and S_4 are described in (2.5) and (2.12) respectively.

In State S_3, there is no contagion since both the anticipated and unanticipated shocks hit Bank A. In addition to reallocating resources between public good and private withdrawals, the bailout also serves as a way to avoid costly bankruptcy. Depending on the parameter D, it could be optimal for the fiscal authority to provide Bank A with a full bailout $b = \varepsilon$, a zero bailout $b = 0$, or a partial bailout $b \in (0, \varepsilon)$. The interesting case is when a partial bailout is optimal. In particular, it is often optimal for the fiscal authority to provide the amount of bailout that is just enough to prevent Bank A to go into bankruptcy.

In State S_4, there is a potential contagion effect from Bank B to bank A. In addition to the two functions that bailouts serve in State S_3, bailouts also serve to prevent financial contagion. Depending on the parameter D, it could be optimal for the fiscal authority to provide the amount of bailout that is just enough to prevent the contagion effect from Bank B to Bank A. With a measure of smaller values of D, it could also be optimal for the fiscal authority to provide the amount of bailout that is enough to prevent the contagion effect *and* Bank B from bankruptcy. Further details on the optimal bailout amount is discussed with the numerical examples in the next section.

2.4.2 Numerical Examples

This section shows a simple numerical example to visualize the optimal bailout policy. The examples that follow assume CRRA utility functions for the utilities of the depositors from withdrawals and public good consumptions:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$
$$v(g) = \frac{g^{1-\gamma}}{1-\gamma}.$$

State $\mathbf{S_3}$:

In State S_3, Bank A is hit by both the anticipated and unanticipated shocks. There is no spillover effect in this state since Bank B is perfectly healthy. Depending on the size of the unanticipated shock, Bank A might or might not go into bankruptcy. If the unanticipated shock is sufficiently large such that Bank A goes bankrupt, on top of allowing for redistribution of resources between public

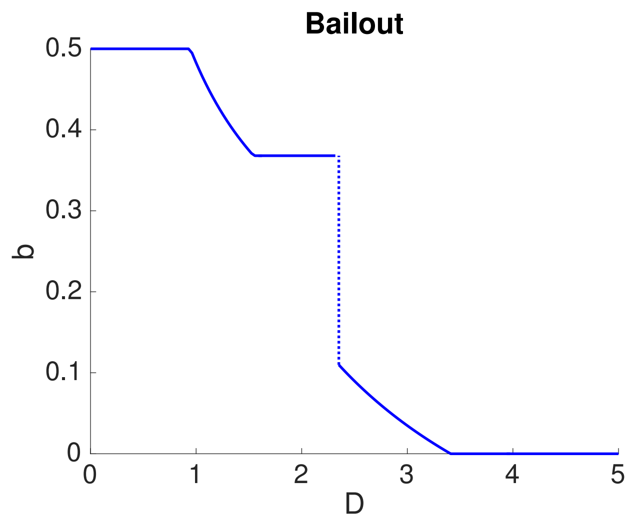

Figure 2.1: Optimal bailout in State S_3

and private consumption, the bailout also serves to avoid costly bankruptcy.

The following parameter values are used:

$$R = 1.5 \qquad \gamma = 1.01 \qquad y = 3 \qquad \pi = 0.5 \qquad \bar{\sigma} = 0.4 \qquad \varepsilon = 0.5 \qquad \tau = 1.5.$$

The parameter values are chosen such that the numerical example shows all the qualitative cases in state S_3. Figure 2.1 shows the plot of the optimal bailout against the value of public good D. Notice that the optimal bailout level is non-increasing in D, which is not a surprising result.

When D is sufficiently small, the value of public good consumption is low. It is optimal for the fiscal authority to provide a full bailout to Bank A since the cost of bailout is low. The cost of bailout increases as D increases. For an intermediate range of D, it is optimal for the fiscal authority to offer a partial bailout to Bank A, or $b \in (0, \varepsilon)$. Notice that there is a horizontal portion and a discontinuity in the plot in Figure 2.1. The horizontal portion of the plot shows the amount of bailout that is just sufficient to prevent Bank A from going into bankruptcy. Since avoiding bankruptcy gives additional welfare to the economy, there is a discountinuity in the welfare function with respect to bailout b. That explains the horizontal portion of the plot such that as D increases, the optimal bailout stays at the same level because of the additional welfare from bankruptcy prevention. However, as D keeps increasing, there is a point in which the cost of bailout overshadows the welfare improvement from bankruptcy prevention. The bailout amount jumps down to a lower level at this point which creates the discontinuity in the plot of bailout b against the value of public consumption D.

This example explains the motivation behind partial bailouts provided by the fiscal authority. In addition to having the advantage of a redistribution of resources between private and public consumption, there is also an additional incentive from bankruptcy prevention. The flat portion of Figure 2.1 highlights the additional motivation such that even as the cost of bailout increases, the optimal level of bailout remains constant because the additional cost is offset by the additional benefit from bankruptcy prevention.

State S_4:

In State S_4, Bank A is hit by the anticipated shock and Bank B by the unanticipated shock. There is a potential spillover effect from Bank B to Bank A for a sufficiently large unanticipated shock ε. In this case, the bailout has an additional benefit, which is spillover prevention. By bailing out Bank B, the fiscal authority can potentially prevent the negative spillover that causes Bank A to go into bankruptcy even when Bank A is not hit by the unanticipated shock.

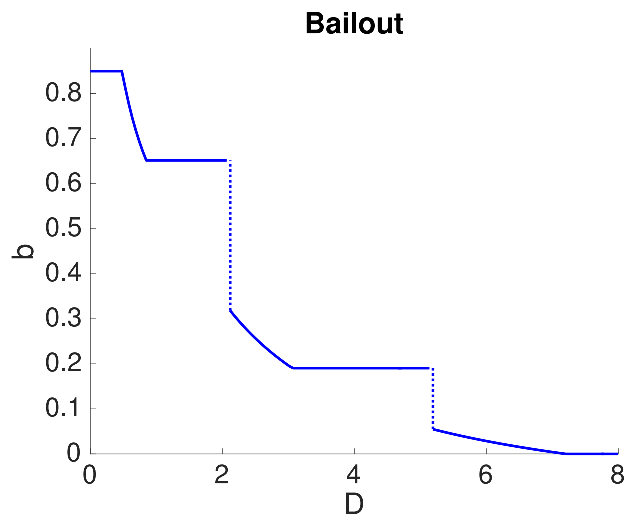

Figure 2.2: Optimal bailout in State S_4

This example uses the following parameter values:

$$R = 1.5 \qquad \gamma = 1.01 \qquad y = 3 \qquad \pi = 0.5 \qquad \bar{\sigma} = 0.4 \qquad \varepsilon = 0.85 \qquad \tau = 1.5.$$

The parameter values are chosen such that the example covers all the qualitative cases in State S_4. Notice that there are two flat portions in the plot of optimal bailout against D in Figure 2.2.

When D is sufficiently small, a full bailout $b = \varepsilon$ is optimal; when D is sufficiently large, a zero bailout $b = 0$ is optimal. The first flat portion of the plot in Figure 2.2 (around $D = 2$) is generated by the additional benefit of bailout from spillover prevention. This is the amount of bailout such that it is exactly sufficiently to prevent Bank A from going bankrupt because of the spillover effect from Bank B. This amount of bailout provides just enough resources to Bank B such that Bank B's payment to Bank A is large enough for Bank A to remain solvent. Notice that there is a discontinuity at the right end of this flat portion of the plot. This is when the benefit from spillover prevention is overshadowed by the increasing cost of bailout as D increases.

The second flat portion of the plot in Figure 2.2 (around $D = 4$) is generated by the additional incentive of the fiscal authority to bailout to prevent Bank B from going into bankruptcy. The explanation behind this segment is similar to State S_3.

This example shows that a partial bailout policy by the fiscal authority can be motivated by spillover prevention. This example also strengthens the argument against the reluctance of the Fed to bailout Lehman Brothers in the 2008 crisis. It was believed that the failure of Lehman Brothers has caused severe negative spillovers to the financial sector and that the financial crisis could have been prevented if the Fed were to bailout Lehman Brothers in the early stage of the crisis.

2.5 Concluding Remarks

This paper intends to highlight the different motivations behind bailouts in an interbank network: the redistribution of resources between private and public consumption, bankruptcy prevention, and spillover prevention. It also stresses the significance of determining the optimal bailout levels to financial institutions based on the goals or motivations behind the bailout payments.

In the 2008 financial crisis, the failure of Lehman Brothers had led to the failures of multiple financial institutions that were closely linked to Lehman. The decision not to bailout Lehman Brothers has been heavily critisized based on the argument that the financial crisis might have been prevented if the government had decided to save Lehman Brothers from failing. As suggested by this model, the motivation to bailout Lehman Brothers could come from the prevention of the institution to go into costly bankruptcy, or the prevention of negative spillovers to other financial institutions. The decision not to bailout Lehman Brothers could be induced by the failure to acknowledge the latter motivation.

The design of this model remains simple to highlight the different motivations behind partial bailouts. Due to the simplicity, the potential moral hazard issues from bailouts is ommitted. It does not allow for the exploitation of bailouts by banks, nor does it allow for the exploitation of banks on the potential bailouts to be received by other banks, as in Eisert and Eufinger (2016). This leaves rooms for potential future works.

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APPENDIX A

APPENDIX FOR CHAPTER 1

A.1 The Post-deposit Game of the Constrained Efficient Economy

The action set of each depositor in the post-deposit game is $\{E, L\}$, where E (L) stands for withdrawing early (late). An impatient depositor always chooses E. Thus, the relevant strategy for a depositor is (E, E) and (L, E), where the first (second) element in the parenthesis stands for the action chosen by the depositor when she is patient (impatient). The expected utilities can be summarized in the payoff matrix below.

	(E, E)	(L, E)
(E, E)	T_1, T_1	T_3, T_2
(L, E)	T_2, T_3	T_4, T_4

where

$$\begin{aligned}
T_1 &= (1-p)\frac{v(c) + v(2(1-\tau)y - c + B)}{2} + p\frac{u(c) + u(2(1-\tau)y - c + B)}{2} \\
T_2 &= (1-p)v((2(1-\tau)y - c)R) + p\frac{u(c) + u(2(1-\tau)y - c + B)}{2} \\
T_3 &= (1-p)\left[(1-p)v(c) + p\frac{v(c) + v(2(1-\tau)y - c + B)}{2}\right] \\
&\quad + p\left[(1-p)u(c) + p\frac{u(c) + u(2(1-\tau)y - c + B)}{2}\right] \\
T_4 &= (1-p)\left[(1-p)v((1-\tau)yR) + pv((2(1-\tau)y - c)R)\right] \\
&\quad + p\left[(1-p)u(c) + p\frac{u(c) + u(2(1-\tau)y - c + B)}{2}\right]
\end{aligned} \tag{A.1}$$

T_1 is the ex-ante expected utility of a depositor for playing (E, E) when the other depositor plays (E, E). With probability $(1 - p)$, the depositor is patient and her utility is given by v. With probability p, the depositor is impatient and her utility is given by u. In both cases, the depositor can either be the first or second in the line with equal probability 1/2. The depositor receives c if she is the first in the line. If she is the second in the line, she receives the leftover resources $2(1-\tau)y-c$. Since both depositors withdraw early, the depositor also receives the bailout payment B. The total withdrawal of the depositor if she is the second in the line is $2(1-\tau)y - c + B$. The derivation of T_2, T_3 and T_4 can be done in similar fashion.

If $T_2 \geq T_1$ and $T_4 \geq T_3$, (L, E) is the dominant strategy for both depositors. It is assumed that a patient depositor always chooses L if she is indifferent between choosing E and L. The only pure strategy Nash equilibrium is that both depositors play (L, E), which constitutes a non-run equilibrium. A patient depositor withdraws late regardless of the action of the other depositor.

If $T_2 < T_1$ and $T_4 \geq T_3$, there are two pure strategy Nash equilibria, either both depositors play (E, E) which gives a run equilibrium or both play (L, E) which gives a non-run equilibrium. The two depositors' actions exhibit strategic complementarity where a patient depositor withdraws early if and only if the other depositor also withdraws early.

If $T_3 > T_4$, the only pure strategy Nash equilibrium is that both depositors play (E, E). The depositors would not have deposited in the bank because the ex-ante expected utility from autarky is higher. It is possible to have $T_2 \geq T_1$ and $T_3 > T_4$. Then, the two pure strategy equilibria are $(L, E), (E, E)$ and $(E, E), (L, E)$. The parameter values are assumed to satisfy (1.1) such that this case never happen since $T_3 > T_4$ implies $T_1 > T_2$.

A.2 Parameter Restriction 1

Lemma 14. *There exists $\bar{\tau}$ such that $\frac{d\widetilde{W}(\widetilde{c},\widetilde{B};s)}{ds} < 0$ for $s \in (0,1)$ and $\tau \leq \bar{\tau}$.*

Notice that $\widehat{W}(c,B) = \widetilde{W}(c,B;0)$. In order for the best panic-based run-proof allocation to always dominate the best panic-based run-permitting allocation, it is sufficient to ensure that $\widetilde{W}(\widetilde{c},\widetilde{B};s)$ is strictly decreasing in s. This happens when the tax rate is low, $\tau \leq \bar{\tau}$.

Higher run probability s could potentially increase the best panic-based run-permitting welfare if tax is high, $\tau > \bar{\tau}$. When $\tau > \bar{\tau}$, it is more desirable to transfer resources from public good provision to private consumption. The only way the planner could do that is through bailouts. However, bailout payments are only made when both depositors withdraw in period 1. When s is high, the probability of having two depositors withdrawing in period 1 is high; the probability of the ability to transfer resources from public good provision to private consumption is also high. Therefore, the best panic-based run-permitting allocation could dominate the best panic-based run-proof allocation.[1]

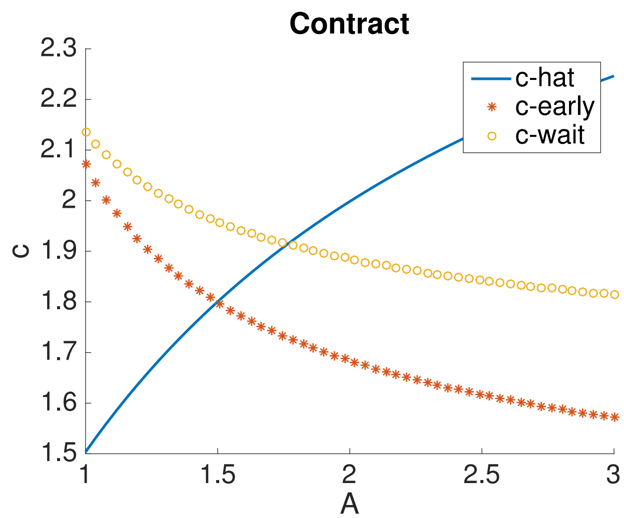

Figure A.1: Unconstrained Efficient Contract

A.3 Constrained Efficient Allocation Numerical Examples

The following parameter values are used for the numerical examples:

$$R = 1.5 \qquad \gamma = 1.01 \qquad D = 1 \qquad y = 3 \qquad p = 0.5 \qquad \tau = 0.5 \tag{A.2}$$

According to Lemma 17, the value of A determines which of the three cases the economy is in. Figure A.1 shows the plot of $\widehat{c}$ together with $\bar{c}^{early}(\widehat{B})$ and $\bar{c}^{wait}(\widehat{B})$. It can be seen from the diagram the range of values of A that corresponds to each of the three cases. Based on the parameter values in (A.2), $A^{early} = 1.49808$ and $A^{wait} = 1.76375$.

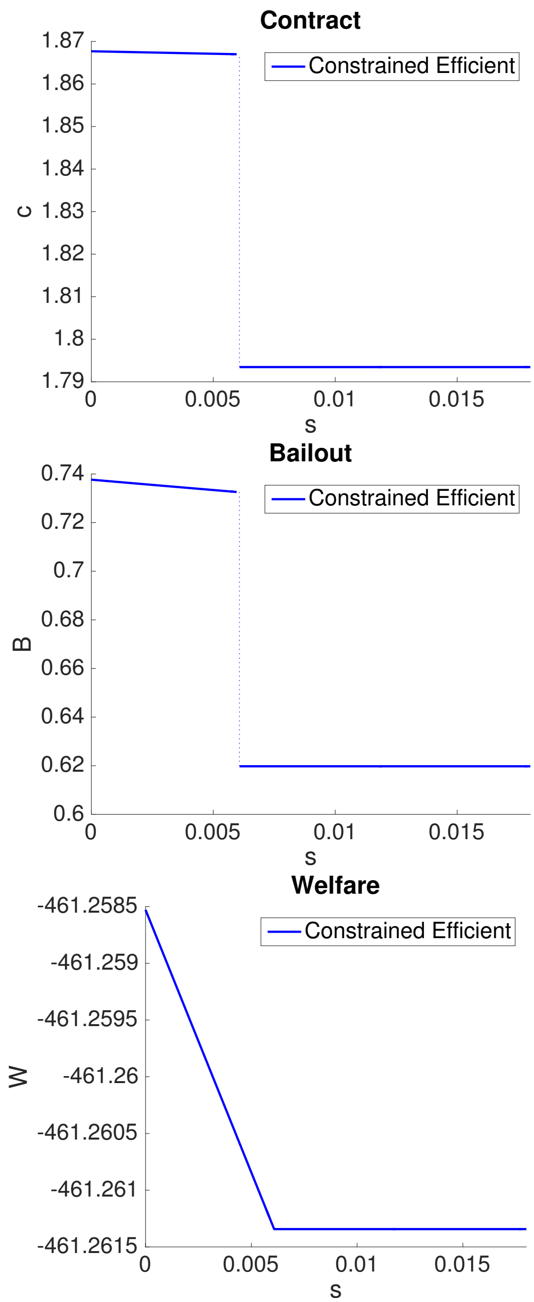

Figure A.2: Constrained Efficient Outcome (Case 2)

A.3.1 Case 1: $A \leq A^{early}$

Set $A = 1.4 \leq 1.49808 = A^{early}$. The unconstrained efficient allocation is always implementable in the constrained efficient economy. The constrained efficient allocation and welfare is given by

$$c^* = 1.75097 \qquad B^* = 0.50421 \qquad W^* = -436.40663 \tag{A.3}$$

and are independent of s.

A.3.2 Case 2: $A^{early} < A \leq A^{wait}$

Set $A = 1.65 \in (A^{early}, A^{wait}]$. Figure A.2 shows the constrained efficient deposit contract and bailout policy. The run probability at which the constrained efficient allocation jumps from panic-based run-permitting to panic-based run-proof is $s_0 = 0.0060746$.

The constrained efficient welfare function is the envelope of two functions: the best panic-based run-proof welfare $\widehat{W}(c^{early}, B^{early})$, which is constant in s, and the best panic-based run-permitting welfare $\widetilde{W}(\widetilde{c}, \widetilde{B}; s)$, which is strictly decreasing in s. Welfare is continuous at s_0 but the optimal contract and bailout are discontinuous. In order for the planner to switch from panic-based run-permitting to panic-based run-proof at s_0, both the contract and the bailout have to be significantly lower to satisfy the DSIC constraint ($c \leq c^{early}(B)$).

[1]It would be interesting to explore the problem when $\tau > \bar{\tau}$. However, the focus is on the more intuitive case where higher run probability always gives a lower welfare under the best panic-based run-permitting allocation.

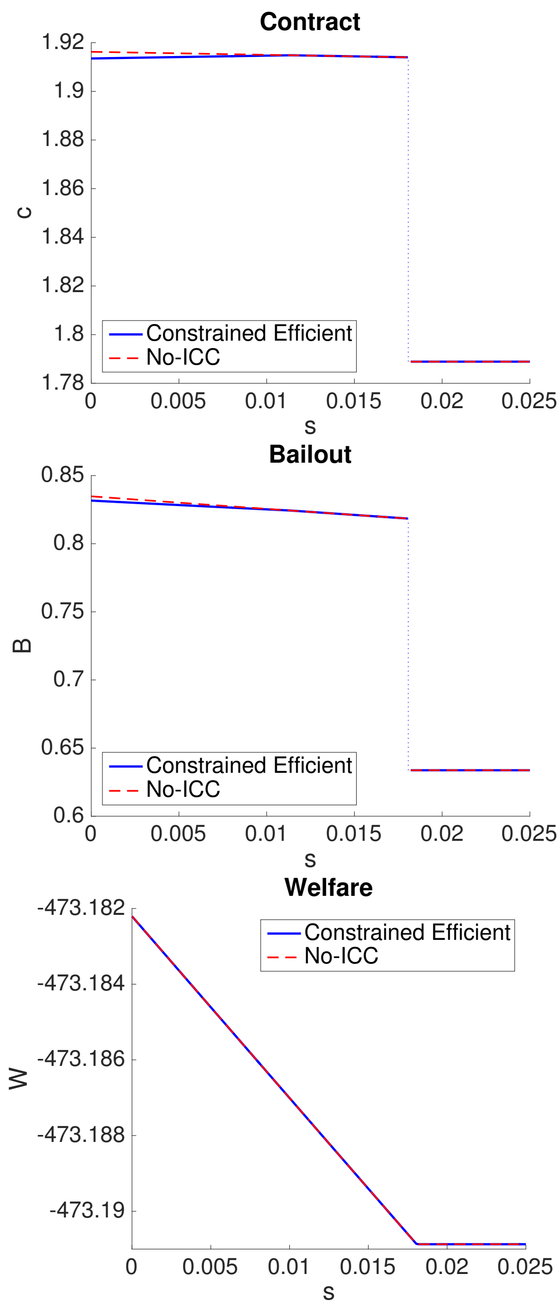

Figure A.3: Constrained Efficient Outcome (Case 3)

A.3.3 Case 3: $A > A^{wait}$

Two examples are presented for Case 3: One with $s_2 < s_1$ where ICC is only binding for a fraction of the measure of s at which the constrained efficient allocation is panic-based run-permitting, and another with $s_1 = s_2$ where ICC is binding for the entire measure.

When $A = 1.77$, the cutoff probabilities are $s_2 = 0.011386$ and $s_1 = 0.018069$. The plots of the constrained efficient allocation and welfare against s are shown in Figure A.3. The red dashed lines show the allocation in the hypothetical case in which there is no ICC. For $s < s_1$, the constrained efficient allocation is panic-based run-permitting. For a subset of this measure, $s \leq s_2$, the ICC is binding. The planner could have achieved a higher welfare by choosing higher c and B, as shown by the dashed lines. However, the planner has to adjust to lower c and B so that the ICC is satisfied.

When $A = 2$, the cutoff probabilities are $s_1 = s_2 = 0.046827$. The ICC binds for the entire measure of s in which the constrained efficient allocation is panic-based run-permitting. Figure A.4 shows the constrained efficient allocation when $A = 2$. Again, the red dashed lines represent the constrained efficient allocation if there was no ICC. For $s < s_1 = s_2 = 0.046827$, the constrained efficient allocation is panic-based run-permitting and the ICC is binding. Similarly, the planner could have achieved a higher welfare by increasing c and B to the levels shown by the dashed lines if there was no ICC. Notice that the constrained efficient allocation switches from panic-based run-permitting to panic-based run-proof at a higher cutoff probability in the case when there was no ICC. This is because the best panic-based run-permitting welfare is strictly higher when there was no ICC. Thus, the intersection between the best panic-based run-permitting welfare and the best panic-based run-proof welfare also occurs at a higher s.

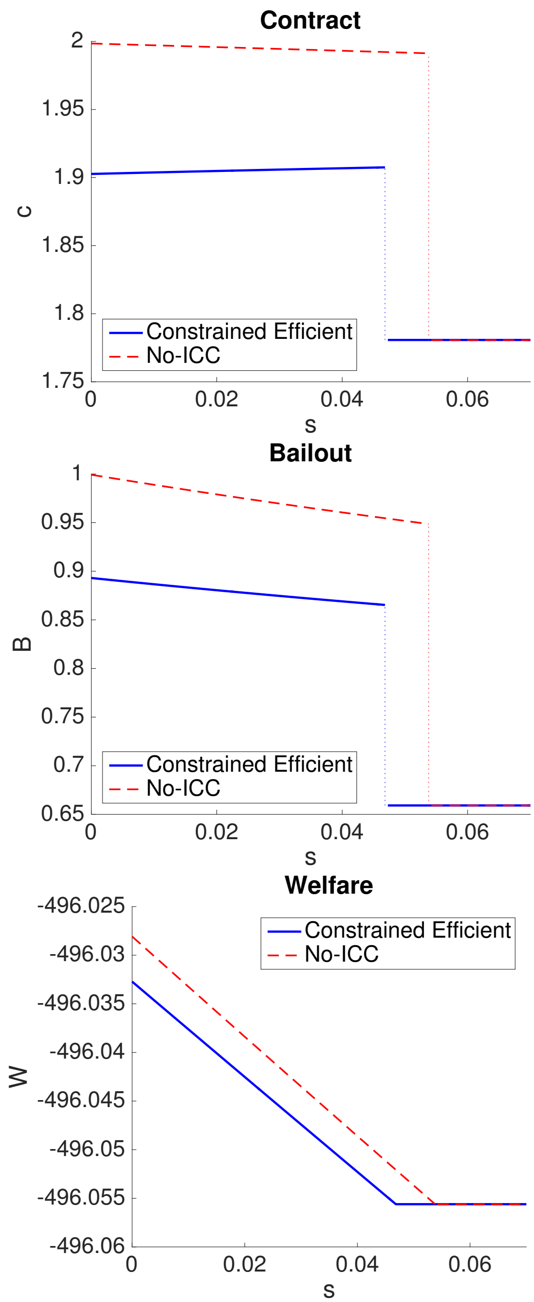

Figure A.4: Constrained Efficient Outcome (Case 3)

A.4 Fiscal Authority with Commitment Numerical Examples

A.4.1 Case 1: $A \leq A^{early}$

The same parameter values from (1.8) are used here. The allocation under the fiscal authority with commitment is identical to the constrained efficient allocation in (A.3) and is independent of s.

$$c_1^* = 1.75097 \qquad B_1^* = 0.50421 \qquad W_1^* = -436.40663$$

A.4.2 Case 3: $A > A^{wait}$

The numerical example for Case 2 is shown in Section 1.4.1. The plots of the equilibrium allocation in Case 3 are qualitatively identical to those in Case 2. Figure A.5 shows the equilibrium allocation and welfare under the fiscal authority with commitment and constrained efficient economy when $A = 2$. The same argument from Case 2 in Section 1.4.1 explains the wedge in welfare between the constrained efficient economy and the economy under the fiscal authority with commitment.

A.5 Fiscal Authority without Commitment Equilibrium Characterization

Proposition 15. *If $p \leq \sqrt{\frac{\gamma-1}{\gamma} K_{good}}$, there exists a threshold $\bar{A}_{sep}$ such that (1) when $A \leq \bar{A}_{sep}$, the equilibrium contract under the fiscal authority without commitment*

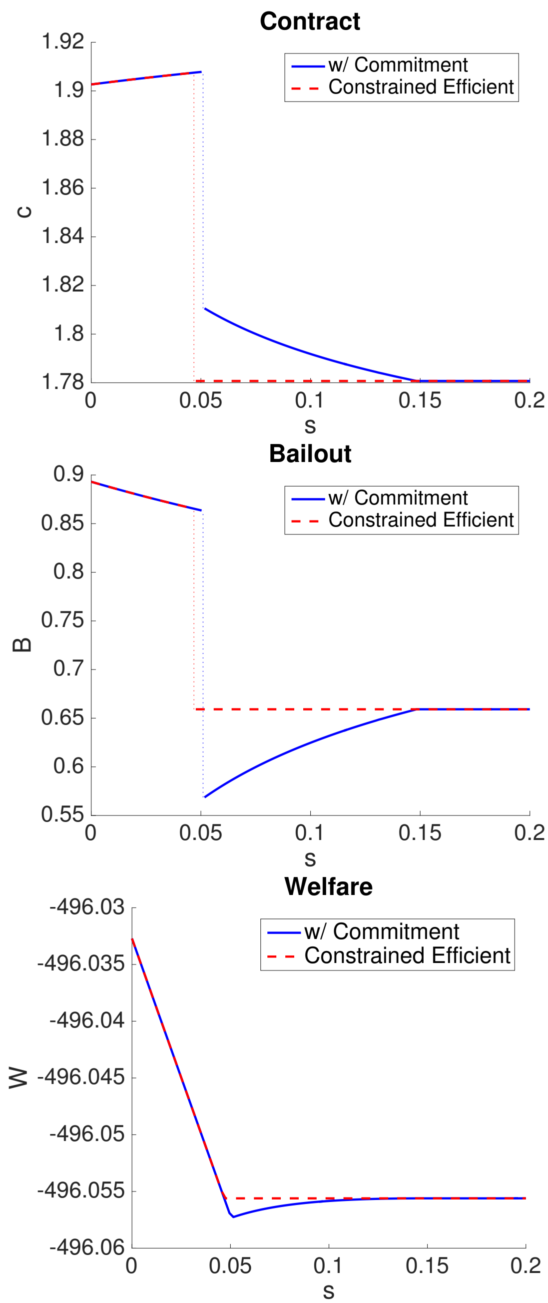

Figure A.5: Equilibrium under Fiscal Authority with Commitment (Case 3) *gives the "good" pooling equilibrium in the post-deposit game for $s \in [0,1]$ and (2) when $A > \bar{A}_{sep}$, the equilibrium contract gives the separating equilibrium in the post-deposit game if $s \in [0, s_3)$ and the "good" pooling equilibrium if $s \in [s_3, 1]$ for some $s_3 \in (0,1]$.*

When the impulse demand is low ($A \leq \bar{A}_{sep}$), the bank does not have the intention to offer high c to channel more resources to the early withdrawals. The bank chooses c that is sufficiently low such that the "good" pooling equilibrium exists in the post-deposit game. When the impulse demand is high ($A > \bar{A}_{sep}$), the bank has high incentive to increase c to channel more resources to the early withdrawals. When s is low ($s < s_3$), the cost of allowing the separating equilibrium in the post-deposit game is low. The bank chooses a high c and tolerates the separating equilibrium in the post-deposit game. As s increases ($s \geq s_3$), the cost of tolerating for the separating equilibrium in the post-deposit game increases and outweighs the benefit of choosing a high c. Therefore, the bank chooses a lower c such that there is the "good" pooling equilibrium in the post-deposit game.

A.6 Parameter Restriction 2

The first restriction is

$$p \leq \sqrt{\frac{\gamma - 1}{\gamma} K_{good}}$$

such that the result from Proposition 15 holds. Next, there exists $\bar{\tau}_1$ such that

$$\frac{d}{ds}\left(\max_{c \in \mathbb{R}_+} \widetilde{W}_{pri}(c, B^*_{sep}(c))\right) < 0$$

as long as $\tau \le \bar{\tau}$. The proof is similar to Lemma 14. To make sure that $c_2^* \notin (c_{good}, \underline{c}_{sep}]$, the only restriction is on A since $\widehat{c}_2$ is strictly increasing in A as shown in the Proof of Proposition 15. There exists A^{good} such that when $A \le A^{good}$, the bank's optimal contract is $c_2^* \in (0, c_{good}]$ which gives the "good" pooling equilibrium in the post-deposit game for $s \in [0,1]$. When $A > \bar{A}_{sep}$, $c_2^* \in (\bar{c}_{sep}, 2(1-\tau)y)$ for $s \in [0, s_3)$. These are the only combinations of A and s that are considered in the analysis.

A.7 Proof of Lemma 1

The inequality $T_2 \ge T_1$ can be written as

$$\frac{((2(1-\tau)y - c)R)^{1-\gamma}}{1-\gamma} - \frac{1}{2}\left[\frac{c^{1-\gamma}}{1-\gamma} + \frac{(2(1-\tau)y - c + B)^{1-\gamma}}{1-\gamma}\right] \ge 0 \tag{A.4}$$

The derivative of the LHS of (A.4) is

$$\begin{aligned}
\frac{\partial LHS}{\partial c} &= -(2(1-\tau)y - c)^{-\gamma}R^{1-\gamma} - \frac{1}{2}c^{-\gamma} + \frac{1}{2}(2(1-\tau)y - c + B)^{-\gamma} \\
&< -(2(1-\tau)y - c)^{-\gamma}R^{1-\gamma} - \frac{1}{2}c^{-\gamma} + \frac{1}{2}(2(1-\tau)y - c)^{-\gamma} \\
&= (2(1-\tau)y - c)^{-\gamma}\left(\frac{1}{2} - R^{1-\gamma}\right) - \frac{1}{2}c^{-\gamma}
\end{aligned}$$

A sufficient condition for the derivative to be negative is that $1/2 - R^{1-\gamma} < 0$, which is equivalent to $\gamma < 1 + \ln 2/\ln R$. The LHS of (A.4) is continuous in c for $c \in (0, 2(1-\tau)y)$. Also, the limit of LHS of (A.4) is $+\infty$ when $c \to 0$ and $-\infty$ when $c \to 2(1-\tau)y$. Therefore, there exists a unique $c^{early} \in (0, 2(1-\tau)y)$ such that $T_1 = T_2$. Also, $T_2 \ge T_1$ as long as $c \in [0, c^{early}]$.

A.8 Proof of Lemma 2

The inequality $T_4 \geq T_3$ can be written as

$$(1-p)\left[\frac{((1-\tau)yR)^{1-\gamma}}{1-\gamma} - \frac{c^{1-\gamma}}{1-\gamma}\right] + p\left[\frac{((2(1-\tau)y-c)R)^{1-\gamma}}{1-\gamma} - \frac{1}{2}\left(\frac{c^{1-\gamma}}{1-\gamma} + \frac{(2(1-\tau)y-c+B)^{1-\gamma}}{1-\gamma}\right)\right] \geq 0 \tag{A.5}$$

The limit of LHS of (A.5) is $+\infty$ when $c \to 0$ and $-\infty$ when $c \to 2(y-\tau)$. The LHS is continuous in c for $c \in (0, 2(y-\tau))$. Also,

$$\begin{aligned}\frac{\partial LHS}{\partial c} &= -(1-\frac{p}{2})c^{-\gamma} - p(2(1-\tau)y-c)^{-\gamma}R^{1-\gamma} + \frac{p}{2}(2(1-\tau)y-c+B)^{-\gamma} \\ &< -(1-\frac{p}{2})c^{-\gamma} - p(2(1-\tau)y-c)^{-\gamma}R^{1-\gamma} + \frac{p}{2}(2(1-\tau)y-c)^{-\gamma} \\ &= -(1-\frac{p}{2})c^{-\gamma} + p(2(1-\tau)y-c)^{-\gamma}\left(\frac{1}{2} - R^{1-\gamma}\right) < 0\end{aligned}$$

as long as $\gamma < 1 + \ln 2/\ln R$.

A.9 Proof of Lemma 3

First, find a sufficient condition for $c^{early}(0) < c^{wait}(0)$. When $B = 0$,

$$c^{early}(0) = \frac{2(y-\tau)}{(\frac{2}{R^{\gamma-1}} - 1)^{1/(\gamma-1)} + 1}$$

In order for $c^{early}(0) < c^{wait}(0)$, it should be true that $T_4 > T_3$ when $c = c^{early}(0)$. This condition can be simplified to

$$\frac{2/R}{(\frac{2}{R^{\gamma-1}} - 1)^{1/(\gamma-1)} + 1} < 1 \tag{A.6}$$

When $\gamma < 1 + \ln 2/\ln R$, $(\frac{2}{R^{\gamma-1}} - 1)^{1/(\gamma-1)}$ is decreasing in γ. Therefore, inequality (A.6) can be further simplified to

$$\gamma < 2$$

Therefore, as long as

$$\gamma < \min\{2, 1 + \ln 2/\ln R\} \tag{A.7}$$

the inequality $c^{early}(0) < c^{wait}(0)$ always holds.

Suppose there exists $B \in (0, 2\tau y]$ such that $c^{wait}(B) \le c^{early}(B)$. Since $c^{early}(B)$ and $c^{wait}(B)$ are continuous functions of B, and $c^{early}(0) < c^{wait}(0)$, there exists $\bar{B} \in (0, 2\tau y]$ such that

$$c^{wait}(\bar{B}) = c^{early}(\bar{B}) \equiv \bar{c}$$

and

$$\frac{\partial c^{wait}(\bar{B})}{\partial B} \le \frac{\partial c^{early}(\bar{B})}{\partial B} \tag{A.8}$$

The derivatives on both sides of the inequality on (A.8) can be obtained by taking implicit derivatives of equations $T_1 = T_2$ and $T_3 = T_4$. The derivatives evaluated at $\bar{c}$ and $\bar{B}$ are given by

$$\frac{\partial c^{early}(\bar{B})}{\partial B} = -\frac{\frac{1}{2}(2(1-\tau)y - \bar{c} + \bar{B})^{-\gamma}}{\frac{1}{2}\bar{c}^{-\gamma} + (2(1-\tau)y - \bar{c})^{-\gamma}R^{1-\gamma} - \frac{1}{2}(2(1-\tau)y - \bar{c} + \bar{B})^{-\gamma}}$$
$$\frac{\partial c^{wait}(\bar{B})}{\partial B} = -\frac{\frac{p}{2}(2(1-\tau)y - \bar{c} + \bar{B})^{-\gamma}}{(1-\frac{p}{2})\bar{c}^{-\gamma} + p(2(1-\tau)y - \bar{c})^{-\gamma}R^{1-\gamma} - \frac{p}{2}(2(1-\tau)y - \bar{c} + \bar{B})^{-\gamma}}$$

Inequality (A.8) evaluated at $\bar{c}$ and $\bar{B}$ can be simplified to

$$0 \ge \bar{c}^{-\gamma}$$

which is a contradiction. Therefore, $c^{early}(B) < c^{wait}(B)$ holds for any $B \in [0, 2\tau]$ as long as (A.7) holds.

A.10 Proof of Lemma 4

$(\widehat{c}, \widehat{B})$ solves the following first-order conditions:

$$\begin{aligned}&p^2\big[A\widehat{c}^{-\gamma} - A(2(1-\tau)y - \widehat{c} + \widehat{B})^{-\gamma}\big]\\&+2p(1-p)\big[A\widehat{c}^{-\gamma} - R^{1-\gamma}(2(1-\tau)y - \widehat{c})^{-\gamma}\big] = 0\end{aligned}$$

$$p^2\big[A(2(1-\tau)y - \widehat{c} + \widehat{B})^{-\gamma} - 2D(2\tau y - \widehat{B})^{-\gamma}\big] = 0$$

Simplifying yields:

$$\widehat{B} = 2\tau y - \frac{(\frac{2D}{A})^{1/\gamma}}{1 + (\frac{2D}{A})^{1/\gamma}}(2y - \widehat{c}) \equiv 2\tau y - K_1(2y - \widehat{c})$$

$$\begin{aligned}(p^2A + 2p(1-p)A)\widehat{c}^{-\gamma} =& p^2A((1-K_1)(2y-\widehat{c}))^{-\gamma}\\&+ 2p(1-p)R^{1-\gamma}(2(1-\tau)y - \widehat{c})^{-\gamma} \qquad \text{(A.9)}\end{aligned}$$

Taking the derivative of (A.9) with respect to A gives:

$$\frac{d\widehat{c}}{dA} = \frac{(p^2 + 2p(1-p))\widehat{c}^{-\gamma} - p^2((1-K_1)(2y-\widehat{c}))^{-\gamma}}{\gamma(p^2A + 2p(1-p)A)\widehat{c}^{-\gamma-1} + p^2A(1-K_1)^{-\gamma}(2y-\widehat{c})^{-\gamma-1} + 2p(1-p)R^{1-\gamma}(2(1-\tau)y-\widehat{c})^{-\gamma-1}} > 0$$

$c^{early}(\widehat{B})$ solves the following equation:

$$\frac{((2(1-\tau)y - c^{early}(\widehat{B}))R)^{1-\gamma}}{1-\gamma} = \frac{1}{2}\left[\frac{c^{early}(\widehat{B})^{1-\gamma}}{1-\gamma} + \frac{(1-K_1)(2y - c^{early}(\widehat{B})))^{1-\gamma}}{1-\gamma}\right]$$

Taking the derivative with respect to A gives:

$$\frac{dc^{early}(\widehat{B})}{dA} = \frac{\frac{1}{2}(1-K_1)^{-\gamma}(2y - c^{early}(\widehat{B}))^{1-\gamma}\frac{dK_1}{dA}}{\frac{1}{2}c^{early}(\widehat{B})^{-\gamma} + R^{1-\gamma}(2(1-\tau)y - c^{early}(\widehat{B}))^{-\gamma} - \frac{1}{2}(1-K_1)^{1-\gamma}(2y - c^{early}(\widehat{B}))^{-\gamma}}$$

Since

$$\begin{aligned}\frac{\partial K_1}{\partial A} &= \frac{-\frac{1}{\gamma}\frac{(2D)^{1/\gamma}}{A^{1/\gamma+1}}(1 + \big(\frac{2D}{A}\big)^{1/\gamma}) + \frac{1}{\gamma}\frac{(2D)^{1/\gamma}}{A^{1/\gamma+1}}\big(\frac{2D}{A}\big)^{1/\gamma}}{(1 + \big(\frac{2D}{A}\big)^{1/\gamma})^2}\\&= \frac{-\frac{1}{\gamma}\frac{(2D)^{1/\gamma}}{A^{1/\gamma+1}}}{(1 + \big(\frac{2D}{A}\big)^{1/\gamma})^2} < 0\end{aligned}$$

it is true that

$$\frac{dc^{early}(\widehat{B})}{dA} < 0$$

$c^{wait}(\widehat{B})$ solves the following equation:

$$\begin{aligned}&(1-p)\frac{((1-\tau)yR)^{1-\gamma}}{1-\gamma} + p\frac{((2(1-\tau)y - c^{wait}(\widehat{B}))R)^{1-\gamma}}{1-\gamma} \\ =&(1-p)\frac{c^{wait}(\widehat{B})^{1-\gamma}}{1-\gamma} + \frac{p}{2}\left[\frac{c^{wait}(\widehat{B})^{1-\gamma}}{1-\gamma} + \frac{((1-K_1)(2y - c^{wait}(\widehat{B})))^{1-\gamma}}{1-\gamma}\right]\end{aligned}$$

Taking the derivative with respect to A gives:

$$\frac{dc^{wait}(\widehat{B})}{dA} = \frac{\frac{p}{2}(1-K_1)^{-\gamma}(2y - c^{wait}(\widehat{B}))^{1-\gamma}\frac{dK_1}{dA}}{\begin{aligned}(1-\tfrac{p}{2})c^{wait}(\widehat{B})^{-\gamma} + pR^{1-\gamma}(2(1-\tau)y - c^{wait}(\widehat{B}))^{-\gamma} \\ - \tfrac{p}{2}(1-K_1)^{1-\gamma}(2y - c^{wait}(\widehat{B}))^{-\gamma}\end{aligned}} < 0$$

Since $\widehat{c}$ is strictly increasing in A and $c^{early}(\widehat{B})$ and $c^{wait}(\widehat{B})$ are strictly decreasing in A, there exist cutoffs A^{early} and A^{wait}. Also, since $c^{wait}(\widehat{B}) > c^{early}(\widehat{B})$ it is also true that $A^{wait} > A^{early}$.

A.11 Proof of Proposition 5

Case 2 is when the unconstrained efficient allocation $(\widehat{c}, \widehat{B})$ is BIC but not DSIC. It is trivial that $(\widetilde{c}, \widetilde{B}) = (\widehat{c}, \widehat{B}) \in S^{wait} \backslash S^{early}$ when $s = 0$. Therefore, the constrained efficient allocation is

$$(c^*, B^*) = (\widetilde{c}, \widetilde{B})$$

for $s = 0$. The run-permitting welfare level evaluated at $(\widetilde{c}, \widetilde{B})$ can be written as

$$\widetilde{W}(\widetilde{c}, \widetilde{B}) = (1-s)\widehat{W}(\widetilde{c}, \widetilde{B}) + sW^{run}(\widetilde{c}, \widetilde{B})$$

Since tax is assumed to satisfy $\tau \leq \bar{\tau}$, it is true that

$$\frac{\widetilde{W}(\widetilde{c}, \widetilde{B})}{ds} < 0$$

for $s \in (0,1)$. It is also easily verified that $\widetilde{W}(\widetilde{c}, \widetilde{B})$ is continuous in s for $s \in [0,1]$.

Notice that when $s = 1$, the panic-based run-permitting welfare is $\widetilde{W}(\widetilde{\tau}, \widetilde{c}, \widetilde{B}) = W^{run}(\widetilde{\tau}, \widetilde{c}, \widetilde{B})$. If $W^{run}(\widetilde{\tau}, \widetilde{c}, \widetilde{B}) < \widehat{W}(\tau^{early}, c^{early}, B^{early})$, there exists $s_w \in (0,1)$ such that

$$\widetilde{W}(\widetilde{\tau}, \widetilde{c}, \widetilde{B}) = (1 - s_w)\widehat{W}(\widetilde{\tau}, \widetilde{c}, \widetilde{B}) + s_w W^{run}(\widetilde{\tau}, \widetilde{c}, \widetilde{B}) = \widehat{W}(\tau^{early}, c^{early}, B^{early}) \tag{A.10}$$

Since $\widetilde{W}(\widetilde{c}, \widetilde{B})$ is continuous and strictly decreasing, the following relations hold:

$$\begin{aligned} \widetilde{W}(\widetilde{c}, \widetilde{B}) &> \widehat{W}(c^{early}, B^{early}) \quad \text{if} \quad s < s_w \\ \widetilde{W}(\widetilde{c}, \widetilde{B}) &< \widehat{W}(c^{early}, B^{early}) \quad \text{if} \quad s > s_w \\ \widetilde{W}(\widetilde{c}, \widetilde{B}) &= \widehat{W}(c^{early}, B^{early}) \quad \text{if} \quad s = s_w \end{aligned}$$

Next is to check if $(\widetilde{c}, \widetilde{B})$ is implementable. Since $\frac{d\widetilde{c}}{ds} < 0$ for $s \in (0,1)$, all is left to prove is that $\frac{dc^{wait}(\widetilde{B})}{ds} > 0$ and $\frac{dc^{early}(\widetilde{B})}{ds} > 0$. Then, there exists $s_c \in (0,1]$ such that $\widetilde{c} > c^{early}(\widetilde{B})$ for $s < s_c$ and $\widetilde{c} \le c^{wait}(\widetilde{B})$ for all $s \in [0,1]$.

Pick $s_0 = \min\{s_w, s_c\}$. Then, when $s < s_0$, the constrained efficient allocation is panic-based run-permitting $(c^*, B^*) = (\widetilde{c}, \widetilde{B})$ and when $s \ge s_0$, the constrained efficient allocation is panic-based run-proof $(c^*, B^*) = (c^{early}, B^{early})$. [2]

If $W^{run}(\widetilde{\tau}, \widetilde{c}, \widetilde{B}) \ge \widehat{W}(\tau^{early}, c^{early}, B^{early})$, there does not exists $s_0 \in (0,1)$ such that (A.10) holds. Therefore, $s_0 = 1$.

Next is to prove that $\frac{dc^{wait}(\widetilde{B})}{ds} > 0$ and $\frac{dc^{early}(\widetilde{B})}{ds} > 0$. Taking the derivative of

[2]Notice that even though when $s = s_0$ the best panic-based run-permitting allocation $(\widetilde{c}, \widetilde{B})$ and the best panic-based run-proof allocation $(\tau^{early}, c^{early}, B^{early})$ give the same level of welfare, it is assumed that the panic-based run-proof option is chosen.

$\widetilde{B}$ with respect to s:

$$\frac{d\widetilde{B}}{ds} = -\frac{dK_2}{ds}(2y - \widetilde{c}) + K_2\frac{d\widetilde{c}}{ds} \tag{A.11}$$

where

$$K_2 = \frac{K_1}{1+K_1}$$
$$K_1 = \left[\frac{\big[(1-s)p^2+s\big]2D}{(1-s)p^2A + s\big[p^2A + p(1-p)(A+1) + (1-p)^2\big]}\right]^{1/\gamma}$$

Taking the derivative of K_1 with respect to s:

$$\frac{dK_1}{ds} = \frac{1}{\gamma}\left[\frac{\big[(1-s)p^2+s\big]2D}{\big[(1-s)p^2A + s\big(p^2A + p(1-p)(A+1) + (1-p)^2\big)\big]^3}\right]^{1/\gamma-1}$$
$$\times\Big[2D(1-p^2)\big[(1-s)p^2A + s\big(p^2A + p(1-p)(A+1) + (1-p)^2\big)\big]$$
$$-2Ds\big[p(1-p)(A+1)+(1-p)^2\big]\big[(1-s)p^2+s\big]\Big]$$

$$sign\left\{\frac{dK_1}{ds}\right\} = sign\Big\{(1-p^2)\big[p^2A + s\big(p(1-p)(A+1)+(1-p)^2\big)\big]$$
$$-\big[p(1-p)(A+1)+(1-p)^2\big]\big[(1-s)p^2+s\big]\Big\}$$
$$= sign\Big\{(1-p^2)p^2A$$
$$+\big(p(1-p)(A+1)+(1-p)^2\big)\big[(1-p^2)s-(1-s)p^2-s\big]\Big\}$$
$$= sign\Big\{(2p(1-p)+(1-p)^2)p^2A - p^2\big(p(1-p)(A+1)+(1-p)^2\big)\Big\}$$
$$= sign\Big\{2p(1-p)A + (1-p)^2A - p(1-p)(A+1) - (1-p)^2\Big\}$$
$$= sign\Big\{p(1-p)(A-1) + (1-p)^2(A-1)\Big\} > 0$$

Therefore, it is also true that

$$\frac{dK_2}{ds} > 0$$

$c^{early}(\widetilde{B})$ solves the following equation:

$$\frac{((2(1-\tau)y - c^{early}(\widetilde{B}))R)^{1-\gamma}}{1-\gamma} = \frac{1}{2}\left[\frac{c^{early}(\widetilde{B})^{1-\gamma}}{1-\gamma} + \frac{(2(1-\tau)y - c^{early}(\widetilde{B}) + \widetilde{B})^{1-\gamma}}{1-\gamma}\right]$$

Taking the derivative with respect to s gives:

$$\frac{dc^{early}(\widetilde{B})}{ds} = -\frac{(2(1-\tau)y - c^{early}(\widetilde{B}) + \widetilde{B})^{-\gamma}\frac{d\widetilde{B}}{ds}}{\frac{1}{2}c^{early}(\widetilde{B})^{-\gamma} - \frac{1}{2}(2(1-\tau)y - c^{early}(\widetilde{B}) + \widetilde{B})^{-\gamma} + R^{1-\gamma}(2(1-\tau)y - c^{early}(\widetilde{B}))^{-\gamma}}$$

$c^{wait}(\widetilde{B})$ solves the following equation:

$$\begin{aligned}
&(1-p)\frac{((1-\tau)yR)^{1-\gamma}}{1-\gamma} + p\frac{((2(1-\tau)y - c^{wait}(\widetilde{B}))R)^{1-\gamma}}{1-\gamma} \\
=&(1-p)\frac{c^{wait}(\widetilde{B})^{1-\gamma}}{1-\gamma} + \frac{p}{2}\left[\frac{c^{wait}(\widetilde{B})^{1-\gamma}}{1-\gamma} + \frac{(2(1-\tau)y - c^{wait}(\widetilde{B}) + \widetilde{B})^{1-\gamma}}{1-\gamma}\right]
\end{aligned}$$

Taking the derivative with respect to s gives:

$$\frac{dc^{wait}(\widetilde{B})}{ds} = -\frac{\frac{p}{2}(2(1-\tau)y - c^{wait}(\widetilde{B}) + \widetilde{B})^{-\gamma}\frac{d\widetilde{B}}{ds}}{(1-\frac{p}{2})c^{wait}(\widetilde{B})^{-\gamma} - \frac{p}{2}(2(1-\tau)y - c^{wait}(\widetilde{B}) + \widetilde{B})^{-\gamma} + pR^{1-\gamma}(2(y-\tau) - c^{wait}(\widetilde{B}))^{-\gamma}}$$

Since $\frac{d\widetilde{c}}{ds} < 0$, it is also true that $\frac{d\widetilde{B}}{ds} < 0$, $\frac{dc^{early}(\widetilde{B})}{ds} > 0$ and $\frac{dc^{wait}(\widetilde{B})}{ds} > 0$.

A.12 Proof of Proposition 6

In Case 3, the unconstrained efficient allocation $(\widehat{c}, \widehat{B})$ is neither DSIC nor BIC. This means that the best panic-based run-permitting allocation when $s = 0$ is (c^{wait}, B^{wait}) instead of $(\widetilde{c}, \widetilde{B})$.

Define $\bar{s}_3 \in (0, 1]$ such that $\widetilde{W}(\widetilde{c}, \widetilde{B}) \leq \widehat{W}(c^{early}, B^{early})$ for $s \in [\bar{s}_3, 1]$ and $\widetilde{W}(\widetilde{c}, \widetilde{B}) > \widehat{W}(c^{early}, B^{early})$ for $s \in [0, \bar{s}_3)$.

It can be shown in a similar fashion as in Lemma 14 that $\widetilde{W}(c^{wait}, B^{wait})$ is strictly decreasing in s for $s \in (0, 1)$. Define $\bar{s}_2 \in [0, 1]$ such that $\widetilde{W}(c^{wait}, B^{wait}) \leq \widehat{W}(c^{early}, B^{early})$ for $s \in [\bar{s}_2, 1]$ and $\widetilde{W}(c^{wait}, B^{wait}) > \widehat{W}(c^{early}, B^{early})$ for $s \in$

$[0, \bar{s}_2)$.

Define $\bar{s}_1 \in (0, 1]$ such that the best run-permitting allocation is (c^{wait}, B^{wait}) if $s \in [0, \bar{s}_1]$ and $(\widetilde{c}, \widetilde{B})$ if $s \in (\bar{s}_1, 1]$.

If $\bar{s}_2 > \bar{s}_1$, pick

$$s_1 = \min\{\bar{s}_3, 1\}$$
$$s_2 = \min\{\bar{s}_1, 1\}$$

If $\bar{s}_2 \leq \bar{s}_1$, pick

$$s_1 = s_2 = \min\{\bar{s}_2, 1\}$$

A.13 Proof of Proposition 7

Let (c_1^*, B_1^*) be the equilibrium allocation under the fiscal authority with commitment and (c^*, B^*) the constrained efficient allocation. The welfare achieved by (c^*, B^*) is always as least as high as the welfare achieved by (c_1^*, B_1^*), it is sufficient to show that the response of the banks given B^* is the similar to the constrained efficient contract, $c_1^*(B^*) = c^*$ to ensure that the economy under the fiscal authority with commitment is constrained efficient. This is shown separately for the three cases.

Define the following response functions:

$$\widehat{c}_1(B) \equiv \arg\max_c \quad \widehat{W}(c, B) \tag{A.12}$$
$$\widetilde{c}_1(B) \equiv \arg\max_c \quad \widetilde{W}(c, B) \tag{A.13}$$

Under the fiscal authority with commitment, for any given by B set by the fiscal authority, $\widehat{c}_1(B)$ is the unconstrained efficient contract and $\widetilde{c}_1(B)$ is the best panic-based run-permitting contract from the bank's perspective.

Case 1:

This is the case in which the unconstrained efficient allocation is DSIC.

$$(c^*, B^*) = (\widehat{c}, \widehat{B})$$

where $(\widehat{c}, \widehat{B})$ satisfies the first-order conditions:

$$\frac{\partial \widehat{W}}{\partial c} = \frac{\partial \widehat{W}}{\partial B} = 0$$

It is straightforward to see that the solution to the bank's problem in (A.12) is $\widehat{c}_1(\widehat{\tau}, \widehat{B}) = \widehat{c}$ because

$$\frac{\partial \widehat{W}_{pri}}{\partial c} = \frac{\partial \widehat{W}}{\partial c} = 0$$

which is also the first-order condition for the bank's problem. Also,

$$(\widehat{c}_1(\widehat{B}), \widehat{B}) = (\widehat{c}, \widehat{B}) \in S^{early}$$

which means that the unconstrained efficient contract is implementable, so $c_1^*(\widehat{B}) = \widehat{c}_1(\widehat{B}) = \widehat{c} = c^*$.

Case 2:

This is the case in which the unconstrained efficient allocation is BIC but not DSIC. Proposition 5 states that there exists $s_0 \in (0, 1]$ such that (Case 2.1) for any $s < s_0$, $(c^*, B^*) = (\widetilde{c}, \widetilde{B})$ and (Case 2.2) for any $s \geq s_0$, $(c^*, B^*) = (c^{early}, B^{early})$.

Case 2.1: $s < s_0$

Suppose the fiscal authority chooses $\widetilde{B}$. If $c_1^*(\widetilde{B}) = \widetilde{c}_1(\widetilde{B})$, then it is true that $c_1^*(\widetilde{B}) = \widetilde{c}_1(\widetilde{B}) = \widetilde{c}$ since

$$\frac{\partial \widetilde{W}_{pri}}{\partial c} = \frac{\partial \widetilde{W}}{\partial c}$$

Suppose, by contradiction, that $c_1^*(\widetilde{B}) \neq \widetilde{c}_1(\widetilde{B})$. Then either $c_1^*(\widetilde{B}) = \widehat{c}_1(\widetilde{B})$ or $c_1^*(\widetilde{B}) = c^{early}(\widetilde{B})$. [3]

If $c_1^*(\widetilde{B}) = \widehat{c}_1(\widetilde{B})$, then $(\widehat{c}_1(\widetilde{B}), \widetilde{B}) \in S^{early}$. It also has to be true that

$$\widehat{W}_{pri}(\widehat{c}_1(\widetilde{B}), \widetilde{B}) \geq \widetilde{W}_{pri}(\widetilde{c}_1(\widetilde{B}), \widetilde{B})$$

which implies that

$$\begin{aligned}
&\widehat{W}(\widehat{c}_1(\widetilde{B}), \widetilde{B}) - 2p^2\Gamma(2\tau y - \widetilde{B}) - 2(1-p^2)\Gamma(2\tau y) \\
&\qquad \geq \widetilde{W}(\widetilde{c}, \widetilde{B}) - 2p^2\Gamma(2\tau y - \widetilde{B}) - 2(1-p)^2\big[s\Gamma(2\tau y - \widetilde{B}) + (1-s)\Gamma(2\tau y)\big]
\end{aligned}$$

Since $\widetilde{B} > 0$, it is true that

$$\widehat{W}(\widehat{c}_1(\widetilde{B}), \widetilde{B}) > \widetilde{W}(\widetilde{c}, \widetilde{B})$$

This means that $(\widetilde{c}, \widetilde{B})$ is not the constrained efficient allocation, which is a contradiction.

If $c_1^*(\widetilde{B}) = c^{early}(\widetilde{B})$, it has to be true that

$$\widehat{W}_{pri}(c^{early}(\widetilde{B}), \widetilde{B}) \geq \widetilde{W}_{pri}(\widetilde{c}_1(\widetilde{B}), \widetilde{B})$$

which implies that

$$\begin{aligned}
&\widehat{W}(c^{early}(\widetilde{B}), \widetilde{B}) - p^2\Gamma(2\tau y - \widetilde{B}) - (1-p^2)\Gamma(2\tau y) \\
&\qquad \geq \widetilde{W}(\widetilde{c}, \widetilde{B}) - p^2\Gamma(2\tau y - \widetilde{B}) - (1-p)^2\big[s\Gamma(2\tau y - \widetilde{B}) + (1-s)\Gamma(2\tau y)\big]
\end{aligned}$$

[3] It is not necessary to consider the possibility that $c_1^*(\widetilde{B}) = c^{wait}(\widetilde{B})$ because $(\widetilde{c}, \widetilde{B}) \in S^{wait}$.

or

$$\widehat{W}(c^{early}(\widetilde{B}), \widetilde{B}) > \widetilde{W}(\widetilde{c}, \widetilde{B}) \tag{A.14}$$

This also means that $(\widetilde{c}, \widetilde{B})$ is not the constrained efficient allocation, which is a contradiction. In the case where (A.14) holds in equality, it is assumed that the panic-based run-proof allocation is chosen, which also means that $(\widetilde{c}, \widetilde{B})$ would not be chosen.

Case 2.2: $s \geq s_0$

Suppose the fiscal authority chooses B^{early}. It is straightforward to see that $c_1^*(B^{early}) \neq \widehat{c}_1(B^{early})$. Otherwise (c^{early}, B^{early}) would not have been the constrained efficient allocation.

Since (c^{early}, B^{early}) is the constrained efficient allocation:

$$\widehat{W}(c^{early}, B^{early}) \geq \widetilde{W}(\widetilde{c}, \widetilde{B}) \geq \widetilde{W}(\widetilde{c}_1(B^{early}), B^{early})$$

which implies that

$$\begin{aligned} &\widehat{W}_{pri}(c^{early}, B^{early}) + p^2\Gamma(2\tau y - B^{early}) + (1-p^2)\Gamma(2\tau y) \\ &> \widetilde{W}_{pri}(\widetilde{c}_1(B^{early}), B^{early}) + p^2\Gamma(2\tau y - B^{early}) \\ &\quad + (1-p^2)\big[s\Gamma(2\tau y - B^{early}) + (1-s)\Gamma(2\tau y)\big] \end{aligned}$$

or

$$\begin{aligned} &\widetilde{W}_{pri}(\widetilde{c}_1(B^{early}), B^{early}) - \widehat{W}_{pri}(c^{early}, B^{early}) \\ &\quad < s(1-p^2)\big[\Gamma(2\tau y) - \Gamma(2\tau y - B^{early})\big] \end{aligned}$$

Notice that the RHS of the inequality is strictly positive. When $s = s_0$, exactly one of the two following conditions has to be true:

(i) $\widetilde{W}_{pri}(\widetilde{c}_1(B^{early}), B^{early}) \leq \widehat{W}_{pri}(c^{early}, B^{early})$

(ii) $\widetilde{W}_{pri}(\widetilde{c}_1(B^{early}), B^{early}) > \widehat{W}_{pri}(c^{early}, B^{early})$

Using the fact that $\widetilde{W}_{pri}(\widetilde{c}_1(B^{early}), B^{early})$ is a strictly decreasing function of s, the constrained efficient allocation is implementable under the fiscal authority with commitment for any $s \geq s_0$ if condition (i) is true. If condition (ii) is true, there exists $\varepsilon_0 > 0$ such that the constrained efficient allocation is not implementable under the fiscal authority with commitment for $s_0 \leq s < s_0 + \varepsilon_0$ but it is implementable for $s \geq s_0 + \varepsilon_0$.

At $s = s_0$ and when condition (ii) is true, even though the constrained efficient allocation (c^{early}, B^{early}) is not implementable, the constrained efficient welfare level can still be achieved since $(\widetilde{c}, \widetilde{B})$ is implementable and gives the same welfare level as (c^{early}, B^{early}). Therefore, the economy under the fiscal authority with commitment is not constrained efficient only when $s \in (s_0, s_0 + \varepsilon_0)$.

Case 3:

This is the case in which the unconstrained efficient allocation is neither BIC nor DSIC. According to Proposition 6, there exist $s_1 \in (0, 1]$ and $s_2 \in (0, s_1]$ such that the constrained efficient allocation is (c^{wait}, B^{wait}) when $s < s_2$, $(\widetilde{c}, \widetilde{B})$ when $s_2 \leq s < s_1$ and (c^{early}, B^{early}) when $s \geq s_1$.

The proof for when $s_2 \leq s < s_1$ is similar to Case 2.1 and $s \geq s_1$ to Case 2.2. It is left to show that when $s < s_2$, the constrained efficient allocation (c^{wait}, B^{wait}) is implementable.

Suppose the fiscal authority chooses B^{wait} when $s < s_2$. The bank's best response contract has to be one of the four: $\widehat{c}_1(B^{wait})$, $c^{early}(B^{wait})$, $\widetilde{c}(B^{wait})$ or

$c^{wait}(\tau^{wait}, B^{wait})$.

If $c_1^*(B^{wait}) = \widehat{c}_1(B^{wait})$, then it has to be true that

$$(\widehat{c}_1(B^{wait}), B^{wait}) \in S^{early}$$

and

$$\widehat{W}_{pri}(\widehat{c}_1(B^{wait}), B^{wait}) \geq \widetilde{W}_{pri}(c^{wait}(B^{wait}), B^{wait})$$

which implies that

$$\begin{aligned}
&\widehat{W}(\widehat{c}_1(B^{wait}), B^{wait}) - p^2\Gamma(2\tau y - B^{wait}) - (1-p^2)\Gamma(2\tau y) \\
&\geq \widetilde{W}_{pri}(c^{wait}, B^{wait}) - p^2\Gamma(2\tau y - B^{early}) \\
&\quad - (1-p^2)\big[s\Gamma(2\tau y - B^{early}) + (1-s)\Gamma(2\tau y)\big]
\end{aligned}$$

or

$$\widehat{W}(\widehat{c}_1(B^{wait}), B^{wait}) \geq \widetilde{W}_{pri}(c^{wait}, B^{wait})$$

which is a contradiction because (c^{wait}, B^{wait}) is the constrained efficient allocation. $c_1^*(B^{wait}) = c^{early}(B^{wait})$ can be eliminated using the same method.

If $c_1^*(B^{wait}) = \widetilde{c}_1(B^{wait})$, then it has to be true that

$$(\widetilde{c}_1(B^{wait}), B^{wait}) \in S^{wait}$$

and

$$\widetilde{W}_{pri}(\widetilde{c}_1(B^{wait}), B^{wait}) \geq \widetilde{W}_{pri}(c^{wait}(B^{wait}), B^{wait})$$

which implies that

$$\widetilde{W}(\widetilde{c}_1(B^{wait}), B^{wait}) \geq \widetilde{W}(c^{wait}, B^{wait})$$

which is a contradiction since (c^{wait}, B^{wait}) is the constrained efficient allocation. Therefore, the only possibility is that $c_1^*(B^{wait}) = c^{wait}(B^{wait}) = c^{wait}$.

A.14 Proof of Lemma 8 and 9

The responses from (1.10) and (1.12) can be written in the following form:

$$B_i^* = 2\tau y - K_i(2y - c)$$

where $K_i \in (0, 1)$ is a constant defined accordingly. The inequality from $T_2 \geq T_1$ becomes

$$\frac{((2(1-\tau)y - c)R)^{1-\gamma}}{1-\gamma} - \frac{1}{2}\left[\frac{c^{1-\gamma}}{1-\gamma} + \frac{((2-K_i)(2y-c))^{1-\gamma}}{1-\gamma}\right] \geq 0 \tag{A.15}$$

The LHS of (A.15) is a continuous function of c on $c \in [0, 2(1-\tau)y]$. The LHS is $+\infty$ when $c = 0$ and $-\infty$ when $c = 2(1-\tau)y$. Again, the remaining proof is just to show that the LHS is strictly decreasing in c.

$$\begin{aligned}
\frac{\partial LHS}{\partial c} &= -(2(1-\tau)y - c)^{-\gamma}R^{1-\gamma} - \frac{1}{2}c^{-\gamma} + \frac{1}{2}(2y-c)^{-\gamma}(2-K_i)^{1-\gamma} \\
&< -(2y-c)^{-\gamma}R^{1-\gamma} - \frac{1}{2}c^{-\gamma} + \frac{1}{2}(2y-c)^{-\gamma}(2-K_i)^{1-\gamma} \\
&= (2y-c)^{-\gamma}\left(\frac{1}{2}(2-K_i)^{1-\gamma} - R^{1-\gamma}\right) - \frac{1}{2}c^{-\gamma}
\end{aligned}$$

A sufficient condition for the derivative to be negative is that

$$\frac{1}{2}(2-K_i)^{1-\gamma} - R^{1-\gamma} < \frac{1}{2} - R^{1-\gamma} < 0$$

which is equivalent to $\gamma < 1 + \ln 2/\ln R$. The proof for when $B_i^* = 0$ is trivial and included in Lemma 12.

The inequality $T_4 \geq T_3$ can be written as

$$\begin{aligned}
&(1-p)\left[\frac{((1-\tau)yR)^{1-\gamma}}{1-\gamma} - \frac{c^{1-\gamma}}{1-\gamma}\right] \\
&\quad + p\left[\frac{((2(1-\tau)y-c)R)^{1-\gamma}}{1-\gamma} - \frac{1}{2}\left(\frac{c^{1-\gamma}}{1-\gamma} + \frac{((2-K_i)(2y-c))^{1-\gamma}}{1-\gamma}\right)\right] \geq 0
\end{aligned} \tag{A.16}$$

The LHS of (A.16) is a continuous function of c on $c \in [0, 2(1-\tau)y]$. The LHS is again $+\infty$ when $c = 0$ and $-\infty$ when $c = 2(1-\tau)y$. The derivative of the LHS

$$\begin{aligned}\frac{\partial LHS}{\partial c} &= -(1-\frac{p}{2})c^{-\gamma} - p(2(1-\tau)y-c)^{-\gamma}R^{1-\gamma} + \frac{p}{2}(2y-c)^{-\gamma}(2-K_i)^{1-\gamma} \\ &< -(1-\frac{p}{2})c^{-\gamma} - p(2y-c)^{-\gamma}R^{1-\gamma} + \frac{p}{2}(2y-c)^{-\gamma}(2-K_i)^{1-\gamma} \\ &= -(1-\frac{p}{2})c^{-\gamma} + p(2y-c)^{-\gamma}\left(\frac{1}{2}(2-K_i)^{1-\gamma} - R^{1-\gamma}\right) < 0\end{aligned}$$

as long as $\gamma < 1 + \ln 2 / \ln R$. The proof for when $B_i^* = 0$ is trivial and included in Lemma 13. It follows from Lemma 16 that $\underline{c}_{sep} < \bar{c}_{sep}$. The proof of the existence of the "good" and "bad" pooling equilibria for $c \leq \underline{c}_{sep}$ and $c > \bar{c}_{sep}$ respectively can be done in a similar fashion.

A.15 Proof of Proposition 10

It is sufficient to show that the constrained efficient allocation is not implementable in the economy under the fiscal authority without commitment. The constrained efficient allocation has to be one of the following:

(i) $(\widehat{c}, \widehat{B})$

(ii) $(\widetilde{c}, \widetilde{B})$

(iii) (c^{early}, B^{early})

(iv) (c^{wait}, B^{wait})

$(\widehat{c}, \widehat{B})$ satisfies the following first-order condition

$$\frac{\partial \widehat{W}(\widehat{c}, \widehat{B})}{\partial c} = \frac{\partial \widehat{W}(\widehat{c}, \widehat{B})}{\partial B} = 0$$

which implies that

$$\frac{\partial \widehat{W}_{pri}(\widehat{c}, \widehat{B})}{\partial c} = 0$$

The first-order derivative of the bank's objective function is

$$\begin{aligned}&\frac{\partial \widehat{W}_{pri}(\widehat{c}, \widehat{B})}{\partial c} + \frac{\partial \widehat{W}_{pri}(\widehat{c}, \widehat{B})}{\partial B} K_{proof} \\ =& p^2(2(1-\tau)y - c + B)^{-\gamma} K_{proof} > 0\end{aligned}$$

which means that $(\widehat{c}, \widehat{B})$ is not implementable. It can be shown in similar way that $(\widetilde{c}, \widetilde{B})$ is not implementable as well.

(c^{early}, B^{early}) has to satisfy the following

$$\begin{aligned}&c^{early} = \bar{c}^{early}(B^{early}) \\ &\frac{\partial \widehat{W}(c^{early}, B^{early})}{\partial c} \frac{\partial \bar{c}^{early}(B^{early})}{\partial B} + \frac{\partial \widehat{W}(c^{early}, B^{early})}{\partial B} = 0 \end{aligned} \tag{A.17}$$

Suppose the bank chooses c^{early}. It is sufficient to show that B^{early} is not the best response of the fiscal authority. From (A.17), one has

$$\frac{\partial \widehat{W}(c^{early}, B^{early})}{\partial B} = -\frac{\partial \widehat{W}(c^{early}, B^{early})}{\partial c} \frac{\partial \bar{c}^{early}(B^{early})}{\partial B} \neq 0$$

This means that B^{early} does not satisfy the first-order condition of the fiscal authority's problem given that the bank chose c^{early}. Therefore, (c^{early}, B^{early}) is not implementable. It can be shown in similar manner that (c^{wait}, B^{wait}) is also not implementable.

A.16 Proof of Proposition 11

A.16.1 The Fiscal Authority with Commitment

From Proposition 7, the constrained efficient allocation is always (c^{early}, B^{early}) when the economy is not constrained efficient. This implies that if the fiscal authority chooses B^{early}, the bank's response is not $\bar{c}^{early}(B^{early}) = c^{early}$. The bank's response c_1^* has to be one of the following: $\widehat{c}_1(B^{early})$, $\widetilde{c}_1(B^{early})$ or $\bar{c}^{wait}(B^{early})$.

Suppose $c_1^* = \widehat{c}_1(B^{early})$. Then $\widehat{c}_1(B^{early}) < \bar{c}^{early}(B^{early})$, and $(\widehat{c}_1(B^{early}), B^{early}) \in S^{early}$. Also, it has to be true that

$$\widehat{W}_{pri}(\widehat{c}_1(B^{early}), B^{early}) > \widehat{W}_{pri}(\bar{c}^{early}(B^{early}), B^{early})$$

Adding the utilities from public good provision on both sides of the inequality gives

$$\widehat{W}(\widehat{c}_1(B^{early}), B^{early}) > \widehat{W}(\bar{c}^{early}(B^{early}), B^{early})$$
$$\widehat{W}(\widehat{c}_1(B^{early}), B^{early}) > \widehat{W}(c^{early}, B^{early})$$

This implies that (c^{early}, B^{early}) is not the constrained efficient allocation, which is a contradiction. Therefore, the bank's optimal contract is either $c_1^* = \widetilde{c}_1(B^{early})$ or $c_1^* = \bar{c}^{wait}(B^{early})$. In both cases, $c_1^* > c^{early}$. If the fiscal authority sets $\bar{c} = c^{early}$, the upper bound is strictly binding in the measure of s and the optimal contract of the bank is c^{early}. The economy is constrained efficient.

A.16.2 The Fiscal Authority without Commitment

Suppose $\bar{A}_{sep} > A^{early}$. Consider $A \in (A^{early}, \bar{A}_{sep}]$. Then both $\widehat{c} > \bar{c}^{early}(\widehat{B})$ and $\widehat{c}_2 \leq \underline{c}_{sep}$ hold. It is shown next that $\widehat{c}_2 > \widehat{c}$. When $s = 0$, $c_2^* = \widehat{c}_2 \in (c^{early}, \underline{c}_{sep}]$, which violates the parameter restrictions stated in Section A.6 of the Appendix. Therefore, it has to be true that $\bar{A}_{sep} \leq A^{early}$.

Case 1: $A \leq \bar{A}_{sep}$

The constrained efficient allocation is $(c^*, B^*) = (\widehat{c}, \widehat{B})$ and the equilibrium allocation is $(\widehat{c}_2, B^*_{good}(\widehat{c}_2))$. Define the following function

$$\Omega_{pri}(c) \equiv \widehat{W}(c, B^*_{good}(c))$$

Ω is strictly concave since it is the composite of a strictly concave function $\widehat{W}_{pri}$ and a weakly concave function B^*_{good}. The equilibrium is never constrained efficient (from Proposition 10), which implies that $(\widehat{c}, \widehat{B}) \neq (\widehat{c}_2, B^*_{good}(\widehat{c}_2))$. Since $B^*_{good}(c)$ is a strictly increasing function of c, either $(\widehat{c}, \widehat{B}) \gg (\widehat{c}_2, B^*_{good}(\widehat{c}_2))$ or $(\widehat{c}, \widehat{B}) \ll (\widehat{c}_2, B^*_{good}(\widehat{c}_2))$.

Suppose $(\widehat{c}, \widehat{B}) \gg (\widehat{c}_2, B^*_{good}(\widehat{c}_2))$. $(\widehat{c}_2, B^*_{good}(\widehat{c}_2))$ has to satisfy the first-order condition of the bank's problem:

$$\frac{\partial \Omega_{pri}(\widehat{c}_2)}{\partial c} = 0 \tag{A.18}$$

Since Ω_{pri} is strictly concave and $\widehat{c} > \widehat{c}_2$,

$$\frac{\partial \Omega_{pri}(\widehat{c})}{\partial c} < \frac{\partial \Omega_{pri}(\widehat{c}_2)}{\partial c} = 0$$

By a simple comparison of the first-order conditions, it can be shown that $\widehat{B} = B^*_{good}(\widehat{c})$. Thus,

$$\frac{\partial \Omega_{pri}(\widehat{c})}{\partial c} = \frac{\partial \widehat{W}_{pri}(\widehat{c}, B^*_{good}(\widehat{c}))}{\partial c} + \frac{\partial \widehat{W}_{pri}(\widehat{c}, B^*_{good}(\widehat{c}))}{\partial B} K_{good} < 0$$

and

$$\frac{\partial \widehat{W}_{pri}(\widehat{c},\widehat{B})}{\partial c} < -\frac{\partial \widehat{W}_{pri}(\widehat{c},\widehat{B})}{\partial B} K_{good} < 0$$

This means that

$$\frac{\partial \widehat{W}(\widehat{c},\widehat{B})}{\partial c} = \frac{\partial \widehat{W}_{pri}(\widehat{c},\widehat{B})}{\partial c} < 0$$

and $(\widehat{c},\widehat{B})$ does not satisfy the first-order condition, which is a contradiction. Therefore, $(\widehat{c},\widehat{B}) \ll (\widehat{c}_2, B^*_{good}(\widehat{c}_2))$. If the fiscal authority set the upper bound at $\bar{c} = \widehat{c}$, the upper bound is binding and the bank responds with $\widehat{c}$, which makes the economy constrained efficient. This is true for $s \in [0,1]$.

Case 2: $\bar{A}_{sep} < A \leq A^{early}$

In this case, $\widehat{c}_2$ is not implementable, or $\widehat{c}_2 > \underline{c}_{sep}$. Also, the constrained efficient allocation is $(\widehat{c},\widehat{B})$, so $\widehat{c} \leq c^{early}$. For $s \in [0,1]$, it is true that

$$\widehat{c}_2 > \underline{c}_{sep} \geq c^{early} \geq \widehat{c}$$

If the fiscal authority sets the upper bound as $\bar{c} = \widehat{c}$, it is always binding for the bank since both $\widehat{c}_2$ and $\underline{c}_{sep}$ are weakly higher than $\widehat{c}$. Therefore, the bank's optimal contract is $\widehat{c}$ and the economy is constrained efficient.

Case 3: $A^{early} < A \leq A^{wait}$

For $s \in [0, s_0)$, the constrained efficient allocation is $(c^*, B^*) = (\widetilde{c}, \widetilde{B})$. For $s \in [0, \bar{s})$, the equilibrium allocation is $(c_2^*, B_2^*) = (\widetilde{c}_2, B^*_{sep}(\widetilde{c}_2))$. Pick, $s_4 = \min\{s_0, \bar{s}\}$. For $s \in [0, s_4)$, it can be shown in similar manner as in Case 1 that $(\widetilde{c}_2, B^*_{sep}(\widetilde{c}_2)) \gg (\widetilde{c}, \widetilde{B})$. If the fiscal authority sets the upper bound as $\bar{c} = \widetilde{c}$, the upper bound is binding. However, it might be true that $\underline{c}_{sep} \leq \widetilde{c}$. The bank could potentially choose $\underline{c}_{sep}$ in response to the upper bound set by the fiscal authority. Since $\widehat{c}_2 > \underline{c}_{sep}$, by the strict concavity of $\widehat{W}_{pri}$, $\widehat{W}_{pri}(\underline{c}_{sep}, B^*_{good}(\underline{c}_{sep}))$ is strictly increasing in s. Also, choosing the upper bound set by the fiscal authority gives $\widetilde{W}_{pri}(\widetilde{c}, B^*_{sep}(\widetilde{c}))$, which is strictly decreasing in s. Therefore, there exists $s_5 > 0$ such that

$$\widetilde{W}_{pri}(\widetilde{c}, B^*_{sep}(\widetilde{c})) > \widehat{W}_{pri}(\underline{c}_{sep}, B^*_{good}(\underline{c}_{sep}))$$

for $s \in [0, s_5)$. Pick $\bar{s} = \min\{s_4, s_5\}$. For $s \in [0, \bar{s})$, the bank's optimal contract is $\widetilde{c}$. The economy is constrained efficient.

A.17 Proof of Lemma 14

$(\widetilde{c}, \widetilde{B})$ solves the following first-order conditions:

$$\begin{aligned}(1-s)\Big\{&p^2\big[A\widetilde{c}^{-\gamma} - A(2(1-\tau)y - \widetilde{c} + \widetilde{B})^{-\gamma}\big] \\ &+ 2p(1-p)\big[A\widetilde{c}^{-\gamma} - R^{1-\gamma}(2(1-\tau)y - \widetilde{c})^{-\gamma}\big]\Big\} \\ +s\Big\{&\big[p^2A + p(1-p)(A+1) + (1-p)^2\big]\big[\widetilde{c}^{-\gamma} - (2(1-\tau)y - \widetilde{c} + \widetilde{B})^{-\gamma}\big]\Big\} = 0\end{aligned} \tag{A.19}$$

$$\begin{aligned}(1-s)\Big\{&p^2\big[A(2(1-\tau)y - \widetilde{c} + \widetilde{B})^{-\gamma} - 2D(2\tau y - \widetilde{B})^{-\gamma}\big]\Big\} \\ +s\Big\{&\big[p^2A + p(1-p)(A+1) + (1-p)^2\big](2(1-\tau)y - \widetilde{c} + \widetilde{B})^{-\gamma} \\ &- 2D(2\tau y - \widetilde{B})^{-\gamma}\Big\} = 0\end{aligned} \tag{A.20}$$

Using (A.20) to solve for $\widetilde{B}$:

$$\widetilde{B} = 2\tau - \frac{K_1}{1+K_1}(2y - \widetilde{c}) \equiv 2\tau - K_2(2y - \widetilde{c}) \tag{A.21}$$

where

$$K_1 = \left[\frac{\big[(1-s)p^2 + s\big]2D}{(1-s)p^2A + s\big[p^2A + p(1-p)(A+1) + (1-p)^2\big]}\right]^{1/\gamma}$$

Plugging (A.21) into (A.19):

$$\begin{aligned}
&\big[(1-s)[p^2A+2p(1-p)A]+s[p^2A+p(1-p)(A+1)+(1-p)^2]\big]\widetilde{c}^{-\gamma} \\
=&(1-s)2p(1-p)R^{1-\gamma}(2(1-\tau)y-\widetilde{c})^{-\gamma} \\
&\quad+\big[(1-s)p^2A+s(p^2A+p(1-p)(A+1)+(1-p)^2)\big]((1-K_2)(2y-\widetilde{c}))^{-\gamma}
\end{aligned} \tag{A.22}$$

Taking the derivative of (A.22) with respect to s:

$$\begin{aligned}
&-\gamma\big[(1-s)[p^2A+2p(1-p)A]+s[p^2A+p(1-p)(A+1)+(1-p)^2]\big]\widetilde{c}^{-\gamma-1}\frac{d\widetilde{c}}{ds} \\
&\quad+\big[p(1-p)(1-A)+(1-p)^2\big]\widetilde{c}^{-\gamma} \\
=&\gamma(1-s)2p(1-p)R^{1-\gamma}(2(y-\tau)-\widetilde{c})^{-\gamma-1}\frac{d\widetilde{c}}{ds}-2p(1-p)R^{1-\gamma}(2(1-\tau)y-\widetilde{c})^{-\gamma} \\
&\quad+\gamma\big[(1-s)p^2A+s(p^2A+p(1-p)(A+1)+(1-p)^2)\big] \\
&\qquad\times(1-K_2)^{-\gamma}(2y-\widetilde{c})^{-\gamma-1}\frac{d\widetilde{c}}{ds} \\
&\quad+\gamma\big[(1-s)p^2A+s(p^2A+p(1-p)(A+1)+(1-p)^2)\big] \\
&\quad\times(1-K_2)^{-\gamma-1}(2y-\widetilde{c})^{-\gamma}\frac{dK_2}{ds} \\
&\quad+\big[p(1-p)(A+1)+(1-p)^2\big]((1-K_2)(2y-\widetilde{c}))^{-\gamma}
\end{aligned}$$

$$\begin{aligned}
sign\left\{\frac{d\widetilde{c}}{ds}\right\}=sign\Big\{&\big[p(1-p)(1-A)+(1-p)^2\big]\widetilde{c}^{-\gamma} \\
&+2p(1-p)R^{1-\gamma}(2(1-\tau)y-\widetilde{c})^{-\gamma} \\
&-\big[p(1-p)(A+1)+(1-p)^2\big]((1-K_2)(2y-\widetilde{c}))^{-\gamma} \\
&-\gamma\big[(1-s)p^2A+s(p^2A+p(1-p)(A+1)+(1-p)^2)\big] \\
&\quad\times(1-K_2)^{-\gamma-1}(2y-\widetilde{c})^{-\gamma}\frac{dK_2}{ds}\Big\}
\end{aligned} \tag{A.23}$$

Find the derivative of $\widetilde{c}$ with respect to τ:

$$
\begin{aligned}
&-\gamma\big[(1-s)[p^2A+2p(1-p)A]+s[p^2A+p(1-p)(A+1)+(1-p)^2]\big]\widetilde{c}^{-\gamma-1}\frac{d\widetilde{c}}{d\tau}\\
&=\gamma(1-s)2p(1-p)R^{1-\gamma}(2(y-\tau)-\widetilde{c})^{-\gamma-1}\left(2+\frac{d\widetilde{c}}{d\tau}\right)\\
&\quad+\gamma\big[(1-s)p^2A+s(p^2A+p(1-p)(A+1)+(1-p)^2)\big]\\
&\qquad\times(1-K_2)^{-\gamma}(2y-\widetilde{c}))^{-\gamma-1}\frac{d\widetilde{c}}{d\tau}
\end{aligned}
$$

$$
sign\left\{\frac{d\widetilde{c}}{d\tau}\right\}=sign\left\{-2\gamma(1-s)2p(1-p)R^{1-\gamma}(2(1-\tau)y-\widetilde{c})^{-\gamma-1}\right\}<0
$$

$$
\frac{d\widetilde{c}}{d\tau}\in(-1,0)
$$

Taking the derivative of the RHS of (A.23) with respect to τ:

$$
\begin{aligned}
\frac{dRHS}{d\tau}&=-\gamma\big[p(1-p)(1-A)+(1-p)^2\big]\widetilde{c}^{-\gamma-1}\frac{d\widetilde{c}}{d\tau}\\
&\quad+2\gamma p(1-p)R^{1-\gamma}(2(1-\tau)y-\widetilde{c})^{-\gamma-1}\left(\frac{d\widetilde{c}}{d\tau}+2\right)\\
&\quad-\gamma\big[p(1-p)(A+1)+(1-p)^2\big](1-K_2)^{-\gamma}(2y-\widetilde{c})^{-\gamma-1}\frac{d\widetilde{c}}{d\tau}\\
&\quad-\gamma^2\big[(1-s)p^2A+s(p^2A+p(1-p)(A+1)+(1-p)^2)\big]\\
&\qquad\times(1-K_2)^{-\gamma-1}(2y-\widetilde{c})^{-\gamma-1}\frac{dK_2}{ds}\frac{d\widetilde{c}}{d\tau}\\
&>\gamma p(1-p)(A-1)\widetilde{c}^{-\gamma-1}\frac{d\widetilde{c}}{d\tau}+4\gamma p(1-p)R^{1-\gamma}(2(1-\tau)y-\widetilde{c})^{-\gamma-1}\\
&\quad+2\gamma p(1-p)R^{1-\gamma}(2(1-\tau)y-\widetilde{c})^{-\gamma-1}\frac{d\widetilde{c}}{d\tau}\\
&\quad-\gamma p(1-p)(A+1)(1-K_2)^{-\gamma}(2y-\widetilde{c})^{-\gamma-1}\frac{d\widetilde{c}}{d\tau}>0
\end{aligned}
$$

There exists $\bar{\tau}$ such that $\tau\leq\bar{\tau}$ implies $\frac{d\widetilde{c}}{ds}\leq 0$ and $\tau>\bar{\tau}$ implies $\frac{d\widetilde{c}}{ds}>0$.

Taking the derivative of $\widetilde{W}(\widetilde{c}, \widetilde{B})$ with respect to s:

$$\begin{aligned}\frac{d\widetilde{W}(\widetilde{c}, \widetilde{B})}{ds} = & \big[(1-s)p^2A + s(p^2A + p(1-p)(A+1) + (1-p)^2)\big] \\ & \qquad \times (2(1-\tau)y - \widetilde{c} + \widetilde{B})^{-\gamma}\frac{d\widetilde{B}}{ds} \\ & + \frac{1}{\gamma-1}\Big\{\big[p(1-p)(A-1) - (1-p)^2\big]\widetilde{c}^{1-\gamma} \\ & - 2p(1-p)((2(1-\tau)y - \widetilde{c})R)^{1-\gamma} \\ & + \big[p(1-p)(A+1) + (1-p)^2\big](2(1-\tau)y - \widetilde{c} + \widetilde{B})^{1-\gamma}\Big\}\end{aligned}$$

If $\frac{d\widetilde{c}}{ds} < 0$ then $\frac{d\widetilde{B}}{ds} < 0$ and $\frac{d\widetilde{W}}{ds} < 0$. Therefore, $\tau \leq \bar{\tau}$ is sufficient.

A.18 Proof of Proposition 15

Define $\widehat{c}_2$ as the optimal contract of the bank assuming that there is always the "good" pooling equilibrium in the post-deposit game, which is given by

$$\widehat{c} \equiv \arg\max_{c \in \mathbb{R}_+} \widehat{W}_{pri}(c, B^*_{good}(c)) \tag{A.24}$$

The first-order condition of (A.24) is

$$\big[p^2A + 2p(1-p)A\big]\widehat{c}^{-\gamma} = p^2A(1-K_{good})^{1-\gamma}(2y-\widehat{c})^{-\gamma} + 2p(1-p)R^{1-\gamma}(2(1-\tau)y-\widehat{c})^{-\gamma}$$

Taking the implicit derivative of the first-order condition with respect to A gives

$$\begin{aligned}& -\gamma\big[p^2A + 2p(1-p)A\big]\widehat{c}^{-\gamma-1}\frac{d\widehat{c}}{dA} + \big[p^2 + 2p(1-p)\big]\widehat{c}^{-\gamma} \\ =& 2p(1-p)R^{1-\gamma}\gamma(2(1-\tau)y - \widehat{c})^{-\gamma-1}\frac{d\widehat{c}}{dA} \\ & + p^2(1-K_{good})^{-\gamma}\Big[A(\gamma-1)\frac{\partial K_{good}}{\partial A} + p^2(1-K_{good})\Big](2y-\widehat{c})^{-\gamma} \\ & + p^2A(1-K_{good})^{1-\gamma}\gamma(2y-\widehat{c})^{-\gamma-1}\frac{d\widehat{c}}{dA}\end{aligned}$$

$$sign\left\{\frac{d\widehat{c}}{dA}\right\} = sign\Big\{\big[p^2 + 2p(1-p)\big]\widehat{c}^{-\gamma} \\ - p^2(1-K_{good})^{-\gamma}\Big[A(\gamma-1)\frac{\partial K_{good}}{\partial A} + p^2(1-K_{good})\Big](2y-\widehat{c})^{-\gamma}\Big\}$$

A sufficient condition for $\frac{d\widehat{c}}{dA} > 0$ is

$$A(\gamma-1)\frac{\partial K_{good}}{\partial A} + p^2(1-K_{good}) \leq 0$$

which is equivalent to

$$p \leq \sqrt{\frac{\gamma-1}{\gamma}K_{good}}$$

Therefore, there exists a threshold $\bar{A}_{sep}$ such that $\widehat{c}_2 \leq \underline{c}$ for $A \leq \bar{A}_{sep}$. Given that the value function $\max_{c\in\mathbb{R}_+}\widetilde{W}_{pri}(c, B^*_{sep}(c))$ is strictly decreasing in s, it is always true that

$$\max_{c\in\mathbb{R}_+}\widehat{W}(c, B^*_{good}(c)) \geq \max_{c\in\mathbb{R}_+}\widetilde{W}(c, B^*_{sep}(c))$$

The optimal contract is $c^*_2 = \widehat{c}_2$ when $A \leq \bar{A}_{sep}$. When $A > \bar{A}_{sep}$, the proof is similar to Proposition 5.

APPENDIX B

APPENDIX FOR CHAPTER 2

B.1 Case 2: $\varepsilon \leq 1 - \pi c_1$ and $\bar{\sigma} \leq 1 - \pi c_1$

To analyze this case fully, a few Lemmas have to be developed.

Lemma 16. *$1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[(\frac{1-\pi}{R} + \pi)c_1 - (1-\bar{\sigma})\big] \geq 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\pi c_1$ and $1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big] \geq 1 - \pi c_1 \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}$ if and only if*

$$\bar{\sigma} \leq 1 - \frac{1-\pi}{R}c_1 \tag{B.1}$$

Lemma 16 says that if (B.1) is satisfied, when bank B goes bankrupt, bank A never goes banrkupt as long as the amount of withdrawal it receives from bank B is sufficient for payments to all its impatient depositors. Notice that (B.1) is always satisfied in Case 2 if and only if $\pi \geq \frac{1}{1+R}$.

Lemma 17. *$(1-\pi c_1)\frac{2\pi c_1 R}{(1-\pi c_1)(1-\pi)+\pi c_1} \geq 1 - \pi c_1$ if and only if*

$$c_1 \geq \frac{1-\pi}{\pi(2R-\pi)} \tag{B.2}$$

Lemma 17 says that (2.9) is always satisfied in Case 2 if and only if (B.2) is true. Lemma 12 to 17 will be used to summarize the outcome in Case 2.

Case 2.2: $1 - \frac{1-\pi}{R}c_1 < \bar{\sigma} \leq (1-\pi c_1)\frac{2\pi c_1 R}{(1-\pi c_1)(1-\pi)+\pi c_1}$

Notice that this case exists only if $\pi < \frac{1}{1+R}$. In this case, if bank B goes bankrupt, it has to be the case that bank A also goes bankrupt. This is because

$$(1-\pi c_1)\Big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2} - \pi c_1)R}\Big] \geq 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\pi c_1$$
$$> \max\Big\{1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\Big[\big(\frac{1-\pi}{R} + \pi\big)c_1 - (1-\bar{\sigma})\Big], 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big]\Big\}$$

The outcome can be summarized as follow:

$$c_1^A = \begin{cases} c_1 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ 1-\bar{\sigma}+\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \end{cases}$$

$$c_2^A = \begin{cases} c_2 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ 1-\bar{\sigma}+\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \end{cases}$$

$$c_1^B = \begin{cases} c_1 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \end{cases}$$

$$c_2^B = \begin{cases} \frac{1}{1-\pi}\frac{(1-\frac{\bar{\sigma}}{2}-\pi c_1)(1-\varepsilon-\pi c_1)R}{1-\pi c_1} & \text{if } \varepsilon \leq (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \end{cases}$$

Case 2.3: $(1-\pi c_1)\frac{2\pi c_1 R}{(1-\pi c_1)(1-\pi)+\pi c_1} < \bar{\sigma} \leq 1 - \frac{1-\pi}{R}c_1$

Notice from Lemma 17 that this case exists only if $c_1 < \frac{1-\pi}{\pi(2R-\pi)}$. In this case, if bank A goes bankrupt, it has to be that bank B also goes bankrupt, but not the another way round. This is because

$$\min\left\{1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\left[(\frac{1-\pi}{R}+\pi)c_1 - (1-\bar{\sigma})\right], 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big]\right\}$$
$$\geq 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\pi c_1 > (1-\pi c_1)\left[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\right]$$

The outcome can be summarized as follow:

$$c_1^A = \begin{cases} c_1 & \text{if } \varepsilon \leq 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1 - (1-\bar{\sigma})\big] \\ 1-\bar{\sigma}+\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1 - (1-\bar{\sigma})\big] \end{cases}$$

$$c_2^A = \begin{cases} c_2 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \frac{1}{1-\pi}\big(1-\bar{\sigma}-\pi c_1+\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}\big)R & \text{if } \begin{array}{l}(1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \qquad < \varepsilon < 1-\pi c_1\frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\end{array} \\ \frac{1}{1-\pi}\big[(1-\bar{\sigma})R+\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}-\pi c_1\big] & \text{if } \begin{array}{l}1-\pi c_1\frac{2c_1+\bar{\sigma}}{\bar{\sigma}} \\ \qquad \leq \varepsilon \leq 1-\frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1-(1-\bar{\sigma})\big]\end{array} \\ 1-\bar{\sigma}+\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > 1-\frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[\big(\frac{1-\pi}{R}+\pi\big)c_1-(1-\bar{\sigma})\big] \end{cases}$$

$$c_1^B = \begin{cases} c_1 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \end{cases}$$

$$c_2^B = \begin{cases} c_2 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > (1-\pi c_1)\big[1-\frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \end{cases}$$

Case 2.4: $\bar{\sigma} > \max\{(1-\pi c_1)\frac{2\pi c_1 R}{(1-\pi c_1)(1-\pi)+\pi c_1}, 1-\frac{1-\pi}{R}c_1\}$

Notice that this case exists only if $c_1 < \frac{1-\pi}{\pi(2R-\pi)}$ and $\pi < \frac{1}{1+R}$. In this case, when bank B goes bankrupt, it is not necessary the case that bank A always depletes the withdrawal from bank B from paying its impatient depositors. The outcome can be summarized as follow:

$$c_1^A = \begin{cases} c_1 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ c_1 & \text{if } \begin{array}{l}(1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \quad < \varepsilon \leq 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big]\end{array} \\ 1-\bar{\sigma}+\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big] \end{cases}$$

$$c_2^A = \begin{cases} c_2 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \frac{1}{1-\pi}\big[(1-\bar{\sigma})R - \pi c_1 + \frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}}\big] & \text{if } \begin{array}{l}(1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \quad < \varepsilon \leq 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big]\end{array} \\ 1-\bar{\sigma}+\frac{(1-\varepsilon)\bar{\sigma}}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big] \end{cases}$$

$$c_1^B = \begin{cases} c_1 & \text{if } \varepsilon \leq (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \end{cases}$$

$$c_2^B = \begin{cases} \frac{1}{1-\pi}\frac{(1-\frac{\bar{\sigma}}{2}-\pi c_1)(1-\varepsilon-\pi c_1)R}{1-\pi c_1} & \text{if } \varepsilon \leq (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \\ \frac{2(1-\varepsilon)c_1}{2c_1+\bar{\sigma}} & \text{if } \varepsilon > (1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] \end{cases}$$

Notice that it is not necessary the case that

$$(1-\pi c_1)\big[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\big] < 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big] \qquad \text{(B.3)}$$

If (B.3) is not satisfied, there does not exist a measure of ε such that bank B goes bankrupt but bank A does not. Either both banks go bankrupt or both do not.

B.2 Proof of Lemma 12

$$(1-\pi c_1)\left[\frac{2}{\bar{\sigma}}\left(1 - \left(\frac{1-\pi}{R}+\pi\right)c_1\right) - 1\right] \geq (1-\pi c_1)\left[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\right]$$

$$\frac{2}{\bar{\sigma}}\left(1 - \left(\frac{1-\pi}{R}+\pi\right)c_1\right) - 1 \geq 1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}$$

$$\big[2 - 2\big(\frac{1-\pi}{R} + \pi\big)c_1 - \bar{\sigma}\big]\big(1 - \frac{\bar{\sigma}}{2} - \pi c_1\big)R$$
$$\geq \big(1 - \frac{\bar{\sigma}}{2} - \pi c_1\big)R\bar{\sigma} - (1-\pi)c_1\bar{\sigma}$$

$$2(1-\pi c_1)R - 2\big(\frac{1-\pi}{R} + \pi\big)(1-\pi c_1)c_1 R + \big(\frac{1-\pi}{R} + \pi\big)Rc_1\bar{\sigma} - R\bar{\sigma}$$
$$\geq 2(1-\pi c_1)R\bar{\sigma} - R\bar{\sigma}^2 - (1-\pi)c_1\bar{\sigma}$$

$$\bar{\sigma}^2 + \left[\left(2\big(\frac{1-\pi}{R}\big) + \pi\right)c_1 - 1 - 2(1-\pi c_1)\right]\bar{\sigma}$$
$$+ \left[2 - 2\left(\frac{1-\pi}{R} + \pi\right)c_1\right](1-\pi c_1) \geq 0$$

Notice that

$$\left(2\big(\frac{1-\pi}{R}\big) + \pi\right)c_1 - 1 - 2(1-\pi c_1)$$

can be rewritten as

$$2\left(\big(\frac{1-\pi}{R}\big) + \pi\right)c_1 - 2 - (1-\pi c_1) < 0$$

which we know is always negative. Using the quadratic formula

$$B^2 - 4AC = \left[\left(2\Big(\frac{1-\pi}{R}\Big) + \pi\right)c_1 - 3\right]^2 - 4\left[2 - 2\left(\frac{1-\pi}{R} + \pi\right)c_1\right](1-\pi c_1)$$
$$= \left[\left(2\Big(\frac{1-\pi}{R}\Big) + \pi\right)c_1 - 1\right]^2 \tag{B.4}$$

the inequality is simplified to

$$\bar{\sigma} \leq \frac{3 - \left(2\big(\frac{1-\pi}{R}\big) + 3\pi\right)c_1 - \left|\big(2\big(\frac{1-\pi}{R}\big) + \pi\big)c_1 - 1\right|}{2}$$

or

$$\bar{\sigma} \geq \frac{3 - \left(2\big(\frac{1-\pi}{R}\big) + 3\pi\right)c_1 + \left|\big(2\big(\frac{1-\pi}{R}\big) + \pi\big)c_1 - 1\right|}{2}$$

If $\big(2\big(\frac{1-\pi}{R}\big) + \pi\big)c_1 - 1 \geq 0$, the condition simplifies to

$$\bar{\sigma} \leq 1 - \pi c_1 \quad \text{or} \quad \bar{\sigma} \geq 2 - 2\left(\frac{1-\pi}{R} + \pi\right)c_1 \tag{B.5}$$

If $\left(2\left(\frac{1-\pi}{R}\right)+\pi\right)c_1 - 1 < 0$, the condition simplifies to

$$\bar{\sigma} \leq 2 - 2\Big(\frac{1-\pi}{R} + \pi\Big)c_1 \quad \text{or} \quad \bar{\sigma} \geq 1 - \pi c_1 \tag{B.6}$$

In both cases the condition is always satisfied as long as $\bar{\sigma} \geq 1 - \pi c_1$

B.3 Proof of Lemma 13

$$(1-\pi c_1)\left[1 - \frac{(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R}\right] \geq 1 - \pi c_1 \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}$$

$$1 - \pi c - 1 - \frac{(1-\pi c_1)(1-\pi)c_1}{(1-\frac{\bar{\sigma}}{2}-\pi c_1)R} \geq 1 - \pi c_1 \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}$$

$$(1-\pi c_1)(1-\pi)c_1\bar{\sigma} \leq 2\pi c_1^2\Big(1 - \frac{\bar{\sigma}}{2} - \pi c_1\Big)R$$

$$\big[(1-\pi c_1)(1-\pi)c_1 + \pi c_1^2\big]\bar{\sigma} \leq 2\pi c_1^2(1-\pi c_1)R$$

$$\bar{\sigma} \leq (1-\pi c_1)\frac{2\pi c_1 R}{(1-\pi c_1)(1-\pi)+\pi c_1} \tag{B.7}$$

B.4 Proof of Lemma 16

First part:

$$1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\Big[\Big(\frac{1-\pi}{R}+\pi\Big)c_1 - (1-\bar{\sigma})\Big] \geq 1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\pi c_1$$

$$\pi c_1 \geq \Big(\frac{1-\pi}{R}+\pi\Big)c_1 - (1-\bar{\sigma})$$

$$\bar{\sigma} \leq 1 - \frac{1-\pi}{R}c_1 \tag{B.8}$$

Second part:

$$1 - \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}\big[c_1 - (1-\bar{\sigma})R\big] \geq 1 - \pi c_1 \frac{2c_1+\bar{\sigma}}{\bar{\sigma}}$$

$$\pi c_1 \geq c_1 - (1-\bar{\sigma})R$$

$$R - (1-\pi)c_1 \geq R\bar{\sigma}$$

$$\bar{\sigma} \leq 1 - \frac{1-\pi}{R} c_1 \tag{B.9}$$

B.5 Proof of Lemma 17

$$(1-\pi c_1)\frac{2\pi c_1 R}{(1-\pi c_1)(1-\pi)+\pi c_1} > 1-\pi c_1$$

$$2\pi c_1 R > (1-\pi c_1)(1-\pi)+\pi c_1$$

$$\big[2\pi R - \pi + \pi(1-\pi)\big]c_1 > 1-\pi$$

$$c_1 > \frac{1-\pi}{\pi(2R-\pi)} \tag{B.10}$$