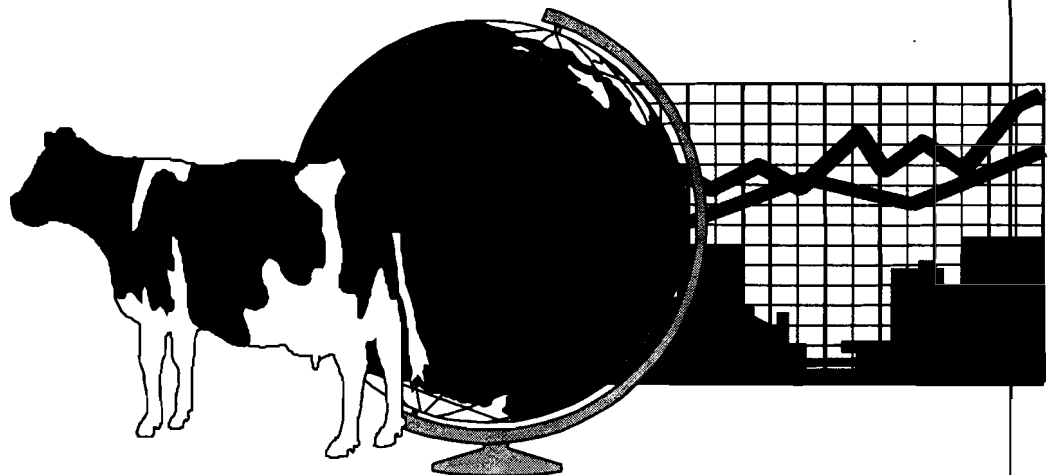


A Spatial Equilibrium Model for Imperfectly Competitive Milk Markets

by
Tsunemasa Kawaguchi
Nobuhiro Suzuki
and
Harry M. Kaiser



Department of Agricultural, Resource, and Managerial Economics
College of Agricultural and Life Sciences
Cornell University, Ithaca, New York 14853-7801

It is the Policy of Cornell University actively to support equality of educational and employment opportunity. No person shall be denied admission to any educational program or activity or be denied employment on the basis of any legally prohibited discrimination involving, but not limited to, such factors as race, color, creed, religion, national or ethnic origin, sex, age or handicap. The University is committed to the maintenance of affirmative action programs which will assure the continuation of such equality of opportunity.

A Spatial Equilibrium Model for Imperfectly Competitive Milk Markets

by

Tsunemasa Kawaguchi, Nobuhiro Suzuki, and Harry M. Kaiser

Abstract

Traditional spatial equilibrium models have assumed that markets are either perfectly competitive or monopolistic. In this paper, a generalized spatial equilibrium model is developed which allows for any degree of market conduct from perfect competition to monopoly. The model incorporates a "dual structure" in which there are oligopolistic consignment sellers (producer marketing boards) and perfectly competitive producers receiving pooled returns.

The usefulness of the model is demonstrated using Kyushu regional milk market data in Japan. Numerous spatial equilibrium solutions are generated for the Kyushu milk market assuming alternative sets of imperfectly competitive behavior with the "dual structure." It is demonstrated that actual interregional milk movements in Japan are better explained by the dual structure imperfect spatial competition model than perfectly competitive or monopolistic spatial competition models. The model solutions generated by the imperfect spatial competition model are useful for analyzing alternative milk marketing organization policies.

Introduction

Spatial equilibrium models have been used frequently to analyze interregional competition problems. Interregional competition issues associated with dairy industries in several countries including Japan have been examined with these models (e.g., Sasaki; Kobayashi; Hayashi; McDowell; Rayner). Originally developed by Enke and Samuelson then refined by Takayama and Judge, spatial price equilibrium models have assumed that markets are either perfectly competitive or

that they're completely monopolistic. However, the structure of dairy markets in most countries are often neither. Therefore, a more plausible model for analyzing interregional milk movements would be a spatial imperfect competition equilibrium model.

The purpose of this paper is to present a generalization of Takayama and Judge's spatial equilibrium model that allows for the incorporation of any degree of market structure from perfect competition to monopoly. The usefulness of the model is demonstrated with an application to interregional milk movements in the Japanese dairy industry with solutions generated and compared for alternative scenarios regarding the degree of market competition.

The Japanese Dairy Industry

Dairy policy in Japan features a quota system in the manufacturing milk market to prevent excess milk production from occurring because of higher than competitive market prices. As a result, the Japanese dairy industry can be divided into three distinct markets: the fluid market, manufacturing market within-payment quotas, and manufacturing market over-payment quotas. Prices in the manufacturing markets are set by the government based on a deficiency payment program. For manufacturing milk sold within-payment quotas, prefectural milk marketing boards (the consignment milk sellers for farmers) receive deficiency payments equal to the difference between the guaranteed price and the standard transaction price for manufacturing milk. Both prices are determined by the national government: the guaranteed price is based on milk production costs, while the standard transaction price is based on dairy product market conditions, and all buyers of manufacturing milk are required to pay this price. To discourage excess production, over-payment quota manufacturing milk receives the lower standard transaction price. Payment quotas for the

guaranteed price are not given to individual producers, but to each prefectural milk marketing board. Individual producers are paid the prefecture-wide uniform pooled price (weighted average prices for milk sold in the fluid and manufacturing milk markets).

Given manufacturing milk prices determined by the government, discriminated price formation for fluid milk occurs through negotiations between each prefectural milk marketing board and the processors it supplies. Since the fluid milk market is more price inelastic than the manufacturing milk market, the fluid market has higher prices. The structure of the Japanese milk market includes an oligopolistic group of consignment milk sellers (prefectural milk marketing boards) who allocate milk to maximize sales revenues, and a large number of perfectly competitive producers who receive pooled returns (blend prices). We refer to this situation as a "dual structure" because dairy farmers are perfectly competitive in producing milk, while they are oligopolistic in selling it through their milk marketing boards. Previous spatial price equilibrium models have not accounted for this "dual structure" in the Japanese milk market.

Conceptual Model

Consider n milk producing and consuming regions with the geographical scope of producing Region i the same as consuming Region i . In each consuming region, there are three administratively different markets: the fluid milk market (FMM_i), the manufacturing milk market within-payment quota (WPQ_i), and the manufacturing milk market over-payment quota (OPQ_i). Unit transportation cost for shipping raw milk from producing Region i to consuming Region j (T_{ij}) is assumed to be the same for both fluid and manufacturing milk.

Buyers of fluid milk in each consuming region are assumed to behave as price takers, which is reasonable since there are many fluid processors in Japan. Within-payment quota milk is traded at the

fixed guaranteed price, $FP1$, and the quantity of milk is limited to the fixed-payment quota. Over-payment quota milk is traded at the lower fixed standard transaction price, $FP2$, and it is assumed that the demand for this milk is perfectly elastic. It is also assumed that each region has a linear marginal raw milk cost function and a linear fluid demand function, with all functions known by all agents (or consignment sellers).

Milk producers in Region i consign their annual milk supply, FS_i , to Agent i . Agent i 's role is to allocate farmers' milk among the $3n$ markets to maximize sales revenues net of transportation costs. The following notation is used based for the variables described above:

D_j : quantity of milk demanded in fluid market j ($j=1, 2, \dots, n$),

FS_i : quantity of raw milk supplied and consigned in Region i ($i=1, 2, \dots, n$),

PS_i : marginal revenue net of transportation costs for each market for Region i ($i=1, 2, \dots, n$),

X_{ij} : quantity of raw milk shipped from Region i to market j ($i=1, 2, \dots, n; j=1, 2, \dots, 3n$),

$X_{i(n+j)}$: quantity of raw milk shipped from Region i to the manufacturing milk market within-payment quotas (WPQ_j) ($i=1, 2, \dots, n; j=1, 2, \dots, n$),

$X_{i(2n+j)}$: quantity of raw milk shipped from Region i to the manufacturing milk market over-payment quotas (OPQ_j) ($i=1, 2, \dots, n; j=1, 2, \dots, n$),

PD_j : demand price in the fluid market j ($j=1, 2, \dots, n$),

PPP_i : producer's pooled (blend) price in Region i ($i=1, 2, \dots, n$),

$D_j = \alpha_j - \beta_j PD_j$: demand function in fluid market j ($j=1, 2, \dots, n$),

$FS_i = -v_i + \eta_i PPP_i$: marginal cost function for raw milk in Region i ($i=1, 2, \dots, n$), where PPP_i means marginal cost.

T_{ij} : unit transportation cost of shipping raw milk from producing Region i to consuming Region j ($i=1, 2, \dots, n$; $j=1, 2, \dots, 3n$),

Q_i : limited quantity (payment-quota) paid the differences between the guaranteed price (FP1) and the standard transaction price (FP2) ($i=1, 2, \dots, n$),

SP_j : shadow price of the right to sell a unit of milk in the manufacturing milk market within-payment quotas (WPQ _{j}) ($i=1, 2, \dots, n$),

R_i : total milk sales revenue net of transportation costs in Region i ($i=1, 2, \dots, n$).

Using the above notation, Agent i 's milk sales revenue maximization problem net of transportation costs can be expressed as:

$$(1) \quad \text{Max: } R_i = \sum_{j=1}^n PD_j X_{ij} + \sum_{j=1}^n FP1 \times X_{i(n+j)} + \sum_{j=1}^n FP2 \times X_{i(2n+j)} - \sum_{j=1}^{3n} T_{ij} X_{ij}.$$

Total revenue maximization problem for all n agents is expressed as:

$$(2) \quad \text{Max. } \sum_{i=1}^n R_i.$$

Agent i 's fluid sales revenue in market j ($PD_j X_{ij}$) can be written as:

$$(3) \quad \begin{aligned} PD_j X_{ij} &= [\alpha_j/\beta_j - (1/\beta_j)D_j]X_{ij} \\ &= [\alpha_j/\beta_j - (1/\beta_j)(\sum_{i=1}^n X_{ij})]X_{ij} \\ &= [\alpha_j/\beta_j - (1/\beta_j)(\sum_{m \neq i} X_{mj} + X_{ij})]X_{ij}, \end{aligned}$$

where m ($m \neq i$) indicates all agents other than i . When Agent i believes that a change in his fluid supply will cause changes in all other agents' fluid supply to market j , Agent i 's "perceived" marginal fluid revenue in market j is:

$$(4) \quad \begin{aligned} \partial(PD_j X_{ij})/\partial X_{ij} &= [\alpha_j/\beta_j - (1/\beta_j)D_j] - (1/\beta_j)(\partial \sum_{m \neq i} X_{mj}/\partial X_{ij} + 1)X_{ij} \\ &= PD_j - (1/\beta_j)(r_{ij} + 1)X_{ij}, \end{aligned}$$

where r_{ij} is Agent i 's conjectural variation regarding changes in all other agents' fluid supply to market j caused by a change in Agent i 's supply.

Using the relationship (4), the total revenue maximization problem for all n agents can be re-specified as the following net social payoff maximization problem adjusted for imperfectly competitive markets (ANSP):

$$(5) \quad \text{Max: ANSP} = \sum_{j=1}^n \int [\alpha_j/\beta_j - (1/\beta_j)D_j]dD_j + \sum_{j=1}^n \sum_{i=1}^n \text{FP1} \times X_{i(n+j)} + \sum_{j=1}^n \sum_{i=1}^n \text{FP2} \times X_{i(2n+j)} \\ - \sum_{j=1}^n \sum_{i=1}^n (1/\beta_j)(r_{ij} + 1) \int X_{ij}dX_{ij} - \sum_{j=1}^n \sum_{i=1}^n T_{ij}X_{ij}$$

$$(6) \quad \text{s.t. } D_j \leq \sum_{i=1}^n X_{ij}, \quad \text{for all } j,$$

$$(7) \quad \sum_{i=1}^n X_{i(n+j)} \leq Q_j, \quad \text{for all } j,$$

$$(8) \quad \sum_{j=1}^n X_{ij} \leq \text{FS}_i, \quad \text{for all } i,$$

$$(9) \quad D_j \geq 0, X_{ij} \geq 0, \quad \text{for all } i \text{ and } j.$$

The difference between ANSP in (5) and the net social payoff (NSP) in the conventional spatial competitive equilibrium model by Takayama and Judge is the term: $-\sum_{j=1}^n \sum_{i=1}^n (1/\beta_j)(r_{ij} + 1) \int X_{ij}dX_{ij}$. When the market is perfectly competitive ($r_{ij} = -1$), the term is zero and (5) is equal to the original Takayama and Judge model. When Cournot-Nash behavior is assumed ($r_{ij} = 0$), the term is equivalent to: $-\sum_{j=1}^n \sum_{i=1}^n (1/\beta_j) \int X_{ij}dX_{ij}$, which is shown in Hashimoto's spatial Nash equilibrium model. Cournot-Nash behavior means that Agent i believes that the other agents will not change their supply in response to the agent's action.

Using the Lagrange function (L) with the multipliers, λ , ω , and θ for the constraints (6), (7), and (8), respectively, the Kuhn-Tucker optimality conditions for the maximization problem can be expressed as:

- (10) $\partial L / \partial D_j = \alpha_j / \beta_j - (1 / \beta_j) D_j - \lambda \leq 0$, $D_j (\partial L / \partial D_j) = 0$, for all j ,
- (11) $\partial L / \partial X_{ij} = -(1 / \beta_j)(r_{ij} + 1)X_{ij} - T_{ij} + \lambda - \theta \leq 0$, $X_{ij}(\partial L / \partial X_{ij}) = 0$, for all i and j ,
- (12) $\partial L / \partial X_{i(n+j)} = FP1 - T_{ij} - \omega - \theta \leq 0$, $X_{i(n+j)}(\partial L / \partial X_{i(n+j)}) = 0$, for all i and j ,
- (13) $\partial L / \partial X_{i(2n+j)} = FP2 - T_{ij} - \theta \leq 0$, $X_{i(2n+j)}(\partial L / \partial X_{i(2n+j)}) = 0$, for all i and j ,
- (14) $\partial L / \partial \lambda = D_j - \sum_{i=1}^n X_{ij} \leq 0$, $\lambda(\partial L / \partial \lambda) = 0$, for all j ,
- (15) $\partial L / \partial \omega = \sum_{i=1}^n X_{i(n+j)} - Q_j \leq 0$, $\omega(\partial L / \partial \omega) = 0$, for all j ,
- (16) $\partial L / \partial \theta = \sum_{j=1}^3 X_{ij} - FS_i \leq 0$, $\theta(\partial L / \partial \theta) = 0$, for all i .

The Lagrange multipliers (or dual variables), λ , ω , and θ , measure the fluid demand price (PD_j), the shadow price for the right to sell milk in the within-payment quota manufacturing market (SP_j), and marginal revenue net of transportation costs for each market (PS_i), respectively. The Kuhn-Tucker conditions represented by (11), (12), and (13) indicates that each agent must equalize marginal revenue net of transportation costs across all markets where it sells milk. The equilibrium values can be calculated by the quadratic programming model solution.

The term, $(1 / \beta_j)(r_{ij} + 1)X_{ij}$, in (11) indicates the difference between the fluid demand price and Agent i 's marginal revenue in market j . The greater the degree of market power by agents, the larger this difference. For example, in the case of perfect competition, the term becomes zero because $r_{ij} = -1$. On the other hand, the term becomes $(1 / \beta_j)X_{ij}$ when Cournot-Nash behavior ($r_{ij} = 0$) is assumed. In this paper, Cournot-Nash behavior is assumed to illustrate the imperfect competition solution, and coalition among agents is treated as follows. To illustrate, consider Cournot-Nash Agent 1 whose "perceived" marginal revenue in fluid market j is $PD_j - (1 / \beta_j)X_{1j}$. If Agent 1 forms a coalition with Agent 2, then marginal revenue for Agents 1 and 2's coalition is $PD_j - (1 / \beta_j)(X_{1j} + X_{2j})$. In the case of

monopoly where Agent 1 forms a coalition with all other agents, marginal revenue for Agent 1 is $PD_j - (1/\beta_j)(\sum_{i=1}^n X_{ij})$. Because any agent can sell the consigned milk individually or in coalition with some other agents, as a price taker or according to Cournot-Nash behavior, many combinations of agents' marketing behavior can be simulated. A tableau formulation and description of the model is presented in Appendix 1.

To complete the model, individual farmers' milk supply needs to be incorporated. Unlike the oligopolistic marketing behavior of agents, individual farmers' milk production is competitively determined. Producers in Region i , as price takers, determine their supply given the producer pooled price. That is, their production level is determined by equating marginal cost to the producer pooled price. Thus,

$$(17) \quad PPP_i = R_i/FS_i \text{ for all } i,$$

$$(18) \quad FS_i = -v_i + \eta_i PPP_i \text{ for all } i.$$

In the comparative static equilibrium, FS_i in (18) must be equal to FS_i given in the above milk sales maximization problem. To solve the model, the following iterative solution process is used to find equilibrium values for FS_i .

First, the quadratic programming model is used to generate equilibrium fluid milk prices and equilibrium quantities of milk shipments in the sales maximization problem expressed by (5) to (16), based on initial values for FS_i and given patterns of behavior of agents in the oligopolistic milk market. Second, producer pooled prices are calculated in (17). Third, new values of FS_i for the next iteration are computed based on the calculated producer pooled prices and marginal cost functions of producing regions, and the assumption that producers behave as price takers in (18). Finally, the quadratic programming problem is solved again with new parameter values for FS_i to obtain new

equilibrium fluid milk prices and quantities of milk shipments. This iteration process is continued until values for FS_i become stationary. For a more detailed explanation of the solution procedures using a tableau format, see Appendix 2.

An Application of the Model to the Japanese Milk Market

This model is applied to the Kyushu area of Japan as a case study. Region 1 includes Fukuoka, Saga, and Nagasaki prefectures, Region 2 is the Kumamoto prefecture, Region 3 is the Oita prefecture, and Region 4 includes Miyazaki and Kagoshima prefectures.

Based on the long run price elasticity of Kyushu milk supply by Ito (0.429), the Kyushu fluid demand price elasticity by Suzuki and Kobayashi (-0.77), and the regional price and quantity observations in Table 1, the linear marginal cost and fluid milk demand functions for each region are:

$$\begin{aligned} FS_1 &= 135.162 + 0.967PPP_1, & D_1 &= 361.434 - 1.438PD_1, \\ FS_2 &= 118.078 + 0.832PPP_2, & D_2 &= 181.071 - 0.666PD_2, \\ FS_3 &= 43.490 + 0.293PPP_3, & D_3 &= 88.146 - 0.324PD_3, \\ FS_4 &= 119.121 + 0.874PPP_4, & D_4 &= 163.371 - 0.639PD_4, \end{aligned}$$

where FS_i and D_j are measured by thousand tons, and PPP_i and PD_j are measured by yen/kg. Unit transportation costs, T_{ij} , are: $T_{12}=T_{21}=4.58$, $T_{13}=T_{31}=3.95$, $T_{14}=T_{41}=7.80$, $T_{23}=T_{32}=4.71$, $T_{24}=T_{42}=6.11$, and $T_{34}=T_{43}=6.00$ yen/kg. Because little milk is traded between Kyushu and other regions of Japan, this milk is treated as exogenous to simplify the model. Payment quotas Q_i for the four regions are: $Q_1=34.0$, $Q_2=32.9$, $Q_3=8.1$, and $Q_4=39.1$ thousand tons. The fixed guaranteed price for within-payment quota is $FP1 = 79.83$ yen/kg, and the fixed standard transaction price for over-payment quota is $FP2 = 67.25$ yen/kg.

To demonstrate how solutions vary based on market structure assumption, the model is solved for perfect competition, monopoly, and imperfect competition scenarios. To represent the perfectly competitive solution, the model is solved assuming that the four agents are all price takers. For the monopoly solution, the model is solved with the assumption that there is a coalition of four agents. To represent imperfect competition, 15 separate combinations with price takers and Cournot-Nash players are solved. In the first case, the four agents are all individual Cournot-Nash players (Cournot-Nash equilibrium). In the next cases, one agent is a price taker and the other three are individual Cournot-Nash players thereby creating four combinations of market structure. For cases six to 11, two agents are price takers and the other two are individual Cournot-Nash players thereby creating six new combinations of market structure. Finally, in the last four cases, three agents are price takers and the other one is a Cournot-Nash player thereby creating four combinations. Although there are other combinations with coalitions, they are not analyzed since the purpose here is to simply demonstrate examples of imperfectly competitive solutions.

The “dual structure” spatial perfect competition solution is shown in Table 2. In this case, virtually all raw milk is allocated to the fluid market, except for a trivial amount shipped to the within-payment quota manufacturing milk market in Region 4. There is also only a small amount of interregional shipments of fluid milk, mostly to Region 1. The amount of milk allocated to the fluid market in the perfect competition solution is substantially higher than the actual amount allocated (see Table 1). This is due to the assumption that agents act as price takers, which results in equality of price across markets net of transportation costs instead of equality across markets of “perceived” marginal revenue net of transportation costs. Consequently, fluid milk prices and producer pooled prices in the perfect competition case are much lower than actual levels.

The "dual structure" spatial monopoly solution is shown in Table 3. In this case, the allocation of raw milk to the fluid market is roughly one-half of the amount allocated under perfect competition, and also less than actual levels (Table 1). Instead, the monopoly solution allocates significant amounts of raw milk to the within- and over-payment quota manufacturing milk markets. The model predicts no interregional shipment of milk in all three markets. Because the own price elasticity of fluid milk demand is inelastic, restricting allocations to the fluid milk market results in higher pooled returns to farmers. In fact, producer pooled prices under monopoly are 30 percent higher than the perfect competition case, as well as 10 percent higher than actual prices. It should be noted that the monopoly distribution of pooled returns to farmers is based on the assumption that the difference in producer pooled price among regions is the same as the differentials generated in the perfect competition solution. Alternatively, one national producer pooled price for all regions could have been allocated. It should also be noted that total milk supply is largest in monopoly equilibrium under the "dual structure." Unless agents have power to control supply, individual producers increase milk supply as higher blend prices are given. Real monopoly rents cannot be realized under the "dual structure."

The Cournot-Nash equilibrium is shown in Table 4. The regional fluid milk and producer pooled prices in this solution are the closest to actual prices for the four regions (Table 1). Not surprisingly, the allocation of raw milk among the three markets in this case is somewhere between the perfect competition and monopoly cases. Unlike the two previous cases, however, the Cournot-Nash equilibrium solution results in the same two regions shipping milk to each other, e.g., Region 2 ships 51,400 tons of fluid milk to Region 1, and Region 1 ships 29,500 tons of fluid milk to Region 2.

While these shipping patterns are unintuitive, they do occur in reality as is clear from Table 1. The

other two spatial competition models did not predict these interregional milk shipment patterns. This suggests that the current complicated interregional milk movements may be caused by imperfectly competitive behavior.

Compared with the other imperfect competition cases where at least one region is assumed to be a price taker (an example is given in Table 5), fluid and producer pooled prices in the Cournot-Nash equilibrium solution in Table 4 are closer to actual prices. Price takers' returns tend to be greater than Cournot-Nash players when both price takers and Cournot-Nash players exist as is shown in Table 5. This is because Cournot-Nash agents try to keep fluid milk prices higher based on their "perceived" marginal revenues, and price takers obtain benefits by moving their milk to the fluid milk markets. In this case, acting as a price taker is like "cheating" in a cartel agreement.

Conclusion

The traditional spatial equilibrium model assumes that market structure is either perfectly competitive or monopolistic. In this paper, a new, generalized "dual structure" spatial imperfect competition equilibrium model was developed which incorporates any degree of market structure from perfect competition to monopoly. The model, which was applied to the Japanese milk market as a case study, incorporated a "dual structure" in which there are oligopolistic consignment sellers and many perfectly competitive small-scale producers with pooled returns given.

Using the model, many spatial equilibrium solutions in the Japanese milk market were demonstrated assuming alternative sets of imperfectly competitive behavior with the "dual structure."

The results indicate that under perfect competition, most milk was shipped to fluid markets, and there were very few interregional milk movements and much lower milk prices than actual price

levels. Under monopoly, much less milk was shipped to fluid markets, there was no interregional milk movement, and milk prices were much higher than the perfect competition solution and actual price levels. The Cournot-Nash equilibrium solutions were the most similar to actual observations, and the actual interregional milk movements could be explained by assuming imperfectly competitive behavior.

References

- Enke, S. "Equilibrium among Spatially Separated Markets: Solution by Electric Analogue." *Econometrica* 19(1951):40-7.
- Hashimoto, H. "A Spatial Nash Equilibrium Model." In *Spatial Price Equilibrium: Advances in Theory, Computation and Application*, edited by Harker, P.T., 20-40. New York: Springer-Verlag, 1985.
- Hayashi, M. "Market Adjustment of Milk in Japan." In *Excessive Supply and Market Adjustment of Agricultural Products*, edited by K. Tsuchiya, 145-65. Tokyo: Norin-Tokei-Kyokai, 1984.
- Ito, F. *A Spatial Equilibrium Analysis on Regional Milk Trade*. Snow Brand Research Institute Report No.54. Sapporo, 1989.
- Judge, G.G., and T. Takayama. *Studies in Economic Planning over Space and Time*. Amsterdam: North-Holland, 1973.
- Kobayashi, K. *Prices and Market Adjustment of Milk*. Tokyo: Taimei-do, 1983.
- McDowell, F.H., *Domestic Dairy Marketing Policy: An Interregional Trade Approach*. Ph.D. Thesis, Univ. of Minnesota, 1982.
- Rayner, A.J. *Regional Price Differentiation Policies for Milk in England and Wales: An Empirical Evaluation Using a Spatial Equilibrium Framework*. Dept. of Agr. Econ. Bull. No. 149, Univ. of Manchester, 1975.
- Samuelson, P.A. "Spatial Price Equilibrium and Linear Programming." *Amer. Econ. Rev.* 42(1952):283-303.
- Sasaki, K. "Spatial Equilibrium in Eastern Japan Milk Market." *J. Rural Econ.* 41(1969):106-16.
- Suzuki, N., and K. Kobayashi. "Equilibrium in Interregional Milk Transportation." *J. Rural Econ.* 64(1993):221-32.
- Takayama, T., and G.G. Judge. "Equilibrium among Spatially Separated Markets: A Reformulation." *Econometrica* 32(1964):510-24.
- Takayama, T., and G.G. Judge. "Spatial Equilibrium and Quadratic Programming." *J. Farm Econ.* 46(1964):67-93.
- Takayama, T., and G.G. Judge. *Spatial and Temporal Price and Allocation Models*. Amsterdam: North-Holland, 1971.

Table 1. Observations in 1989 (unit: 1,000 tons and yen/kg)

\To From\	Fluid Milk Market				Manufacturing Milk Market								Total
					(Within Quota)				(Over Quota)				
	1	2	3	4	1	2	3	4	1	2	3	4	
1	128.4	19.1	12.0	1.7	34.0	0	0	0	10.9	0	0	0	206.1
2	33.1	74.7	1.5	1.6	0	32.9	0	0	0	0	0	0	143.8
3	31.4	0	34.3	0	0	0	8.1	0	0	0	1.2	0	75.1
4	11.3	8.5	2.0	89.0	0	0	0	39.1	0	0	0	8.3	158.0
Total	204.2	102.3	49.8	92.3	34.0	32.9	8.1	39.1	10.9	0	1.2	8.3	583.0

Region i or j	Fluid Milk Price PD _j	Producer's Pooled Price PPP _i
1	109.35	101.62
2	118.22	103.21
3	118.22	107.75
4	111.20	99.07
Average	112.74	102.11

Source: Suzuki and Kobayashi.

Table 2. "Dual-Structure" Spatial Perfect Competition Equilibrium (unit: 1,000 tons and yen/kg)

\To From\	Fluid Milk Market				Manufacturing Milk Market (Within Quota)				Manufacturing Milk Market (Over Quota)				Total	
	1	2	3	4	1	2	3	4	1	2	3	4		
1	193.2	0	0	0	0	0	0	0	0	0	0	0	0	193.2
2	4.8	125.8	0	0	0	0	0	0	0	0	0	0	0	130.5
3	7.0	0	61.0	0	0	0	0	0	0	0	0	0	0	68.0
4	30.5	0	0	112.4	0	0	0	1.4	0	0	0	0	0	144.3
Total	235.4	125.8	61.0	112.4	0	0	0	1.4	0	0	0	0	0	536.0

Region i or j	Fluid Milk Price	Agent's Marginal Revenue ^a	Producer's Pooled Price ^b
	PD _i	PS _i	PPP _i
1	87.63	87.63	88.23
2	83.05	83.05	87.19
3	83.68	83.68	83.68
4	79.83	79.83	83.32
Average	84.46	84.46	86.07

^aPS_i is Agent i's "perceived" marginal revenue (net of transportation costs) equalized in each market (marginal revenue = market price in perfect competition) (Same in Tables 3 to 5).

^bExogenously given milk shipments from each region to the outside of Kyushu are taken into account in calculating PPP_i (Same in tables 3 to 5).

Table 3. "Dual-Structure" Spatial Monopoly Equilibrium (unit: 1,000 tons and yen/kg)

\To From\	Fluid Milk Market				Manufacturing Milk Market (Within Quota)				Manufacturing Milk Market (Over Quota)				Total
	1	2	3	4	1	2	3	4	1	2	3	4	
1	132.4	0	0	0	34.0	0	0	0	51.8	0	0	0	218.1
2	0	68.1	0	0	0	32.9	0	0	0	50.9	0	0	152.0
3	0	0	33.2	0	0	0	8.1	0	0	0	34.3	0	75.6
4	0	0	0	60.2	0	0	0	39.1	0	0	0	67.5	166.8
Total	132.4	68.1	33.2	60.2	34.0	32.9	8.1	39.1	51.8	50.9	34.3	67.5	612.5

Region i or j	Fluid Milk Price PD_i	Agent's Marginal Revenue PS_i	Producer's Pooled Price ^a PPP_i
1	159.30	67.25	114.03
2	169.56	67.25	112.99
3	169.65	67.25	109.48
4	161.46	67.25	109.13
Average	163.29	67.25	111.87

^aEstimated PPP differentials in the perfect competition equilibrium in Table 2 are used to allocate monopoly pooled returns and to calculate PPP_i of each region.

Table 4. "Dual-Structure" Spatial Cournot-Nash Equilibrium (unit: 1,000 tons and yen/kg)

\To From\	Fluid Milk Market				Manufacturing Milk Market (Within Quota)				Manufacturing Milk Market (Over Quota)				Total
	1	2	3	4	1	2	3	4	1	2	3	4	
1	64.6	29.5	14.5	24.7	34.0	6.6	8.1	0	20.5	0	0	0	202.6
2	51.4	29.5	12.8	22.9	0	26.3	0	0	0	0	0	0	142.9
3	33.5	17.6	10.1	14.6	0	0	0	0	0	0	0	0	75.7
4	50.6	27.2	13.2	28.5	0	0	0	39.1	0	0	0	0	158.6
Total	200.1	103.8	50.6	90.7	34.0	32.9	8.1	39.1	20.5	0	0	0	579.7

Region i or j	Fluid Milk Price PD_j	Agent's Marginal Revenue PS_i	Producer's Pooled Price PPP_i
1	112.18	67.25	97.94
2	116.10	71.83	102.09
3	115.99	84.96	109.91
4	113.76	69.20	99.70
Average	113.85	70.73	100.89

Table 5. "Dual-Structure" Spatial Equilibrium in the Case Where Agent 1 is a Price taker and the Others are Individual Cournot-Nash Players (unit: 1,000 tons and yen/kg)

\To From\	Fluid Milk Market				Manufacturing Milk Market (Within Quota)				Manufacturing Milk Market (Over Quota)				Total	
	1	2	3	4	1	2	3	4	1	2	3	4		
1	107.9	47.8	24.1	28.1	0	0	0	0	0	0	0	0	0	207.9
2	36.0	22.8	9.4	20.0	16.3	32.9	0	0	0	0	0	0	0	137.4
3	30.6	16.7	9.5	17.3	0	0	0	0	0	0	0	0	0	74.0
4	36.0	20.9	10.0	26.0	13.0	0	8.1	39.1	0	0	0	0	0	153.0
Total	210.6	108.2	52.9	91.4	29.2	32.9	8.1	39.1	0	0	0	0	0	572.3

Region i or j	Fluid Milk Price PD_j	Agent's Marginal Revenue PS_j	Producer's Pooled Price PPP_j
1	104.89	104.89	103.45
2	109.47	75.25	95.45
3	108.84	79.68	104.16
4	112.69	72.03	93.37
Average	107.95	88.60	98.43

Appendix 1 The Quadratic Programming Model in Tableau Form

A convenient way to explain a quadratic programming model is using Tableau form.

Appendix Table 1 briefly expresses the following concave quadratic programming problem. The problem is to find values for primal variables X_j ($j=1, 2, \dots, s$) and dual variables $U_i \geq 0$ ($i=1, 2, \dots, r$) which maximize the objective function F :

$$\text{Max: } F = \sum_{j=1}^s (c_j X_j - (1/2) w_j X_j^2),$$

subject to:

$$a_{11}X_1 + a_{12}X_2 + \dots + a_{1s}X_s \leq b_1 ; U_1$$

$$a_{21}X_1 + a_{22}X_2 + \dots + a_{2s}X_s \leq b_2 ; U_2$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$a_{r1}X_1 + a_{r2}X_2 + \dots + a_{rs}X_s \leq b_r ; U_r$$

$$w_j, X_j \geq 0 \quad (j=1, 2, \dots, s),$$

where a_{ij} , b_i , c_j , and w_j ($i=1, 2, \dots, r$; $j=1, 2, \dots, s$) are constants.

The quadratic programming model in (5) to (16) in the text is expressed in quantity formulation with quantities as the primal variables. The model can be also expressed in price formulation with PD_j , SP_j , and PS_i as primal variables. It is advantageous to solve the model under the price formulation because it is easier to solve computationally. The price formulation using a tableau format defined in Appendix Table 1 is shown in Appendix Table 2.

The following vectors of constants are used in Appendix Table 2: $\alpha = (\alpha_1 \ \alpha_2 \ \dots \ \alpha_n)$, $\beta = (\beta_1 \ \beta_2 \ \dots \ \beta_n)$, $FS = (FS_1 \ FS_2 \ \dots \ FS_n)$, $h = (1 \ 1 \ \dots \ 1)$ (the dimension is k), $Q = (Q_1 \ Q_2 \ \dots \ Q_n)$, $T_{.j} = (T_{1j} \ T_{2j} \ \dots \ T_{nj})'$ ($j=1, 2, \dots, n$), $e = (1 \ 1 \ \dots \ 1)'$, where ' indicates transpose of a matrix. The following

vectors of variables are used in Appendix Table 2: $PD=(PD_1 PD_2 \dots PD_n)$, $PS=(PS_1 PS_2 \dots PS_n)$, $q_j=(q_{1j} q_{2j} \dots q_{kj})$ ($j=1, 2, \dots, n$), $SP=(SP_1 SP_2 \dots SP_n)$, $X_j=(X_{1j} X_{2j} \dots X_{nj})'$ ($j=1, 2, \dots, 3n$), where q_{ij} ($i=1, 2, \dots, k$) means the difference between PD_j and marginal revenue of coalition i in market j ($j=1, 2, \dots, n$), which is equal to $(1/\beta_j)(r_{ij} + 1)X_{ij}$ in the relationship (11) and $q_{ij} = (1/\beta_j)X_{ij}$ in the following cases where Cournot-Nash behavior is assumed. E_j is an $n \times n$ matrix, and any element of j th column is 1, but any element of i th ($i \neq j$) column is 0, where $j=1, 2, \dots, n$.

$$E_j = \begin{array}{c} \left[\begin{array}{cccccc} 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \\ \left| \begin{array}{cccccc} 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right| \\ \left| \begin{array}{cccccc} 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right| \\ \left[\begin{array}{cccccc} 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right] \\ : \\ \text{-----} \rightarrow \text{jth column} \end{array}$$

I is an $n \times n$ identity (unit) matrix.

Coalition among agents can be expressed by matrix A . There are k ($k \leq n$) coalitions behaving as Cournot-Nash players. Any coalition may be composed of only one agent, but agents behaving as price takers do not belong to any coalitions. The patterns of behavior for agents in the oligopolistic milk market are summarized below by matrix A , which has n rows corresponding to each agent and k columns corresponding to each coalition. The element, A_{ij} , of matrix A corresponding to the i th row and the j th column is either -1 or 0 ($i=1,2, \dots, n; j=1,2, \dots, k$). A_{ij} is -1 if and only if Agent i belongs to coalition j , and A_{ij} is 0 for any j if and only if Agent i behaves as a price taker.

Appendix 2 The Solution Process for the "Dual Structure" Model for two Agents

[Step 1] Specify matrix A, values for payment quota $Q_1, Q_2, \dots,$ and Q_n , and initial values for $FS_1, FS_2, \dots,$ and FS_n . To illustrate, consider a simple case of $n=2$ as shown in Appendix Table 3.

The $n=2$ case corresponds to the following matrix A:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \equiv A1.$$

Matrix A1 illustrates the case where there are two coalitions: one is composed of Agent 1, and the other is composed of Agent 2. In this instance, Agents 1 and 2 behave individually as Cournot-Nash players. Another possibility is the case where there is a coalition composed of Agent 1, who behaves as a Cournot-Nash player, but Agent 2 behaves as a price taker. In this case, A1 would be replaced by the following matrix A:

$$A = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \equiv A2.$$

In this case, the columns for variables q_{21} and q_{22} in Appendix Table 3 should be deleted. If Agents 1 and 2 behave as price takers, then matrix A has no columns and is empty, therefore columns for variables $q_{11}, q_{21}, q_{12},$ and q_{22} in Appendix Table 3 should be deleted. Another case would be where Agent 1 behaves in coalition with Agent 2 as a Cournot-Nash player. Under this situation, the following matrix A would be used:

$$A = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \equiv A3.$$

This matrix shows that there is a coalition composed of Agents 1 and 2, therefore columns for variables q_{21} and q_{22} in Appendix Table 3 should be deleted, and also columns for variables q_{11} and q_{12} should be revised according to matrix A3.

[Step 2] Solve the quadratic programming model to get equilibrium fluid milk demand prices PD_j ($j=1, 2, \dots, n$) and equilibrium quantities of milk shipments X_{ij} ($i=1, 2, \dots, n; j=1, 2, \dots, 3n$) corresponding to the given specification. To illustrate, consider the following smaller two-region example. In this case, there are two regions, and producers in producing Region 1 (2) consign their milk supply FS_1 (FS_2) to Agent 1 (2). Agents 1 and 2 sell the consigned milk individually as Cournot-Nash players in six ($=3 \times 2$) oligopolistic milk markets: FMM_1 (market 1), FMM_2 (market 2), WPQ_1 (market 3), WPQ_2 (market 4), OPQ_1 (market 5), and OPQ_2 (market 6). Milk is traded in market 3 (4) at the fixed guaranteed price FP_1 within the limit of payment quota Q_1 (Q_2), and milk is also traded in markets 5 and 6 at the fixed standard transaction price FP_2 without any limit of demand quantity. The Kuhn-Tucker optimality conditions for the $n=2$ problem are the following: each agent's opportunity costs moving a unit of milk from market to market (or marginal revenue net of transportation costs for each market) PS_1 and PS_2 must satisfy the following conditions:

$$X_{11}+X_{12}+X_{13}+X_{14}+X_{15}+X_{16} \leq FS_1, \quad (\text{RES.})PS_1=0.$$

$$X_{21}+X_{22}+X_{23}+X_{24}+X_{25}+X_{26} \leq FS_2, \quad (\text{RES.})PS_2=0,$$

where (RES.) means the difference between the right and left hand sides of the corresponding inequality.

The fluid milk demand prices PD_1 and PD_2 must satisfy the following:

$$\alpha_1 - \beta_1 PD_1 \leq X_{11} + X_{21}, \quad (\text{RES.})PD_1=0.$$

$$\alpha_2 - \beta_2 PD_2 \leq X_{12} + X_{22}, \quad (\text{RES.})PD_2=0.$$

The differences q_{11} , q_{21} , q_{12} , and q_{22} must satisfy the following:

$$X_{11} \leq \beta_1 q_{11}, \quad (\text{RES.})q_{11}=0.$$

$$X_{21} \leq \beta_1 q_{21}, \quad (\text{RES.})q_{21}=0.$$

$$X_{12} \leq \beta_2 q_{12}, \quad (\text{RES.})q_{12}=0.$$

$$X_{22} \leq \beta_2 q_{22}, \quad (\text{RES.})q_{22}=0.$$

The shadow prices SP_1 and SP_2 must satisfy the following:

$$X_{13}+X_{23} \leq Q_1, \quad (\text{RES.})SP_1=0.$$

$$X_{14}+X_{24} \leq Q_2, \quad (\text{RES.})SP_2=0.$$

The shipment quantities X_{11} , X_{21} , X_{12} , and X_{22} must satisfy the following:

$$(PD_1-q_{11})-PS_1 \leq T_{11}, \quad (\text{RES.})X_{11}=0.$$

$$(PD_1-q_{21})-PS_2 \leq T_{21}, \quad (\text{RES.})X_{21}=0.$$

$$(PD_2-q_{12})-PS_1 \leq T_{12}, \quad (\text{RES.})X_{12}=0.$$

$$(PD_2-q_{22})-PS_2 \leq T_{22}, \quad (\text{RES.})X_{22}=0.$$

The shipment quantities X_{13} , X_{23} , X_{14} , and X_{24} must satisfy the following:

$$(FP_1-SP_1)-PS_1 \leq T_{11}, \quad (\text{RES.})X_{13}=0.$$

$$(FP_1-SP_1)-PS_2 \leq T_{21}, \quad (\text{RES.})X_{23}=0.$$

$$(FP_1-SP_2)-PS_1 \leq T_{12}, \quad (\text{RES.})X_{14}=0.$$

$$(FP_1-SP_2)-PS_2 \leq T_{22}, \quad (\text{RES.})X_{24}=0.$$

The shipment quantities X_{15} , X_{25} , X_{16} , and X_{26} must satisfy the following:

$$FP_2-PS_1 \leq T_{11}, \quad (\text{RES.})X_{15}=0.$$

$$FP_2-PS_2 \leq T_{21}, \quad (\text{RES.})X_{25}=0.$$

$$FP_2-PS_1 \leq T_{12}, \quad (\text{RES.})X_{16}=0.$$

$$FP_2-PS_2 \leq T_{22}, \quad (\text{RES.})X_{26}=0.$$

We can get the following relations between the optimal values for q_{ij} and X_{ij} ($i=1,2; j=1,2$)

from the Kuhn-Tucker optimality condition for the problem.

$$X_{ij} \leq \beta_j q_{ij}, (\beta_j q_{ij} - X_{ij}) q_{ij} = 0, X_{ij} \geq 0, q_{ij} \geq 0,$$

Or more conveniently, using slack variables v_{ij} ,

$$X_{ij} + v_{ij} = \beta_j q_{ij}, v_{ij} q_{ij} = 0, v_{ij} \geq 0,$$

$$\text{and therefore } v_{ij} X_{ij} + v_{ij}^2 = \beta_j v_{ij} q_{ij} = 0.$$

Thus, we get

$$v_{ij} = 0, q_{ij} = (1/\beta_j) X_{ij}.$$

Let the total quantity of milk shipped from agents other than Agent i (coalition i) to market j be denoted by x . Then marginal revenue MR_{ij} of Agent i in market j can be written as follows.

$$\begin{aligned} MR_{ij} &= d[(\alpha_j/\beta_j) - (1/\beta_j)(X_{ij} + x)] X_{ij} / dX_{ij} \\ &= [(\alpha_j/\beta_j) - (1/\beta_j)(X_{ij} + x)] - (1/\beta_j) X_{ij} \\ &= PD_j - (1/\beta_j) X_{ij}, \end{aligned}$$

where X_{ij} is the quantity of milk shipped from Agent i to market j , equation $PD_j = (\alpha_j/\beta_j) - (1/\beta_j) D_j$ is the inverse demand function in market j , and $i=1, 2; j=1, 2$.

The difference between demand price PD_j and the marginal revenue MR_{ij} is equal to $(1/\beta_j) X_{ij}$ and to the optimal value of q_{ij} , therefore MR_{ij} is equal to $(PD_j - q_{ij})$.

The Kuhn-Tucker condition further shows the following relations: (a) MR_{ij} is not larger than the sum of Agent i 's opportunity cost moving a unit of milk from market to market PS_i and unit transportation costs T_{ij} , and shipment quantity X_{ij} can be positive if and only if MR_{ij} is equal to the sum of PS_i and T_{ij} ; (b) shadow price SP_j can be positive if and only if demand quantity in WPQ_j is equal to payment quota Q_j , and a positive value of SP_j means a premium for milk sold in WPQ_j , also, $(FP1 - SP_j)$ is not larger than $(PS_i + T_{ij})$, and $X_{i(2+j)}$ can be positive if and only if $(FP1 - SP_j)$ is equal to

(PS_i+T_{ij}) ; and (c) FP2 is not larger than (PS_i+T_{ij}) , and $X_{i(4+j)}$ can be positive if and only if FP2 is equal to (PS_i+T_{ij}) , where $i=1, 2; j=1, 2$.

All these and other relations derived from the Kuhn-Tucker condition show that we can get a static equilibrium solution which corresponds to the given specification by solving the $n=2$ problem. Similarly, we can get a static equilibrium solution which corresponds to the given general specification by solving the model in Appendix Table 2. It should be noted that in the general case, the following relations hold between optimal values of vectors q_j and X_j ($j=1, 2, \dots, n$).

$$q_j' = (1/\beta_j)(-A)'X_j \text{ or } q_j = (1/\beta_j)X_j'(-A)$$

Based on this relation, we can show that the optimal value of q_{ij} ($i=1, 2, \dots, k$) is equal to the difference between demand price PD_j and marginal revenue of coalition i in market j .

[Step 3] Calculate PPP_i ($i=1, 2, \dots, n$), based on the equilibrium solution computed in Step 2.

[Step 4] Calculate the new "equilibrium" supply quantity FS_i ($i=1, 2, \dots, n$) of milk in each producing region with its marginal cost function ($FS_i=-v_i+\eta_iPPP_i$) and the value of PPP_i calculated in Step 3 (the quantity FS_i may be revised according to exogenously given trade conditions if necessary).

[Step 5] Compare the values of FS_i calculated in Step 4 with the values of FS_i currently given.

If $\max\{ | \text{the new } FS_i - \text{the last } FS_i | ; i=1, 2, \dots, n \}$ is not larger than some specified absolute stopping error, then stop computation and use the equilibrium solution computed in Step 2 as an equilibrium solution of the "dual structure" model. Otherwise, go to the next step.

[Step 6] Calculate $FS_i' = \text{the last } FS_i + (1/2)(\text{the new } FS_i - \text{the last } FS_i) = (1/2)(\text{the last } FS_i + \text{the new } FS_i)$ ($i=1, 2, \dots, n$) as the value of the "actual" supply quantity of milk in each producing region in the next iteration (it is assumed here that adjustment speed of milk supply is 0.5 per

iteration), then replace the fixed value of FS_i with the value of FS_i' in the quadratic programming model, and go to Step 2.

Appendix Table 1. Expression Formula for Quadratic Programming Problem

	C_1	C_2	...	C_s	
	w_1	w_2	...	w_s	
	X_1	X_2	...	X_s	
b_1	a_{11}	a_{12}	...	C_{1s}	U_1
b_2	a_{21}	a_{22}	...	C_{2s}	U_2
			...		
b_r	a_{r1}	a_{r2}	...	C_{rs}	U_s

Appendix Table 2. Quadratic Programming Problem for the Japanese Milk Market (Price Formulation)

	α	-FS	0	0	.	0	-Q	
	β	0	β_1h	β_2h	.	β_nh	0	
	PD	PS	q_1	q_2	.	q_n	SP	
$T_{.1}$	E_1	-I	A	0	.	0	0	$X_{.1}$
$T_{.2}$	E_2	-I	0	A	.	0	0	$X_{.2}$
.
$T_{.n}$	E_n	-I	0	0	.	A	0	$X_{.n}$
$T_{.1}-(FP1)e$	0	-I	0	0	.	0	$-E_1$	$X_{.(n+1)}$
$T_{.2}-(FP1)e$	0	-I	0	0	.	0	$-E_2$	$X_{.(n+2)}$
.
$T_{.n}-(FP1)e$	0	-I	0	0	.	0	$-E_n$	$X_{.2n}$
$T_{.1}-(FP2)e$	0	-I	0	0	.	0	0	$X_{.(2n+1)}$
$T_{.2}-(FP2)e$	0	-I	0	0	.	0	0	$X_{.(2n+2)}$
.
$T_{.n}-(FP2)e$	0	-I	0	0	.	0	0	$X_{.3n}$

OTHER A.R.M.E. RESEARCH BULLETINS

- No. 96-04 Commodity Promotion Economics: A Symposium in Honor of Olan Forker's Retirement Jennifer Ferrero
Harry M. Kaiser
(eds.)
- No. 96-05 The Magnitude of and Factors Influencing Product Losses in 141 Fluid Milk Plants in the United States Mark W. Stephenson
Jay Mattison
Richard D. Aplin
Eric M. Erba
- No. 96-06 Dairy Department Procurement Dynamics The Role of the Supermarket Buyer Edward W. McLaughlin
Debra J. Perosio
- No. 96-07 Integrating Knowledge to Improve Dairy Farm Sustainability Caroline N.
Rasmussen, ed.
- No. 96-08 A Descriptive Analysis of the Characteristics and Financial Performance of Dairy Farms in Michigan, New York, Ontario, Pennsylvania and Wisconsin S. Ford
R. Gardner
S. Gripp
S. Harsh
W. Knoblauch
A. Novakovic
L. Putnam
M. Stephenson
A. Weersink
R. Yonkers
- No. 96-09 The Feasibility of a Mid-Hudson Valley Wholesale Fresh Product Facility: A Buyer Assessment Craig Robert Kreider
Edward W. McLaughlin
- No. 96-10 Impact of National Dairy Advertising on Dairy Markets, 1984-95 Harry M. Kaiser
- No. 96-11 Dairy Farm Management Business Summary New York State 1995 Stuart F. Smith
Wayne A. Knoblauch
Linda D. Putnam