ASSET INVESTMENT AND PORTFOLIO MANAGEMENT OF SUSTAINABLE INFRASTRUCTURE SYSTEMS: OPTIMIZATION AND REAL-OPTIONS APPROACHES

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Active asset-investment and portfolio-construction strategies for infrastructure systems are developed. Two primary questions are explored: how to allocate a limited budget among the assets to enhance resilience of an infrastructure portfolio against extreme events and how to decide the most appropriate time to invest in a portfolio of infrastructure assets under information uncertainty.

A portfolio optimization model with the objective of minimizing the economic loss from extreme events under the budget resource constraint is developed. Because of the network effect of the assets in a large-scale portfolio, two types of algorithms are developed to improve the computational efficiency of portfolio selection: an algorithm that restructures the objective function as a monotonic non-increasing function with a Taylor series expansion and uses the first-order term as an approximation, which does not consider the effect of simultaneous investment in two or more assets, and a heuristic algorithm that systematically considers the network effect of the assets in the portfolio via consecutive iteration. The results show that the investment decisions given by the heuristic algorithm reduce the expected value of the economic loss given by the approximation algorithm, significantly increase the expected return on the investment portfolio, and improve the system resilience under extreme events.

A real-options-based approach is used to determine the optimal investment

time and investment criteria for a portfolio of infrastructure assets under information uncertainty. First, a single-option framework for portfolio investment under a single uncertainty is developed, and applied under different growth scenarios. The resulting investment criteria are compared to the net present value (NPV) break-even point, and illustrate the merits of the advanced real-options-based investment model in both the deterministic and stochastic cases. Theoretical considerations and real-life cases show why the NPV rule cannot yield the optimal investment decision in either scenario.

The single-option model is then extended to a multi-option framework for a portfolio of interdependent infrastructure assets under multidimensional uncertainties, with the aim of deciding the selection of assets for investment and the optimal time to invest in each of them, that is, whether investment should be made immediately or postponed to maximize the return on the portfolio. The portfolio planning analysis begins with a static NPV framework to quantify the marginal investment payoff of interacting assets and then considers the value of a growth option contingent on the information uncertainty. An algorithm based on dynamic programming and least-squares Monte Carlo simulation to search for the optimal investment decision and the compound option value of the portfolio is proposed. The results show that the multi-option model increases the investment value of the portfolio in both the deterministic and stochastic cases by allowing flexible investment timelines for individual assets. The stochastic scenario further reveals the advantage of the multi-option model as volatility increases, and shows that the model could serve as an effective dynamic adaptive decision support tool for multi-period investment in balancing the return on a portfolio versus risk by incorporating new information as it becomes available.

BIOGRAPHICAL SKETCH

Yan Deng grew up in Fangchenggang, Guangxi, China, where she finished her primary and secondary education. Later, she left her hometown to pursue an undergraduate program in transportation engineering at Southeast University in Nanjing, Jiangsu, where she received the award for the best undergraduate thesis. Based on her love for research and her desire to help render life on Earth more sustainable, she decided to continue her studies and enrolled in a graduate program at Cornell University under the supervision of Prof. Oliver Gao, in the field of infrastructure asset investment and portfolio management for sustainability.

To my mother Zhongling Yan, father Weiyin Deng

and my husband, Dr. Yu Su

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Motivation

Transportation infrastructure investments are the most critical ones for sustainable economic growth and social development, as they enable increased communication and commerce between cities, and they reduce the extent of economic loss resulting from extreme events that are due to natural and human causes. Without robust transportation infrastructure systems, socio-economic loss due to such events is inevitable.

One of the main sources of socio-economic loss is the lack of functionality of infrastructure systems under natural disaster disruption. Many regions suffer significant economic losses from natural disasters (Pelling and Uitto, 2001). For instance, the average annual loss from tropical cyclones is estimated at $835 million in the Caribbean and $178 million in the Pacific. A single disaster can even have a devastating impact on the economy of a relatively small region (SOPAC, 2009; OPCS, 2016). For example, Fijis 2015 Cyclone Pam caused damage and losses estimated at $450 million, the equivalent of 64% of GDP (OPCS, 2016).

The transportation infrastructure sector plays an important role in vulnerability to natural disasters (Hallegatte and Przyluski, 2010). In many of the developing countries, with a less developed economy and a low degree of redundancy in the transportation network, disruptions in transportation occur frequently under extreme events such as natural disasters and cause significant socio-economic loss. Damage to the transport infrastructure is very expensive to repair—and, more importantly, such disruptions sometimes isolate communities for months, and thus have a significant detrimental impact on their local economies. For example, Fiji is affected by river floods and storm surges that repeatedly damage the transport infrastructure and disrupt its use for almost half of the time each year (Iimi, 2017). When a disaster hits, critical pieces of the transport network often get destroyed, thereby generating huge economic and social impacts, as road users may have no way to get from their point of origin to their destination. For example, people may not be able to get to hospitals or shelters, basic supplies may not be able to get to customers, and workers may not be able to get to their work place. Transport disruptions can also have long-lasting negative impacts if they affect tourism and reduce the attractiveness of the country.

The infrastructure system can be treated as a network. The functionality of the network is critical. For regions with frequent disasters such as floods and landslides, pre-disaster infrastructure investment is crucial, since it improves network resilience and increases the chance that the system network will respond effectively to a disaster and remain operational. Pre-disaster transport investment plays an important role in improving national security and economic development. For instance, it will improve farmers' connectivity to markets, and consequently increase their revenues and reduce poverty, while also improving the national economy, by reducing food import needs for instance (Bell and Van Dillen, 2012). The aim of the first part of this study is to develop a pre-disaster investment model and to explore the important question of how to select a portfolio of infrastructure projects in which to invest, given a limited budget, in order to strengthen and enhance the resilience of the infrastructure network against disasters.

Another main source of socio-economic loss is human interaction. For instance, according to the California High Speed Rail Authority, California's economy loses \$18.7 billion a year because of traffic congestion. In addition, the socio-economic loss due to excess travel time and the lack of reliability is significant. Public infrastructure investment is one of the most critical activities, as it increases the efficiency of mobility and thus reduces the human-related socio-economic loss. Investment in a public infrastructure system can connect various metropolitan areas, along the busiest corridors, into an integrated system. For instance,the U.S. government has recently launched a number of public high-speed transport infrastructure investment projects. As reported by the U.S. High Speed Rail Association, a 17,000-mile national high-speed rail system will be completed by 2030 as the initial stage of planning. Upon completion, Los Angeles will be connected to San Francisco by advanced rail transport, as will New York City to Washington D.C., Chicago to St. Louis, and Seattle to Portland.

There is no denying that investment in infrastructure systems will greatly accelerate social development and sustainable economic growth; however, there are huge investment costs associated with such an undertaking. For instance, the high-speed rail segment from Los Angeles to San Francisco, which is projected to be completed in 2029, is estimated to cost approximately \$68 billion. Investment expenditures have two very important characteristics. First, the investment costs are at least partly irreversible; in other words, the sunk costs cannot be recovered after the funds are invested. Second, the investment can be delayed, so the investment authority has the opportunity to wait for new information about the price, demand, costs, and other market and travel conditions before committing the resources. Given that such investment projects are irreversible, it's crucial to carry out a thorough investment planning and feasibility analysis aimed at determining when the investment should be made, depending on the specific market and socio-economic conditions, so as to maximize the social welfare.

Investment planning and feasibility analysis depends heavily on the estimation of human behavior and social and economic factors. Demand is one of the most important economic factors. The demand for travel between the points of origin and destination directly governs the passenger levels for a new transport system, and consequently the return on investment and social welfare. However, the travel demand between cities is uncertain, and the volatility which is driven by migration as well as by business and technology development, increases the risk of investment. An investment made too early may result in a low passenger level and thus low utilization, revenue shortfalls, and capital waste, whereas an investment made too late may result in large social costs, since a delay in investment that leaves the demand unmet for too long will result in considerable additional travel time, higher fares, and environmental pollution. Therefore, the aim of the second part of the study is to propose an adaptive approach that can be used by the investment authority to explore the critical question, What is the most appropriate time to invest in a new infrastructure system under the demand uncertainty?

1.2 Infrastructure Investment Portfolio under Extreme Events

The first type of intervention that can increase the resilience of road networks is improvement in maintenance. Generally, two types of maintenance are performed on roads: routine and periodic. Routine maintenance involves all sorts of small interventions on a road and can be carried out once a year or more often, depending on the weather, the materials used, and the traffic. Periodic maintenance generally occurs every five years and includes major rehabilitation. Even in the absence of natural hazards, frequent maintenance has considerable economic benefits (Burningham and Stankevich, 2005; Harral and Faiz, 1988). It should be noted that every dollar of routine maintenance that is deferred will end up costing $5 in repairs—or ultimately $25 in rehabilitation or replacement as the asset declines over time. Maintenance management strategies for infrastructure systems have been discussed in previous research. For example, we proposed a model to decide the optimal maintenance interval and prioritize the infrastructure projects to be undertaken under the budget constraints so as to minimize the economic and environmental costs (Deng, 2016). Here we will focus on the second type of intervention: investment in and upgrading of the critical infrastructure so as to minimize post-disaster socio-economic loss under network effects.

There is no doubt that investment in transportation plays an essential role in strengthening the network and enhancing survivability. However, because of the large expenditure on construction, investment places a huge burden on the government, especially in poor developing countries. In a budget-constrained scenario, and with many roads in need of rehabilitation or repair, it is important to prioritize the infrastructure interventions in such a way as to realize the greatest socio-economic benefits and ensure reliability of the transport network under disruptive weather events. The approach that's currently used most commonly in tackling this problem is a model that's based on a weighted combination of factors. For example, the Fiji Road Association (FRA) used a factor-based model to select the infrastructure investment portfolio with the condition of the existing infrastructure used as the selection criterion. This method prioritizes the infrastructure systems that are in the worst condition, but it does not consider network effects.

Some system-level models that are intended to perform better in investment decisions under disaster conditions have been proposed. For example, Espinet et al. (2018) of the World Bank developed a methodology to identify the critical and vulnerable roads, and then to combine criticality and risk to identify which ones should take priority in the way of investment. The analysis of criticality and vulnerability helps the decision maker identify the most important links in the network system as well as the links that are of highest vulnerability and would incur the greatest repair cost in case of a disaster. However, that model does not have the capability to measure the expected improvement in network performance that would result, in case of a disaster, from investment in the infrastructure. Peeta et al. (2010) introduced a model of pre-disaster investment decisions intended for strengthening of a highway network and solved the investment optimization using an approximation algorithm. Du and Peeta (2014) developed a stochastic optimization model which is solved by a two-stage heuristic algorithm for making decisions on investment in links in a network and enhancing network survivability subject to budget constraints. Because of limitations of these investment models, however, they cannot measure the network effects of simultaneously investing in two or more projects. In addition, all the infrastructure projects in the investment portfolio must be of the same type, while in real-world situations the disaster response and recovery time vary with the type of structure. In view of these limitations, this study focuses on developing an optimization model that will capture how investments would alter the performance of the post-disaster network and provides a system-level analysis of heterogeneous types of infrastructure.

1.3 Investment under Uncertainty using Real Options

In this section, we will discuss the background for the second component, investment decision-making under uncertainty. The investment planning and feasibility analysis which is applied in current practice relies mainly on static valuation, such as cost–benefit analysis that uses the net present value to evaluate the social welfare resulting from investment in public infrastructure. For instance, Adler et al. (2010) identified the main factors that determine the economic success of high-speed rail (HSR), developed a cost–benefit analysis model, and examined a variety of possible scenarios to determine circumstances under which benefits could exceed costs. De Rus and Nombela (2007) estimated the minimum level of demand required for HSR investment in Europe for it to be considered profitable, by evaluating the costs and social benefits using the net present value (NPV) method. Chester and Horvath (2010) created an environmental life-cycle assessment model using the NPV method to compare the environmental effects of HSR in California to those of the alternatives (automobiles, heavy rail, and aircraft) in terms of vehicle operation, infrastructure, and fuel.

The results of the aforementioned studies were based mainly on the traditional method of NPV-based cost–benefit analysis. However, it has been shown that it does not suffice to support an investment decision on the basis of a positive net present value, since the traditional NPV method ignores the opportunity cost of investment that arises from forgoing the option to wait for more information in an uncertain environment. There is growing interest on the part of public-sector entities such as state and local governments and the U.S. Department of Transportation to adopt a more active style of project management (LLP, 2010). Efficient allocation of resources in investment and timely adaptation are therefore becoming increasingly important. The real-options approach is an improved valuation tool for capturing the value of flexibility, because of the irreversible and uncertain nature of investment (Dixit and Pindyck, 1994; Trigeorgis, 1996).

1.3.1 Real options with a single source of uncertainty

The term real options is often used to describe investment situations involving non-financial (i.e., real) assets together with some degree of optionality. There are usually options available to the decision-maker. More generally, real-options problems are usually control problems where the decision-maker can partially control some of the quantities under consideration. The basic real-options model with a single source of uncertainty, which was introduced by McDonald and Siegel (1986), can be described as follows:

There is a project whose value V evolves according to the following geometric Brownian motion:

$$dV = \alpha V dt + \sigma V dz, \tag{1.1}$$

where dz is the increment of a Wiener process. This implies that the current value of the project is known but that the change in its value in the future is log-normally distributed. The change in the value of the project can be observed over time, but the future value is uncertain. There is a key question that has to be answered: At what point is it optimal to invest by paying a sunk cost I in return for being awarded the project? The investment opportunity is equivalent to a perpetual call option, like the right but not the obligation to buy a share of stock at a pre-specified price. Thus making the investment is equivalent to deciding when to exercise such an option. The investment problem can be viewed as a problem of option valuation.

The value of the investment opportunity is denoted by $F(V)$. The optimal investment strategy consists in essentially finding a rule to maximize the value of the investment opportunity:

$$F(V) = \max E[(V_T - I)e^{-\rho T}], \tag{1.2}$$

where T is the unknown future time at which the investment will be made, ρ is the discount rate, and the maximization is subject to the condition on V given in Equation (1.2). The optimal investment criterion shown in Equation (1.2) can be solved either analytically or via a numerical method.

Emerging from the financial market, real options have attracted increasing attention for infrastructure investment, since instead of investing now or never, this evaluation method allows for capture of the inherent value of flexibility. In the area of transportation infrastructure investment, Zhao, Sundararajan, and Tseng (2004) Zhao et al. (2004) developed a real options model, based on Monte Carlo simulation and regression, for highway development and operation under the uncertainty of the highway traffic demand. Couto, Nunes, and Pimentel (2015) Couto et al. (2015) used the real options method to derive a closed-form expression for the optimal investment policy of HSR investment, by assuming that the HSR demand is the main source of uncertainty and representing it by geometric Brownian motion. Bowe and Lee (2004) Bowe and Lee (2004) evaluated the HSR project in Taiwan using actual project data, with the uncertainty in the project value simulated by the log-transformed binomial numerical analysis method. Ashuri et al. (2011) Ashuri et al. (2011) proposed a model for a new build–operate–transfer highway project to help the public and private sectors analyze the economic risk of the project investment under the uncertainty; the objective of their model was to maximize the producer's revenue from the investment. Li and Guo (2015) studied on transit technology investment issues under urban population volatility, they investigated which transit technology should be selected and when to introduce and suggested the population threshold for shifting from a transit technology to another Li et al. (2015). Galera and Solino (2010) evaluated the highway concessions with operational flexibility and suggested a highway project value at different traffic guarantee level Galera and Soliño (2010).

The uncertainty variables in the studies cited above are mainly those that affect the direct travel demand for a new system of transport—or, even more generally, the value of an investment project. However, the demand or passenger level and the value of investment in a new transport system depend strongly on the consumer surplus and utility, which is driven by the fare, travel time, service level, etc., under the market competition with existing transport systems. Moreover, no historical data on the travel demand for a new transport system are available, which makes the sources of uncertainty unpredictable and consequently weakens the assumption of stochasticity. Sheffi (1985) Sheffi (1985), Train (2009) Train (2009), Yang and Sung (2010)Yang and Sung (2010), and Adler, Pels, and Nash (2010) Adler et al. (2010) have shown that introduction of a new transport mode could induce a traffic reassignment and mode selection under the competition among multiple transport types and eventually reach a transport equilibrium state. With the equilibrium outcome, the social welfare can be defined as the sum of the consumer surplus, the producer surplus, etc. Compared to using the passenger level as the source of uncertainty for a new transport system, the overall origin–destination (OD) demand which includes travel between pairs of specific cities or regions to be served by the system should therefore be a more traceable metric. In view of the above, in Chapter 3, we first develop the methodological framework to address issues of high-speed transport investment under the intercity travel demand uncertainty based on the real options approach, with the objective of determining the optimal timing for high-speed rail under the demand uncertainty.

1.3.2 A portfolio of real options with multiple uncertainties

In the previous section, we introduced infrastructure investment issues in the context of uncertainty of demand for regional intercity travel, with the goal of maximizing social welfare using a model with a single real option. Although that model provides useful insights into the decision on investment in an infrastructure system and policy evaluation under uncertainty, there are limitations in that model, as it cannot treat each project in the system as a separate investment decision. For large infrastructure systems with huge construction and labor costs, the investment decision is typically multi-stage, where investment is made every year or every couple of years with a long-term plan in mind. For instance, according to the report of the U.S. High Speed Rail Association for 2015, a 17,000-mile national high-speed rail system is planned to be built in four phases and will be completed by the year 2040. There is growing interest on the part of public-sector entities such as state and local governments and the U.S. Department of Transportation to adopt a more active style of project management (LLP, 2010). As the travel demand for individual origin–destination (OD) pairs is uncertain—and driven by migration as well as by business and technology development—the travel demand for different OD pairs may not grow at the same rate or even in the same direction. In addition, since the uncertainties fluctuate with time, the estimation in the early planning phase may not be consistent with demand in the future, thus what is needed is development of an adaptive tool that can account for the time-dependent uncertainties. In reality, in most cases the transportation authority has to manage a portfolio of interdependent projects, hence there should be a truly flexible infrastructure investment model that has the capability of addressing the interaction of investments in different infrastructure projects under non-stationary uncertainty. This requires an adaptive decision-making framework with multiple embedded options for a portfolio of projects. Efficient allocation of resources in multi-stage investment and timely adaptation are therefore becoming increasingly important.

A portfolio of real options often exhibits higher complexity than a single option, as real projects exhibit intricate sets of interacting components and decision choices, as well as flexible investment timelines for individual projects. This renders the evaluation very complicated. In order to determine the optimal investment threshold with a flexible timeline, which has been called "American options," Longstaff and Schwartz (2001) presented a simple but powerful approach for approximating the option values: by least-squares Monte Carlo simulation (LSM). Later, Gamba (2002) extended this approach to the valuation of compound options for capital investment by considering the interaction and interdependence among them. The multiple-option problem has become an active area of research in many fields (Hemmati et al., 2013). Blanco et al. (2011) evaluated the investment time for an electric transmission network under power market uncertainties by modeling the investment in transmission lines connecting the interacting components as compound multiple options, and used the LSM approach to solve the optimal installation and abandon times for the components of the power system.

In view of the above, this chapter is an extension of prior research on investment in a single infrastructure project using the real-options approach. We have developed an infrastructure investment model for a portfolio of transportation projects under network effects with multiple compound investment options. The model treats each project in the portfolio as a separate and interacting option.

1.3.2.1 Classification of a portfolio of real options

Before presenting the infrastructure investment model, we will review and summarize (in the next section) a valuation framework for a portfolio of real options. The investment decision for a portfolio of multiple options is essentially a capital budgeting problem. Gamba (2002) proposed an extension of the LSM method to a valuation framework for embedded real options with interaction and strategic independence. They classified portfolios of real options into three categories based on the relationship between the options: independent options, compound options, and mutually exclusive options.

Independent options. For a portfolio of independent options, the value of the portfolio is the sum of the values of the single options, which can be computed by the classic LSM method. The term "independent" refers to strategic independence, not stochastic independence. The underlying uncertainty for a portfolio of independent options may be dependent, but the investment decision for each individual option will have no impact on any of the others. According to Gamba, the option value of a portfolio of independent options can be expressed as

$$G(t, X_t) = \sum_{h=1}^{H} F_h(t, X_t), \tag{1.3}$$

where $G(t, X_t)$ is the value of a portfolio of independent options, $F_h(t, X_t)$ is the value of option h, and there are H independent real options in the portfolio.

Mutually exclusive options. A set of options are mutually exclusive when the exercise of one of them eliminates the opportunity for execution of the others. The expansion and abandon options are common examples of mutually exclusive options. Thus the problem is extended to find both the optimal stopping time and the optimal option to be exercised. It is obvious that the mutually exclusive options framework is not appropriate for a portfolio of infrastructure investments, because the aim of our study of a portfolio of infrastructure investments is to find the optimal investing time for each project rather than to select one project and eliminate the others.

Compound options. For a portfolio of compound options, exercising the option h creates the right to subsequently exercise option $h + 1$. The exercise of a product introduction option creates the right to exercise the first expansion option, the exercise of the first expansion option creates the right to exercise the second expansion option, and so on. The exercise of this type of option usually takes place in a sequence of staged investments, in which each installment is an option in the subsequent stages. In this case, the value of the previous claim depends on the value of the subsequent one. For the exercise of option h, in addition to its own payoff the value of that option takes the value of the-subsequent option into account, since it gives the right to exercise option $h+1$. Let $\Pi_h(t, X_t)$ and $F_h(t, X_t)$ denote the payoff and value, respectively, of option h. When evaluating the current option, which is based on a backward recursion algorithm, we assume that the value of option $h+1$ is already known. Assuming that the expiration times $T_1, T_2, \ldots T_H$ for the H options in the portfolio satisfy $T_1 \leq T_2 \leq \ldots \leq T_H$, the value of an option drops to 0 when it expires, thus $F_h(t, X_t) = 0$ when $t > T_h$. For $t < T_h$, the option value is

$$F_h(t, X_t) = \max_{\tau \in [t,T]} \{e^{-\rho(\tau - t)} \mathbb{E}_t[\Pi_h(\tau, X_\tau) + F_{h+1}(\tau, X_\tau)]\} \tag{1.4}$$

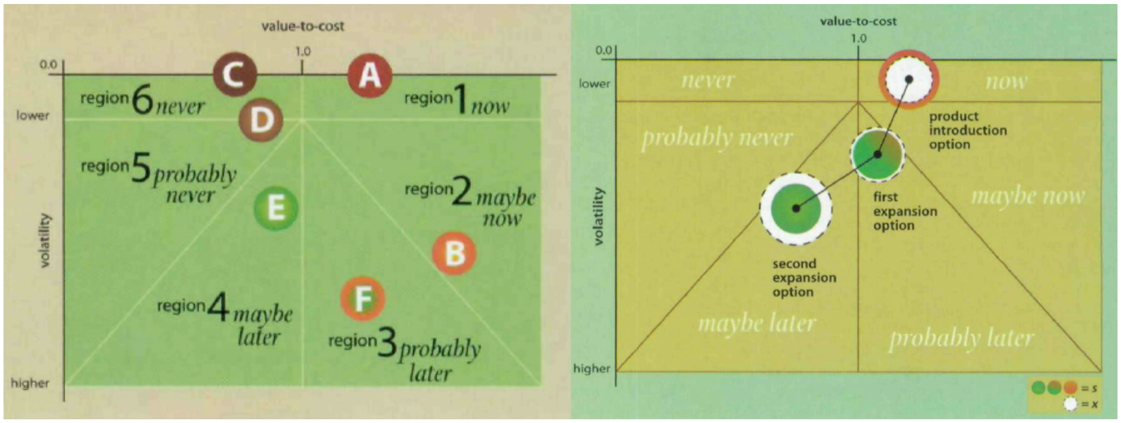
On comparing Equation (1.4) to Equation (1.3), the formula for independent option valuation, we see that the difference is that in the case of compound options there is an additional term, $F_{h+1}(\tau, X_\tau)$, since upon investing in the current option, we get a payoff and the right to invest in the subsequent options.

1.3.2.2 Valuation framework for a portfolio of real options

The traditional financial valuation tool most commonly used in estimating the value of a project is the assumption that it follows a predetermined plan, regardless of how information is revealed or changes that may occur in the future. One of the dominant tools is the net present value (NPV) framework with discounted cash flow valuation (Brealey et al., 1995). The NPV method is used to calculate the present value of a project, with the principle of investing in the project when the present value of its expected cash flow is at least as large as its cost. The investment decision with the NPV method is either invest now or invest never. However, that principle ignores the investment cost of making a commitment now, thereby relinquishing the option of waiting for new information. In strategic planning, one important question is how much should be invested in projects with short-term realized profitability vs. in projects with long-term growth potential, where the latter depends on uncertain information. A proper balance between the two metrics is necessary for success of a firm (Smit and Trigeorgis, 2006). In the following section, we present two frameworks for valuation of a portfolio of compound options.

1. Luehrman framework for a portfolio of real options

In the 1970s, the Boston Consulting Group (BCG) developed a growth–share matrix for portfolio planning, which is one of the first portfolio planning approaches based on the trade-off between current profitability and future growth (Morrison and Wensley, 1991; Hax and Majluf, 1983). For active management of a portfolio of projects, it is critical to develop a valuation framework that addresses the renewed uncertainty and builds a degree of flexibility and adaptability into strategic planning. Later, in 1998, Luehrman (1998) proposed an options-based framework on the portfolio level. He treated investments as options instead of commitments and expanded the space of two regions, "now" and "never," to six regions, as shown in Figure 1.1(a). There are two valuation metrics in the framework. The first metric is value-to-cost, the ratio of the net present value to the present value of the investment cost, which incorporates the



(a) (b)

Figure 1.1: Luehrman's diagram for locating projects in decision space

time value of being able to defer the investment. The second metric is volatility, which measures how much things can change before an investment decision must finally be made. For example, if the project expires immediately, as shown in regions 1 and 6, there will be no uncertainty in its value, and the investment decision is either invest now or invest never. This has been shown to be identical to the NPV rule (Dixit and Pindyck, 1994). Luehrman further extended the framework for a portfolio of projects to one with a series of compound options that are explicitly designed to affect one another. An example of this is shown in Figure 1.1(b), with an order to invest now as the product introduction option, then proceeding from that to the first expansion option, and from that to the second expansion option. The value of a portfolio of nested options can be generalized as follows:

$$PV\{P_1 + V[P_2 + V[P_3 + ... + V(P_N)]]\}, \tag{1.5}$$

where PV is the discounted present value, P_N is the last project in the portfolio to be invested in, and V is the option value function of a project. The option value of the innermost project, P_N, must be estimated first, because its value is part of the underlying asset value (S) for the next option in the nest, P_{N-1}. This formulation represents the option value of the portfolio with a sequence of investment contingencies. In Figure 1.1(b), each solid circle is proportional to the underlying asset value S, and each dashed circle is proportional to the investment cost of that project. A dashed circle inside a solid circle indicates that the option is "in the money": The cost of exercising it is less than the amount that the assets are worth (i.e., $NPV > 0$). The line segments connecting the centers of circles indicate that the options are nested. The first expansion option is acquired if and only if the product introduction option is exercised. Thus by Equation (1.5) the underlying asset for the product introduction option S_1 includes both the value of the operating cash flows associated with P_1 itself and the present value of the expansion options S_2 and S_3. When the product introduction option is in the money and is about to expire, it is located in the "invest now" region.

2. Smit framework for a portfolio of real options

The framework developed by Luehrman incorporates the trade-off between immediate profitability and cumulative volatility, proxying for the option value. However, this framework has no metric with which to measure the value of a real option directly, thus it is difficult to use it in complicated system valuation. Smit and Trigeorgis (2006) extended the framework developed by Luehrman to one that can be better adapted to the real-option valuation metrics using the expanded NPV criterion. The expanded NPV is the value of the real option and has two components:

$$\text{Expanded NPV} = \text{base NPV} + \text{PVGO} \tag{1.6}$$

This model embeds dynamic options-based valuation as part of two main dimensions of portfolio planning analysis: The first dimension, base NPV, is the value of the stream of cash flow expected from the existing asset under steady-state growth, while the second dimension is the present value of growth option (PVGO), which incorporates not only the impact from an uncertain environment, such as a change in price or demand, but also the managerial flexibility/adaptability to respond to change in the environment. This valuation framework is shown in Figure 1.2(a). Similar to Luehrman's diagram, the space is divided into six regions: invest now; profitable projects with low growth potential: maybe now; profitable projects with high growth potential: probably later; maybe later; probably never; and never invest. A solid black circle represents the value of the project, while a dashed circle represents the investment cost. For the nested-option example shown in Figure 1.2(b), the investment option for project II is acquired if and only if project I is exercised; project I expires at time 0, and project II expires at time 1. At time 0, project I has a negative NPV (-30), but investing in project I would enable the opportunity to invest in project II. The total strategic option value (the expanded NPV) of project II is 37, with a base NPV of 26 and a PVGO of 11. Thus the total strategic value of project I is: expanded NPV $= -30 + 37 = 7$. Given that the option to invest in project I will expire if not invested in at time 0, and that its expanded NPV is greater than 0, the best investment strategy is to invest in project I immediately.

Compared to Luehrman's diagram, the merit of this framework is that the value of the present value of the growth option, which may be a complicated

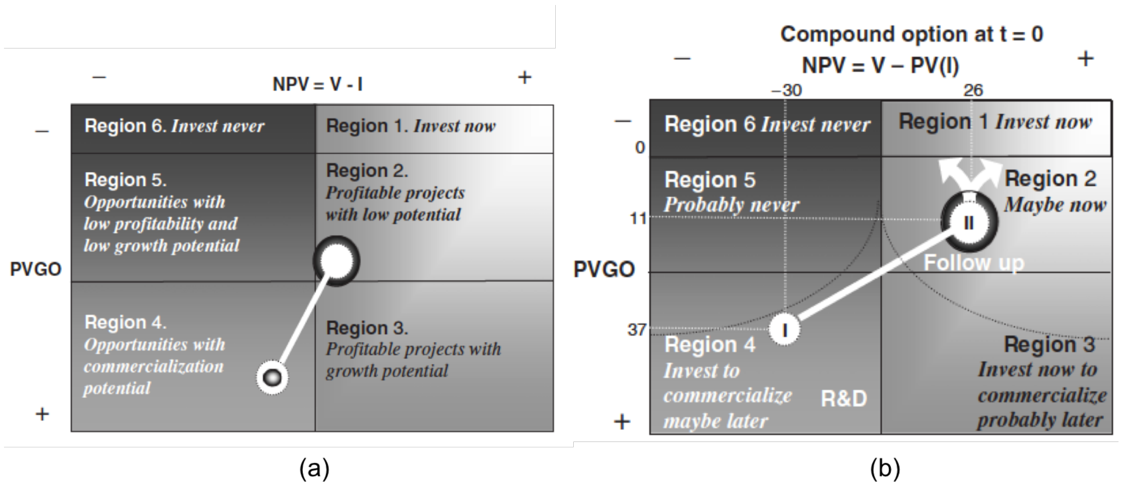


Figure 1.2: Smit's diagram for locating projects in decision space

function of underlying uncertainty and strategic flexibility, can be shown explicitly in the diagram. This feature gives this model the capability to be integrated with real-options valuation methodology and to be presented in a systematic way. However, Smit and Trigeorgi's framework has some limitations. This framework doesn't consider the interacting options in a network setting (Chow and Regan, 2011a). In planning for investment in an infrastructure network, such as in transportation planning, the passenger level and thus the value of investment in a new transport system depend strongly on the network interdependent effects and the market competition with existing transport systems. Thus it would be desirable to extend the existing portfolio management framework to one that is more comprehensive and adapted to facilitate investment decision-making on the part of the transportation infrastructure planning authority.

1.4 Major Contribution

This dissertation addresses infrastructure system investment and management strategies with the goals of reducing the socio-economic loss and improving the social welfare and resilience. It has three main components: (a) extension of the previous research on system-level infrastructure asset management to a network-based resource allocation strategy for an infrastructure system under natural disaster, (b) development of a real-options-based model to determine the optimal timing of infrastructure investment with strategic flexibility, beginning with a single-option investment, and then (c) proceeding from a single-option investment to multiple compound options selected from a portfolio of interdependent projects under network effects.

In general, there are four main highlights of the dissertation:

1. Multiple objectives. The proposed comprehensive asset management approach not only deals with financial goals but also explicitly considers the social welfare dimensions.

2. Life-cycle view and systemic scope. The research that has gone into this dissertation is not limited to an annual cycle but instead incorporates analysis and decision-making over the entire life cycle of the infrastructure. Thus it comprises not just a traditional maintenance program, but also a means of operating, managing, and optimizing infrastructure assets in a smart way, with the goal of reducing both agency costs and user costs.

3. Advanced valuation method. Traditional NPV methods are compared to a real-options valuation approach that considers future uncertainty and strategic flexibility.

4. Risk recognition and network resilience. The need to optimize risk-adjusted returns in infrastructure asset management is built in, with a view to securing a maximum return at a given acceptable level of risk.

To be specific, we highlight the main contributions of each component:

In the first component of the dissertation, we develop a pre-disaster investment model to optimally construct an investment portfolio and allocate the limited budget among a selected subset of links in the transportation network, to strengthen and enhance the resilience of the network against disasters. The main contributions of this component are: First, an optimization model that utilizes system-level analysis to capture how investments would alter the performance of the post-disaster network is presented. The model measures resource allocation decisions under the network effects of simultaneously investing in two or more projects under the budget constraints. Second, the study develops a dynamic recovery model for multiple types of infrastructure to measure the recovery from a disaster and the subsequent reconstruction of the system. Investment decisions on different types of infrastructure are considered, whereas most of the existing literature limits the infrastructure projects in the investment portfolio to all be of the same type. In real-world cases, the disaster response and recovery time varies with the type of structure. The proposed model is more flexible, and it can be applied in practice since it incorporates the heterogeneous characteristics of the infrastructure. Third, an economic demand model is developed to evaluate the socio-economic loss due to disaster. Most of the existing studies in this area treat the extra cost to travelers as a socio-economic loss without considering their behavior and their response to the increment in cost. However, when the cost of a trip is increased as a result of a detour or loss of road travel altogether, some users will still travel, while others will cancel a trip if they don't consider its value to be worth the increased cost. As the travel behavior of the users varies, and depends on their willingness to pay as well as on the travel cost, we have built a transport economic model with a cost–demand curve for each OD pair that relates the number of trips to the cost per trip. Last and most importantly, we propose a heuristic algorithm which incorporates the network effects of the investment projects. The algorithm reduces the expected socio-economic loss on the objective function given by the local optimum solution in the existing study (Peeta et al., 2010). Our model has the ability to capture the effect of simultaneous investment in two or more links, which overcomes the drawback of this approximation approach used in the existing literature.

In the second component, we present the optimal infrastructure investment strategy under uncertainty using single option framework. The contribution of this component is as follows: First, the study investigates the optimal investment criteria for high-speed transport with the goal of maximizing the social welfare, which includes the benefits for multiple agencies and stakeholders. Second, using the overall intercity travel demand as the source of uncertainty and taking into consideration travelers' choices and the market competition between a new system and existing transport can give a better representation of reality and provide guidance for government officials and decision makers. Third, the optimal investment policy and investment threshold are discussed for different demand growth scenarios, including deterministic and stochastic cases. The optimal investment criteria are compared with the net present value break-even point to illustrate the merits of investment flexibility. Fourth, the effects of environmental benefits of a new transport system are considered, and optimal policy decisions are discussed by comparison of the new system with and without consideration of environmental benefits. Fifth, sensitivity analyses are carried out to determine the effects of the parameters in terms of both analytic deduction and physical interpretation; the key input parameters include the travel demand growth rate, growth volatility, the discount rate, investment duration, etc. Investment strategies and policy implications are discussed under various scenarios.

The third component is an extension of a single-option investment to multiple compound options from a portfolio of interdependent projects under network effects, as stated in regard to the second component. The following are the main contributions of this component: A general dynamic decision support tool for a portfolio of interdependent projects with nested options under network effects is proposed. The investment decision is extended from a single option to multiple options. We further illustrate the practicality of this model through application of it to a portfolio of multiple high-speed transportation investment projects with several compound investment options that treat each link or project investment as a separate and interacting option. Second, the model has the flexibility to incorporate the investment benefits for multiple players; the demand for and value of newly introduced projects will be quantified via consumer behavior modeling in the market competition with existing transport systems. Third, the valuation metrics are completely general and allow for multiple groups of stakeholders. When estimating the net present value of the investment benefit for the project, we not only consider the financial revenue as most of the traditional valuation models do, but make social welfare the objective, including the benefits for multiple agencies and stakeholders, from both economic and environmental perspectives. Fourth, the investment policies for different demand growth scenarios are discussed. Our dynamic decision support tool for use by the investment authority is effective: Accounting, as it does, for flexibility and uncertainty, it provides real-time strategies, such as how to leverage the limited resources to develop the portfolio in an optimal investment sequence, and which projects to invest in, which to postpone, which to reject altogether in an uncertain environment.

CHAPTER 2

PRE-DISASTER INVESTMENT AND PORTFOLIO OPTIMIZATION FOR INTERDEPENDENT INFRASTRUCTURE SYSTEMS

This chapter develops a pre-disaster investment model for optimal allocation of a limited budget among a selected subset of transportation infrastructure projects to strengthen the transportation network and enhance its resilience against disasters. The objective of the investment is to minimize the expected post-disaster socio-economic loss. The chapter is organized as follows: Section 2.1 defines the methodology used in the evaluation of the criticality and vulnerability of roads. We analyze the criticality of roads via metrics of network performance and socio-economic attributes. Then we assess the exposure and vulnerability of road networks to floods, to calculate the risk of disruption. Section 2.2 presents a mathematical model and solution algorithm for the investment optimization problem, that is, the pre-disaster strategic planning problem to facilitate the link investment decisions that will minimize the expected post-disaster socio-economic loss. Section 2.3 presents a case study of transportation infrastructure investment in Fiji. We present the investment decision under various levels of budget restriction, analyze the ratio of socio-economic benefits to the investment cost, compare the network performance with the factor-based and system-level investment optimization models, and discuss policy implications for budget planning and resource allocation. Section 2.4 concludes and summarizes the study, highlights the contribution, reveals the limitations of our approach, and discusses possibilities for future work.

2.1 Infrastructure Network Reliability Assessment

Criticality of the links in infrastructure systems and the condition of the infrastructure are some of the main factors in infrastructure network reliability assessment. In the next section, we discuss assessment of criticality of an infrastructure system.

2.1.1 Criticality assessment of infrastructure by link

Identifying the most critical links in a road network is essential for decision makers so that they can prioritize their investment. In this section, we present a criticality assessment model that can be applied to each link of an infrastructure system. Criticality is defined here as the loss of network performance if this link is removed from the network. It can be evaluated by the effect of edge removal in graph theory. The metrics employed in evaluating the network performance include the road user cost, defined in units of dollars per vehicle, and the monetary value of the total travel time when road users choose the shortest path for their trip. We assumed free flow of traffic in the entire network; traffic congestion was not considered, as traffic volume in the study area is very low (Espinet et al., 2018).

The shortest-path tree (SPT) model is a well-known model used in finding the shortest travel path in a transportation network (Pallottino and Scutella, 1998). Let $G = (N, A)G = (N, E)$ be a directed graph, where N is the set of nodes, which is of cardinality m, and E is the set of edges. In addition, let c_{ij} be the travel time for origin–destination (OD) pair (i, j), which assigns a weight to edge (i, j). The well-known problem of finding the shortest path from node 1 to node m can be mathematically stated as the determination of the optimal values of the binary variables $x_{i,j}$ for $(i, j) \in E$; those variables are defined below. It has been shown that a finite solution exists for SPT if and only if there is no directed cycle in G with a negative cost (Pallottino and Scutella, 1998). This holds for transportation networks, since the travel time in a link must be non-negative. Using linear programming notation (Handler and Zang, 1980), the problem and its dual can be formulated as follows:

$$\min \sum_{(i,j)\in E} c_{i,j} x_{i,j} \tag{2.1}$$

subject to

$$\sum_j x_{i,j} - \sum_i x_{j,i} = \begin{cases} 1 & \text{if } i = O \\ -1 & \text{if } i = D \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

$$x_{i,j} \in \{0, 1\}, (i, j) \in E, \tag{2.3}$$

where $x_{i,j} = 1$ if edge (i, j) lies on the solution path, and $x_{i,j} = 0$ otherwise. Dijkstra's algorithm, which was proposed by Dijkstra (1959), is one of the most widely used algorithms for solving the shortest-path model with non-negative edge costs. That algorithm is essentially an example of dynamic programming. The strategy of Dijkstras algorithm is: In each iteration, find the node that's closest to node 1.

The details of the steps used in Dijkstra's algorithm to find the shortest path between two nodes (which are called the source and the destination node, respectively) can be summarized as follows: 1) Create the shortest-path tree set (sptSet) that keeps track of the nodes in the network which are included in the shortest-path tree, that is, those whose minimum distance from the source has already been calculated and finalized. Initially, this set is empty. 2) Assign a weight to every OD pair in the transportation network graph. Also, initialize a distance value to every node. The distance value of the source is initialized to 0, and the distance value of each of the other nodes is initialized to ∞, so that the source will be chosen as the "current node" first. 3) As long as the tree set doesn't include all the nodes, choose as the current node a node u which is not in sptSet and and such that there is a path from the source to u whose length is a minimum, where the minimum is taken over all paths from the source to all nodes not yet in sptSet. Add u to sptSet. Then for every node v which is adjacent to u but is not in sptSet, do the following: If the sum of the length of the indicated path from the source to u and the weight of the edge between u and v is less than the current distance value of v, then update the distance value of v to that sum. 4) Go to Step 3.

For each OD pair (i, j), we implement the shortest-path algorithm to find the optimal travel path, that is, the path with the minimum travel cost, which is represented by a subset of edges, $E^*_{i,j}$, such that $x_e = 1$ for all $e \in E^*_{i,j}$. Then the criticality of link e, C_e, is the difference between the network performance when all the links in the network are included and the performance when link e is removed from the network:

$$C_e = \sum_{i=1}^{P-1} \sum_{j=i+1}^{P} f(\tilde{E}^*_{i,j}|_{E \setminus e}, N_{i,j}) - f(E^*_{i,j}|_E, N_{i,j}), \tag{2.4}$$

where P is the number of nodes in the network, f is the network performance function that, for OD pair (i, j), represents the total user travel cost for travel from i to j given that the road users choose the shortest path to travel from i to j via the links in $E^*_{i,j}$, $\tilde{E}^*_{i,j}$ is the set of links in the shortest path from i to j when e is removed from the network, and $N_{i,j}$ is the demand for travel between i and j. It should be noted that we assume that the network is an undirected graph, and that the travel time and network performance for travel from j to i are the same as for travel from i to j.

2.2 Infrastructure Investment Optimization under Network Effects

In this section, we introduce a mathematical model for the investment problem: a portfolio optimization model for the strategic pre-disaster planning problem which facilitates the link investment decisions that will minimize the post-disaster socio-economic loss.

2.2.1 Transportation network disaster vulnerability analysis

We will begin with analysis of the vulnerability of a transportation infrastructure network to disaster, as that serves as the objective of the optimization model. The purpose of the vulnerability assessment is to estimate the infrastructure damage and the socio-economic loss under disaster. The cost of damage to infrastructure is the expected cost to repair or rebuild it. The socio-economic loss comes from the post-disaster extra road user cost that stems from trip detours and non-connectivity of communities as a result of disruption of the road network. The economic demand curve is used to quantify the expected socio-economic loss due to the extra road user cost. The socio-economic loss is the objective of the optimization model discussed in the next section.

2.2.1.1 Infrastructure damage cost

After consulting with the local authorities in regard to an increase in the water level that occurs on roads as a result of a disaster, we decided to model the cost of damage to the infrastructure as a percentage of the total replacement cost. This cost is based on the difference between the observed water level and the water level the structure was designed for (Espinet et al., 2018). We use the following simplistic damage equation:

$$D_i^k = \sum_i \max\left\{\frac{WLcc_i^k - WLd_i}{WLd}, 1\right\} \cdot S \cdot Rc, \tag{2.5}$$

where D_i^k is the cost of damage to infrastructure i under flood type k and WLd_i is the water level designed for infrastructure i, that is, the threshold that it was designed to withstand, such as in a flood. The design standard is evaluated on the basis of local design criteria and adjusted by the condition of the infrastructure. $WLcc_i^k$ is the water level of infrastructure i under flood type k, which is represented by the return period, S is the area of the infrastructure in square feet, and Rc is the replacement cost of the infrastructure per square foot which results from complete damage.

2.2.1.2 Road user costs

The extra road user costs due to flooding consist of the difference between the (total) user costs that correspond to the flood map and those that correspond to the baseline map (the one without flood effects). We overlaid the road network with flood maps corresponding to different return-period events. After a flood, the links could be either operational or non-operational. The functionality of the links in the system will be a function of the magnitude of the flood type k. We use a binary variable η_e^k to indicate the functionality of link e under disaster type k, where $\eta_e^k = 1$ if the link is operational and $\eta_e^k = 0$ otherwise. The vector of the variables η_e for all the links in the road network is denoted by $\boldsymbol{\eta^k}$. Traditionally, the increase in the road user cost has been calculated by using the network model for the shortest-path algorithm described in the previous section under the functionality of the links. If an origin and destination are isolated from each other because of a disaster, that is, if there is no road path connecting them, a fixed penalty is incurred. Thus the total extra cost from a disaster is the sum of the detour costs and the fixed penalties for the isolated OD pairs.

That valuation approach has limitations. When the cost of a trip increases as a result of a detour, some users will still take the trip, while others will cancel the travel if they don't consider the value of the trip to be worth the increased cost. Even if an origin and destination are isolated from each other in terms of road travel, there might be options to switch to alternative modes of transport, such as boats or helicopters. Even where that is possible, however, not all users will switch to other modes, either because of capacity or cost constraints, or because of the value of the trip. Some trips, such as those to hospitals, have a very high socio-economic value in life-threatening situations. People will want to complete those trips at any cost (including, for instance, using a helicopter). Conversely, some trips may be canceled or postponed if the main means of transportation is unavailable. An alternative model is essential to represent the fact that for a given OD pair, different trips have different values for different users and/or different purposes, thus the travel behavior of users varies, and depends on their willingness to pay and the travel cost. We have built a transport economic model with a cost–demand curve for each OD pair which relates the number of trips to the cost of the trip. Following the existing study on estimation of the elasticity of road traffic demand (Graham and Glaister, 2004), we used a classic demand function for a single OD pair, which shows that for a fixed demand elasticity, the logarithm of the change in demand is proportional to the logarithm of the change in price. The demand function for a single OD pair is expressed as

$$N_1 = N_0 \cdot e^{-\beta \ln \frac{C_1(\boldsymbol{\eta})}{C_0}}, \tag{2.6}$$

where N_1 is the transport demand after the disruption, N_0 is the original transport demand (before the disruption), and β is the elasticity of travel demand to travel cost, which was calibrated using the elasticity of demand to fuel prices and the share of fuel prices in road user costs (Avner et al., 2014). $C_1(\boldsymbol{\eta})$ is the travel cost under the disruption, and C_0 is the original travel cost. It is assumed that the most expensive alternative option is directly proportional to the original cost when travel on a road is disrupted, hence we assume that there is a constant K such that $K \cdot C_0 > C_{\max}$, where $C_{\max}$ is the maximum acceptable travel cost from O to D in the network. This assumption implies that if the cost of any alternative route in the network is greater than $K \cdot C_0$, the road user will choose an alternative travel mode. For a fixed elasticity parameter, if there is an alter-

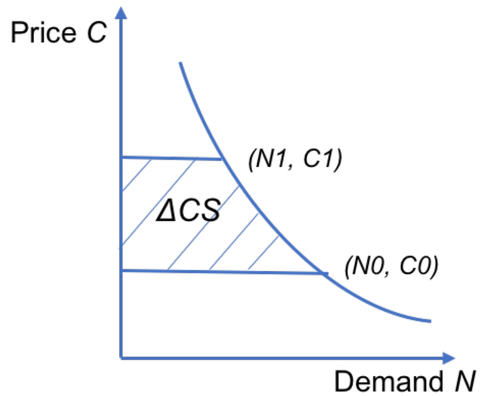


Figure 2.1: Change in road user (consumer) surplus for a single OD pair

native route with a cost lower than that for the alternative mode, the number of trips taken will depend on the increase in the road user cost due to the detour, $C_1(\boldsymbol{\eta})-C_0$; if there is no alternate route or the cost of the alternate route is greater than that of the alternate mode, only a few trips will be taken and travelers will have to pay K times as much to reach their destination: $C_1(\boldsymbol{\eta}) = K \cdot C_0$. The socio-economic loss for the system which is associated with disruption from a flood of type k is the difference between the road user consumer surplus for the baseline scenario and that for the scenario with a flood disruption, as shown in Figure 2.1, summed over all OD pairs and integrated over all times up to the post-disaster impact time horizon T:

$$S(\boldsymbol{\eta}_k) = \int_0^T \sum_{i=1}^{P-1} \sum_{j=i+1}^{P} \left\{ \left[\int_{N_1^{i,j}}^{N_0^{i,j}} C_0^{i,j} * e^{-\frac{1}{\beta_{i,j}} \ln \frac{n}{N_0^{i,j}}} dn \right] - C_0^{i,j} N_0^{i,j} + C_1^{i,j}(\boldsymbol{\eta}_k(t)) N_1^{i,j} \right\} dt, \quad (2.7)$$

where P is the number of OD in the system. As the recovery time varies with the infrastructure, the network realization $\boldsymbol{\eta}_k(t)$ is a function of time, as the functionality of a link depends on the recovery time of the associated infrastructure. T is the time horizon for total restoration of service. $N_0^{i,j}$ and $N_1^{i,j}$ are the daily demands, before and after the flood, for travel on link (i, j).

2.2.1.3 Post-disaster socio-economic loss

The final step is to combine the socio-economic losses from all flood types k to estimate the total socio-economic loss and the annual flood risk. The type of a flood is usually measured by its severity, which is indicated by its return period (Gumbel, 1941). The return period of a flood is defined as the reciprocal of the probability that one or more floods of the same severity (discharge of water) will occur in any given year: $T = \frac{1}{p}$; p is called the annual exceedance probability (AEP). For example, a river flood with a 100-year return period is defined as the annual maximum river flood discharge that is exceeded with an annual exceedance probability of 1% (Cooley, 2013). The expected annual loss (EAL) is the expected value of the loss which occurs, considering floods of all possible severities, which can be calculated by integrating the loss over floods with all probabilities p (Olsen et al., 2015):

$$D_e = \int_0^1 D(p)dp \tag{2.8}$$

It's impossible to simulate the cost of repairing the damage to a road network from a flood and restoring service on it to normal for all flood events i.e., for floods with all values of p between 0 and 1); therefore, some approximation must be made to calculate the EAL. Several different methods for numerical integration exist; however, the trapezoidal rule is often used in practice (Espinet et al., 2018). The expected annual damage to the infrastructure is then calculated for some finite number K of different flood types. Individual flood events are then aggregated into types for purposes of calculating their expected annual losses. To calculate the expected annual loss, we quantify the loss associated with each event by using the probability of its occurrence, that is, the reciprocal of its return period:

$$S(\boldsymbol{\eta}) = \frac{1}{2}\sum_{k=1}^{K-1}\left(\frac{1}{T_k} - \frac{1}{T_{k+1}}\right)[S(\eta_k) + S(\eta_{k+1})], \tag{2.9}$$

where T_k is the return period for a flood of type k and $S(\eta_k)$ is the socio-economic loss from a flood of type k. $\boldsymbol{\eta}$ is a K-component vector, with a variable for each flood type, that indicates the network realization.

2.2.2 Investment optimization model

Preventive investment in infrastructure plays an important role in enhancing the network functionality and consequentlyreducing the expected socio-economic loss. The investment decision is denoted by $\boldsymbol{y} = (y_h)$, where $y_h = 1$ if we invest in infrastructure h and $y_h = 0$ otherwise. The network realization, $\eta(\boldsymbol{y})$, is a function of investment and the flood type, since the investment changes the network realization by improving the links' survivability from floods. For example, $\eta_h(y_y = 0) = 0$ and $\eta_h(y_h = 1) = 1$, thus the shortest path for a given OD pair may change with the investment.

The following describes the optimization model we used to construct an investment portfolio with infrastructure projects that minimize the socio-economic loss from a flood. The investment optimization is modeled as a two-stage problem. The Stage I problem is a system-level optimization with the objective of minimizing the expected socio-economic loss under the budget constraints. The Stage II problem comprises the micro-level design of the network, to determine the shortest travel path for each OD pair in the infrastructure network under the investment decisions resulting from the first stage. The two-stage optimization problem for a network with link set E and the number of infrastructures being considered for investment as H, can be represented as follows (for the sake of simplicity, H is used for both the number of infrastructures considered for investment and the set $\{1, 2, \ldots, H\}$):

Stage I:

$$\min_{\boldsymbol{y}} S(\boldsymbol{\eta}(\boldsymbol{y})) \tag{2.10}$$

s.t.

$$\sum_{h} I_h y_h \leq B \tag{2.11}$$

$$y_h \in \{0, 1\}, h \in H \tag{2.12}$$

Stage II:

$$\min \sum_{e \in E} c_e x_e(\eta_{y_e}) \tag{2.13}$$

s.t.

$$\sum_{e=(i,j) \in E} x_e(\eta_{y_e}) - \sum_{e=(j,i) \in E} x_e(\eta_{y_e}) = \begin{cases} 1 & \text{if } i = O \\ -1 & \text{if } i = D \\ 0 & \text{otherwise} \end{cases} \tag{2.14}$$

$$x_e(\eta_{y_e}) \leq \eta_{y_e}, e \in E \tag{2.15}$$

$$x_e(\eta_{y_e}) = \{0, 1\}, e \in E, \tag{2.16}$$

where $S(\boldsymbol{\eta}(\boldsymbol{y}))$ is the annual expected socio-economic loss for all OD pairs in the system under all possible types of flood occurrences as a result of the investment decision and $\boldsymbol{y}$ is the investment vector, where y_h is the investment decision for infrastructure h. I_h is the investment cost for infrastructure h. Equation (2.11) means that the investment in the selected infrastructure cannot be greater than the budget, B, and Equation (2.12) means that the investment decision is a binary variable, that is, to invest or not invest. In Stage II, c_e is the travel cost of using link e and η_{y_e} is the decision variable from Stage I: $x_e(\eta_{y_e}) = 1$ if there is flow from O to D via link e in the network, and 0 otherwise, as shown in Equation (2.16). A link e can be selected only if it is operational, as shown in Equation (2.15). Equation (2.14) is the flow conservation constraint. It should be noted that the Stage II problem is the shortest-path model for a single OD pair, and that all OD pairs follow the same model in the search for the shortest path.

2.2.3 Solution algorithms

The next step is the development of an algorithm for the two-stage optimization problem. The complexity of computing $S(\boldsymbol{\eta}(\boldsymbol{y}))$ is an NP-hard problem, as there are 2^H possible network realizations in the computation of $S(\boldsymbol{\eta}(\boldsymbol{y}))$: two for every h (one for $y_h = 0$ and one for $y_h = 1$). In addition, the evaluation under multiple disaster scenarios further increases the computational complexity. An approximation algorithm is essential for this type of problem with a large network(Peeta et al., 2010). We relax the integrality restrictions on $\boldsymbol{y}$ and restructure the objective function as a monotonic non-increasing function using Taylor series expansion in the neighborhood of $\boldsymbol{y_0}$:

$$S(\boldsymbol{y}) = S(\boldsymbol{y_0}) + \sum_h g_h(\boldsymbol{y_0})(y_h - y_0) + \frac{1}{2!}\sum_{h_1}\sum_{h_2} g_{h_1,h_2}(\boldsymbol{y_0})(y_{h_1} - y_{0_{h_1}})(y_{h_2} - y_{0_{h_2}}) + ... \quad (2.17)$$

where $g_h(\boldsymbol{y_0}) = \frac{\partial S(y)}{\partial y_h}|_{\boldsymbol{y}=\boldsymbol{y_0}}$ is the first order derivative of investment in h, it is proved to be the change of disaster loss by investing infrastructure h alone. We use the first order term as approx, let $\boldsymbol{y_0} = \boldsymbol{0}$ when none infrastructure is invested, we have the approximation of the objective:

$$S(\boldsymbol{y}) \approx S(\boldsymbol{0}) + \sum_h g_h(\boldsymbol{0}) y_h. \quad (2.18)$$

where $g_h(\boldsymbol{0}) = S(y_h = 1) - S(\boldsymbol{0})$. Ignore the constant term $S(\boldsymbol{0})$, and denote $g_h(\boldsymbol{0})$ as M_h, which is the change of system-level socio-economic loss by investing in infrastructure h alone, $M_h \leq 0$. With the first-order term of the monotonic function, the objective function from Stage I is decomposed as the sum of the change of economic loss from investing on the individual links, under the investment budget constraint in Equation (2.11). Stage II remains the same as the original formulation. The approximation formulation for Stage I is as follows:

Model I approximation algorithm:

$$\min_{\boldsymbol{y}} \sum_h M_h y_h \quad (2.19)$$

s.t.

$$\sum_h I_h y_h \leq B \quad (2.20)$$

$$y_h \in \{0, 1\}, h \in H, \tag{2.21}$$

The decision outcome of the approximation optimization model is a selected set of infrastructures, represented by the vector $\boldsymbol{y^*} = (y_h^*)$ (that is, the ones with $y_h^* = 1$), which have been proved to be local optimum solution for the original optimization problem (Peeta et al., 2010). The Stage I approximation model that uses the first-order term of the original function leads to a classic solvable knapsack problem.

The approximation significantly improves the computational efficiency, as it reduces the network realization from size 2^H to size H. However, the drawback of this approximation approach is that by disregarding the second- and higher-order terms, we cannot capture the effect of simultaneous investment in two or more links. To overcome this limitation, we propose a heuristic algorithm to further reduce the expected value of the post-disaster economic loss given by the local optimum solution from Stage I.

We run the heuristic algorithm iteratively, and the optimal investment portfolio in iteration n is $\boldsymbol{y_n}$. The first-order term in the original objective function in iteration $n + 1$ is computed contingent on the investment of infrastructures that were indicated for investment in iteration n, thus the network effect of the investment infrastructure assets identified in iteration n will be considered in iteration $n + 1$, $M_{h,n+1} = g_h(\boldsymbol{y_n})$. Therefore, we may get additional infrastructures which generate marginal benefits in iteration $n + 1$ as a result of the investments made in iteration n, i.e. $M_{h,n+1} < 0$ but $M_{h,n} = 0$. The candidate set of assets evaluated by the knapsack optimization in iteration $n + 1$ includes all assets in which investment is made in iteration n and the assets which gener-

Algorithm 1: Model II: A heuristic algorithm to improve the performance on the Stage I approximate solution

Input initialization ;

$n = 0$;

$\boldsymbol{y} = \emptyset$;

while $\boldsymbol{y}_{n+1} \neq \boldsymbol{y}_n$ **do**

  $n = n + 1$;

  for $i \in H \setminus k, \forall y_{k,n} = 1$ **do**

    Compute project marginal investment payoff $M_{i,n+1} = g_i(\boldsymbol{y_n})$;

  end

  $M_{h,n+1} = \min\{M_{h,n}, M_{h,n+1}\}, \forall h \in H;$

  Run knapsack optimization: $\min_{\boldsymbol{y}} \sum_h M_{h,n+1} y_h$;

  s.t. $\sum_{h \in H} I_h y_h \leq B, y_h \in \{0, 1\}, \forall h \in H$;

  $\boldsymbol{y_{n+1}} = \boldsymbol{y}^*$;

  $M_{i,n+1} = 0, \forall i \notin \boldsymbol{y}^*$;

end

ate marginal benefits in iteration $n + 1$ contingent of the investment of iteration n, $M_{h,n+1} = \min\{M_{h,n}, M_{h,n+1}\}$. We used the Knapsack programming to find the optimum investment decision $\boldsymbol{y}^*$ for iteration $n + 1$ using Equation(2.19) to Equation(2.21). If the infrastructure is not indicated for investment, we assigned its benefit coefficient to zero at this iteration $M_{i,n+1} = 0, \forall i \notin \boldsymbol{y}^*$, but re-evaluate it at the next iteration. The iteration process will end, as it converges if the assets in the investment portfolio are the same in consecutive iterations or the threshold of maximum number of iterations is exceeded. The performance of the Model II heuristic algorithm is illustrated in the next section.

2.3 Applications

We applied the infrastructure reliability assessment model and the pre-disaster investment model to the transportation infrastructure system of Fiji, an island nation located in the Pacific. We first evaluated the existing infrastructure network, including the links that are critical and vulnerable to damage from a flood. Furthermore, we compared the investment decisions from the local government with those based on the investment strategy and investment decisions from the system-level optimization model proposed in this chapter, and we measured the increment in the road network resilience against the socio-economic loss from a natural disaster.

Based on consultation with the local authorities, we compiled the model input inventory and other information used in our assessment of the Fiji network and the investment valuation:

Population centers. There were ten population centers included in the study: Suva, Lami, Lautoka, Sigatoka, Nausori, Nasinu, Ba, Rakiraki, Nadi, and Tavua Town.

Road user cost (RUC). This was defined as dollars per vehicle mile calculated on the basis of road conditions (IRI) using the Fourth Highway Development and Management Model (HDM-4) road-use costs model (Bennett and Greenwood, 2001). The share of the fuel cost in the RUC is 7%. The value of time is \$11.72/hr.

Flood map. We overlaid the road network with flood maps corresponding to different return-period events, ranging from 5 to 1000 years. The flood map layers were derived from the Fathom (formerly SSBN) Global Flood Model (GFM). GFM is a large-scale hydrodynamic modeling structure designed to enable flood modeling in data-scarce regions.

Infrastructure type and investment cost. The goal of the study was to invest in the weakest infrastructures in the road network to increase resilience. The types of infrastructure were bridges, culverts, and crossings. The investment (upgrade) cost of these three was \$13000, \$2500, and \$1250 per square kilometer (km^2).

Infrastructure failure criteria. The chance of failure of infrastructure depends on its design standard, condition, and the flood return period. The local authority based the priority of the infrastructure on its condition, that is, higher priority meant a worse condition. We assumed that bridges with high priority, medium priority, and low priority could accommodate floods with a maximum return period of 10 years, 20 years, and 50 years, respectively; that culverts with high priority, medium priority, and low priority could accommodate floods with a maximum return period of 5 years, 10 years, and 20 years, respectively; and that all crossings could accommodate floods with a maximum return period of 5 years. The investment would upgrade the design standards of the infrastructure. We defined the upgrades in such a way that if investments were made, bridges could accommodate floods with a maximum return period of 100 years, culverts 50 years, and crossings 10 years.

Infrastructure repair duration. It usually takes a year to repair a bridge, a month to repair a culvert, and a week for a crossing. The government has bought Bailey bridges to replace bridges that were washed away. The installation time for a Bailey bridge is about one month. A Bailey bridge reduces road capacity but does not disrupt traffic completely. We assumed that the travel cost for using a Bailey bridge is twice as much as the cost of travel on the original path.

Alternative travel modes for pairs of isolated communities. All of the population centers have access to jetties. People will use boats as an alternative travel mode when roads are completely disrupted and there's no way to get to their destination. Based on a survey by the local authorities, the travel cost by boat is about 10 times as high as that by road vehicle. Thus it's reasonable to assume that the most expensive alternative option is 10 times the original cost, and that if the cost of a detour via the road network is greater than the cost of travel by boat, people will either cancel their trip or travel by boat.

The elasticity of travel demand to travel cost β. The elasticity parameter β used in Equation (2.6) was calibrated to 2.9, using the elasticity of demand to fuel prices and the 7% share of fuel prices in road user costs in Fiji.

2.3.1 Fiji infrastructure network assessment

Criticality of a road network is measured by the increased road user cost when the road segment is removed from the network. As shown in Figure 2.2, there are two road segments of high criticality in Fiji. Both are critical because the average increase in road user cost per trip is high when trips are disrupted and because of the high traffic volume.

Empirically, the resilience of the transport network was also tested by hundreds of simulations in which several links were randomly removed at the same

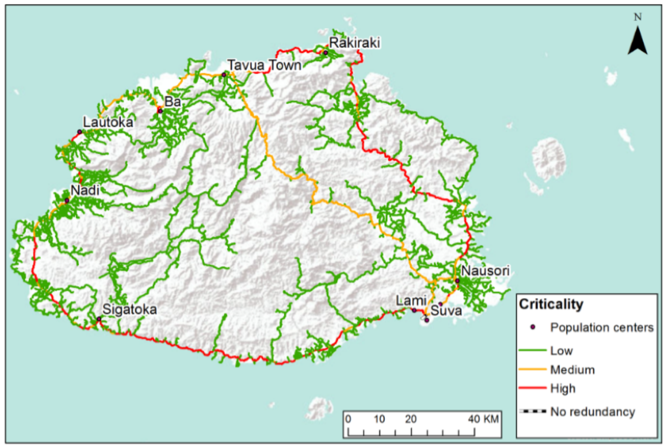


Figure 2.2: Critical road segments in Fiji

time. Many trips took a longer time and had higher travel costs, or could not be completed at all, so we estimated the socio-economic loss due to the change in road user surplus given by Equation (2.9) as a function of the percentage of links damaged. The results are presented in Figure 2.3. The blue crosses show the results for individual simulations, and the red curve is the mean of all the simulations. It shows that if 0 to 20% of the links are disrupted, the economic loss depends heavily on which links are damaged. If the damaged links are not critical, only a small percentage of socio-economic value is lost. But if critical links are damaged, in some cases almost 100% of trips are disrupted even if the percentage of disrupted links is less than 20%. This simulation reveals the importance of incorporating network effects in investment modeling, as the socio-economic loss is highly dependent on the importance of the links in the network. Figure 2.3 also shows that when more than 20% of the links are disrupted, the road network is completely paralyzed, with nearly 100% socio-economic loss. It should be noted, however, that because of the existence of alternative modes of travel, the percentage of economic loss is close to but less than 100%.

Transport network reliability was also evaluated under the impact of disas-

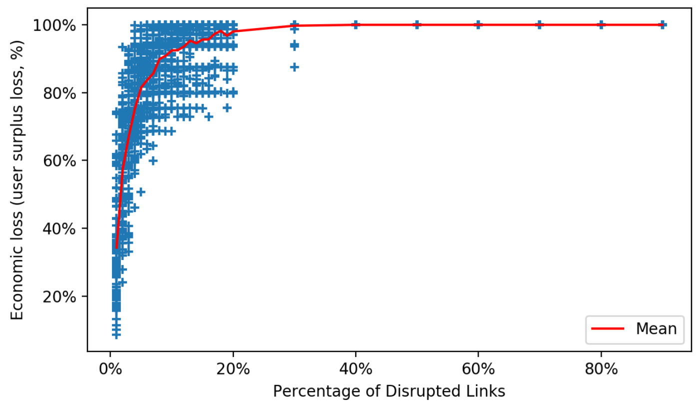


Figure 2.3: Socio-economic loss vs. percentage of disrupted links

Return period (year)	Infrastructure Damage Loss			Connectivity Loss		Detour Loss		Totals	
	Disrupted structures (%)	Value loss ($ million)	Value loss (%)	Unconnected trips (million)	Economic loss ($ million)	Detour trips (million)	Economic loss ($ million)	Economic loss ($ million)	Economic loss (%)
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	21.8	6.0	0.5	1.6	122.0	1.5	31.6	153.6	1.5
20	29.7	60.0	5.0	52.3	736.4	634.7	2516.8	3253.2	30.8
50	47.1	289.1	24.2	71.7	887.1	763.7	4505.6	5392.7	51.0
75	56.6	447.3	37.5	71.7	887.1	763.7	6048.3	6935.3	65.6
100	58.5	518.3	43.4	71.7	887.1	763.7	6082.1	6969.2	65.9
200	60.6	620.3	52.0	71.7	887.1	763.7	6162.5	7049.6	66.6
250	61.2	635.7	53.3	71.7	887.1	763.7	6181.4	7068.4	66.8
500	63.9	730.0	61.2	71.7	887.1	763.7	6201.1	7088.2	67.0
1000	66.2	798.5	66.9	71.7	887.1	763.7	6240.1	7127.1	67.4
AEL	**4.60**	**16.85**	**1.40**	**4.65**	**68.77**	**51.47**	**284.75**	**353.52**	**3.30**

Table 2.1: Transport-related economic losses from flood events

ter. Table 2.1 and Figure 2.4 show the results for a selection of flood events in terms of damage to the infrastructure and the socio-economic loss due to transport service disruption. The results show that for any return period, the socio-economic loss from service disruption (in %) is larger than the infrastructure asset loss (in %). This implies that the loss of transport service has a greater impact on the economy than damage to the infrastructure. This can also be seen in Figure 2.3, which shows that about 20% of disrupted links can completely paralyze the network. However, as shown in Figure 2.4, the damage to the infras-

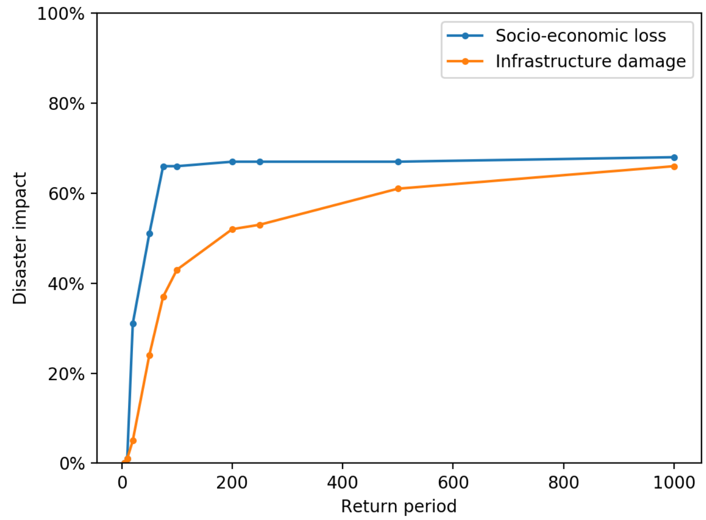


Figure 2.4: Service losses and infrastructure damage due to pluvial flood in Fiji

tructure keeps increasing steeply, while the service loss increases more slowly for a flood with a return period greater than 100 years. Since the system will be nonfunctional under a flood with severity greater than or equal to a return period of 100 years, the socio-economic loss will not change much for events with longer return periods. In addition, some trips have the possibility of being taken by other travel modes, provided that the jetties and the airport runway are not damaged. However, the value of damage to the infrastructure is proportional to the severity of the flood. It should also be noted that the road network is not necessarily completely paralyzed even under the severest floods, because some links in the transportation network may still be available, depending on their location and the structures in them. The last row of Table 2.1 shows the annual expected loss (AEL) computed from Equation (2.9). The annual expected infrastructure damage is $16.85 million, while the socio-economic loss is much greater, at $353.52. Compared to the loss from damage to the infrastructure, the social effects of the infrastructure damage are much more significant. Thus we need a robust and effective decision-making tool for investing in and upgrading of the infrastructure, so that the expected socio-economic loss can be minimized.

2.3.2 Investment strategy with factor-based and optimization models

Investment in the form of upgrading is the intervention that could increase the infrastructure design standards, which would reduce the disruptions due to less frequent but more harmful events, such as storms and floods that happen every 50–100 years. As shown in the previous section, however, the transport network could still fail under extreme hazards, because even with the upgrade, the infrastructure cannot resist flood events with a return period greater than 100 years. Against other types of events, there are ways to reduce the dependence of the economy on transport services, for instance by increasing inventories, improving logistics, and protecting telecommunications to allow telecommuting. However, there is room for improving the resilience of road networks against small- to medium-size events, for example floods with a return period under 100 years, because the investments improve infrastructure failure criteria in these disaster scenarios.

- Factor-based investment portfolio from FRA

The factor-based approach is the dominant investment decision method currently in use. Infrastructure condition is one of the most important criteria the authorities use to select the priority investment projects. The factor-based investment approach would reduce the socio-economic loss in a disaster, since investing in upgrading of infrastructure that is in poor condition would increase its resilience against floods. For example, the investment could upgrade a bridge that's in poor condition and fails as a result of events with a 10-year return period so that it is resistant to floods with a 100-year return period.

In Fiji, many structures have been identified by the Fiji Road Authority (FRA) as having high priority for rehabilitation because of a lack of maintenance performed in the past, as shown in Figure 2.5(a). The purpose of most investments is to renew and strengthen existing roads and bridges (including culverts, crossings, and footbridges) so they can better serve users in normal times and cope with floods. Using the socio-economic loss evaluation model described in Section 2.2, we estimated the priorities that had been set and found that the condition-based investments identified by FRA could reduce the value of the annual infrastructure damage (and thus emergency repair costs) by 47% and the loss of service by 29%. This is a resilience co-benefit of about $111.05 million per year for these rehabilitation investments, of which only $7.84 million comes from avoided infrastructure repair costs needed after natural hazards, while the other $103.21 million comes from reducing the socio-economic loss from $353.52 million to $250.31 million after investment by avoiding the the need for detours and preventing the isolation of communities from one another. Although those investments would have some impact on reducing post-disaster losses in the country, the capital expenditure needs have been estimated at $612.35 million. Therefore, the economic return on investment is not very promising, as the ratio of the annual benefit to the cost is less than 0.17. In addition, the investment expenditure is far higher than the planning budget of the Fiji government. This is due in part to the limitations of the factor-based model, as it does not take network effects into consideration, and therefore cannot maximize the investment benefit in terms of reducing the socio-economic loss.

- System-level investment portfolio with optimization model

Although the best solution would be to fully protect the entire network against all possible disruptions, in a resource-constrained world, governments have a limited budget and need to prioritize a subset of assets to invest in. The Fiji government should maximize the return on its budget and ensure that its road network will deliver the best level of service after disaster strikes. The government could then identify the best system-level investment portfolio that would protect the high-priority assets against potential disruptions. By making the network more resistant to extreme events in strategic places, major transport service losses could be avoided and the socio-economic loss could be reduced to the lowest level. We used the investment optimization model discussed in Section 3.3 to identify a portfolio of infrastructure upgrades that could minimize the expected socio-economic loss. Essentially, the goal of the investment was to increase the survivability, and hence connectivity, of key links in the road network. In a real-world case, there are multiple infrastructures in a link, hence we assumed that the design standards are identical for all structures that are of the same type and in the same condition. For example, we assumed that all bridges with a low priority can resist floods with a 50-year return period. Because of this assumption, we grouped together all the structures within a link that are of the same type and in the same condition, and imposed the condition that either investment should be made in all the structures in a group or that investment should be made in none of them; otherwise, the investment would not generate

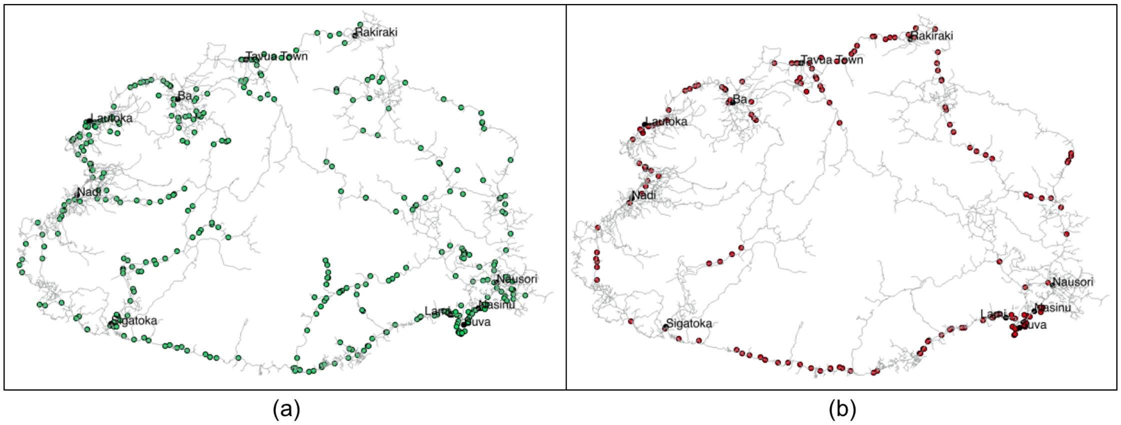


Figure 2.5: Condition-based investment portfolio of FRA (a), system-based investment portfolio (b)

`		No investment	FRA portfolio	Our portfolio	Invest all
Investment cost ($M)		N/A	612.35	476.85	1194.99
Infrastructure damage ($M)		16.85	9.01	8.35	3.81
Socio-economic loss ($M)	Detour loss	284.75	186.10	55.09	41.15
	Isolation loss	68.77	64.20	27.66	16.52
	Total	353.52	250.31	82.75	57.67
Economic loss reduction (%)		0.00	29.2	76.6	83.7
B/C		N/A	0.17	0.57	0.25

Table 2.2: Model comparison of investment benefit and cost

any benefits, since the link would be disrupted if at least one structure in the link became nonfunctional. Therefore, in the following analysis the investment decisions were made for all infrastructures in the same group instead of for each infrastructure separately.

Following use of the algorithm developed in Section 2.2 for the first part of the model, Model I, we used the approximation algorithm to find the first-order term of the marginal investment cost reduction but refrained from incorporating the effect of simultaneous investment in two or more links under Knapsack programming. Based on the financial plan of the Fiji government, we set the budget for this infrastructure investment plan at \$500 million. The portfolio of structures we identified that could minimize the socio-economic loss is shown in Figure 2.5(b). This is a smaller set of structures, and has a lower investment cost, than FRAs condition-based portfolio; however, we found that this investment portfolio significantly reduces the annual socio-economic loss in Fiji. After the investment, the expected annual socio-economic loss is \$82.75 million, which is a decrease of 76.6% from the case of no investment. In terms of the socio-economic loss, therefore, the network-based system-level approach to investment decision-making is much more efficient than the FRA condition-based approach, as the FRA portfolio saves only 29% of the annual socio-economic loss, as shown in Table 2.2. The system-level investment portfolio increases the benefit–cost (B/C) ratio from 0.17 to 0.57, and it satisfies the budget constraint, with a construction expenditure of only \$476.85 million. As shown in Figure 2.5, The locations of the infrastructure in the FRA portfolio tend to be dispersed throughout more of the road network, since that portfolio was generated by a condition-based investment approach, while the infrastructures in our system-level investment portfolio tend to be concentrated in links that connect the population centers. In addition, a larger proportion of the infrastructures in the system-level portfolio are located in the "high-criticality" links shown in Figure 2.2. These results are logical, since the benefit of investment comes primarily from improvement in network connectivity, given that connectivity reduces the travel costs for road users traveling between population centers. Thus the infrastructures located in links which are less likely to be used for commuting become less valuable to invest in, hence it is better to prioritize the infrastructures located in the critical links, that is, those whose recovery can generate greater social benefits during disasters.

Infrastructure ID	Structure type	Marginal benefit ($M)	Invest cost ($M)	Link criticality ($M)	Link criticality
215	Bridge	79.49	23.43	12.22	High
232	Bridge	18.28	10.62	4.21	High
318	Bridge	9.77	15.85	1.50	Medium
514	Bridge	6.90	1.04	0.02	Low
224	Bridge	4.25	1.42	0.02	Low
110	Bridge	3.41	6.56	1.31	Medium
588	Bridge	3.16	0.52	1.75	Medium
80	Culvert	3.03	0.08	1.31	Medium
135	Bridge	2.79	5.50	3.56	High
129	Bridge	2.62	7.58	3.56	High

Table 2.3: Groups of infrastructures with greatest marginal investment benefit

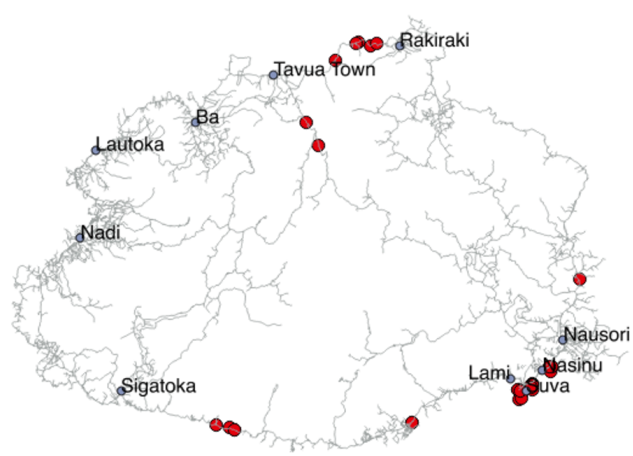
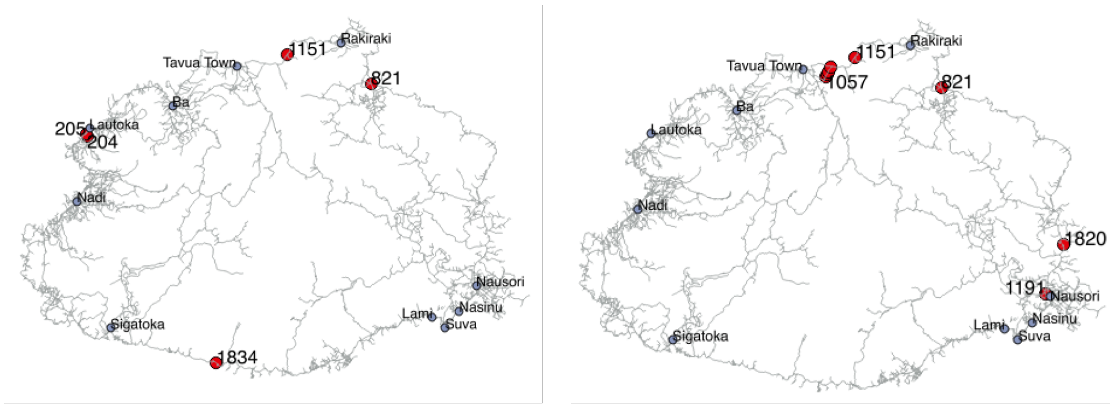


Figure 2.6: Infrastructures in the groups of greatest marginal investment benefit

In Table 2.3, we list the ten groups of infrastructures with the greatest marginal investment benefits. Figure 2.6 shows the locations of those infrastructures with red dots. The marginal investment benefit is the absolute value of coefficient in the objective given in Equation (2.19), which shows the marginal system-level benefit of investing in that link alone in reducing the post-disaster economic loss. The infrastructure listed in Table 2.3 generate a significant benefit in terms of reducing the socio-economic loss from the perspective of road users, as it reduces their travel cost. Most of the ten infrastructure groups are located in metropolitan areas along the coast of Fiji that have a high population density. For example, the top three infrastructure groups (215, 232, and 318) are located in the Lami–Suva–Nausori coastal triangle area, which is the economic center of Fiji. As the demand for short-distance travel in these population centers is significant and coastal regions are more vulnerable to floods, upgrading the weakest structural elements in that region could ensure functionality of the highway system and maximize accessibility for a larger group of users after a flood. Although many of the infrastructures in the system-level investment portfolio are located along high-criticality roads, Table 2.3 and Figure 2.6 show that there could be a significant benefit from investment in infrastructures along some low-criticality roads. For example, infrastructure groups 514 and 224 are located in links with low criticality. The target infrastructures wouldn't necessarily be located along high-criticality roads, because we measured criticality by removing a link from the original network under conditions of no disaster, while in our optimization model the benefit of investment was evaluated from the flood maps. Thus some links could be selected as high priority, since they have greater value during a disaster and increase the network connectivity.

The system-level investment portfolio constructed by Model I performs much better than the FRA portfolio. It reduces the expected economic loss by 76.6%, from $353.52 million to $82.75 million, while the FRA portfolio reduces the loss by only 29.2%. However, the solution algorithm for Model I is based on an approach that does not consider the effects of simultaneous investment in two or more infrastructure groups. To overcome this limitation, we developed Model II, using the heuristic optimization algorithm to improve the performance on the solution given by Model I. We used a budget of $1 million



(a)

(b)

Figure 2.7: Update of investment portfolio from Model I to Model II for a budget of $1 million

to illustrate the updated investment decisions given by Model II. Figure 2.7(a) shows the portfolio constructed by Model I; the updated investment portfolio constructed by Model II is shown in Figure 2.7(b). Because of the network effect of investing in assets 1151 and 821, the marginal benefit increases by investing in additional assets. To maximize the total marginal benefits in the budget, some new assets with positive marginal benefits (e.g., assets 1057, 1820, and 1191) were selected for investment by the knapsack model, while some of the assets identified for investment by Model I (e.g., assets 204, 205, and 1834) were removed from the investment portfolio.

Table 2.4 compares the performance of Model I and Model II under different budget levels. The speed of convergence of the heuristic algorithm in Model II varied with the budget. In all cases shown in Table 2.4, the maximum number of iterations needed for convergence was 17 and the computation time for one iteration with a 2.2 GHz Intel core i7 processor was about 350 seconds. The table shows that for all budget levels, Model II increases the investment benefit by up-

Budget ($M)	Model I		Model II				
	Investment cost ($M)	Socio-economic loss ($M)	Investment cost ($M)	Socio-economic loss ($M)	Performance increase (%)	Economic loss reduction (%)	B/C
0	0	353.52	0.00	353.52	0	0.00	N/A
1	0.97	344.59	0.97	332.94	3.4	5.8	21.15
5	4.81	326.52	4.98	325.93	0.2	7.8	5.54
10	9.97	321.90	9.96	321.21	0.2	9.1	3.24
20	19.84	304.49	19.75	302.84	0.5	14.3	2.57
50	49.99	155.30	49.95	147.02	5.3	58.4	4.13
100	99.84	125.99	99.84	114.37	9.2	67.6	2.40
200	199.91	109.82	199.89	87.73	20.1	75.2	1.33
300	299.94	91.64	299.90	72.97	20.4	79.4	0.94
400	399.84	84.28	399.91	69.40	17.7	80.4	0.71
500	476.85	82.75	485.53	68.11	17.7	80.7	0.59

Table 2.4: Results of investment under different budget levels

dating the original investment portfolio given by Model I. For a budget of $500 million, for example, the investment benefit given by Model II decreases the socio-economic loss from $82.75 million to $68.11 million, a 17.1% improvement in the objective value given by Model I. The expected annual socio-economic loss is reduced by 80.7%, which is merely 3 percentage points less than the socio-economic loss from investing in all the structures in the network which is given in Table 2.2, while at the same time reducing the investment cost from $1195 million to $485 million and keeping within the budget level stipulated by the local authority. Thus we have shown that the heuristic model is significantly valuable for investment decision-making.

There are interesting policy implications for the choice of investment strategy under different budget levels, as shown in Table 2.4. First, as the budget increases, the socio-economic loss decreases monotonically but the benefit–cost ratio fluctuates. The socio-economic loss decreases monotonically as the budget increases, as expected, since having more resources enables upgrades of more infrastructures, thereby improving the connectivity of the road network, reducing the time spent in detouring, and increasing the accessibility to road users after a disaster. The benefit–cost ratio is an important indicator for infrastructure managers, as it reveals the marginal benefits of the budget and the priority of projects under different budgets.

The last column of Table 2.4 shows the benefit–cost ratio for the investment under different budget levels.It was computed as the ratio of the annual socioeconomic cost reduction to the investment cost. It should be noted that the benefit value used in the benefit–cost ratio is typically defined as the present discounted value of the long-term return on investment. In our model, the investment benefit is evaluated on a yearly basis; we used the annual benefit value to show the relationship between the benefit of the investment and its cost for different budget levels. As the budget increased, the B/C first decreased and then increased. The B/C was high at the lowest budget level, because we first prioritized the investment on infrastructures with a high marginal benefit but a low investment cost, and those infrastructures are the ones that yield the best return consistent with the budget constraints. The infrastructures with a significant investment benefit but a huge investment cost will not be invested in until the budget increases to a certain level. For example, asset group 215 has the highest marginal benefit shown in Table 2.3 ($79.49 million), but it will not be invested in for some budget levels because it has a huge investment cost ($23.43 million). This explains why the B/C ratio would increase as the budget increases. For example, the B/C increased from 2.57 to 4.13 as the budget increased from $20 million to $50 million. In terms of the B/C ratio, the system-level approach to investment decision-making is much more efficient than the condition-based approach.

2.4 Summary

Investing in resilient transportation infrastructures has many economic and social benefits. More resilient transportation infrastructures directly help to avert asset losses from natural disasters, reducing the size of the investment needed for reconstruction and rehabilitation. Reduced transportation disruptions further reduce interruptions to business activity and mitigate the long-term social and economic impacts of natural disasters. This chapter explored the optimal resource allocation strategies for infrastructure asset investment and quantified the benefits of reducing the socio-economic loss from disaster that flow from such investment.

We began with an introduction to traditional economic analysis of investment in transportation. We analyzed the criticality of roads based on metrics of network performance and socio-economic attributes, and presented a model to assess the exposure and vulnerability of a road network to floods in terms of damage to infrastructure and socio-economic loss. Furthermore, we presented a mathematical model for the investment problem, that is, the pre-disaster strategic planning problem to construct an investment portfolio that will minimize the post-disaster socio-economic loss. Two types of models were developed to solve this NP-hard problem. We first proposed an approximation algorithm (Model I) to find the local optimum solution; however, the limitation of that approximation is that it does not consider the effects of simultaneous investment in two or more projects. To overcome that limitation and improve the performance on the approximate solution, we developed an iterative heuristic algorithm (Model II) that systematically considers the network effects of the projects in the optimal portfolio from consecutive iterations.

We applied the optimization model to a case study of transportation infrastructure investment in Fiji. We evaluated the criticality and vulnerability of the Fiji network during flood events with different return periods. Also, we compared the investment portfolio constructed by a factor-based model from the Fiji Road Authority (FRA) to the system-level investment portfolio proposed by the optimization model developed in this chapter. Furthermore, we presented the investment decision under different budget levels, analyzed the ratio of the socio-economic benefit to the investment cost, and discussed policy implications for budget planning and resource allocation. We found that the investment portfolio constructed by the advanced heuristic algorithm reduced the socio-economic loss by 80.7%—and that it did so with an economic rate of return of 59%. This decision-making approach is much more efficient than the condition-based approach proposed by FRA, which reduced the socio-economic loss by only 29%. The investment benefit of the portfolio constructed by the model we developed is only 3 percentage points less than the benefit of investing in all the infrastructures, but it reduced the construction cost from $1195 million to $485 million and kept within the budget stipulated by the local authority.

The current study provides a robust approach to system-level investment decision-making and portfolio construction, though it does have limitations. First, because of the population and travel conditions in the country where it was applied (Fiji), traffic congestion did not need to be considered in the network analysis; instead, we assumed free flow of traffic. However, there is a need to address the user equilibrium in the choice of travel routes in order to expand the model to regions with traffic congestion. In addition, the current study focuses on medium- to long-term disaster recovery and reconstruction. The investment benefits were evaluated for a time horizon of one year. From a life-cycle perspective, the investment benefits were undervalued, since the investment will continue to create benefits for years into the future, protecting the socio-economic loss from disaster. However, the evaluation of investment benefits in the future is not trivial, because of the need to consider infrastructure deterioration and potential damage from severe disasters. A model that integrates the deterioration of infrastructure and its survivability in a disaster is needed for assessment of the long-term investment value. Furthermore, while the heuristic model we proposed in this study has been tested and shown to significantly improve the performance on the local optimal solution given by the approximation algorithm, there is room for improvement in terms of performance of the algorithm. This is one direction for future research—one that could incorporate travel pattern analysis, long-term deterioration of infrastructure and its resistance to disaster, and algorithm improvement.

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CHAPTER 3

INFRASTRUCTURE SYSTEM INVESTMENT STRATEGIES FOR SOCIAL WELFARE UNDER DEMAND UNCERTAINTY: AN ANALYTIC REAL OPTIONS-BASED APPROACH

In this chapter, we develop an investment decision model and present an optimal infrastructure investment strategy under uncertainty using single option framework. The objective of the public infrastructure investment is to maximize the social welfare. The chapter is organized as follows. In Section 3.1, the social welfare modeling under the market competition of an new investment with existing modes is described, and the benefits of investment are quantified. Section 3.2 presents a model for the optimal investment in the infrastructure system using the real options approach. In Section 3.3, a case study is used to illustrate the application of the real options model, and sensitivity analysis is implemented with policy implications. The primary conclusions and recommendations for future extensions of this work are discussed in Section 3.4.

3.1 Computation of Investment Benefits and Social Welfare

The objective of this section is to analyze the infrastructure characteristics, the cost and revenue structure, and the passenger market behavior after introduction of a new transport system, and then from there to analyze the social welfare from the investment. The social welfare in this study includes the consumer, producer, and environmental benefits. The consumer benefits depend on various characteristics of the alternatives, including the fare, travel time, distance, routing, service level, etc. (Train, 2009 Train (2009)). The main modes of travel are airlines, high-speed rail transport, and automobiles. In the following section, we will first discuss the characteristics of those transport types, including the operating cost and the profit model. Then the consumer benefits, the producer profit, and the environmental benefits are quantified based on the resulting market share. Finally, the social welfare function will be computed based on the benefits for multiple parties. There are some important assumptions: First, we assume that the total intercity travel demand is not impacted by introduction of a new transport operator. The new investment changes only the market share by way of trip reassignment. In reality, it's likely that the new investment will encourage trip generation(De Rus and Nombela, 2007 De Rus and Nombela (2007)), but in this study we haven't considered this effect. Second, the intercity travel demand is assumed to be stochastically fluctuating over time and to follow a geometric Brownian motion process (Zhao, Sundararajan, and Tseng, 2004 Zhao et al. (2004); Gao and Driouchi, 2013 Gao and Driouchi (2013)), which will be discussed in depth later.

3.1.1 Transport characteristics and cost analysis

The annual project cost for investment in transport includes mainly the infrastructure investment cost and the cost of operation of the vehicles. The infrastructure investment cost can be broken down into fixed costs and variable costs. The investment cost can be expressed as

$$C_I = \lambda_1 + \lambda_2 L, \tag{3.1}$$

where λ_1 is the fixed cost, λ_2 is the variable cost per mile, and L is the length of the transport system. Because the construction period for large-scale infrastructure such as high-speed rail could be more than 10 years, we can use the annual construction cost in the net present value cost–benefit analysis.

The estimation of the cost of operation for a transport project varies, depending on the type of transport. The access charge for the infrastructure is not addressed in this chapter, since for simplicity we assume that the parties responsible for operation and investment are one and the same. Generally, the operation cost includes fixed and variable costs, which can be expressed as

$$C_O = C_f + C_v(d, S)F, \tag{3.2}$$

where C_f is the fixed operation cost, such as the infrastructure maintenance cost at a decision horizon; $C_v(d, S)$ is the variable operation cost, which depends on the distance traveled (d) and the number of seats (S); and F is the frequency of the transport mode at the decision horizon. The fixed and variable operation cost structures for different types of transport can be found in project operation cost reports and from other sources. For example, according to Swan and Adler (2006) Swan and Adler (2006), the operation cost of aircraft is a function of two main factors: the great circle distance and the number of seats on the aircraft. For the medium- to short-haul market, the cost function can be expressed as

$$C_{v,air} = \$0.019(GCD + 722)(S + 104), \tag{3.3}$$

where GCD is the great circle distance in miles and S is the number of seats.

3.1.2 Computation of social benefits

The social benefits of transportation include the producer benefits, the consumer benefits, and the environmental benefits. In what follows, we will describe each of these.

- Producer profit function

The profit of a transport carrier depends on the revenue and the cost of operation. The revenue depends on the market share, the origin–destination demand, and the relevant fare. The cost of operation includes the fixed cost and variable costs discussed earlier. The profit function for transport operator k in a given operation region is

$$\pi_k = \sum_i \sum_j M_{ijk} D_{ij} P_{ijk} - C_{f,k} - \sum_i \sum_j C_{v,k}(d_{ij}, S_k) F_{ijk}, \tag{3.4}$$

where P_{ijk} is the fare from city i to city j with operator k in U.S. dollars, $C_{f,k}$ is the annual fixed operation cost for operator k, $C_{v,k}(d_{ij}, S_k)$ is the variable operation cost discussed in Section 3.1.1, and F_{ijk} is the service frequency of operator k from city i to city j, represented by the number of vehicles that it uses in a year. For a given average occupancy rate, the service frequency can be approximated by using the ratio of the total number of passengers in the decision time frame to the number of passengers on board with operator k. In that case, the profit function in Equation (3.4) can be expressed as

$$\pi_k = \sum_i \sum_j M_{ijk} D_{ij} P_{ijk} - C_{f,k} - \sum_i \sum_j C_{v,k}(d_{ij}, S_k) \frac{M_{ijk} D_{ij}}{S_k \eta_k}, \tag{3.5}$$

where S_k is the number of seats provided by operator k and η_k is the average occupancy rate. We can see that $M_{ijk}D_{ij}$ is the number of passengers traveling from city i to city j using operator k per year, and $S_k\eta_k$ is the average number of passengers on board with operator k.

- Consumer benefits

The consumer benefits consist of the sum of the consumers' maximum expected surpluses defined in monetary terms. By definition, a person's consumer surplus is the utility, in terms of dollars, that the person receives in the situation at hand. In general, the consumer benefits can be expressed as

$$\zeta_v = \sum_i \sum_j \sum_k M_{ijk} D_{ij} X_{ijkv} \alpha_v, \tag{3.6}$$

where ζ_v is the monetary value of the consumer benefit of attribute v, where the attributes may include travel time, the fare, and the level of service; M_{ijk} is the market share from city i to city j for operator k, $0 \leq M_{ijk} \leq 1$; D_{ij} is the annual intercity travel demand from i to j; X_{ijkv} is the utility for operator k of providing attribute v for travel from city i to city j, such as the travel time and fare; and α_v is the monetary value of attribute v. If an attribute such as travel time or the fare has negative utility, we assign a negative value to it. The market share can be found by discrete choice model, which permits passengers to choose between the available modes in order to maximize their utility given the attributes such as travel time, fare, and operational frequency (Train, 2009 (Train, 2009)).

- Environmental emissions

An environmental emissions model is developed to evaluate the environmental performance of transportation. The environmental emissions from transport mode k can be expressed as

$$\gamma_a = \sum_i \sum_j \sum_k M_{ijk} D_{ij} d_{ijk} E_{k,a}(\eta_k) \xi_a, \tag{3.7}$$

where γ_a is the monetary value of the expected total cost due to emissions of pollutant a; M_{ijk} is the market share; D_{ij} is the intercity travel demand; $E_{k,a}(\eta_k)$ is the mass of emissions of pollutant a per passenger mile from mode k for a certain occupancy rate; d_{ijk} is the travel distance for mode k from i to j; and ξ_a is the negative of the monetary value of emissions of pollutant a, such as the carbon price.

- Calculation of social benefits

The social benefits include the total consumer benefit, the producer benefits, and the environmental benefits, as shown in the following equation:

$$W(D) = \sum_k \pi_k(D) + \sum_v \zeta_v(D) + \sum_a \gamma_a(D), \tag{3.8}$$

where $W(D)$ comprises the social benefits for demand level $D = \sum_i \sum_j D_{ij}$, that is, the sum of the travel demands for all OD pairs in the region for which service is available. In this study, we assumed that the total travel demand D is the only source of uncertainty, and that $D_{ij} = \varphi_{ij} D$, where φ_{ij} is the demand share for travel from city i to city j.

The project benefits (i.e., the value of investment in the project) is the change in the expected discounted social welfare due to the introduction of the project, which can be expressed as the difference, over the life-cycle decision horizon, between the social benefits with the investment and the social benefits without the investment. Here we assume an infinite decision horizon. Mathematically, it can be represented as the integral of the expected discounted social welfare over the decision years. The investment benefit is the difference between the project benefits and the expected discounted project investment cost. Therefore, by Equation (3.8), the infrastructure investment benefit can be expressed as

$$\Psi = \mathbb{E}\int_{\Delta}^{\infty} W'(D(t))e^{-\rho t}dt - \mathbb{E}\int_{\Delta}^{\infty} W(D(t))e^{-\rho t}dt - \mathbb{E}\int_{0}^{\Delta} C_I(t)e^{-\rho t}dt, \tag{3.9}$$

where Ψ is the investment benefit, $W'(D(t))$ is the social benefit at time t if the investment is made, and $W(D(t))$ is the social benefit at time t without the investment. $C_I(t)$ is the investment cost at time t during construction of the system, Δ is the construction duration, and ρ is the social discount rate.

Combining Equations (3.6), (3.5), (3.7), (3.8) and (3.9), we find that the investment value in terms of the social benefits when invest immediately is

$$\Psi = \mathbb{E}\int_{\Delta}^{\infty} (\tilde{\zeta} + \tilde{\pi} + \tilde{\gamma})D(t)e^{-\rho t}dt - \mathbb{E}\int_{\Delta}^{\infty} C_f(t)e^{-\rho t}dt - \mathbb{E}\int_{0}^{\Delta} C_I(t)e^{-\rho t}dt, \tag{3.10}$$

where $\tilde{\zeta}$ is the monetary value of the expected traveler benefits per unit increase in demand, $\tilde{\pi}$ is the expected operator's profit per unit increase in demand, and $\tilde{\gamma}$ is the monetary value of the expected environmental benefits per unit increase in demand. Thus

$$\tilde{\zeta} = \sum_{v}\sum_{i}\sum_{j}[\sum_{k'} M_{ijk'}X_{ijk'v} - \sum_{k} M_{ijk}X_{ijkv}]\varphi_{ij}\alpha_v \tag{3.11}$$

$$\tilde{\pi} = \sum_i \sum_j [\sum_{k'} (M_{ijk'} P_{ijk'} - \frac{M_{ijk'} C_{O,k'}}{S'_k \eta_k}) - \sum_k (M_{ijk} P_{ijk} - \frac{M_{ijk} C_{O,k}}{S_k \eta_k})] \varphi_{ij} \tag{3.12}$$

$$\tilde{\gamma} = \sum_a \sum_i \sum_j [\sum_{k'} M_{ijk'} E_{k',a}(\eta_k) d_{ijk'} - \sum_k M_{ijk} E_{k,a}(\eta_k) d_{ijk}] \xi_a \varphi_{ij}, \tag{3.13}$$

where k is the set of operators before investment and k' is the set of operators after investment.

- Discount rate selection

Social discount rate (SDR) is the discount rate used in computing the value of funds spent on social projects. Benefits and costs are typically valued in constant dollars to avoid having to forecast future inflation and project the future value of benefits and costs accordingly. The use of constant dollars requires the use of the real discount rate to discount the present value. The real discount rate ρ measures the risk-free interest rate that the market places on the time value of resources after accounting for inflation (CHSRA, 2014). Choosing an appropriate discount rate is essential to appropriately assessing the costs and benefits of a project. The higher the discount rate, the lower the present value of future cash flows. For typical investments, with costs concentrated in early periods and benefits following in later periods, raising the discount rate tends to reduce the net present value or economic feasibility of the investment. According to a World Bank report, SDR is a reflection of a societys relative valuation on todays well-being versus well-being in the future. Standard economic analysis links social discount rates to the long-term growth prospects of the country where the project takes place. This is because future benefits and costs should be valued at their marginal contribution to welfare, which will be lower the higher the growth rate and the wealthier the future project beneficiaries. Higher (lower) growth prospects would normally imply a higher (lower) discount rate for a particular country. Given reasonable parameters for the other variables in the standard Ramsey formula linking discount rates to growth rates, a 3% per capita growth rate translates into a 6% discount rate, and per capita growth rates of 1–5% yield discount rates of 2–10% (World Bank, 2016 OPSPQ (2016)). It's recommended that the social discount rate for developed countries be lower than for developing countries. The OECD member state governments provided the default discount rate in various countries, for example, the default discount rates for the United States, Japan, and Mexico are 3–7%, 4%, and 12%, respectively. In addition, SDR should represent the opportunity cost of what else the firm could accomplish with those same funds. Another source for the discount rate for projects in the United States is USDOT guidance. In this study, we used the discount rate of 7% which is recommended by USDOT guidance for cost–benefit analysis of federal programs; this is also within the recommended range given in the World Bank report. This rate approximates the marginal pretax rate of return on an average investment in the private sector in recent years (White House Circular No. A-94). Regardless of the social discount rate chosen, it is good practice to undertake sensitivity analysis that calculates the return on investment of a project for a range of discount rates, which we have done in this study and is discussed in detail later in the section.

3.2 Infrastructure Investment Decision with Real Options

Urban infrastructure projects, such as roads, rail, and airports, are long-term projects with large sunk costs. And there is no doubt that the infrastructure system is subject to many uncertainties over its life cycle. One of the most important uncertainties comes from travel demand volatility. The travel demand is closely related to the return on investment and social benefits, as shown in the previous section. Because of the demand uncertainty, it's rational to allow for flexibility in infrastructure investment decisions. The flexibility allows the investor to adapt to future changes in the uncertainty, such as demand, to make rational investment decisions at appropriate times by waiting for new information.

The traditional discounted cash flow or net present value (NPV) method was used to calculate the present value of a project, with the principle of investing in a project when the present value of its expected cash flow is at least as large as its cost. However, that principle ignores the investment cost of making a commitment now, thereby relinquishing the option of waiting for new information. Dixit and Pindyck (1994) Dixit and Pindyck (1994) show that this NPV rule is incorrect, since there is a value in the opportunity for investment, which is equal to the opportunity cost of investing now instead of waiting Dixit and Pindyck (1994). Real Options (RO) is an alternative method of real asset valuation, as it allows for flexibility.

In finance, an option is defined as the right, but not the obligation, to buy or sell an asset under the specified terms (Ruppert, 2011 Ruppert (2011)). An option that gives the right to purchase assets is called a call option; an option that gives the right to sell assets is called a put option. Infrastructure investment is similar to exercising a financial call option. First, the cost of an irreversible fixed amount of money in infrastructure investment is similar to the exercise price of an option. Second, the present value of an asset being acquired is similar to the stock price. Third, just as with the volatility in a stock price, there are sources of uncertainty in real options in regard to the future value of a project; those uncertainties translate to the standard deviation. In addition, the decisions made in exercising options are similar to those made in the real options approach, which entail determining the optimal time/criteria for which the option should be exercised in order to maximize the investment benefits (Martins, 2015 Martins et al. (2013)).

3.2.1 Underlying uncertainties

The return on transportation infrastructure investment is uncertain because of the uncertainty in the travel demand, which governs the passenger level. To describe the change in the travel demand in the region where the infrastructure investment is made, we will denote the demand at time t by $D(t)$. The demand represents the total number of passengers traveling in that region. Because of the wide variability in the traffic flow over time, we model $D(t)$ as a stochastic process. A stochastic process is defined as "a variable that evolves over time in a way that is at least in part random" (Dixit and Pindyck, 1994 Dixit and Pindyck (1994)). A continuous-time stochastic process is represented by a generalized Brownian motion defined as follows:

$$dx = a(x,t)dt + b(x,t)dz \tag{3.14}$$

A continuous-time stochastic process $x(t)$ represented by Equation (3.14) is called an Ito process, where dz is the increment in a Wiener process and $a(x,t)$ and $b(x,t)$ are known (non-random) functions. This is a Markov process which implies that the probability distribution for all future values of the process are independent of the history of values. The increments in a Wiener process are independent and can be expressed as

$$dz = \epsilon_t \sqrt{dt}, \tag{3.15}$$

where ϵ_t is normally distributed with a mean of 0 and a standard deviation of 1. If the known functions are constant, with $a(x,t) = a$ and $b(x,t) = b$, the process becomes a Brownian motion; therefore, over any time interval Δt, the change in x is normally distributed, with expected value $\mathbb{E}[x(\Delta t)|a = \alpha, b = \sigma] = \alpha\Delta t$ and variance $\mathbb{V}[x(\Delta t)|a = \alpha, b = \sigma] = \sigma^2\Delta t$. The usual formulation of Brownian motion doesn't work well with the travel demand variable, since the demand variable is non-negative and has a compound or exponential growth which is driven by population growth.

Two common forms of Ito processes are geometric Brownian motion and mean-reverting processes. In a mean-reverting process, it is assumed that the stochastic variable will tend to drift toward its long-term mean Dixit and Pindyck (1994), which may work well with the travel demand variable over a short period of time, such as operations over the course of a single day, as the demand may fluctuate, but it will tend to drift toward the mean Guo et al. (2017). However, the mean-reverting process doesn't work well with the demand variable in the long run, such as over the life cycle of an infrastructure project, because the demand over a long period of time is driven by population growth, which is commonly modeled as a compound or exponential growth Saphores and Boarnet (2006). Geometric Brownian motion (GBM) is defined by

$$dx = axdt + bxdz, \tag{3.16}$$

where the percentage changes in x are normally distributed. The distribution in each increment is log normal, with expected value $\mathbb{E}[x(t)|a = \alpha, b = \sigma, x(0) = x_0] = x_0 e^{\alpha t}$ and variance $V[x(t)|a = \alpha, b = \sigma, x(0) = x_0] = x_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1)$. GBM is commonly used for population variables such as OD demand that have a compound or exponential growth rate. Zhao, Sundararajan, and Tseng (2004) Zhao et al. (2004) used GBM for travel demand, Chow and Regan (2011) Chow and Regan (2011c) used standard GBM for each OD pair, and Saphores and Boarnet (2006) Saphores and Boarnet (2006) used a variation of GBM for population. Empirically, the time series process can be appropriately modeled as GBM if the following two conditions are satisfied: 1) normality of the log ratio $\ln(\frac{x_{k+t}}{x_k})$, with constant mean μt and variance $\sigma^2 t$; 2) independence of the current values of the log ratios from their past values. Marathe and Ryan (2005) Marathe and Ryan (2005) implemented an empirical study and validated the growth in demand for services to be GBM using historical data from airline passenger emplacements. The study tested 20-year historical monthly data from the U.S. Aeronautical Board. A California High Speed Rail System technical report shows that the intercity travel demand in California grows by 2.17% per year on average.

In this study, we relied on theoretical considerations and assumed that the intercity travel demand follows a GBM. In addition, we will discuss other types of evolution of travel demand such as deterministic growth. If the intercity travel demand $D(t)$ follows geometric Brownian motion, then the travel demand satisfies the equation

$$dD(t) = \alpha D(t)dt + \sigma D(t)dz(t), \tag{3.17}$$

where α is the rate of growth of the travel demand, $\mathbb{E}[D(t)] = D_0 e^{\alpha t}$, σ is the volatility rate of the travel demand, dt is an infinitesimal time increment, and $dz(t)$ is a standard Wiener process.

3.2.2 Optimal investment criteria

In this section, we first use the traditional net present value (NPV) evaluation rule to find the investment threshold, and then we develop the optimal investment criteria using a real options approach and compare the two sets of results.

3.2.2.1 Net present value evaluation rule

According to the net present value principle, the investment threshold for an infrastructure project is the point at which the difference in the expected social benefit from the discount rate with and without the project just exceeds the expected investment cost. By Equation (3.10), we have

$$\mathbb{E}\int_0^{\Delta} C_I e^{-\rho t}dt = \mathbb{E}\int_{\Delta}^{\infty} (\tilde{\zeta} + \tilde{\pi} + \tilde{\gamma})D(t)e^{-\rho t}dt - \mathbb{E}\int_{\Delta}^{\infty} C_f e^{-\rho t}dt \tag{3.18}$$

If $D(t)$ has a compound growth rate, we have

$$\mathbb{E}[D(t)|D_0 = D] = De^{\alpha t} \tag{3.19}$$

Solving Equation (3.18), we obtain the NPV break-even point

$$\Psi(D_{NPV}) = \frac{D_{NPV}}{\rho - \alpha} e^{(\alpha-\rho)\Delta}[\tilde{\zeta} + \tilde{\pi} + \tilde{\gamma}] - \frac{1}{\rho} C_f(e^{-\rho\Delta}) - \frac{1}{\rho} C_I(1 - e^{-\rho\Delta}) = 0 \tag{3.20}$$

$$D_{NPV} = \frac{(\rho - \alpha)[\frac{1}{\rho} C_f(e^{-\rho\Delta}) + \frac{1}{\rho} C_I(1 - e^{-\rho\Delta})]}{e^{(\alpha-\rho)\Delta}[\tilde{\zeta} + \tilde{\pi} + \tilde{\gamma}]} \tag{3.21}$$

As explained in the previous section, $\tilde{\zeta}$ is the monetary value of the expected traveler benefits per unit increase in demand, $\tilde{\pi}$ is the expected operator's profit per unit increase in demand, and $\tilde{\gamma}$ is the expected monetary value of environmental benefits per unit increase in demand.

3.2.2.2 Optimal investment criteria using real options

Because of the irreversible investment sunk cost, the real options approach is used to determine the optimal investment timing for a given infrastructure investment project. Suppose that the present value of the investment cost is $I(T)$ and the expected present value of the project is $V(D_T)$ if the investment is made at time T. The decision maker aims to maximize the value of the investment opportunity, which is the value of the option to invest. We want to find an unknown future investment exercise time T, and invest immediately if $T = 0$, so that we can maximize the expected present value of the payoff.

$$F(D) = \max_{T} \mathbb{E}[V(D_T) - I(T)] \tag{3.22}$$

- deterministic case

Before discussing the stochastic case, we will first assume that the volatility of demand growth is 0. In this deterministic case, we will have a non-random investment time T. By Equation (3.10), the expected value of the project is

$$\begin{aligned}\mathbb{E}[V(D_T)] =&\mathbb{E}\int_{\Delta+T}^{\infty}(\tilde{\zeta}+\tilde{\pi}+\tilde{\gamma})D(t)e^{-\rho t}dt - \mathbb{E}\int_{\Delta+T}^{\infty}C_f e^{-\rho t}dt\\ =&(\tilde{\zeta}+\tilde{\pi}+\tilde{\gamma})\frac{e^{-(\rho-\alpha)\Delta}}{\rho-\alpha}De^{-(\rho-\alpha)T} - C_f\frac{e^{-\rho\Delta}}{\rho}e^{-\rho T}\end{aligned} \tag{3.23}$$

and the present value of the investment cost is

$$I(T) = \int_{T}^{T+\Delta} C_I e^{-\rho t}dt = \frac{C_I(1-e^{-\rho\Delta})}{\rho}e^{-\rho T} \tag{3.24}$$

Substituting Equations (3.23) and (3.24) into Equation (3.22), we obtain the value of the option:

$$F(D) = \max_{T}\{(\tilde{\zeta}+\tilde{\pi}+\tilde{\gamma})\frac{e^{-(\rho-\alpha)\Delta}}{\rho-\alpha}De^{-(\rho-\alpha)T} - (C_f\frac{e^{-\rho\Delta}}{\rho} + \frac{C_I(1-e^{-\rho\Delta})}{\rho})e^{-\rho T}\} \tag{3.25}$$

By Equation (3.25), we must assume that $\alpha < \rho$; otherwise, the expected value of the project will increase with T, and become infinite as $T \to \infty$. This means that if the travel demand growth rate α is larger than the discount rate, there is always a value in waiting and the optimum investment time does not exist. Here we first discuss the optimal investment decision when the travel demand evolves deterministically.

Scenario 1. The travel demand is constant or decreasing, $\alpha \leq 0$. In this case, $V(D_T)$ is constant or decreasing in T over time. The optimal investment strategy is to invest immediately if the expected present value of the life-cycle benefit of the project is greater than the present value of the investment cost. Otherwise, we will never invest. This principle is the same as for the NPV rule.

Scenario 2. The travel demand increases at a rate which is positive but lower than the discount rate, $0 < \alpha < \rho$. Taking the derivative of the expression for $F(D)$ which is given in Equation (3.25), we have

$$T^* = \max\{\frac{1}{\alpha}\ln[\frac{\rho}{\rho-\alpha}\frac{(C_f\frac{e^{-\rho\Delta}}{\rho} + C_I\frac{(1-e^{-\rho\Delta})}{\rho})}{(\tilde{\zeta}+\tilde{\pi}+\tilde{\gamma})\frac{e^{-(\rho-\alpha)\Delta}}{\rho-\alpha}D}], 0\} \tag{3.26}$$

Substituting the travel demand break-even point under NPV from Equation (3.21), we have

$$T^* = \max\{\frac{1}{\alpha}\ln[\frac{\rho}{\rho-\alpha}\frac{D_{NPV}}{D}], 0\} \tag{3.27}$$

Equation (3.27) shows that the time at which the travel demand just reaches the NPV investment threshold D_{NPV} is not the optimal time to invest, because $\frac{\rho}{\rho-\alpha}$ is greater than 1 and thus $T^* > 0$. The optimal time to invest is when the travel demand increases to $D_{NPV}\frac{\rho}{\rho-\alpha}$:

$$D^* = \frac{\rho}{\rho-\alpha}D_{NPV} \tag{3.28}$$

Equation (3.28) shows that even if the demand growth is deterministic and the demand growth rate is positive, it is still worth waiting when the demand reaches the NPV break-even point. By observing Equation (3.25), we can see that the benefit of investment decreases by a smaller factor, $e^{-(\rho-\alpha)T}$, than the cost of investment, $e^{-\rho T}$. Therefore, an increase in demand growth rateleads to a positive value from waiting and an increase in the option value.

Scenario 3. The rate of increase in demand is greater than the discount rate, $\alpha > \rho$. In this case, the optimum investment time does not exist. The value of waiting is always a greater than the value of receiving an immediate payoff from investment.

- stochastic case

In the general case when the travel demand evolves stochastically with a positive travel demand volatility, the investment time T is uncertain, which cannot be expressed analytically, but we can find an optimal investment threshold. In the stochastic case, the decision on the timing of investment is a standard stopping problem: Stopping corresponds to making the investment, and continuation corresponds to waiting. Continuation does not generate any benefit/return within that period (Dixit and Pindyck, 1994 Dixit and Pindyck (1994)). Using the rule of dynamic programming by assuming infinite horizon for the decision problem, we can break down a large problem into a set of smaller, overlapping, and easily solvable problems using Bellman equation.

$$F(D_t) = \max_u [\pi(D_t, u)\Delta t + \frac{1}{1+\rho\Delta t}\mathbb{E}(F(D_{t+\Delta t})] \tag{3.29}$$

Before the investment, that is, $u = 0$, no social benefit is generated in the continuation region, so $\pi(D_t, u) = 0$. Thus the Bellman equation reduces to the follow:

$$\rho F(D_t)dt = \mathbb{E}[dF(D_t)] \tag{3.30}$$

Applying Ito's Lemma to $dF(D_t)$, which is a function of the travel demand process D_t that follows a geometric Brownian motion:

$$dF(D_t) = F'(D_t)dD_t + \frac{1}{2}F''(D_t)(dD_t)^2, \tag{3.31}$$

Substituting Equation(3.17) for dD_t to Equation(3.31), and noting that $(dD_t)^2 = \sigma^2 D_t^2 dt$, and $\mathbb{E}(dz) = 0$, the Bellman equation becomes the following stochastic differential equation with the condition of $\alpha < \rho$ to ensure the existing of optimum solution as discussed in deterministic case:

$$\frac{1}{2}\sigma^2 D_t^2 F''(D_t) + \alpha D_t F'(D_t) - \rho F(D_t) = 0 \tag{3.32}$$

Here we discuss the boundary conditions on the value of the investment opportunity. The first boundary condition is that when the traffic demand is 0, the value of the investment project equals 0 and the investment option will stay at 0:

$$F(0) = 0 \tag{3.33}$$

From the first boundary condition, we can obtain the solution of Equation (3.32):

$$F(D_t) = aD_t^{\beta} \tag{3.34}$$

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}} \tag{3.35}$$

The second threshold boundary is a value-matching condition. At the travel demand threshold for investment, the investment authority is indifferent between investing immediately and waiting. The value of continuing to wait is equal to the value of investing immediately. The value of the project when investing immediately is equal to the return on investment. As discussed earlier, the analytic form of the return on investing immediately is

$$\Psi(D) = \frac{D}{\rho - \alpha} e^{(\alpha-\rho)\Delta}[\tilde{\zeta} + \tilde{\pi} + \tilde{\gamma}] - \frac{1}{\rho} C_f(e^{-\rho\Delta}) - \frac{1}{\rho} C_I(1 - e^{-\rho\Delta}) \tag{3.36}$$

and the second threshold boundary condition becomes

$$F(D^*) = \Psi(D^*) \tag{3.37}$$

The last boundary condition is a "smooth-pasting" condition Dixit and Pindyck (1994). The function $F(D)$ should be smooth at the critical exercise point D^*; otherwise, one could do better at a different exercise point. Mathematically, this condition is

$$\frac{dF(D)}{dD}\Big|_{D=D^*} = \frac{d\Psi(D)}{dD_t}\Big|_{D=D^*} \tag{3.38}$$

Combining Equations (3.34), (3.35), (3.36), (3.37), and (3.38), we find that the threshold travel demand D^* for investing is

$$D^* = \frac{\beta(\rho - \alpha)[\frac{1}{\rho}C_f(e^{-\rho\Delta}) + \frac{1}{\rho}C_I(1 - e^{-\rho\Delta})]}{(\beta - 1)e^{(\alpha-\rho)\Delta}[\tilde{\zeta} + \tilde{\pi} + \tilde{\gamma}]} \tag{3.39}$$

$$a = (\frac{e^{(\alpha-\rho)\Delta}(\tilde{\zeta} + \tilde{\pi} + \tilde{\gamma})}{\beta(\rho - \alpha)})^{\beta}(\frac{\beta - 1}{\frac{1}{\rho}C_f(e^{-\rho\Delta}) + \frac{1}{\rho}C_I(1 - e^{-\rho\Delta})})^{\beta-1} \tag{3.40}$$

Comparing the investment threshold with the NPV break-even point from Equation (3.21), we have

$$D^* = \frac{\beta}{\beta - 1}D_{NPV} \tag{3.41}$$

The optimal investment demand criteria shown in Equations (3.41) indicates that the demand threshold from the real options approach is always greater than or equal to the demand threshold from the traditional NPV approach, since $\beta \geq 1$ shown at Equation (3.35). This is because the real options valuation approach considers the opportunity cost of infrastructure investment, and the total cost of infrastructure investment includes the investment sunk cost and the opportunity cost of investing while forgoing the option of waiting. To understand the physical meaning of the optimal investment criteria, it's important to examine how the underlying parameters drive the change in the magnitude of $\frac{\beta}{\beta-1}$. As shown in Equation (3.35), the underlying parameters include travel demand volatility, the demand growth rate, and the discount rate.

3.2.3 Infrastructure investment policy discussion

In this section, we explain how the parameters change the investment threshold, which would be critical for infrastructure investors and planners.

- Travel demand volatility σ

By Equation (3.32), (3.34) and (3.35), $\frac{\partial\beta}{\partial\sigma} < 0$. As the volatility of travel demand increases, β decreases, and $\frac{\beta}{\beta-1}$ increases.

Scenario 1. As the volatility approaches infinity, β approaches 1, and thus D^* approaches infinity. From the investment policy perspective, as the uncertainty in the travel demand increases, the larger excess return the investment authority will demand before it's willing to make an irreversible investment. When the travel demand becomes infinite, the authority will never make the investment. In the NPV approach, however, the optimal investment N^* doesn't change with volatility, since NPV methods quantify only the expected return.

Scenario 2. When the volatility approaches 0, the investment scenario becomes deterministic. If the travel growth rate α is negative or 0, β approaches infinity, and thus $D^* = N^*$. These results imply that when the travel demand growth is constant or deterministically decreasing in the future, the optimal demand threshold D^* using the real options approach is identical to the break-even threshold N^* from NPV. The authority will invest immediately if the current demand is greater than or equal to N^*; otherwise, it will never invest.

Scenario 3. As the volatility approaches 0, if the travel growth rate α is positive with upper barrier at ρ, β approaches $\frac{\rho}{\alpha}$, and thus the investment threshold D^* approaches $\frac{\rho}{\rho-\alpha}N^*$. These results show that when the future travel demand increases deterministically, the investment demand threshold from the real options approach is greater than that from NPV, which means it is still worth waiting rather than investing as soon as the demand reaches the NPV break-even threshold. Thus these results for scenarios 2 and 3 are identical with what we found for the corresponding scenarios in the deterministic case.

- Travel demand growth rate α

By Equation (3.32), (3.34) and (3.35), we find that $\frac{\partial \beta}{\partial \alpha} < 0$, hence a higher demand growth rate implies a higher magnitude, $\frac{\beta}{\beta - 1}$, for the NPV break-even point at the optimal investment threshold. Just as in the deterministic case, this result can easily be explained: the option value increases because the investment benefit decreases slower, by a factor of $e^{-(\rho-\alpha)}$, as α increases, but the cost of investment decreases by a fixed factor, $e^{-\rho}$, which makes it more worthwhile to wait. However, the optimum investment threshold D^* decreases, because the NPV break-even threshold D_{NPV} decreases as the demand growth rate α increases, as shown in Equation (3.21).

- Discount rate ρ

As the discount rate decreases, $\frac{\beta}{\beta - 1}$ increases. It's easy to show that $\frac{\partial \beta}{\partial \rho} > 0$, and that $\frac{\beta}{\beta - 1}$ increases as ρ decreases. This is because a reduction in the discount rate makes the future generally more important and increases the value of waiting (the option value), which is the ability to reduce the prospect of future loss. However, the NPV break-even threshold D_{NPV} decreases as ρ decreases as shown in Equation (3.21); as a consequence, the optimum investment threshold D^* decreases as ρ decreases.

- Construction duration Δ

The construction duration will not impact the value of β, and $\frac{\beta}{\beta-1}$ keeps constant. However, an increase in the duration of construction duration increases the NPV threshold; therefore, it increases the optimum investment threshold accordingly. The sensitivity analysis of all parameters discussed above will now be illustrated in a case study with numerical examples.

3.3 Model Application

In this section, a case study is used to illustrate the application of the proposed model and the contributions of the study. The case we present is for high-speed rail investment strategies in California. The California high-speed rail (HSR) system will connect the major California cities, from San Diego and the Los Angeles (LA) Basin in southern California, to the San Joaquin Valley in central California, and the San Francisco Bay Area and Sacramento in northern California. Figure 3.1 shows a map of the planned California High Speed Rail system. In this section, we will first discuss the benefits of investment in the California high-speed rail system with various attributes. Then we investigate the optimal investment criteria for the net present value (NPV) approach vs. those for the real options (RO) valuation approach under different demand growth scenario. This will further illustrate the difference in the investment decision that stems from using social welfare as the objective and the investment decision that uses HSR revenue as the objective. Investment policy with and without consideration of environmental benefits will be discussed. In addition, we will examine the key model parameters, such as travel demand volatility, the

Figure 3.1: Map of planned California High Speed Rail system

travel demand growth rate, the discount rate, and the construction duration, as well as their effects on the optimal investment threshold. Finally, we will make a recommendation based on California's current investment policy.

The major operation parameters for transport are travel distance, fare, and travel time. Table 3.1 shows the values of the key parameters for the California HSR system compared to those for travel by air and auto, according to official documents of the California High-Speed Rail Authority (hsr.ca.gov). In 2015, the total travel demand, from all modes of transportation, between the regions to be served by HSR was 330 million (CAHSR, March 2012). The percentage of the demand for each pair of regions is given in Table 3.1 CAHSR (March 2012). For airline and HSR, the travel time between regions includes the in-vehicle and out-of-vehicle travel times, where the latter comprises the access time and the waiting time. For example, the flight time between the Bay Area and the LA

Basin is 75 minutes. The average travel times from the centers of Los Angeles and San Francisco to their airports are 24 minutes and 15 minutes, respectively. Assuming that a passenger arrives at the airport 1 hour before a flight and takes 30 minutes to disembark from the aircraft and leave the airport, this amounts to a total average travel time of 3 hours and 24 minutes. The travel time by HSR is 177 minutes on average (Parsons Brinckerhoff et al., 2015), and the travel times to the Los Angeles and San Francisco stations from the centers of those cities are 6 and 8 minutes, respectively. Assuming an average wait in the station of 10 minutes, and 5 minutes to leave the station at the end of a trip, this amounts to a total travel time of around 206 minutes. In Table 1, we show the on-mode travel time (left) and the total travel time (right) by air and HSR. The travel time from LA to San Francisco by HSR is comparable to that by air, while the fare is much lower. Also, the travel time is greatly reduced, from around 6 hours by automobile to around 3.5 hours by HSR; consequently, a large proportion of passengers is expected to choose HSR. Table 3.2 shows the shares projected by the California High-Speed Rail Peer Review Group California HSR Peer Review Group (2012) for the three intercity modes before and after investment in HSR. Baseline values of the other input parameters used in our model, which were taken from government reports, are given in Table 3.3.

3.3.1 Analysis of investment benefits

The calculation of the benefits of HSR investment is based mainly on the share of the HSR travel mode, which shows how people switching from other modes of travel to HSR, driven by the travel time, fare, safety, reliability, etc. As described in Section 2, the investment benefits in our analysis include operator

OD pairs		Ridership %	Distance (mile)			Fare ($)			Travel time (min)		
			air	HSR	auto	air	HSR	auto	air	HSR	auto
Bay Area	LA Basin	5.37%	318.33	520	382	165	81	50.26	75/204	177/206	336
Bay Area	San Diego	2.11%	410.83	640	493	180	81	64.87	85/214	231/261	440
Bay Area	Central Valley	17.49%	152.5	240	183	168	73	24.08	60/189	94/124	180
Bay Area	Sacramento	17.61%	65.67	120	78.8	-	21	10.37	-	48/77	92
LA Basin	Central Valley	16.61%	182.5	280	219	410	69	28.82	60/189	110/140	210
LAB	Sacramento	1.83%	321.67	520	386	162	81	50.79	80/209	173/203	330
LA Basin	San Diego	33.12%	100	160	120	-	33	15.79	-	65/95	125
San Diego	Sacramento	0.43%	420	660	504	107	81	66.32	90/219	237/266	450
San Diego	Central Valley	0.07%	282.5	400	339	420	72	44.61	75/204	163/192	310
Sacramento	Central Valley	5.37%	142.5	230	171	102	78	22.5	60/189	83/112	158

Sources: California High Speed Rail Ridership and Revenue Forecast (CAHSR & Cambridge Systems)

Table 3.1: Intercity travel times and fares

OD pairs		share (with HSR)			share (withouth HSR)	
		Air	HSR	Auto	Air	Auto
Bay Area	LA Basin	34%	37%	29%	54%	46%
Bay Area	San Diego	64%	22%	14%	82%	18%
Bay Area	Central Valley	1%	9%	90%	1%	99%
Bay Area	Sacramento	0%	5%	95%	0%	100%
LA Basin	Central Valley	2%	7%	91%	2%	98%
LA Basin	Sacramento	30%	23%	47%	39%	61%
LA Basin	San Diego	0%	6%	94%	0%	100%
San Diego	Sacramento	93%	5%	2%	98%	2%
San Diego	Central Valley	26%	18%	56%	32%	68%
Sacramento	Central Valley	1%	3%	96%	1%	99%

Sources: California High Speed Rail Ridership and Revenue Forecast (CAHSR & Cambridge Systems)

Table 3.2: Market shares of intercity travel modes

benefits, consumer benefits, and environmental benefits. The operator benefits come mainly from the profits of the airlines vs. the profit of HSR. The consumer benefits include savings in the travel time and fare, reliability, productivity, and safety. CO_2 emissions are the main indicator of the environmental benefits. The expected benefits of the travel time and fare can easily be calculated from the data in Tables 3.1 and 3.2. We will now explain the benefits of reliability, productivity, and safety in detail.

- Reliability benefits

Reliability in travel times is an important component of user benefits from HSR investment. Compared to the highway network, where random delays due to congestion occur frequently, HSR has been proved to operate extremely reliably. Road users tend to plan extra lead time into their trip to compensate for the additional time spent on the road due to unexpected delays. We used a "planning time index" to measure the amount of actual time spent on a trip after incorporating a certain buffer period above and beyond the standard travel time. According to a report on urban mobility by the Texas Transportation Institute, the planning time index (PTI) for different cities in California ranges from 1.25 to 1.46. In this analysis, we used 1.35 as the average PTI to quantify the reliability benefit. A PTI of 1.35 means that for the average road trip, users incorporate 35 percent extra time into their trip to account for the unreliability of the highway network. Similarly, because of frequent delays in air travel, the travel time reliability increases as the traveler shifts from air to HSR. According to the Bureau of Transportation Statistics and the Federal Aviation Administration, the average delay for flights departing from California airports in 2010 was 10.7 minutes. Also, a study by NEXTOR found that there are 1.06 minutes of "non-disrupted passenger" delay per minute of flight delay and 31.19 minutes of "disrupted passenger" delay per minute of flight delay. By assuming 10% of disrupted passengers and 90% of non-disrupted passengers on a flight (Brinckhoffs, April 2014), we estimated the expected delay time for the air mode.

- Productivity benefits

Productivity benefits refer to the idea that travelers can be productive while traveling on HSR, whereas they are incapable of being productive while driving, and less likely to be productive on an aircraft Brinckhoffs (April 2014). We

Categories	Description	Parameters	HSR	Air	auto
Project Parameters	Social discount rate	rho	0.07		
	Annual travel growth rate	alpha	2.17%		
	Annual travel volatility rate	sigma	5%		
	Value of time	VOT $/hour	43.70		
	Number of seat	s	118	450	5
	Load rate	η	0.8	0.8	0.38
	Construction period (year)	delta	14	-	-
Operation Cost	Infrastructure fixed operation costs (per year route mile)	C_f	$200,000	-	-
	Variable operation costs	C_o	$20 per trainset mile	0.019*(d+722)(s+104)	$0.25 per mile
Investment Cost	Total investment cost YOE$ (billion)	-	$64.2	-	-

Sources:
2014 California High speed rail benefit – cost analysis; 2014 California high speed rail ridership and revenue forecast; 2016 California high-speed rail business plan; USDOT VOT guidance

Table 3.3: Model input parameters

assumed that 0% of automobile travelers, 33% of airline travelers, and 50% of HSR travelers are productive in transit. By estimating the percentages of travelers switching from other modes to HSR from Table 3.2 and using the total in-transit travel times from Table 3.1, the productivity benefits before and after investment in HSR can be calculated.

- Safety benefits

As a proportion of people switches from traveling by auto to using HSR, investment in HSR will reduce the vehicle miles traveled (VMT), which consequently lowers the incidence of traffic crashes. The expected safety benefits will be evaluated by the changes in the crash rate and the crash loss using the change in highway VMT. There are three car crash types: fatal crashes, injury crashes, and crashes with property damage only. We computed the safety benefits of car crash cost savings from 2010 statewide crash data reported by the California Highway Patrol (CHP). The CHP reports aggregated injury crashes, and in our analysis we disaggregated the injury crash rates into maximum injury abbre-

Category	Crash Rate (per million VMT)	Crash loss ($)
MAIS 6 (fatal)	0.0084	$9,233,293
MAIS 6 (critical)	0.0013	$5,475,343
MAIS 6 (severe)	0.0048	$2,456,056
MAIS 6 (serious)	0.0167	$969,496
MAIS 6 (moderate)	0.0579	$433,965
MAIS 6 (minor)	0.619	$27,700
Property Damage Only	0.7715	$3,476

Sources: U.S. Department of Transportation, 2013; California Highway Patrol, 2011

Table 3.4: Crash rates and economic losses

viated scale (MAIS) categories based on nationwide crash data reported by the National Highway Traffic Safety Administration Patrol (2011). Table 3.4 shows the crash rates and crash losses for all damage categories.

- Environmental benefits of CO_2 reduction

The CO_2 emissions inventory for the different modes of transport can be determined by life-cycle analysis. Based on the study of Chester and Horvath (2010) Chester and Horvath (2010), the emission intensities for each mode, which depend on the occupancy rate, were normalized per passenger kilometer of travel (PKT). HSR has great potential to be the lowest greenhouse-gas (GHG) emitter at high occupancy rates: For an 80% occupancy rate, GHG emissions are approximately 65g CO_2/PKT. When the occupancy rate decreases to 10%, GHG emissions increase to 720g CO_2/PKT. With an 80% occupancy rate on aircraft, GHG emissions are approximately 100g CO_2/PKT. When the occupancy rate decreases to 15%, GHG emissions on HSR increase to 490g CO_2/PKT. With 5 people per auto, GHG emissions are approximately 64g CO_2/PKT, and this

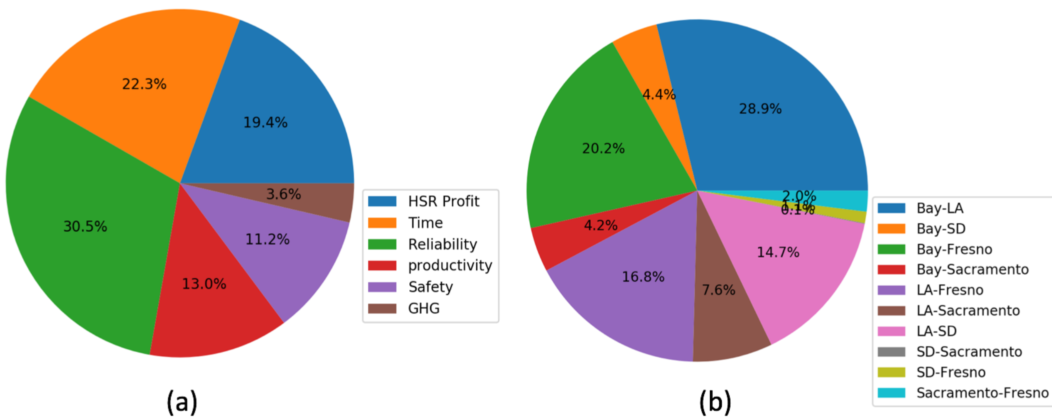


Figure 3.2: Benefits of investment in high-speed rail

increases to 320g CO_2/PKT as the occupancy rate decreases to 1 person per auto. From these data, at a high occupancy rate (80–100%) with the emissions distributed over a large number of passengers, the auto shows the best performance. However, based on DOT statistics, 1.9 is the average number of passengers per auto, with a corresponding 143g CO_2/PKT. From the market share data given in Table 3.2, we can expect that a large proportion of travelers will switch to HSR, and hence that the average occupancy rate for HSR could be 80% or more with effective service frequency control and revenue management strategies. From that assumption and statistics, HSR has great potential for decreasing CO_2 emissions. Travel from LA to San Francisco by air and automobile will generate 54kg CO_2 and 100.4kg CO_2 per passenger, respectively, while HSR will generate only 46kg CO_2 per passenger.

Using the input data and model outlined in Section 2, we present a pie chart in Figure 3.2(a) to show the shares of the individual benefits of investment in HSR as determined by their monetary values. HSR profit is the main component of the producer surplus; from the traveler's perspective, the main benefit is related with time: 22.3% comes from travel time savings, 30.5% comes from reliability, 13% comes from productivity, and 11.2% comes from improvements in safety. The savings in the fare are negative, which means that the expected fare increases when HSR is an available option for travelers. The fare decreases as travelers switch from air travel to HSR but increases as travelers switch from driving to HSR. However, the decrease in utility from the fare is small compared to the increase in utility from the other benefits. HSR investment also has a significant impact on CO_2 emissions, which are expected to be reduced by 18.4 kg per passenger after investment in HSR. If we use a carbon price of $40 per tonne in our baseline scenario, the share of GHG benefits (3.6%) is small compared to the shares of the other benefits. Figure 3.2(b) presents a pie chart of the shares of the total investment benefits for individual OD pairs. The share of expected social benefits for a given OD pair depends on the total travel demand for that pair and the percentage of people who would switch to HSR after investment. As shown in the chart, the benefits are greatest for travel between the Bay Area and Los Angeles (Bay–LA: 28.9%), between the Bay Area and Fresno (Bay–Fresno: 20.2%), between Los Angeles and Fresno (LA–Fresno: 16.8%), and between Los Angeles and San Diego (LA–SD: 14.7%).

3.3.2 Optimal investment threshold

3.3.2.1 Deterministic case

We first take a look at the deterministic case by assuming that there is no volatility in the travel demand growth in California, an assumption that we will use to illustrate the return on investment under different demand levels. The investment benefit is defined as the difference between the social benefits

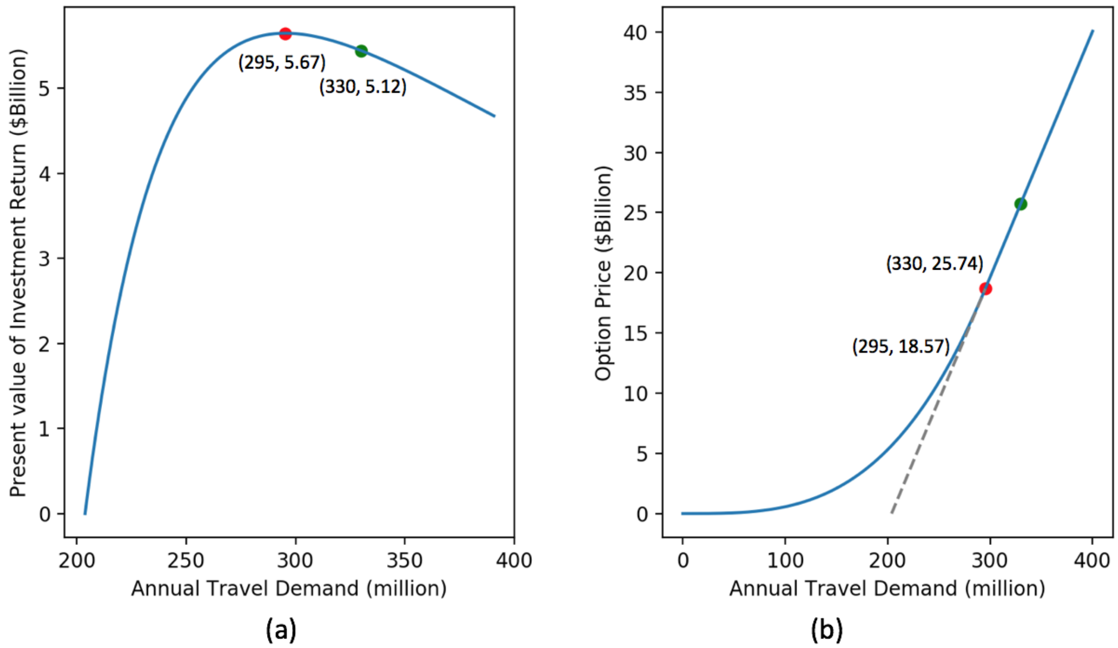


Figure 3.3: Optimal investment threshold in case of deterministic demand

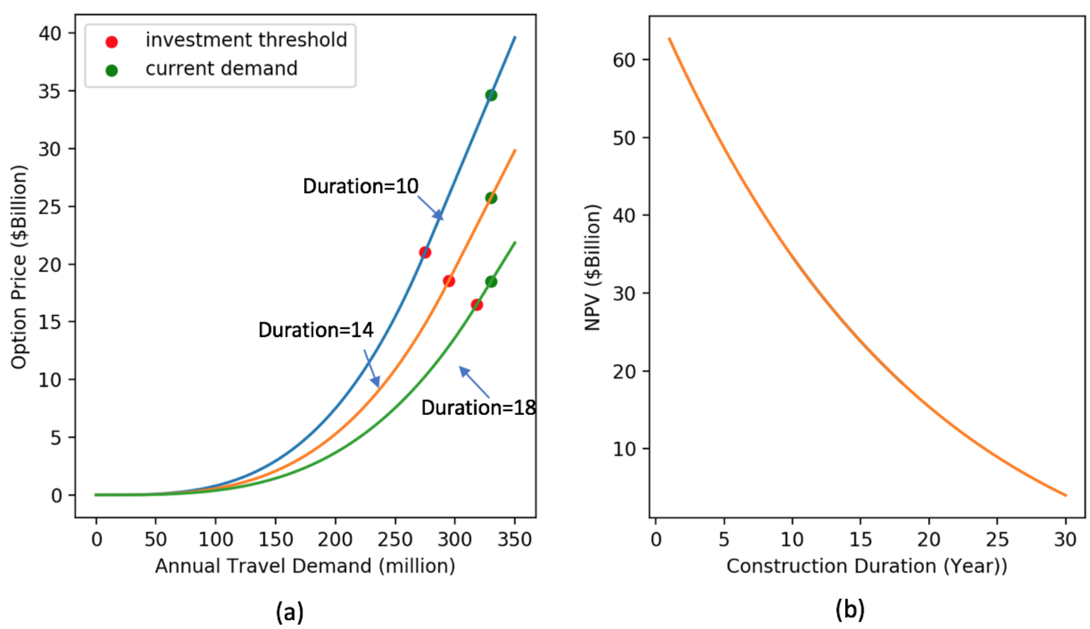


Figure 3.4: Change in investment threshold and NPV with project construction duration

and the investment cost, as shown in Equation (3.9). Figure 3.3(a) shows the present value of the return on investment discounted to the break-even year when the net present value is 0, with break-even threshold as 203 million passengers per year. The present value of the return on investment increases with the demand growth, and it reaches a maximum value of \$5.67 billion when the demand grows to 295 million passengers per year. This example shows that the NPV rule is not correct, even in the deterministic scenario: The break-even point from the NPV approach is not the optimal threshold for investment; this is consistent with the theoretical explanation given in Section 3.2. Figure 3.3(b) shows the relationship between the option price and the optimal demand threshold. The option price is the adjusted project value which is found by incorporating the value of the investment opportunity, provided that the investment decision is made optimally. When the demand is lower than the investment threshold, which is 295 million passengers per year in the deterministic case, the real options price lies on a curve above the NPV line. This means that there is a deferral premium: When the demand is above the optimal investment threshold, the real options price is equal to the NPV value, which means there is no value in waiting and the best strategy is to invest immediately. The regional intercity travel demand in 2015, when the investment began, was around 330 million passengers per year based on cost–benefit analysis and a revenue forecasting report California HSR Peer Review Group (2012) Brinckhoffs (April 2014). By assuming a deterministic demand with a 2.17% annual growth rate, the investment is indicated to be made with a delay of about 5 years beyond the optimal investment time, and the best strategy is to invest immediately.

Accelerating the project construction process is one of the most effective ways to increase the return on investment. Figure 3.4(a) shows the change in

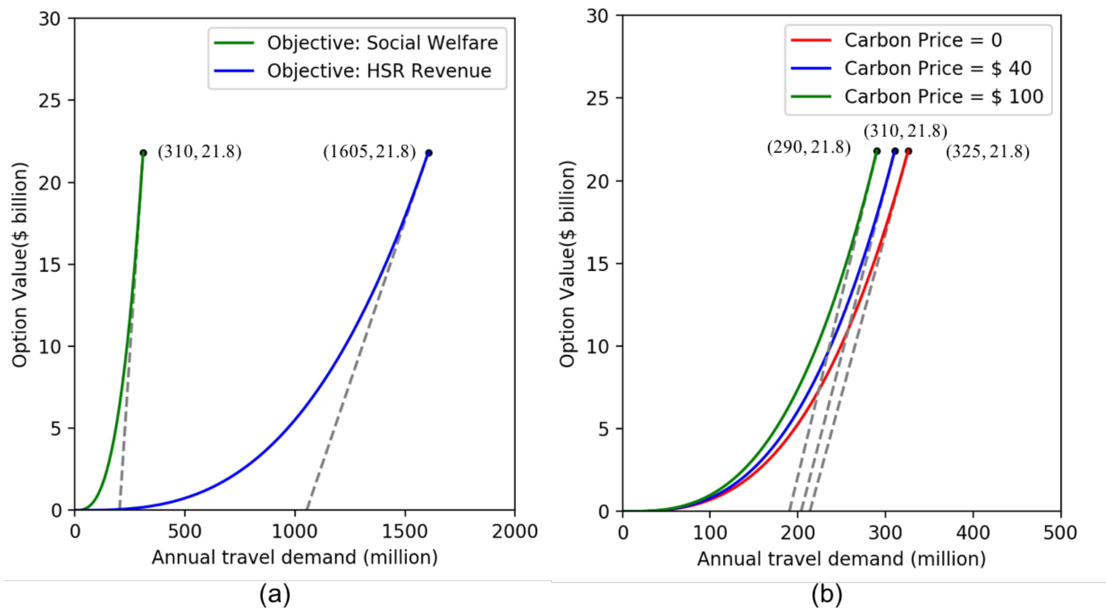


Figure 3.5: HSR investment threshold

the investment threshold, and Figure 3.4(b) shows the net present value (NPV), when investment is made immediately after a change in project construction duration. As the construction duration decreases, the NPV of investment increases, since the increment in the net present benefit of investment outweighs the present value of the increment in the cost ofconstruction. When the construction duration increases to 30 years, the net present value of investment drops almost to 0. As explained in the previous section, the construction duration will not change the magnitude of the NPV threshold relative to the real options threshold. Figure 3.4(a) shows that the real options threshold changes in accord with the NPV threshold. As the construction duration increases, a higher threshold is required to trigger investment.

3.3.2.2 Stochastic case

In the stochastic case, we assume that the travel demand follows a geometric Brownian motion, as described earlier. Based on existing research Zhao et al. (2004), we first assume that the demand volatility is 5% as the baseline scenario. Later we will use sensitivity analysis to further illustrate how the optimal investment criteria change with volatility. Figure 3.5(a) shows the investment threshold under the objective of maximizing the social welfare compared with the objective of maximizing the HSR revenue. Solid lines are used for the real options curves, and dashed lines are used for the NPV curves. The points of tangency of the real options value and NPV curves indicate the investment threshold, which means that the value of waiting is equal to the value of investing immediately. When the travel demand is lower than the trigger value, the real options value curve lies above the NPV curve, which means that the value of keeping this investment option alive is greater than the payoff from investing immediately. We can also call the real options value the adjusted NPV, as it is equal to the sum of the static NPV and the deferral premium. When the demand is lower than the investment threshold, the deferral premium is positive, so the optimal investment strategy is to wait rather than to invest. If we compare the NPV curves with the real options value curves, we see that the NPV approach indicates that the investment threshold is much lower than the solution from the real options approach, hence the NPV rule should be modified because it doesn't include the opportunity cost of investing now rather than waiting. Also, we see that for both objectives, the social welfare investment threshold is much lower than the HSR revenue threshold. Given the current travel demand in California, we are now at the optimal stage of investment under the objective of maximizing the social welfare, but the investment threshold for maximizing the revenue from HSR operation is much higher than the current demand. At this time, therefore, the HSR investment is socially profitable but not financially profitable because of the huge investment sunk cost.

Figure 3.5(b) compares the investment threshold under different carbon prices. Carbon pricing is the method favored by many economists for reducing global-warming emissions. The carbon price, which represents the importance of controlling greenhouse-gas emissions, is the amount that must be paid for the right to emit one tonne of CO_2 into the atmosphere. It can also be explained as the means of assigning a monetary value to CO_2 emissions and integrating that with the other social benefits. According to a report of the World Bank, the recommended range of carbon prices is \$25 to \$100 per tonne. The plot in Figure 3.5(b) shows that ignoring the environmental benefits of HSR investment will lead to late investment, and that the higher the carbon price, the lower the investment threshold. Compared with the baseline scenario of \$40 per tonne of CO_2, it shows that the trigger annual travel demand increases to 325 million when the carbon price is 0. If the carbon price increases to \$100 per tonne, the annual travel demand threshold decreases to 290 million. Under a high carbon price, ignoring the environmental benefits will lead to late investment because of underestimating the social benefits from the climate-change perspective.

3.3.3 Policy implications and investment strategies

In this section, we will discuss the characteristics of the optimal investment rule, with sensitivity analysis of various input parameters in the real options model. The parameters include travel demand volatility, the travel demand

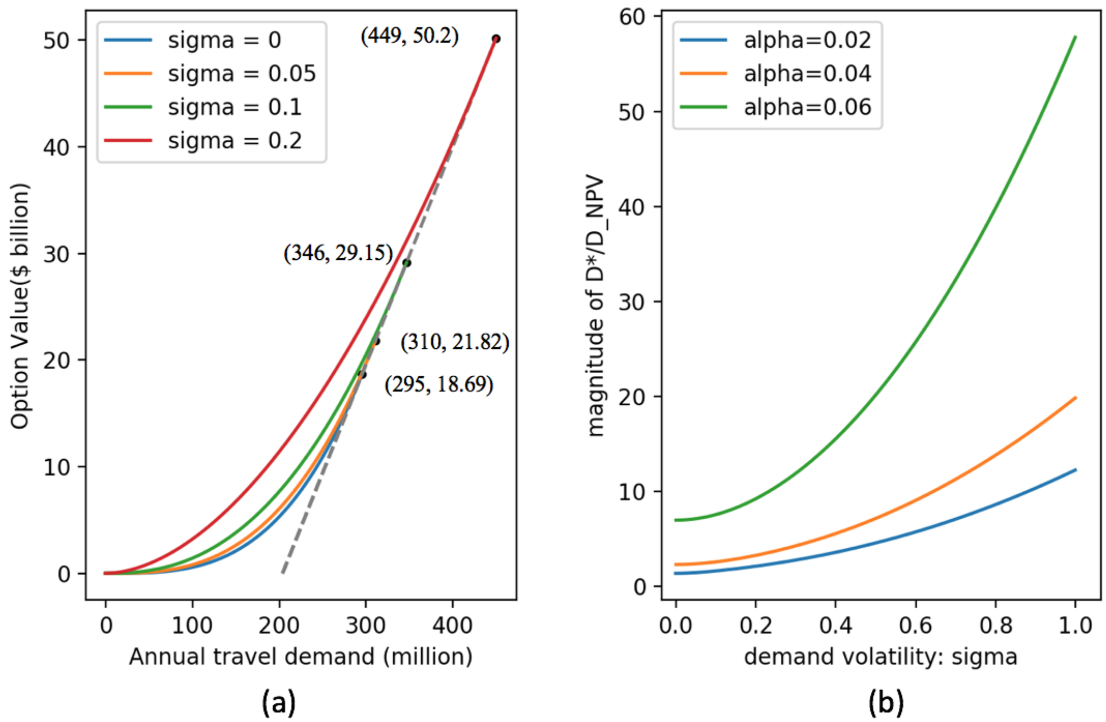


Figure 3.6: Value of investment opportunity under demand volatility rate

Project valuation result, with demand growth rate 2.17% , discount rate 7%

Demand Volatility	Current Demand (Million Passenger)	Static NPV ($B)	Option value ($B)	Deferral Premium($B)	Investment Threshold	Decision	Expected Deferral Years
0	330	25.74	25.74	0	295	Invest	0
5.0%	330	25.74	25.74	0	310	Invest	0
8.0%	330	25.74	25.74	0	330	Invest	0
10.0%	330	25.74	29.03	3.29	346	Defer	2.5
15.0%	330	25.74	38.65	12.91	393	Defer	9.7
20.0%	330	25.74	50.05	24.31	449	Defer	17

Table 3.5: Project valuation results

growth rate, and the social discount rate. The numerical results will illustrate the dependence of changes in the optimal investment threshold on the various parameters.

- Travel demand volatility σ

Figure 3.6(a) shows the relationship between travel demand volatility and the investment threshold for HSR investment. We varied the annual travel demand volatility from 0 to 20% and kept the other parameters at the baseline. As the volatility increases, the investment threshold increases. The NPV break-even point doesn't change with the volatility, but the magnitude of $\frac{\beta}{\beta-1}$ increases with the volatility, as shown in Figure 3.6(b). This is because the increased volatility means a higher risk, and thus a higher return is required as risk premium compensation. Therefore, it increases the optimal investment threshold and postpones the investment. However, volatility increases the opportunity value of the investment, since the real options approach can avoid the downside risk, but a larger volatility increases the return on investment with a larger upward demand fluctuation. It should be noted that when the volatility decreases to 0, the investment demand threshold is equal to 295 million passengers per year, which is still larger than the NPV break-even point. This result is identical to that in the deterministic case. As explained in Section 3.2, a positive demand growth rate makes it worthwhile to wait, because the investment cost decreases by a larger factor than the investment benefit as time goes on. Only when the growth rate is 0 or negative will the optimal investment rule from the real options approach be equal to that from the NPV rule. In Table 3.5, we show the project valuation results and evaluate the investment decision for California in 2015 as the construction was beginning. The intercity annual travel demand was 330 million, and the net present value was \$25.74 billion. We analyzed multiple scenarios for volatility from 0 to 20%. When the annual demand volatility is lower than 8.0%, there is no deferral premium and the current demand is greater than the investment threshold, hence the optimal investment strategy is to invest immediately. If the demand volatility is greater than 8.0%, there is a premium to wait, so the optimal investment decision is to defer instead of

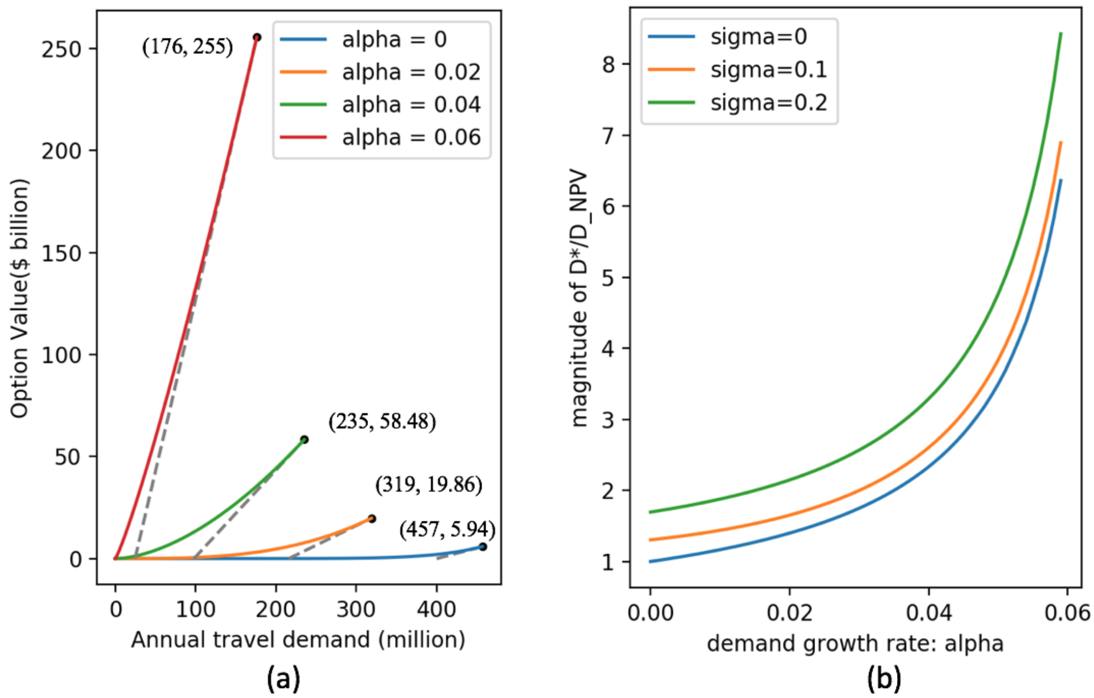
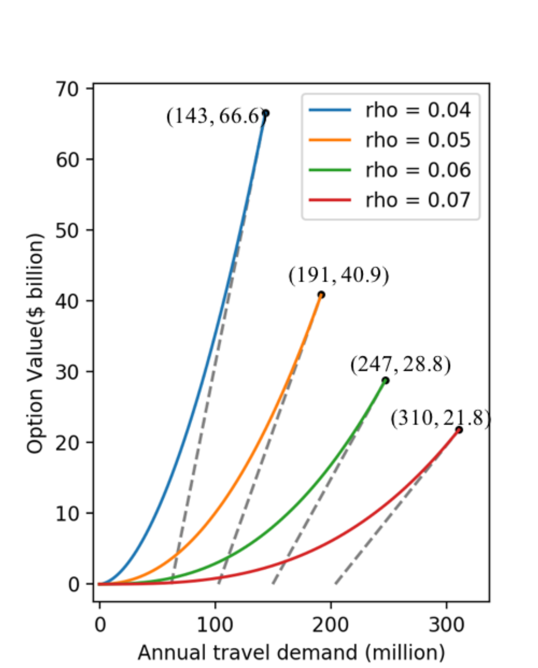


Figure 3.7: Value of investment opportunity under demand growth rate

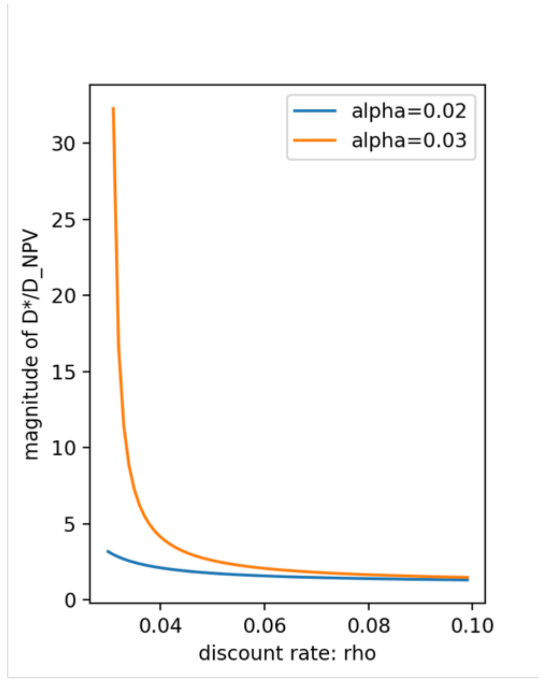
investing immediately.

- Travel demand growth rate α

Figure 3.7 shows the relationship between the travel demand growth rate and the investment threshold. Note that as discussed in Section 3.2, the upper bound of the demand growth rate should be smaller than the social discount rate $\rho = 0.07$. Figure 3.7(a) shows that the investment threshold decreases as the demand growth rate increases. By Equation (39), the investment threshold is $\frac{\beta}{\beta-1}$ multiplied by the NPV break-even point. Figure 3.7(b) shows that as the growth rate increases, the magnitude increases; however, because the NPV threshold decreases at a higher rate than the increment in the magnitude of $\frac{\beta}{\beta-1}$, the investment threshold decreases with the growth rate. The reason for the decrement in the NPV threshold is straightforward: The higher the demand



(a)



(b)

Figure 3.8: Value of social discount rate

growth rate, the higher the expected life-cycle investment benefits, hence investment is encouraged and the demand threshold is smaller. The magnitude of $\frac{\beta}{\beta-1}$ increases with the growth rate, because the growth makes it worthwhile to wait, as discussed in Section 3.2, which increases the value of the investment opportunity and postpones the investment.

- Social discount rate ρ

Figure 3.8 shows the the relationship between the social discount rate and the investment threshold. Figure 3.8(a) shows that the investment threshold increases as the discount rate increases, and Figure 3.8(b) shows that the magnitude of $\frac{\beta}{\beta-1}$ decreases as the social discount rate increases. The increment in the

NPV threshold is straightforward: A higher discount rate makes the future relatively less important, hence it depreciates the value of the investment benefits because the costs are concentrated in the early periods and the benefits accrue in the later periods. The magnitude of $\frac{\beta}{\beta-1}$ decreases as the discount rate increases, simply because a high discount rate decreases the value of exercising the option in the future and thus encourages investment. Because the NPV threshold increases at a larger rate than the decrements in the $\frac{\beta}{\beta-1}$, the investment demand threshold increases with the discount rate.

3.4 Summary

In this chapter, we developed a methodological framework to address transport infrastructure investment issues under regional intercity travel demand uncertainty based on the real options approach, with the goal of maximizing the social welfare, which includes the benefits from the consumer surplus, the producer surplus, and the environmental surplus. We derived a mathematical formulation of the optimal investment criteria using the real options method for both the deterministic and stochastic scenarios, and the merits of the real options approach are illustrated by comparison with the NPV investment breakeven point. Furthermore, we discussed the characteristics of the optimal investment rule under changes in the demand volatility, the demand growth rate, and the discount rate. In a case study, we used high-speed rail to investigate the optimal investment criteria and policy for California. The study used the regional intercity travel demand as the source of uncertainty, and considered travelers' choices and market competition between the new and existing transport modes to provide guidance for both government officials and private decision makers. The study also compared the social welfare investment threshold to the revenue threshold to validate whether HSR investment is socially profitable or revenue profitable under the current circumstances in the U.S. In addition, we evaluated the effects of environmental benefits of investment in HSR transport, and analyzed the investment policy decision with and without consideration of the environmental benefits. Furthermore, we discussed the difference in the optimal investment criteria estimated by the net present value and real options methods. Sensitivity analyses were also carried out to analyze the effects of key parameters and the corresponding changes in the optimal investment thresholds.

Although the proposed model provides useful insights for the HSR investment decision and policy evaluation under uncertainty, it does have several shortfalls, and some important extensions of this study could be undertaken in future work.

The parameters of the geometric Brownian motion cannot be calibrated without the time series OD data. However, this is an issue for transportation and urban planning, because the sporadic surveys were not available in the past decades thus there is no way to require such OD time series historic data. However, new technologies and sensor instruments have become widely used in this recent years, thus such OD data may be more readily available for the future research.

The social benefits of investment were underestimated in this study. In addition to the consumer surplus, the producer surplus, and the environmental surplus, there are other benefits resulting from HSR investment. According to CAHSR reports, for example, the annual economic loss of about $18.7 billion due to traffic congestion would be reduced with the use of HSR, and the price of land would change as a result of HSR investment. Future studies will explore methods that incorporate indirect economic benefits from the investment.

In this chapter, HSR is the main transport technology we used for illustration of our methodology, and the variation in the intercity travel demand was considered the major source of uncertainty affecting the HSR investment decision. In recent years, booming new transport technologies have captured public attention, for instance, hyperloop and Uber on-demand air transport. The uncertainties surrounding these new transport technologies are much larger than the uncertainties associated with mature technologies such as HSR. In addition to the demand uncertainty, the technological uncertainty cannot be ignored. Future studies will explore the appropriate model to capture multiple uncertainties in the investment decision for new modes of transport. We may model these uncertainties as a mix of Brownian motion and Poisson jump processes to simulate the technological innovation and competition.

The model proposed in this chapter was investigated for a single project in isolation, without considering budget constraints. A more realistic case for the investment decision making of government or investors should include a portfolio of projects together with investment prioritization under budget constraints. A future study could expand the current model to a portfolio optimization problem under a knapsack formulation, with the nature of the decision changing from what is the optimal timing of the investment to what is the optimal timing to invest in which projects.

The current study posited the government as the investor in transit projects with the objective of maximizing the expected social welfare. Recently, because of the huge capital requirements and the size of the federal budget deficit, the source of infrastructure financing has become a crucial issue. The mechanism of issuing revenue bonds for construction of high-speed transportwould be an interesting extension. In addition, the private sector has become increasingly involved in public infrastructure investment via other means, such as build–operate–transfer (BOT) and public–private partnership (PPP). The objective of private investors is to maximize profit and minimize investment risk. It would be worthwhile to extend the current model to consider investment strategies from the perspective of private investors , and to construct regulatory regimes to minimize the investment risk, such as by determination of the appropriate minimum revenue guarantee which is needed to attract private investors.

CHAPTER 4

INVESTMENT STRATEGY AS A PORTFOLIO OF INFRASTRUCTURE UNDER UNCERTAINTY: A SIMULATION BASED ON THE REAL-OPTIONS METHOD

Chapter 4 is an extension of the previous research on the optimal investment decision for a single infrastructure project to a dynamic decision support tool for investment in a portfolio of interdependent projects. The objective of this research is to address the following important issues in active infrastructure portfolio management: use of the most updated data and information for each candidate project in the portfolio, and whether to invest or not; which projects should be given priority and more attention; how the value of the portfolio changes when some of the options are exercised, and what the optimal managerial adaptable strategies are, contingent on the uncertain environment and previous investment.

The remaining sections of this chapter are organized as follows: In Section 4.1, we present a static investment benefit framework to quantify the marginal social welfare return on the interdependent projects in the portfolio. Section 4.2 presents the transportation infrastructure investment model using single and compound options to determine the investment strategy under time-dependent uncertainty. Section 4.3 illustrates the practicality of the multi-compound real-options model with case studies. The investment strategies are analyzed in the deterministic and stochastic cases. To show the merits of the proposed model, the results for the multi-option model are compared to those for the NPV and single-option models. The conclusions and recommendations for future extensions are given in Section 4.4.

4.1 Infrastructure Network Portfolio Management Model

The objective of this section is to quantify the investment payoff value. For investment in public infrastructure by an authority such as a state or local government or the U.S. Department of Transportation (DOT), the benefit is usually measured in terms of the social welfare instead of the flow of cash from operating revenue. Without loss of generality, we extended the social welfare model from investment in a single project in Chapter 3 to a portfolio of projects. The social welfare is defined as the sum of consumer benefits, producer benefits, and environmental benefits. The infrastructure investment cost and the operating cost are consistent with the models in Chapter 3. As discussed there, the marginal benefit of investment is defined as the difference in the social welfare with and without the investment, measured via the transport graph with the network effects from the existing transportation system. The assumptions made in the previous chapter are maintained in this chapter: that the total intercity travel demand is not impacted by the introduction of a new transport operator, and that the new investment changes only the market share by way of trip reassignment (De Rus and Nombela, 2007); and that the intercity travel demand is stochastically fluctuating over time and follows a geometric Brownian motion process (Zhao et al., 2004; Gao and Driouchi, 2013). In addition, all travelers in the region that would be served by the network are assumed to be homogeneous, and all operators are assumed to be homogeneous within the same type of transport mode.

4.1.1 Social welfare

In this work, the social welfare function is defined as the sum of the consumer benefits, including the fare, travel time, reliability, and safety benefits, in monetary terms; the producer surplus, which is the total profit of all operators; and the environmental benefits. Similar to Section 3.1.2, given the market share M_{ijk} of travel mode k for each OD pair (i, j), the producer surplus, consumer benefits, and environmental benefits can be expressed as

$$\pi_{i,j,k} = M_{ijk}D_{ij}P_{ijk} - C_{v,k}(d_{ij}, S_k)\frac{M_{ijk}D_{ij}}{S_k\eta_k} - C_{f,k}, \tag{4.1}$$

$$\zeta_{i,j,k} = \sum_v M_{ijk}D_{ij}X_{ijkv}\alpha_v, \tag{4.2}$$

$$\gamma_{i,j,k} = \sum_a M_{ijk}D_{ij}d_{ijk}E_{k,a}(\eta_k)\xi_a, \tag{4.3}$$

respectively, where D_{ij} is the travel demand for OD pair (i, j); P_{ijk} is the fare on mode k; and $C_{v,k}(d_{ij}, S_k)$ is the operating cost per trip, which is a function of the travel distance d_{ij} and the capacity (represented by the number of seats, S_k). $\frac{M_{ijk}D_{ij}}{S_k\eta_k}$ estimates the expected number of trips via mode k for (i, j), where η_k is the load rate. $C_{f,k}$ is the annual fixed operating cost. X_{ijkv} is the consumer benefit per person using mode k which derives from attribute v; the attributes include travel time, fare, reliability, productivity, safety, etc. $E_{k,a}$ denotes the emissions of substance a per person per unit distance, such as emissions of CO_2 per person per mile, on mode k. α_v is the monetary value of attribute v, such as value of time (VOT), and ξ_a is the monetary value of emissions of a.

For an OD pair (i, j), the changes in benefits for the producer, the consumer, and the environment which are due to the investment can be expressed as

$$\Delta\pi_{i,j} = \sum_{k}^{K^1} \pi^1_{i,j,k} - \sum_{k}^{K^0} \pi^0_{i,j,k}, \tag{4.4}$$

$$\Delta\zeta_{i,j} = \sum_{k}^{K^1} \zeta^1_{i,j,k} - \sum_{k}^{K^0} \zeta^0_{i,j,k}, \tag{4.5}$$

$$\Delta\gamma_{i,j} = \sum_{k}^{K^1} \gamma^1_{i,j,k} - \sum_{k}^{K^0} \gamma^0_{i,j,k}, \tag{4.6}$$

where $\pi^1_{i,j,k}$, $\zeta^1_{i,j,k}$, and $\gamma^1_{i,j,k}$ are the benefits after investment, and $\pi^0_{i,j,k}$, $\zeta^0_{i,j,k}$, and $\gamma^0_{i,j,k}$ are the benefits before investment. For each of the three categories of stakeholders, the total change in their benefits from all travel modes k is the sum of their benefits over all OD pairs. The infrastructure project investment could change the market share M_{ijk}, and thus change the benefits of the stakeholders. Therefore, the investment payoff of project h is the life-cycle benefit due to the change in social welfare from the OD pairs $(i, j) \in P_h$ that benefit from the investment in project h, less the investment construction cost C_h. It is reasonable to assume that the life cycle of a large infrastructure project is infinite (Chow and Regan, 2011b). Then the investment payoff can be expressed as follows:

$$\Pi_h = \mathbb{E} \int_0^\infty \sum_{(i,j)\in P_h} (\Delta\zeta_{i,j}(t) + \Delta\pi_{i,j}(t) + \Delta\gamma_{i,j}(t))e^{-\rho t} dt - C_h \tag{4.7}$$

The benefits of the different categories of stakeholders are a function of travel demand $D_{i,j}(t)$. The demand is a stochastic variable that evolves with time; details on this are presented in the next section. It should be noted that we define the investment decision time as the time at which the construction is completed and the project starts to generate benefits. The present value of the investment payoff shown in Equation (4.7) is the objective that we are aiming to maximize, by choosing an unknown optimal investment time. In the following section, we discuss an algorithm for finding the investment exercise time that will maximize the discounted present value of the investment payoff using the real-options method.

4.2 Investment Flexibility using Real Options

There is no doubt that the investment payoff is uncertain, since there are many sources of uncertainty over the life cycle of an infrastructure system. For example, the uncertainties may come from population growth, and from business and technology development activities, which can lead to changes in the travel demand and therefore in the investment benefits. The traditional discounted cash flow (DCF) valuation framework cannot address the uncertainty, since that framework assumes that once the decision is made, future stages of the project will take place as planned. In reality, however, the uncertain factors may not evolve as expected. In addition, the infrastructure investors can adjust their investment times contingent on new information. The option of a decision maker is the right, but not the obligation, to take an action in the future. As discussed in detail in Chapter 3, real options is an investment evaluation method derived from financial options for corporate finance, with the physical asset as the underlying option, as opposed to the financial instrument in financial options. Real options is an advanced valuation tool compared to the net present value method (NPV), as it allows for investment flexibility (Dixit and Pindyck,

1994) by allowing the decision maker to make use of the latest data to maximize the value of the decision. In option theory, we consider the investment decision as a right that can expire, as opposed to a static obligation in the NPV method. In Chapter 3, we showed that our modeling approach to demand uncertainty uses an Ito process, specifically a geometric Brownian motion. Then we derived a closed-form solution for the optimal investment criterion for a single project. In this chapter, we extend the valuation framework of a single project to a portfolio of interdependent projects.

4.2.1 Uncertainty modeling with multiple options

There are many sources of uncertainty in the life cycle of a large public infrastructure project, for example the interest rate, price and technology innovation on the part of market players, and the mode choice of the customers. However, for long-term transportation planning purposes, the OD demand is generally the source of greatest uncertainty (Chow and Regan, 2011a; Li et al., 2015; Zhao et al., 2004). We will begin from a model with a one-dimensional stochastic variable such as demand uncertainty, and then expand that to a model with a multi-dimensional stochastic variable. As discussed in Chapter 3, the intercity travel demand can be formulated as a geometric Brownian motion (GBM). GBM is commonly used for population variables such as OD demand that have a compound or exponential growth rate (Saphores and Boarnet, 2006; Chow and Regan, 2011b). Under this assumption, the travel demand satisfies

$$dD(t) = \alpha D(t)dt + \sigma D(t)dz(t), \tag{4.8}$$

where $D(t)$ is the travel demand at time t, α is the growth rate of travel demand, σ is the volatility rate of travel demand, dt is an infinitesimal time increment, and $dz(t)$ is a standard Wiener process, $dz = \epsilon_t \sqrt{dt}$, where ϵ_t is normally distributed with a mean of 0 and a standard deviation of 1. The geometric Brownian motion (GBM) shows that the percentage change in $D(t)$ is normally distributed. The distribution of the increment is log normal, the expected value is $\mathbb{E}[D(t)|D(0) = D_0] = D_0 e^{\alpha t}$, and the variance is $V[D(t)|D(0) = D_0] = D_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1)$. Using Ito's lemma, the $F(D) = \log D$ follows the simple Brownian motion with drift:

$$dF = (\alpha - \frac{1}{2}\sigma^2)dt + \sigma\epsilon_t \sqrt{dt} \tag{4.9}$$

For the infrastructure portfolio management model with multiple options, we will extend the one-dimensional geometric Brownian motion to a multi-dimensional geometric Brownian motion:

$$dD(t) = \alpha I D(t)dt + \sigma D(t)dz(t), \tag{4.10}$$

where $D(t)$ is the travel demand at time t for all OD pairs, $\alpha \in \mathbb{R}^{|N|*|N|}$ is the diagonal drift matrix, which can be rewritten as the drift vector $\alpha \in \mathbb{R}^{|N|^2}$, and I is the identity matrix. The diagonal elements of the diffusion matrix $\sigma \in \mathbb{R}^{|N|^2}$ represent the volatility of demand per OD pair, and the non-diagonal elements represent the covariance. If the demands for different OD pairs are independent, the diffusion matrix can be simplified as a volatility vector $\sigma \in \mathbb{R}^{|N|^2}$.

Real options are contingent claims on real investment projects. They are contingent on the state variables with the information uncertainty. To maximize the investment value, the real-options approach is used to determine the optimal investment time for infrastructure projects with an irreversible investment sunk cost. We will first define the valuation algorithm for a single option for a portfolio of infrastructure investments, and then expand that to the case of multiple compound options.

4.2.2 Portfolio of a single option under multiple uncertainties

For investment in a single project in a transportation network, the investment benefits is a function of the stochastic demand vector for all origin–destination pairs. We want to find the unknown future time τ of investment so that we can maximize the value of the investment option, $F(t, D_t)$, on a single project at time t:

$$F(t, D_t) = \max_{\tau \in [t,T]} \mathbb{E}_t\{e^{-\rho(\tau - t)}\Pi(\tau, D_\tau)\}, \tag{4.11}$$

where $\mathbb{E}_t$ represents the expectation conditional on the information which is available at time t, τ is the optimal exercise time ($\tau \in [t, T]$), and ρ is the adjusted-risk discount rate for continuous compounding. As in Chapter 3, we use the adjusted-risk discount rate instead of the risk-free rate commonly used in financial options, because the fundamental principle underlying financial option pricing is the assumption that no arbitrage exists in the markets. This condition would allow us to price financial options as the expected values under risk-neutral probabilities of their future payoffs at the risk-free rate (Meier et al., 2001). However, given the fact that in our case the underlying asset is the travel demand, which is not traded in financial markets, we cannot use the risk-free rate; instead, we use the risk-adjusted rate to discount the future value (Zhao et al., 2004). The selection of the risk-adjusted rate ρ was discussed in detail in

Chapter 3.

Because of the stochastic evolution of uncertainties, the optimal investment time τ is uncertain and cannot be expressed analytically. To solve the problem numerically, a Bellman equation is constructed to determine the optimal decision at any time step t_n, which is based on backward recursion from the final time step. The decision on the timing of infrastructure investment is a standard stopping problem: Stopping corresponds to making the investment, and continuation corresponds to waiting. Continuation does not generate any benefit/return within that period (Dixit and Pindyck, 1994). The Bellman equation for the investment problem is

$$F(t_n, D_{t_n}) = \max_{\mu_{t_n} \in \{0,1\}} [\Pi(D_{t_n}, \mu_{t_n}) + e^{-\rho(t_{n+1}-t_n)} \mathbb{E}_{t_n}(F(t_{n+1}, D_{t_{n+1}})], \tag{4.12}$$

where $F(t_n, D_{t_n})$ is the option value of an investment with a multi-dimensional underlying stochastic uncertainty, and $\Pi(D_{t_n}, \mu)$ is the investment payoff at time step t_n under the decision at time step t_n: $\mu_{t_n} \in \{0, 1\}$. $\mu_{t_n} = 1$ means invest, in which case the investment payoff will be quantified by Equation (4.7). Let $\Pi(D_{t_n}, 1) = \Pi(D_{t_n})$. If the investment is made, the option will no longer be alive, in which case $F(t_{n+1}, D_{t_{n+1}}) = 0$. If $\mu_{t_n} = 0$, we defer the investment and keep the option alive. Since no social benefits are generated in continuation, $\Pi(D_{t_n}, 0) = 0$. Thus Equation (4.12) can be rewritten as

$$F(t_n, D_{t_n}) = \max_{\mu_{t_n} \in \{0,1\}} [\Pi(D_{t_n}), e^{-\rho(t_{n+1}-t_n)} \mathbb{E}_{t_n}(F(t_{n+1}, D_{t_{n+1}})] \tag{4.13}$$

Equation (4.13) is the Bellman equation for investment in a single project under multi-dimensional uncertainty. At each time step, the decision of the process is whether to stop deferring and invest immediately (if $\mu_{t_n} = 1$) with value $\Pi(D_{t_n})$, or to continue deferring with value $e^{-\rho(t_{n+1}-t_n)}\mathbb{E}_{t_n}(F(t_{n+1}, D_{t_{n+1}})$. The algorithm that finds the investment decision is relatively simple in the case of one-dimensional uncertainty and can easily be dealt with via financial options. The problem can be solved using differential equations if the underlying variable of the option is an Ito process, as defined in Equation (4.8). For details of a closed-form formulation derived from Ito's Lemma, see Chapter 3, where we proposed an algorithm to identify the optimal investment criterion for infrastructure investment with one-dimensional uncertainty. However, for the more complicated case of multiple uncertainties, the underlying value of the option is no longer an Ito process, but rather a function of variables that evolve as Ito processes. In our case, for example, the underlying value of the investment option is a function of multiple state variables such as the demands, each of evolves as an Ito process. The underlying asset value has no closed form (Beaulieu and Xie, 2004). In addition, the network effect with a portfolio of multiple interacting options renders the model more complicated than the closed-form solution and thus has limitations for this type of problem (Vergara-Alert, 2007; Li et al., 2015). Numerical methods have advantages for valuation of a portfolio of options with uncertainty driven by several stochastic variables.

There are three numerical methods that are commonly used for option valuation: finite difference, binomial tree, and Monte Carlo simulation (Hull and Basu, 2016). The finite difference method requires establishing a differential equation to relate the option value to the stochastic variables, and then using discrete finite difference methods to numerically estimate the solution. This method is not suitable for portfolio investment in a network, where the differential equation would be difficult to specify. Binomial lattice methods assume that the probability distribution at each time step can be divided into two groups. It has been proved that the solution of a binomial lattice problem converges to the actual solution when the number of intervals increases within the time horizon. This method can be used to deal with multiple options simultaneously; however, it cannot handle multidimensional variables very well, because of its inherent method of simplifying the probability distribution into branches. The third method, Monte Carlo simulation, entails simulation of multiple paths of realization for the stochastic processes and backward dynamic programming.

The Monte Carlo simulation method has an advantage over the finite difference and binomial tree techniques when there are multiple uncertainty factors. The difficulty of using Monte Carlo simulation is that at any time step, the backward dynamic programming requires estimation of the conditional expected continuation value at the next time step, as shown in Equation (4.13). Longstaff and Schwartz (2001) developed a least-squares Monte Carlo (LSM) simulation method for estimation of the conditional expectation from cross-sectional information in the simulation. The LSM regresses the realized payoffs from continuation on a function of the values of the state variables. The fitted value from this regression provides a direct estimation of the conditional expectation function. By estimating the conditional expectation function for each time step, we obtain a complete specification of the optimal exercise strategy along each simulation path.

4.2.3 Solution algorithm for single-option investment

For the infrastructure investment problem of a single project for a transportation network, the investment benefit is a function of a multidimensional stochastic variable, D_{t_n}. As a result and because of the complexity of the differential equation, it is difficult to apply the binomial lattice method or the finite difference method to solve the problem. We developed an algorithm for a single option based on the Monte Carlo simulation algorithm to estimate the option value given in Equation (4.13). To determine the optimal path policy, we compare the continuation value with the payoff. Denote the continuation value at time step t_n for simulation path i by $\Phi(t_n, D_{t_n}(i))$. That is,

$$\Phi(t_n, D_{t_n}(i)) = e^{-\rho(t_{n+1}-t_n)}\mathbb{E}_{t_n}(F(t_{n+1}, D_{t_{n+1}}(i)) \tag{4.14}$$

For simulation path i, the optimal investment time can be found by backward recursion, beginning at the last time step of the decision horizon T. The decision horizon is defined as the period of time within which the investment option is alive, while the option value is 0 after the expiration time T. For each time step in the backward recursion, the following condition holds:

$$\text{if } \Phi(t_n, D_{t_n}(i)) \leq \Pi(D_{t_n}(i)), \text{then } \tau(i) = t_n \tag{4.15}$$

Since the option is expired for all $t > T$, the continuation value at time T is 0: $\Phi(T, D_T) = 0$. Then as long as the payoff value is positive, Equation (4.15) holds and $\tau(i) = T$. For each simulation path, the decision is evaluated backwards from time T. For time steps t_n prior to T, the decision maker should compare the payoff obtained from immediate exercise at t_n, $\Pi(D_{t_n}(i))$, to the continuation value, $\Phi(t_n, D_{t_n}(i))$. Thus in order to apply the rule of Equation (4.15), the problem reduces to finding the payoff and expected continuation value at each time step.

The payoff is the present value of the investment, which is a function of the stochastic variables for travel demand, can be obtained by Equation (4.7). In the payoff model, we omit the notation for simulation path i, as the rule applies to all simulation paths. If we denote the demand growth rate for OD pair (i, j) by $\alpha_{i,j}$, then the expectation of the demand is

$$\mathbb{E}[D(t_n + t)] = D_{t_n} e^{\alpha_{i,j} t}, t > 0 \tag{4.16}$$

Thus the investment payoff at time step t_n is

$$\begin{aligned}\Pi(D_{t_n}) =& \mathbb{E} \int_0^\infty \sum_{(i,j)} [\Delta\zeta_{i,j}(D_{t_n}) + \Delta\pi_{i,j}(D_{t_n}) + \Delta\gamma_{i,j}(D_{t_n})] e^{\alpha_{i,j} t} e^{-\rho t} dt - \int_0^\infty C_f e^{-\rho t} dt - C_I \\ =& \sum_{(i,j)} \frac{1}{\rho - \alpha_{i,j}} [\Delta\zeta_{i,j}(D_{t_n}) + \Delta\pi_{i,j}(D_{t_n}) + \Delta\gamma_{i,j}(D_{t_n})] - \frac{1}{\rho} C_f - C_I\end{aligned} \tag{4.17}$$

In Equation (4.17), the first term is the present value of the investment benefit once construction is completed and the project is in service, which is the sum of the present values of the social welfare from all the OD pairs that stand to benefit from the investment project. The second term is the present value of the fixed maintenance cost, and the third term is the investment cost. Equation (4.17) is a general formulation for both the single-option model and the multi-option model. The single-option model is a special case where investment is made in all projects at the same time.

The expectation value of the continuation can be estimated by the LSM method developed by Longstaff and Schwartz (2001). The LSM method regresses the discounted future option value $\Phi(t_{n+1})$ on a set of polynomials that form an orthonormal basis for the current state variables X_{t_n}. We could choose any set of measurable basis functions to approximate the conditional expectation function for the option value, $\mathbb{E}_{t_n}(F(t_{n+1}, D_{t_{n+1}}))$. It has been shown that the conditional expectation is an element of the L^2 space of square-integrable functions relative to some measure (Longstaff and Schwartz, 2001). There are various choices for the orthonormal basis functions. For example, one possible choice is the Laguerre polynomials:

$$L_n(X) = \exp(-X/2)\frac{e^X}{n!}\frac{d^n}{dX^n}(X^n e^{(-X)}) \tag{4.18}$$

L_n is the orthonormal basis for the state variables X_{t_n} that we used as the regressors to determine the present value on simulation path i, where the dependent variable is the discounted option value at the next time step, $F(t_{n+1}, D_{t_{n+1}}(i))$, which is known at time step t_n by the backward recursion algorithm. Thus the least-squares regression is equivalent to solving the following optimization problem:

$$\min_{\beta} \sum_{i=1}^{I} \| e^{-\rho(t_{n+1}-t_n)} F(t_{n+1}, D_{t_{n+1}}(i)) - \sum_{n=1}^{N} \beta_n L_n(X_{t_n}(i)) \|^2 \tag{4.19}$$

The state variables X_{t_n} in Equation (4.19) are defined as the values of the assets underlying the options (Longstaff and Schwartz, 2001). In our case, the value of the underlying asset of an investment option is the present value of the investment. This is a function of the demand vector D_{t_n}, which evolves as an Ito process, thus $X_{t_n}(i) = \Pi(D_{t_n}(i))$. Then the expected continuation value $\Phi^*(t_n, D_{t_n}(k))$ conditioned on the current information available at time step t_n will be estimated by the coefficients β_n^* that are found from solving Equation (4.19):

$$\Phi(t_n, D_{t_n}(i)) \approx \Phi^*(t_n, D_{t_n}(i)) = \sum_{n=1}^{N} \beta_n^* L_n(D_{t_n}(i)) \tag{4.20}$$

By comparing the estimated continuation value in Equation (4.20) with the immediate exercise value and choosing the larger one of the two for each time step, and working backwards until $t = t_0$, we can determine the optimal investment time $\tau(i)$ for simulation path i. When the decision given in Equation (4.15) holds, the decision is updated. Finally, the value of the investment option is calculated as the mean of the values over all simulation paths:

$$F(t_0, D_{t_0}) = \frac{1}{I} \sum_{i=1}^{I} e^{-\rho(\tau(i))} \cdot \Pi(D_\tau(i)), \tag{4.21}$$

where I is the number of simulation paths.

The value of $F(t_0, D_{t_0})$ given in Equation (4.21) is the option value of the decision horizon at time t_0, which is the value of keeping the opportunity open instead of investing immediately. If $F(t_0, D_{t_0}) > \Pi(D_{t_0})$, then by the condition in Equation (4.15), the option is worth keeping and the better strategy is to defer the investment, whereas if $F(t_0, D_{t_0}) \leq \Pi(D_{t_0})$ the better strategy is to invest immediately.

4.2.4 Portfolio of multiple options under multidimensional uncertainty

The framework in Section 4.2 provides an effective algorithm for single-option valuation under multidimensional uncertainty, and the numerical simulation algorithm enables the decision maker to take the investment flexibility into consideration. However, this framework cannot handle a portfolio of interdependent options, while for investment in infrastructure systems it is critical that the capital budgeting model have the capability to treat each link or project separately but as one that interacts with the other options in the network system as well, and to recommend the optimal investment strategy for each project.

For the problem of a portfolio of interdependent transportation infrastructure network investments, our goal is to build a dynamic adaptive decision support tool, to decide the best time to invest in each individual project in the portfolio based on all the current information we have, and to make an adaptive decision as more information is revealed in the future under a stochastic setting. The real-options principle provides a great solution method for this problem. There are some challenges, as it involves a portfolio of infrastructure investments in a network setting. First, the investment payoff of a certain project h depends on what has been invested before, since the payoff is the marginal benefit of investment, which is defined as the difference in the social welfare with and without the investment, as shown in Equations (4.7) and (4.17). However, the dynamic programming in the real-options model requires a backward recursion algorithm in which the option value and the investment decision for project h at time step t_n depend on the option value and investment decision at time step t_{n+1}. This feature makes the problem intractable without a defined investment sequence. Thus for each investment candidate project, we have to define what has been invested before and what is to be invested at subsequent time steps. Therefore, an investment sequence has to be defined in such a way that it serves as an instrument of the valuation model.

There are three characteristics of the investment sequence. First, assuming that we have a sequence of projects $1, 2, \ldots, h, h+1, \ldots H$, project $h+1$ cannot be invested in unless all projects from 1 through h have already been invested in. Second, the investment payoff of project $h+1$ is the life-cycle social welfare from the investment in projects 1 through $h+1$ less the life-cycle social welfare from the investment in projects 1 through h. (See the payoff model given in Section 4.1.) Third, the total number of permutations for a portfolio of H projects is $H!$, and our goal is to search for the sequence that gives the highest initial option value, since the initial option value captures the option values of all projects in the sequence, as shown in Chapter 1. It should be noted that the sequence of the projects is defined according to their construction completion times, not their construction starting times. As the infrastructure investment cannot generate any benefit until construction is completed, the value of the marginal benefit from project h depends on its construction completion time. If we defined the investment according to the construction starting time, we could not decide whether projects $1, ..., h-1$ have been completed or are still under construction, hence the investment payoff of project h would be intractable if we used dynamic programming. Under this definition, the option value will be evaluated at the point of construction completion, and the investment decision for the best construction completion time will be proposed. Since the construction duration is predefined, we can estimate the expected optimal time to begin the construction.

The investment sequence we have constructed follows the features of the compound options described in Gamba's portfolio of options framework (Gamba, 2002). Using dynamic programming to break the multi-stage problem into time steps with the backward recursion algorithm, we obtain the option values given by the following Bellman equation for project h in a given sequence of ordered investment projects:

$$F_h(t_n, D_{t_n}) = \max[\Pi_h(D_{t_n}) + F_{h+1}(t_n, D_{t_n}), e^{-\rho(t_{n+1}-t_n)}\mathbb{E}_{t_n}(F_h(t_{n+1}, D_{t_{n+1}}))], \tag{4.22}$$

where $F_h(t_n, D_{t_n})$ is the option value of project h at time step t_n, which is the maximum value of two components: the investment payoff and the continuation value. In contrast to the Bellman equation for a single option from Equation (4.13), the investment payoff in the multi-option model is the sum of two parts, as the benefit of investing in project h includes the investment payoff for project h and the option value of investing in project $h+1$. $\Pi_h(D_{t_n})$ is the present value of the investment payoff for project h at time step t_n, which can be calculated by Equation (4.17), and $F_{h+1}(t_n, D_{t_n})$ is the option value for project $h+1$ at time step t_n. The second component of Equation (4.22) is the continuation value, $\mathbb{E}_{t_{n+1}}(F_h(t_{n+1}, D_{t_{n+1}}))$, which is the expectation function of all future investment benefits from the contingent claim optimally exercised at a certain time, and hence is essentially the option value for project h at time step $t+1$, where the expectation is taken conditional on the information which is available at time t_n. The value of $\mathbb{E}_{t_{n+1}}(F_h(t_{n+1}, D_{t_{n+1}}))$ can be estimated by the LSM method, as described in Section 4.2.2.

4.2.5 Solution algorithm for a portfolio of multiple options

The LSM-based solution algorithm for the multi-option model entails two-dimensional dynamic programming: We add a backward option recursion to the backward time recursion for the single-option model. The solution algorithm for the model with a portfolio of options is given below. If the number of options is reduced to 1, it is identical to the single-option solution algorithm discussed in Section 4.2.2.

The solution algorithm for the multi-option investment model searches for the investment sequence that offers the highest option value for the initial project, and uses that sequence as the reference for the optimal investment decision. This portfolio of multiple options sets the framework and estimates which future decision will be made in which time frame, but it also has the flexibility to learn from ongoing development and allows for discretion to act based on what is learned. However, it should be noted that the investment decision can be updated and the sequence can be changed in the future in order to incorporate any investments that are not made initially. Therefore, the best fixed sequence is not necessarily the final investment strategy, but it serves as an instrument for finding the lower bound on the true solution. As additional information becomes available in the future, the dynamic adaptive decision support model can be evaluated by incorporating the updated information to recommend updated investment decisions. A case study is used in the next section to illustrate the feasibility and practicality of the dynamic adaptive decision support tool.

Algorithm 2: Algorithm for a portfolio of options in infrastructure system investment

Demand simulation with I paths, T steps, and S $(= H!)$ project sequences ;
while $s < S$ **do**
  $t = T$: $F_h(D_T) = \max\{\Pi_h(D_T), 0\}, h \in H$;
  while $t > t_1$ **do**
    while $h > 0$ **do**
      Compute $\Pi_h(D_{t_n})$;
      Least-squares estimation: $\mathbb{E}_{t_n}(F_h(t_{n+1}, D_{t_{n+1}}))$;
      if $\Pi_h(D_{t_n}) + F_{h+1}(t_n, D_{t_n}) > e^{-\rho(t_{n+1}-t_n)}\mathbb{E}_{t_n}(F_h(t_{n+1}, D_{t_{n+1}})$ **then**
        $\mu_{t,h} = 1$, invest; $F_h(t_n, D_{t_n}) = \Pi_h(D_{t_n}) + F_{h+1}(t_n, D_{t_n})$;
      else
        $\mu_{t,x} = 0$, defer; $F_x(t_n, D_{t_n}) = e^{-\rho(t_{n+1}-t_n)}(F_x(t_{n+1}, D_{t_{n+1}}), x \in \{h, \ldots, H\}$;
      end
      $h = h - 1$;
    end
    $t = t - 1$;
  end
  $t = t_0$: $F_h(t_0, D_{t_0}) = \frac{1}{I}\sum_{i=1}^{I} e^{-\rho(\tau_h(i))} \cdot \Pi_h(D_\tau(i)), h \in H$;
  for $h = H : 1$ **do**
    if $\Pi_h(t_0, D_{t_0}) + F_{h+1}(t_0, D_{t_0}) > F_h(t_0, D_{t_0})$ **then**
      invest: $\mu_{t_0,h} = 1, F_h(t_0, D_{t_0}) = \Pi_h(t_0, D_{t_0}) + F_{h+1}(t_0, D_{t_0})$
    else
      defer: $\mu_{t_0,x} = 0, x \in \{h, \ldots, H\}$
    end
  end
end
Value of portfolio of investment = $\max_s\{F_{s,h=1}(t_0, D_{t_0})\}$

4.2.6 Algorithm convergence

The convergence properties of the LSM algorithm have been studied in previous research. Clément et al. (2002) pointed out that there are two types of approximations in the algorithm: (1) approximation of the continuation value by its projection on the linear space of a finite set of basis functions, and (2) use of a finite number of Monte Carlo simulation paths, together with ordinary least-squares (OLS) regression, to estimate the coefficients of the basis functions. Longstaff and Schwartz (2001) proved that for a basis with a sufficiently large number of polynomials, as the numbers of simulation paths and time steps approach infinity, the option value is within ϵ of the true value. They partially proved that the LSM converges asymptotically to the unbiased estimator of the true option value with some fixed number of basis functions. This conclusion is applicable for a regression with one dependent variable. In our case, we have multiple uncertainty variables, but the regressor, which is a one-dimensional variable that represents the net present value of investment, is a function of the uncertainties. Garcia et al. (2003) validated that there is a tendency to have a low bias for small numbers of basis functions and a high bias for small numbers of simulation paths. Glasserman et al. (2004) studied the convergence rate of the algorithm when the number of basis functions and the number of paths increase simultaneously. They demonstrated that in certain cases, in order to guarantee convergence the number of paths must grow exponentially with the number of polynomial basis functions when the underlying state variable follows a Brownian motion, or faster than exponentially when the underlying state variable follows a geometric Brownian motion. According to Glasserman et al. (2004), the number of basis functions, M, and the number of simulation paths, N, are related as in Equations (4.23) and (4.24) for the worst-case convergence in the one-dimensional case. Jonen (2009) tested this and validated that it works for the case of a multi-dimensional geometric Brownian motion.

$$M = \mathcal{O}(\sqrt{\log(N)}) \tag{4.23}$$

$$N = \mathcal{O}(\exp(M^2)) \tag{4.24}$$

Based on the previous research, we will first choose a fixed number of basis functions following the relation given in Equation (4.24), and then increase the number of simulation paths until the maximum relative error is within some tolerance. The performance of a Monte Carlo estimator $\bar{V}$ for a quantity V can be measured by the root mean square error (RMSE) (Jonen, 2009), which is a scale-dependent indicator defined by

$$\mathrm{RMSE}(\bar{V}) = \sqrt{\mathbb{E}(\bar{V} - V)^2} = \sqrt{\mathrm{Bias}(\bar{V})^2 + \mathrm{Variance}(\bar{V})} \tag{4.25}$$

Clement showed that, in practice, LSM converges to an approximation of the true value which is asymptotically Gaussian distributed (Clément et al., 2002) and with $\mathrm{Bias}(\bar{V})^2 = 0$. We calculate the standard deviation of the estimator under different numbers of simulation paths until the option value stops increasing beyond some tolerance.

4.3 Applications

In the case study, we continue to use the case of investment in the California High Speed Rail system shown in Chapter 3 to illustrate the investment strategies for a portfolio of infrastructure investments under the network effect using the methods proposed in this chapter.

4.3.1 Project background

The California High Speed Rail (CAHSR) system will connect five clusters of major population density regions in California, from San Diego and the Los Angeles (LA) Basin in southern California, to the San Joaquin Valley in central California (the "Central Valley"), and the San Francisco Bay Area and Sacramento in northern California. A map of the planned system with the connected clusters is shown in Figure 4.1(a) (Brinckhoffs, April 2014). The baseline values of the model input parameters are given in Table 4.1. As in Chapter 3, we use the values of the parameters, such as the social discount rate, the expected annual travel growth rate, the volatility rate, and the value of time, which were taken from government reports. As our aim in this chapter is to develop advanced investment strategies for a portfolio of projects, we further divide the California High Speed Rail system into four projects, as shown in Figure 4.1(b). Project 1 connects the San Francisco Bay Area to the Central Valley, project 2 connects the Central Valley to Los Angeles, project 3 connects Los Angeles to San Diego, and project 4 connects the Central Valley to Sacramento. The construction costs of the projects are given in Table 4.1 (California HSR Peer Review Group, 2012). We estimated the cost of each project from the duration of its construction. The

(a) (b)

Figure 4.1: Planned California High Speed Rail

Categories	Description	Parameters	HSR	Air	auto
Model Parameters	Social discount rate	ρ	0.07		
	Annual travel growth rate	α	2.17%		
	Value of time ($/hour)	VOT	43.70		
	Carbon Price ($/ton)	ξ	40		
	Number of seats	S	118	450	5
	Load rate	η	0.8	0.8	0.38
Operation Cost	Infrastructure fixed operation costs (per year per route mile)	C_f	$200,000	-	-
	Variable operating costs	C_v	$20 per train mile	0.019*(d+722)*(S+104)	$0.25 per mile
Investment Cost	Project 1 ($ billion)	C	$23.14	-	-
	Project 2 ($ billion)		$42.34		
	Project 3 ($ billion)		$22.38		
	Project 4 ($ billion)		$22.58		

Table 4.1: Baseline parameters of planned California High Speed Rail system

construction cost in Table 4.1 is the year-of-expenditure (YOE) cost for the year of project completion. We can quantify the project investment cost at completion using the product of the actual annual expenditure and the real discount rate for the cost in the future.

The benefits of investment in the California High Speed Rail system include operator benefits, consumer benefits, and environmental benefits derived from various attributes such as the fare, travel time, reliability, productivity, safety, and reduction in emissions of greenhouse gases (GHG). The analysis of investment benefits was presented in Section 3.4. The cost of investment includes both the investment construction cost and the operating cost over the service life cycle of the infrastructure. We keep the main input parameters such as investment cost, travel distance, fare, travel time, and the share of travel modes before and after the investment in CAHSR consistent with those in Chapter 3. We present and compare the two cases, the single option under multiple uncertainties and a portfolio of multiple options under multiple uncertainties, to illustrate the investment decision under different model strategies.

The first case is the investment model with a single option. The single-option model allows us to take the four projects as an integrated system to determine the optimal investment time of the system. The decision of the single-option model is similar to that in the case shown in Chapter 3, but with some differences. In Chapter 3, we assumed that the total travel demand over all origin–destination (OD) pairs is the only source of uncertainty, and that the demand for each OD pair is proportional to the total demand: $D_{i,j} = \theta_{i,j}D$. D evolves as a stochastic process, and $\theta_{i,j}$ is the given value of demand share for travel from city i to city j. However, it is less likely that the demand for all OD pairs will grow in a fixed proportion to the total demand. Thus the single-option model with multiple uncertainties increases the flexibility, as the demand for each OD pair can be modeled as a separate stochastic process. The second case is the investment model for a portfolio of multiple compound options. In practice, it is critical that we treat each investment project in the system separately and

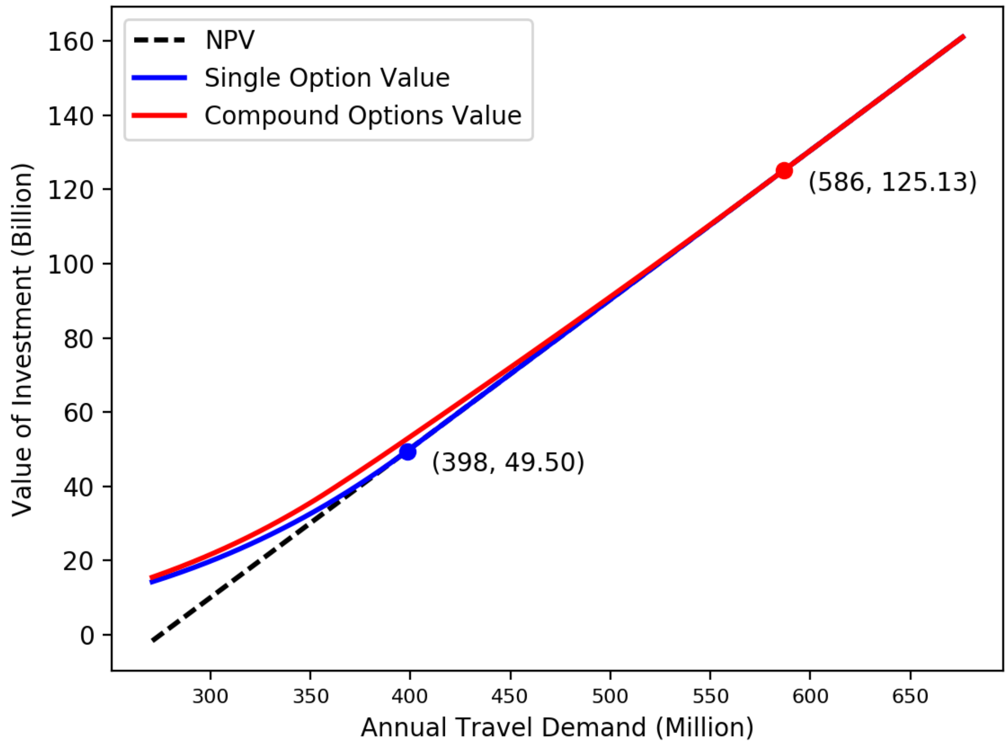


Figure 4.2: Option values and NPV in the deterministic case

recommend the optimal investment time for each of them. Building upon case 1, we add extra flexibility to the second case by treating the investment in each project as a separate option, and proposing the optimal investment time and strategy for each project in the portfolio. In the following section, we begin with the deterministic case, and then extend that to the stochastic case.

4.3.2 Deterministic case

The deterministic approach is a generally accepted practice for many planning agencies. The growth in travel demand can be represented as $D(t_1) = e^{\alpha(t_1-t_0)}D(t_0)$, where α is the annual compounding growth rate. Figure 4.2 shows the value of the investment portfolio under different demand levels upon valuation with three models: NPV, a single option, and a portfolio of multiple options. The dashed black line shows the net present value of social welfare under the annual travel demand at completion of construction, the blue line is the option value with a single option, and the red line is the option value with multiple compound options. The dashed line shows that the break-even threshold under NPV is about 275 million passengers per year at completion of construction, which is identical to the 203 million passenger level at the start of construction with a 14-year construction duration, as shown in Chapter 3. Table 4.2 shows that at the break-even threshold, the net present value of investment is 0, while the option value with a one-time investment is \$14.99 billion, and the option value with multiple compound options is \$16.29 billion. The option value is the expanded net present value, which includes the base NPV and the present value of growth option (PVGO). At the break-even threshold, the base NPV is 0, while it is worth \$14.99 billion to defer the one-time investment in the system, and \$16.29 billion if there is flexibility to invest in each project in the portfolio separately under the multi-option model. When the demand is lower than the single-option investment threshold, which is 398 million passengers per year, the option value lies on a curve above the NPV line, which means that there is a deferral value and the best strategy is to keep the option alive. Furthermore, the multi-option value lies on a curve above the single-option value until the demand reaches 586 million, which is the optimal investment threshold for project 3.

The deterministic results have some implications: First, similarly to the results shown in Chapter 3, the NPV rule is incorrect even in the deterministic scenario. There is still a value of waiting in the deterministic case, because the

Model	Project	Investment year	Demand threshold (Million/yr)	Investment criteria	Value of investment ($ billion)
NPV	All projects	0	275.06	NPV>=0	0
Single Option	All projects	17.1	398.65	NPV>=V	14.99
Multi-Option (best sequence)	1	12.5	361.69	NPV1+V2>=V1	16.29
	2	12.5	361.69	NPV2+V3>=V2	
	3	16.3	391.7	NPV3+V4>=V3	
	4	34.9	586.46	NPV4>=V4	

Table 4.2: Investment strategies in the deterministic case

benefit of investment driven by the positive demand growth rate α decreases by a smaller factor ($e^{-(\rho-\alpha)T}$) than the cost of investment ($e^{-\rho T}$), which leads to an optimal investing time at which the discounted social welfare of the investment reaches a maximum. Second, the multi-option model reveals its advantages over the single-option rule by treating each project in the portfolio as a separate option. The investment criterion for the single-option case is to invest if the net present value of the investment payoff is at least as large as the value of waiting; for the multiple-option case, we will invest if the sum of the value of the payoff and the option value of the subsequent project is at least as large as the value of deferring the current project. Table 4.2 shows the investment time, demand threshold, and option value for all three models. Assigning the break-even year when NPV is equal to 0 as the initial year, the optimal time to invest in the system is in year 17.1, when the annual demand grows to 398.65 million passengers. With the multi-option model, however, the best investment strategy is to invest in both project 1 and project 2 in year 12.5, to invest in project 3 in year 16.3, and to invest in project 4 in year 34.9, when the annual demand grows to 586.46 million. The multi-option model allows the flexibility of investing in each project in its optimal year. This flexibility increases the value of the investment portfolio.

There are a couple of reasons to accelerate investment in projects 1 and 2 while deferring investment in project 4, as shown in the multi-option model. First, the investment benefits of projects 1 and 2 are large, while the benefits of project 4 are smaller. As shown in Figure 3.2 in Chapter 3, the investment benefits are greatest for travel between San Francisco and Los Angeles (28.9%), between San Francisco and the Central Valley (20%), and between the Central Valley and Los Angeles (16.8%). These OD pairs are connected by projects 1 and 2. Project 3 benefits the most from travel between Los Angeles and San Diego (14.7%). The benefits of project 4 come from travel between Sacramento and San Francisco, the Central Valley, Los Angeles, and San Diego. However, based on the mode share data in Table 3.1, the percentage of travelers who switch from travel by auto or airline to high-speed rail is very small. The investment benefits are least for travelers between Sacramento and the other cities, with only 14% in total, while the investment cost for project 4 is relatively the same as for the other projects because the construction cost depends mainly on the duration of construction. Thus project 4 is the least favorable one, because of its huge construction cost and relatively smaller investment benefits.

Second, the "backbone" of the high-speed rail network is project 2, as it connects northern California to southern California. Investment in projects 3 and 4 wouldn't provide as much in the way of benefits as project 2. Therefore, there is value in deferring the option so that the investment benefits which are driven by the demand level can increase to the level of compensation for the large investment cost. The investment strategy proposed by our model supports the current investment policy and practices. According to the California High Speed Rail Authority (Brinckhoffs, April 2014), the system is planned to be built in two phases. Phase I connects San Francisco, Los Angeles, and the Central Valley, which corresponds to projects 1 and 2 in our model. Phase II extends that service by connecting the Central Valley to Sacramento, and Los Angeles to San Diego, which corresponds to projects 3 and 4 in our model.

4.3.3 Stochastic case

In this section, we will present the investment strategies under stochastic scenarios, where we assume that the travel demand follows a geometric Brownian motion (GBM). Stochastic growth means that the demand increment for each OD pair is log-normally distributed such that $\ln D_{t+\Delta t} - \ln D_t \sim N((\alpha - \frac{1}{2}\sigma^2)\Delta t, \sigma^2 \Delta t)$. Thus we simulated the demand, $D_t = D_{t-1} \exp\{(\alpha - \frac{\sigma^2}{2})dt + \epsilon_t \sigma \sqrt{dt}\}$, where α is the drift rate and σ is the volatility of the GBM. To intuitively visualize the demand growth under different volatilities, we simulated 10 sample paths over a period of 30 years under four different volatilities: $\sigma = 0.05$, $\sigma = 0.1$, $\sigma = 0.15$, and $\sigma = 0.2$. As the time time increment dt increases, the uncertainty in the forecast of the demand increases. In the simulations, we divided each year in the 30-year period into 1000 time steps. We used the fixed random seed in all uncertain scenarios in order to compare the impact of the change in σ. In Figure 4.3, we compare the growth of stochastic demand under each of the four different volatilities with the expected growth shown in the solid black line.

4.3.3.1 Number of simulation paths and polynomial basis

To test the convergence of the algorithm, we ran least-squares Monte Carlo simulations to estimate the option value with different numbers of paths and different numbers of basis functions. Based on Equation (4.24), we first ran

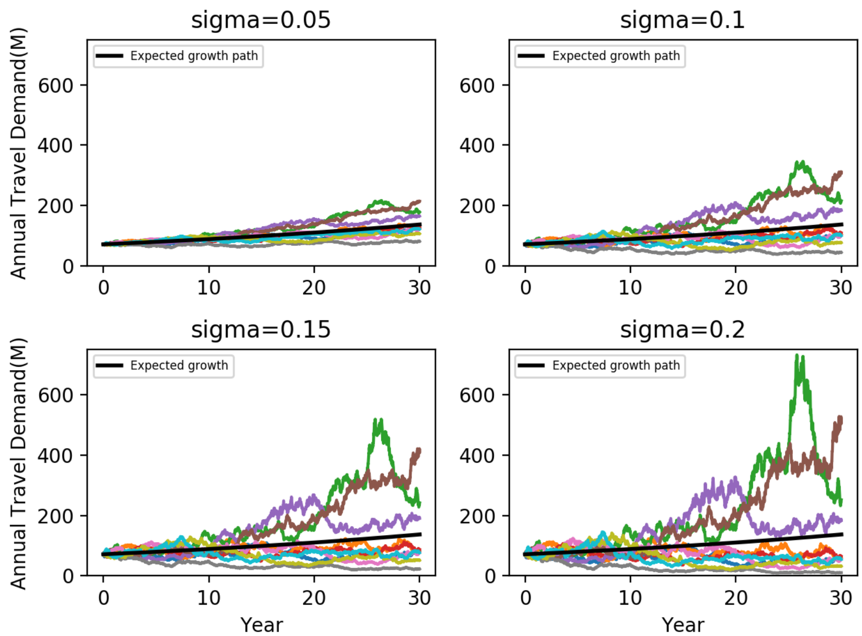


Figure 4.3: Travel demand simulation under different volatilities

a least-squares fit of Laguerre polynomials to our data with a basis of three polynomials, and compared the option value in the models with 1,000, 10,000, and 100,000 simulation paths and 10% demand volatility. Table 4.3 shows the means, standard errors, and 95% confidence intervals of the resulting option values with 20 trials. As the number of simulation paths increases, the standard deviation decreases, as expected (Longstaff and Schwartz, 2001). For a given number of simulation paths, the standard errors of the single-option model and the multi-option model are very similar. The standard error of the 20 samples decreases from about 0.8% to about 0.3% when the number of simulation paths increases from 1,000 to 10,000, with an increase in the computation time from 10 seconds to 45 seconds. With 100,000 simulation paths, the standard error is about 0.1%, while the computation time is 1 hour 53 minutes.

Num. of Paths N	Model	mean ($B)	s.e. ($B)	s.e. (%)	95% Confident Intervals	
					Lower	Upper
1000	Single-Option	16.595	0.141	0.85	16.543	16.647
	Multi-Option	18.823	0.133	0.71	18.775	18.872
10000	Single-Option	16.523	0.044	0.27	16.507	16.539
	Multi-Option	18.649	0.045	0.24	18.633	18.666
100000	Single-Option	16.495	0.019	0.11	16.488	16.501
	Multi-Option	18.614	0.019	0.10	18.607	18.621

Table 4.3: LSM performance vs. number of simulation paths

We then tested the model performance with different numbers of basis functions. Table 4.4 shows the means, standard errors, and 95% confidence intervals as we increased the number of basis functions from 1 to 6. The results presented in Table 4.4 and Figure 4.4 show that the option value increases significantly as the number of basis functions increases from 1 to 3, while using more than three basis functions does not change the option value, which is consistent with the conclusion reported by Longstaff and Schwartz (2001). This shows that it suffices to use three basis functions to obtain effective convergence of the algorithm in this case. Since we needed 10,000 simulation paths with three basis functions to satisfy the convergence criterion, we use this combination of number of basis functions and number of simulation paths in the analysis that follows.

4.3.3.2 Optimal investment rule

The real-options valuation framework is an advanced tool for use in investment under uncertainty. Here we discuss the characteristics of the optimal investment rule. As discussed in Chapter 3, there are various parameters, such as demand volatility, the demand growth rate, and the discount rate, that could change the investment decision. The discount rate and travel growth rate are

Num. of Basis Functions M	Model	mean ($B)	s.e. ($B)	s.e. (%)	95% Confident Intervals	
					Lower	Upper
1	Single-Option	16.192	0.038	0.23%	16.178	16.206
	Multi-Option	18.307	0.041	0.22%	18.292	18.322
2	Single-Option	16.411	0.046	0.28%	16.395	16.428
	Multi-Option	18.528	0.031	0.16%	18.517	18.539
3	Single-Option	16.523	0.044	0.27%	16.507	16.539
	Multi-Option	18.649	0.045	0.24%	18.633	18.666
4	Single-Option	16.535	0.038	0.23%	16.521	16.549
	Multi-Option	18.686	0.045	0.24%	18.670	18.702
5	Single-Option	16.519	0.042	0.26%	16.504	16.535
	Multi-Option	18.713	0.043	0.23%	18.697	18.728
6	Single-Option	16.549	0.053	0.32%	16.530	16.569
	Multi-Option	18.768	0.045	0.24%	18.752	18.785

Table 4.4: LSM performance vs. number of basis functions

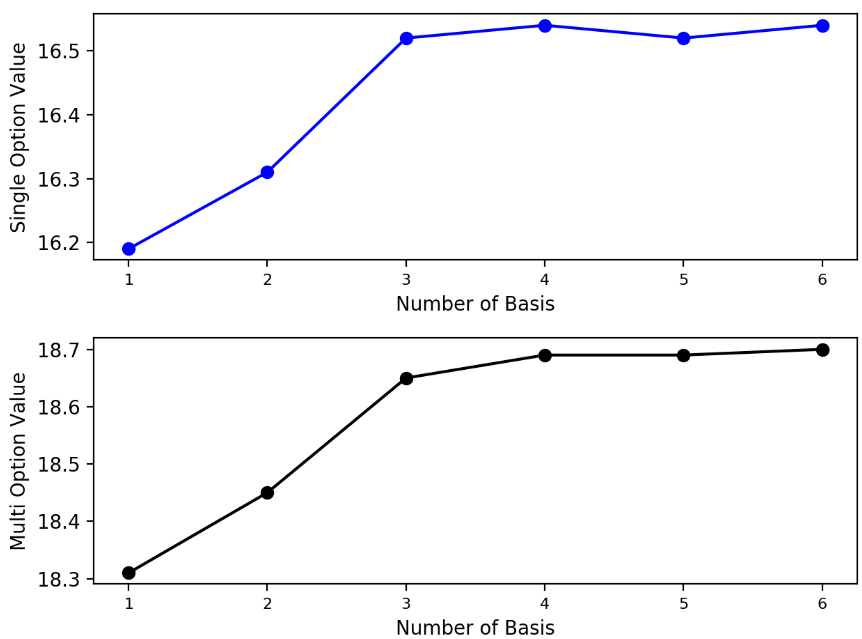


Figure 4.4: LSM performance vs. number of basis functions

relatively fixed according to the government report, hence we will illustrate the dependence of the optimal investment rule on the main uncertain parameter, demand volatility.

Table 4.5 shows the demand volatility and option value of the investment for both a single option and multiple options. We again used the break-even point when the NPV of investment equals 0 as the benchmark, and we varied the annual travel demand volatility from 0 to 20% while keeping the other parameters at the baseline levels. The results in Table 4.5 show that, as expected, the option value increases in both the single-option model and the multi-option model as the volatility increases. The single-option value increases from $14.99 billion to $20.26 billion as the volatility increases from 0 to 20%. As discussed in Chapter 3, the volatility increases the opportunity value of investment, since keeping the option alive can avoid the downside risk, as the investment will be abandoned if market conditions decline, while higher volatility increases the return on investment. On the other hand, increased volatility means greater risk, which requires risk premium compensation. Thus a higher return will result in postponement of the investment. Table 4.5 also shows the advantage of the multi-option model over the single-option model as volatility increases: The benefit increases from 9% to 17% as volatility increases from 0 to 20%. This makes sense because greater volatility means a wider range of possible demand scenarios for each OD pair, and hence the extra flexibility which stems from treating each project as a separate option creates higher value.

Volatility	Single option model (V_s)			Multi option model (V_m)			$(V_m-V_s)/V_s$ (%)
	mean	s.e. (%)	95% CI	mean	s.e. (%)	95% CI	
0	14.99	0	N/A	16.29	0	N/A	9
0.05	15.40	0.13	[15.36, 15.43]	16.93	0.09	[16.92, 16.94]	10
0.10	16.52	0.27	[16.51, 16.54]	18.65	0.24	[18.63, 18.67]	13
0.15	18.12	0.39	[18.09, 18.15]	20.91	0.39	[20.87, 20.94]	15
0.20	20.26	0.68	[20.20, 20.32]	23.67	0.47	[23.63, 23.72]	17

Table 4.5: Option value under different volatility levels

4.3.4 Investment adaptive decision tool

To illustrate the feasibility and practicality of using the real-options-based dynamic decision tool to find the optimal investment decision for the system and for each project in the portfolio, we simulated the OD demands for three different projections. The deterministic case shows that the earliest project investment demand threshold is 362 million. In Chapter 3 and in the previous section of Chapter 4, we showed that the uncertainty will only increase the investment deferral value, and hence that the project investment time can only be postponed. For purposes of illustration, for all three projections we used the earliest demand threshold as the initial demand in our simulation. For each projection, the annual demand growth rate was assumed to be continuous, and equal to 2.17%, with 10% demand volatility. In Figure 4.5, the dashed black lines show the expected growth, and the black dots on the simulation curves are the demands for the individual OD pairs at 5-year intervals.

Table 4.6 shows the results of the investment decisions for all three projections. At year 0, the investment value using the single-option model is \$34.51 billion, with a positive deferral premium of \$3.5 billion, thus the investment strategy is to defer investment in the infrastructure system. The investment value using the multi-option model is \$38.18 billion. The best sequence of in-

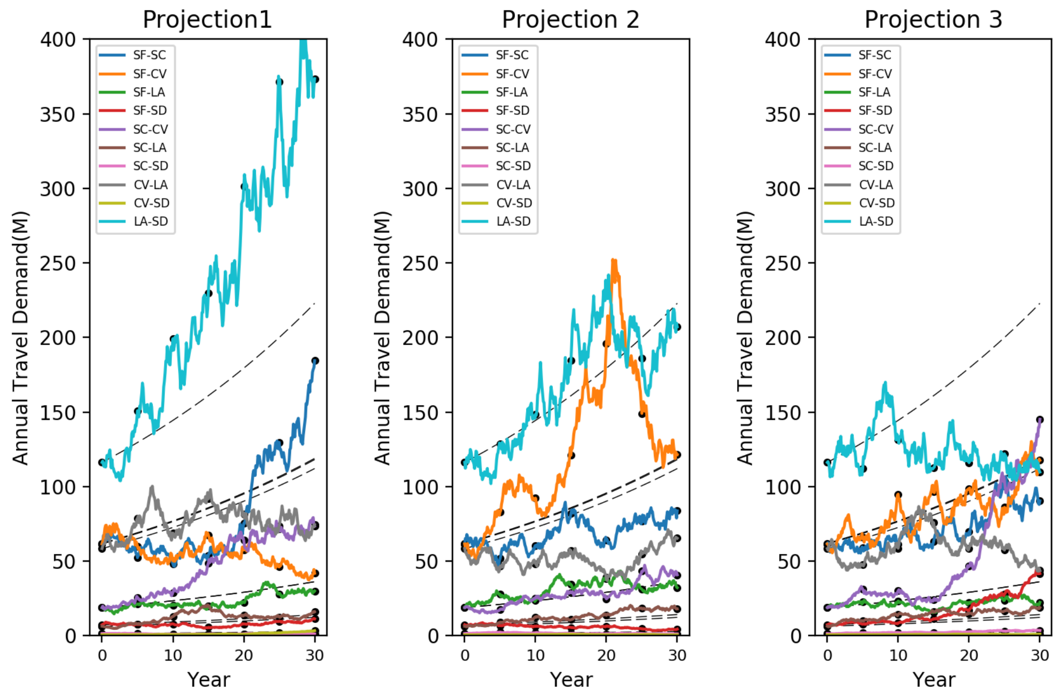


Figure 4.5: Travel demand projection scenarios

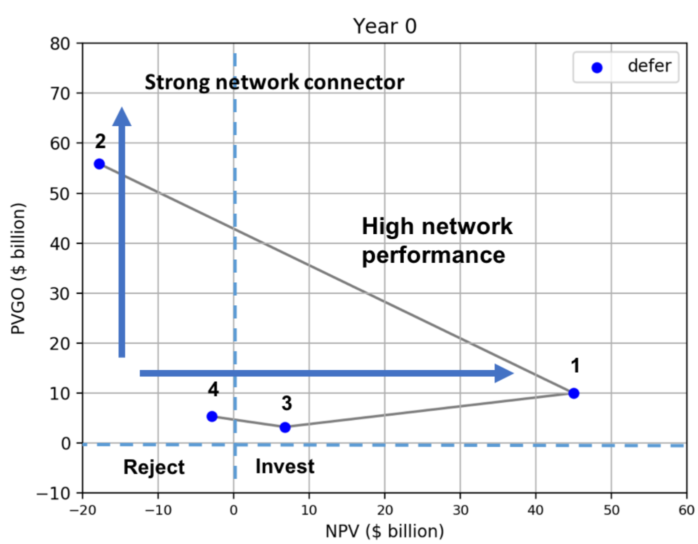


Figure 4.6: Real-options matrix for the portfolio of projects at year 0

	Model	Project order	Invest cost ($B)	Static Investment value ($B)	NPV ($B)	Staging premium ($B)	Deferral Premium ($B)	Option value ($B)	Investment strategy
Year 0	Single option	All projects	110.44	141.45	31.01	N/A	3.50	34.51	Defer
	Multiple options	2	42.34	24.52	-17.82	54.99	0.92	38.09	Defer
		1	23.14	68.12	44.98	10.02	0.00	54.99	Defer
		3	22.38	29.15	6.77	2.43	0.81	10.02	Defer
		4	22.58	19.66	-2.92	0.00	5.35	2.43	Defer
				Projection 1					
Year 5	Single option	All projects	110.44	163.64	53.20	N/A	0.05	53.25	Defer
	Multiple options	1	23.14	27.06	3.92	53.27	0.00	57.20	**Invest**
		2	42.34	78.27	35.93	17.34	0.00	53.27	**Invest**
		3	22.38	36.39	14.01	3.34	0.00	17.34	**Invest**
		4	22.58	21.91	-0.67	0	4.00	3.34	Defer
Year 10	Single option	All projects	110.44	171.34	60.90	N/A	0.00	60.90	**Invest**
		3	22.58	29.94	7.36	0	0.79	8.15	Defer
Year 15	Multiple options	3	22.58	44.07	21.49	0	0.00	21.49	**Invest**
				Projection 2					
Year 5	Single option	All projects	110.44	170.63	60.19	N/A	0.00	60.19	**Invest**
	Multiple options	2	42.34	19.59	-22.75	86.95	0.00	64.20	**Invest**
		1	23.14	97.08	73.94	13.01	0.00	86.95	**Invest**
		3	22.38	31.17	8.79	3.80	0.42	13.01	Defer
		4	22.58	22.79	0.21	0.00	3.59	3.80	Defer
Year 10	Multiple options	3	22.38	34.98	12.60	8.19	0.00	20.79	**Invest**
		4	22.58	30.07	7.49	0	0.71	8.19	Defer
Year 15	Multiple options	4	22.58	30.19	7.61	0	0.62	8.23	Defer
Year 20	Multiple options	4	22.58	32.32	9.74	0	0.22	9.96	Defer
Year 25	Multiple options	4	22.58	43.65	21.07	0	0.00	21.07	**Invest**
				Projection 3					
Year 5	Single option	All projects	110.44	153.96	43.52	N/A	0.83	44.35	Defer
	Multiple options	1	23.14	29.86	6.72	39.49	0.15	46.36	Defer
		2	42.34	68.63	26.29	13.20	0.00	39.49	Defer
		3	22.38	28.83	6.45	5.74	1.02	13.20	Defer
		4	22.58	26.64	4.06	0.00	1.67	5.74	Defer
Year 10	Single option		110.44	183.97	73.53	N/A	0.00	73.53	**Invest**
	Multiple options	1	23.14	42.90	19.76	54.22	0.00	73.99	**Invest**
		2	42.34	74.39	32.05	22.17	0.00	54.22	**Invest**
		3	22.38	33.14	10.76	11.00	0.41	22.17	Defer
		4	22.58	33.53	10.95	0	0.05	11.00	Defer
Year 15	Multiple options	4	22.58	35.36	12.78	9.81	0.00	22.58	**Invest**
		3	22.38	31.86	9.48	0	0.32	9.81	Defer
Year 20	Multiple options	3	22.38	36.88	14.50	0	0.00	14.50	**Invest**

Table 4.6: Adaptive investment decision for infrastructure network based on real-options approach

vestments at year 0 is project 2, project 1, project 3, project 4, as the first project in this order generates the highest value in all permutations. Since there is a deferral value for the first project, the investment strategy at year 0 is to defer all investment in the system. Thus at year 0 the investment decision from the single-option model is the same as that from the multi-option model. The static investment value (NPV) of project 2 is negative, because its investment sunk cost is huge—and greater than its investment benefits for people who benefit directly from project 2. However, its staging premium is as large as $54.99 billion. Since a compound option is essentially an option of an option, the staging premium is the option value of the rest of the investment if investment is made in project 2.

The large staging premium reveals that project 2 serves as a strong network connector in the system, as expected. As shown in Figure 4.1, project 2 serves as the "backbone" of the high-speed rail (HSR) system, as it connects northern California to southern California. The investment in project 2 significantly increases the NPV of project 1. Project 1 has a strong network performance, as it provides connections where people benefit the most from the HSR investment: between San Francisco and the Central Valley, and between the Central Valley and Los Angeles. In Figure 4.6, we generalize the characteristics of projects according to their NPV and the present value of their growth option (PVGO). At the top of the infrastructure investment option matrix, the present value of the growth option (PVGO) is large. The projects located in this region usually serve as a strong network connector for the system if the PVGO is dominated by the staging premium, such as project 2. The PVGO could also stem from a large deferral premium, which implies that there is great uncertainty. At the right side of the matrix, the static NPV of investment is large. The projects located in this region have great network performance, as there are huge benefits for travelers if investment is made. At the bottom of the matrix, the growth option value is 0, meaning that there is no compound option value and no value of waiting. If the NPV is positive, the project should be invested in immediately; otherwise, the project should be rejected.

It should be noted that the output of the multi-option model is the portfolio option value and the investment decision for each project: invest or defer. As mentioned earlier, in order to find the maximum option value of a portfolio, we need to construct and enumerate all sequences of projects; however, the option value of a fixed sequence is actually a lower bound on the true option value, because any investments that are not made immediately can be reordered in the future. Therefore, a given sequence is not the final output of the investment decision, but rather an instrument of the model. This model serves as a dynamic adaptive tool, as information that becomes available in the future can be used to update the investment decision. We used the adaptive investment decision support tool to evaluate the portfolio of projects at 5-year intervals, and we observed some interesting results:

Comparing the three projections at year 5, investment is made in projects 1, 2, and 3 in projection 1, investment is made only in projects 1 and 2 in projection 2, and investment is deferred in all three projects in projection 3. As shown in Chapter 3 (Figure 3.3), the investment benefits of project 3 stem mainly from travel between Los Angeles and San Diego (LA-SD). The travel demand for LA-SD increases much faster in projection 1 than in the other two projections, since projection 1 provides the largest investment payoff and triggers the earliest investment time. In projection 3, project 3 is postponed until year 20, since the growth in demand for LA-SD is much slower in this projection, thus the investment payoff is too low to exceed the value of waiting before year 20.

At year 5, the value of the portfolio is much larger for projection 2. This is because the investment benefits of the system stem mainly from the demands for travel between San Francisco and the Central Valley (SF-CV), and between San Francisco and Los Angeles (SF-LA), and those demands are much higher in projection 2 than in the other projections. At year 5, the demand growth is slower for projection 3 and the investment payoff does not exceed the value of waiting, thus the best strategy is to defer all projects by keeping the options alive, to avoid the downside risk.

In projection 2, project 4 is postponed until year 25 because the benefits of travel between Sacramento and Los Angeles (SC-LA) contributes more than 80% of the total investment payoff of project 4, but the demand for this OD pair is much lower in projection 2. Therefore, there is value in deferring project 4 so that the investment benefits driven by the demand can increase to compensate for the large investment cost.

By applying the framework to large infrastructure planning, the decision maker could dynamically manage the portfolio by comparing the immediate investment payoff with the risk. The model is flexible and can be modified at any time in the planning horizon. For example, if the travel growth rate or growth uncertainty increases for some OD pairs, the investor could incorporate the updated information and adjust the investment decisions accordingly. As new information becomes available, the infrastructure manager could monitor the projects by identifying the NPV and option values in the option space, to optimize the investment in a portfolio of projects and maximize the social welfare.

4.4 Summary

In this chapter, we extended the investment model from a single option under single uncertainty in Chapter 3 to a multi-option portfolio under multiple uncertainties. For transportation investment, the advanced valuation framework treats each infrastructure project as a separate and interacting option under network effects. The study first introduced the marginal social welfare stemming from project investment under steady-state growth. Furthermore, we derived the value of the growth option in portfolio planning for infrastructure projects contingent on the demand variables with information uncertainty. We proposed a valuation algorithm for a portfolio of infrastructure investments with a single option, and later expanded it to compound multiple options. The case study on investment in the California High Speed Rail system in Chapter 3 continued to be used as an example to illustrate the advantages of the valuation framework in this chapter. We compared the optimal investment timing of infrastructure projects and the option value of the portfolio for all types of valuation approaches: NPV, the single-option model, and the multiple-option model in both the deterministic and stochastic cases. The results show the merits of the multi-option model, as it allows the flexibility to invest in each project in its optimal year. This flexibility increases the value of the investment portfolio. In addition, we illustrated the feasibility and practicality of the proposed model as a dynamic decision tool for use in updating the investment decision adapted to new information.

Although the multi-option model provides useful insights for a portfolio of infrastructure investments, as it provides extra flexibility for adjusting the investment return and risk, it does have several limitations, hence there is room for future work. First, the model cannot address the abandon option after the beginning of infrastructure construction. For a large infrastructure system with a huge sunk cost and long construction duration, the flexibility to abandon the investment would create extra option value for the portfolio. Second, the infrastructure investment framework doesn't address the problem from a game-theoretic perspective. The investment benefits in terms of social welfare are quantified according to the market share of new investment projects; however, the pricing, bidding, and competition between different transportation modes such as air travel and high-speed rail may change the investment payoff and consequently the investment strategies. A future study could expand the real-options model to consider competitive strategies in a game setting. Third, the computational complexity of the proposed multi-option model is $O(N!)$. As a result, it cannot be applied to an investment portfolio with a large number of projects. Future research should investigate more advanced algorithms, approximations, and heuristic methods to improve the computational performance.

CHAPTER 5

CONCLUSION

In this dissertation, we developed an asset investment and portfolio management model for the infrastructure system, to improve the system resilience and reduce the socio-economic loss. This dissertation is an attempt to explore the optimal resource allocation strategies and optimal investment timing and criteria for infrastructure asset investment, and quantify its socio-economic investment benefits.

Chapter 2 extends the previous study of portfolio investment and asset management of an independent infrastructure system to a network-based interdependent system under the impact of extreme events. As failures in an interconnected infrastructure system stem, at least in part, from the vulnerability of tightly coupled facilities, it is necessary to consider mutually dependent network properties in designing a resilient system and a robust investment portfolio. As the investment upgrades the standard infrastructure, it decreases the chance of infrastructure failure and hence increases post-disaster network connectivity, and it reduces the social economic loss. We developed a portfolio optimization model with the objective of minimizing the post-disaster socio-economic loss under the budget resource constraint. Two types of algorithms are developed to find the optimal investment portfolio. We first proposed an approximation algorithm to find the local optimum solution; however, the limitation of the approximation algorithm is that it cannot consider the effect of simultaneous investment in two or more projects. To overcome that limitation and improve the lower bound on the approximate solution, we developed an iterative heuristic algorithm to systematically consider the network effect of the projects in the optimal portfolio via consecutive iteration. We applied the model to a case of transportation infrastructure investment in Fiji. We compared the investment portfolio constructed by the traditional factor-based model to our proposed system-level model with two types of algorithms. The results show that the system-level model significantly reduces the socio-economic loss compared to that of the factor-based model, while the type II heuristic algorithm further improves the lower bound on the local optimum solution from that of the type I algorithm and increases the return on the investment portfolio. The results show that the investment decision pro-posed by FRA can merely reduce the socio-economic loss by 29%. In comparison, the system-level investment model solved by type I approximate algorithmreduces the socio-economic loss by 76.6%, while the type II heuristic algorithmimproves the lower bound of the local optimum solution from type I algorithm,with 80.7% socio-economic losses reduction.

The second part of the dissertation consists of finding the optimal investment time and investment criteria for an infrastructure system under uncertainty, with the goal of maximizing the social welfare. Chapter 3 explores the social benefits of infrastructure investment and identifies the optimal investment criteria using a real-options-based approach. The optimal investment policy is further discussed under different travel demand growth scenarios, including both the deterministic and stochastic cases. We compare the investment thresholds to the net present value (NPV) break-even point, from which the merits of the advanced real-options-based investment model in both the deterministic and stochastic cases are illustrated. We use both theoretical proof and real-life cases to illustrate why the NPV rule cannot yield the optimal investment decision in either scenario. The model is applied to investment in high-speed rail

(HSR) in the state of California in the U.S. We analyze the social welfare benefits of such investment from the standpoint of the consumer, the producer, and the environmental performance. The results provide practical insights into the California High Speed Rail system: We show that by assuming deterministic growth in demand, the current practice is a delay of about five years beyond the optimal time of investment, while if the demand uncertainty increases, the current demand level could be lower than the optimal investment threshold if we still have a deferral premium. In addition, we show that HSR investment at the current demand level would be profitable in terms of the social welfare, but that it is difficult for a project like this to be financially profitable, because of the huge investment sunk cost. Furthermore, we find that ignoring the effects of environmental benefits will lead to late investment. Finally, we propose investment policy implications by examining the characteristics of the optimal investment rules under the effects of key uncertainty parameters.

Chapter 4 extends the study of infrastructure investment with a single option in Chapter 3 to a multi-option framework for a portfolio of interdependent infrastructure projects under network effects. We developed methods to address the optimal investment strategies for a portfolio of interdependent assets under uncertainty. The underlying assets of the multi-option model are the individual and interacting projects of the system. The aim of the multi-option model is to decide the project selection and the time of investing under demand uncertainty, where investment in each project can be made immediately or postponed to maximize the social welfare of the portfolio. The chapter begins from the first dimension of portfolio planning analysis: a static investment benefit framework to quantify the marginal net present value of project investment social welfare. Furthermore, the second dimension of portfolio planning is discussed in terms of the value of a growth option contingent on the state variables and the information uncertainty. An algorithm based on dynamic programming and least-squares Monte Carlo simulation is proposed to search for the optimal investment decision and the lower bound on the option value of the portfolio. The model is then applied to investment in a high-speed transport network. In order to illustrate the merits and practicality of the models, we compared the investment strategies and the investment social welfare from the traditional net present value model, the single-option model, and the multi-option model. The results show that the multi-option model increases the investment value of the infrastructure portfolio in both the deterministic and stochastic cases, by allowing a progressive adaptation of project deferral, contingent on the changing scenarios under uncertainty. In the deterministic scenario, the infrastructure portfolio is worth \$14.99 billion using the strategy of the single-option model and \$16.29 billion with the strategies from the multi-option model at the break-event point when NPV is equal to 0. The stochastic scenario further reveals the advantage of the multi-option model as volatility increases: Compared to the single-option portfolio, the value of the multi-option portfolio increases from 9% to 17% as volatility increases from 0 to 20%. The model could serve as an effective dynamic adaptive decision support tool for infrastructure planners to use in updating investment decisions by incorporating new observed uncertainty information. The infrastructure planning authority could apply this model to monitor the return on a portfolio versus risk and maximize the investment social welfare.

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