FINAL REPORT ON NSF RESEARCH GRANT

AUTOMATA AND COMPUTATIONAL COMPLEXITY,

1968 - 1972

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ABSTRACT: This report summarizes the results obtained in research supported by the National Science Foundation Grant AUTOMATA AND COMPUTATIONAL COMPLEXITY. The report lists the problem areas considered, the publications resulting from this work and gives an outline of the more recent research results which have not yet been published.
I. INTRODUCTION

This is the final report of research supported by the National Science Foundation Grant AUTOMATA AND COMPUTATIONAL COMPLEXITY at Cornell University. The research grant was in effect from September 1968 until June 1972 and the goal of this research was to contribute to the general development of a mathematical theory dealing with the quantitative aspects in the theory of computing. The main research areas attacked are listed in the next section which also references the publications that have resulted from this work. The following section outlines in somewhat more detail the newer research results which have not yet been published and indicates some promising areas for further research.
II. RESEARCH PROBLEMS CONSIDERED

The main motivation of this work is the fact that at this time we still have little understanding of the quantitative aspects of computing and that such an understanding is needed to develop a comprehensive theory of computing. Thus almost all the problems considered in this research effort deal with the complexity of computations and the main expectation is that from the study of a diversity of such complexity problems eventually will emerge the much needed understanding to develop a quantitative theory of computing. The following list indicates the research problems which have been considered and indicates the resulting publications as well as the collaborators who work with the principal investigator on these problems. For the sake of brevity, only the results in the last two problem areas and the work on linearly bounded automata are discussed in more detail since the listed publications give a reasonably good summary of the results in the first four problem areas.

1. The study of what makes language theory problems undecidable, how lists of problems can be used in the study of undecidable problems in automata theory and how problems change from decidable to undecidable problems as the complexity of the languages is increased [1,2,3,4].

Collaborators: Prof. John E. Hopcroft [1,3],

2. The study of very specific computational problems and their complexity in hope to gain new techniques and insights; for example, the recognition of prime numbers by tape bounded Turing machines; the speed differences between one-tape and many-tape Turing machines; the computational power of one-way and two-way automata, as well as the relation between deterministic and non-deterministic context-sensitive languages [5,6,7,8]. The work on prime recognition was done in collaboration with H. Shank who received his Ph.D. in Applied Mathematics from Cornell University in 1970.

3. Investigation of computational complexity measures of random access stored program machines, RASP's, and the differences of running times of fixed and self-modifying programs in such computing devices [9].

4. Preparing a comprehensive Overview of the theory of computational complexity which summarized the main results and techniques that have been developed in complexity theory during the last eight to ten years. This work was done in close collaboration with John E. Hopcroft of Cornell University [10] and besides a summary of known results it contains considerable
simplifications of previous work as well as some unifying concepts which permit an easier presentation of the existing theory.

5. The use of group theoretic methods to characterize linearly realizable permutation automata and the characterization of homorphism of linear machines which preserve linearity [12]. This work was done jointly with Professor H. Walter and is outlined in more detail in the next section.

6. A study of the complexity of formal translations from one universal language into another and investigation of the optimality of programs produced by these translations. The initial work on the translation problem as well as a new speed-up result (solution of a problem posed by M. Blum) was done in collaboration with Professor R.L. Constable of Cornell University. This phase of the work is summarized in [11]. Some more recent work is described in the next section.
III. SUMMARY OF MOST RECENT WORK

In this section we discuss shortly the newer research results which have not yet been published in technical journals, though two papers dealing with this work have already been accepted for publication [8,12].

A. In the study of specific complexity problems the principal investigator considered the classic problem about deterministic and non-deterministic linearly bounded automata. To simplify the problem we considered one-tape Turing machines with read only-heads. The input is placed on the tape between end-markers and the Turing machine has k read only heads which can move in both directions on the input tape but cannot leave the input. We refer to such machines as k-head automata. The main result asserts the following: If every set accepted by 3-head, non-deterministic automaton can be accepted by a deterministic k-head automaton, then the deterministic and non-deterministic context sensitive languages are identical [8].

This shows that if for a 3-head non-deterministic, read-only automaton we can eliminate the non-determinism in the automaton by using more read-only heads then we can eliminate the non-determinism in all linearly bounded automata. Unfortunately, even the simple looking problem of determining whether the non-determinism in 3-head automata can be eliminated by using more heads appears to be very difficult and we have made no further
progress on this problem until now.

B. During the winter of 1971-72 the principal investigator and Professor H. Walter of the Technical University of Darmstadt occupied themselves for a time with a completely different problem area and were able to obtain some interesting new results about linear automata.

We recall that for a long time we have known that the group generated by a permutation automaton plays a very important role in the decomposition theory of finite automata [13,14]. On the other hand, very often results about the group of automata could not be directly translated into results about automata since the group could be generated by an automaton in many different ways. Now the study of linearly realizable automata [15,16] have provided a good illustration how group theoretic concepts can be used to solve automata theory problems. By using some recent work of Ecker [17] we have been able to give a complete characterization in group theoretic terms of the permutation automata which are linearly realizable over GF(p). Furthermore from this characterization we obtain easily the characterization of all homomorphisms of a linear automaton whose image automata are again realizable. The last problem had been discussed previously [18] but only the group theoretic approach permitted a complete solution of this problem and gave a very good understanding of the nature of the results.
We outline the main results. First, observe that any connected permutation automaton \( M \) which generates the group \( G = G(M) \) is completely characterized by \( H \) and \( I \), where \( H \) is a subgroup of \( G \) such that no subgroup of \( H \), besides \( \{ e \} \), is normal in \( G \), and \( I \) is a subset of \( G \): namely the input permutations. We know furthermore that we can consider the cosets of \( H \) as states of \( M \) and thus we will simply write

\[ M = M_{G,H,I}. \]

**Theorem:** The automaton \( M = M_{G,H,I} \) is linearly realizable, say over \( GF(2) \), if and only if

A. \( G \) has a normal, abelian subgroup \( N \) whose elements, except \( e \), have order two and such that \( G/N \) is cyclic.

B. \( N \cap H = \{ e \} \) and \( I \subseteq Na \) for some \( a \) in \( G \).

This result can be used to give a complete characterization of all homomorphisms of linear machines which are again linearly realizable. We recall that for a permutation machine \( M \) every homomorphism is characterized by a subgroup of \( G(M) \). Note that these subgroups do not have to be normal.

**Theorem:** Let \( M = M_{G,\{ e \},I} \) be linearly realized using normal subgroup \( N \) of \( G \). Then, the homomorphic image \( M' \) of \( M \), defined by the subgroup \( K \) of \( G \), is again linearly realizable if and only if

\[ K \cap N = \text{normal subgroup of } G. \]
These results give an easy understanding how linear realizability can be destroyed by homomorphic mappings and yields several interesting applications [12]. It is our conviction that the explicit characterization of how an automaton generates its group will permit us to gain further insights in automata theory by use of group theoretic arguments.

C. Finally, the principal investigator has continued the study of the computational complexity of formal translations from one universal language into another. This work was initiated in collaboration with Professor R.L. Constable [11] and continued by the principal investigator during his sabbatic year in Germany. The research in this area is aimed at a better understanding of the theoretical foundations of translations between different (programming) languages. The approach taken is axiomatic in order that the results would have a general validity for translations between all languages and it is hoped that in later research one will be able to identify what properties in languages facilitate "easy and good" translations and that then specializations of languages can be incorporated in the theory.

The work in this area is basically concerned with two problem areas. The first one deals with the complexity of the translator (compiler) and the second with the quality of the translated programs. Quite a few results have been already obtained on the
problem of quality of programs and during the last few months of this research period the principal investigator has obtained several new results about the complexity of translators. One of the more novel ones was the observation that there exist a Gödel numbering $G$ such that any other Gödel numbering $H$ can be translated into $G$ by a finite automaton. From this result it immediately follows that if we fix the target language $L_T$ and the complexity of the recognition problem for the source language $L_S$ (is $w$ in $L_S$?), then the complexity of the translators can be bounded independently of the semantics defined for the language $L_S$. It also is interesting to note that many of the languages encountered in computer science are such that finite automata can translate any Gödel numbering to them and that these languages form a subclass of the Optimal Gödel Numberings defined and investigated by Schnorr [19].

These recent insights in the complexity of translations and relations to other fields have suggested many new problems and possible connections and are being intensively investigated by the principal investigator. It is hoped that a paper covering this research will be finished during the coming winter.
IV. REFERENCES


