On Scheduling Meetings

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Abstract:

We comment on the problem of assigning papers to time-slots to minimize the time needed for a technical meeting. We claim that this problem is, in abstract, the problem of discovering a minimal coloring of a conflict graph. This approach contrasts with that of Joseph E. Grimes in [1].

Key words and phrases: allocation, conflict matrix, connected component, scheduling, spanning tree, undirected linear graph, graph node coloring, graph vertex coloring.

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ON SCHEDULING MEETINGS

In a recent paper [1], Joseph E. Grimes reported on a technique for scheduling meetings on minimum time, while reducing the number of paper "conflicts" which arise. A "conflict" arises when the same person wants to hear two papers, both of which are presented at the same time. Grimes suggests polling the attendees, and deriving a "conflict matrix" M.

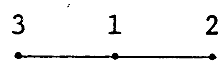
$M[i,j] = 1$ if more than T people wish to hear both paper i and paper j,

$M[i,j] = 0$ otherwise.

Unfortunately, Grimes' suggested handling of matrix M does not reduce the meeting-time to its ultimate.

The matrix M can be viewed as the incidence matrix of a graph, G . Nodes in G correspond to papers; an arc connects two nodes i and j if and only if $M[i,j] = 1$. Grimes suggests using an algorithm due to Gottlieb and Corneil [2]. This algorithm is used by Grimes to partition G into disjoint subgraphs. Grimes claims that papers in such a subgraph must be scheduled at different times, to satisfy the constraint that no two papers i and j whose $M[i,j]$ entry = 1 be scheduled at the same time. However, this is false, as the following example shows.

Suppose three papers are to be given at a meeting. Every attendee wants to hear paper 1; the first $T + 1$ attendees wish to hear paper 2, while the last $T + 1$ attendees want to hear paper 3. If more than $T + 2$ people are attending the meeting, less than T people wish to hear both papers 2 and 3. The corresponding graph is:



Grimes' algorithm would require that 3 time-slots be used; however, papers 2 and 3 can obviously be presented at the same time. Thus, only 2 time-slots need be allotted.

The scheduling problem as abstracted by Grimes should be viewed as a *graph-coloring problem* rather than a problem of discovering disjoint subgraphs. A *coloring* of the nodes of a graph G is an assignment of integers to the nodes of G in such a way that two nodes connected by an arc of G are assigned distinct colors. A *minimal coloring of G* if a coloring of G which uses fewest integers. [3]

In the current problem, distinct integers correspond to distinct time-slots. Minimizing the meeting-time (total number of time-slots) can be achieved by constructing a minimal coloring for the conflict graph.

Constructing minimal colorings is difficult. The fastest algorithms seem to require time which grows exponentially with the number of nodes of the graph. For example, a straightforward combinatorial search which assigns nodes to colors in all possible valid combinations, has this time requirement. However, a method which achieves near-minimal colorings rapidly is available. [4] Furthermore, it seems important that the true character of this problem not be obscured.

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