SECTION V.

STATICS OF FLUID BODIES.

CHAPTER I.

ON THE EQUILIBRIUM AND PRESSURE OF WATER IN VESSELS.

§ 272. *Fluidity.*—We regard *fluid bodies* as systems of material points, whose cohesion is so feeble, that the smallest forces are sufficient to effect a separation, and to move them amongst each other (§ 59). Many bodies in nature, such as air, water, &c., possess this property of fluidity in a high degree; other bodies, on the contrary, such as oil, fat, soft earth, &c., are fluid in a low degree. The former are called *perfectly fluid*, the latter *imperfectly fluid bodies*. Certain bodies, as, for instance, paste, are intermediate between solid and fluid bodies.

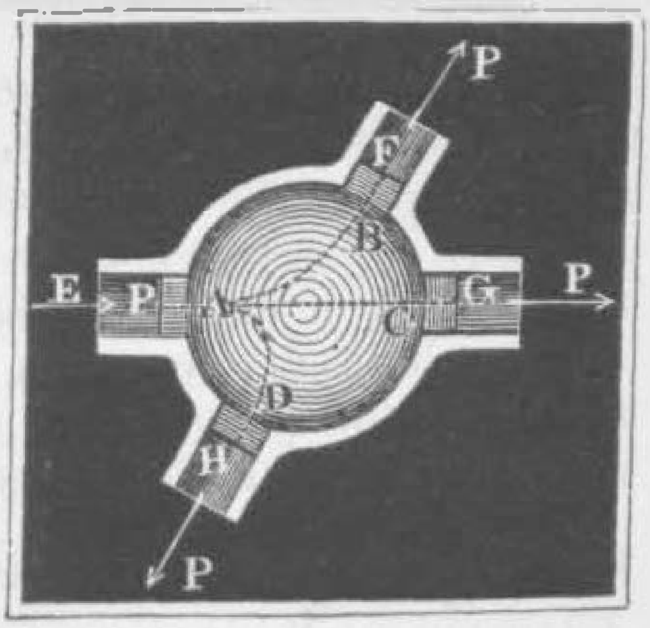
Perfectly fluid bodies, of which only we shall subsequently speak, are at the same time perfectly elastic, *i. e.*, they may be compressed by external forces, and will perfectly resume their former volume after the withdrawal of these forces. The amount of the change of volume corresponding to a certain pressure, is different for different fluids; in *liquid bodies* this is scarcely perceptible, while in *aëriform bodies*, which, on this account, are also called elastic fluids, it is very great. This slight degree of compressibility of liquid bodies is the reason why in most investigations in hydrostatics (§ 63) they are considered and treated as incompressible or inelastic. As water, of all liquids, is the one most generally diffused, and the most useful for the purposes of life, it is taken as the representant of all these fluids, and in the investigations of the mechanics of fluids, water only is spoken of, whilst it is tacitly understood that the mechanical properties of other liquids are the same as those of water.

From a similar reason in the mechanics of the elastic fluid bodies ordinary atmospheric air is only spoken of.

Remark.—A column of water of one square inch transverse section is compressed by a weight of 15 lbs., which corresponds to the atmospheric pressure, by about 0,00005 or 50 millionths of its volume, while the same column of air under this pressure would be compressed to one half of its original volume.

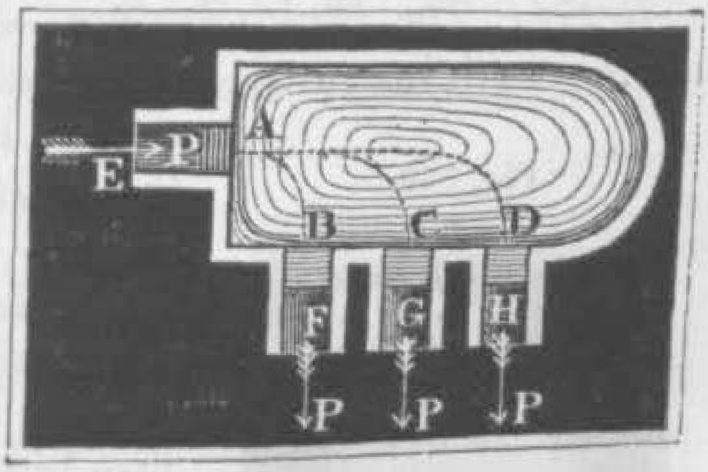
§ 273. *Principle of Equality of Pressures.*—The characteristic property of fluids, which essentially distinguishes them from solid bodies, and which serves as a basis of the laws of the equilibrium of fluid bodies, *is the capability of transmitting the pressure which is exerted upon a part of the surface of the fluid in all directions unchanged.* The pressure on solids is transmitted only in its proper direction (§ 83); while, on the other hand, when water is pressed on one side, a tension takes place in the entire mass, which exerts itself on all sides, and may be observed at all parts of the surface. To satisfy ourselves of the correctness of this law, we may make use of an apparatus filled with water, as is shown in the horizontal section in Fig. 335. The tubes AE and BF, &c., equally distant, and at an equal height above the horizontal base, are closed by perfectly movable and accurately fitting pistons; the water presses, therefore, by its weigbt, as strongly against the one piston as against the other. Let us do away with this pressure, and regard the water as devoid of weight. Let us press the one piston with a certain pressure P against the water, this pressure will then be transmitted by the water to the other pistons B, C, D, and for the restoration of equilibrium, or to prevent the pushing back of these pistons, it is requisite that an equal and opposite pressure P act against each of these pistons. We are, therefore, justified in assuming, that the pressure P, acting upon a point A of the surface of the mass of water, produces in it a tension, and not only transmits this in the straight line AC, but also in every other direction BF, DH, &c., to every equal area of the surface C, B, D.

Fig. 335.



If the axes of the tubes BF, CG, &c., Fig. 336, are parallel to each other, the pressures which act upon their pistons may be united by addition into a single pressure; if n be the number of the pistons, then the aggregate pressure upon these amounts to $P_1 = nP$, and in the case represented in the figure $P_1 = 3P$. But now the areas F_1 of the pressed surfaces B, C, D, are equal to n times the pressed surface F of the one piston, hence n may not only be put $= \frac{P_1}{P}$, but also $= \frac{F_1}{F}$, therefore $\frac{P_1}{P} = \frac{F_1}{F}$.

Fig. 336.



If the tubes B, C, D, form a single one, as in Fig. 337, and if we close it by a single piston, F_1 then becomes a single surface, and P_1

is the pressure acting upon it, hence there follows this general law, *the pressure which a fluid body exerts upon different parts of the sides of a vessel, is proportional to the area of these parts.*

Fig. 337.

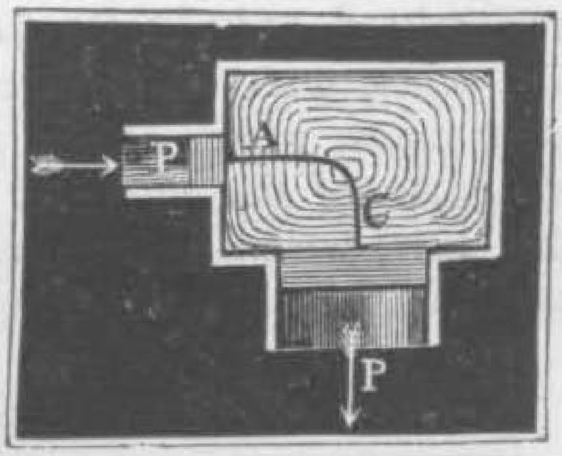
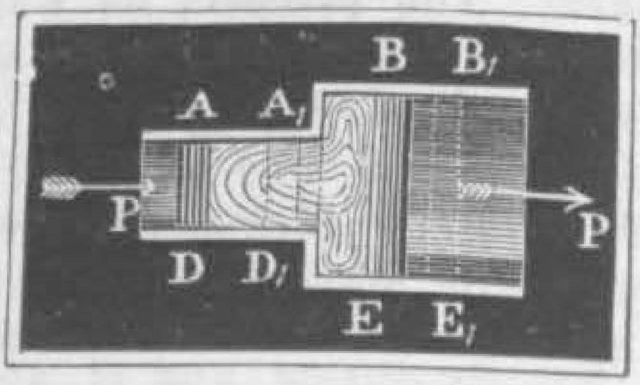


Fig. 338.



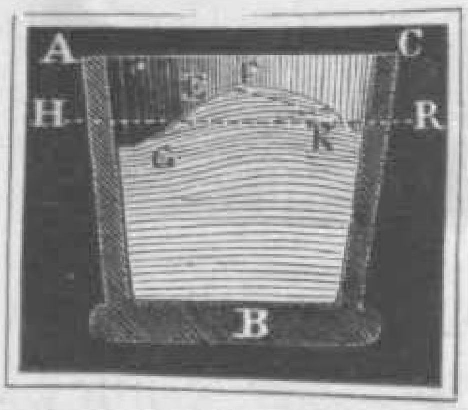
This law corresponds also to the principle of virtual velocities. If the piston $AD = F$, Fig. 338, moves inwards through a space $AA_1 = s$, it then presses the column of water Fs from its tube, and if the piston $BE = F_1$, it passes outwards through the space $BB_1 = s_1$, it then leaves a space $F_1 s_1$ behind. But since we have supposed that mass of water neither allows of expansion nor compression, its volume then by this motion of the piston must remain unaltered, that is, the increase Fs must be equal to the decrease $F_1\, s_1$. But the equation $F_1\, s_1 = Fs$ gives $\frac{F_1}{F} = \frac{s}{s_1}$, and by combining this proportion with the proportion $\frac{P_1}{P} = \frac{F_1}{F}$, it follows that $\frac{P_1}{P} = \frac{se}{s_1}$, hence, therefore, the mechanical effect $P_1 s_1$ = mechanical effect Ps (§ 80).

Example. If the piston AD has a diameter of 1½ inches, and the piston BE one of 10 inches, and each is pressed by a force P of 36 lbs. upon the water, this piston exerts a pressure $P_1 = \frac{F}{F_1} P = \frac{10^2}{1{,}5^2} \cdot 36 = 1600$ lbs. If the first piston is pushed forwards 6 inches, the second will only go back by $s_1 = \frac{F}{F_1} s = \frac{9 \,.\, 6}{400} = \frac{27}{200} = 0{,}135$ in.

Remark. Numerous applications of this law will come before us in the hydraulic press, or water column machines, in pumps, &c.

§ 274. *The Fluid Surface.*—The gravity inherent in water causes all its particles to tend downwards, and they would actually so move unless this motion were prevented. In order to obtain a coherent mass of water, it is necessary to enclose it in vessels. The water in the vessel ABC, Fig. 339, is then only in equilibrium if its free surface HR is perpendicular to the direction of gravity, and therefore horizontal, for so long as this surface is curved or inclined to the horizon: then there are elementary portions E, F, &c., lying higher, which, from their extreme mobility in virtue of their gravity, slide down on those below them, as if it were on an inclined plane GK.

Fig. 339.

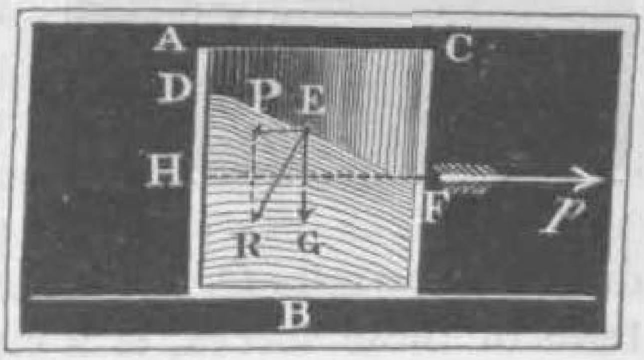


Since the directions of gravity for great distances can no longer be regarded as parallel, we must, therefore, consider the free surface, or the level of water in a large vessel, as for example, in a great lake, no longer as a plane, but as part of a spherical surface.

If any other force than that of gravity act upon the particles of water, the fluid surface in the state of equilibrium, will be perpendicular to the direction of the resultant arising from gravity and the concurrent force.

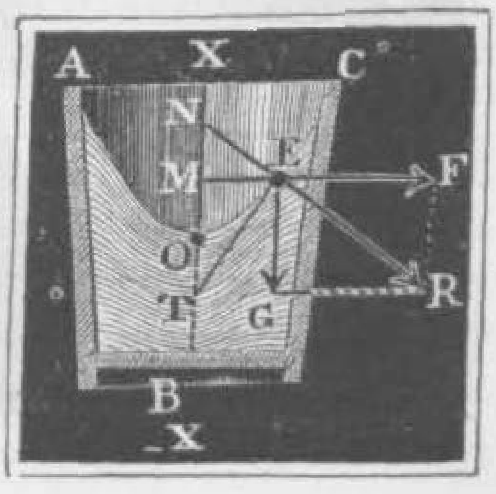
If a vessel ABC, Fig. 340, is moved forward horizontally by a uniformly accelerating force p, the free surface of the water in it will form an inclined plane DF, for in this case every element E of this surface will be impelled downwards by its weight G, and horizontally by its inertia $P = \frac{p}{g} G$, there will then be a resultant R, which will make with the direction of gravity a uniform angle $REG = \alpha$. This angle is at the same time the angle DFH which the surface of the water makes with the horizon. It is determined by $tang. \alpha = \frac{P}{G} = \frac{p}{g}$.

Fig. 340.



If, on the other hand, a vessel ABC, Fig. 341, rotates uniformly about its vertical axis $\overline{XX}$, the surface of the water then forms a hollow surface AOC, whose sections through the axis are parabolic. If ω be the angular velocity of the vessel and the water in it, G the weight of an element of water E, and y its distance ME from the vertical axis, we shall then have for the centrifugal force of this element $F = \omega^2 \frac{Gy}{g}$ (§ 231), and hence for the angle $REG = TEM = \phi$, which the resultant R makes with the vertical or the tangent to the water profile with the horizone

Fig. 341.

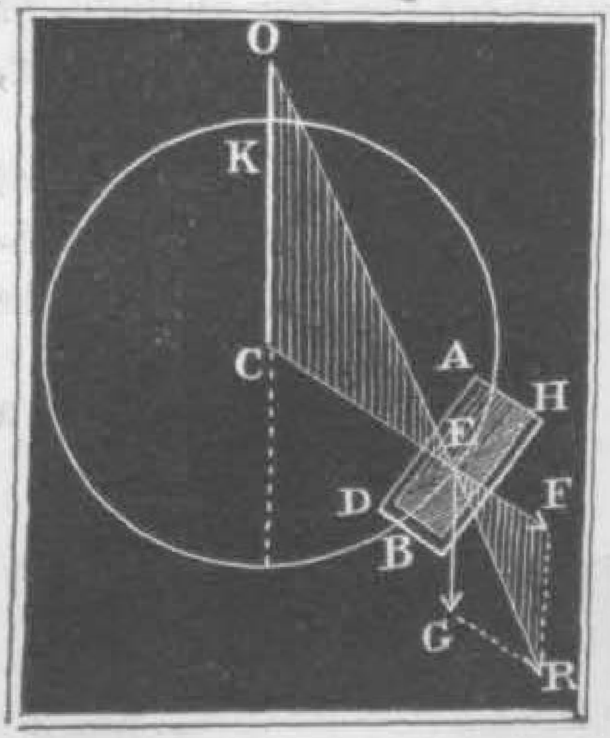


$$tang. \phi = \frac{F}{G} = \frac{\omega^2 y}{g}.$$

From this, therefore, the tangent of the angle which the line of contact makes with this ordinate, is proportional to the ordinate. As this property belongs to the common parabola (§ 144), the vertical section AOC of the surface of water is also a parabola whose axis coincides with the axis of revolution $\overline{XX}$.

If a vessel ABH be moved in a vertical circle, Fig. 342, uniformly about a horizontal parallel axis C, the surface of the water will form in it a cylindrical surface with circular sections DEH. If we prolong the direction of the resultant R of the gravity G, and the centrifugal force F of an element E to the intersection O with the vertical CK passing through the centre of revolution, we shall then

Fig. 342.



obtain the similar triangles ECO and EFR, for which

$$\frac{OC}{EC} = \frac{FR}{EF} = \frac{G}{F},$$

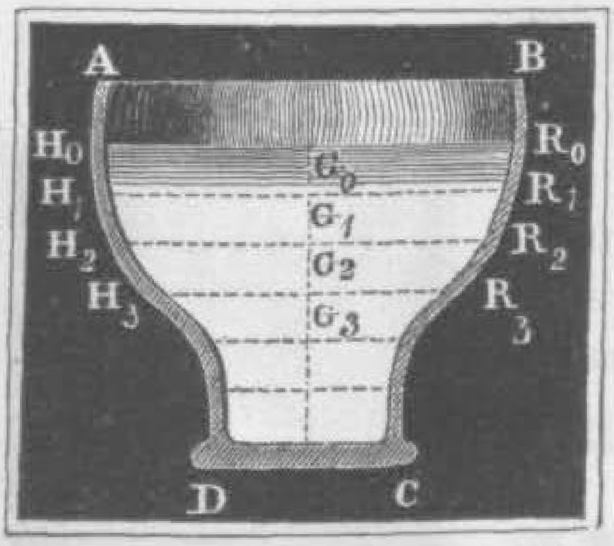
but now, if we put the radius of gyration $EC = y$, and retain the last notation, $F = \frac{\omega^2 Gy}{g}$, it follows that the line

$$CO = \frac{g}{\omega^2} = \frac{32,2}{(3,1416)^2} \left(\frac{30}{u}\right)^2 = \frac{2936}{u^2}$$

if u represents the number of revolutions per minute. As this value of CO is one and the same for all the particles of water, it follows that the components of all the particles forming the section DEH are directed towards O, and hence the section perpendicular to the directions of these forces, is a circle described from O as a centre. According to this, the surfaces of water in the buckets of an overshot wheel form perfect cylindrical surfaces, corresponding to one and the same horizontal axis.

§ 275. *Pressure on the Bottom.*—The pressure of water in a vessel $ABCD$, Fig. 343, immediately under the water level is the least, but becomes greater and greater in proportion to the depth, and is greatest immediately above the bottom. To prove this generally, let us assume that the level of the water H_0R_0, whose area may be F_0, is uniformly pressed by a force P_0, for example, by the superincumbent atmosphere, or by a piston, and let us suppose the whole mass of water divided by horizontal planes, as H_1R_1, H_2R_2, into equally thick strata of water. If now

Fig. 343.



λ be the thickness or the height of such a stratum, and γ the density of water, we shall then have the weight of the first stratum $G_1 = F_0\lambda\gamma$, and hence the entire pressure on the subjacent water: $P_1 = P_0 + F_0\lambda\gamma$. If we divide this pressure by the area F_1 of the following horizontal section H_1R_1, we shall obtain the pressure for each unit of this surface: $p_1 = \frac{P_0}{F_1} + \frac{F_0}{F_1}\lambda\gamma$, or, since F_1, on account of the infinitely small distance between H_0R_0 and H_1R_1, is infinitely little different from F_0, and may be substituted for this: $p_1 = p_0 + \lambda\gamma$, where p_0 represents the external pressure on the unit of surface. The pressure of the succeeding horizontal section H_2R_2 may be determined as exactly as the pressure of the stratum H_1R_1, if we take into consideration that the initial pressure upon the unit is now $p_1 = p_0 + \lambda\gamma$, whilst it was then only p_0. The pressure in the horizontal stratum H_2R_2 then follows: $p_2 = p_1 + \lambda\gamma = p_0 + \lambda\gamma + \lambda\gamma = p_0 + 2\lambda\gamma$; likewise the pressure in the third stratum $H_3R_3 = p_0 + 3\lambda\gamma$, in the fourth $= p_0 + 4\lambda\gamma$, and in the nth $= p_0 + n\lambda\gamma$. But now $n\lambda$ is the depth $G_0Gn = h$ of the nth stratum below the level of the water, hence the pressure upon each unit of surface in the nth horizontal stratum may be put: $p = p_0 + h\gamma$.

The depth h of an element of surface below the water level, is called the *head of water*, and the pressure of water upon any unit of surface may from this be found, if the externally acting pressure be increased by the weight of a column of water whose base is this unit, and whose height is the head of water.

The bead of water h on a horizontal surface, for instance, on the bottom CD, is at all places one and the same; hence the area of this surface $= F$, and the pressure of water against it is $P = (p_0 + h\gamma) F = Fp_0 + Fh\gamma = P_0 + Fh\gamma$, or if we abstract the outer pressure $P = Fh\gamma$. *The pressure of water against a horizontal surface is therefore equivalent to the weight of the superincumbent column of water Fh.*

This pressure of water against a horizontal surface—a horizontal bottom, for instance—or against a horizontal part of a lateral wall, is independent of the form of the vessel; whether, therefore, the vessel AC, Fig. 344, be prismatic as a, or wider above than below as b, or wider below than above, as c, or inclined as d, or bulging out as e, &c., the pressure on the bottom will be always equal to the weight of a column of water whose base is the bottom and whose height is the depth of the bottom below the level of the water. As the pressure of water transmits itself one all sides, this law is therefore applicable when the surface, as BC, Fig. 345, is pressed upon from below upwards. Every unit of surface in the stratum lying in BC is pressed by a column of water of the height $HB = RK = h$; consequently, the pressure against $CB = Fh\gamma$, F being the area of the surface.

Fig. 344.

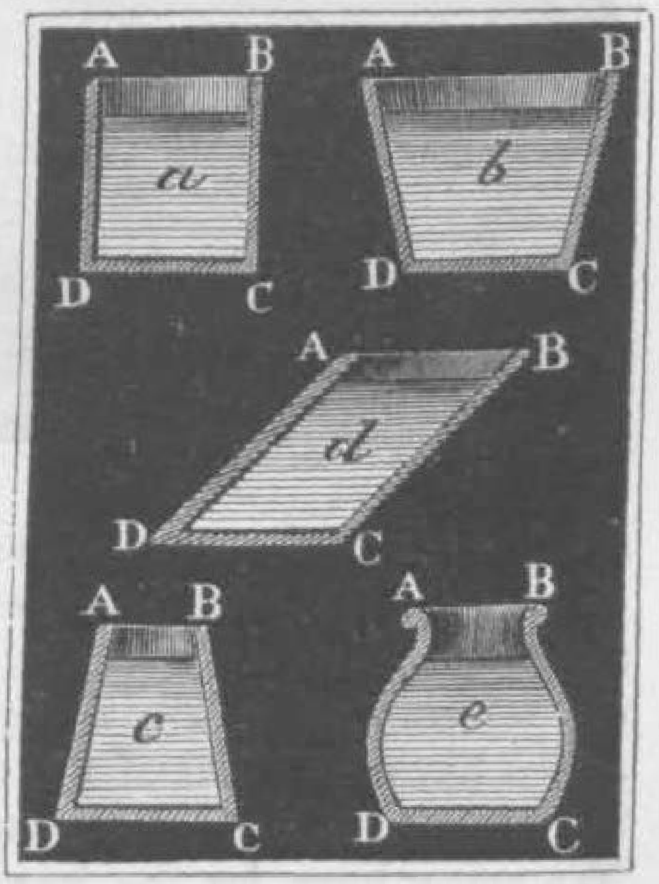


Fig. 345.

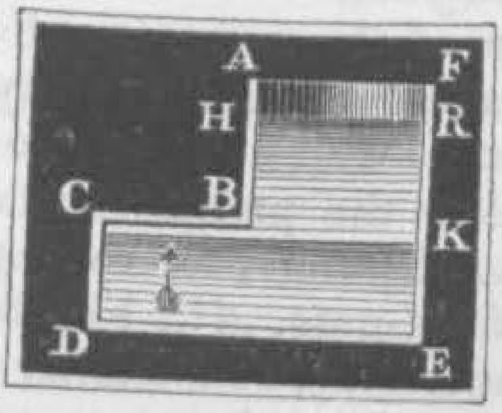
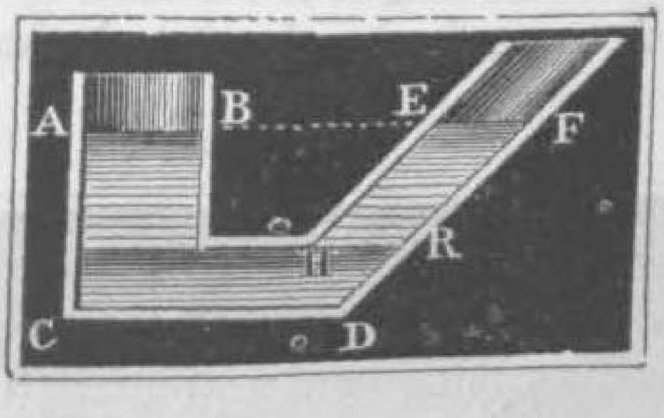


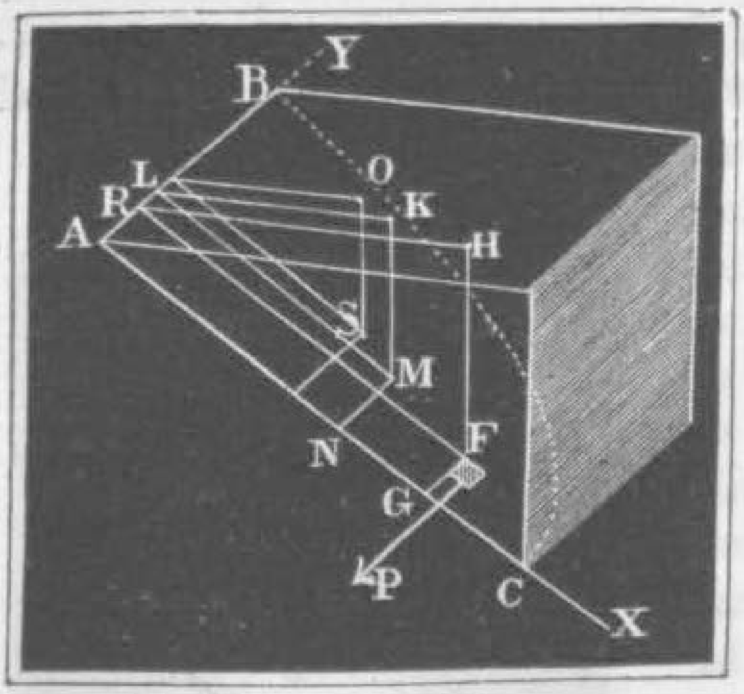
Fig. 346.



It further follows from this, that the water in tubes communicating with each other ABC and DEF, Fig. 346, when equilibrium subsists, stands equally high, or that the two levels AB and EF are in one and the same horizontal plane. For the subsistence of equilibrium, it is requisite that the stratum of water HR be as forcibly pressed downwards by the superincumbent column of water ER, as pressed upwards by the mass of water lying below it. But as in both cases the surface pressed is one and the same, so must the head of water in both cases be one and the same, therefore the level AB must stand as high above HR as the level EF.

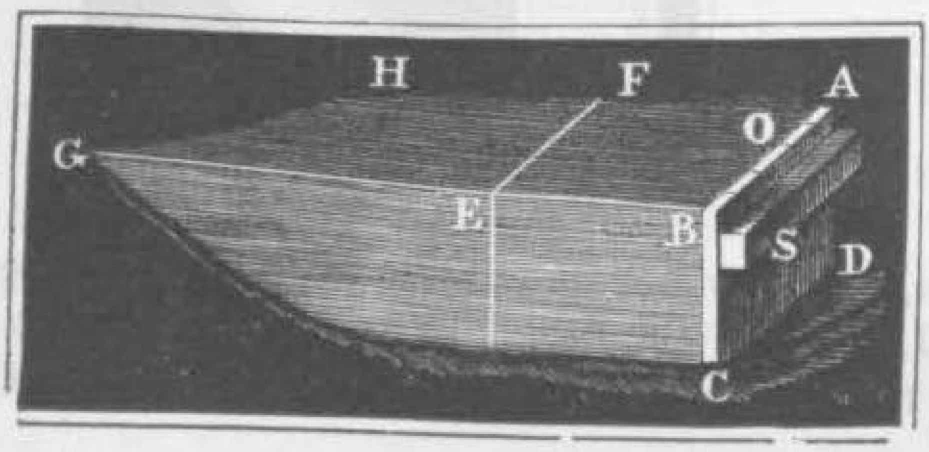
§ 276. *Lateral Pressure.*—The laws found above for the pressure of water against a horizontal surface, are not directly applicable to a plane surface inclined to the horizon; for in this case the heads of water at different places are different. The pressure $p = h\gamma$ on each unit of surface within the horizontal stratum of water, which lies a depth h below the level, acts in all directions (§ 273), and consequently also perpendicular to the fixed lateral walls of the vessel, which (from § 128) perfectly counteract it. If now F_1 be the area of an element of a lateral surface ABC, Fig. 347, and h_1 its head of water FH, we shall then have the normal pressure of the water against it: $P_1 = F_1 \cdot h_1\gamma$; if F_2 be a second element of the surface, and h_2 its head of water, we shall then have the normal pressure on it: $P_2 = F_2h_2\gamma$; and for a third element $P_3 = F_3h_3\gamma$, &c. These normal pressures form a system of parallel forces, whose resultant P is the sum of these pressures; therefore $P = (F_1h_1 + F_2h_2 + \ldots)\gamma$. But now, further, $F_1h_1 + F_2h_2 + \ldots$ is the sum of the statical moments of F_1, F_2, &c., with respect to the surface OHR of the water, and $= Fh$, F representing the area of the whole surface, and h the depth SO of its centre of gravity below the level; hence, the aggregate normal pressure against the plane surface is $P = Fh\gamma$. We mean here, by the head of water of a surface, the depth SO of its centre of gravity below the level of the watere the general rule, therefore, is true thate *the pressure of water against a plane surface is equivalent to the weight of a column of water whose base is the surface and whose height is the head of water of the surface.*

Fig. 347.



It must further be stated, that this pressure of the water is not dependent on the quantity of water which is before or below the pressed surface, that therefore, for example, a flood-gate, AC, Fig. 348, under otherwise similar circumstances, has to sustain the same pressure, whether the water to be dammed up be that of a small sluice

Fig. 348.



$ACEF$, or that of a larger dam $ACGH$, or that of a great reservoir. From the breadth $AB = CDB = b$ and height $AD = BC = a$ of a rectangular flood-gate, $F = ab$, and the head of water $SO = \frac{a}{2}$; hence the pressure of water

$$P = ab \cdot \frac{a}{2} \gamma = \tfrac{1}{2} a^2 b \gamma.$$

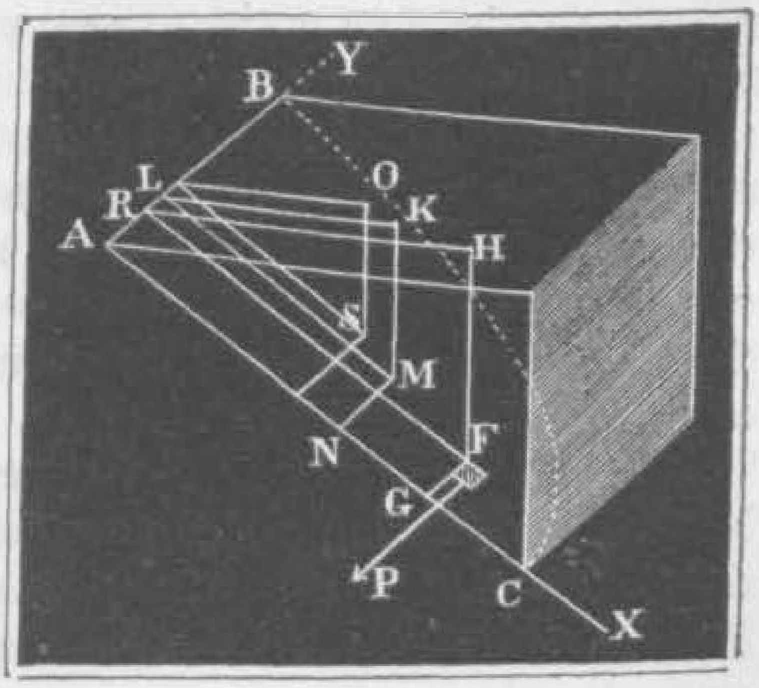
Therefore the pressure increases as the breadth, or as the square of the height of the pressed surface.

Example. If the water stand $3\frac{1}{2}$ feet high before a board of oak 4 feet broad, 5 feet high, and $2\frac{1}{2}$ inches thick, what will be the force required to draw it up? The volume of the board is $4 \cdot 5 \cdot \frac{5}{24} = \frac{25}{6}$ cubic feet. If now we take the density of oak saturated with water from § 58 at $62{,}5 \times 1{,}11 = 67{,}3$ lbs., the weight of this board will be: $G = \frac{25}{6} \cdot 67{,}3 = 280{,}5$ lbs. The pressure of the water against the board, and also the pressure of this last against the guides will be:

$P = \frac{1}{2} \cdot \left(\frac{7}{2}\right)^2 \cdot 4 \cdot 62{,}5 = 49 \cdot 30{,}25 = 1531{,}25$ lbs.; if now we take the co-efficient of friction for wet wood from § 161, $f = 0{,}68$, the friction of this board against its guides will be $F = f\,P = 0{,}68 \cdot 1531{,}25 = 1041{,}25$ lbs. If to this be added the weight of the board, we shall obtain the force required to pull it up $= 1041{,}25 + 67{,}3 = 1108{,}55$ lbs.

§ 277. *Centre of Pressure.*—The resultant $P = Fh\gamma$ of the collective elementary pressures $F_1 h_1 \gamma$, $F_2 h_2 \gamma$, &c., has, like every other system of parallel forces, a definite point of application, which is called the *centre of pressure.* Equilibrium will subsist for the whole pressure of the surface, if this point be supported. The statical moments of the elementary pressures $F_1 h_1 \gamma$, $F_2 h_2 \gamma$, &c., with respect to the plane of the level OHR, Fig. 349, are: $F_1 h_1 \gamma \cdot h_1 = F_1 h_1^2 \gamma$, $F_2 h_2^2 \gamma$, &c.; therefore, the statical moment of the whole pressure with respect to this plane is: $(F_1 h_1^2 + F_2 h_2^2 + \ldots)\,\gamma$. If we put the distance KM of the centre M of this pressure from the level of the water $= z$, we shall then have the moment of pressure $= Pz = (F_1 h_1 + F_2 h_2 + \ldots)\,z\gamma$, and by equating both moments, the depth in question of the centre M below the surface:

Fig. 349.



$$1.\ z = \frac{F_1 h_1^2 + F_2 h_2^2 + \ldots}{F_1 h_1 + F_2 h_2 + \ldots}, \text{ or } = \frac{F_1 h_1^2 + F_2 h_2^2 + \ldots}{Fh},$$

if, as before, F represent the area of the whole surface, and h the depth of its centre of gravity below the surface. To determine this pressure completely we must know further its distance from another plane or line. If we put the distances $F_1 G_1$, $F_2 G_2$, &c., of the elements of the surface $F_1 F_2$, &c., from the line AC which determines the angle of inclination of the plane $= y_1 y_2$, &c., we shall then have the moments of the elementary pressures with respect to this line $= F_1 h_1 y_1 \gamma$, $F_2 h_2 y_2 \gamma$, &c., therefore, the moment of the whole surface $= (F_1 h_1 y_1 + F_2 h_2 y_2 + \ldots)\, \gamma$; and if we represent the distance *MN* of the centre *N* from this line by v, we shall then have the moment also $= (F_1 h_1 + F_2 h_2 + \ldots)\, v\gamma$; if, lastly, we make both moments equal, we shall obtain the second ordinate:

$$2.\ v = \frac{F_1 h_1 y_1 + F_2 h_2 y_2 + \ldots}{F_1 h_1 + F_2 h_2 + \ldots}, \text{ or } = \frac{F_1 h_1 y_1 + F_2 h_2 y_2 + \ldots}{Fh}.$$

If α be the angle of inclination of the plane *ABC* to the horizon, and x_1, x_2, &c., the distances $F_1 R_1$, $F_2 R_2$, &c., of the elements F_1, F_2, &c., as likewise u the distance of the centre of pressure *M* from the line of intersection *AB* of the plane with the level of the water, we shall then have:

$h_1 = x_1 \sin. \alpha$, $h_2 = x_2 \sin. \alpha$, &c., as well as $z = u \sin. \alpha$;

and if these values be put into the expressions for z and v, we shall then obtain:

$$u = \frac{F_1 x_1^2 + F_2 x_2^2 + \ldots}{F_1 x_1 + F_2 x_2 + \ldots} = \frac{\text{moment of inertia}}{\text{statical moment}}, \text{ and}$$

$$v = \frac{F_1 x_1 y_1 + F_2 x_2 y_2 + \ldots}{F_1 x_1 + F_2 x_2 + \ldots} = \frac{\text{centrifugal moment}}{\text{statical moment}}.$$

We may, therefore, find the distances u and v of the centre of pressure from the horizontal axis *AY*, and from the axis *AX* formed by the line of fall, if we divide the statical moment of the surface with respect to the first axis, once by its moment of inertia with respect to the same axis, and a second time by its centrifugal moment with respect to both axes.e The first distance is at once the distance of the centre of suspension from the line of intersection with the line of the water (§ 251). It is easy to see that the centre of pressure coincides perfectly with the centre of percussion, determined in § 270, if the line of intersection *AY* of the surface with the level, be regarded as the axis of revolution.

If the pressed surface is a rectangle *AC*, Fig. 350, with horizontal base *CD*, the centre of pressure *M* will be found in the line *LK* let fall upon *CD* bisecting the basis, and will be distant $\frac{2}{3}$ of this line from the side *AB* in the surface of water. If this rectangle does not

Fig. 350.

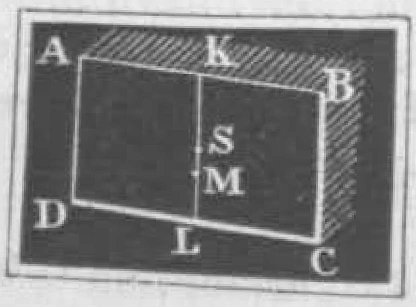


Fig. 351.

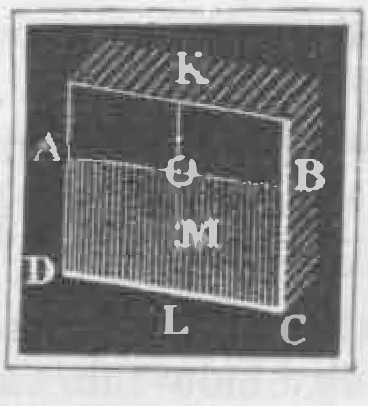
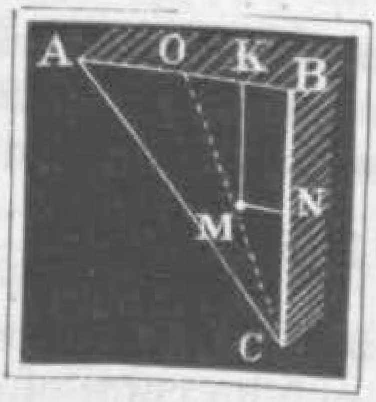


Fig. 352.



reach the surface as in Fig. 351, if further the distance *KL* of the lower base *CD* from the surface be l_1, and that of the upper base

$AB = l_2$, we then have the distance KM of the centre of pressure from the fluid surface:

$$u = \tfrac{2}{3} \cdot \frac{l_1^{\,3} - l_2^{\,3}}{l_1^{\,2} - l_2^{\,2}}.$$

For the case of a right-angled triangle ABC, Fig. 352, whose base AB lies in the fluid surface, the distance KM of the centre of pressure M from AB (§ 223), $u = \frac{\frac{1}{6} F\,l^2}{\frac{1}{3} F\,.\,l} = \frac{1}{2}\,l$, if l represent the height BC of the triangle, and the distance of the same point from the other leg, as this point in every case lies in the line CO bisecting the triangle, which passes from the point ● to the middle point of the base, $NM = v = \frac{1}{4}\,b$, where b represents the base AB.

If the point C lies in the surface, as in Fig. 353, therefore, the base AB below this point, we have

$$KM = u = \frac{\frac{1}{2} F l^2}{\frac{2}{3} F l} = \tfrac{3}{4}\,l \text{ and } NM = v = \tfrac{3}{4} \cdot \frac{b}{2} = \tfrac{3}{8}\,b.$$

Fig. 353.

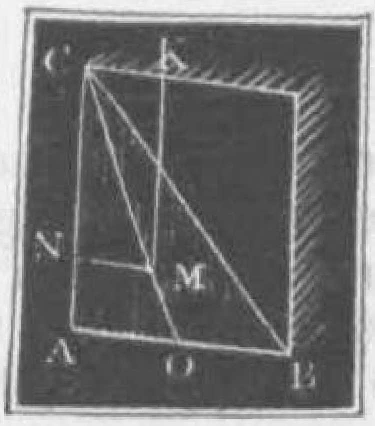
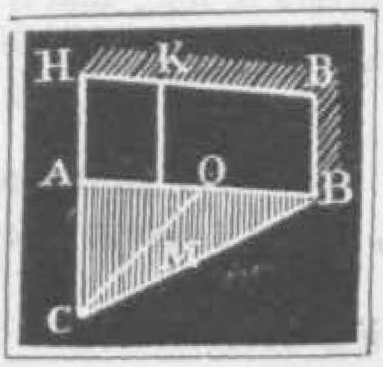


Fig. 354.



If the whole triangle ABC, Fig. 354, be under water, if the base AB is at a distance $AH = l_2$, and the point a distance $CH = l_1$ from the surface HR, we then have the distance MK from the surface HR:

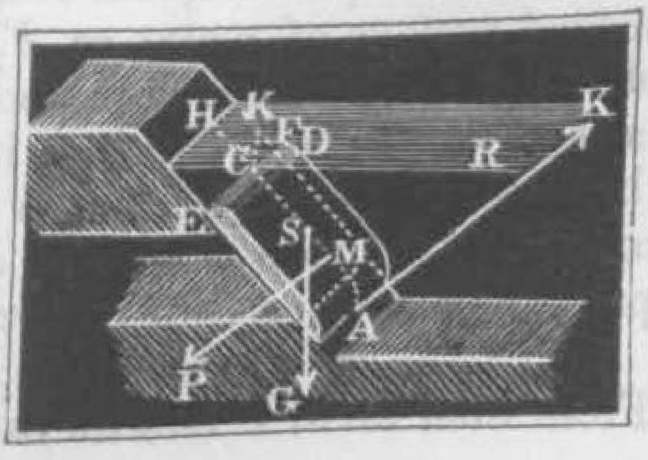
$$u = \frac{\frac{1}{18} F (l_1 - l_2)^2 + F\left(l_2 + \frac{l_1 - l_2}{3}\right)^2}{F\left(l_2 + \frac{l_1 - l_2}{3}\right)}$$

$$= \frac{\frac{1}{18}(l_1 - l_2)^2 + \frac{1}{9}(2\,l_2 + l_1)^2}{\frac{1}{3}(2\,l_2 + l_1)}$$

$$= \frac{l_1^2 + 2\,l_1\,l_2 + 3\,l_2^2}{2\,(l_1 + 2\,l_2)}.$$

In a similar manner the centres of pressure may be determined for other figures.

Fig. 355.



Example. What force K must be expended to draw up a trap-door AC turning about an axis EF, Fig. 355? Let its length $CA = 1\frac{1}{4}$ feet, its breadth $EF = 1\frac{1}{2}$ feet, its weight $= 35$ lbs.; further, the distance CK of the axis of revolution C from the surface HR, measured in the plane of the door, $= 1$ foot, and the angle of inclination of this plane to the horizon $= 68°$.

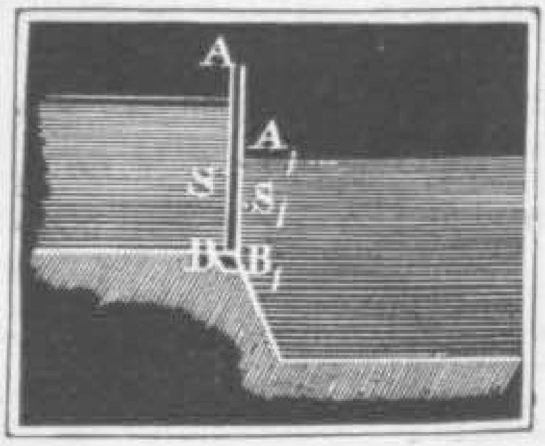
The pressed surface is $F = \frac{3}{2} \cdot \frac{5}{4} = \frac{15}{8}$ square feet, and the head of water or the depth of its centre of gravity below the surface, $h = HS\,sin.\,\alpha = (HC + CS)\,sin.\,\alpha = (HC + \frac{1}{2}.AC)\,sin.\,\alpha = \left(1 + \frac{1}{2} \cdot \frac{5}{4}\right) sin.\,68^\circ = \frac{13}{8}\,sin.\,68^\circ = \frac{13\,.\,0{,}92718}{8} = 1{,}5067$ feet; hence, the pressure of water on the surface is: $P = Fh\gamma = \frac{15}{8}\,.\,1{,}5067\,.\,66 = 186{,}45$ lbs. The arm of this force about the axis of revolution is the distance CM of the centre of pressure M from this axis; therefore $= HM - HC$

$$= \frac{2}{3} \cdot \frac{l_1^3 - l_2^3}{l_1^2 - l_2^2} - l_2 = \frac{2}{3} \cdot \frac{\left(\frac{9}{4}\right)^3 - \left(\frac{4}{4}\right)^3}{\left(\frac{9}{4}\right)^2 - \left(\frac{4}{4}\right)^2} - 1 = \frac{1}{6} \cdot \frac{729 - 64}{81 - 16} - 1$$

$= 0{,}705$ ft.; hence the statical moment of the pressure of water $= 186{,}45\,.\,0{,}705 = 131{,}46$ ft. lbs. If the centre of gravity S of the trap-door lies about half the length $CS = \frac{1}{2} \cdot \frac{5}{4} = \frac{5}{8}$ feet from the axes of revolution, the arm CD of the weight of the revolving door will be $= CS\,cos.\,\alpha = \frac{5}{8}\,.\,cos.\,68^\circ = \frac{5}{8}\,.\,0{,}3746 = 0{,}2341$ ft., and hence the statical moment of this weight $= 35\,.\,0{,}2341 = 8{,}19$ ft. lbs. By the addition of both moments, we obtain the whole moment for drawing up the trap-door $= 131{,}46 + 8{,}19 = 139{,}65$ ft. lbs.; and if the force K for this effect act at the arm $CA = 1{,}25$ feet, its amount will be $= \frac{139{,}65}{1{,}25} = 112$ lbs.

§ 279. If water presses against both sides of a plane surface AB, Fig. 356, there arises from the resultant forces corresponding to the two sides a new resultant, which is obtained by the subtraction of the former, because these two act oppositely to each other.

Fig. 356.



If F is the area of the pressed portion on the one side of the surface AB, and h the depth AS of its centre of gravity below the level of the water; further, F_1 the area of the portion $A_1 B_1$ on the other side of the surface, and h_1 the depth $A_1 S_1$ of its centre of gravity below the corresponding level of the water, we then have for the resultant sought, $P = Fh\gamma - F_1 h_1 \gamma = (Fh - F_1 h_1)\,\gamma$.

If the moment of inertia of the first portion of the fluid surface with respect to the line in which the plane of the surface intersects that of the water, $= Fx^2$, the statical moment of the pressure of water of the one side is, therefore, $= Fx^2\,.\,\gamma$; if, further, the moment of inertia of the second portion with respect to the line of intersection with the second surface of water $= F_1 x_1^2$, the statical moment of the pressure of water of the other side about the axis lying on the second surface is then $= F_1 x_1^2 \gamma$. Further, if the distance AA_1 of the axes $= a$, we then obtain the augmentation of the last moment in its transit from the axis A_1 to the axis A, $= F_1 h_1\,a\,\gamma$, and hence the statical moment of the pressure of water with respect to the axis in the first surface

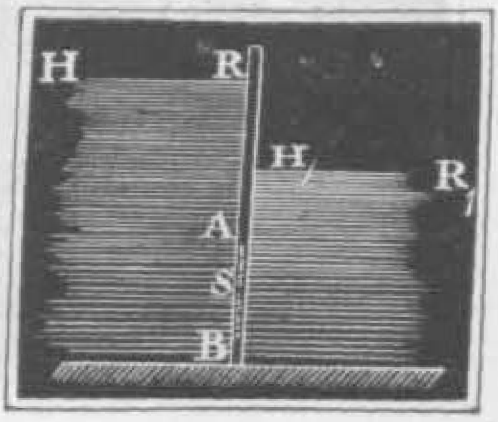
$$= F_1 x_1^2 \gamma + F_1 h_1\,.\,a\,.\,\gamma = (F_1 x_1^2 + F_1 a h_1)\,\gamma.$$

From this, then, it follows that the statical moment of the difference of both mean pressures $= (Fx^2 - F_1x_1^2 - a\,F_1h_1)\,\gamma$, and the arm of this latter force, or the distance of the centre of pressure from the axis in the first surface of water is:

$$u = \frac{Fx^2 - F_1x_1^2 - a\,F_1h_1}{Fh - F_1h_1}$$

If the portions of surface pressed are equal to one another, which takes place when, as Fig. 357 represents, the entire surface AB is below the water, we have then more simply $P = F(h - h_1)\,\gamma$ and $u = h$; the last, because $h - h_1 = a$, and $x_1^2 = x^2 - 2\,a\,h + a^2$ (§ 217). In the last case, therefore, the pressure is equivalent to the weight of a column of water, whose base is the surface pressed, and whose height is the difference of altitude RH_1 of both surfaces of water, and the centre of pressure coincides with the centre of gravity S of the surface. This law is also further correct if both surfaces of water are besides further pressed by equal forces, for example, by a piston or by the atmosphere. For this pressure upon each unit of surface $= p$, and therefore the corresponding height of a column of water $x = \frac{p}{\gamma}$ (§ 275), we have then to substitute for h, $h + x$, and for h_1, $h_1 + x$; and by subtraction, we have the residuary force $P = (h + x - [h_1 + x])\,F\gamma = (h - h_1)\,F\gamma$. For this reason, the pressure of the atmosphere in hydrostatic investigations is generally left out of consideration.

Fig. 357.

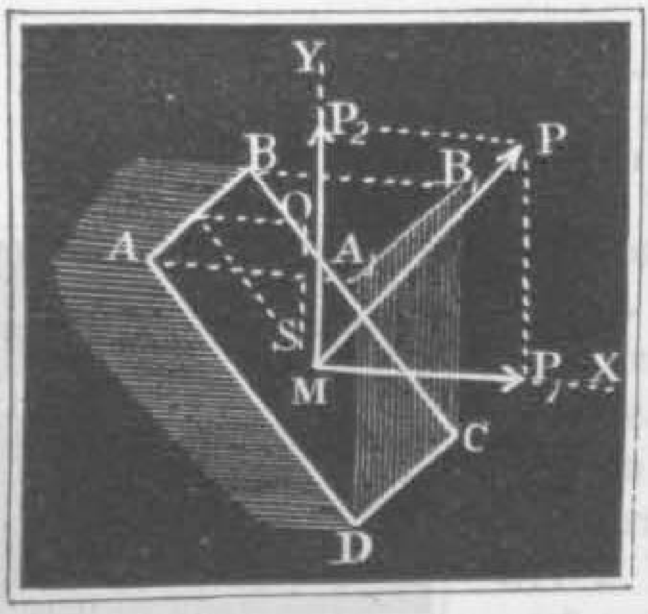


Example. The height AB of the upper surface of water in a canal, Fig. 356, amounts to 7 feet, the water in the lock stands 4 feet high at the sluice-gate, and the breadth of the canal and of the lock measure 7,5 feet, what mean pressure has the sluice-gate to sustain? It is $F = 7\cdot 7{,}5 = 52{,}5$, and $F_1 = 4\cdot 7{,}5 = 30$ square feet. Further, $h = \frac{1}{2}\cdot 7 = \frac{7}{2}$ and $h_1 = \frac{4}{2} = 2$ feet, $a = 7 - 4 = 3$ feet, $x^2 = \frac{1}{3}\cdot 7^2 = \frac{49}{3}$ and $x_1^2 = \frac{1}{3}\cdot 4^2 = \frac{16}{3}$; hence it follows, that the mean pressure sought ise $P = (Fh - F_1h_1)\,\gamma = (52{,}5\cdot\frac{7}{2} - 30\cdot 2)\cdot 62{,}5 = 123{,}75\cdot 62{,}5 = 7734{,}375$ lbs.; and the depth of its point of application below the surface of the water is:

$$u = \frac{52{,}5\cdot\frac{49}{3} - 30\cdot\frac{16}{3} - 3\cdot 60}{52{,}5\cdot\frac{7}{2} - 60} = \frac{517{,}5}{123{,}75} = 4{,}182 \text{ feet.}$$

§ 280. ***Pressure in a Definite Direction.***—In many cases it is of importance to know only one part of the pressure acting in a definite direction upon a surface. In order to find this component, we resolve the normal pressure $MP = P$ of the surface $AC = F$, Fig. 358, in the given direction MX, and in the direction MY perpendicular to it into two component pressures $MP_1 = P_1$, and $MP_2 = P_2$. Let a be the angle PMX, which the normal in the given direction MX makes with the component, we shall then obtain for the components; $P_1 = P$

Fig. 358.

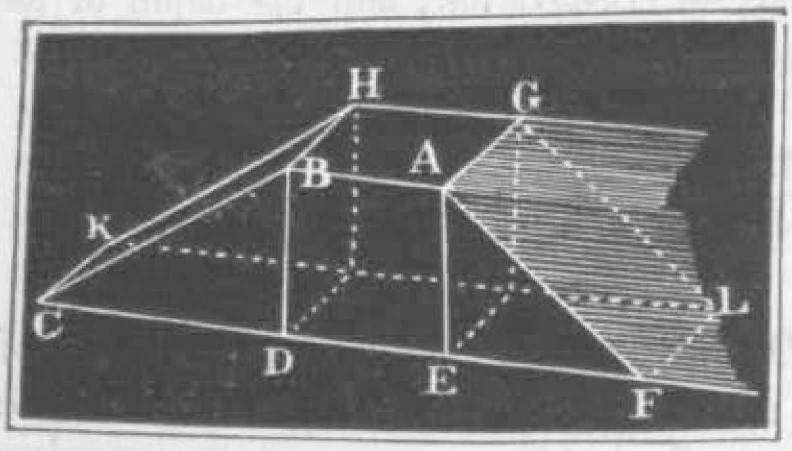


cos. α and $P_2 = P\, sin.\, \alpha$. Let a projection $A_1 B_1 CD$ of the surface AB be made on a plane at right angles to the given direction MX, we shall then have for its area F_1, the formula $F_1 = F\,.\,cos.\,ADA_1$, or since the angle of inclination ADA_1 of the surface from its projection is equal to the angle $PMX = \alpha$ between the normal pressure P and its component P_1, we then have $F_1 = F\, cos.\, \alpha$, or inversely: $cos.\, \alpha = \frac{F_1}{F}$, and hence $P_1 = P\,.\,\frac{F_1}{F}$. But as the normal pressure $P = Fh\gamma$, it follows finally that $P_1 = F_1 h\gamma$, *i. e. the pressure with which water presses against a surface in a given direction, is equal to the weight of a column of water which has for base the projection of the surface perpendicular to the given direction, and for height, the depth of the centre of gravity of the surface below that of the water.*

It is important, in most cases of application, to know only the vertical or the horizontal component of the pressure of water against a surface. Since the projection at right angles to the vertical direction is the horizontal, and the projection at right angles to the horizontal direction, a vertical projection, the vertical pressure of water against a surface may be found, if the horizontal projection or its trace be considered as the surface pressed, and on the other hand the horizontal pressure of the water in any direction may be also found, if the vertical projection or the elevation of the surface at right angles to the given direction be considered as the surface pressed, but in both cases the depth of the centre of gravity of the surface below that of the water taken as the head of water.

For a prismatic dam ACH, Fig. 359, the longitudinal profile EG for the horizontal pressure of the water and the horizontal projection EL of the surface of water for the vertical pressure must be regarded as the surfaces pressed. Hence, if the length AG of the dam $= l$, the height $AE = h$, and the front slope $EF = a$, we have then the horizontal pressure of the water $= lh\,.\,\frac{h}{2}\gamma = \frac{1}{2} h^2 l\gamma$,

Fig. 359.



and its vertical pressure $= al\,.\,\frac{h}{2}\gamma = \frac{1}{2} alh\gamma$. If now, further, the upper breadth of the top of the dam $= b$, the slope at the back $CD = a_1$, and the density of the mass of the dam $= \gamma_1$, we then have the weight of the dam $= \left(b + \frac{a + a_1}{2}\right) hl\gamma_1$, and the whole vertical pressure of this against the horizontal bottom

$$= \frac{1}{2} alh\gamma + \left(b + \frac{a + a_1}{2}\right) hl\gamma_1 = \left[\frac{1}{2} a\gamma + \left(b + \frac{a + a_1}{2}\right)\gamma_1\right] hl.$$

If we put the co-efficient of friction $= f$, then the friction or force to push the dam forward is:

$$F = \left[\frac{1}{2} a\gamma + \left(b + \frac{a + a_1}{2}\right)\gamma_1\right] fhl.$$

In the case where the horizontal pressure of the water is to effect this displacement, we have:

$$\frac{1}{2} h^2 l\gamma = \left[\frac{1}{2} a\gamma + \left(b + \frac{a + a_1}{2}\right)\gamma_1\right] fhl, \text{ or more simply:}$$

$$h = f\left[a + \left(2b + a + a_1\right)\frac{\gamma_1}{\gamma}\right].$$

Therefore, in order that the dam may not be pushed away by the water, we must have:

$$h < f\left[a + \left(2b + a + a_1\right)\frac{\gamma_1}{\gamma}\right], \text{ or,}$$

$$b > \frac{1}{2}\left[\left(\frac{h}{f} - a\right)\frac{\gamma}{\gamma_1} - \left(a + a_1\right)\right].$$

For safety we assume that the base of the dam is quite permeable, on which account there is further a counter pressure from below upwards $= (b + a + a_1)\, lh\gamma$ to abstract, and we may put

$$h < f\left[\left(2b + a + a_1\right)\left(\frac{\gamma_1}{\gamma} - 1\right) - a_1\right].$$

Example. The density of the mass of a clay dam is nearly twice as great as that of water, therefore, $\frac{\gamma_1}{\gamma} = 2$ and $\frac{\gamma_1}{\gamma} - 1 = 1$; hence, for such a dam we may put simply $h > f(2b + a)$. According to experience, a dam will resist a long time if its height, slope and breadth at the top are equal to one another; if in the last formula we put $h = b = a$, then $f = \frac{1}{3}$, whence we must in other cases put:—

$h = \frac{1}{3}\left[(2b + a + a_1)\left(\frac{\gamma_1}{\gamma} - 1\right) - a_1\right]$, and for clay dams especially, $h = \frac{1}{3}(2b + a)$, and inversely, $b = \frac{3h - ae}{2}$. If the height of the dam be 20 feet, and the angle of slope $\alpha = 36°$, the slope a will be

$$= h \,cotg.\, \alpha = 20 . cotg.\, 36° = 20 . 1{,}3764 = 27{,}53 \text{ feet},$$

and hence the upper breadth of the dam $b = \frac{60 - 27{,}53}{2} = 16{,}24$ feet.

§ 281. *Pressure on Curved Surfaces.*—The law found in the last paragraph on the pressure of water in a definite direction is true only for plane surfaces, or for the separate elements of curved surfaces, but not for curved surfaces in general. The normal pressures on the separate elements of a curved surface may be resolved into lateral components parallel to a given direction, and into others acting in the plane normal to it; these components form a system of parallel forces, whose resultant gives the pressure in the given direction, and these components may be reduced to a resultant, but the two resultants admit of no further composition when their directions do not intersect. It is not possible in general to reduce the aggregate pressures against the elements of a curved surface to a single force, but particular cases present themselves where this composition is possible.

Let G_1, G_2, G_3, &c., be the projections, and h_1, h_2, h_3, &c., the heads of water of the elements F_1, F_2, F_3, &c., of a curved surface, we then have the pressure of water in the direction perpendicular to the plane of projection :

$$P_1 = (G_1 h_1 + G_2 h_2 + G_3 h_3 + \ldots)\gamma,$$

and its moment with respect to the plane of the surface of water.

$$P_1 u = (G_1 h_1^2 + G_2 h_2^2 + G_3 h_3^2 + \ldots)\gamma.$$

If the curved surface pressed upon can be decomposed into elements which have a uniform ratio to their projections, we may then put

$$\frac{F_1}{G_1} = \frac{F_2}{G_2} = \frac{F_3}{G_3}, \text{ \&c.}, = n, \text{ we then have:}$$

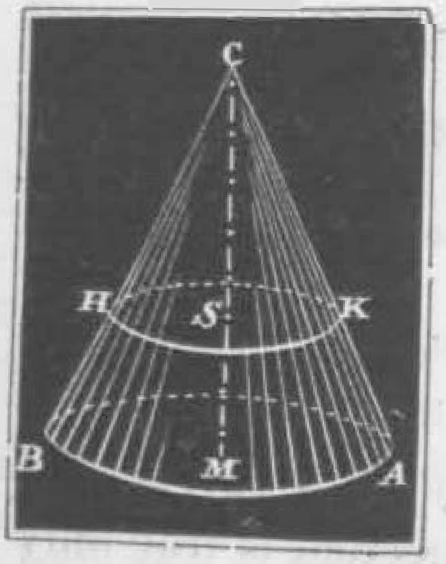
$$P_1 = \left(\frac{F_1 h_1}{n} + \frac{F_2 h_2}{n} + \ldots\right)\gamma = \left(\frac{F_1 h_1 + F_2 h_2 + \ldots}{n}\right)\gamma,$$

or, since the ratio of the entire curved surface F to its projection G, *i. e.* $\frac{F}{G}$ is $= n$,

$P_1 = \frac{Fh}{n}\gamma = Gh\gamma$; in this case we have, as for every plane surface, the pressure in any direction equivalent to the weight of a prism of water, whose basic surface is at right angles to the projection of he curved surface in the given direction, and whose height is equal to the depth of the centre of gravity of the curved surface below the surface of water.

So, for example, the vertical pressure of water against the envelope of a conical vessel ACB, filled with water, Fig. 360, is equal to the weight of a column of water which has the bottom for its base, and two-thirds of the length of the axis CM for height, because the horizontal projection of the envelope of a right cone upon its base, as likewise the envelope, may be resolved into exactly similar triangular elements, and because the centre of gravity S of the surface of the cone is distant two-thirds of the height of the cone from the vertex (§ 110). If r be the radius of the base, and h the height of the cone, we shall then have the pressure against the bottom $= \pi r^2 h\gamma$, and the vertical pressure against the envelope $= \frac{2}{3}\pi r^2 h\gamma$, but as the bottom is rigidly connected with the sides, and both pressures act opposed to each other, the force with which the vessel is pressed downwards by the water is :

Fig. 360.

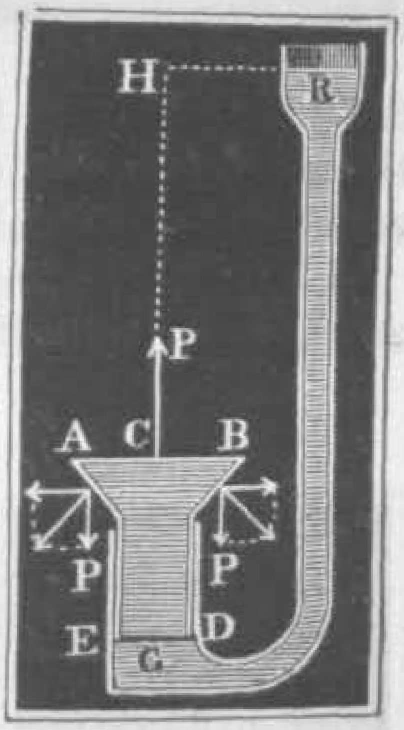


$$= \left(1 - \frac{2}{3}\right) \pi r^2 h \gamma = \frac{1}{3} \pi r^2 h \gamma =$$

to the weight of the whole mass of water. If the bottom be separated by a fine cut from the envelope, this will then press with its full force $\pi r^2 h \gamma$ downwards, or on its support, and on the other hand it would be necessary to hold down the envelope with a force $\frac{2}{3} \pi r^2 h \gamma$ to prevent its being raised off.

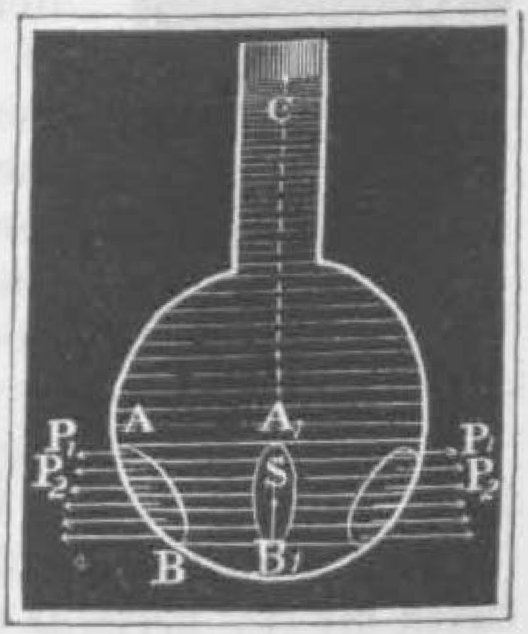
Remark. From this the force which the steam of a steam-engine or the water of a water-column machine exerts on the piston, is independent of the form of the piston. Whether the surface of pressure be augmented by being hollowed out or rounded, the pressure with which the steam or water pushes forward the piston is equivalent to the product of the cross section or horizontal projection of the piston and the pressure on a unit of surface. The pressure on the larger surface of a funnel-shaped piston AB, Fig. 361, whose greater radius $CA = CB = r$ and lesser radius $GD = GE = r_1$, is $= \pi r^2 p$, and the reaction upon the envelope $= \pi (r^2 - r_1^2) p$; hence, the residuary effective pressure is $= \pi r^2 p - \pi (r^2 - r_1^2) p = \pi r_1^2 p$ = the cross section of the cylinder multiplied by the pressure on a unit of surface.

Fig. 361.



§ 282. *Horizontal and Vertical Pressure.*—Whatever may be the form of a curved surface, AB, Fig. 362, the horizontal pressure of the water against it is always equivalent to the weight of a column of water, whose base is the vertical projection A_1B_1 of the surface perpendicular to the given direction of pressure, and whose height of pressure is the depth CS of the centre of gravity S of the projection below the surface of water. The correctness of this follows directly from the formula $P_1 = (G_1h_1 + G_2h_2 + \ldots)\gamma$, when we consider that the height of pressure h_1, h_2, &c., of the elements of the surface are also the heights of pressure of their projections, that, therefore, $G_1h_1 + G_2h_2 + \ldots$ is the statical moment of the whole projection, *i. e.* the product Gh of the vertical projection G and the depth h of its centre of gravity below the surface of water. We have here, therefore, again to put $P_1 = Gh\gamma$, and to consider h as the height of pressure of the vertical projection.

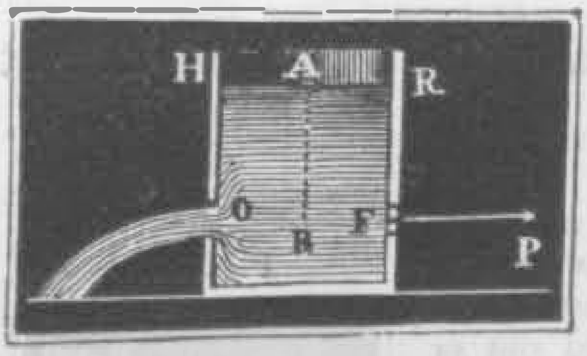
Fig. 362.



The vertical section which divides a vessel containing water into two equal or unequal parts, is at once the vertical projection of the two parts, but the horizontal pressure on one part of the wall of the vessel is proportional to the product of its vertical projection and to the depth of its centre of gravity below the surface of the water, consequently the horizontal pressure on a part of the wall of the vessel is exactly equal in amount to the oppositely acting horizontal pressure on the part opposite, and consequently the two forces balance each other in the vessel; the whole vessel is therefore equally pressed by the enclosed water in all horizontal directions.

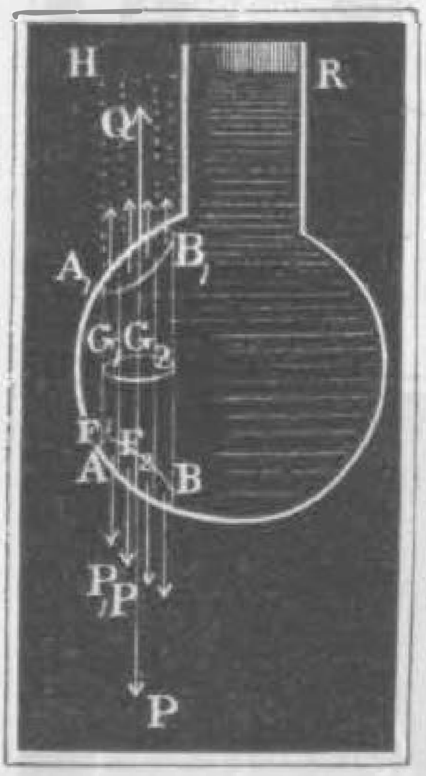
If an opening O be made in the side of a vessel HBR, Fig. 363, the part of the pressure corresponding to the section of this opening disappears, and the pressure on the oppositely situated portion of the surface F now comes into action. Whilst, therefore, the water flows out at the lateral aperture, an equal distribution of the horizontal pressure no longer takes place over the whole extent, and there ensues a reaction opposite to the motion of the flowing water: $P = Fh\gamma$, F being the projection of the aperture, and h the height of pressure of its projection. By this reaction the vessel may be set into motion.

Fig. 363.



The vertical pressure of water is $P_1 = G_1 h_1 \gamma$ against an element of surface F_1, Fig. 364, of the side of the vessel, since the horizontal projection G_1 may be regarded as the transverse section, and the height of pressure h_1 as the height, and therefore $G_1 h_1$ as the volume of a prism, equivalent to the weight of a column of water HF_1 incumbent on the element, and reaching the surface of the water. The elements of the surface which make up a finite part of the bottom, or side of the vessel, hence suffer a vertical pressure which is equivalent to the weight of all the incumbent columns of water, *i. e.* to the weight of a column of water incumbent on the whole portion. Let this volume $= V_1$, we then obtain for the vertical pressure $P = V_1\gamma$. For another portion A_1B_1, which lies vertically above the former, we have the vertical pressure opposed to it $Q = V_2\gamma$; but if both portions are rigidly connected with each other, there results from the two forces the force acting vertically downwards $R = (V_1 - V_2)\gamma = V\gamma =$ *to the weight of the columns of water contained between the two portions of the surface.* If, lastly, we apply this law to the whole vessel, it follows that *the aggregate vertical pressure of the water against the vessel is equivalent to the weight of the enclosed mass of water.*

Fig. 364.

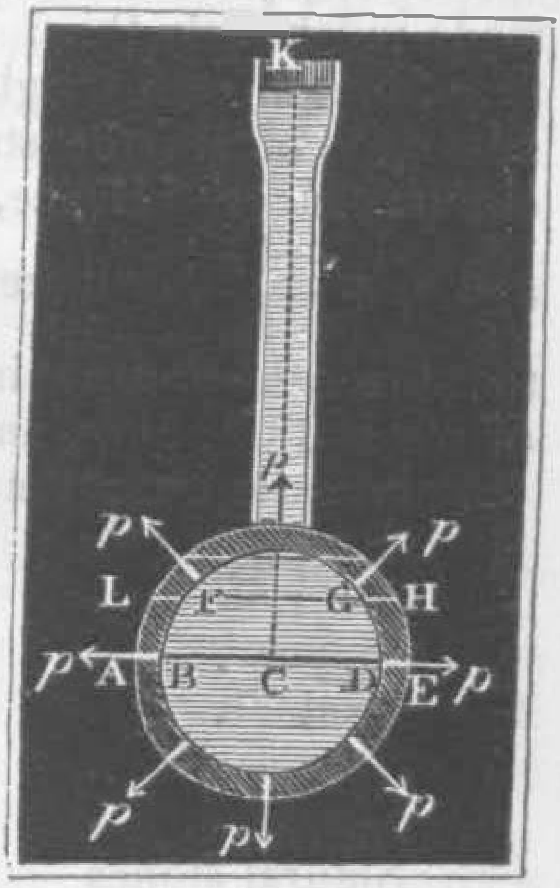


§ 283. *Thickness of Pipes.*—The application of the laws of the pressure of water to pipes, boilers, &c., is of particular importance. That these vessels may adequately resist the pressure, and be prevented bursting from its effect, we must give a certain thickness to their sides, corresponding to the head of water and the internal width. The bursting of a pipe may take place in various ways, viz., transversely or longitudinally; the latter happens more frequently than the former, as will be soon understood from what follows.

The width of the pipe $BD = 2r$, Fig. 365, and the head of water

$CK=h$, therefore the pressure on a unit of surface $p=h\gamma$, we then have the whole pressure in the direction of the axis of the pipe $=\pi r^2 p=\pi r^2 h\gamma$; if the thickness of the side $AB=DE=e$, we then have the transverse section of the mass of the pipe $e=\pi(r+e)^2 - \pi r^2 = 2\pi r e + \pi e^2 = 2\pi r e\left(1+\frac{e}{2r}\right)$, and if lastly we put the modulus of elasticity $=K$, we then have the pressure for rupture over the whole section of the pipe

Fig. 365.



$$=2\pi r e\left(1+\frac{e}{2e}\right)K,$$

for this reason we have now to put:

$$2\pi r e\left(1+\frac{e}{2e}\right)K=\pi r^2 p,$$

or approximately and more simply $2eK=rp$, and hence the thickness of the pipe $e=\frac{rp}{2K}=\frac{ph\gamma}{2K}$. In order, therefore, to avoid any transverse rent in the pipe or in the boiler, the thickness of the sides must be made $e>\frac{rp}{2K}$. Of all longitudinal rents, AE, LH, &c., those running diametrically, such as AE, take place the most easily, because they have the smallest area, whence we must only take these into account. Let us consider a portion of a pipe of the length l, and let us have regard to the occurrence of a rent of the length l, we then obtain a transverse section of the surface of separation $=le$, and hence the force for rupture in this surface leK. For two oppositely situated rents this force is $=2\,leK$, whilst the pressure of water for each half of the pipe is proportional to the transverse section $2\,rl$, and hence is $=2\,rlp$. By equating the two expressions, it follows that $2\,leK=2\,rlp$, i. e. $eK=rp$, therefore the thickness $e=\frac{rp}{K}$. To provide against longitudinal rents, the sides must be made as thick again, as to provide against transverse rents.

From the formula $e=\frac{rp}{K}=\frac{rh\gamma}{K}$, it follows that *the strength of similar pipes is as the widths and as the heads of water or pressures upon a unit of surface.* A pipe three times the width of another, which has five times the pressure to sustain on each unit of surface that the other has, must have its sides fifteen times as thick.

Hollow spheres, which have to sustain a pressure p from within on each unit of surface, require a thickness $e=\frac{rp}{2K}$, because here the projection of the surface of pressure is the greatest circle πr^2, and the surface of separation the ring $2\pi r e\left(1+\frac{e}{2r}\right)$, or approximately for a smaller thickness $=2\pi r e$.

The formula found give for $p=o$, also $e=o$, for this reason, therefore, pipes which have no internal pressure to sustain, may be made indefinitely thin; but as each pipe must sustain a certain pressure from its own weight, we must still give to it a certain thickness e_1, to obtain the strength of a tube which will resist under all circumstances. Hence, for cylindrical pipes or boilers we must put $e = e_1 + \frac{rh\gamma}{K}$, or more simply, if d represents the interior diameter of the pipe, n the pressure in atmospheres, each corresponding to a column of water 33 ft. high, and μ a number from experiment $e = e_1 + \mu nd$.

From experiments made we must take for pipes of

Iron plate . . .	$e = 0{,}00086\ nd + 0{,}12$	inches
Cast iron . . .	$e = 0{,}00238\ nd + 0{,}33$	"
Copper	$e = 0{,}00148\ nd + 0{,}16$	"
Lead	$e = 0{,}00242\ nd + 0{,}20$	"
Zinc	$e = 0{,}00507\ nd + 0{,}16$	"
Wood	$e = 0{,}0323\ nd + 1{,}04$	"
Natural stones . .	$e = 0{,}0369\ nd + 1{,}15$	"
Artificial stones . .	$e = 0{,}0538\ nd + 1{,}53$	"

Example. If a perpendicular water-column machine has cast-iron pipes of 10 inches inner width, how thick must these be at 100, 200, and 300 feet depths? From the formula, for 100 feet pressure, this thickness is:

$$= 0{,}00238 \cdot \frac{100}{33} \cdot 10 + 0{,}33 = 0{,}07 + 0{,}33 = 0{,}40 \text{ inches};$$

for 200 feet, $e = 0{,}14 + 0{,}33 = 0{,}47$ inches; and for 300 feet pressure, $= 0{,}22 + 0{,}33 = 0{,}55$ inches. Cast iron conducting pipes are commonly proved at 10 atmospheres, for which reason, $e = 0{,}0238 \cdot d + 0{,}33$ inches; therefore, for pipes of 10 inches width the thickness $e = 0{,}24 + 0{,}33 = 0{,}57$ inches must be given.

Remarks. The thickness of the sides of steam-boilers will be considered in the Second Part. Concerning the theory of the strength of pipes, a treatise by Brix, in the "Verhandlungen des Vereins zur Beförderung des Gewerbfleiszes in Preuszen," Jahrgang, 1834, may be consulted. The technical relations and the proving of pipes are fully treated of in Hagen's "Handbuch der Wasserbaukunst," vol. i., and in Genieys' "Essai sur les moyens de conduire, &c., les eaux."

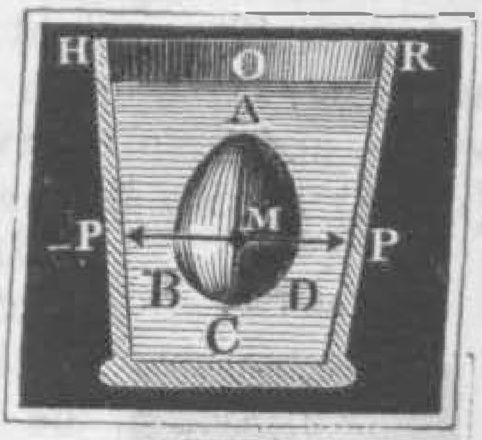
[For a view of the general principles governing the construction and strength of cylindrical steam-boilers, the editor may refer to his paper on that subject read before the Franklin Institute, July 26, 1832, and published in its Journal, in which the relation stated in the text, between the strength required in the direction of the curvature and that in the direction of the length of the tube or boiler, was pointed out, accompanied by a table of diameters and thicknesses of boilers, with the tenacities per inch of iron required in each direction for a given pressure. See, likewise, American Journal of Science and Arts, vol. xxiii. No. 1.]

CHAPTER II.

ON THE EQUILIBRIUM OF WATER WITH OTHER BODIES.

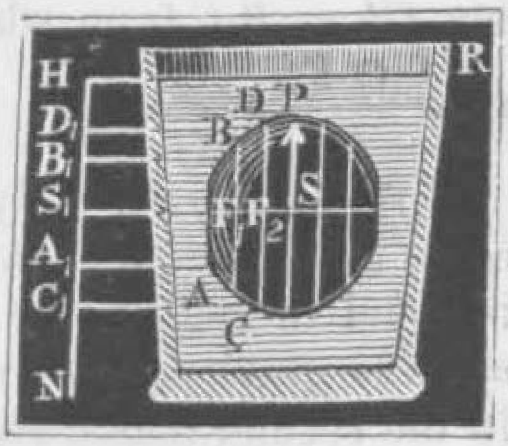
§ 284. *Buoyancy.*—A body immersed under water is pressed upon by the water on all sides, and now the question arises as to the amount, direction and point of application of the resultant of all these pressures. Let us imagine this resultant to consist of a vertical and a horizontal component, and determine these forces according to the rules of § 282. The horizontal pressure of the water against a surface is equivalent to the horizontal pressure against its vertical projection, but now every projection of a body, AC, Fig. 366, is at the same time the projection of the fore part ADC and the back part ABC of its surface; hence, also, the horizontal pressure of water against the back portion of the surface of a body is equal in amount to that of the front portion, and as both pressures are exactly opposite, their resultant $= 0$. As this relation takes place for every arbitrary horizontal direction, and the vertical projection corresponding to this, it follows that the resultant of all the horizontal pressures is nothing; that, therefore, the body AC below the water is equally pressed in all horizontal directions, and for this reason exerts no effort to move forward in a horizontal direction.

Fig. 366.



To find the vertical pressure of the water against the body BCS, Fig. 367, let us suppose it made up of vertical elementary prisms, AB, CD, &c., and determine the vertical pressures on their terminating surfaces A and B, C and D. If the lenghts of these prisms are l_1, l_2, &c., the depths of their upper extremities B, D, &c., below the surface of water HR : h_1, h_2, &c., and the horizontal transverse sections $F_1 F_2$, &c., we then have for the vertical pressures acting from above downwards against the extremities, B, D, &c., $= F_1 h_1 \gamma$, $F_2 h_2 \gamma$, &c.; on the other hand, the pressures acting from below upwards and against the extremities A, C, &c., $= F_1 (h_1 + l_1) \gamma$, $F_2 (h_2 + l_2) \gamma$, &c.; and it now follows, from a composition of these parallel forces, that the resultant P

Fig. 367.



$$= F_1 (h_1 + l_1) \gamma + F_2 (h_2 + l_2) \gamma + \ldots - F_1 h_1 \gamma - F_2 h_2 \gamma - \ldots$$
$$= (F_1 l_1 + F_2 l_2 + \ldots) \gamma = V\gamma,$$

if V represents the volume of the immersed body or the water displaced.

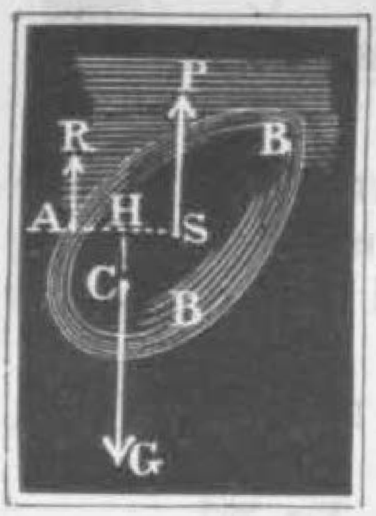
Therefore the buoyancy or the force with which the water strives to push a body immersed from below upwards, is equivalent to the weight of water displaced, or to a quantity of water which has the same volume as the submerged body.

Further, to find the point of application of this resultant, let us put the distances AA_1, CC_1, &c., of the elementary columns AB, CD, &c., from a vertical plane HN : a_1, a_2, &c., and determine the moments of the forces with respect to this plane. If S is the point of application of the upward pressure, and $SS_1 = x$ its distance from that principal plane, we shall then have:

$V\gamma . x = F_1 l_1 \gamma h. a_1 + F_2 l_2 \gamma a_2 + \ldots$; and hence,
$x = \frac{F_1 l_1 a_1 + F_2 l_2 a_2 + \ldots}{F_1 l_1 + F_2 l_2 + \ldots} = \frac{V_1 a_1 + V_2 a_2 + \ldots}{V_1 + V_2 + \ldots}$, if V_1, V_2, &c., represent the contents of the elementary columns. Since (from § 100) the centre of gravity is accurately determined by the same formula, it follows *that the point of application S of the upward pressure coincides with the centre of gravity of the water displaced.*

§ 285. The weight G of the body acting in an opposite direction associates itself with the buoyancy of the body immersed or under water, and from the two there arises a resultant $R = G - V\gamma$ or $= (\varepsilon - 1) V\gamma$, if ε be the specific gravity of the body.

Fig. 368.

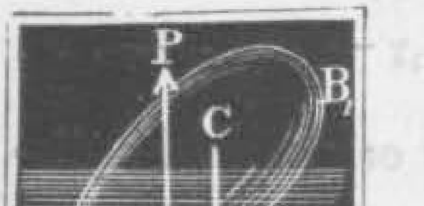


If the mass of the body be homogeneous, the centre of gravity of the displaced water will coincide with that of the body, and hence this point will be the point of application of the resultant R; but if there be not homogeneity, then these centres of gravity do not coincide, and the point of application of the resultant R deviates from both centres of gravity. Let us put the horizontal distance SH, Fig. 368, of both centres of gravity from each other, $= b$, and the horizontal distance SA of the point of application A sought from the centre of gravity S of the displaced water $= a$, we shall have the equation $Gb = Ra$, from which is given:

$$a = \frac{G\,b}{R} = \frac{G\,b}{G - P}.$$

If the immersed body be left to its own gravity, the three following cases may present themselves. Either the specific gravity of the body is equal to that of the water, or it is greater, or it is less than the specific gravity of the water. In the first case the buoyancy is equal, in the second it is less, and in the third it is greater than the weight of the water. Whilst, in the first case, equilibrium subsists between the weight and the buoyancy, the body must in the second case sink with the force $G - V\gamma = (\varepsilon - 1) V\gamma$, and, in the third case, rise with the force $V\gamma - G = (1 - \varepsilon) V\gamma$. The rising goes on only as long as the mass of water V_1, cut off from the plane of the surface and displaced by the body, has the same weight as the entire body. The weight $G = V\varepsilon\gamma$ of the body BB_1, Fig. 369, and the buoyancy $P = V_1\gamma$ now constitute a couple, by which the body is made to revolve until the directions of both coincide, or until the centre of gravity of the body lies in one and the same vertical line with the centre of gravity of the displaced water.

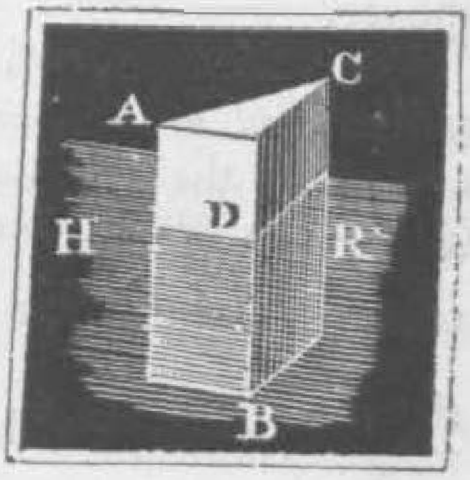
Fig. 369.



The line passing through the centre of gravity of the floating body and through that of the displaced water, is called *the axis of floatation*; and on the other hand, the section of the body formed by the plane of the surface of the water, *the plane of floatation*. Every plane which divides a body, so that one part is to the whole as the specific gravity of the body to that of the fluid, and that the centres of gravity of the two parts lie in a line normal to this plane, is a plane of floatation of the body.

§ 286. *Depth of Floatation.* If the figure and weight of a floating body be known, the depth of immersion may be calculated beforehand, with the help of the previous rule. If G be the weight of the body, we may then put the volume of the displaced water $V = \frac{G}{\gamma}$; if we combine with it the stereometrical formula for the volume V, we shall obtain the equation of condition. Hence, for the prism ABC, Fig. 370, with vertical axis, for example, $V = Fy$, if F represent the section and y the depth BD of immersion, $Fy = \frac{G}{\gamma}$ and $y = \frac{G}{F\gamma}$. For a pyramid ABC, Fig. 371, whose vertex floats under the water, $V = \frac{1}{3} f y^3$, if f represents the section at the distance of unity from the vertex; hence it follows, that:

Fig. 370.



$$\tfrac{1}{3} f y^3 = \frac{G}{\gamma}, \text{ and hence the depth } CE = y = \sqrt[3]{\frac{3\,G}{f\,\gamma}}.$$

Fig. 371.

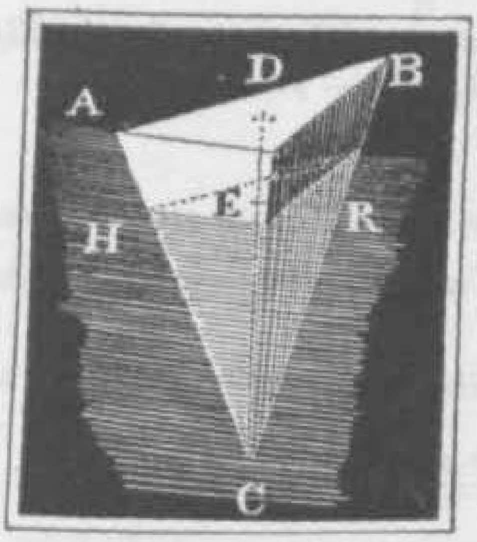
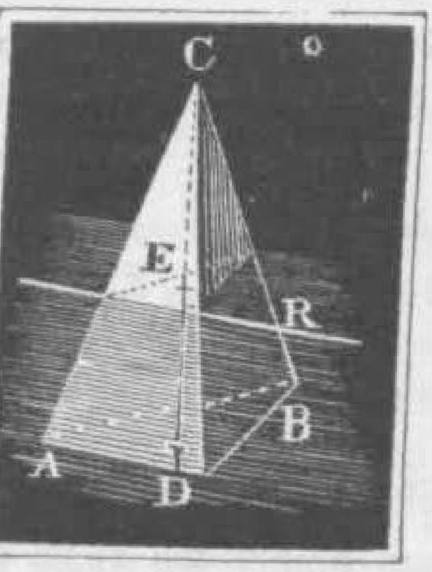


Fig. 372.



For a pyramid ABC, Fig. 372, floating with its base below the water, the distance is given $CE = y_1$ of the vertex from the surface, from the height h of the entire pyramid, if we put

$$V = \tfrac{1}{3} f (h^3 - y_1{}^3),\ y_1 = \sqrt{h^3 - \frac{3\,G}{f\,\gamma}}.$$

For a sphere AB, Fig. 373, with the radius $CA = r$,

Fig. 373.

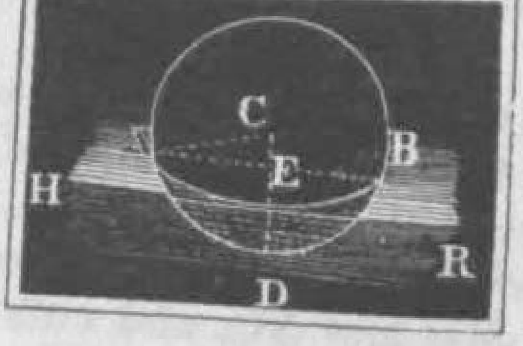
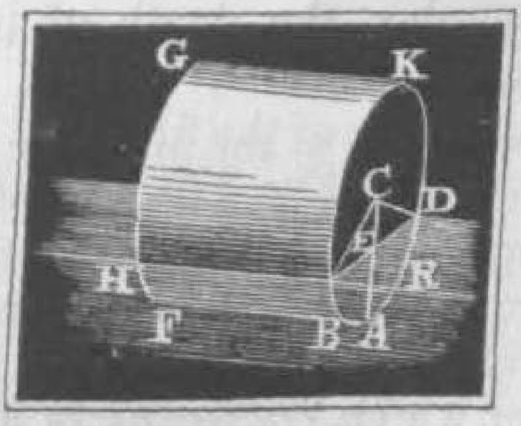


Fig. 374.



$V = \pi y^2\left(r - \frac{y}{3}\right)$, hence we shall have to solve the cubic equation $y^3 - 3 r y^2 + \frac{3 G}{\pi \gamma} = 0$, to find the depth of immersion DE of the sphere.

For a floating cylinder AG, with horizontal axis, Fig. 374, of a radius $BC = DC = r$, if α^0 be the angle BCD subtended at the centre by the arc immersed, the depth of immersion $AE = y = r(1 - \cos. \frac{1}{2} \alpha)$, but to find the arc immersed, we must put the volume of the water displaced = to the segment $\frac{r^2 \alpha}{2}$ less the triangle $\frac{r^2 \sin. \alpha}{2}$, multiplied by the length $GKB = l$ of the cylinder; therefore, $(\alpha - \sin. \alpha)\frac{lr^2}{2} = \frac{G}{\gamma}$, and solve the equation $\alpha - \sin. \alpha = \frac{2\,G}{lr^2 \gamma}$, by approximation, with respect to α.

Examples.—1. A wooden sphere, of 10 inches diameter, floats $4\frac{1}{2}$ inches deep, the volume of water displaced by it is then:

$$V = \pi \left(\frac{9}{2}\right)^2 \left(5 - \frac{9}{6}\right) = \frac{\pi \,.\, 81 \,.\, 7}{8} = \frac{567 \,.\, \pi}{8} = 222{,}66 \text{ cubic inches},$$

whilst the solid contents of the sphere are $\frac{\pi d^3}{6} = \frac{\pi \,.\, 10^3}{6} = 523{,}6$ cubic inches.

From this, 523,6 cubic inches of the mass of the sphere weigh as much as 222,66 cubic inches of water, and it follows that the specific gravity of the former is:

$$\varepsilon = \frac{222{,}66}{523{,}6} = 0{,}425 \,..$$

2. How deep will a wooden cylinder of 10 inches diameter and specific gravity $\varepsilon = 0{,}425$ sink? $\frac{\alpha - \sin.\alpha}{2} = \frac{\pi r^2 l \,.\, \varepsilon \gamma}{l r^2 \gamma} = \pi \varepsilon = 0{,}425 \,.\, \pi = 1{,}3352$; now a table of segments gives for the area $\frac{\alpha - \sin.\alpha}{2} = 1{,}32766$ of a circular segment, the angle subtended at the centre by the arc $\alpha^0 = 166^0$, and for $\frac{\alpha - \sin.\alpha}{2} = 1{,}34487$, the same angle $= 167^0$; hence, simply, the angle subtended at the centre corresponding to the slice 1,3352 is:

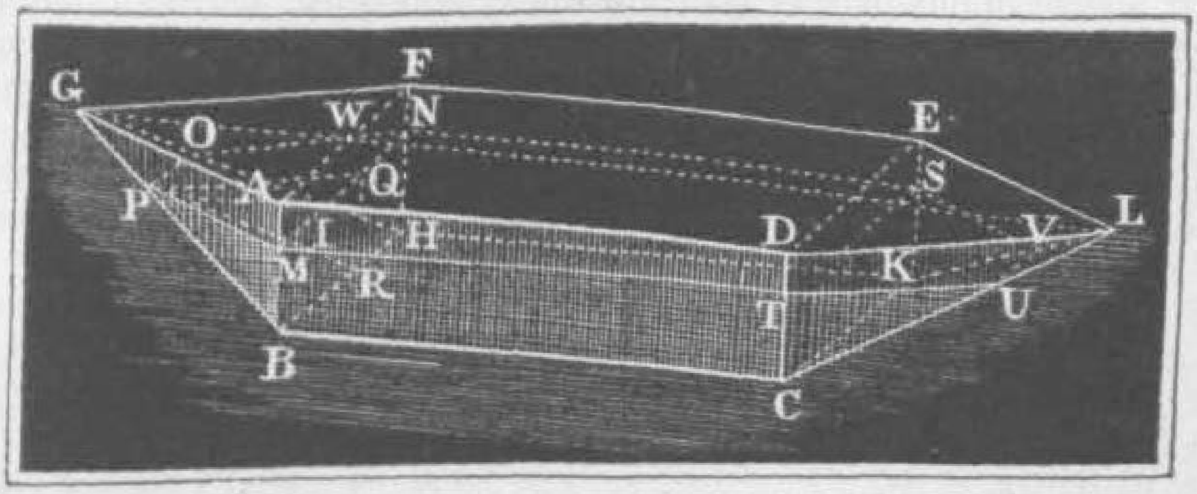
$$\alpha^0 = 166^0 + \frac{1{,}3352 - 1{,}32766}{1{,}34487 - 1{,}32766} \,.\, 1^0 = 166^0 + \frac{754^0}{1721} = 166^0\, 26'$$; therefore the depth of immersion:

$$y = r\,(1 - \cos. \tfrac{1}{2}\alpha) = 5\,(1 - \cos. 83^0\, 13') = 5 \,.\, 0{,}8819 = 4{,}41 \text{ inches.}$$

§ 287. The determination of the depth of immersion occurs chiefly in the case of ships, boats, &c. If these have a regular form, the depth may be calculated from geometrical formulæ; but if this regularity fails, or the law of configuration is not known, or if the form is very complex, the depth of immersion must then be determined by experiment.

An example of the first case is in the body $ACLEG$ (a pointed scow), represented in Fig. 375, bounded by plane surfaces. It consists of a parallelopiped ACE, and of two four-sided pyramids BFG and CEL, forming the head and the stern, and its plane of floatation is composed of a parallelogram MS, and two trapeziums MO and SU, and cuts off a bulk of water, consisting of a parallelopiped MCS, and two triangular prisms PNR, and two quadrilateral pyramids BQP. If we put the length AD of the middle portion $= l$, the breadth $AF = b$, and the

Fig. 375.



depth $AB = h$; further, the length GW of each of the two ends $= c$, and the depth of immersion, *i. e*, $BMB = CT = y$, the immersed part MCS of the middle portion will be: $= \overline{MN} \times \overline{MT} \times \overline{MB} = lby$. The base of the quadrilateral pyramid BQP is $B\overline{M}.\overline{BR}$, and the height PJ, hence the solid contents of this pyramid $= \frac{1}{3}\,\overline{BM}.\overline{BR}$ $.\overline{PJ}$. But now:

$$BM = y,\ BR = \frac{BP}{BG}\cdot BH = \frac{BM}{BA}B\,BH = \frac{y}{h}\cdot b = \frac{by}{h},$$

and likewiseh

$$PJ = \frac{BM}{BA}\cdot GW = \frac{y}{h}c = \frac{cy}{h},$$

hence the contents of both pyramids areh

$$= 2\,.\,\tfrac{1}{3}\,.\,y\,.\,\frac{by}{h}\,.\,\frac{cy}{h} = \tfrac{2}{3}\frac{bcy^3}{h^2}.$$

The transverse section of the triangular prism

RNO is $= \frac{1}{2}\,\overline{RQ}\,.\,\overline{PJ} = \frac{1}{2}\,y\,.\,\frac{cy}{h} = \frac{cy^2}{2h}$, and the side

$$RH = QN = b - \frac{by}{h} = b\left(1 - \frac{y}{h}\right),$$

hence the solid contents of both prisms are:

$$= 2\,.\,\frac{cy^2}{2h}\,.\,b\left(1 - \frac{y}{h}\right) = \frac{bcy^2}{h}\left(1 - \frac{y}{h}\right).$$

By addition of the three volumes found, the volume of the water displaced is knownh

$$V = bly + \tfrac{2}{3}\,\frac{bcy^3}{h^2} + \frac{bcy^2}{h} - \frac{bcy^3}{h^2} = \left(l + \frac{cy}{h} - \tfrac{1}{3}\,.\,\frac{cy^2}{h^2}\right)by.$$

Now the gross weight of the boat $= G$, we then have to put:

$$\left(l + \frac{cy}{h} - \tfrac{1}{3}\,.\,\frac{cy^2}{h^2}\right)by\gamma = G, \text{ or,}$$

$$y^3 - 3\,hy^2 - \frac{3\,lh^2}{c}\,.\,y + \frac{3\,h^2G}{bc\gamma} = 0.$$

The depth of immersion y is determined from the loading by the solution of the last cubic equation.

Examples.—1. If the length of the middle portion $l = 50$ feet, the length of each end $c = 15$ feet, the breadth $b = 12$ feet, and the depth $h = 4$ feet, with a depth of immersion $y = 2$ feet, the whole weight amounts to

$$G = [50 + 15 \cdot \tfrac{1}{2} - \tfrac{1}{3} \cdot 15 \cdot (\tfrac{2}{4})^2] \cdot 12 \cdot 2 \cdot 62{,}5 = (50 + 7{,}5 - 1{,}25) \cdot 24 \cdot 62{,}5 = 87235 \text{ lbs.}$$

2. If the clear weight of the former boat amount to 50000 lbs., we shall have for the depth of immersion: $y^3 - 12y^2 - 160y + 202{,}02 = 0$. By trial, it is easily found that this equation may be answered pretty accurately by $y = 1{,}17$, whence the depth of immersion sought may be taken as great.

Remark. To know the weight of the load of a ship, a scale is attached to both sides, which is called a water-gauge. The divisions are made from experiment, while it is observed what loads correspond to definite immersions.

§ 288. *Stability.*—The floating of bodies takes place either in an upright or an oblique position; and further, with or without stability. A body, a ship, for example, floats uprightly, if one plane through the axis of symmetry is a plane of symmetry of the body; and a body floats obliquely if it is not divided by any of the planes, which may be carried through the axis of floatation into two congruent halves. A body floats with stability, if it strives to maintain its state of equilibrium (compare § 130); if, therefore, mechanical effect is to be expended to bring it out of this position, or if it returns of itself into a position of equilibrium after having been drawn out of one. On the other hand, a body floats without stability if it passes into a new position of equilibrium after having been brought out of one by a shock or blow.

If a body ABC, Fig. 376, floating at first uprightly, is brought into an inclined position, the centre of gravity S of the water displaced passes from the plane of symmetry EF, and assumes a position S_1 on the larger half immersed. The buoyancy applied at S: $P = V\gamma$, and the weight applied at the centre of gravity C of the body, viz., $G = -P$ form a couple by which (§ 90) a revolution is produced. About whatever point this revolution may take place, the point C, yielding to the weight G, will always go down, and S_1, or another point M of the vertical S_1P, obedient to the force P, will rise, therefore the plane of symmetry, or of the axis EF of the ship, will be drawn downwards at C, and upwards at M, and hence it will remain upright if M, as in the figure, lie above C, or incline itself still more

Fig. 376.

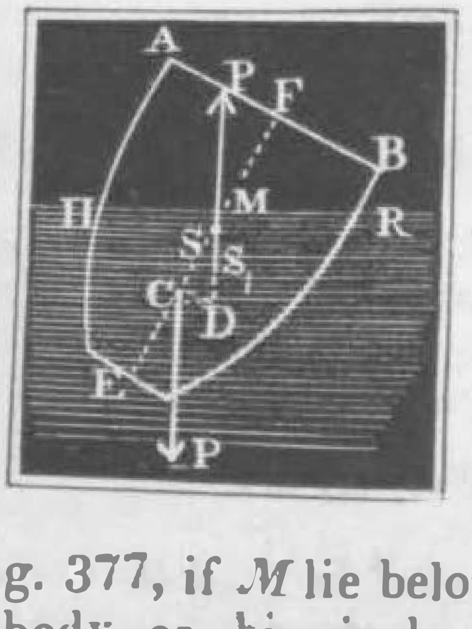
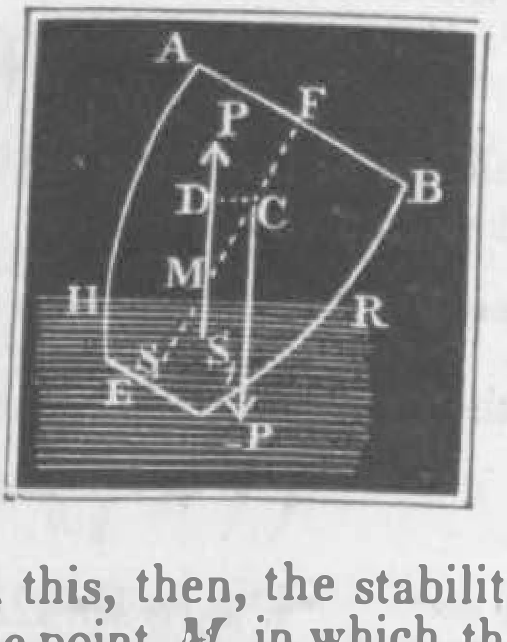


Fig. 377.



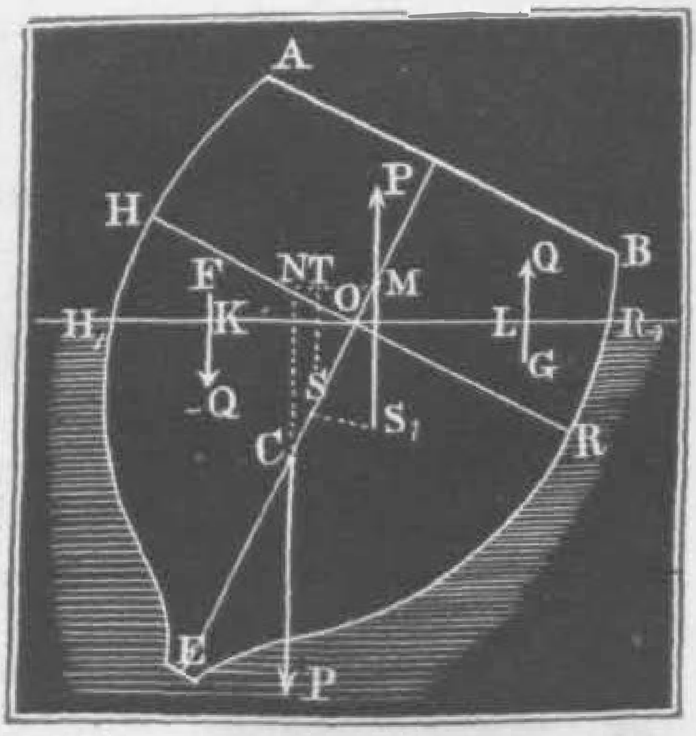
as in Fig. 377, if M lie below C. From this, then, the stability of a floating body, or ship, is dependent on the point M, in which the vertical through the centre of gravity S_1 of the displaced water intersects the plane of symmetry. This point is called the *metacentre.*

It follows, therefore, from this that a ship or other body floats with stability if its metacentre lies above the centre of gravity of the ship, and without stability if it lies below, lastly if the two points coincide, it is in a state of indifferent equilibrium.

The horizontal distance CD of the metacentre M from the centre of gravity C of the ship, is the arm of the force of the couple constituted of P, and $G = -P$, and hence the moment of the last is the measure of its stability $= P\,.\,\overline{CD}$. If we represent the distance CM by c, and the angle of revolution SMS_1 of the ship, or of the plane of its axis, by $\phi°$, we obtain for the measure of stability $S = Pc \sin \phi$; and this is, therefore, the greater, the greater the weight, the greater the distance of the metacentre from the centre of gravity of the ship, and the greater the angle of inclination of this last.

§ 289. In the last formula, $S = P\,c\,sin.\,\phi$, the stability of the ship depends principally on the distance of the metacentre from the centre of gravity of the ship; it is hence of importance to obtain a formula for the determination of this distance.

Fig. 378.



In the transit of the ship ABE, Fig. 378, from the upright into the inclined position, the centre of gravity S advances to S_1, the space HOH_1 is drawn out of the water, and that of ROR_1 sinks below, and the buoyancy on the one side is thereby diminished by a force Q acting at the centre of gravity F of the space HOH_1, and on the other side increased by an equal force Q applied at the centre of gravity G of the space ROR_1. Therefore, the buoyancy P applied at S_1 replaces the buoyancy P, originally applied at S, and the couple $(Q,-Q)$, or what comes to the same thing, an opposite force applied at S_1 keeps in equilibrium a force $-P$ applied at S together with a couple $(Q,-Q)$, or more simply, the couple $(P,-P)$ is in equilibrium with the couple $(Q,-Q)$. If now the transverse section $HER = H_1ER_1$ of the part of the ship immersed $= F$, and the section $HOH_1 = ROR_1$ of the space by which the ship is drawn up on the one side, and down on the other $= F_1$; if, further, the horizontal distance KL of the centre of gravity of these spaces $= a$, and that of MT of the centres of gravity S and S_1, or the horizontal projection of the space SS_1 which S describes during the rolling $= s$, we have then from the conditions of equilibrium of the two couples:

$$Fs = F_1 a, \text{ hence } s = \frac{F_1}{F} a \text{ and } SM = \frac{MT}{sin.\,\phi} = \frac{s}{sin.\,\phi} = \frac{F_1 a}{F sin.\,\phi}.$$

The line $CM = c$, appearing as factor in the measure of the stability is $= CS + SM$; hence, if further, we represent the distance CS

of the centre of gravity C of the ship from the centre of gravity of the displaced water by e, we obtain the measure of the stability

$$S = Pc \, sin. \, \phi = P \left(\frac{F_1 a}{F} + e \, sin. \, \phi \right).$$

If the angle of revolution be small, the transverse sections HOH_1 and ROR_1 may be regarded as equally small triangles; if we represent the breadth $HR = H_1R_1$ of the ship at the place of immersion by b, we may then put

$$F_1 = \tfrac{1}{2} \,.\, \tfrac{1}{2} b \,.\, \tfrac{1}{2} b \phi = \tfrac{1}{8} b^2 \phi, \text{ and } KL = a = 2 \,.\, \tfrac{2}{3} \frac{b}{2} = \tfrac{2}{3} b,$$

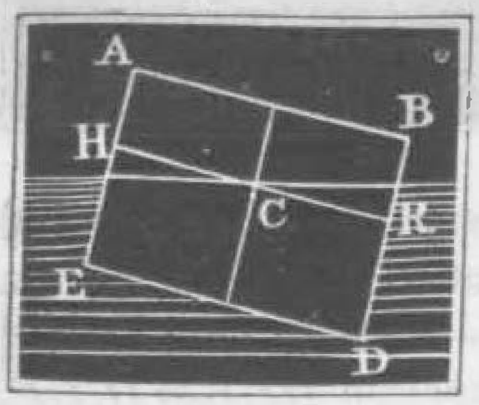
as also $sin. \, \phi = \phi$, from which the stability is:

$$S = P \left(\tfrac{1}{12} \frac{b^3 \phi}{F} + e \phi \right) = \left(\frac{b^3}{12 \, F} + e \right) P \phi.$$

If the centre of gravity C of the ship coincides with the centre of gravity S of the displaced water, we then have $e = 0$, hence: $S = \dfrac{b^3}{12 \, F} \,.\, P \phi$, and if the centre of gravity of the ship lies below that of the displaced water, we then have e negative; hence $S = \left(\dfrac{b^3}{12 \, F} - e \right) P \phi$. It follows also that the stability of a ship is nothing, if e be negative and at the same time $e = \dfrac{b^3}{12 \, F}$.

It is seen from the results obtained, that the stability comes out greater, the broader the ship is, and the lower its centre of gravity lies.

Fig. 379.



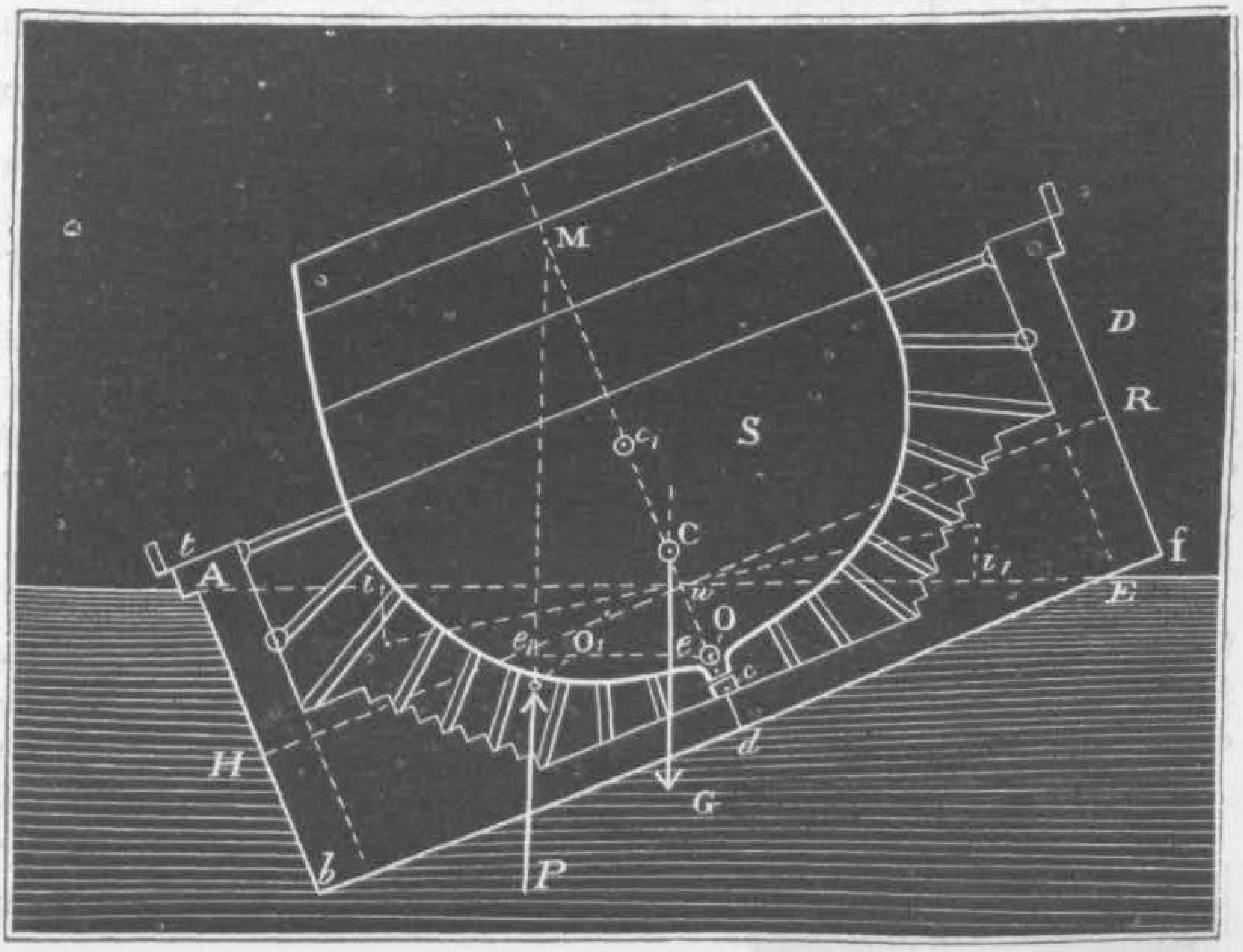
Example. A rectangular figure AD, Fig. 379, of the breadth $AB = b$, height $AE = h$, and depth of immersion $EH = y$, $F = by$, and $e = -\dfrac{h-y}{2}$; hence, the amount of stability is: $S = P\phi \left(\dfrac{b^3}{12 \, by} - \dfrac{h}{2} + \dfrac{y}{2} \right)$, or if the specific gravity of the mass of the body be put $= \varepsilon$,

$$S = P \phi \left(\frac{b^2}{12 \, h\varepsilon} - \frac{h}{2} (1 - \varepsilon) \right).$$

Hence, the stability ceases if $b^2 = 6 \, h^2 \varepsilon \, (1 - \varepsilon)$, *i. e.*, if $\dfrac{b}{h} = \sqrt{6 \, \varepsilon \, (1 - \varepsilon)}$. For $\varepsilon = \tfrac{1}{2}$, $\dfrac{b}{h} = \sqrt{\tfrac{6}{2} \cdot \tfrac{1}{2}} = \sqrt{\tfrac{3}{2}} = 1{,}225$; if, therefore, the breadth is not 1,225 of the height, the body will float without any stability.

[The principles explained in this section apply not only to the construction and use of vessels of every description, and to the ballasting and lading of ships themselves, but likewise to the loading of floating docks with vessels, including their cargoes or armaments. The form of the floating mass, and the position of its centre of gravity, together with its absolute weight, must be taken into account, as well as the density of the liquid in which it floats. Thus, a floating dock D, Fig. 379_1, in the form of a rectangular prism, capable of being closed at the ends after having received the ship S, and of being freed from water, will be subject to exact calculation, if its weight and centre of gravity be known, together with the weight of the vessel and its centre of gravity. *Example.*—Admitting that *D* is 90 feet wide, 36 feet

Fig. 379_1.



high from *b* to *t*, and 250 feet in length—that its weight, including ballast, is 3860 tons, and that its centre of gravity *c* is four feet above the bottom *d*; also that the ship *S*, weighing 5200 tons, has been received and securely shoared in place, having its centre of gravity c_1 25 feet vertically above the bottom of the keel, supposed to be 1 foot above *c*, then the common centre of gravity *C* is $\dfrac{26 \times 5200}{9060} = 14{,}92$ ft. above the point *c*, and 18,92 feet above the bottom of the dock. The total displacement in sea water (64 lbs. per cubic foot) will be 337500 cubic feet; and, consequently, in a state of repose, it will sink to the depth of $bH = \dfrac{337500}{250{,}90} = 15$ feet. *HR* will be the "load line," and the common centre of gravity of ship and dock *C* will be 3,92 feet above it. Should any force acting in a horizontal direction careen the dock so as to make the angle $AwH = \phi = 22° 25'$, depressing the side *AH*, 18 feet, what force must be applied 25 above the water line to keep the dock in this position, *i. e.* what is now the *restoring power*?

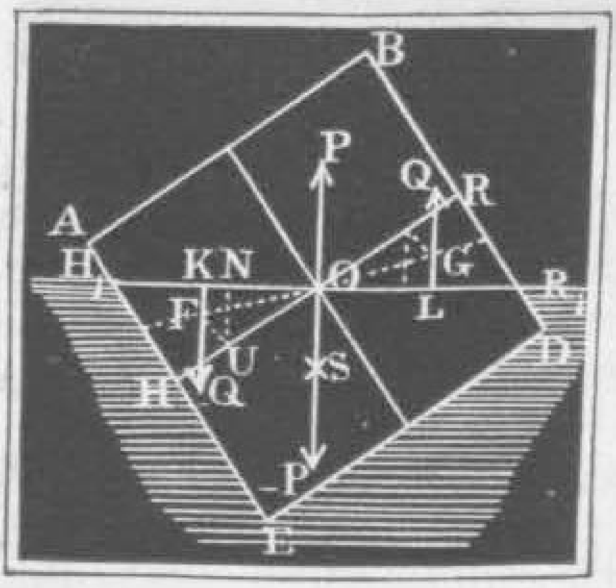
The centre of gravity of the original *quadrangular* prism of displacement *HRfb* is at *O*, that of the new *triangular* prism *EAb* is at O_1. The weight of ship and dock *C* acting at the point *C* is 9060 tons, which is also the force *P* of the prism of water acting upward at

O_1—tending to restore the position of the dock. The original centre of displacement O is $18{,}92 - 7{,}5 = 11{,}42$ feet below the centre of gravity of ship and dock.

The area of the immersed section which is *transferred* by the careening from one side to the other is 401,5 sq. feet, and the distance transferred $ii_1 = 58{,}5$ feet, hence $\frac{401{,}5 \,.\, 58{,}5}{90 \,.\, 15} = 17{,}4 = e_1 O =$ the horizontal distance, the centre of the whole displacement has been removed by the inclination supposed; and $sin.\ 22^\circ\, 25' : 17{,}4 = rad. : 45{,}63$ ft. = height of the metacentre above the original centre of buoyancy O. Again, putting $r_1 = 11{,}42$, we have $r_1 : sin.\ \phi = 11{,}42 \,.\, 4{,}35$ feet; hence, ee_1, the "equilibrating lever," or distance apart of CG and PO_1 is $17{,}4 - 4{,}35 = 13{,}05$ feet, and the statical moment is, therefore, $9060 \,.\, 13{,}05 = 116058{,}6$ ft.–tons; which, for a distance of 25 feet from the centre of oscillation, C gives a stability or restoring power of 4642 tons.]

§ 290. *Oblique Floatation.*—The formula $S = P\left(\frac{F_1 a}{F} \pm e\ sin.\ \phi\right)$ for the stability of a floating body may be also applied to find the different positions of floating bodies, for if we put $S = 0$ we obtain the equation for a second position of equilibrium, whose solution leads to the determination of the corresponding angle of inclination. The equation, therefore, $\frac{F_1 a}{F} \pm e\ sin.\ \phi = 0$, must be solved with respect to ϕ.

Fig. 380.



The transverse section of a parallelopiped AD, Fig. 380, is $F = HRDE = H_1R_1DE = by$, if b be the breadth $AB = HR$, and y the perpendicular depth $EH = DR$; further, the transverse section $F_1 = HOH_1 = ROR_1$ as a rectangular triangle with the leg $OH = OR = \frac{1}{2}\, b$, and the leg:

$$HH_1 = RR_1 = \tfrac{1}{2}\, b\ tang.\ \phi,\ F_1 = \tfrac{1}{8}\, b^2\ tang.\ \phi.$$

If, further, the centre of gravity F is distant from the base $FU = \frac{1}{3}\, HH_1 = \frac{1}{6}\, b\ tang.\ \phi$, and if from O about $OU = \frac{2}{3}\, OH = \frac{1}{3}\, b$, it follows that the horizontal distance of the centre of gravity F from the middle O, $= OK = ON + NK = OU\ cos.\ \phi + FU\ sin.\ \phi = \frac{1}{3}\, b\ cos.\ \phi + \frac{1}{6}\, b\ tang.\ \phi\ sin.\ \phi$, and the arm:

$$a = KL = 2\, OK = \tfrac{2}{3}\, b\ cos.\ \phi + \tfrac{1}{3}\, b\, \frac{sin.\ \phi^2}{cos.\ \phi}.$$

According to this the equation for the oblique position of equilibrium is:

$$\frac{\frac{1}{8}\, b^2\ tang.\ \phi\ (\frac{2}{3}\, b\ cos.\ \phi^2 + \frac{1}{3}\, b\ sin.\ \phi^2)}{by\ cos.\ \phi} - e\ sin.\ \phi = 0,$$

or, $\frac{sin.\ \phi}{cos.\ \phi} = tang.\ \phi$ being substituted,

$$\sin.\,\phi\,[(\tfrac{1}{12} + \tfrac{1}{24}\,tang.\,\phi^2)\,b^2 - ey] = 0;$$
which equation will be satisfied by:

$$\sin.\,\phi = 0 \text{ and by } tang.\,\phi = \sqrt{2}\sqrt{\frac{12\,e\,y}{b^2} - 1}.$$

The first equation, when $\phi = 0$, corresponds to upright, and the second to oblique floatation. The possibility of the latter requires that $\frac{ey}{b^2} > \frac{1}{12}$. If now h be the height of the parallelopiped, and ϵ its specific gravity, we then have:

$$y = \epsilon\,h \text{ and } e = \frac{h-y}{2} = (1-\epsilon)\frac{h}{2}, \text{ hence it follows that}$$

$$tang.\,\phi = \sqrt{2}\sqrt{\frac{6\,\epsilon\,(1-\epsilon)\,h^2}{b^2} - 1},$$

and the equation of condition of oblique floatation is:

$$\frac{h}{b} > \sqrt{\frac{1}{6\,\epsilon\,(1-\epsilon)}}.$$

Examples.—1. If the floating parallelopiped is as high as it is broad, and has a specific gravity $\epsilon = \frac{1}{2}$, then the *tang.* ϕ is $= \sqrt{2}\sqrt{3\,.\,\frac{1}{2} - 1} = \sqrt{3 - 2} = 1$; hence, $\phi = 45^\circ$.

2. If the height $h = 0{,}9$ of the breadth b, and the specific gravity $\frac{1}{2}$, we have then *tang.* $\phi = \sqrt{3\,.\,0{,}81 - 2} = \sqrt{0{,}43} = 0{,}6557$; hence, $\phi = 33^\circ\ 15'$.

§ 291. *Specific Gravity.*—The law of buoyancy of water may be applied to the determination of the density, or the specific gravity of bodies. From § 284, the upward pressure of water is equal to the weight of liquid displaced; hence if V is the volume of a body and γ_1 the density of the liquid, we then have the buoyancy $P = V\gamma_1$. If now γ_2 be the density of the mass of the bodies, we then have the weight of the body $G = V\gamma_2$; hence the ratio of the densities $\frac{\gamma_2}{\gamma_1} = \frac{G}{P}$, i. e. *the density of the body immersed is to the density of the fluid as the absolute weight of the body to the buoyancy or loss of weight by immersion.*

Therefore, $\gamma_2 = \frac{G}{P}\gamma_1$, and $\gamma_1 = \frac{P}{G}\gamma_2$; or if γ be the density of water, ϵ_1 the specific gravity of the liquid, and ϵ_2 that of the body, then will $\gamma_1 = \epsilon_1\gamma$, and $\gamma_2 = \epsilon_2\gamma$, $\epsilon_2 = \frac{G}{P}\epsilon_1$, and $\epsilon_1 = \frac{P}{G}\epsilon_2$. If, therefore, the weight of a body or its loss of weight by immersion is known, then the density or the specific gravity of the mass of a body may be found from the density or specific gravity of the liquid, and inversely, the density or specific gravity of the first, from the density or specific gravity of the last.

If the fluid in which the solid body is weighed is water, we then have $\epsilon_1 = 1$, and $\gamma_1 = \gamma = 1000$ kilogrammes, or 62,5 lbs., according as we take the cubic metre or cubic foot for unit of volume, hence for this case the density of the body is:

$$\gamma_2 = \frac{G}{P}\gamma = \frac{\text{absolute weight}}{\text{loss of weight}} \text{ times the density of water,}$$

and the specific gravity ε

$$\varepsilon_2 = \frac{G}{P} = \frac{\text{absolute weight}}{\text{loss of weight}}.$$

To estimate the buoyancy or loss of weight, as well as to determine the weight C, we make use of an ordinary balance, only that below one of the scale pans of this balance there is appended a hook, to which the body may be suspended by a fine thread or fine wire, whilst it dips into the water contained in a vessel underneath. A balance arranged for the weighing of bodies in water is commonly called a *hydrostatic balance.*

If the body whose specific gravity we wish to determine is lighter than water, we may connect it mechanically with another heavy body, so as to make it sink. If this heavy body loses the weight P_2, and the system the weight P_1, the loss of weight of the lighter body: $P = P_1 - P_2$, now if G represents the loss of weight of the lighter body, we have then its specific gravity:

$$\varepsilon_2 = \frac{G}{P} = \frac{G}{P_1 - P_2}.$$

If the specific gravity of a mechanical combination, or a composition of two bodies, and the specific gravities of their constituents ε_1 and ε_2 are known, from the weight of the whole, the weights G_1 and G_2 may be estimated. In every case $G_1 + G_2 = G$, and also the volume $\frac{G_1}{\varepsilon_1\gamma}$ + volume $\frac{G_2}{\varepsilon_2\gamma}$ = volume $\frac{G}{\varepsilon\gamma}$, therefore:

$\frac{G_1}{\varepsilon_1} + \frac{G_2}{\varepsilon_2} = \frac{G}{\varepsilon}$. By combining these equations we have:

$$G_1 = G\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_2}\right) \div \left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2}\right), \text{ and}$$

$$G_2 = G\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_1}\right) \div \left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1}\right).$$

Examples.—1. If a piece of limestone, weighing 310 grains, becomes 121,5 grains lighter when under water, its specific gravity is $\varepsilon = \frac{310}{121,5} = 2,55$.—2. To find the specific gravity of a piece of oak, round which a piece of lead has been wrapped, and which has lost by being weighed in water 10,5 grains; if now the wood itself weighed 426,5 grains, and the system under water was 484,5 grains lighter than in the air, the specific gravity of the mass of wood would be:

$$\varepsilon = \frac{426,5}{484,5 - 10,5} = \frac{426,5}{474} = 0,9.$$

3. An iron vessel, completely filled with quicksilver and perfectly closed, has a net weight of 500 lbs., and has lost 40 lbs. in the water; if now the specific gravity of cast-iron = 7,2, and that of quicksilver is 13,6, the weight of the empty vessel is:

$$G_1 = 500\left(\frac{40}{500} - \frac{1}{13,6}\right) \div \left(\frac{1}{7,2} - \frac{1}{13,6}\right)$$

$$= 500\,(0,08 - 0,07353) \div (0,1388 - 0,0735)$$

$$= \frac{500\,.\,0,00647}{0,0653} = \frac{3235}{65,3} = 49,5 \text{ lbs.},$$

and the weight of the enclosed quicksilver:

$$G_2 = 500\,(0{,}08 - 0{,}1388) : (0{,}07353 - 0{,}1388) = \frac{500 \,.\, 0{,}0588}{0{,}0653}$$

$$= \frac{2940}{6{,}53} = 450{,}2 \text{ lbs.}$$

Remark 1. For the determination of the specific gravities of liquids, meal, corn, &c., the mere weighing in open air is sufficient, because we may give to the bodies any volume at will, by filling vessels with them. If an empty bottle weighs $= G$, and the same filled with water G_1, and the weight G_2 if it contain any other substance, we shall then have the specific gravities of masses of these : $\varepsilon = \frac{G_2 - G}{G_1 - G}$. For example, to find the specific gravity of rye (not rye grains), a bottle is filled with the grains, and after much shaking, then weighed. After deduction of the weight of the empty bottle, the weight of the rye was $= 120{,}75$ grms., and the weight of an equal quantity of water $= 155{,}65$; the weight of the rye is accordingly $= \frac{120{,}75}{155{,}65} = 0{,}776$; and, therefore, 1 cubic foot of this grain weighs

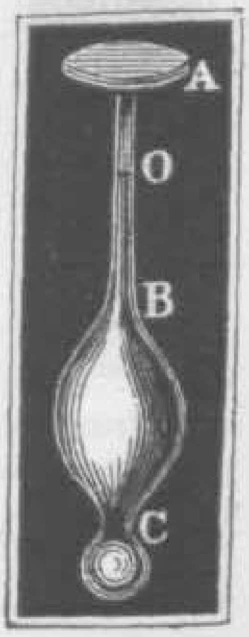
$$= 0{,}776 \,.\, 62{,}5 = 48{,}5 \text{ lbs.}$$

Remark 2. The problem solved by Archimedes of finding the ratio of the constituents from the specific gravity of a mixture, and from the specific gravity of its constituents, admits only of a limited application to chemical combinations, metallic alloys, &c., because a contraction or expansion of the mass generally takes place, so that the volume of the mixture is no longer equal to the sum of the volumes of the constituents.

Remark 3. The further extension of this subject, namely, its application to the measurement of volume, &c., belongs to physics and chemistry.

§ 292. *Areometer.*—Areometers are principally used to determine the density of liquids. These instruments are hollow bodies, formed about a symmetrical axis, whose centres of gravity lie very low, and by floating perpendicularly in liquids, give their density. They are made of glass, brass, &c., and are called, according to the various purposes for which they are intended, hydrostatic balances, salzometers, hydrometers, alcoholometers, &c. There are two kinds of hydrometers, viz., the weight and the scale hydrometer. The first are often used for the determination of the weights, as was the specific gravity of solid bodies.

Fig. 381.



1. If V be the volume of the portion of a hydrometer ABC, Fig. 381, floating freely, and immersed up to a certain mark O in the water, G the weight of the whole balance, P the weight placed upon the plate while floating in the water, whose density may be $= \gamma$, and P_1 the weight required to be put on to make it float in any other liquid of the density γ_1, we shall then have

$$V\gamma = P + G \text{ and } V\gamma_1 = P_1 + G\text{; hence,}$$

$$\frac{\gamma_1}{\gamma} = \frac{P_1 + G}{P + G}.$$

2. If P be the weight which must be put upon the plate to make the hydrometer ABC, Fig. 382, sink up to a mark O, and P_1 the weight which must be put upon A, together with the body to be weighed, to obtain the same immersion, we shall then have simply the weight of this body $G_1 = P - P_1$. But if P_1 must be augmented by P_2, when the body to be weighed is put into the cup D under the

Fig. 382.

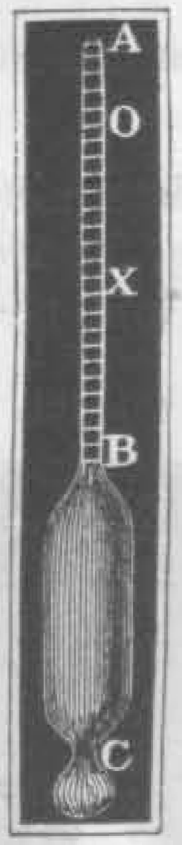
surface, to preserve the depth of immersion unchanged, P_2 will then be the buoyancy, and hence the specific gravity of the body:

$$s = \frac{G_1}{P_2} = \frac{P - P_1}{P_2}.$$

Those hydrometers which have a cup suspended below for the determination of the specific gravities of solid bodies, minerals for instance, are called Nicholson's hydrometers.

3. Let the weight of a hydrometer ABC, Fig. 383, $= G$, and the volume immersed, if this balance floats in water, $= V$, then $G = V\gamma$. If the balance rise by $OX = x$, when immersed in a heavier liquid, for the transverse section F of the stem, the volume immersed is $= V - FX$, and hence $G = (V - Fx)\,\gamma_1$; the two formulæ, divided by one another, give the density of the liquid:

Fig. 383.



$$\gamma_1 = \frac{V}{V - Fx}\,\gamma = \gamma \div \left(1 - \frac{F}{V}x\right) = \gamma \div (1 - \mu x).$$

If the liquid in which the hydrometer is immersed, be lighter than the water, it will sink in it to a depth x, for which reason, $G = (V + Fx)\,\gamma$, and hence we must put $\gamma_1 = \gamma \div (1 + \mu x)$.

To find the co-efficient $\mu = \frac{F}{V}$, the balance is loaded with a weight P of quicksilver, which is poured in and takes the lowest position, so that while floating in water, a considerable length l of the stem to which the scale is applied, sinks lower down. If now we put $P = Fl\gamma$, we shall then obtain:

$$\mu = \frac{F}{V} = \frac{P}{Vl\gamma} = \frac{P}{Gl}.$$

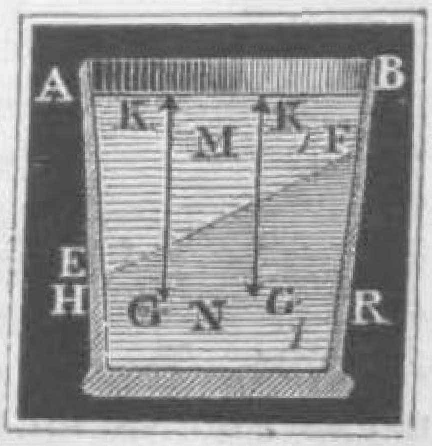
Examples.—1. If a Nicholsoo's hydrometer weighs 65 grains, 13,5 grains must be taken off the plate, that it may sink to the same depth in alcohol as it does in water; the specific gravity of alcohol is $\sigma = \frac{65 - 13.5}{65} = 1 - 0{,}208 = 0{,}792$.—2. The normal weight of a Nicholson's balance is 1500 grains, *i.e.* 1500 grains require to be put on to make the instrument sink to 0; from this 1030 grains must be taken by the weighing of a piece of brass placed upon the upper plate, and 121,5 to be added if this body is placed on the lower plate. The absolute weight of this piece of brass is therefore = 1030 grains, and its specific gravity $\sigma = \frac{1030}{121{,}5} = 8{,}47$.—3. A scale areometer, of 1162 grains weight, after having been lightened by 465 grains, rises 6 inches, and has therefore the co-efficient $\mu = \frac{465}{1162.6} = \frac{465}{6772} = 0{,}00686$. After complete filling and restoration of the weight of 1162 grains, it ascends, when floating in a saline solution, $2\frac{5}{12}$ inches; hence, the specific gravity of this is:

$$= 1 \div \left(1 - 0{,}00686 \times \frac{29}{12}\right) = 1 \div 0{,}983 = 1{,}02.$$

Remark. The further extension of this subject belongs to physics, chemistry, and technology.

§ 293. *Liquids of Different Densities.*—If several liquids, of different densities, are in the same vessel, without their exerting any chemical action upon each other, from the ready displacement of their particles, they arrange themselves above each other, according to their specific gravities, viz. the densest below, then the less dense, and then the lightest. The limiting surfaces are also in a state of equilibrium, as likewise the free surface horizontal; for as long as the surface of limitation EF between the masses M and N, Fig. 384, is inclined, columns of fluid, of different densities, like GK, G_1K_1, rest on the horizontal stratum HR, and hence the pressure on this stratum will not be everywhere the same; and lastly, no equilibrium will subsist.

Fig. 384.



In communicating tubes AB and CD, Fig. 385, the liquids arrange themselves one above the other, according to their densities, only their surfaces A and D do not lie in one and the same level. If F be the area HR of the transverse section of a piston, Fig. 386, in the one branch AB of two communicating tubes, and the height of pressure or

Fig. 385.

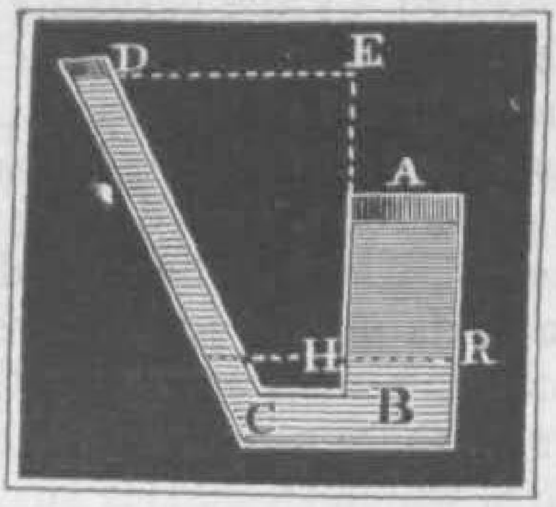
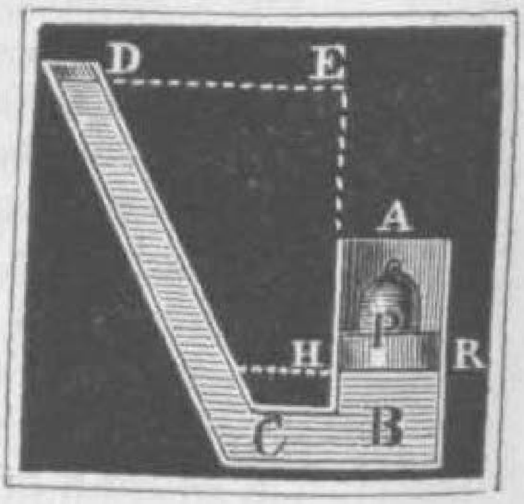


Fig. 386.



the height EH of the surface of the water in the second tube CD above HR, $= h$, we then have the pressure against the surface of the piston $P = F h\gamma$. On the other hand, if we replace the pressure of the piston by a column of liquid AH, Fig. 386, of the height $AH = h_1$ and the density γ_1, we then have $P = Fh_1 \gamma_1$; and equating both expressions, we obtain the equation $h_1 \gamma_1 = h \gamma$ or the proportion $\frac{h_1}{h} = \frac{\gamma}{\gamma_1}$.

Therefore, the heights of pressure in communicating tubes, for the subsistence of equilibrium between two different liquids, or the heights of the columns of liquid measured from the common plane of contact, are inversely as the densities or specific gravities of these liquids.

As mercury is of about 13,6 times the density of water, a column of mercury, in communicating tubes, will hold in equilibrium a column of water of 13,6 times the height.

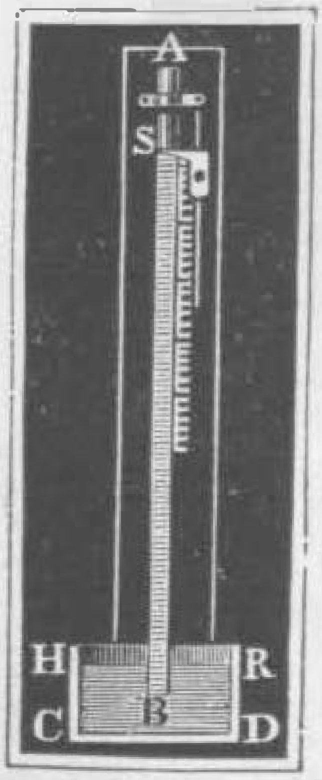
29

CHAPTER III.

ON THE EQUILIBRIUM AND PRESSURE OF AIR.

§ 294. *Tension of Gases.*—The atmospheric air which surrounds us, as well as all kinds of air or gases, possesses, in virtue of the repulsive force of its parts or molecules, a tendency to occupy a greater and greater space; hence, we can obtain a limited mass of air only by confining it in perfectly closed vessels. The force with which gases endeavor to dilate themselves is called their *elasticity*, *tension*, or *expansive force*. It exhibits itself by pressure against the sides of the vessels which enclose it, and so far differs from the elasticity of solids and liquids, that it manifests its action in every condition of density, while the elasticity of the last-mentioned bodies in a certain state of expansion, is nothing. The pressure or tension of air and other gases is measured by the *barometer*, the *manometer*, and the *valve*. The barometer is chiefly used for determining the pressure of the atmosphere. The common, or as it is called, the *cistern barometer*, Fig. 387, consists of a glass tube, closed at one end *A* and open at the other *B*, which, when filled with mercury, is inverted, and its open end immersed in a cistern likewise containing mercury. By the inversion of this instrument, there remains in the tube a column of mercury *BS*, which (§ 393) is sustained in equilibrium by the pressure of the air on the surface of mercury *HR*. The space *AS* above the mercurial column is deprived of air, or a vacuum; hence, there is no pressure on this column from above, for which reason, the height of the mercurial column above the surface of mercury *HR* in the cistern, serves for a measure of the air's pressure.

Fig. 387.



To measure this height with precision and convenience, an accurately divided scale is appended, which runs lengthwise along the tube. A more particular description of the different barometers, and an explanation of their uses, &c., belong to the department of physics.

§ 295. It has been found by the barometer, that for a certain mean state of the atmosphere, and at places very little above the level of the sea, the air's pressure is held in equilibrium by a column of mercury, 76 centimetres, or about 28 Paris inches = 29 Prussian inches = 30 English inches nearly, (29,994 exactly.) As the specific gravity of mercury is nearly 13,6 (13,598), it follows that the pressure of the air is equivalent to the weight of a column of water,

$0{,}76 \cdot 13{,}6 = 10{,}336$ metres $= 31{,}73$ Paris feet $= 32{,}84$ Prussian feet $= \frac{13{,}598 \cdot 29{,}998}{12} = 33{,}988$ English feet.

The tension of the air is very often measured by the pressure it exerts upon a unit of surface. Since a cubic centimetre of mercury weighs 0,0136 kilogrammes, the pressure of the atmosphere, or the weight of a column of mercury 76 centimetres high on a base of 1 centimetre square, $= 0{,}0136 \cdot 76 = 1{,}0336$ kilogrammes, and since a cubic inch of mercury weighs $\frac{66 \cdot 13{,}6}{1728} = 0{,}5194$ Prussian lbs., or $\frac{62{,}5 \cdot 13{,}6}{1728} = 0{,}491$ lbs. English, the mean pressure of the atmosphere is then $= 29 \cdot 0{,}5194 = 15{,}05$ Prussian lbs. on the square inch, $= 2167$ lbs. on the square foot, and in English measure $= 30 \cdot 0{,}491 = 14{,}73$ on the square inch, $= 2131{,}12$ lbs. avd. on the square foot. 14,76 lbs. per square inch is the standard usually adopted.

In mechanics, the mean pressure of the atmosphere is commonly taken as unity, and other expansive forces referred to this and assigned in atmospheric pressures, or *atmospheres*. Hence, to a pressure of n atmospheres corresponds a mercurial column of $30 \cdot n$ inches, or a weight of 14,73 lbs. on each square inch; and inversely, to a mercurial column of h inches corresponds a tension of $\frac{h}{28} = 0{,}03571\, h$ or $= \frac{h}{30} = 0{,}0333\, h$ atmospheres, and to a pressure of p lbs. on the square inch, a tension of

$$\frac{h}{15{,}05} = 0{,}0644\, p \text{ or } \frac{h}{14{,}73} = 0{,}0678 \text{ atmospheres.}$$

The equation $\frac{h}{28} = \frac{p}{15{,}05}$ or $\frac{h}{30} = \frac{p}{14{,}73}$ give the formulæ of reduction $h = 1{,}8604\, p$ inches and $p = 0{,}5375\, h$ lbs., or $h = 2{,}036\, p$ inches, and $p = 0{,}491\, h$ lbs. English. For a tension h inches $= p$ lbs., the pressure against a plane surface of F square inches: $P = Fp = 0{,}491\, Fh$ lbs. English, or $= 0{,}5375\, Fh$ lbs. Prussian.

Examples.—1. If the water in a water-pressure engine stands 250 feet above the surface of the piston, the pressure against the surface will then be $= \frac{250}{33{,}988} = 7.35$ atmospheres.—2. If the blast of a cylindrical bellows has a tension of 1,2 atmospheres, its pressure on every square inch $= 1{,}2 \cdot 14{,}73 = 17{,}676$ lbs., and on the surface of the piston of 50 inches ~~diameter~~ $= \frac{\pi \cdot 50^2}{4} \cdot 17{,}676 = 34707$ lbs. As the atmosphere exerts a counter-pressure $\frac{\pi \cdot 50^2}{4} \cdot 14{,}73 = 28922{,}3$ lbs., the pressure on the piston is $= 34695 - 28922{,}3 = 5784{,}7$ lbs.

§ 296. *Manometer.*—To find the tension of gases or vapors enclosed in vessels, instruments similar to the barometer are made use of, which are called *manometers*. These instruments are filled with mercury or water, and are either open or closed; but in the latter case, the upper part is either a vacuum or full of air. The vacuum manometer, Fig. 388, differs little from the ordinary barometer. To measure by this instrument the tension of air in a reservoir, a tube *GK* is fitted in, one end of which *G* passes into the reservoir, and the other *K* projects above the surface of mercury *CE* in the cistern of the instrument. The space *EFHR* above the mercury is hereby put into communication with the air-holder, and the air in it assumes the tension of the air in the holder, and forces into the tube a column of mercury *OS*, which sustains in equilibrium the pressure of the air which is to be measured.

The siphon manometer, *ABC*, Fig. 389, open above, gives the

Fig. 388.

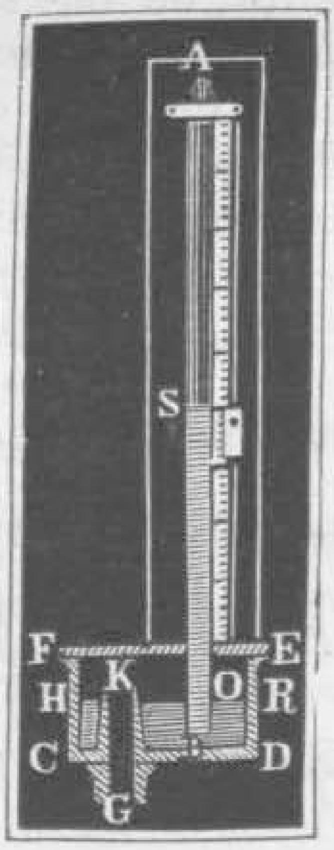


Fig. 389.

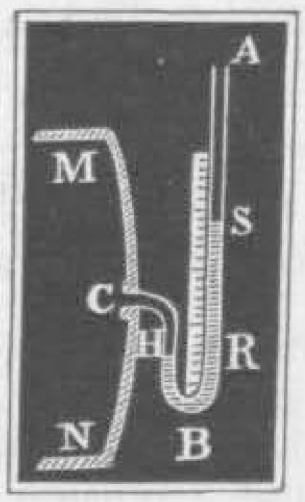
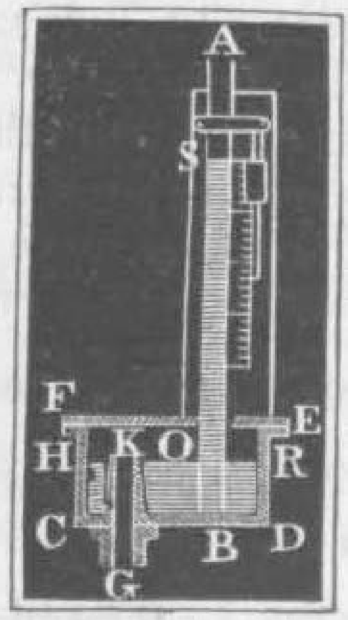


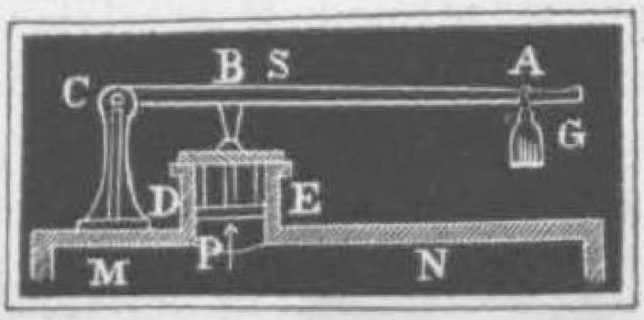
Fig. 390.



excess of tension above the pressure of the atmosphere in the vessel *MN*, because the pressure of the atmosphere on *S*, joined to that of the mercurial column *RS*, is in equilibrium with the tension. If b be the height of the barometer, and h that of the manometer, or the difference of heights *RS* of the surfaces of mercury in both branches of the manometer, we shall then have the tension of the air communicating with the shorter branch measured by the height of a column of mercury: $b_1 = b + h$, or the pressure measured on a square inch $p = 0{,}491\ (b + h)$ lbs.; or if b be the mean height of the barometer, $p = 14{,}73 + 0{,}491\ h$ lbs.

Cistern manometers, Fig. 390, *ABCE*, are more common than siphon manometers. As the air here acts through a greater quantity of mercury or water, as it may be, upon the column of fluid, its oscillations do not so quickly affect the column of fluid, and its measurement, when thus at rest, is rendered both easier and more accurate. For the sake of convenience of measuring by, or reading off from the scale, a float is not unfrequently attached to it, which rests on the mercury, and is connected with an index hand, accompanying the scale by means of a thread passing over a small roller.

Fig. 391.



The expansive force of a gas or vapor enclosed in *MN* may be likewise determined, but with less accuracy, by the help of a valve *DE*, Fig. 391, if the sliding weight is so placed that it is in equilibrium with the pressure of the air or vapor. If $CS = s$ be the distance of the centre of gravity of the lever from the fulcrum *C*, $CA = a$ the arm of the weight, and *Q* the weight of the lever with its valve, we then have the statical moment with which the valve is pressed down by the weight $= Ga + Qs$;* if now the pressure of the gas or vapor from below $= P$, the pressure of the atmosphere from above $= P_1$, and lastly, the arm *CB* of the valve $= d$, we then have the statical moment with which the valve strives to lift itself up $= (P - P_1) d$, and by equating the moments of both:

$$Pd - P_1 d = Ga + Qs, \text{ and } P = P_1 + \frac{Ga + Qs}{d}.$$

If r represent the radius $\frac{1}{2}$ *DE* of the valve, p the internal and p_1 the external tension, measured by the pressure on a square inch, we then have: $P = \pi r^2 p$ and $P_1 = \pi r^2 p_1$; hence, $p = p_1 + \dfrac{Ga + Qs}{\pi r^2 d}$.

Examples.—1. If the height of mercury of a manometer, open above, is 3,5 inches, but that of the barometer 27 inches, the corresponding expansive force is then $h = b + h_1 = 27 + 3{,}5 = 30{,}5$ inches, or $p = 0{,}491\, h = 0{,}491 \cdot 30{,}5 = 14.97$ lbs.—2. If the height of a water-manometer is 21 inches, the expansive force corresponding to this, with the height of the barometer at 27 inches, is:

$$h = 27 + \frac{21}{13{,}6} = 28{,}54 \text{ inches} = 14{,}01 \text{ lbs. English.}$$

3. If the statical moment of an unloaded safety-valve is 10 inch lbs., the statical moment of a 10 lbs. sliding weight $15 \cdot 10 = 150$ inch lbs., the arm of the valve measured, from the valve to the fulcrum, 4 inches, and the radius of the valve 1,5 inches, then the difference of the pressures on both surfaces of the valve is:

$$p - p_1 = \frac{150 + 10}{\pi\, (1{,}5)^2 \cdot 4} = \frac{160}{9\, \pi} = 5{,}66 \text{ lbs.}$$

Were the pressure of the atmosphere $p_1 = 14$ lbs., the tension of the air below the valve would from this amount to $p = 19{,}66$ lbs.

§ 297. *Law of Mariotte.*—The tension of gases increases with their density; the more a certain quantity of air is compressed or condensed, the greater is its tension; and the greater its tension, the more it is allowed to expand or become rarefied, the less does its expansive force exhibit itself. The ratio in which the tension and the density, or the volume of the gases, stand to each other, is expressed by the law discovered by Mariotte, and named after him. This law assumes that *the density of one and the same quantity of air or gas is proportional to its tension;* or, as the spaces which are occupied by one and the same mass are inversely proportional to the densities,

* If the weight of the lever and valve be counterpoised by a weight attached to a cord, passing upwards and over a pulley above *S*, the statical moment is reduced to *Ga*.—Am. Ed.

that the volume of one and the same mass of gas is inversely as its expansive force. Accordingly, if a certain quantity of air becomes compressed to one-half its original volume, its density is therefore doubled, its tension is also as great again as at first; and on the other hand, if a certain quantity of air be expanded to one-third of its original bulk, therefore, its density reduced one-third, its elasticity will be equal to one-third only of its original tension. If atmospheric air, for example, under the piston EF of a cylinder AC, Fig. 392, be supposed to press with 15 lbs. on every square inch, it will press on the piston with a force of 30 lbs., if this piston be pushed to E_1F_1, and the enclosed air compressed to one half its original volume, and this force will amount to 3 . 15 = 45 lbs., if the piston come to E_2F_2, and describes two-thirds of the whole height. If the area of the piston be 1 square foot, the pressure of the atmosphere against it will amount to = 144e 15 = 2160 lbs.; hence, to press down the piston one-half the height of the cylinder, it will require 2160 lbs., and to push it down two-thirds of this height 2 . 2160 = 4320 lbs. to be exerted.

Fig. 392.

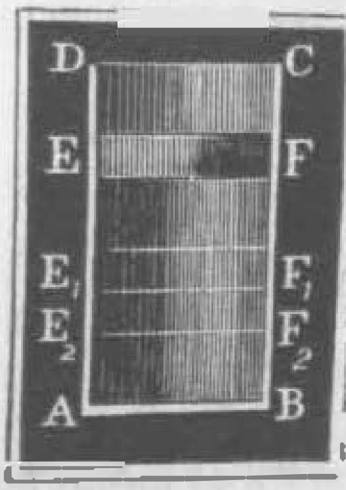
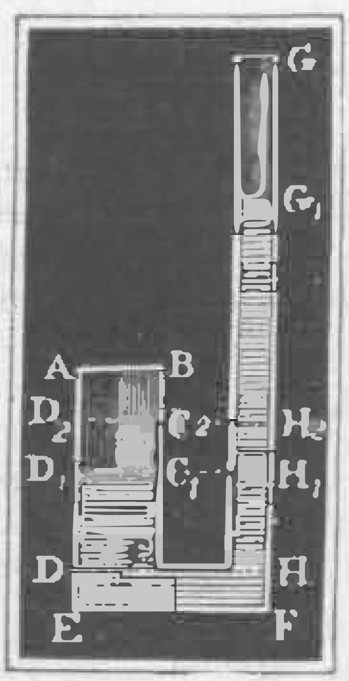


Fig. 393.



The law of Mariotte may be likewise proved by pouring mercury into the tube GH communicating with the air of a cylinder AC, Fig. 393. If a column of air AC be originally enclosed by the quantity of mercury $DEFH$, which has the same tension as the external air, and afterwards be compressed to one-half or one-fourth its volume by the addition of fresh mercury, we shall then find that the distances of the surfaces of G_1H_1, G_2H_2, &c., of mercury are equivalent to the single and treble height of the barometer b, &c., that, therefore, if we add to this the single height, corresponding to the external pressure of the air, the tension will be twice or four times as great as that due to its original volume.

The tensions are h and h_1 or p and p_1, γ and γ_1 the corresponding densities, and V and V_1 the volumes appertaining to one and the same quantity of air, we then have, according to the law laid down:

$$\frac{\gamma}{\gamma_1} = \frac{V_1}{V} = \frac{h}{h_1} = \frac{p}{p_1}; \text{ hence}$$

$$\gamma_1 = \frac{h_1}{h}\gamma = \frac{p_1}{p}\gamma \text{ and } V_1 = \frac{h}{h_1}V = \frac{p}{p_1}V.$$

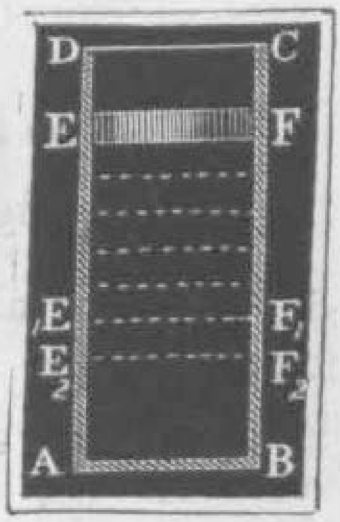
From this the density and also the volume may be reduced from one tension to another.

Examples.—1. If in a blowing machine, the manometer stand at 3 inches, whilst the barometer is at 28 inches, the density of the wind is $= \frac{28 + 3}{28} = \frac{31}{28} = 1{,}107$ times that of the external air.*—2. If a cubic foot of atmospheric air, with the barometer at 28 inches, weighs $\frac{62,5}{770}$ lbs.; with the barometer at 34 inches it will weigh:

$$\frac{62.5}{770} \cdot \frac{34}{28} = \frac{21250}{21560} = 0{,}985 \text{ lbs.}$$

§ 298. The mechanical effect which must be expended to condense a certain quantity of air to a certain degree, and the effects which the air by its expansion will produce, cannot be directly assigned, because the expansive force varies at every moment of condensation or extension; we must therefore endeavor to find a special formula for the calculations of this value. Let us imagine a certain quantity of air AF, enclosed in a cylinder AC, Fig. 394, by a piston EF, and let us inquire what effect must be expended to push forward the piston through a certain space $EE_1 = FF_1$. If the original tension $= p$, and the original height of the capacity of the cylinder $= s_0$, and the tension after describing the space $EF_1 = p_1$, and the residuary volume of air $= s_1$, the proportion $p_1 : p = s_0 : s_1$ then holds true, and gives $p_1 = \frac{s_0}{s_1} p$.

Fig. 394.



While describing a very small space $E_1E_2 = x$, the tension p_1 may be regarded as invariable, and hence the mechanical effect to be expended is $= Ap_1x = \frac{Aps_0x}{s_1}$, when A represents the surface of the piston.

It follows from the properties of logarithms, that a very small magnitude $y = hyp.\ log.\ (1 + y) = 2{,}3026\ Log.\ (1 + y)$, if $hyp.\ log.$ represents the hyperbolic, and $Log.$ the common logarithms; we may consequently put

$$Aps_0 \frac{x}{s_1} = Aps_0\ hyp.\ log. \left(1 + \frac{x}{s_1}\right)$$

$$= 2{,}3026\ Aps_0\ log. \left(1 + \frac{x}{s_1}\right)$$

But now:

$$hyp.\ log. \left(1 + \frac{x}{s_1}\right) = hyp.\ log. \left(\frac{s_1 + x}{s_1}\right) = hyp.\ log.\ (s_1 + x) - hyp.\ log.\ s_1;$$

hence the elementary mechanical effect ise

$$= Aps_0\ [hyp.\ log.\ (s_1 + x) - hyp.\ log.\ s_1].$$

Let us imagine the whole space EE_1 to be made up of very small parts, such as x, and therefore put $EE_1 = nx$, we shall find the mechanical effects corresponding to all these parts, if in the last formula we substitute for

$s_1, s_1 + x, s_1 + 2\,x, s_1 + 3\,x, \ldots$ to $s_1 + (n-1)\,x$, and for
$s_1 + x, s_1 + 2\,x, s_1 + 3\,x$, &c., to $s_1 + nx$, or s_0,

* For comparisons with the manometer, the division of a barometer ought not to be into either inches or metres, but into 1000th parts of 1 *atmosphere.*—Am. Ed.

and by summation the whole expenditure of mechanical effect in describing the space $s_0 — s_1$:

$$L = \mathfrak{A}ps_0 \begin{cases} hyp.\ log.\ (s_1 + x) — hyp.\ log.\ s_1 \\ hyp.\ log.\ (s_1 + 2x) — hyp.\ log.\ (s_1 + x) \\ hyp.\ log.\ (s_1 + 3x) — hyp.\ log.\ (s_1 + 2x) \\ \quad\cdot \qquad\qquad \cdot \\ \quad\cdot \qquad\qquad \cdot \\ \quad\cdot \qquad\qquad \cdot \\ hyp.\ log.\ (s_1 + nx) — hyp.\ log.\ [s_1 + (n-1)x] \end{cases}$$

$$= \mathfrak{A}ps_0\ [hyp.\ log.\ (s_1 + nx) — hyp.\ log.\ s_1]$$

$$= \mathfrak{A}ps_0\ (hyp.\ log.\ s_0 — hyp.\ log.\ s_1) = \mathfrak{A}ps_0\ hyp.\ log.\ \left(\frac{s_0}{s_1}\right),$$

since one member in the one line always cancels one member in the following one.

Since $\frac{s_0}{s_1} = \frac{h_1}{h} = \frac{p_1}{p}$, this mechanical effect may be put:

$$L = \mathfrak{A}ps_0\ hyp.\ log.\left(\frac{h_1}{h}\right) = \mathfrak{A}ps_0\ hyp.\ log.\left(\frac{p_1}{p}\right).$$

If we put the space described by the piston $s_0 — s_1 = s$, we shall hence find that the mean force of the piston p condensing the air is in the proportion

$$\frac{h_1}{h} = \frac{p_1}{p},\ P = \frac{L}{s} = \mathfrak{A}p\ \frac{s_0}{s}\, hyp.\ log.\left(\frac{p_1}{p}\right).$$

Let $\mathfrak{A} = 1$ (square foot) and $s_0 = 1$ (foot), we obtain the mechanical effect produced

$$L = p\ hyp.\ log.\left(\frac{p_1}{p}\right) = 2{,}3026\ p\ log.\left(\frac{p_1}{p}\right).$$

This formula gives the mechanical effect which must be expended to convert a unit or cubic foot of air of a lower pressure or tension p into a higher tension p_1, and to reduce it thereby to the volume $\left(\frac{p}{p_1}\right)$ cubic feet. On the other hand:

$$L = p_1\ hyp.\ log.\left(\frac{p_1}{p}\right) = 2{,}3026\ p_1\ log.\left(\frac{p_1}{p}\right)$$

expresses the effect which a unit of volume of gas gives out or produces when it passes from a higher pressure p_1 to a lower p.

To reduce by condensation a mass of air of the volume V, and tension p to the volume V_1, and the tension $p_1 = \frac{V}{V_1}\, p$, the mechanical effect requisite to be expended is $V p\ hyp.\ log.\ \left(\frac{V}{V_1}\right)$, and when, inversely, the volume V_1 at a tension p_1 is converted by expansion into the volume V, and into the tension $p = \frac{V_1}{V}\, p_1$, the effect:

$$V\ p\ hyp.\ log.\ \left(\frac{V}{V_1}\right) = V_1 p_1\ hyp.\ log.\ \left(\frac{V}{V_1}\right)$$

will be given out.

Examples.—1. If a blast converts 10 cubic feet of air per second, of 28 inches tension, into air of 30 inches tension, the effect to be expended upon this for every second will be $= 17280e.\ 0{,}491e.\ 28$ *hyp. log.* $\left(\frac{30}{28}\right)e= 237565$ (*hyp. log.* 15 — *hyp. log.* $14) = 237565$ $(2{,}708050 - 2{,}639057) = 237565\ .\ 0{,}068993 = 16390$ inch lbs. $= 1365.8$ ft. lbs.— 2. If a mass of vapor in a steam engine below the surface of a piston $A = \pi 8^2 = 201$ square inches, stands 15 inches high, and with a tension of three atmospheres, pushes up theepiston 25 inches, the mechanical effect evolved, and which is expended on the piston, is:

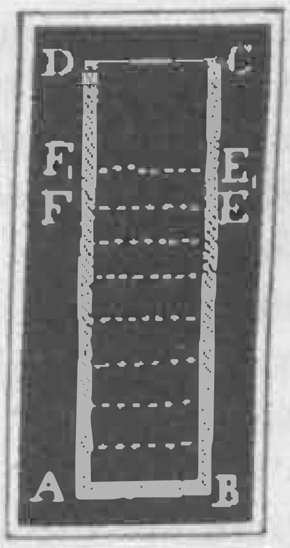
$$L = 201e\ 3\ .\ 14{,}73e\ 15\ \textit{hyp. log}\ \left(\frac{15+25}{15}\right) = 133232\ \textit{hyp. log.}\ \frac{8}{3}$$

$= 133232\ .\ 0{,}98083 = 130567$ inch lbs. $= 10881$ feet lbs., and the mean force of the piston, without regard to its friction and the counter pressure, is:

$$= \frac{130567}{25} = 5222 \text{ lbs.}$$

§ 299. *Strata of Air.*—Air enclosed in a vessel is at different depths of different density and tension, for the upper strata press together the lower on which they rest, so that there are only one and the same density and tension in one and the same horizontal stratum, and both increase with the depth. But in order to discover the law of this increase of density downwards, or the decrease upwards, we must adopt a method very similar to that of the former paragraph.

Fig. 395.



Let us imagine a vertical column of air AE, Fig. 395, of the transverse section $AB = 1$, and of the height $AF = s$. Let the density of the lower stratum $= \gamma$, and the tension $= p$, and the density of the upper stratum $EF = \gamma_1$, and the tension $= p_1$, we shall then have $\frac{\gamma_1}{\gamma} = \frac{p_1}{p}$. If x is the height EE_1 of the stratum E_1F, we have its weight, and hence also the diminution of its tension corresponding to:

$$y = 1\ .\ x\ .\ \gamma_1 = \frac{x\gamma p_1}{p}, \text{ and inversely,}$$

$$x = \frac{p}{\gamma}\ .\ \frac{y}{p_1}, \text{ or as in the former paragraph:}$$

$$x = \frac{p}{\gamma}\ \textit{hyp. log.}\left(1 + \frac{y}{p_1}\right) = \frac{p}{\gamma}\left[\textit{hyp. log.}\ (p_1 + y) - \textit{hyp. log.}\ p_1\right].$$

Let us put for p_1, successively

$$p_1 + y,\ p_1 + 2y,\ p_1 + 3y, \text{ \&c., to } p_1 + (n-1)y,$$

and add the corresponding heights of the strata or values of x, and we shall then obtain the height of the entire column of air, as in the former §.

$$s = \frac{p}{\gamma}\left(\textit{hyp. log. } p - \textit{hyp. log. } p_1\right) = \frac{p}{\gamma}\ \textit{hyp. log.}\left(\frac{p}{p_1}\right), \text{ also}$$

$$s = \frac{p}{\gamma}\ \textit{hyp. log.}\left(\frac{b}{b_1}\right) = 2{,}302\frac{p}{\gamma}\ \textit{log.}\left(\frac{b}{b_1}\right),$$

if b and b_1 are the heights of the barometer corresponding to the tensions p and p_1.

If, inversely, the height s is given, the expansive force and density of the air corresponding to it may be calculated. It is:

$$\frac{p}{p_1} = \frac{\gamma}{\gamma_1} = e^{\frac{s\gamma}{p}}, \text{ therefore } \gamma_1 = \gamma e^{\frac{-s\gamma}{p}}, \text{ where } e = 2{,}71828$$

is the base of the hyperbolic system of logarithms.

Remark. This formula is applicable to the measurement of heights. Leaving the temperature out of consideration, we may put $s = 58604 \,.\, Log. \left(\frac{b}{b_1}\right)$; for the English measure $= 60000 \, Log. \left(\frac{b}{b_1}\right)$.

Examples.—1. If the height of the barometer at the foot of a mountain be 28 inches, and at the top 25 inches, the height of this mountain will be:

$s = 58604 \,.\, Log. \left(\frac{30}{25}\right) = 58604 \,.\, 0791813 = 4640$ Prussian feet.—2. The density of the air on a mountain 10,000 feet high is: $Log. \frac{\gamma}{\gamma_1} = \frac{10000}{58604} = 0{,}1706$; hence, $\frac{\gamma}{\gamma_1} = 1{,}481$, and $\frac{\gamma_1}{\gamma} = \frac{1}{1{,}481} = 0{,}675$; it is therefore only $67\frac{1}{2}$ per cent. of the density of that at the foot.

§ 300. *Gay-Lussac's Law.*—Heat or temperature has an essential influence on the density and expansive force of gases. The more air enclosed in a vessel becomes heated, the greater does its expansive force exhibit itself, and the higher that the temperature of the air enclosed by a piston in a vessel is raised, the more it expands, and pushes against the piston. From the experiments of Gay-Lussac, which in later times have been repeated by Rudberg, Magnus and Regnault, it results that for equal densities the expansive force, and for equal expansive forces the volume of one and the same quantity of air increases as the temperature. We may place this law by the side of that of Mariotte, and name it, for distinction's sake, Gay-Lussac's law.

According to the latest experiments, the expansive force of a definite volume of air increases by being heated from the freezing to the boiling point, by 0,367 of its original value, or for this increase of temperature the volume of a definite quantity of air increases, the tension remaining the same, by 36,7 per cent. If the temperature is given in centigrade degrees, of which there are 100 between the freezing and boiling point, it follows that the expansion for each degree is = 0,00367, and for t degrees temperature = $0{,}00367 \,.\, t$; if we make use of Fahrenheit's thermometer, which contains between the freezing and boiling point 180°, for each degree the expansion is = .002039, and for t degrees temperature = $.002039 \,.\, t$. This co-efficient is true only for atmospheric air; slightly greater values correspond to other gases, and even for atmospheric air, this co-efficient increases slightly with the temperature.

If a mass of air of the original volume V_0, and of the temperature (centigrade) 0°, be heated t degrees without assuming a different tension, the new volume is then $V = (1 + 0{,}00367\, t)\, V_0$, and if it acquire the temperature t_1, it will then assume the volume: $V_1 = (1 + 0{,}00367\, t_1)\, V_0$, and by dividing the ratio of the volumes:

$$\frac{V}{V_1} = \frac{1 + 0{,}00367\, t}{1 + 0{,}00367\, t_1},$$

on the other hand, the corresponding ratio of densitye

$$\frac{\gamma}{\gamma_1} = \frac{V_1}{V} = \frac{1 + 0,00367\, t_1}{1 + 0,00367\, t}.$$

If, moreover, a change take place in the tensions, if p_0 is the tension at zero, p that at the temperature t, and p_1 that at t_1, we then have:

$$V = (1 + 0,00367\, t)\frac{p_0}{p}\, V_0, \text{ further } V_1 = (1 + 0,00367\, t_1)\, \frac{p_0}{p_1}\, V_0,$$

hencee

$$\frac{V}{V_1} = \frac{1 + 0,00367\, t}{1 + 0,00367\, t_1} \cdot \frac{p_1}{p}, \text{ and } \frac{\gamma}{\gamma_1} = \frac{1 + 0,00367 \,.\, t_1}{1 + 0,00367 \,.\, t} \cdot \frac{p}{p_1}, \text{ or,}$$

$$\frac{\gamma}{\gamma_1} = \frac{1 + 0,00367 \,.\, t_1}{1 + 0,00367 \,.\, t} \cdot \frac{b}{b_1}.$$

Example. If a mass of air, of 800 cubic feet, and of 10 lbs. tension, and 10° (centigrade) temperature, is raised by the blast, and by the warming apparatus of a blast-furnace to a tension of 19 lbs. and to a temperature of 200°*, it will at length assume the greater volume:

$$V_1 = \frac{1 + 0,00367 \,.\, 200}{1 + 0,00367 \,.\, 10} \cdot \frac{15}{19} \,.\, 800 = \frac{1,734}{1,0367} \cdot \frac{12000}{19} = 1056 \text{ cubic feet (Prussian).}$$

§ 301. *Density of the Air.*—By aid of the formula at the end of the former paragraph, γ may now be calculated by the density corresponding to a given temperature and tension of the air. By accurate weighings and measurements we have the weight of a cubic metre of atmospheric air at a temperature of 0°, and 0,76 metre height of barometer = 1,2995 kilogrammes. Since a cubic foot (Prussian) = 0,030916 cubic metre and 1 kilogramme = 2,13809 lbs. The density of the air for the relations given is: = 0,030916 . 2,13809 . 1,2995 = 0,08590 lbs. If now the temperature is = t^0 cent., the density for the French measure: $\gamma = \dfrac{1,2995}{1 + 0,00367\, t}$ kilogrammes; and for the Prussian measure $\gamma = \dfrac{0,08590}{1 + 0,00367 \,.\, t}$ lbs., and for the English: $\gamma = \dfrac{0,081241}{1 + 0,00204\, t}$ lbs. If now the expansive force varies from the mean, if, for example, the height of the barometer is not 0,76 metres, but b, we shall obtain:

$$\gamma = \frac{1,2995}{1 + 0,00367 \,.\, t} \cdot \frac{b}{0,76} = \frac{1,71 \,.\, b}{1 + 0,00367\, t} \text{ kilog.}$$

or if b, as is commonly the case, be given in Paris inches:

$$\gamma = \frac{0,003058e. \, b}{1 + 0,00367\, t} \text{ lbs.}$$

Very often the expansive force is expressed by the pressure p, on a square centimetre or square inch, for this reason the factor

$\dfrac{p}{1,0336}$, or $\dfrac{p}{14,73}$, or $\dfrac{p}{15,05}$ must be introduced, and it then follows that:

* 10° C. = 50 F., and 200° C. = 392° F.,—the co-efficient will then be .002039.—Am. Ed.

$$\gamma = \frac{1,2995}{1+0,00367\,.\,t} \cdot \frac{p}{1,0336} = \frac{1,2572p}{1+0,00367\,t} \text{ kilog. or}$$

$$\gamma = \frac{0,08565}{1+0,00367\,.\,t} \cdot \frac{p}{15,05} = \frac{0,005691p}{1+0,00367\,t} \text{ lbs. Prussian.}$$

For the same temperature and tension, the density of steam is $\frac{5}{8}$ of the density of atmospheric air, for which reason we have for steam:

$$\gamma = \frac{0,8122}{1+0,00367\,t} \cdot \frac{p}{1,0336} = \frac{0,78577}{1+0,00367\,t} \text{ kilog. or}$$

$$\gamma = \frac{0,05353}{1+0,00367\,.\,t} \cdot \frac{p}{15,05} = \frac{0,003557p}{1+0,00367\,t} \text{ lbs. Prussian.}$$

$$= \frac{0,050775}{1+0,00204\,t} \cdot \frac{p}{14,73} = \frac{0,003447p}{1+0,00204\,t} \text{ lbs. English.}$$

Examples.—1. What weight has the wind contained in a cylindrical regulator of 40 feet length and 6 feet width, at a temperature of 10° and 18 lbs. pressure? The density of this wind is:

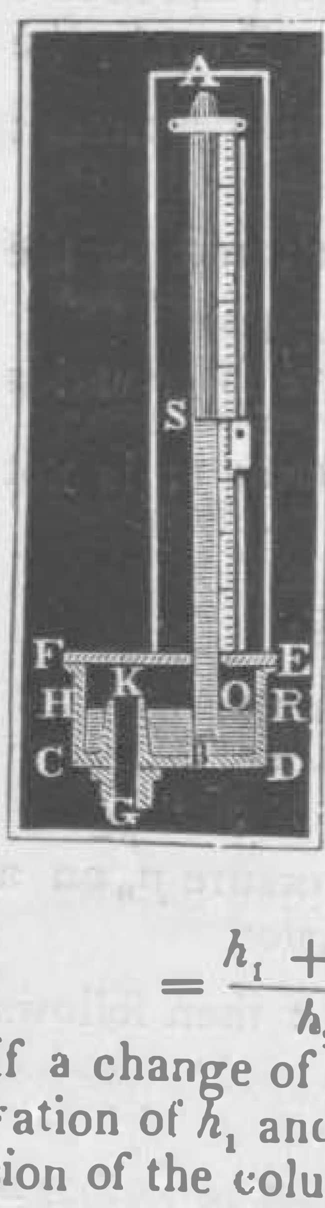
$$= \frac{0,005691\,.\,18}{1,0367} = \frac{0,10244}{1,0367} = 0,0988 \text{ lbs. (Prussian)};$$

the capacity of the regulating vessel is $= \pi\,.\,3^2\,.\,40 = 1131$ cubic feet; hence, the quantity of wind $= 0,0988\,.\,1131 = 112$ lbs.—2. A steam engine uses per minute 500 cubic feet of vapor, of 107° C. temperature and 36 inches pressure, how many pounds of water are required for the generation of this steam? The density of this steam is:

$$= \frac{0,05353}{1+0,00367.107^0} \cdot \frac{36}{28} = \frac{0,05353\,.\,36}{1,393\,.\,28} = 0,0494 \text{ lbs.};$$

hence, the weight of 500 cubic feet, or the weight of the corresponding quantity of water, $= 500\,.\,0,0494 = 24,7$ lbs.

Fig. 396.



§ 302. By aid of the results obtained in the last paragraph, the theory of the air manometer may be explained. This instrument consists of a barometer tube of uniform bore AB, Fig. 396, filled above with air and below with mercury, and of a vessel CE likewise containing mercury, which is put in communication with the gas or vapor whose tension we wish to find. From the height of the columns of mercury and of air, the expansive force may be estimated as follows. The instrument is commonly so arranged, that the mercury in the tube stands at the same level as the mercury in the cistern, when the temperature of the enclosed air $t = 50°$ (10° C.), and the tension in the space EH equal to the mean atmospheric pressure $b = 0,76$ metres $= 30$ inches.

But if for a height of the barometer b, from EH a column of mercury h_1 has ascended into the tube, and the length of the column of the residuary air is h_2, we have then its tension

$$= \frac{h_1 + h_2}{h_2} b, \text{ and hence } b_1 = h_1 + \frac{h_1 + h_2}{h_2} b.$$

If a change of temperature takes place, the temperature from observation of h_1 and h_2 is not as at first $= t$, but t_1, we then have the tension of the column of air:

$$.AS = \frac{h_1 + h_2}{h_2} \cdot \frac{1+0{,}00367 \,.\, t_1}{1+0{,}00367 \,.\, t} \,.\, b,$$

and hence the height of the barometer in question:

$$b_1 = h_1 + \frac{h_1 + h_2}{h_2} \cdot \frac{1+0{,}00367 \,.\, t_1}{1+0{,}00367 \,.\, t} \,.\, b.$$

For $b = 28$ inches (Paris), and $t = 10^0$ C., it follows that

$$b_1 = h_1 + 27\,(1 + 0{,}00367\, t_1)\,\frac{h}{h_2}, \text{ whereby } h = h_1 + h_2,$$

represents the whole length of the tube measured to the surface of mercury HR.

From the height of the barometer b_1, it follows that the pressure on the square inch (Prussian) is

$$p = \frac{15{,}6}{28}\,h_1 + 15{,}6 \,.\, \frac{27}{28}\,(1 + 0{,}00367\, t_1 \,.\, \frac{h}{h_2} = 0{,}538\, h_1$$

$$+ 14{,}51\,(1+0{,}00367\, t)\,\frac{h}{h_2} \text{ lbs.}$$

Example.—If an air manometer, of 25 inches length, at 21° temperature C., indicates a column of air of 12 inches in length, then the corresponding height of the barometer is:

$b_1 = 25 - 12 + 27\,(1 + 0{,}00367e\; 21)\,\frac{25}{12} = 13 + 9 \,.\, 1{,}07707e\,\frac{25}{4}$

$= 13 + 60{,}58 = 73{,}58$ inches, and the pressure on a square inch
$= 0{,}538 \,.\, 73{,}58 = 39{,}59$ lbs.