CHAPTER XX.

BELTING.

§ 276.

Self-Guiding Belting.

Belt pulleys are indirect acting friction wheels (§ 191) and the belt itself is a tension organ combining the functions of driving and guiding (§ 201). Those belts which act without requiring the use of special guiding devices may be called self-guiding belts. This action is attained by the use of cylindrical pulleys when the edge of the prismatic belt runs in a plane at right angles to the axis of the pulley; or in other words, when the middle line of the advancing side of the belt lies in the plane of the middle of its pulleys.

When a belt runs upon a conical pulley in a direction normal to its axis, its tendency will be to describe a conical spiral path upon the pulley, as will readily be seen upon the examination of the development of the surface of the cone, Fig. 840.

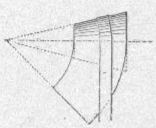

FIG. 840.

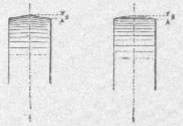

FIG. 841.

If the pulley is made with a double cone face or a rounded face, Fig. 841, the tendency will be for the belt to run at the middle of the face even when the direction of the belt is not exactly correct.

For leather belting, with a height of the crowning or curvature of the face $s = \frac{1}{10}$ of the width of face, the belt may deviate from the plane of the pulley by $2\frac{1}{2}°$ (tan = four per cent), while for cotton belting, on account of the lesser elasticity of the material, the crowning s should not exceed $\frac{1}{15}$ of the face, thus reducing very materially the permissible deviation. In ordinary circumstances at least one of a pair of pulleys should be made with rounded force.

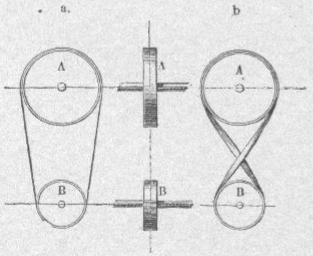

FIG. 842.

The simplest arrangement of self guiding belting is that for parallel axes, Fig. 842 *a* and *b*, *a* being for open belt and *b* for crossed belt, either arrangement being suitable to run in either direction.

For inclined and intersecting axes self-guiding belts are not suitable, except in the case of inclined axes in which the trace *S S*, Fig. 843, of the intersection of the planes of the two pulleys passes through the points at which the belt leaves the pulleys. The leading line then falls in the middle plane of each pulley, but the following side of the belt does not, hence such systems can only be run in one direction. The leaving points in the figures are at a and b_1. The arrangement gives an open belt when the angle β between the planes of the pulleys = 0°, and a crossed belt when $\beta = 180°$. In the intermediate positions a partial crossing of the belt is produced. If $\beta = 90°$, the belt is half crossed (or as commonly called, quarter twist); if $\beta = 45°$, it is quarter crossed.*

The leading off angle may be made as much as 25°, which occurs when the distance between the axes is equal to twice the

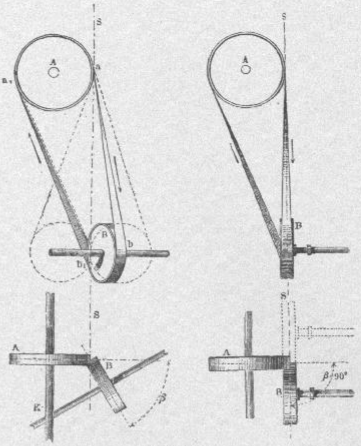

FIG. 843.

diameter of the largest pulley. Another rule for the minimum distance between shafts for quarter-twist belts is to make the distance never less than $\sqrt{b\,D}$.

§ 277.

Guide Pulleys for Belting.

When a belt transmission is arranged with guide pulleys, the proper guiding action is obtained when each guide pulley is placed at the point of departure of its plane with that of the next following pulley.†

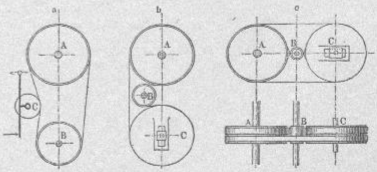

FIG. 844.

In Fig. 844 examples are given of guide pulleys for parallel axes, all three pulleys lying in the same plane.

At *a* is shown a belt transmission with tightening pulley, *b* is a device for transmitting motion when great difference of speed is desired. In this case the guide pulley *C* is as large as the driver *A*, and if desired may also be arranged to act as a tightener.‡ At *c* is Weaver's device for similar uses.§ In this case two belts are used, and the device has been used for driving circular saws. The pulleys should be fitted to run very smoothly in such devices.

The cases in Fig. 845–846 have parallel axes with two guide pulleys. In the first case the guide pulleys are placed in planes tangent to both operating pulleys, and hence driving may occur in either direction. Usually, however, it is required to provide

* The above geometrical construction is only approximate; for an exact solution see a paper by Prof. J. B. Webb, Trans. Am. Soc. Mech. Eng'rs, Vol. IV., 1883, p. 165.

† See also the paper of Prof. Webb, referred to in the preceding note.

‡ Eckert's patent (German) for driving the drum of a threshing machine.

§ See Cooper's Use of Belting. Phila., 1878, p. 171.

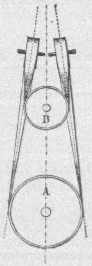

FIG. 845.

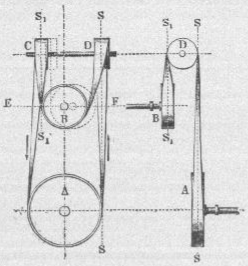

FIG. 846.

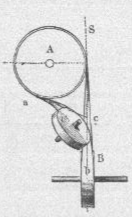

FIG. 840.

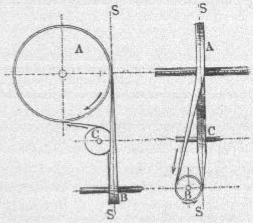

FIG. 850.

for motion in but one direction, in which case the second form is used as being simpler of installation. The pulley B may be used as one of the guide pulleys, in which case it may be placed loose upon the same shaft as A, and C or D be made drivers or driven.

By placing the guide pulleys between the axes of A and B, instead of beyond them, they will revolve in the same direction, and may be made fast upon one shaft, as in Fig. 847; this arrangement admitting of motion in only one direction.

In Fig. 848 is an arrangement for inclined axes, which is a modification of Fig. 846, as will be seen by the dotted lines. The guide pulleys run in opposite directions, but may conveniently be placed upon the same shaft.

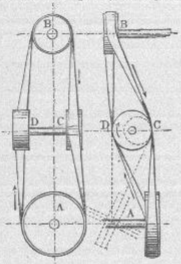

FIG. 847.

In Fig. 849 is shown an arrangement of quarter-twist belts with guide pulleys. One side of the belt is placed in the intersection $S\,S$ of the planes of the two pulleys. From any point c on $S\,S$, the tangents $c\,a$ and $c\,b$ are drawn, and in the plane of these the guide pulley C is placed. This arrangement permits of rotation in either direction.

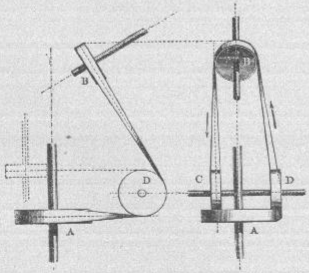

FIG. 848.

Another arrangement for the same purpose is shown in Fig. 850. The side of the belt leading off from A is inclined towards B, the other side passing over the guide pulley C, which is in the same plane as A and $S\,S$. This arrangement is well adapted for driving a number of vertical spindles from one horizontal shaft.*

Fig. 851 shows the general case for inclined axes. Two points c and c_1 are chosen on the line of intersection $S\,S$ of the planes of the two pulleys, and the tangents $c\,a$, $c\,b$, $c_1\,a_1$, $c_1\,b_1$

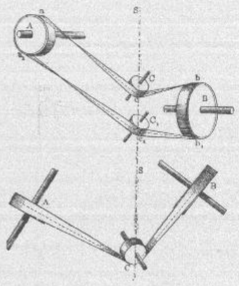

FIG. 851.

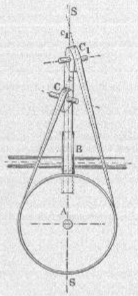

FIG. 852.

drawn, and in the planes of these tangents the guide pulleys C and C_1 are placed. Under these conditions the rotation may be in either direction. The arrangement shown in Fig. 852 occurs when the line $S\,S$ passes through the middle of one of the pulleys.

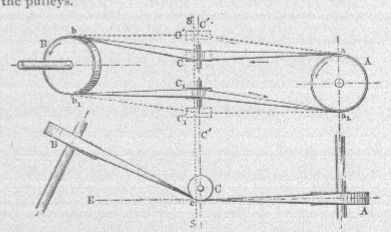

FIG. 853.

A simplification of the general case occurs when, as in Fig. 853, the guide pulleys fall upon one and the same geometrical axis which is parallel to the axes of both transmitting pulleys. In this case the only inclination of the belt is that given to it by the guide pulleys. The rotation can be in but one direction, viz: that shown by the arrows; if the reverse is desired, the guide pulleys must be placed as shown in the dotted lines. If the inclination of the shafts is too great the belt will be liable to drop off when the pulleys come to rest. The use of guide pulleys involves special hangers, a practical form for which is shown in Fig. 854.†

* An example is Jacob's grinding mill with 40 sets of stones; see Uhland's Praks. Masch. Konstr., 1868, p. 83, 1869, p. 242.

† Patented in Germany by the Berlin-Anhaltischen Maschinenbau-Aktien-Gesellschaft.

The vertical axis is provided with an oil hole, and is fitted by a ball and socket bearing to the bracket *D*. The flange on the lower edge of the pulley keeps the belt from falling off the

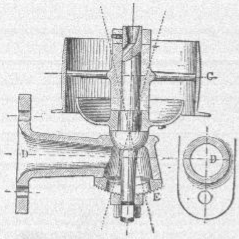

FIG. 854.

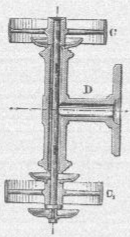

FIG. 855.

pulley when at rest. The form in Fig. 855 was designed by the author for the arrangement of Fig. 848, both pulleys being loose upon the wrought iron shaft.

If the position of the shafts can be so chosen that the line *S S* touches at least one of the pulleys, the very practical arrangement shown in Fig. 856 can be applied. If the distance

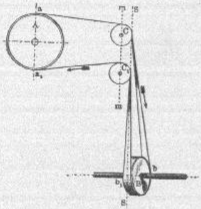

FIG. 856.

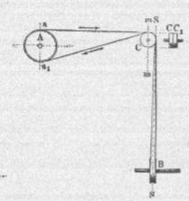

FIG. 857.

A C is great in comparison with the width of belt, the pulleys *C* and C_1 can be placed side by side instead of over each other, Fig. 857, in which case round face pulleys should be used.

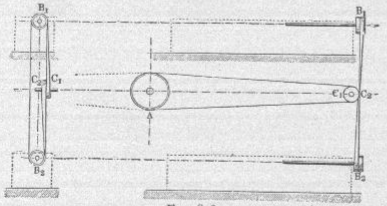

FIG. 858.

By the use of a fifth pulley the preceding arrangement may be so modified that two pulleys, B_1 and B_2, can be driven from one driver, *A*. This is shown in Fig. 858 as applied in a spinning mill, in which the pulleys B_1 and B_2 are on different floors of the building, and are also provided with loose pulleys.*

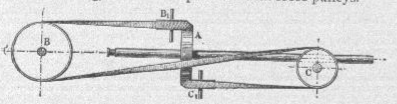

FIG. 859.

In the arrangement of Fig. 859 the pulley *A* drives two parallel shafts, one of which intersects its axis at right angles, the other passing beneath.

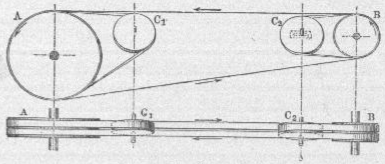

FIG. 860.

Another arrangement, devised by the author, is given in Fig. 860. In this case the following side of the belt is passed over an idler pulley, C_1 or C_2, and a second time around the driver (see also Fig. 795) by which the angle of contact a is doubled, and the modulus of friction $e^{f\alpha}$ (§ 264) increased. This may be called a double-acting transmission. The cross section of belt may be made $\frac{1}{10}$ of a single acting transmission, so that in spite of the increase of length an economy of belting is obtained. One of the guide pulleys may also be used for a tightener. These devices will also be considered in connection with rope transmission (Chapter XXI.) to which they are especially applicable.

§ 278.

FAST AND LOOSE PULLEYS.

Fast and loose, or tight and loose pulleys, as they are sometimes called, are generally used in connection with another belt transmission in order to throw the latter in and out of action, the belt being guided by a belt shifter, which by the means of forks or finger-bars, enables the moving belt to be shifted. These shifting devices may properly be regarded as guide pulleys, and are sometimes fitted with rollers, as shown dotted in Fig. 861, at c and c_1.†

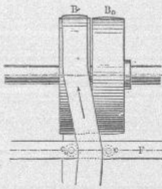

FIG. 861.

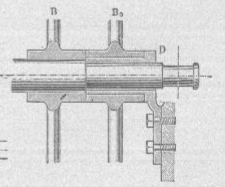

FIG. 862.

It is preferable to have the loose pulley upon the driven shaft, since the belt then can be shifted with a gradual spiral action by the shifter *F*, Fig. 861. It is best for the driving pulley to be made straight face, or if two fast pulleys are used side by side on the driving shaft, these should have very slightly rounded faces, if the belt is to be shifted promptly and readily, and for the same object the shifter should be placed as close to the driven pulleys as possible. The loose pulley should be kept thoroughly lubricated, and for this purpose numerous oiling devices have been made. The friction between the hub and shaft acts as a driving force upon the loose pulley, and this has been a source of numerous accidents. This action is avoided in the arrangement in Fig. 862, in which the loose pulley is carried on a consecutive and stationary sleeve *D*.‡

A variety of mechanical belt shifting devices have been made,§ the desire being to prevent the action of the belt from moving the shifter. A useful form is Zimmermann's Shifter, Fig. 863.

* See Fairbairn, Mills and Millwork, II., London, 1863, p. 103. For the theoretical discussion of these various arrangements, see § 302.

† Such rollers as especially necessary for shifting cotton belts, which are liable to catch on the shifter fingers, and even larger rollers are best in such cases.

‡ See Berliner Verhandlungen, 1869, p. 127. This has been used by the Society for Prevention of Accidents, of Mülhouse.

§ See Berliner Verhandlungen, 1868, p. 171, Rittershaus, Belt Shifters.

The shifter bar F, to which the fork G can be clamped at any desired point, is operated by the lever H, which turns upon an

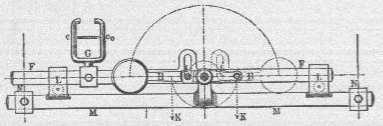

FIG. 863.

axis at I, forming a "dead" ratchet mechanism. The similarity to the ratchet devices of Figs. 754 and 755 will be observed. The movement of the bar is effected by connection at K or K_1.

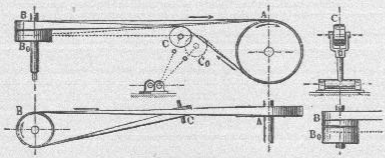

FIG. 864.

Fig. 864 shows a shifter for quarter-twist belt. In this form, devised by the author, the guide pulley, which is required to support the belt, also serves as a shifter to move the belt to and from the belt pulley B, and loose pulley B_2. If these pulleys are given greater width than that of the belt, as shown on the right, a vertical adjustment can be given to the upright shaft; a condition sometimes required in grinding mills and similar machines.

§ 279.

CONE PULLEYS.

When a number of pulleys are placed side by side in order to enable varied speeds to be obtained with belt transmission, and are united together in one member, we obtain what is called a cone pulley, such pulley being used in pairs. This construction involves the problem of determining the proper radii for the various pulleys, so that the same belt shall serve for all the changes, *i. e.*, so that the length of the belt shall be the same for each pair of pulleys in the set. The problem may be solved as follows:

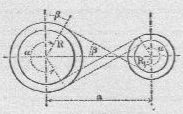

FIG. 865.

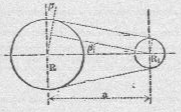

FIG. 866.

a. Crossed Belts, Fig. 865. The belt makes the angle β with the centre line of the pulleys R and R_1; and the half length of the belt, $l = R\left(\frac{\pi}{2}+\beta\right) + R_1\left(\frac{\pi}{2}+\beta\right) + a\cos\beta$, a being the distance from centre to centre of the pulley. We then have:

$$l = (R+R_1)\left(\frac{\pi}{2}+\beta\right) + a\sqrt{1-\frac{(R+R_1)^2}{a^2}} \quad . \quad . \quad (260)$$

This value is constant when $R + R_1$ is constant; that is, when the increase to the radius of one pulley is equal to the decrease in the radius of the other. Crossed belts are seldom used for this service, however, because of the injurious friction between the rubbing parts of the belt.

b. Open Belts, Fig. 866. In this case we have:

$$l = (R+R_1)\frac{\pi}{2} + (R-R_1)\beta + a\cos\beta,$$

and also $a\sin\beta = R - R_1$, which gives:

$$\left.\begin{aligned} R &= \frac{l}{\pi} + \frac{a}{\pi}(\beta\sin\beta+\cos\beta) + \frac{a}{2}\sin\beta \\ R_1 &= \frac{l}{\pi} - \frac{a}{\pi}(\beta\sin\beta+\cos\beta) - \frac{a}{2}\sin\beta \end{aligned}\right\} \quad . \quad . \quad (261)$$

This function is transcendental, but may be graphically represented in the following manner, Fig. 867. In the rectangle $A\,B\,B'\,A'$, with a radius $A\,B = a$, strike the quadrant $B\,M\,C$ about the centre A. Within this arc will fall all the values of

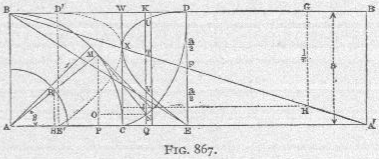

FIG. 867.

β which can occur. For any value of $\beta = C\,A\,M$, draw $M\,N$ perpendicular to $M\,A$ and make $M\,N =$ the arc $M\,C = a\,\beta$. Drop the perpendicular $M\,P$ to $A\,C$, and draw $N\,O$ perpendicular to $M\,P$. $N\,O$ will then $= a\,\beta\sin\beta$. Through N draw $Q\,N\,K$ parallel to $A\,B$, and we have $A\,Q = P\,Q + A\,P = a(\beta\sin\beta+\cos\beta)$. By taking successively all the values of β between 0° and 90° in this manner, we can determine the path of the point N, which will be the evolute of a circle, $C\,N\,D$ $B\,D$ being equal to the length of the arc $B\,M\,C = \frac{\pi}{2}a$. If we now draw $D\,E$ parallel to $B\,A$, and take its middle point F, we have $D\,F = E\,F = \frac{a}{2}$, and hence the proportion:

$$D\,F : D\,B = \frac{a}{2} : \frac{\pi}{2}a = a : \pi, \text{ and by similar triangles:}$$

$$T\,K = \frac{a}{\pi}\,Q\,A = \frac{a}{\pi}(\beta\sin\beta+\cos\beta).$$

This value is dependent upon $\frac{l}{\pi}$. If we prolong $B\,F$ until it intersects $A\,C$ prolonged, the resulting length $A\,A' = B\,B'$ will bear to $A'\,B'$ the ratio $\frac{\pi}{1}$. By then working $B\,G = l$, and drawing $G\,H$ parallel to $A'\,B'$, we have $G\,H = \frac{l}{\pi}$. This length being transferred to $I\,K$ gives $I\,T = \frac{l}{\pi} - \frac{a}{\pi}(\beta\sin\beta + \cos\beta)$. We then have only to use $\pm\frac{a}{2}\sin\beta$ to solve the problem.

Make $A\,R = \frac{a}{2}$, and we have the perpendicular $R\,S = \frac{a}{2}\sin\beta$. By laying this length off above and below T on $Q\,K$, we obtain the points U and V, and this finally gives $I\,U$ for the radius R of the larger cone pulley and $I\,V = R_1$, the radius of the corresponding smaller cone pulley.

By solutions for successive values of β, we obtain the curve $D\,U\,X\,V\,E$, which can be used for the determination of the radii of any desired pair of pulleys, each pair of ordinates measured from $H\,I$ belonging to corresponding pulley on each cone.

In practice it is usual to find one of the cone pulleys given and the dimensions of the other required. In this case $V\,U$ may be taken as the difference $R - R_1$, between the radii, were the steps uniform. By taking this difference $R - R_1$ in the dividers, and finding the equivalent ordinate $U\,V$ on the curve, and then adding $V\,T = R_1$, the axis $H\,I$ is found.

In order to use the curve conveniently, it may also be laid off left-handed, as shown in the dotted lines $D'\,X\,E'$.

The use of the diagram will be rendered still more convenient if we omit the unnecessary value l. This enables us to distort the curve in the direction of the abscissas to any desired extent.

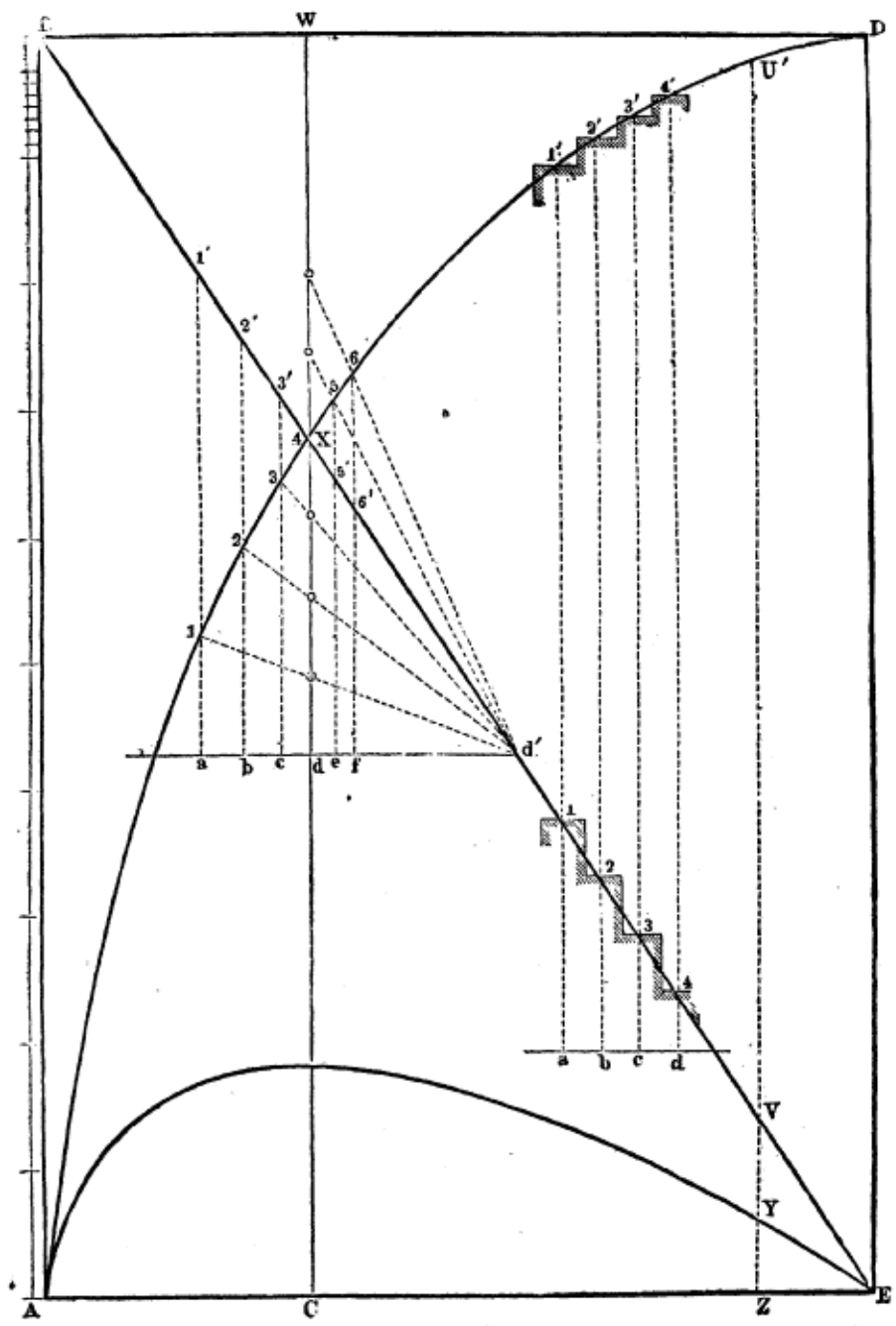

FIG. 868.

This has been done in the proportional diagram for cone pulleys, Fig. 868.

The method of using the diagram is as follows:

The sides $A\,B$ and $D\,E$ of the rectangle represent the distance a between the centres of the pulleys; all radii are given in proportional parts of a, for which reason $A\,B$ is sub-divided, the size of the diagram being selected so that $A\,B = 18$ to 20 inches. If, then, $1\,a$ and $1'\,a$ are two given radii for a pair of pulleys on a pair of cones, we take the vertical chord of the curve which $= 1'\,a - 1\,a$, prolong the chord downward until its length $= 1\,a$, and draw the axis $a\,b\,c\,d$ parallel to $A\,E$. Then for the other pairs of pulleys on the cones, we have $b\,2$ and $b\,2'$, $c\,3$ and $c\,3'$, etc., which can be taken directly from the diagram with the dividers. If the given pair of radii to which the cones are to be made equal, the chord $R - R_1 = o$, and the axis will pass through X at right angles to $C\,X$.

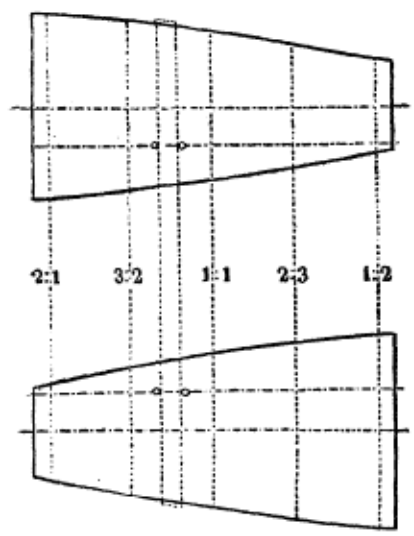

FIG. 869.

If it is desired to construct a pair of cone pulleys to any given speed ratio, this can readily be done. If, for example, the given ratio is 1 : 1, we lay off toward C, the corresponding radius $X\,d$ and prolong the axial line $d\,d'$ to its intersection d' with $B\,E$. Then lay off the given geometric ratio on $C\,X$, considering $X\,d$ as 1 (shown in the diagram by the small circles for the ratios $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{4}$), and draw rays from d' through the points of division, and these rays will intersect the curve at the correspouding points for the pulley radii R_1. We then have for the radii:

a 1	and a 1′	for the ratio	1 : 4
b 2	" b 2′	"	2 : 4
c 3	" c 3′	"	3 : 4
$d\,X$	" $d\,X'$	"	4 : 4
e 5	" c 5′	"	5 : 4
c 6	" c 6′	"	6 : 4.

Cone pulleys may also be made continuous, thus becoming conoids upon which the belt can be shifted to any point by an adjustable guide or shifter. Such conoids are used for driving the rollers in spinning machinery. Such a pair of conoids are shown in Fig. 869, the proportions having been determined by the graphical scale. The angular velocity varies in an arithmetical ratio as shown.

The curve $E\,Y\,A$ in the scale shows the limit to which the axial line may approach $A\,E$; this distance must not be less than $R + R_1 = a$, from which $V\,Y = \frac{1}{2}(A\,B - V\,U)$.

§ 280.

CROSS SECTION AND CAPACITY OF BELTS.

A belt of rectangular cross section of width b, and thickness δ, will be subjected to a tension T on the tight side (see § 264), which it must be proportioned to sustain. If S is the permissible stress for the unit of cross section, we have, therefore, $T = b\,\delta\,S$.

The minimum ratio which T bears to the transmitted force P is dependent upon the stress modulus τ, since $T = \tau\,P$ (§ 264). But $\tau = \frac{\rho}{\rho - 1}$, in which ρ represents the modulus of friction e^{fa}. Hence, if N is the horse power transmitted for a belt speed of v feet per minute, we have: $N = \frac{P\,v}{33000} = \frac{b\,\delta\,S\,v}{33000\,\tau}$.

This enables us to determine the cross section of the belt, but in practice the width of the belt is the variable factor, the thickness usually being determined by commercial considerations, and limited to few definite sizes.

If we let q represent the cross section of the belt in square inches, we have:

$$N = \frac{q\,v\,S}{33000\,\tau}$$

This formula is very useful, since it may be used to determine the capacity of a belt from its cross section and velocity. If we put $N_o = \frac{N}{q\,v}$ we have:

$$N_o = \frac{1}{33000} \cdot \frac{S}{\tau} \quad \ldots\ldots\ldots\ldots \quad (262)$$

The value depends upon the material and stress modulus, the latter including the arc of contact a, and upon f, which itself depends upon the material of both belt and pulley; it may also be considered as dependent upon a, independent of the material, in the same manner as was the subject of specific weight. The author has called this value N_o, the *specific capacity* of a belt. It will be seen that when this specific capacity is determined for any kind of belt, the proper cross section for the transmission of a given horse power N can readily be found, since the velocity v can be chosen, and we have at once

$$q = \frac{N}{N_o\,v} \quad \ldots\ldots\ldots\ldots \quad (263)$$

For the determination of the specific capacity of any kind of belt it is necessary to find the constants S and τ.

The materials used for belting are:

Tanned leather,
Cotton, woven and treated with oil,
Rubber, interlaid with linen or cotton webbing.

In practice the value of S to be used must depend much upon

judgment, the value being governed to a great extent by the quality of the material. Customary values are for:

Leather . . . . . $S = 4000$ to 6000 lbs.
Cotton . . . . . $S = 3000$ to 4000 lbs.
Rubber . . . . . $S = 3500$ to 5000 lbs.

The thickness δ for single leather belts varies from $\frac{3}{16}''$ to $\frac{1}{2}''$; double, triple, quadruple, and even quintuple thicknesses being sometimes used, the thicknesses being secured by cement, and sewed or rivetted together. Cotton belts are usually from $\frac{1}{16}''$ to $\frac{11}{64}''$ thick, while rubber belts are made of any desired thickness, a web of canvas being interlaid between the successive thicknesses of rubber.

The stress modulus τ depends upon a and f, and the latter coefficient varies with the age of the belt, being greater with belts which have been used some time than with quite new belts. It is advisable, however, to make all calculations as for new belts, in which case we have for smooth iron pulleys, for:

Leather and cotton, $f = 0.16$ to 0.75*, $\rho = 1.6$ to 2.1
Rubber, $f = 0.20$ to 0.25, $\rho = 1.8$ to 2.1

These give as approximate values for.

Leather and cotton, $\dfrac{T}{P}$ or $\tau = 2.5$ to 1.9

Rubber, $\dfrac{T}{P}$ or $\tau = 2.2$ to 1.9

By using these values together with those given for S, in (262) we get for the *specific capacity* for belting:

$$\left.\begin{array}{ll} \text{Leather,} & N_o = 0.0062 \text{ to } 0.0098 \\ \text{Cotton,} & N_o = 0.0036 \text{ to } 0.0068 \\ \text{Rubber} & N_o = 0.0050 \text{ to } 0.0082 \end{array}\right\} \quad (265)$$

These are based upon low and moderate speeds; say up to 3000 feet per minute, and the variations between the limits given are those due to the differences in strength of various kinds of leather and canvas used.

The resistance to bending or stiffness of a belt must be taken into account, and the ratio of thickness δ to pulley radius R, must not be too great. Practical experience has shown that $\dfrac{\delta}{R} = \dfrac{1}{50}$ should not be exceeded to obtain best results.§

From the known stress and the thickness of the belt the superficial pressure p, between belt and pulley may be calculated. We have only to substitute in (241) for the width b' of the surface of contact, the width b of the belt itself, and $\sin \varphi = \delta \div d$, we get the simple relation:

$$\frac{p}{S} = \frac{\delta}{R} \quad \ldots\ldots\ldots\ldots (264)$$

Example 1.—Required a leather belt to transmit 100 H. P. and the speeds of pulleys to be $n = 80$ $n_1 = 150$ revolutions. Taking the specific capacity at 0.007 and the lineal velocity of belt at 3000 feet, we have $q = \frac{100}{3000 \times 0.007} = 4.8$ sq. in. cross section.

If we use a double belt 0.4" thick, the width should be $\frac{4.8}{0.4} = 12$ inches.

For the driving pulley we have: $\frac{2\pi R n}{12} = v$, and $R = \frac{3000 \times 12}{2\pi n} = 71.7$ say 72", or 154 inches. For the driven pulley we have $R_1 = \frac{80 \times R}{150} = 38.4$".

For the superficial pressure p, we have $P = \frac{33000 \times 100}{3000} = 1100$ lbs. Also $T = 2.5\ P = 2750$, hence $S = \frac{2750}{0.4 \times 72} = 573$. We have also $t = 1.5\ P = 1650$, which gives $S_1 = 343$, or a mean of 458 lbs., which in (264) gives a mean value $\rho = \frac{458 \times 0.4}{72} = 2.5$ lbs. on the large pulley, and $\rho = \frac{458 \times 0.4}{38.4} = 4.98$, or nearly 5 pounds. This is verified since, if $f = 0.16$:

$$72 \times 3.14 \times 12 \times 2.5 \times 0.16 = 1700,$$

which is the value of P as above.

Example 2.—What horse power can be transmitted by a cotton belt 4 inches wide and 0.25" thick, at a velocity of 2000 feet per minute? Taking the specific capacity at 0.006, which has been found satisfactory in practice, we have from (262) $N = q\ V N_o = 4 \times 0.25 \times 2000 \times 0.006 = 12$ H. P.

Example 3.—A rubber belt is required to drive a centrifugal pump (rubber being especially adapted for damp locations). $N = 20$, the pump vane to make 300 revolutions, and the driving shaft 80 revolutions per minute, and the belt speed 2000 feet. Taking the specific capacity at 0.007, we have $20 = q \times 2000 \times 0.007$, hence $q = 1.43$ sq. in., and if we make $\delta = 0.2$ we have a width $b = 7.1$". For the driven pulley we have $R_1 = \frac{2000 \times 12}{2\pi 300} = 12.7$, say $12\frac{3}{4}$", and for the driver $R = \frac{12.75 \times 300}{80} = 47.8$". A mean value of S is 425 lbs., whence $\rho = \frac{425 \times 0.2}{47.8} = 1.78$ on the large pulley and $\frac{425 \times 0.2}{12.75} = 6.7$ on the small pulley.

For extraordinary cases the fundamental formula should always be applied. For double-acting belts, as in Fig. 860, in which $a = 2\pi$ instead of π, the value $f a = 1$, and the modulus of stress is only 0.5 of the preceding value, hence q is reduced in the same proportion. If the belt velocity v is very high, it is no longer permissible to neglect the influence of centrifugal force. For a speed $v = 5000$ feet and a stress $S = 568$ pounds (see § 264) the exponent in the friction modulus becomes $0.84\ f a$ instead of $f a$, which for $f = 0.16$ and $a = \pi$, gives $f' a = 0.84 \times 0.16\ \pi = 0.42$. This gives $\tau = 2.91$ or about $\frac{2}{3}$ of the normal value, which requires one-sixth greater cross section q for the belt. The highest limit of belt speed in ordinary practice appears to be about 6000 feet per minute.†

§ 281.

EXAMPLES OF BELT TRANSMISSION.

The table of existing examples of belt transmission on next page will serve to furnish data for comparison with calculated results.

The great variations in the values of S and N_o, in the following table are not surprising when the differences in the quality of material, and the various conditions are considered. Many leather belts are working under high stresses which are only practicable because of the excellence of the material. Some such belting can be operated under stresses as high as 2000 pounds, which enables much lighter sections to be used. Many belts which appear to have been excessively heavy have simply been calculated to work at a moderate stress.

The plausible but erroneous idea that the pressure of the atmosphere influences belt action cannot be admitted. It is contradicted not only by the fact that the same coefficient of friction exists for ropes as for belts, but also by the recent and careful experiment made in a vacuum by Leloutre which confirmed the theory of the modulus of friction.

§ 282.

BELT CONNECTIONS.

The various methods of connecting the ends of belts generally give a greater stress at the point of connection than in the body of the belt. The attempts to reduce this weakness and also provide for the greatest facility in the making of the joint, has caused a great variety of methods to be proposed; some of the best of these are here given:

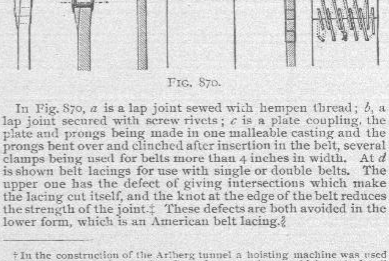

FIG. 870.

In Fig. 870, *a* is a lap joint sewed with hempen thread; *b*, a lap joint secured with screw rivets; *c* is a plate coupling, the plate and prongs being made in one malleable casting and the prongs bent over and clinched after insertion in the belt, several clamps being used for belts more than 4 inches in width. At *d* is shown belt lacings for use with single or double belts. The upper one has the defect of giving intersections which make the lacing cut itself, and the knot at the edge of the belt reduces the strength of the joint.‡ These defects are both avoided in the lower form, which is an American belt lacing.§

* For cotton the thinner belts from 0.25" to 0.4" are preferable.

† In the construction of the Arlberg tunnel a hoisting machine was used in which the belt had a velocity of 4700 feet per minute, which worked well for fourteen months.

‡ Leloutre has used the upper form of lacing for a belt of 26" wide, 0.56" thick with excellent performance and durability.

§ See Cooper, Use of Belting, p. 189.

EXAMPLES OF BELT TRANSMISSION.

No.	Horse Power N	n	R	v	P	b	δ	S	N_o	REMARKS.
1	624	$\frac{40}{100}$	$\frac{271.8}{108}$	2887	7114	105	0.67	512	.0062	Leather, 2 belts side by side.
2	200	$\frac{52}{182}$	$\frac{137.8}{39.37}$	3749	1731	24	0.47	388	.0046	Leather.
3	190	$\frac{65}{223}$	$\frac{71.8}{22.6}$	2440	2528	21	0.24	1222	.0147	Leather.
4	175	$\frac{120}{228}$	$\frac{56.9}{30}$	3561	1573	29	0.35	388	.0046	Leather.
5	153	$\frac{120}{160}$	$\frac{63}{47.25}$	3955	1256	12.6	0.52	483	.0059	Leather.
6	130	$\frac{36}{94}$	$\frac{128}{45.3}$	2410	1544	10	0.40	981	.0121	Leather.
7	90	$\frac{65}{182}$	$\frac{83.8}{30}$	2833	1034	12	0.35	612	.0075	Leather.
8	81	$\frac{55}{137.5}$	$\frac{98.4}{39.37}$	2833	928	9.8	0.52	455	.0056	Leather.
9	60	$\frac{100}{100}$	$\frac{59}{59}$	1535	631	12.2	0.47	270	.0033	Leather.
10	54	$\frac{45}{125}$	$\frac{98.4}{35.4}$	2318	660	17.3	0.24	400	.0092	Leather.
11	42	$\frac{60}{90}$	$\frac{70.8}{47.25}$	2224	614	11.8	0.20	654	.0082	Leather.
12	40	$\frac{66}{102}$	$\frac{49.2}{38.9}$	2066	630	13.8	0.24	483	.0059	Leather.
13	530	$\frac{60}{262}$	$\frac{144}{27}$	5156	3337	38	0.72	313	.0036	Cotton.
14	497	$\frac{70}{144.4}$	$\frac{99}{48}$	3620	4457	30	0.72	526	.0065	Cotton.
15	470	$\frac{62.5}{114}$	$\frac{96}{49.5}$	3130	4877	32	0.72	540	.0065	Cotton.
16	413	$\frac{48}{120}$	$\frac{120}{48}$	3000	4453	30	0.72	512	.0105	Cotton.
17	325	$\frac{125}{172.4}$	$\frac{60}{43.5}$	3915	2583	22	0.72	412	.0049	Cotton.
18	134	$\frac{125}{133.3}$	$\frac{48}{45}$	3130	1390	10	0.72	412	.0049	Cotton.
19	60	$\frac{70}{175}$	$\frac{73.8}{29.6}$	2706	722	16.5	0.47	228	.0049	Cotton.
20	35	$\frac{81.3}{99.3}$	$\frac{38}{31}$	1633	704	5	0.72	498	.0062	Cotton.
21	66	$\frac{165}{243}$	$\frac{55}{37.5}$	4763	451	11.8	0.52	185	.0023	8-ply Rubber.

Fig. 871 *a*, shows Bötter's belt fastening. This is a form of belt hook which has been found very serviceable, reducing the strength of the belt but little, and permitting easy renewal. Another form is Moxon's belt fastening,* shown at *b*, is a pin

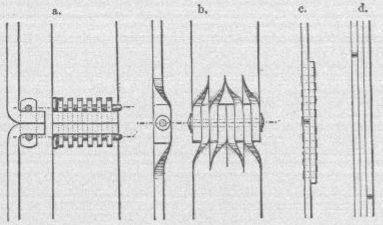

FIG. 871.

point, the ends of the pin being riveted over, and from its construction should be very strong. At *c* is a butt joint with a reinforcement piece especially suited for cotton belts. When a belt is made for special service it can be in several layers as at *d*; the joints overlapping, but thus giving no opportunity for change of length.

The stretching and joining of heavy belts is a matter requiring much care in order to secure the desired tension, $= \frac{1}{2}(T + t)$.† Belts which are subjected only to light tensions may be cemented by scarfing the ends and using a cement composed of common glue mixed with fish glue, or of rubber dissolved in bisulphide of carbon.

§ 283.

THE PROPORTIONS OF PULLEYS.

Pulleys are usually made of cast iron and of single width, *i. e.*, one set of arms. The arms, which formerly were made curved, in order to resist the stresses due to contraction, are now made straight, and for wide face pulleys two or even three parallel sets of arms are used.

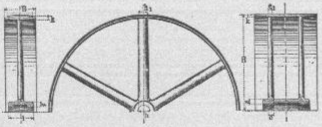

FIG. 872.

Fig. 872 shows both single and double arms. The dimensions of arms and rim have been determined by experience, based upon practical considerations. For the number A of arms for a single set, we get serviceable values from:

$$A = \tfrac{1}{2}\left(5 + \frac{R}{b}\right) \quad \ldots\ldots\ldots (266)$$

which gives, for:

$\frac{R}{b}$ =	1	2	3	4	5	6	7	8	9	10	11	12	13
A =	3		4		5		6		7		8		9.

The width h of the arm, if prolonged to the middle of the hub, may be obtained from:

$$h = 0.25'' + \frac{b}{4} + \frac{1}{10}\frac{R}{A} \quad \ldots\ldots\ldots\ldots (267)$$

The width h_1 of the arm at the rim is equal to 0.8 h, and the corresponding thicknesses are $e = \frac{1}{2} h$, and $e_1 = \frac{1}{2} h_1$.

Pulleys with two or three sets of arms may be considered as two or three separate pulleys combined in one, except that the proportions of the arms should be 0.8 or 0.7 times that of single arm pulleys, or in the proportion of $\sqrt[3]{\frac{1}{2}}$ and $\sqrt[3]{\frac{1}{3}}$.

The thickness of the rim may be made: $k = \frac{1}{8}$ to $\frac{1}{4}$ h, this being frequently turned much thinner. The width of face should be from $\frac{2}{3}$ to $\frac{3}{4}$ the width of the belt.

The thickness of metal in the hub may be made $W = h$, to $\frac{3}{4}$ h. The length of hub may $= b$, for single arm pulleys and $2\,b$ for double arm pulleys. Light pulleys are usually secured to the shaft by means of set screws, as in Fig. 875 and 877; heavier ones are keyed as in Fig. 191, either with or without set screws.*

For many purposes pulleys are made in two parts, such being commonly called "split pulleys. The forms of split pulleys are shown in Figs. 873 to 875.

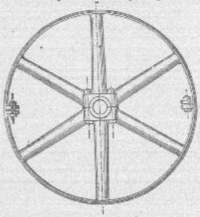

FIG. 873.

The arrangement of the two halves is clearly shown, that of Fig. 874 with hollow clamping section, being especially good.†

The form in Fig. 875 is the design of the Walker Mfg. Co. of Cleveland, Ohio, the clamps being made of malleable iron or steel. In all three cases there is no especial method of fastening to the shaft. In England and America pulleys are frequently made with wrought iron rims and cast iron hubs. This construction greatly simplifies the casting of the arms, and at the same time gives pulleys 25 to 60 per cent. lighter than those of cast iron, which in large transmissions greatly reduces the friction

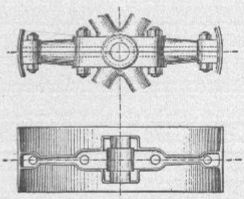

FIG. 874.

at the bearings of the shafting. Fig. 876 shows the Medart pulley. The rim is curved in bending rolls, and also given a rounding face, and is countersunk for the rivets at the attachment of the arms. The pads on the arms are truly finished, as is also the rim after it is riveted on, thus giving an accurate and useful pulley.‡

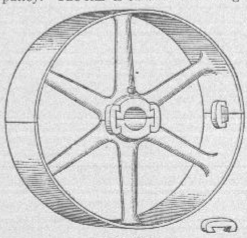

FIG. 875.

A metal pulley by the Hartford Engineering Company 60″ diameter and 16″ face weighed 320 pounds. A cast iron pulley of the same dimensions made by the Berlin-Anhalt Works, weighed 700 pounds, and one by Briegleb, Hansen & Co., a little narrower face weighed 528 pounds.

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* See Chronique industrielle, 1882, Vol. 5, p. 97; also Mechanical World, 1882, Vol. 12, p. 56.

† Leloutre has used a dynamometric belt-stretcher for tensions of $\frac{1}{2}(T + t) = 8800$ pounds.

*In order to determine the necessary friction to secure a pulley to the shaft, the force p on the belt will serve. In ordinary cases, assuming a coefficient of friction on the key of one-half that on the belt, there should be a pressure P on the key of about 400 times that on the belt, which, according to § 20 will not give more than 5000 to 7000 lbs. for P.

† This is the construction of the Berlin-Anhalt Machine Works.

‡ Made in England by George Richards & Co., Manchester.