CHAPTER IV.

OF THE EFFICIENCY AND COUNTER-EFFICIENCY OF PIECES, COMBINATIONS, AND TRAINS, IN MECHANISM.

370. **Nature and Division of the Subject.**—The terms *Efficiency* and *Counter-efficiency* have already been explained in Article 335, page 377; and the laws of friction, the most important of the wasteful resistances which cause the efficiency of a machine to be less than unity, have been stated in Articles 309 to 311, pages 348 to 355. In the present Chapter are to be set forth the effects of wasteful resistance, and especially of friction, on the efficiency and counter-efficiency of single pieces, and of combinations and trains of pieces, in Mechanism. In practical calculations the *counter-efficiency* is in general the quantity best adapted for use; because the useful work to be done in an unit of time, or *effective power*, is in general given; and from that quantity, by multiplying it by the counter-efficiency of the machine—that is, by the continued product of the counter-efficiencies of all the successive pieces and combinations by means of which motion is communicated from the driving-point to the useful working-point—is to be deduced the value of the expenditure of energy in an unit of time, or *total power*, required to drive the machine. In symbols, let U be the useful work to be done per second; c, c', c'', &c., the counter-efficiencies of the several parts of the train; T, the total energy to be expended per second; then

$$T = c \cdot c' \cdot c'' \cdot \&c. \dots U. \quad \text{(1.)}$$

When the mean effort required at the driving-point can conveniently be computed by reducing each resistance to the driving-point, and adding together the reduced resistances (as in Article 324, page 369, and Article 338, page 379), the ratio in which the actual effort required at the driving-point is greater than what the required effort would be, in the absence of wasteful resistance, is expressed by the continued product of the counter-efficiencies of the parts of the train, as follows: let P_0 be the effort required, in the absence of wasteful resistance; P, the actual effort required; then

$$P = c \cdot c' \cdot c'' \cdot \&c. \dots P_0; \quad \text{(2.)}$$

and in determining the efficiency or the counter-efficiency of a single piece, the most convenient method of proceeding often consists in comparing together the efforts required to drive that piece, with and without friction; and thus finding the ratios

$$\frac{P_0}{P} = \text{efficiency}; \quad \frac{P}{P_0} = \text{counter-efficiency}. \quad \text{.........(3.)}$$

In the ensuing sections of this Chapter, the efficiency of single primary pieces is first treated of, and then that of the various modes of connection employed in elementary combinations.

Section I.—*Efficiency and Counter-efficiency of Primary Pieces.*

371. **Efficiency of Primary Pieces in General.**—A primary piece in mechanism, moving with an uniform velocity, is balanced under the action of four forces, viz. :—

I. The re-action of the piece which it drives: this may be called the *Useful Resistance*, and denoted by R;

II. The *weight* of the piece itself: this may be denoted by W.

III. The *effort* by which the piece is driven: this may be denoted by P; and its values with and without friction by P_1 and P_0 respectively.

IV. The resultant pressure at the bearings, or *bearing-pressure*, which may be denoted by Q; and which of course is equal and directly opposed to the resultant of the first three forces.

In the absence of friction, the bearing-pressure would be normal to the bearing-surface. The effect of friction is, that the line of action of the bearing-pressure becomes oblique to the bearing-surface, making with the normal to that surface the angle of repose (ϕ), whose tangent ($f = \tan \phi$) is the co-efficient of friction (see Article 309, page 349); and the amount of the friction is expressed by Q sin ϕ, or very nearly by f Q, when the co-efficient of friction is small.

In the class of problems to which this Chapter relates, the first two forces—that is, the useful resistance R, and the weight W—are given in magnitude, position, and direction; and in most cases it is convenient to find their resultant, in magnitude, position, and direction, by the rules of statics: that is to say, if the line of action of R is vertical, by Rule I. of Article 280, page 322; and if inclined, by the Rules given or referred to in Article 278, page 319. In what follows, the resultant of the useful resistance and weight will be called the *given force*, and denoted by R′.

The third force—that is, the effort required in order to drive the piece at an uniform speed—is given in position and direction; for its line of action is the line of connection of the piece under consideration with the piece that drives it. The magnitude of the effort is one of the quantities to be found.

The fourth force—that is, the bearing-pressure—has to be found in position, direction, and magnitude. The general principles according to which it is determined are the following:—

First, That if the lines of action of the given force and the effort are parallel to each other, the line of action of the resultant bearing-pressure must be parallel to them both; and that if they are inclined to each other, the line of action of the resultant bearing-pressure must traverse their point of intersection.

Secondly, That at the centre of pressure, where the line of action of the resultant bearing-pressure cuts the bearing-surfaces, it makes an angle with the common normal of those surfaces equal to their angle of repose, and in such a direction that its tangential component (being the friction) is directly opposed to the relative sliding motion of that pair of surfaces over each other.

Thirdly, That the given force, the effort, and the bearing-pressure, form a system of three forces that balance each other; and are therefore proportional to the three sides of a triangle parallel respectively to their directions.

371 A. **Conditions Assumed to be Fulfilled.**—In all the problems treated of in this section, the following conditions are assumed to be fulfilled:—*First*, that except when otherwise specified, the forces other than bearing-pressures which are applied to the piece under consideration—that is, the useful resistance, the weight, and the effort—act either in parallel directions, or exactly or nearly in one plane, parallel to the planes of motion of the particles of the piece; *secondly*, that the acting parts of the piece do not *overhang* the bearings; and *thirdly*, that the bearing-surfaces fit each other easily without any grasping or pinching. As to the object of the fulfilment of such conditions, and the effects of departure from them, the following explanations have to be made:—

I. The bearing-surface of many primary pieces, and especially of rotating pieces, is in general divided into two parts; for example, an axle is very often supported by two journals. If the forces other than bearing-pressures which are applied to the moving piece, are parallel to each other, the parts of the bearing-pressure will also be parallel to them and to each other; and the sum of the frictional resistances due to the two parts of the bearing-pressure will be simply equal to the frictional resistance due to the whole bearing-pressure treated as one force. The same will be the case when the forces other than bearing-pressures act in one plane, parallel to the planes of motion of the particles of the piece; and will be nearly the case when, although those forces act in different planes, the transverse distance between their planes of action is small compared with the distance between the planes of action of the two components into which the bearing-pressure is divided.

But when that condition is not fulfilled, the friction at the bearings, being proportional to the sum of the two components into which the bearing-pressure is divided, will be greater than the friction due simply to the resultant bearing-pressure considered as one force; and the efficiency of the piece will be diminished.

II. The effect upon the friction, and upon the work lost in overcoming it, produced when the acting parts of a moving piece *overhang* its bearings, may be approximately calculated and allowed for in the following manner:—

Suppose that the bearing-surface of a primary piece, whether sliding or turning, is divided into two parts; and that the transverse distance between the centres of those two parts—that is, the distance in a direction perpendicular to the planes of motion of the particles of the piece—is denoted by c. Let the plane of action of the forces other than bearing-pressures be situated *outside* the space between the two parts of the bearing-surface, and at the transverse distance z from the centre of the nearer of those parts; and consequently at the distance $z + c$ from the centre of the further of them. Let Q be the resultant bearing-pressure. The two components of that resultant pressure, exerted at the two parts of the bearing-surface, will be contrary to each other in direction; and their values will be respectively,

at the nearer part, $\frac{Q(z + c)}{c}$;

and at the further part, $\frac{-Qz}{c}$.

The total friction will be the sum of two components exerted at the two parts of the bearing-surface respectively, and will be proportional to the *arithmetical sum* of the two components of the bearing-pressure; that is, to the force

$$\frac{Q(c + 2z)}{c};$$

whereas, if the plane of action of the resultant of the given force and the effort had not overhung the bearings, the friction would have been simply proportional to Q. Hence the effect of that plane's overhanging the bearings by the distance z, is to increase the friction approximately in the ratio of

$$1 + \frac{2z}{c} : 1.$$

III. As to the condition that the bearing-surfaces should fit each other easily, it is necessary in order that the bearing-pressure may not contain, to any appreciable extent, pairs of components which balance each other, being transverse to the direction of the resultant bearing-pressure; for such components cause an unnecessary addition to the friction. The ratio in which the friction of a *tight-fitting bearing* exceeds that of an easy-fitting bearing of the same dimensions and figure, is very nearly equal to that in which the whole area of the bearing-surface exceeds the area of the projection of that surface on a plane normal to the direction of the resultant bearing-pressure.

When the use of bearing-surfaces in pairs, oblique to the plane of the pressure and motion, is unavoidable (as, for example, in the case of the V-shaped bearings of a planing machine), their effect may be allowed for by increasing the co-efficient of friction in the ratio above-mentioned; which is expressed by the secant of the equal angles which the normals to the bearing-surfaces make with that plane.

372. **Efficiency of a Straight-sliding Piece.**—In fig. 263, let A A be a straight guiding-surface, upon which there slides, in the direc-

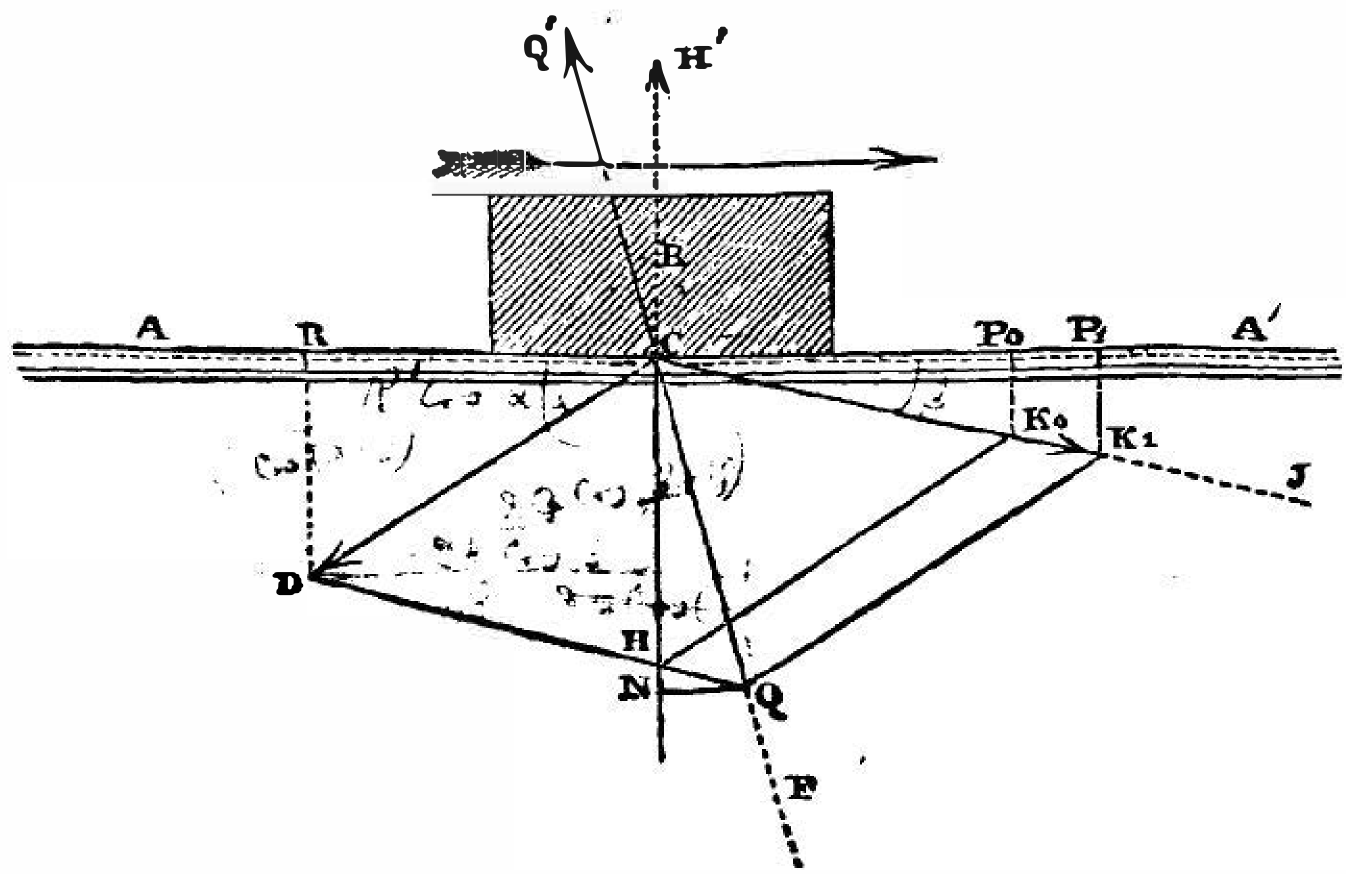

Fig. 263.

tion marked by the feathered arrow, the moving piece B. Let C D represent the *given force*, being the resultant of the useful resistance and of the weight of the piece B. (The figure shows the motion of B as horizontal; but it may be in any direction.) Let C J be the line of action of the effort by which the piece B is driven.

Draw C N perpendicular to A A; and C F making the angle N C F = the angle of repose. Through D, parallel to C J, draw the straight line D H Q, cutting C N in H, and C F in Q; and through H and Q, and parallel to D C, draw H K_0 and Q K_1, cutting C J in K_0 and K_1 respectively. Produce H C to H', and Q C to Q', making C H' = H C, and C Q' = Q C.

Then, in the absence of friction, C H′ will represent the resultant bearing-pressure exerted upon B by A A; and $CK_0 = DH$ will represent the force in the given direction C J required to drive B at an uniform speed; and when friction is taken into account, C Q′ will represent the resultant bearing-pressure, and CK_1 the actual driving force required; and we shall have

$$\text{the efficiency} = \frac{CK_0}{CK_1}; \text{ and the counter-efficiency} = \frac{CK_1}{CK_0}.$$

If from D, K_0, and K_1 there be let fall upon A A the perpendiculars D R, $K_0 P_0$, and $K_1 P_1$, C R will represent the direct resistance to the advance of B; CP_0, the direct effort in the absence of friction; and CP_1, the direct effort taking friction into account; so that the distance $P_0 P_1$ will represent the friction itself; which is also represented by Q N perpendicular to C N.

To express these results by symbols, let $CD = R'$ (the given force); let the acute angle A C D be denoted by α, and the acute angle A′ C J by β; and let ϕ denote the angle of repose N C Q.

Then, in the triangle C D H, we have $\angle DCH = \frac{\pi}{2} - \alpha$, and $CHD = \frac{\pi}{2} - \beta$; and in the triangle C Q D, we have $\angle DCQ = \frac{\pi}{2} - \alpha + \phi$, and $\angle CQD = \frac{\pi}{2} - \beta - \phi$; consequently

$$DH = R' \frac{\cos \alpha}{\cos \beta}; \; DQ = R' \cdot \frac{\cos (\alpha - \phi)}{\cos (\beta + \phi)};$$

whence it follows that the efficiency and counter-efficiency are given by the following equations:—

$$\text{Efficiency} = \frac{P_0}{P_1} = \frac{DH}{DQ} = \frac{\cos \alpha \cdot \cos (\beta + \phi)}{\cos \beta \cdot \cos (\alpha - \phi)} = \frac{1 - f \tan \beta}{1 + f \tan \alpha} \quad (1.)$$

$$\text{Counter-efficiency} = \frac{P_1}{P_0} = \frac{1 + f \tan \alpha}{1 - f \tan \beta}. \quad \ldots\ldots\ldots\ldots \quad (2.)$$

It is to be remarked, that the efficiency diminishes to nothing when $\cot \beta = f$; that is to say, when β is the complement of the angle of repose, ϕ. In other words, if the oblique effort is applied in the direction C Q, no force, how great soever, will be sufficient to keep the piece B in motion.

373. **Efficiency of an Axle.**—In fig. 264, let the circle A A A represent the trace of the bearing-surface of an axle on a plane perpendicular to its axis of rotation, O—in other words, the transverse section of that surface. Let the arrow near the letter N represent the direction of rotation. Let C D be the given force;

that is, as before, the resultant of the weight of the whole piece that rotates with the axle, and of the useful resistance or re-action exerted on that piece by the piece which it drives; C J, the line of action of the effort by which the rotating piece is driven.

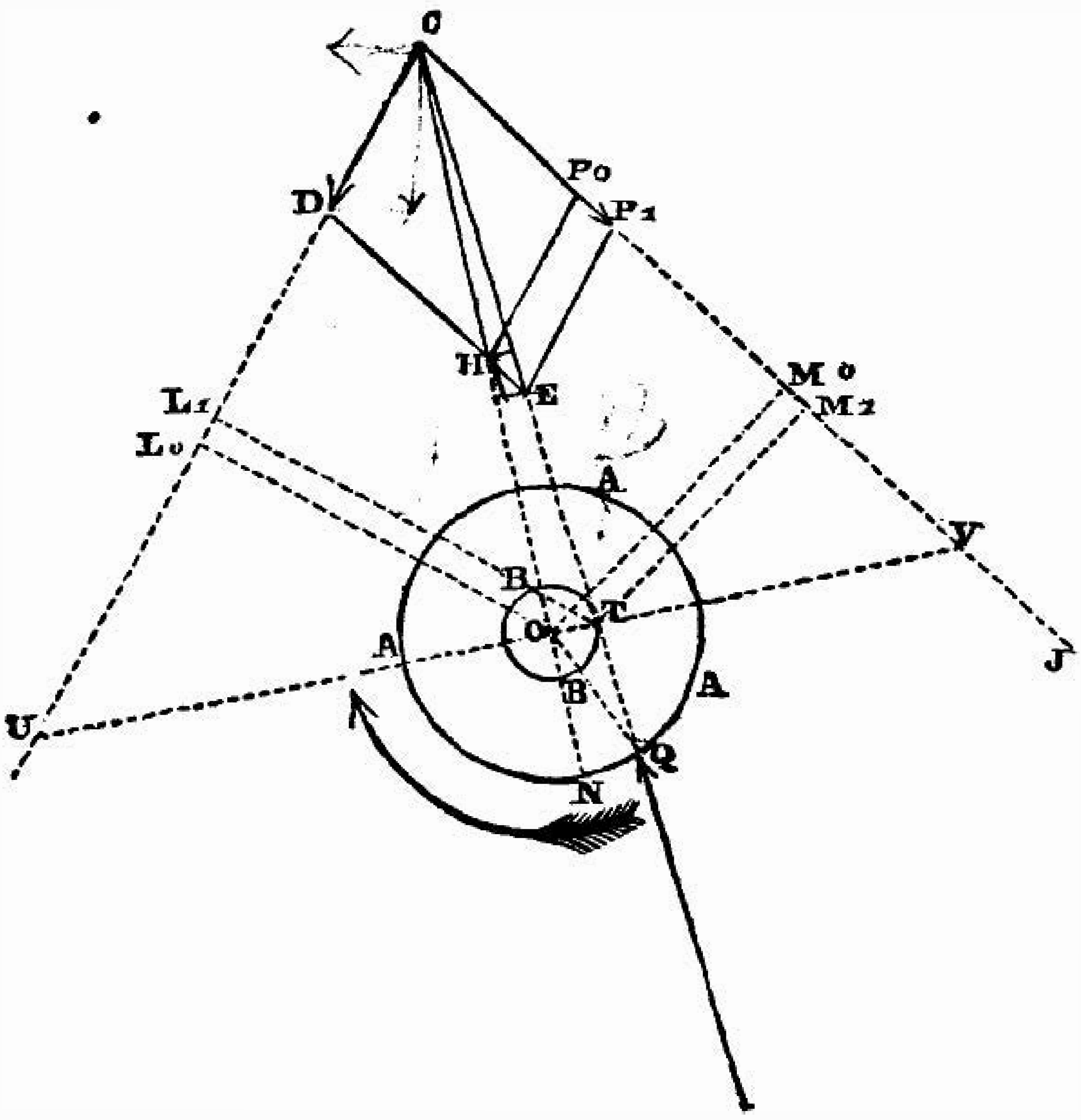

Fig. 264.

In toothed wheel-work the lines of action of the useful resistance and of the effort may be taken as coinciding with the lines of connection of the rotating piece with its follower and with its driver respectively. In pulleys connected with each other by bands, special principles have to be attended to, which will be explained in the ensuing Article.

Let r denote the radius of the bearing-surface.

About O describe the small circle B B, with a radius $= r \sin \varphi = fr$, very nearly. Draw the line of action, C T Q, of the resultant bearing-pressure, touching the small circle at that side which will make the bearing-pressure resist the rotation. In the case in which C D and C J intersect each other in a point, C, as shown in the figure, C T Q will traverse that point also; and in the case in which the lines of action of the given force and the effort are parallel to each other, C T Q will be parallel to both. The centre of bearing-pressure is at Q; and O Q T $= \varphi$, the angle of repose.

In the former case the efficiency may be found by parallelograms of forces, as follows:—Draw the straight line C O N; this would be the line of action of the resultant bearing-pressure in the absence of friction, and N would be the centre of bearing-pressure. Through D, parallel to C J, draw D H E, cutting C O N in H, and C T Q in E. Through ~~D~~ and E, parallel to D C, draw H P_0 and ~~H~~ P_1. Then, in the absence of friction, H C would represent the bearing-pressure, and C P_0 = D H the effort; the actual bearing-pressure is represented by E C, and the actual effort by C P_1 = D E. Hence the efficiency and counter-efficiency are as follows:—

$$\frac{P_0}{P_1} = \frac{\mathrm{D\,H}}{\mathrm{D\,E}}; \quad \frac{P_1}{P_0} = \frac{\mathrm{D\,E}}{\mathrm{D\,H}} \text{.....................(1.)}$$

Another method, applicable whether the forces are inclined or parallel, is as follows:—From the axis of rotation O, let fall O L_0 and O M_0 perpendicular respectively to the lines of action of the given force and of the effort. Then, by the balance of moments, the effort in the absence of friction is

$$P_0 = R' \cdot \frac{\mathrm{O\,L_0}}{\mathrm{O\,M_0}}.$$

From a convenient point in the actual line of action, C Q, of the bearing-pressure (such, for example, as T, where it touches the small circle B B), let fall T L_1 and T M_1 perpendicular respectively to the same pair of lines of action; then the actual effort will be

$$P_1 = R' \cdot \frac{\mathrm{T\,L_1}}{\mathrm{T\,M_1}}.$$

Hence the efficiency and the counter-efficiency have the following value:—

$$\left.\begin{aligned} \frac{P_0}{P_1} &= \frac{\mathrm{O\,L_0 \cdot T\,M_1}}{\mathrm{O\,M_0 \cdot T\,L_1}}; \\ \frac{P_1}{P_0} &= \frac{\mathrm{O\,M_0 \cdot T\,L_1}}{\mathrm{O\,L_0 \cdot T\,M_1}}. \end{aligned}\right\} \text{.....................(2.)}$$

The same results are expressed, to a degree of approximation sufficient for practical purposes, by the following trigonometrical formulæ:—Let O $L_0 = l$; O $M_0 = m$; $\angle$ C O $L_0 = \alpha$; $\angle$ C O $M_0 = \beta$. Then we have, very nearly, ?

$$\frac{P_0}{P_1} = \frac{l}{m} \cdot \frac{m - f r \sin \beta}{l + f r \sin \alpha} = \frac{1 - \frac{f r}{m} \cdot \sin \beta}{1 + \frac{f r}{l} \cdot \sin \alpha} \text{. ....(3.)}$$

In making use of the preceding formula, it is to be observed that the *contrary algebraical signs* of sin α and sin β apply to those cases in which the two angles α and β lie at contrary sides of O C. In the cases in which those angles lie at the same side of O C, their algebraical signs are the same; and in the formula they are to be made *both positive* or *both negative*, according as β is *less* or *greater* than α; so that the efficiency may be always expressed by a fraction less than unity. That is to say,

$$\text{If } \beta > \alpha; \ \frac{P_0}{P_1} = \frac{1 - \frac{f r}{m} \sin \beta}{1 - \frac{f r}{l} \sin \alpha}; \quad \text{(3 A.)}$$

$$\text{If } \beta < \alpha; \ \frac{P_0}{P_1} = \frac{1 + \frac{f r}{m} \sin \beta}{1 + \frac{f r}{l} \sin \alpha} \quad \text{(3 B.)}$$

When the lines of action intersect, let O C be denoted by c; then $l = c \cos \alpha$, and $m = c \cos \beta$; and consequently the three preceding equations take the following form:—

$$\beta \text{ and } \alpha \text{ of contrary signs}; \ \frac{P_0}{P_1} = \frac{1 - \frac{f r}{c} \tan \beta}{1 + \frac{f r}{c} \tan \alpha}; \quad \text{(4.)}$$

β and α of the same sign;

$$\beta > \alpha; \ \frac{P_0}{P_1} = \frac{1 - \frac{f r}{c} \tan \beta}{1 - \frac{f r}{c} \tan \alpha}; \quad \text{(4 A.)}$$

$$\beta < \alpha; \ \frac{P_0}{P_1} = \frac{1 + \frac{f r}{c} \tan \beta}{1 + \frac{f r}{c} \tan \alpha} \quad \text{(4 B.)}$$

When the lines of action of the forces are parallel, we have sin β and sin $\alpha = +1$ or -1, as the case may be; and the formulæ take the following shape:—

When l and m lie at contrary sides of O, the piece is a "lever of the first kind;" and

$$\frac{P_0}{P_1} = \frac{1 - \frac{f r}{m}}{1 + \frac{f r}{l}}. \quad \text{(5.)}$$

When l and m lie at the same side of O;

If $m > l$, the piece is a "lever of the second kind;" and

$$\frac{P_0}{P_1} = \frac{1 - \frac{fr}{m}}{1 - \frac{fr}{l}} \text{ ....(5 A.)}$$

If $m < l$, the piece is a "lever of the third kind;" and

$$\frac{P_0}{P_1} = \frac{1 + \frac{fr}{m}}{1 + \frac{fr}{l}} \text{ ....(5 B.)}$$

(As to levers of the first, second, and third kinds, see Article 221, page 233.)

The following method is applicable whether the forces are inclined or parallel; in the former case it is approximate, in the latter exact. Through O, perpendicular to O C, draw U O V, cutting the lines of action of the given force and of the effort in U and V respectively. The point where this transverse line cuts the small circle B B coincides exactly with T when the forces are parallel, and is very near T when they are inclined; and in either case the letter T will be used to denote that point. Then

$$\frac{P_0}{P_1} = \frac{OU}{OV} \cdot \frac{TV}{TU} \text{ ....(6.)}$$

It is evident that with a given radius and a given co-efficient of friction, the efficiency of an axle is the greater the more nearly the effort and the given force are brought into direct opposition to each other, and also the more distant their lines of action are from the axis of rotation.

374. **Axles of Pulleys connected by Bands.**—When the rotating piece which turns with an axle consists of a pair of pulleys, one receiving motion from a driving pulley, and the other communicating motion to a following pulley, regard must be had to the fact that the useful resistance and the driving effort are each of them the *difference* of a pair of tensions; and that it is upon the *resultant* of each of those pairs of tensions (being their sum, if they act parallel to each other) that the axle-friction depends.

The principles according to which the tensions required at the two sides of a band for transmitting a given effort are determined, have been stated in Article 310 A, pages 351, 352.

The belt which drives the first pulley may be called the *driving belt;* that which is driven by the second pulley, the *following belt.*

The tensions on the two sides of the following belt are given; and the moment of the useful resistance is that of their difference, acting with a leverage equal to the effective radius of the second pulley. Let p be that radius; T_1 and T_2 the two tensions; then the moment of the useful resistance is

$$p\,R = p\,(T_1 - T_2).$$

For the actual useful resistance there is to be substituted a force equal to the resultant of T_1 and T_2, and exerting the same moment. That is to say, let γ denote the angle which the two sides of the band make with each other; then for the actual useful resistance is to be substituted a force,

$$R'' = \sqrt{\{T_1^2 + T_2^2 + 2\,T_1\,T_2 \cos\gamma\}}, \quad \text{.............(1.)}$$

acting at the following perpendicular distance from the axis of rotation:—

$$k = \frac{p\,(T_1 - T_2)}{R''}. \quad \text{.....................(2.)}$$

And this is to be compounded with the weight of the rotating piece, to find the *given force* R' of the rules in the preceding Article, whose perpendicular distance from the axis will be

$$l = \cancel{R} = \frac{p\,(T_1 - T_2)}{R'}. \quad \text{.....................(3.)}$$

The value of k may be expressed in terms of the ratio of the tensions to each other, and independently of their absolute values, as follows:—Let $N = \frac{T_1}{T_2}$ be the ratio of the two tensions found by the rules of Article 310 A, page 351. Then

$$k = \frac{p\,(N - 1)}{\sqrt{\{N^2 + 1 + 2\,N \cos\gamma\}}}. \quad \text{.............(4.)}$$

In like manner, for the actual line of action of the effort by which the first pulley is driven is to be substituted the line of action of a force exerting the same moment, and equal to the resultant of the tensions of the two sides of the driving-band. The perpendicular distance m of this line of action from the axis of rotation is given by the following formula:—Let p' be the effective radius of the pulley; N', the ratio of the greater to the lesser tension; γ', the angle which the two sides of the band make with each other; then

$$m = \frac{p'\,(N' - 1)}{\sqrt{\{N'^2 + 1 + 2\,N' \cos\gamma'\}}}. \quad \text{............(5.)}$$

There are many cases in practice in which the two sides of each of the bands may be treated as sensibly parallel; and then we have simply,

$$\left.\begin{aligned} R'' &= \frac{R(N+1)}{N-1}; \\ k = \frac{p(N-1)}{N+1}; &\quad m = \frac{p'(N'-1)}{N'+1}. \end{aligned}\right\} \ldots\text{a}\ldots(6.)$$

And if, moreover, as frequently happens, the weight of the pulleys and axle is small compared with the tensions, we may neglect it, and make $R' = R''$ and $l = k$, preparatory to applying the rules of the preceding Article to the determination of the efficiency.

375. **Efficiency of a Screw.**—The efficiency of a screw acting as a primary piece is nearly the same with that of a block sliding on a straight guide, which represents the development of a helix situated midway between the outer and inner edges of the screw-thread; the block being acted upon by forces making the same angles with the straight guide that the actual forces do with that helix. As to the development of a helix, see Article 63, page 40; and as to the efficiency of a piece sliding along a straight guide, see Article 372, page 426.

376. **Efficiency of Long Lines of Horizontal Shafting.**—In a line of horizontal shafting for transmitting motive power to long distances in a mill, a great part of the wasted work is spent in overcoming the friction produced simply by the weight of the shaft resting on its bearings; and the efficiency and counter-efficiency as affected by this cause of loss of power can be considered and calculated separately.

For reasons connected with the principles of the strength of materials, to be explained further on, the *cube of the diameter* of a shaft of uniform diameter must be made to bear a certain proportion to the driving moment exerted upon it to keep up its rotation. That is to say, let M_1 denote that moment; h, the diameter of the shaft; then

$$M_1 = A h^3; \ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

A being a co-efficient whose values in practice range, according to circumstances to be explained in the Third Part of this treatise,

for forces in lbs. and dimensions in inches, from 300 to 1,800;

and for forces in kilogrammes and dimensions in millimètres,

from 0·21 to 1·26.

Let w denote the heaviness of iron; f, the co-efficient of friction; then the weight of an unit of length of the shaft is

$$\frac{\pi}{4} w h^2;$$

the friction per unit of length is, very nearly,

$$\frac{\pi}{4} f w h^2;$$

and the *moment of friction* per unit of length is

$$\frac{\pi}{8} f w h^3 = \cdot 3927 f w h^3 \text{ nearly.} \ldots\ldots\ldots\ldots(2.)$$

Let L be the length of a shaft of uniform diameter, such that the whole driving moment is exhausted in overcoming its own friction. This may be called the *exhaustive* length. Then we must have

$$M_1 = A h^3 = \cdot 3927 f w h^3 L; \text{ and therefore}$$

$$L = \frac{A}{\cdot 3927 f w} \ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

For lengths in feet, and diameter in inches, we have $w = \frac{10}{3}$; being the weight in pounds of a rod of iron a foot long and an inch square. For lengths in mètres, and diameters in millimètres, we have $w = \cdot 0077$ nearly; being the weight of a rod of iron one mètre long and one millimètre square. Let $f = 0 \cdot 051$; then the following are the values of the exhaustive length L corresponding to different values of A :—

A, British measures,	300	600	1,200	1,800
„ French,	0·21	0·42	0·84	1·26
L, feet	4,500	9,000	18,000	27,000
„ mètres	1,365	2,730	5,460	8,190

It is obvious that the efficiency of a length, l, of shafting of uniform diameter is given by the expression

$$\frac{M_0}{M_1} = 1 - \frac{l}{L}; \ldots\ldots\ldots\ldots\ldots\ldots(4).$$

M_0 being the driving moment in the absence of friction; M_1, the actual driving moment; and $\frac{l}{L}$, the fraction of that moment expended on friction; also, that the counter-efficiency is

$$\frac{M_1}{M_0} = \frac{L}{L - l} \ldots\ldots\ldots\ldots\ldots\ldots(5.)$$

When, besides its own weight, the shaft is loaded with the weights of pulleys and tensions of belts, the effect of such additional load may be allowed for, with a degree of accuracy sufficient for practical purposes, in the following manner:—Find the magnitude of the resultant of the weight of the shaft and additional load; and let m be the ratio which it bears to the weight of the shaft. Then the modified value of the exhaustive length is to be found by putting $m\,w$ instead of w in the denominator of the expression (3.): that is to say

$$L = \frac{A}{\cdot 3927\, f\, m\, w}. \qquad \text{(6.)}$$

The waste of work in a long line of shafting may be diminished, and the efficiency increased, by causing it to taper, so that the cube of the diameter shall at each cross-section be proportional to the moment exerted there. The most perfect way of fulfilling that condition is to make the diameter diminish continuously in geometrical progression; the generating line or longitudinal section of the shaft being a logarithmic curve. Let h be the diameter at the driving end, x the distance of a given cross-section from that end, and y the diameter at that cross-section; then

$$y = h\, e^{-\frac{x}{3\,L}}; \qquad \text{(7.)}$$

in which $e^{-\frac{x}{3\,L}}$ is the reciprocal of the natural number, or anti-logarithm, corresponding to the hyperbolic logarithm $\frac{x}{3\,L}$, and to the common logarithm $\frac{0{\cdot}4343\, x}{3\,L}$. Let l be the total length of such a tapering shaft, and M_0 the useful working moment exerted at its smaller end; then we have

2)

$$\left.\begin{array}{ll} \text{Efficiency,} & \dfrac{M_0}{M_1} = e^{-\frac{l}{L}}; \\[2ex] \text{Counter-efficiency,} & \dfrac{M_1}{M_0} = e^{\frac{l}{L}}. \end{array}\right\} \qquad \text{(8.)}$$

This cannot be perfectly realized in practice; but it can be approximated to by making the shaft consist of a series of lengths, or divisions, each of uniform diameter, and increasing in diameter step by step.

Let $\frac{l}{n}$ now denote the length of one of those divisions; the number of divisions being n. The counter-efficiency of each division is expressed by

$$\frac{L}{L - \frac{l}{n}}; \qquad \text{(9.)}$$

and consequently, the counter-efficiency of the whole shaft is,

$$\frac{M_1}{M_0} = \left\{ \frac{L}{L - \frac{l}{n}} \right\}^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10.)$$

The diameters of the lengths of shafting, beginning at the driving-end, form a diminishing geometrical progression, of which the common ratio is

$$\left\{ 1 - \frac{l}{n\,L} \right\}^{\frac{1}{3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11.)$$

Section II.—*Efficiency and Counter-efficiency of Modes of Connection in Mechanism.*

377. **Efficiency of Modes of Connection in General.**—In an elementary combination consisting of two pieces, a driver and a follower, there is always some work lost in overcoming wasteful resistance occasioned by the mode of connection; the result being that the work done by the driver at its working-point is greater than the work done upon the follower at its driving-point, in a proportion which is *the counter-efficiency of the connection*; and the reciprocal of that proportion is *the efficiency of the connection.* In calculating the efficiency or the counter-efficiency of a train of mechanism, therefore, the factors to be multiplied together comprise not only the efficiencies, or the counter-efficiencies, of the several primary pieces considered separately, but also those of the several modes of connection by which they communicate motion to each other.

378. **Efficiency of Rolling Contact.**—The work lost when one primary piece drives another by rolling contact is expended in overcoming the *rolling resistance* of the pitch-surfaces, a kind of resistance whose mode of action has been explained in Article 311, page 353; and the value of that work in units of work per second is given by the expression $a\,b\,N$; in which N is the normal pressure exerted by the pitch-surfaces on each other; b, a constant arm, of a length depending on the nature of the surfaces (for example, 0·002 of a foot = 0·6 millimètre for cast iron on cast iron, see page 354); and a, the relative angular velocity of the surfaces.

The useful work per second is expressed by $u\,f\,N$, in which f is the coefficient of friction of the surfaces, and u the common velocity of the pitch-lines. Hence the *counter-efficiency* is

$$c = 1 + \frac{a\,b}{u\,f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

Let p_1 and p_2 be the lengths of two perpendiculars let fall from the two axes of rotation on the common tangent of the two pitch-lines; if the pieces are circular wheels, those perpendiculars will be the radii. Then the absolute angular velocities of the pieces are respectively $\frac{u}{p_1}$ and $-\frac{u}{p_2}$; and their relative angular velocity is therefore

$$a = u\left(\frac{1}{p_1} + \frac{1}{p_2}\right);$$

which value being substituted in equation (1), gives for the counter-efficiency the following value:—

$$c = 1 + \frac{1}{f}\left(\frac{b}{p_1} + \frac{b}{p_2}\right). \quad\text{..................(2.)}$$

It is assumed that the normal pressure is not greater than is necessary in order to give sufficient friction to communicate the motion.

It is evident, from the smallness of b, that the lost work in this case must be almost always a very small fraction of the whole.

379. **Efficiency of Sliding Contact in General.**—In fig. 265, let T be the point of contact of a pair of moving pieces connected by sliding contact. Let the plane of the figure be that containing the directions of motion of the two particles which touch each other at the point T; and let T V be the velocity of the driving-particle, and T W the velocity of the following-particle; whence V W will represent the velocity of sliding, and T U, perpendicular to V W, the common component of the velocities of the two particles along their line of connection R T P. C T C, parallel to V W, and perpendicular to R T P, is a common tangent to the two acting surfaces at the point T; the arrow A represents the direction in which the driver slides relatively to the follower; and the arrow B, the direction in which the follower slides relatively to the driver.

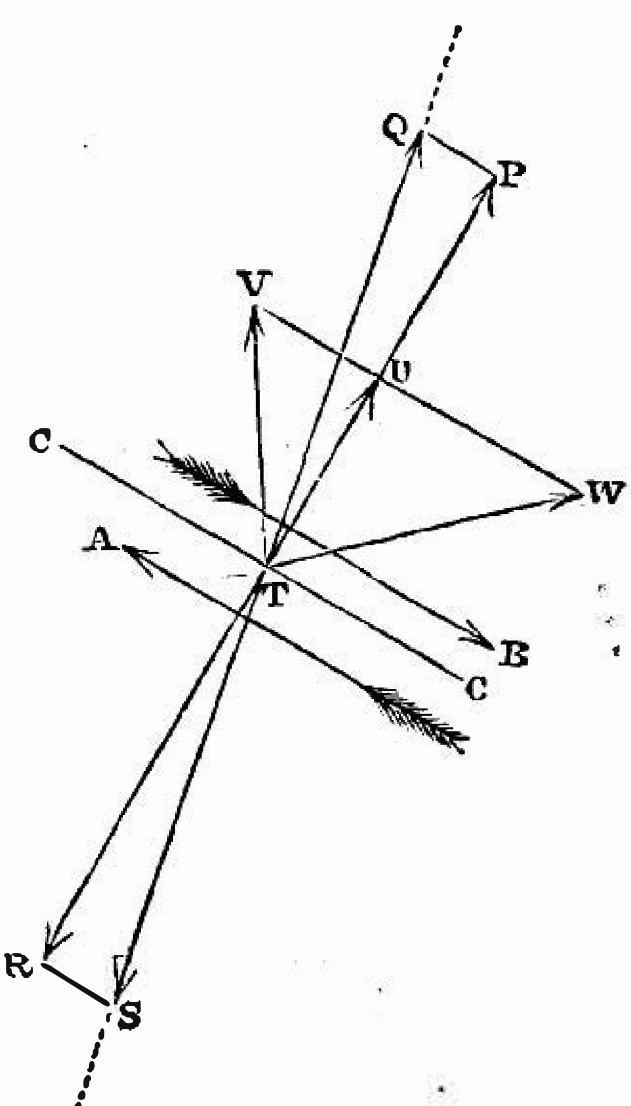

Fig. 265.

Along the line of connection, that is, normal to the acting surfaces at T, lay off T P to represent the effort exerted by the driver on the follower, and T R (= − T P) to represent the equal and opposite useful resistance exerted by the follower against the driver. Draw S T Q, making with R T P an angle equal to the angle of repose of the rubbing surfaces (see Article 309, page 349), and inclined in the proper direction to represent forces opposing the sliding motion; draw P Q and R S parallel to C C. Then T Q will represent the resultant pressure exerted by the driver on the follower, and T S (= − T Q), the equal and opposite resultant pressure exerted by the follower against the driver, and P Q = − R S will represent the friction which is overcome, through the distance V W, in each second; while the useful resistance, T R, is overcome through the distance T U. Hence the useful work per second is T U · T R; the lost work is V W · R S; and the counter-efficiency is

$$c = 1 + \frac{\mathrm{V\,W} \cdot \mathrm{R\,S}}{\mathrm{T\,U} \cdot \mathrm{T\,R}}. \qquad (1.)$$

Let the angle U T V = α, the angle U T W = β, and let f be the co-efficient of friction. Then we have—

$$\frac{\mathrm{V\,W}}{\mathrm{T\,U}} = \tan\alpha + \tan\beta; \quad \frac{\mathrm{R\,S}}{\mathrm{T\,R}} = f;$$

and consequently

$$c = 1 + f(\tan\alpha + \tan\beta). \qquad (2.)$$

380. **Efficiency of Teeth.**—It has already been shown, in Article 127, page 118, that the relative velocity of sliding of a pair of teeth in outside gearing is expressed at a given instant by

$$(a_1 + a_2)\, t;$$

where t denotes the distance at that instant of the point of contact from the pitch-point. (In inside gearing the angular velocity of the greater wheel is to be taken with the negative sign.)

The distance t is continually varying from a maximum at the beginning and end of the contact, to nothing at the instant of passing the pitch-point. Its *mean value* may be assumed, with sufficient accuracy for practical purposes, to be sensibly equal to *one half* of its greatest value; and in the formulæ which follow, the symbol t stands for that mean value.

Let P be the mutual pressure exerted by the teeth; f, the co-efficient of friction; then the work lost per second through the friction of the teeth is

$$(a_1 + a_2)\, t f \mathrm{P}.$$

Let u be the common velocity of the two pitch-circles; θ, the *mean obliquity* of the line of connection to the common tangent of the pitch-circles; then $u \cos \theta$ is the mean value of the common component of the velocities of the acting surfaces of the teeth along the line of connection; and the useful work done per second is expressed by

$$P u \cos \theta$$

so that the counter-efficiency is

$$c = 1 + \frac{(a_1 + a_2)\, t f}{u \cos \theta}. \quad \text{.....................(1.)}$$

Let r_1 and r_2 be the radii of the two pitch-circles; then we have

$$a_1 = \frac{u}{r_1};\ a_2 = \frac{u}{r_2};$$

and consequently

$$c = 1 + f t \sec \theta \left\{ \frac{1}{r_1} + \frac{1}{r_2} \right\}. \quad \text{..............(2.)}$$

If two pairs of teeth at least are to be in action at each instant (as in the case of involute teeth, and of some epicycloidal teeth), and if the pitch be denoted by p, we have $t \sec \theta = \frac{p}{2}$; and therefore

$$c = 1 + \frac{f p}{2} \left\{ \frac{1}{r_1} + \frac{1}{r_2} \right\} = 1 + \pi f \left\{ \frac{1}{n_1} + \frac{1}{n_2} \right\}; \ldots(3.)$$

where n_1 and n_2 are the number of teeth in the two wheels.

In many examples of epicycloidal teeth, especially where small pinions are used, the duration of the contact is only $\frac{2}{3}$ or $\frac{3}{4}$ of that assumed in equation (3); and the work lost in friction is less in the same proportion.

The preceding rules have been stated in the form applicable to spur-wheels. In order to make them applicable to bevel-wheels, all that is necessary is to understand that the measurements of radii, distances, and obliquity, are to be made, not on the actual pitch-circles, but on the pitch-circles as shown *on the development of the normal cones;* as to which, see Article 144, page 144.

When there is a *transverse component* in the relative velocity of sliding (as in gearing-screws, Article 154, page 160), the fractional value of the work lost in friction is to be first computed as if for a pair of spur-wheels whose pitch-circles are the osculating circles of the normal screw-lines (see Article 154, pages 161, 162; and Article 155, page 163). Then find in what ratio the velocity of sliding is increased by compounding the transverse component with the direct component $(a_1 + a_2)\, t$; and increase the fraction of work lost through friction in the same proportion.

381. **Efficiency of Bands.**—A band, such as a leather belt or a hempen rope, which is not perfectly elastic, requires the expenditure of a certain quantity of work—first to bend it to the curvature of a pulley, and then to straighten it again; and the quantity of work so lost has been found by experiment to be nearly the same as would be required in order to overcome an additional resistance, varying directly as the sectional area of the band, directly as its tension, and inversely as the radius of the pulley. In the following formulæ for leather belts, the stiffness is given as estimated by Reuleaux (*Constructionslehre für Maschinenbau*, § 307).

Let T be the mean tension of the belt; S, its sectional area; r, the radius of the pulley; b, a constant divisor determined by experiment; R′, the resistance due to stiffness; then

$$R' = \frac{S\,T}{b\,r}. \qquad \text{(1.)}$$

b (for leather) = 3·4 inches = 87 millimètres.

To apply this to an endless belt connecting a pair of pulleys of the respective radii r_1 and r_2, let T_1 and T_2 be the tensions of the two sides of the belt, as determined by the rule of Article 310 A, page 351. Then the useful resistance is $T_1 - T_2$; the mean tension is $\frac{T_1 + T_2}{2}$; and the additional resistance due to stiffness is

$$\frac{T_1 + T_2}{2} \cdot \frac{S}{b} \left\{ \frac{1}{r_1} + \frac{1}{r_2} \right\};$$

consequently the counter-efficiency is

$$\left.\begin{aligned} c &= 1 + \frac{T_1 + T_2}{2\,(T_1 - T_2)} \cdot \frac{S}{b} \left\{ \frac{1}{r_1} + \frac{1}{r_2} \right\}; \\ &= 1 + \frac{N + 1}{2\,(N - 1)} \cdot \frac{S}{b} \left\{ \frac{1}{r_1} + \frac{1}{r_2} \right\}; \end{aligned}\right\} \qquad \text{(2.)}$$

N denoting $\frac{T_1}{T_2}$, as in Article 383, page 432. The sectional area, S, of a leather belt is given by the formula

$$S = \frac{T_1}{p}; \qquad \text{(3.)}$$

where p denotes the safe working tension of leather belts, in units of weight per unit of area; its value being, according to Morin,

0·2 kilogramme on the square millimètre, or
285 lbs. on the square inch.

The ordinary thickness of the leather of which belts are made is about 0·16 of an inch, or 4 millimètres; and from this and from the area the breadth may be calculated. A double belt is of double thickness, and gives the same area with half the breadth of a single belt.

When a band runs at a high velocity, the *centrifugal tension*, or tension produced by centrifugal force, must be added to the tension required for producing friction on the pulleys, in order to find the total tension at either side of the band, with a view to determining its sectional area and its stiffness. The centrifugal tension is given by the following expression :—

$$\frac{w \mathrm{S} v^2}{g}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

in which w is the heaviness (being, for leather belts, nearly equal to that of water); S, the sectional area; v, the velocity; and g, gravity (= 32·2 feet, or 9·81 metres per second).

When centrifugal force is taken into consideration, the following formula is to be used for calculating the sectional area; T_1 being the tension at the driving-side of the belt, as calculated by the rules of Article 310 A, page 351, *exclusive of centrifugal tension*:—

$$\mathrm{S} = \frac{\mathrm{T}_1}{p - \frac{w v^2}{g}}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.)$$

and the following formula for the counter-efficiency:—

$$c = 1 + \frac{\mathrm{T}_1 + \mathrm{T}_2 + \frac{2 w v^2}{g}}{2(\mathrm{T}_1 - \mathrm{T}_2)} \cdot \frac{\mathrm{S}}{b} \cdot \left\{ \frac{1}{r_1} + \frac{1}{r_2} \right\} \ldots\ldots(6.)$$

The questions of areas of bands and centrifugal tension will be further considered in the part of this treatise relating to the strength of machinery.

For calculating the efficiency of hempen ropes used as bands, it is unnecessary in such questions as that of the present Article to use a more complex formula than that of Eytelwein—viz.,

$$\mathrm{R}' = \frac{\mathrm{D}^2 \mathrm{T}}{b' r}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7.)$$

where D is the diameter of the rope, and b' = 54 millimètres = 2·125 inches.

In all the formulæ, $\frac{D^2}{b'}$ is to be substituted for $\frac{S}{b}$. The proper value of D^2 is given by the formula

$$D^2 = \frac{T_1}{p'}; \dots\dots\dots\dots\dots\dots\dots\dots\dots (8.)$$

where $p' = 1000$ for measures in inches and lbs.; and
$p' = 0{\cdot}7$ for measures in millimètres and kilogrammes.

382. **Efficiency of Linkwork.**—In fig. 266, let $C_1 T_1$, $C_2 T_2$ be two levers, turning about parallel axes at C_1 and C_2, and connected with each other by the link $T_1 T_2$; T_1 and T_2 being the connected points.

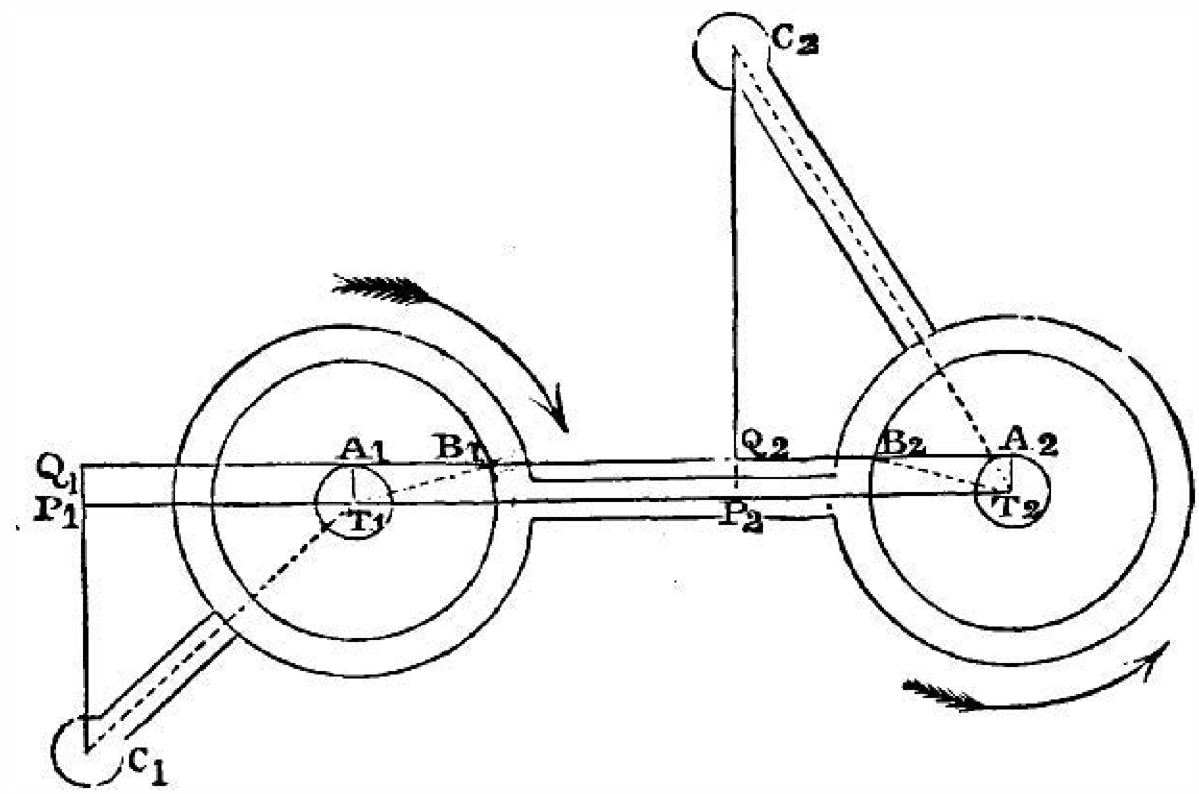

Fig. 266.

The pins, which are connected with each other by means of the link, are exaggerated in diameter, for the sake of distinctness. Let $C_1 T_1$ be the driver, and $C_2 T_2$ the follower, the motion being as shown by the arrows. From the axes let fall the perpendiculars $C_1 P_1$, $C_2 P_2$, upon the line of connection. Then the angular velocities of the driver and follower are inversely as those perpendiculars; and, in the absence of friction, the driving moment of the first lever and the working moment of the second are directly as those perpendiculars; the driving pressure being exerted along the line of connection $T_1 T_2$. Let M_2 be the working moment; and let M_0 be the driving moment in the absence of friction; then we have

$$M_0 = \frac{M_2 \cdot C_1 P_1}{C_2 P_2}.$$

To allow for the friction of the pins, multiply the radius of each pin by the sine of the angle of repose; that is, very nearly by the co-efficient of friction; and with the small radii thus computed, $T_1 A_1$ and $T_2 A_2$, draw small circles about the connected points. Then draw a straight line, $Q_1 A_1 B_1 Q_2 B_2 A_2$, touching both the small circles, and in such a position as to represent the line of action of a force that resists the motion of both pins in the eyes of the link. This will be the line of action of the resultant force exerted through the link. Let fall upon it the perpendiculars $C_1 Q_1$, $C_2 Q_2$; these will be proportional to the actual driving moment and working moment respectively; that is to say, let M_1 be the driving moment, including friction; then

$$M_1 = \frac{M_2 \cdot C_1 Q_1}{C_2 Q_2}.$$

Comparing this with the value of the driving moment without friction, we find for the counter-efficiency

$$c = \frac{M_1}{M_0} = \frac{C_1 Q_1 \cdot C_2 P_2}{C_2 Q_2 \cdot C_1 P_1}; \qquad (1.)$$

and for the efficiency

$$\frac{1}{c} = \frac{M_0}{M_1} = \frac{C_2 Q_2 \cdot C_1 P_1}{C_1 Q_1 \cdot C_2 P_2}. \qquad (2.)$$

(See page 449.)

383. **Efficiency of Blocks and Tackle.** (See Articles 200, 201, pages 214 to 216.)—In a tackle composed of a fixed and a running block containing sheaves connected together by means of a rope, let the number of plies of rope by which the blocks are connected with each other be n. This is also the collective number of sheaves in the two blocks taken together, and is the number expressing the *purchase*, when friction is neglected.

Let c denote the counter-efficiency of a single sheave, as depending on its friction on the pin, according to the principles of Article 373, page 427. Let c' denote the counter-efficiency of the rope, when passing over a single sheave, determined by the principles of Article 381, the tension being taken as nearly equal to $\frac{R}{n}$; where R is the useful load, or resistance opposed to the motion of the running block. $R \div n$ is also the effort to be exerted on the hauling part of the rope, in the absence of friction. Then the counter-efficiency of the tackle will be expressed approximately by

$$(c\, c')^n; \qquad (1.)$$

so that the actual or effective purchase, instead of being expressed by n, will be expressed by

$$n\, (c\, c')^{-n}. \qquad (2.)$$

384. **Efficiency of Connection by means of a Fluid.**—When motion is communicated from one piston to another by means of an intervening mass of fluid, as described in Articles 207 to 210, pages 221 to 224, the efficiencies and counter-efficiencies of the two pistons have in the first place to be taken into account; which quantities are to be determined by means of the principles stated at page 399; that is to say, with ordinary workmanship and packing, the efficiency of each piston may be taken at 0·9 nearly; while with a carefully made cupped leather collar the [redacted] efficiency of a plunger may be taken at the following value:—

$$1 - \frac{4\,b}{d};\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

in which d is the diameter of the plunger; and b a constant, whose value is from 0·01 to 0·015 of an inch, or from 0·25 to 0·38 of a millimètre. For if c be the circumference of the plunger, and p the effective pressure of the liquid, the whole amount of the pressure on the plunger is $\frac{p\,c\,d}{4}$; and the pressure required to overcome the friction is $p\,c\,b$.

The efficiency and counter-efficiency of the intervening mass of fluid remain to be considered; and if that fluid is a liquid, and may therefore be regarded as sensibly incompressible, these quantities depend on the work which is lost in overcoming the resistance of the passage which the liquid has to traverse.

To prevent unnecessary loss of work, that passage should be as wide as possible, and as nearly as possible of uniform transverse section; and it should be free from sudden enlargements and contractions, and from sharp bends, all necessary enlargements and contractions which may be required being made by means of gradually tapering conoidal parts of the passage, and all bends by means of gentle curves. When those conditions are fulfilled, let Q be the volume of liquid which is forced through the passage in a second; S, the sectional area of the passage; then,

$$v = \frac{\mathrm{Q}}{\mathrm{S}},\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.)$$

is the velocity of the stream of fluid. Let b denote the wetted border or circumference of the passage; then,

$$m = \frac{\mathrm{S}}{b},\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

is what is called the *hydraulic mean depth* of the passage. In a cylindrical pipe, $m = \frac{1}{4}$ diameter. Let l be the length of the passage, and w the heaviness of the liquid. Then the *loss of pressure* in overcoming the friction of the passage is

$$p' = \frac{f\,l}{m} \cdot \frac{w\,v^2}{2\,g}; \qquad (4.)$$

in which g denotes gravity, and f a co-efficient of friction whose value, for water in cylindrical cast-iron pipes, according to the experiments of Darcy, is

$$f = 0{\cdot}005 \left(1 + \frac{1}{12\,d}\right);* \qquad (5.)$$

d being the diameter of the pipe in feet.

Let p be the pressure on the driven or following piston; then the pressure on the driving piston is $p + p'$; and the *counter-efficiency of the fluid* is

$$1 + \frac{p'}{p}; \qquad (6.)$$

which, being multiplied by the product of the counter-efficiencies of the two pistons, gives the *counter-efficiency of the intervening liquid.*

When the intervening fluid is *air*, there is a loss of work through friction of the passage, depending on principles similar to those of the friction of liquids; and there is a further loss through the escape by conduction of the heat produced by the compression of the air.

The friction which has to be overcome by the air, and which causes a certain loss of pressure between the compressing pumps and the working machinery, consists of two parts, one occasioned by the resistance of the valves, and the other by the friction along the internal surface of pipes.

To overcome the resistance of valves, about *five per cent.* of the effective pressure may be allowed.

The friction in the pipes depends on their length and diameter, and on the velocity of the current of air through them. It is nearly proportional to the square of the velocity of the air.

A velocity of about *forty feet per second* for the air in its compressed state has been found to answer in practice. The diameter of pipe required in order to give that velocity can easily be computed, when the dimensions of the cylinders of the machinery to be driven, and the number of strokes per minute, are given.

When the diameter of a pipe is so adjusted that the velocity of the air is 40 feet per second, the pressure expended in overcoming

* When the diameter is expressed in millimètres, for $\frac{1}{12\,d}$ substitute $\frac{25{\cdot}4}{d}$.

and when diameter is [illegible]

[illegible]

its friction may be estimated at *one per cent. of the total or absolute pressure of the air, for every five hundred diameters of the pipe that its length contains.*

Although the abstraction from the air of the heat produced by the compression involves a certain sacrifice of motive power (say from 30 to 35 per cent.) still the effects of the heated air are so inconvenient in practice, that it is desirable to cool it to a certain extent during or immediately after the compression. This may be effected by injecting water in the form of spray into the compressing pumps; and for that purpose a small forcing pump of about $\frac{1}{100}$th of the capacity of the compressing pumps has been found to answer in practice. The air may be thus cooled down to about 104° Fahr. or 40° Cent.

The factor in the counter-efficiency due to the loss of heat expresses the ratio in which the volume of air as discharged from the compressing pump at a high temperature is greater than the volume of the same air when it reaches the working machinery at a reduced temperature; which ratio may be calculated approximately by taking *two-sevenths of the logarithm of the absolute working pressure of the compressed air in atmospheres, and finding the corresponding natural number.* That is to say, let p_0 denote one atmosphere (= at the level of the sea 14·7 lbs. on the square inch, or 10333 kilogrammes on the square mètre); let p_1 be the absolute working pressure of the air, so that $p_1 - p_0$ is the effective pressure; then the counter-efficiency due to the escape of heat is,

$$c = \left(\frac{p_1}{p_0}\right)^{\frac{2}{7}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.)$$

From examples of the practical working of compressed air, when used to transmit motive power to long distances, it appears that in order to provide for leakage and various other imperfections in working, the capacity of the compressing pumps should be very nearly double of the net volume of uncompressed air required; and it has also been found necessary, in working the compressing pumps, to provide from three to four times the power of the machinery driven by the compressed air.

ADDENDUM TO ARTICLE 343, PAGE 386.

Rotatory Dynamometers—Epicyclic-Train Dynamometer.—The term of "epicyclic-train dynamometers" may be applied to those instruments in which the power to be measured is transmitted through an epicyclic train, and the effort exerted is measured by means of the force required to *hold the train-arm at rest.* In King's dynamometer, for example, there is a train of wheel-work of which the principle (though not the details) is sufficiently well represented by fig. 176, page 245. The bevel-wheel B is driven by the prime mover; and through the bevel-wheels (or bevel-wheel, there being usually only one) carried by the arm A, it drives the bevel-wheel C, which drives the working machinery. The train-arm A is kept steady by a weight, or by a spring; and it is obvious that the moment of that force relatively to the common axis of rotation of B, C, and A, must be *double* of the moment transmitted from B to C; which latter moment—that is, half the moment of the weight or spring that holds A steady, being multiplied by $2\pi \times$ the number of turns in a given time, gives the work done in that time. This apparatus may be made to record its results on a travelling strip of paper, like other kinds of dynamometers.

Addenda to Article 381, Page 440.

I. **Use of Relieving Rollers between Pulleys.**—When a pair of pulleys connected with each other by means of a band are near together, the bearings of their shafts may be relieved from the pressure due to the tension of the band by placing between the pulleys a smooth idle wheel or roller, turning in rolling contact with them both. The axis of rotation of the roller should be in the same plane with those of the pulleys; and two out of the three shafts should have their bearings so fitted up as to be capable of a small extent of motion in a direction perpendicular to the axes of rotation, in order that the distances of those axes from each other may adjust themselves when the band is tightened, and that the tension of the band and the pressure transmitted through the roller may balance each other without the aid of pressures at the bearings.

II. **Efficiency of Telodynamic Transmission.**—The phrase "Telodynamic Transmission" is used to denote Mr. C. F. Hirn's method of transmitting motive power to long distances by means of an endless wire rope, connecting a pair of large pulleys, and moving at a high speed. The pulleys are made of cast iron; and each of them has at the bottom of its groove a dovetail-shaped recess filled with gutta-percha, which is driven in and rammed tight by means of a mallet; the wire rope bears against the gutta-percha bottom of the groove; and this is found both to transmit an effort better, and to ensure greater durability of the rope and pulleys, than when the rope bears against a cast-iron surface.

The ordinary speed of the rope is from 50 to 80 feet per second; and with wrought-iron pulleys, it is considered that it might be increased to 100 feet per second. The effort to be transmitted is calculated from the power to be transmitted, by expressing that power in units of work per second, and dividing by the speed. The available tensions at the driving and returning sides of the rope are calculated by the rules of Article 310 A, page 351; in practice it is considered sufficiently accurate to make the former *twice*, and the latter *once*, the effort to be transmitted. To each of those tensions is to be added the centrifugal tension (see Article 381, page 441) in order to obtain the total tensions. The transverse dimensions of the rope are adapted to the *total tension at the driving side of the rope*, by the application of rules to be given in the Part of this Treatise relating to strength.

In order that the rope may not be overstrained by the bending of the wires of which it consists, in passing round the driving and following pulleys, the diameter of each of those pulleys should not be less than 140 times the diameter of the rope, and is sometimes as much as 260 times.

The distance between the driving and following pulleys is not made less than about 100 feet; for at less distances shafting is more efficient; nor is it made more than 500 feet in one span, because of the great depth of the catenary curves in which the rope hangs. When the distance between the driving and following pulleys exceeds 500 feet, the rope is supported at intermediate points by pairs of bearing pulleys, so as to divide the whole distance into intervals of 500 feet or less.

The bearing pulleys are constructed in the same way with the driving and following pulleys, and of about half the diameter.

The loss of work due to the stiffness of the rope may be regarded as insensible; because when the diameters of the pulleys are sufficient, the wires of which the rope is made straighten themselves by their own elasticity after having been bent.

It has been found by practical experience that the losses of power in this apparatus are nearly as follows, in fractions of the whole power transmitted:—

Overcoming the axle-friction of the driving and following pulleys, about $\frac{1}{40}$, or	0·0250
Overcoming the axle-friction of each pair of bearing pulleys, about $\frac{1}{900}$, or	0·0011

Hence the efficiency of telodynamic transmission may be estimated at

$$0{\cdot}975 - \frac{N}{900};$$

N being the number of pairs of intermediate bearing pulleys.*

* For detailed information on the subject of Telodynamic Transmission, see the following authorities:—*Notice sur la Transmission Telodynamique*,

ADDENDUM TO ARTICLE 382, PAGE 442.

Effect of Obliquity of a Connecting-Rod on Friction.—The alternate thrust and tension along the connecting-rod is almost always an important component, and sometimes the most important component, of the force which is balanced by the pressure of the bearings of a crank-shaft; and the lateral component of that alternate thrust and tension is the cause of the friction of the guides by which the head of the piston-rod is made to move in a straight line, when there is no parallel motion.

The direction of the connecting-rod is continually changing between certain limits; and this causes a continual change in the ratio borne by the whole force exerted along that rod, and by its lateral component, to its direct component.

Let r be the crank-arm, c the length of the connecting-rod; then the *mean* value of the ratio which the lateral component bears to the direct component is very nearly as follows:—

$$\frac{Q}{P} = \frac{0{\cdot}7854\, r}{\sqrt{(c^2 - 0{\cdot}617\, r^2)}}; \qquad (1.)$$

and if f be the co-efficient of friction of the guides, the counter-efficiency of the ~~piston-rod~~ head will be nearly

$$c = 1 + \frac{fQ}{P}. \qquad (2.)$$

The mean ratio borne by the total force (T) exerted along the connecting-rod to its direct component (P) is nearly as follows:—

$$\frac{T}{P} = \frac{c}{\sqrt{(c^2 - 0{\cdot}617\, r^2)}}; \qquad (3.)$$

and the axle-friction of the crank-shaft is increased nearly in that ratio, beyond what it would be if the obliquity of the connecting-rod were insensible.*

par C. Ft Hirn (Colmar, 1862). Reuleaux, *Constructionslehre für Maschinenbau* (Braunschweig, 1854 to 1862), §§ 324 to 342.

* The exact solution of these questions is given by the aid of elliptic functions; but for practical purposes the approximate solution in the text is sufficient.