CHAPTER III.

OF REGULATING APPARATUS.

349. **Regulating Apparatus Classed—Brake—Fly—Governor.**—The effect of all regulating apparatus is to control the speed of machinery. A regulating instrument may act simply by consuming energy, so as to prevent acceleration, or produce retardation, or stop the machine if required; it is then called a *brake;* or it may act by storing surplus energy at one time, and giving it out at another time, when energy is deficient: in this case it is called a *fly;* or it may act by adjusting the power of the prime mover to the work to be done, when it is called a *governor.* The use of a brake involves waste of power. A fly and a governor, on the other hand, promote economy of power and economy of strength.

SECTION I.—*Of Brakes.*

350. **Brakes Defined and Classed.**—The contrivances here comprehended under the general title of *Brakes* are those by means of which friction, whether exerted amongst solid or fluid particles, is purposely opposed to the motion of a machine, in order either to stop it, to retard it, or to employ superfluous energy during uniform motion. The use of a brake involves waste of energy, which is in itself an evil, and is not to be incurred unless it is necessary to convenience or safety.

Brakes may be classed as follows:—

I. *Block-brakes,* in which one solid body is simply pressed against another, on which it rubs.

II. *Flexible brakes,* which embrace the periphery of a drum or pulley (as in Prony's Dynamometer, Article 341, page 383).

III. *Pump-brakes,* in which the resistance employed is the friction amongst the particles of a fluid forced through a narrow passage.

IV. *Fan-brakes,* in which the resistance employed is that of a fluid to a fan rotating in it.

351. **Action of Brakes in General.**—The work disposed of by a brake in a given time is the product of the resistance which it produces into the distance through which that resistance is overcome in a given time.

To *stop* a machine, the brake must employ work to the amount of the whole actual energy of the machine, as already stated in

Article 334. To *retard* a machine, the brake must employ work to an amount equal to the difference between the actual energies of the machine at the greater and less velocities respectively.

To *dispose of surplus energy*, the brake must employ work equal to that energy; that is, the resistance caused by the brake must balance the surplus effort to which the surplus energy is due; so that if n is the ratio which the velocity of rubbing of the brake bears to the velocity of the driving point, P, the *surplus effort* at the driving point, and R the resistance of the brake, we ought to have—

$$R = \frac{P}{n}. \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

It is obviously better, when practicable, to store surplus energy, or to prevent its exertion, than to dispose of it by means of a brake.

When the action of a brake composed of solid material is long-continued, a stream of water must be supplied to the rubbing surfaces, to abstract the heat that is produced by the friction, according to the law stated in Article 311, page 354.

352. **Block-Brakes.**—When the motion of a machine is to be controlled by pressing a block of solid material against the rim of a rotating drum, it is advisable, inasmuch as it is easier to renew the rubbing surface of the block than that of the drum, that the drum should be of the harder, and the block of the softer material—the drum, for example, being of iron, and the block of wood. The best kinds of wood for this purpose are those which have considerable strength to resist crushing, such as elm, oak, and beech. The wood forms a facing to a frame of iron, and can be renewed when worn.

When the brake is pressed against the rotating drum, the direction of the pressure between them is obliquely opposed to the motion of the drum, so as to make an angle with the radius of the drum equal to the *angle of repose* of the rubbing surfaces (denoted by φ; see page 349). The component of that oblique pressure in the direction of a tangent to the rim of the drum is the friction (R); the component perpendicular to the rim of the drum is the normal pressure (N) required in order to produce that friction, and is given by the equation

$$N = \frac{R}{f}; \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

f being the co-efficient of friction, and the proper value of R being determined by the principles stated in Article 351.

It is in general desirable that the brake should be capable of effecting its purpose when pressed against the drum by means of the strength of one man, pulling or pushing a handle with one hand or one foot. As the required normal pressure N is usually considerably greater than the force which one man can exert, a lever, or screw, or a train of levers, screws, or other convenient mechanism, must be interposed between the brake block and the handle, so that when the block is moved towards the drum, the handle shall move at least through a distance as many times greater than the distance by which the block *directly* approaches the drum, as the required normal pressure is greater than the force which the man can exert.

Although a man may be able occasionally to exert with one hand a force of 100 lbs., or 150 lbs., for a short time, it is desirable that, in working a brake, he should not be required to exert a force greater than he can keep up for a considerable time, and exert repeatedly in the course of a day, without fatigue—that is to say, about 20 lbs. or 25 lbs.

353. The **Brakes of Carriages** are usually of the class just described, and are applied either to the wheels themselves or to drums rotating along with the wheels. Their effect is to stop or to retard the rotation of the wheels, and make them slip, instead of rolling on the road or railway. The resistance to the motion of a carriage which is caused by its brake may be less, but cannot be greater, than the friction of the stopped or retarded wheels on the road or rails under the load which rests on those wheels. The distance which a carriage or train of carriages will run on a level line during the action of the brakes before stopping, is found by dividing the actual energy of the moving mass before the brakes are applied, by the sum of the ordinary resistance and of the additional resistance caused by the brakes; in other words, that distance is as many times greater than the height due to the speed as the weight of the moving mass is greater than the total resistance.

The *skid*, or *slipper-drag*, being placed under a wheel of a carriage, causes a resistance due to the friction of the skid upon the road or rail under the load that rests on the wheel.

354. **Flexible Brakes.** (*A. M.*, 678.)—A flexible brake embraces a greater or less arc of the rim of a drum or pulley whose motion it resists. In some cases it consists of an iron strap, of a radius naturally a little greater than that of the drum; so that when left free, the strap remains out of contact with the drum, and does not resist its motion; but when tension is applied to the ends of the strap, it clasps the drum, and produces the required friction. The rim of the drum may be either of iron or of wood. In other cases the brake consists of a chain, or jointed series of iron bars, usually faced with wooden blocks on the side next the drum. When tension is applied to the ends of the chain, the blocks clasp the drum and produce friction; when that tension is removed, the blocks are

drawn back from the drum by springs to which they are attached, and the friction ceases.

The following formulæ are exact for perfectly flexible continuous bands, and approximate for elastic straps and for chains of blocks. Their demonstration has already been given in Article 310 A, page 35[illegible].

In fig. 254, let A B be the drum, and C its axis, and let the direction of rotation of the drum be indicated by the arrow. Let T_1 and T_2 represent the tensions at the two ends of the strap, which embraces the rim of the drum throughout the arc A B. The tension T_1 exceeds the tension T_2 by an amount equal to the friction between the strap and drum, R; that is,

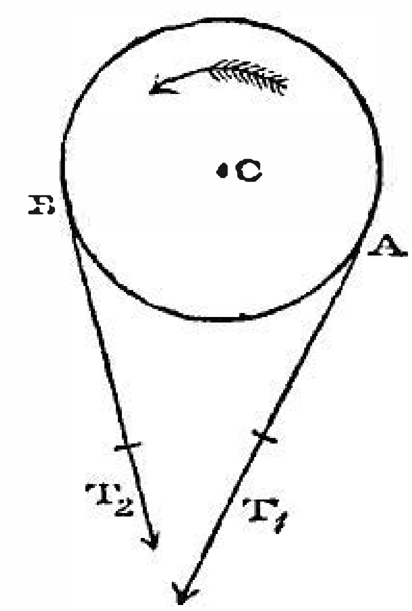

Fig. 254.

$$R = T_1 - T_2.$$

Let c denote the ratio which the arc of contact, A B, bears to the *circumference* of the drum; f, the co-efficient of friction between the strap and drum; then the ratio $T_1 : T_2$ is *the number whose common logarithm is* $2{\cdot}7288\,fc$, or

$$\frac{T_1}{T_2} = 10^{2\cdot7288\,fc} = N; \quad\ldots\ldots\ldots\ldots\ldots\ldots(1.)$$

which number having been found, is to be used in the following formulæ for finding the tensions, T_1, T_2, required in order to produce a given resistance, R :—

Backward or greatest tension, $T_1 = R \cdot \dfrac{N}{N-1}; \ldots\ldots\ldots\ldots(2.)$

$T_1 = N T_2$

Forward or least tension, $T_2 = R \cdot \dfrac{1}{N-1}. \ldots\ldots\ldots\ldots(3.)$

The following cases occur in practice :—

I. When it is desired to produce *a great resistance compared with the force applied to the brake*, the backward end of the brake, where the tension is T_1, is to be fixed to the framework of the machinery, and the forward end moved by means of a lever or other suitable mechanism; when the force to be applied by means of that mechanism will be T_2, which, by making N sufficiently great, may be made small as compared with R.

II. When it is desired that *the resistance shall always be less than a certain given force*, the forward end of the brake is to be fixed, and the backward end pulled with a force not exceeding the given force. This will be T_1; and, as the equation 2 shows, how great soever N may be, R will always be less than T. This is the principle of the brake applied by Sir William Thomson to apparatus for paying out submarine telegraph cables, with a view to limiting the resistance within the amount which the cable can safely bear.

In any case in which it is desired to give a great value to the ratio N, the flexible brake may be coiled spirally round the drum, so as to make the arc of contact greater than one circumference.

355. **Pump-Brakes.**—The resistance of a fluid, forced by a pump through a narrow orifice, may be used to dispose of superfluous energy; as in the "cataract," or "dash-pot."

The energy which is expended in forcing a given weight of fluid through an orifice is found by multiplying that weight into the height due to the greatest velocity which its particles acquire in that process, and into a factor greater than unity, which for each kind of orifice is determined experimentally, and whose excess above unity expresses the proportion which the energy expended in overcoming the friction between the fluid and the orifice bears to the energy expended in giving velocity to the fluid.

The following are some of the values of that factor, which will be denoted by $1 + F$:—

For an orifice in a thin plate, $1 + F = 1{\cdot}054$ .... (1.)

For a straight uniform pipe of the length l, and whose *hydraulic mean depth*, that is, the area divided by the circumference of its cross-section, is m,

$$1 + F = 1{\cdot}505 + \frac{fl}{m} \qquad \text{(2.)}$$

For cylindrical pipes, m is one-fourth of the diameter.

The factor f in the last formula is called the *co-efficient of friction* of the fluid. For *water in iron pipes*, the diameter d being expressed in feet, its value, according to Darcy, is

$$f = 0{\cdot}005\left(1 + \frac{1}{12d}\right) \qquad \text{(3.)}$$

For *air*, $f = 0{\cdot}006$ nearly .... (4.)

The greatest velocity of the fluid particles is found by dividing the volume of fluid discharged in a second by the area of the outlet at its most contracted part. When the outlet is a cylindrical pipe, the sectional area of that pipe may be employed in this calculation; but when it is an orifice in a thin plate, there is a *contracted vein* of the issuing stream after passing the orifice, whose area is on an average about 0·62 of the area of the orifice itself; and that contracted area is to be employed in computing the velocity. Its ratio to the area of the orifice in the plate is called the *co-efficient of contraction.*

The computation of the energy expended in forcing a given quantity of a given fluid in a given time through a given outlet, is expressed symbolically as follows :—

Let V be the volume of fluid forced through, in units of volume per second.

D, the heaviness of the fluid (see page 326).

A, the area of the orifice.

c, the co-efficient of contraction.

v, the velocity of outflow.

R, the resistance overcome by the piston of the pump in driving the water.

u, the velocity of that piston.

Then

$$v = \frac{V}{c\,A}; \qquad (5.)$$

and

$$R\,u = D\,V\,(1 + F)\,\frac{v^2}{2\,g}; \qquad (6.)$$

the factor $1 + F$ being computed by means of the formulæ 1, 2, 3, 4.

To find the intensity of the pressure (p) within the pump, it is to be observed, as in Article 302, that if A′ denotes the area of the piston,

$$V = A'\,u;\ R = p\,A'; \qquad (7.)$$

consequently,

$$p = \frac{R}{A'} = D\,(1 + F)\cdot\frac{v^2}{2\,g}; \qquad (8.)$$

that is, the *intensity of the pressure is that due to the weight of a vertical column of the fluid, whose height is greater than that due to the velocity of outflow in the ratio* $1 + F : 1$.

To allow for the friction of the piston, about *one-tenth* may in general be added to the result given by equation 6. (See page 399.)

The piston and pump have been spoken of as single; and such may be the case when the velocity of the piston is uniform. When a piston, however, is driven by a crank on a shaft rotating at an uniform speed, its velocity varies; and when a pump-brake is to be applied to such a shaft, it is necessary, in order to give a sufficiently near approximation to an uniform velocity of outflow, that there should be at least either three single acting pumps, driven by three cranks making with each other angles of 120°, or a pair of double-acting pumps, driven by a pair of cranks at right angles to each other; and the result will be better if the pumps force the fluid into one common air vessel before it arrives at the resisting orifice.

That orifice may be provided with a valve, by means of which its area can be adjusted so as to cause any required resistance.

A pump-brake of a simple kind is exemplified in the apparatus called the "*cataract*," for regulating the opening of the steam valve in single-acting steam engines. It is fully described in most special treatises on those engines.*

356. **Fan-Brakes.**—A fan, or wheel with vanes, revolving in water, oil, or air, may be used to dispose of surplus energy; and the resistance which it causes may be rendered to a certain extent adjustable at will, by making the vanes so as to be capable of being set at different angles with their direction of motion, or at different distances from their axis.

Fan-brakes are applied to various machines, and are usually adjusted so as to produce the requisite resistance by trial. It is, indeed, by trial only that a final and exact adjustment can be effected; but trouble and expense may be saved by making, in the first place, an approximate adaptation of the fan to its purpose by calculation.

The following formulæ are the results of the experiments of Duchemin, and are approved of by Poncelet in his *Mécanique Industrielle*:—

For a thin flat vane, whose plane traverses its axis of rotation, let

A denote the area of the vane;

l, the distance of its centre of area from the axis of rotation;

s, the distance from the centre of area of the entire vane to the centre of area of that half of it which lies nearest the axis of rotation;

v, the velocity of the centre of area of the vane ($= a\, l$, if a is the angular velocity of rotation);

D, the heaviness of the fluid in which it moves;

R l, the moment of resistance;

k, a co-efficient whose value is given by the formula

$$k = 1{\cdot}254 + 1{\cdot}6244 \frac{\sqrt{\mathrm{A}}}{l - s}; \quad \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

then

$$\mathrm{R}\, l = l\, k\, \mathrm{D}\, \mathrm{A} \cdot \frac{v^2}{2\, g}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.)$$

When the vane is oblique to its direction of motion, let i denote

* Pump-brakes have been applied to railway carriages by Mr. Laurence Hill. Hydraulic buffers, which act on the same principle, have been applied to railway carriages by Colonel Clark, R.A.

the acute angle which its surface makes with that direction; then the result of equation 2 is to be multiplied by

$$\frac{2 \sin^2 i}{1 + \sin^2 i} \text{..............a............(3.)}$$

It appears that the resistance of a fan with several vanes increases nearly in proportion to the number of vanes, so long as their distances apart are not less at any point than their lengths. Beyond that limit the law is uncertain.

SECTION II.—*Of Fly-Wheels.*

357. **Periodical Fluctuations of Speed** in a machine (*A. M.*, 689) are caused by the alternate excess and deficiency of the energy exerted above the work performed in overcoming resisting forces, which produce an alternate increase and diminution of actual energy, according to the law explained in Article 330, page 373.

To determine the greatest fluctuation of speed in a machine moving periodically, take A B C, in fig. 255, to represent the motion of the driving point during one period; let the effort P of the prime mover at each instant be represented by the ordinate of the curve D G E I F; and let the sum of the resistances, reduced to the driving point as in Article 305, at each instant, be denoted by R, and represented by the ordinate of the curve D H E K F, which cuts the former curve at the ordinates A D, B E, C F. Then the integral,

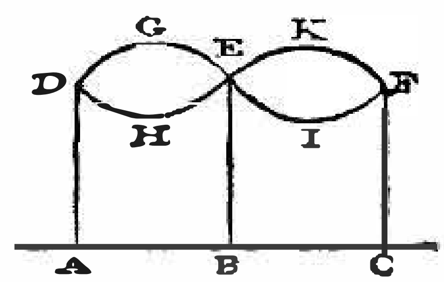

Fig. 255.

$$\int (\mathrm{P} - \mathrm{R})\, d\, s,$$

being taken for any part of the motion, gives the excess or deficiency of energy, according as it is positive or negative. For the entire period A B C, this integral is nothing. For A B, it denotes an *excess of energy received*, represented by the area D G E H; and for B C, an equal *excess of work performed*, represented by the equal area E K F I. Let those equal quantities be each represented by Δ E. Then the actual energy of the machine attains a maximum value at B, and a minimum value at A and C, and Δ E is the difference of those values.

Now let v_0 be the mean velocity, v_1 the greatest velocity, v_2 the least velocity of the driving point, and $\Sigma \cdot n^2$ W the *reduced inertia* of the machine (see Article 315, page 362); then

$$\frac{v_1^2 - v_2^2}{2g} \cdot \Sigma \cdot n^2 \mathrm{W} = \Delta \mathrm{E}; \text{.....a..........(1.)}$$

which, being divided by the *mean actual energy*,

$$\frac{v_0^2}{2g} \cdot \Sigma \cdot n^2 \mathrm{W} = \mathrm{E}_0,$$

gives

$$\frac{v_1^2 - v_2^2}{v_0^2} = \frac{\Delta \mathrm{E}}{\mathrm{E}_0}; \qquad \text{(2.)}$$

and observing that $v_0 = (v_1 + v_2) \div 2$, we find

$$\frac{v_1 - v_2}{v_0} = \frac{\Delta \cdot \mathrm{E}}{2\,\mathrm{E}_0} = \frac{g \,\Delta\, \mathrm{E}}{v_0^2 \,\Sigma \cdot n^2 \mathrm{W}}; \qquad \text{(3.)}$$

a ratio which may be called the *co-efficient of fluctuation of speed* or of *unsteadiness*.

The ratio of the periodical excess and deficiency of energy $\Delta\,\mathrm{E}$ to the whole energy exerted in one period or revolution, $\int \mathrm{P}\, d\, s$, has been determined by General Morin for steam engines under various circumstances, and found to be from $\frac{1}{10}$ to $\frac{1}{4}$ for single-cylinder engines. For a pair of engines driving the same shaft, with cranks at right angles to each other, the value of this ratio is about one-fourth, and for three engines with cranks at 120°, one-twelfth of its value for single-cylinder engines.

The following table of the ratio, $\Delta\,\mathrm{E} \div \int \mathrm{P}\, d\, s$, for *one revolution* of steam engines of different kinds is extracted and condensed from General Morin's works:—

NON-EXPANSIVE ENGINES.

$\frac{\text{Length of connecting rod}}{\text{Length of crank}}$	=	8	6	5	4
$\Delta\,\mathrm{E} \div \int \mathrm{P}\, d\, s$	=	·105	·118	·125	·132

EXPANSIVE CONDENSING ENGINES.

Connecting rod = crank × 5.

Fraction of stroke at which steam is cut off,	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
$\Delta\,\mathrm{E} \div \int \mathrm{P}\, d\, s$ =	·163	·173	·178	·184	·189	·191

EXPANSIVE NON-CONDENSING ENGINES.

Steam cut off at	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\Delta E \div \int P\, ds =$	·160	·186	·209	·232

For double-cylinder expansive engines, the value of the ratio $\Delta E \div \int P\, ds$ may be taken as equal to that for single-cylinder non-expansive engines.

For *tools working at intervals*, such as punching, slotting, and plate-cutting machines, coining presses, &c., ΔE is nearly equal to the whole work performed at each operation.

358. **Fly-Wheels.**—(*A. M.*, 690.)—A fly-wheel is a wheel with a heavy rim, whose great moment of inertia being comprehended in the reduced moment of inertia of a machine, reduces the co-efficient of fluctuation of speed to a certain fixed amount, being about $\frac{1}{32}$ for ordinary machinery, and $\frac{1}{50}$ or $\frac{1}{60}$ for machinery for fine purposes.

Let $\frac{1}{m}$ be the intended value of the co-efficient of fluctuation of speed, and ΔE, as before, the fluctuation of energy. If this is to be provided for by the moment of inertia, I, of the fly-wheel alone, let a_0 be its mean angular velocity; then equation 3 of Article 357 is equivalent to the following :—

$$\frac{1}{m} = \frac{g \Delta E}{a_0^2 I}; \qquad (1.)$$

$$I = \frac{m g \Delta E}{a_0^2}; \qquad (2.)$$

the second of which equations gives the requisite moment of inertia of the fly-wheel.

The fluctuation of energy may arise either from variations in the effort exerted by the prime mover, or from variations in the resistance, or from both those causes combined. When but one fly-wheel is used, it should be placed in as direct connection as possible with that part of the mechanism where the greatest amount of the fluctuation originates; but when it originates at two or more points, it is best to have a fly-wheel in connection with each of those points.

For example, let there be a steam engine which drives a shaft that traverses a workshop, having at intervals upon it pulleys for driving various machine-tools. The steam engine should have a fly-wheel of its own, as near as practicable to its crank, adapted to that value of Δ E which is due to the fluctuations of the effort applied to the crank-pin above and below the mean value of that effort, and which may be computed by the aid of General Morin's tables, quoted in Article 357; and each machine-tool should also have a fly-wheel, adapted to a value of Δ E equal to the whole work performed by the tool at one operation.

As the rim of a fly-wheel is usually heavy in comparison with the arms, it is often sufficiently accurate for practical purposes to take the moment of inertia as simply equal to the weight of the rim multiplied by the square of the mean between its outside and inside radii—a calculation which may be expressed thus:—

$$\mathrm{I} = \mathrm{W} r^2; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.)$$

whence the weight of the rim is given by the formula—

$$\mathrm{W} = \frac{m g \Delta \mathrm{E}}{a_0^2 r^2} = \frac{m g \Delta \mathrm{E}}{v'^2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.)$$

if v' be the velocity of the rim of the fly-wheel.

In millwork the ordinary values of the product $m g$, the unit of time being the second, lie between 1,000 and 2,000 feet, or approximately between 300 and 600 mètres. In pumping-machinery it is sometimes only about 300 feet, or 90 mètres.

The rim of the fly-wheel of a factory steam engine is very often provided with teeth, or with a belt, in order that it may directly drive the machinery of the factory.

Section III.—*Of Governors.*

359. The **Regulator** of a prime mover is some piece of apparatus by which the rate at which it receives energy from the source of energy can be varied; such as the sluice or valve which adjusts the size of the orifice for supplying water to a water-wheel, the apparatus for varying the surface exposed to the wind by windmill sails, the throttle-valve which adjusts the opening of the steam pipe of a steam engine, the damper which controls the supply of air to its furnace, and the expansion gear which regulates the volume of steam admitted into the cylinder at each stroke of the piston.

In prime movers whose speed and power have to be frequently and rapidly varied at will, such as locomotives and winding engines for mines, the regulator is adjusted by hand. In other cases the regulator is adjusted by means of a self-acting instrument driven by the prime mover to be regulated, and called a Governor.

The special construction of the different kinds of regulators is a subject for a treatise on prime movers. In the present treatise it is sufficient to state that in every governor there is a moving piece which acts on the regulator through a suitable train of mechanism, and which is itself made to move in one direction or in another according as the prime mover is moving too fast or too slow.

The object of a governor, properly so called, is to preserve a certain uniform speed, either exactly or approximately; and such is always the case in millwork. There are other cases, as in marine steam engines, where it may be considered sufficient to prevent sudden variations of speed, without preserving an uniform speed; and in those cases an apparatus may be used possessing only in part the properties of a governor: this may be called a *fly-governor*, to distinguish it from a governor proper.

Governors proper may be distinguished into *position-governors*, *disengagement-governors*, and *differential-governors*: a position-governor being one in which the moving piece that acts on the regulator assumes positions depending on the speed of motion, as in the common steam engine governor, which consists of a pair of revolving pendulums acting directly on a train of mechanism which adjusts the throttle-valve: a disengaging-governor being one which, when the speed deviates above or below its proper value, throws the regulator into gear with one or other of two trains of mechanism which move it in contrary directions so as to diminish or increase the speed, as the case may require, as in water-mill governors; and a differential-governor being one which, by means of an aggregate combination, moves the regulator in one direction or in another with a speed proportional to the difference between the actual speed and the proper speed of the engine.

In almost all governors the action depends on the centrifugal force exerted by two or more masses which revolve round an axis. By another classification, different from that which has already been described, governors may be distinguished into *gravity-governors*, in which gravity is the force that opposes the centrifugal force; and *balanced governors*, in which the actions of gravity on the various moving parts of the governor are mutually balanced, and the centrifugal force is opposed by the elasticity of a spring.

Governors may be further distinguished into those which are truly isochronous—that is to say, which remain without action on the regulator at one speed only; and those which are nearly isochronous—that is to say, which admit of some variation of the permanent or steady speed when the resistance overcome by the engine varies; and lastly, governors may be distinguished into those which are specially adapted to one speed, and those which can be adjusted at will to different speeds.

360. **Pendulum-Governors.**—A pendulum-governor is the simplest kind of gravity-governor. It has a vertical spindle, driven by the engine to be regulated; and from that spindle there hang, at

opposite sides, a pair of revolving pendulums, which, by the positions that they assume at different speeds, act on the regulator.

The relation between the height of a simple revolving pendulum and the number of turns which it makes per second has already been stated in Article 319; but for the sake of convenience it may here be repeated:—Let h denote the height or *altitude* of the pendulum ($=$ O H in fig. 256), and T the number of turns per second; then

$$h = \frac{g}{4\pi^2\,\mathrm{T}^2} = \frac{\cdot 815 \text{ foot}}{\mathrm{T}^2} = \frac{9\cdot 78 \text{ inches}}{\mathrm{T}^2} = \frac{0\cdot 248 \text{ mètre}}{\mathrm{T}^2}. \quad (1.)$$

If the rods of the revolving pendulums are jointed, as in fig. 257, not to a point in the vertical axis, but to a pair of points,

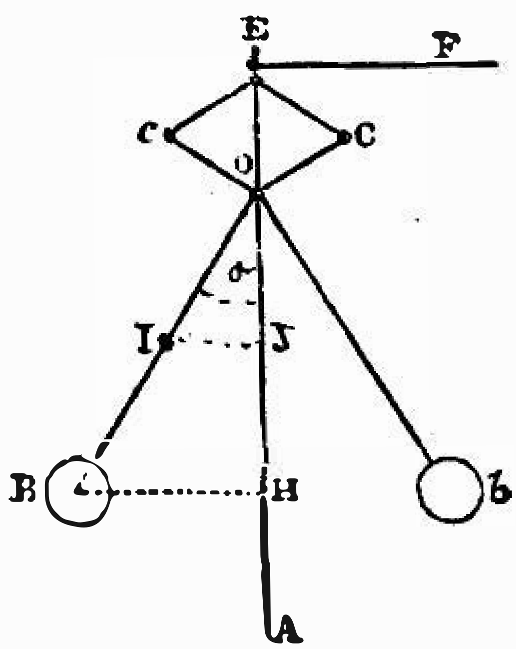

Fig. 256.

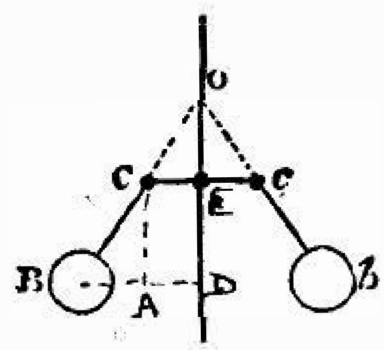

Fig. 257

such as C, c, in arms projecting from that axis, the height is to be measured to the point O, where the lines of tension of the rods cut the axis.

In most cases which occur in practice, the balls are so heavy, as compared with the rods, that the height may be measured without sensible error from the level of the centres of the balls to the point O, where the lines of suspension cut the axis. This amounts to neglecting the effects both of the weight and of the centrifugal force of the rods. These effects may, if required, be taken into account approximately, as follows:—Let B be the weight, and b the radius, of a ball; let R be the weight of a rod, and r the length from O to the centre of B; let h be the height from the centre of B to O, and h' the corrected height; then

$$h' = h\left(1 + \frac{\mathrm{R}\,(r - b)}{2\,\mathrm{B}\,r}\right) \div \left(1 + \frac{\mathrm{R}\,(r - b)^2}{3\,\mathrm{B}\,r^2}\right); \ldots\ldots (2.)$$

and the number of revolutions per second will correspond nearly to this corrected height.

The ordinary steam engine governor invented by Watt, which is represented in fig. 256, is a position-governor, and acts on the regulator by means of the variation of its altitude, through a train of levers and linkwork. That train may be very much varied in detail. In the example shown in the figure, the lever O C forms one piece with the ball-rod O B, and the lever O *c* with the ball-rod O *b*; so that when the speed falls too low, the balls B, *b*, by approaching the spindle, cause the point E to rise; and when the speed rises too high, the balls, by receding from the spindle, cause the point E to fall. At the point E there is a collar, held in the forked end of the lever E F, which communicates motion to the regulator.

The ordinary pendulum-governor is not truly isochronous; for when, in order to adapt the opening of the regulator to different loads, it rotates with its revolving pendulums at different angles to the vertical axis, the altitude h assumes different values, corresponding to different speeds.

As in Article 357, let the utmost extent of fluctuation of the speed of the engine between its highest and lowest limits be the fraction $\frac{1}{m}$ of the mean speed; let h be the altitude of the governor corresponding to the mean speed; and let k be the utmost extent of variation of the altitude between its smaller limit, when the regulator is shut, and its greater limit, when the regulator is full open. Then we have the following proportion:—

$$1 : \left(1 + \frac{1}{2m}\right)^2 - \left(1 - \frac{1}{2m}\right)^2 :: h : k;$$

and consequently

$$1 : \cancel{1} + \frac{1}{m} + \cancel{\frac{1}{4m^2}} - \cancel{1} + \frac{1}{m} - \cancel{\frac{1}{4m^2}} :: h : k$$

$$\frac{k}{h} = \frac{2}{m} \qquad 1 : \frac{2}{m} :: h : k \qquad (3.)$$

361. Loaded Pendulum-Governor.—From the balls of the common governor, whose collective weight is (say) A, let there be, hung by a pair of links of lengths equal to the ball-rods, a load B, capable of sliding up and down the spindle, and having its centre of gravity in the axis of rotation. Then the centrifugal force is that due to A alone; and the effect of gravity is that due to A + 2 B; for when the ball-rods shift their position, the load B moves through twice the vertical distance that the balls move through, and is therefore equivalent to a double load, 2 B, acting directly on the balls. Consequently the altitude for a given speed is greater than that of a simple revolving pendulum, in the ratio $1 + \frac{2\,B}{A}$; a given *absolute* variation of altitude in moving the regulator produces a proportionate variation of speed smaller than in the common governor, in the ratio $\frac{A}{A + 2B}$; and the governor is said to be *more sensitive* than a common governor, in the ratio of $A : A + 2B$. Such is the construction of Porter's governor.

The links by which the load B is hung may be attached, not to the balls themselves, but to any convenient pair of points in the ball-rods; the links, and the parts of the ball-rods to which they are jointed, always forming a rhombus, or equilateral parallelogram. Let q be the ratio borne by each of the sides of that rhombus to the length on the ball-rods from the centre of a ball to the point where the line of suspension cuts the axis; then in the preceding expressions $2qB$ is to be substituted for $2B$.

In the one case $2B$, and in the other $2qB$, is the weight, applied directly at A, which would be *statically equivalent* to the load B, applied where it is.

362. **Parabolic Pendulum-Governors.**—In fig. 258, let B X be the axis of the spindle, and E the centre of one of the balls, which, as it moves towards or from the spindle, is guided so as to describe a parabolic arc, K E, with the vertex at K. Let E F be a normal to the parabola, cutting the axis in F. The vertical height of F above E is constant,* being equal to twice the focal distance of the parabola; hence this governor is absolutely isochronous. That is to say, the balls cannot remain steady in any position except at one particular speed of rotation; being that corresponding to an altitude equal to twice the focal distance of the parabola; and any deviation of the speed above or below that value causes the balls to move continuously outwards and upwards, or inwards and downwards, as the case may be, until their action on the regulator restores the proper speed. The force with which the balls tend to shift their position *vertically*, when a deviation of speed occurs, is expressed very nearly by $\frac{2A \Delta n}{n}$; in which A is the collective weight of the balls, n the proper number of revolutions in a given time, and Δn the deviation from that number. The balls may be guided in various ways, *viz*:—

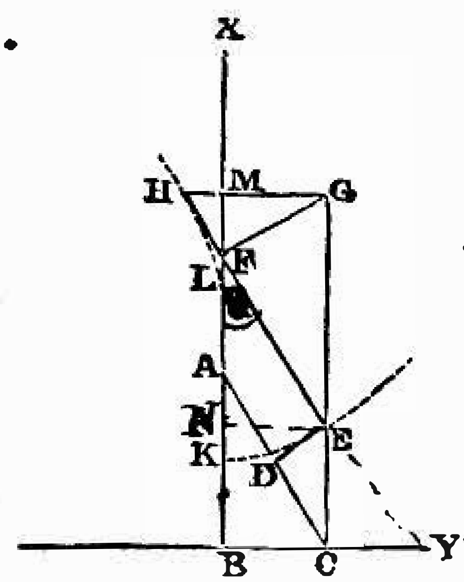

Fig. 258.

I. By hanging each of them by means of a flexible spring from a cheek, L H, of the form of the evolute of the parabola. To find a series of points in the parabola and its evolute, let h be the altitude; then from the vertex K lay off $KA = KB = \frac{1}{4}h$; A will be the focus, and the horizontal line B Y the directrix. Draw A C parallel to an intended position of the ball-rod; bisect it in D;

* Subnormal = Const.

draw D E perpendicular to A C, and C E parallel to B X; the intersection E will be a point in the parabola, and E D a tangent. Then parallel to C A, draw E F; this will be a normal, and a position of the ball-rod. From F, parallel to D E, draw F G, cutting C E produced in G; and from G, parallel to B Y, draw G, cutting E F produced in H; this will be a point in the evolute. To express this algebraically, let B C = y and C E = z be the co-ordinates of the parabola; and let B M = z' and M H = $-y'$ be those of its evolute. Then we have

$$z = \frac{1}{2}\left(h + \frac{y^2}{h}\right); \; z' = 3z; \; -y' = \frac{y^3}{h^2}.$$

See p. 416+

II. Another method of guiding the balls is to support them by means of a pair of properly curved arms, on which they slide or roll. On the top of the balls there rests a horizontal plate or bar, which communicates their vertical movements to the regulator.

III. *Approximate Parabolic Governor.*—In Farcot's governor, the rod E H, in its middle position, is hung from a joint, H, at the end of an arm, M H; this gives approximate isochronism. The co-ordinates of the point H are found by the rules already given. 1

362A. **Loaded Parabolic Governor.**—When the balls of a parabolic governor are guided in the second manner described in the preceding article, and support above them a plate or bar, to which their vertical movements are communicated, an additional load may be applied to them by means of that plate. Let A be the collective weight of the balls; B, the additional load; then the altitude corresponding to a given speed is greater than in the unloaded governor, in the ratio of A + B : A; and the speed corresponding to a given altitude is greater, in the ratio of $\sqrt{(A + B)} : \sqrt{A}$; and by varying the load, the speed of the governor may be varied at will.

363. **Isochronous Gravity-Governor** (*Rankine's*).—In this form of governor (see fig. 259) the four centrifugal balls marked B are balanced, as regards gravity, about the joint A, on the spindle ~~A M.~~ D, D are sliders on the ball-rods; D C, D C, levers jointed to the sliders, and centred on a point in the spindle at C, and of a length D C = C A; G G, a loaded circular platform hung from the levers C D, C D, by links E F, E F; H, an easy-fitting collar, jointed to the steelyard lever H K, whose fulcrum is at K; L, a weight adjustable on this lever. This governor is truly isochronous; the altitude h of a revolving pendulum of equal speed is given by the equation

$$h = \frac{B \cdot A B^2}{2 D \cdot C D}; \quad = \frac{B \cdot A B}{2 D \cdot A C}$$

(?)

in which B is the collective weight of the centrifugal masses, and

D the load, suspended directly at D, to which the actual load is statically equivalent. The load D, and consequently the altitude

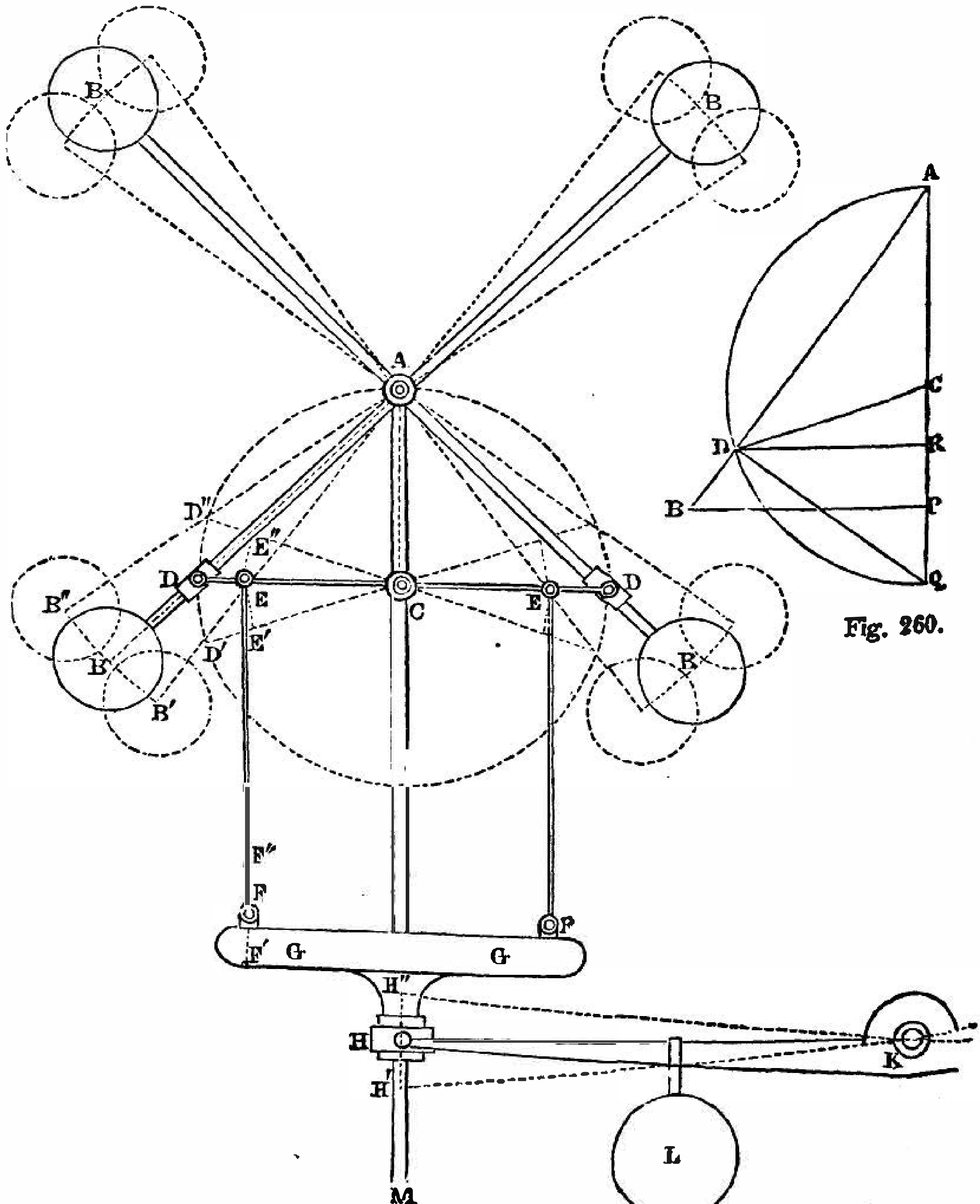

Fig. 259.

Fig. 260.

and the speed, can be varied at will, by shifting the weight L; which can be done either by hand or by the engine itself. The regulator may be acted on by the other end of the lever H K. The levers C D, C D should be horizontal when in their middle position; and then the ball-rods will slope at angles of 45°. Two positions of the parts of the governor when the rods deviate from their middle position, are shown by dotted lines and accented letters. If convenient, the links E F, E F may be hung directly from the slides D, D.

The theory of this governor is illustrated by fig. 260. In any position of the parts, let A C be the axis of rotation; A B, a ball-rod carrying a ball at B; C, the point at which the lever C D = C A is jointed to the spindle; D, the central point of the slider at the end of that lever. About C draw the circle A D Q, cutting the axis of rotation in Q; join D Q; and draw D R and B P perpendicular to A Q.

Then when the position of the parts varies, and the speed is constant, the moment of the centrifugal force of the balls relatively to A varies proportionally to B P · P A, and therefore proportionally to the area of the right-angled triangle A P B; and the moment relatively to A of the load which acts on the point D varies proportionally to D R, and therefore to the area of the right-angled triangle A D Q; but the areas of the triangles A B P and A D Q bear a constant ratio to each other—viz., that of AB^2 to AQ^2; therefore the moment of the centrifugal force at a constant speed, and the moment of load, bear a constant ratio to each other in all positions of the parts of the governor; and if they are equal in one position, they are equal in every position; and if unequal in one position, they are unequal in every position. Therefore the governor is truly isochronous.

To express algebraically the relations between the dimensions, the revolving mass, the load, and the speed; let B be the collective weight of the four balls; D, the total load which is actually or virtually applied at the points D, D; let the length of each ball-rod A B $= b$; and let the length of each of the levers C D $= c$. In any position of the governor, let the angle Q A B $= \theta$. Then, because A C D is an isosceles triangle, we have the angle Q C D $= 2\theta$. It is also evident that B P $= b \sin\theta$; A P $= b \cos\theta$; D R $= c \cdot \sin 2\theta = 2c \cdot \cos\theta \sin\theta$.

Let n, as before, be the number of revolutions per second. Then the centrifugal moment of the balls relatively to A is

$$B \cdot \frac{4\pi^2 n^2}{g} \cdot BP \cdot PA = \frac{B b^2 \sin\theta \cos\theta}{h};$$

and the statical moment of the load relatively to A is

$$D \cdot DR = 2 D c \cos\theta \sin\theta;$$

which two moments, being equated to each other, and common factors struck out, give the following equation:—

$$\frac{B b^2}{h} = 2 D c;$$

and therefore

$$h = \frac{B\,b^2}{2\,D\,c} = \frac{B \cdot A\,B^2}{2\,D \cdot A\,C};$$

as has already been stated.*

364. **Fluctuations of Isochronous Governors.**—When a truly isochronous governor is rapid in its action on the regulator, and meets with little resistance from friction, it may sometimes happen that the momentum of the moving parts carries them beyond the position suited for producing the proper speed; so that a deviation from the proper speed takes place in the contrary direction to the previous deviation, followed by a change, in the contrary direction, in the position of the governor, which again is carried too far by momentum; and so on; the result being a series of periodical fluctuations in the speed of the engine. When this is found to occur, it may be prevented by the use of a piston working in an oil-cylinder or dash-pot; which will take away the momentum of the moving parts, and cause the regulating action of the governor to take place more slowly, without impairing its accuracy.

365. **Balanced, or Spring Governors,** (*Silver's, Weir's, Hunt's, Sir W. Thomson's, &c.*)—In this class of governors, often called
2 *Marine Governors,* as being specially suited for use on board ship, the action of gravity on the balls is either self-balanced, or made, by rapid rotation, so small compared with the centrifugal force as to be unimportant. The centrifugal force is opposed by springs. To make such a governor isochronous, the springs ought to be so arranged as to make the elastic force exerted by them vary in the simple ratio of the distance from the centres of the balls to the axis.

In order that the action of gravity on the balls may be self-balanced, if there are two balls only, they must move in opposite directions, in a plane perpendicular to the axis of rotation: which axis may have any position, but is usually horizontal. They might be guided by sliding on rods perpendicular to the spindle; but they are more frequently guided by combinations of linkwork, different forms of which are exemplified in Weir's governor and in Hunt's governor. If there are four balls, they are carried by a pair of arms like the letter X, as in fig. 259 (but with the spindle usually horizontal instead of vertical), and such is the arrangement in Silver's Marine Governor. The springs in balanced governors are seldom fitted up with a view to perfect isochronism; but for marine engines this is unimportant, as the principal object of applying governors to them is to prevent changes of speed so great

* It has been pointed out by Mr. Edmund Hunt that this form of governor is virtually a parabolic governor; for the common centre of gravity of the balls and of the load moves in a parabola, of a focal distance equal to half the altitude given by the formula.

and sudden as to be dangerous; such as those which tend to occur when the screw-propeller of a vessel pitching in a heavysea is alternatelytlifted out of and plungedtintothetwater.

Rules showing the relation between the deflection of a straight spring, or the extension of a spiral spring, and the elastic force exerted by the spring, have already been given in Article 342, page 386, and Article 345, page 389.

366. **Disengagement-Governors.**—The most complete example of a disengagement-governor is that commonly used for water-wheels, and sometimes also for the steam engine. The peculiar parts of this governor are represented in fig. 261. A A is part of the spindle of a pairtoftrevolving pendulumstsimilar to those in an ordinarygovernor; B, a cylindrical slider, hung from the ball-rods by links whose lower endsare shown at C, C. D ista tooth or cam projecting from the slider, and sweeping round as the spindle and pendulums rotate.

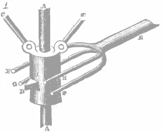

Fig.261.

To makethe slider rotate truly withthe spindle, the part of the spindle on which it slides may either be made square, or may have a projecting longitudinal feather fitting easily a groove in thetinside ofthe slider.

E is one end of a lever capable of turning about a vertical axis (not shown), and provided with a fork of four prongs, F, F, G, H. The prongs F, F are just far enough apart to clear the tooth D, as it sweeps round, when the spindle is turning at its proper speed, and the ball-rods and slider in their middle position; and the lever E is thenin its middle position also. The prong G is below, and the prong H above, the level of the prongs F, F; and when the lever is in its middle position, the clear distance of G and H from the cylindricaltsurface ofthe slider B is one-half of thetdistance of F, F from that surface. When the spindle begins to fall below its proper speed, the slider moves downwards until the tooth D strikes the prong G, and drives the lever E to one side. Should the spindle begin to turn faster than the proper speed, the slider rises until the tooth D strikes the prong H, and drives the lever E to the contrary side. The lever E acts through any convenient train of mechanism upon thetclutch of a setof reversinggear, like the combination shown in fig. 214, Article 263, page 299. The driving-shaft of that combination is continually driven by the engine. When the lever E is in its middle position, the following shaft is disengaged from the driving-shaft, and remains at rest. When the lever E is shifted to one side or to the other, the reversing-gear drives that following shaft in one direction or in the other; and its motion, being transmitted by a suitable train to the regulator, corrects the deviation of speed. So soon as the spindle resumes its proper speed, the tooth D, by striking one or other of the prongs F, F, replaces the lever E in its middle position, and disengages the regulating train.

367. In **Differential Governors** the regulation of the prime mover is effected by means of the difference between the velocity of a wheel driven by it and that of a wheel regulated by a revolving pendulum. This class of governors is exemplified by fig. 262, representing Siemens's differential governor as applied to prime movers. A is a vertical dead-centre or fixed spindle about which the after-mentioned pieces turn; C is a pulley driven by the prime mover, and fixed to a bevel-wheel, which is seen below it; E is a bevel-wheel similar to the first, and having the same apex to its pitch-cone. To this wheel are hung the revolving masses B, of which there are usually four, although two only are shown. Those masses form sectors of a ring, and are surrounded by a cylindrical casing, F. When the masses revolve with their proper velocity, they are adjusted so as nearly to touch this casing; should they exceed that velocity, they fly outwards and touch the casing, and are retarded by the friction. Their centrifugal force may be opposed either by gravity or by springs. For practical purposes, their angular velocity of revolution about the vertical axis may be considered constant. G, G are horizontal arms projecting from a socket which is capable of rotation about A, and carrying vertical bevel-wheels, which rest on E and support C, and transmit motion from C to E. There are usually four of the arms G, G, with their wheels, though two only are shown. H is one of those arms which projects, and has a rod attached to its extremity to act on the regulator of the prime mover, of what sort soever it may be.

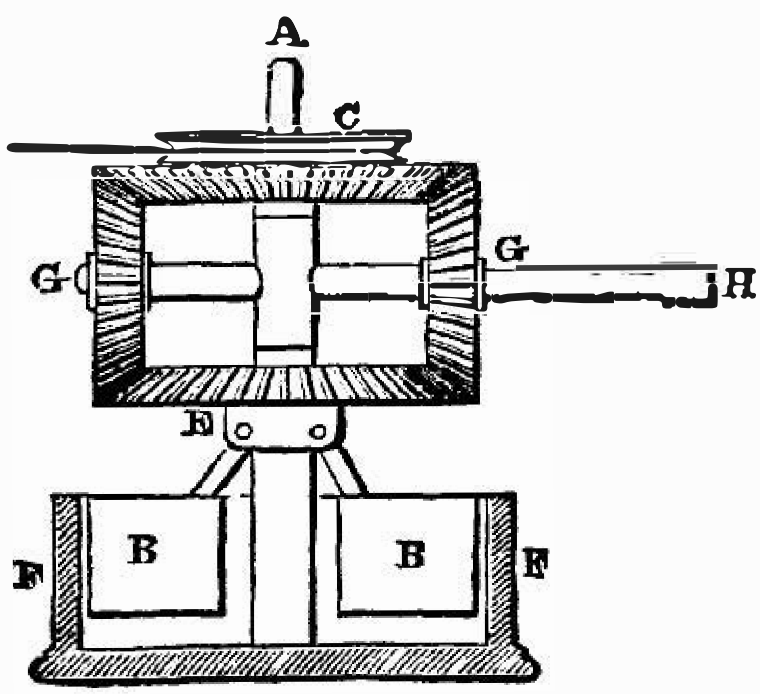

Fig. 262.

When C rotates with an angular velocity equal and contrary to that of E with its revolving pendulums, the arms G, G remain at rest; but should C deviate from that velocity, those arms rotate in one direction or the other, as the case may be, with an angular velocity equal to one-half of the difference between the angular velocity of C and that of E (see Article 234, page 245), and continue in motion until the regulator is adjusted so that the prime mover imparts to C an angular velocity exactly equal to that of the revolving masses B, B.

There are various modifications of the differential governor, but they all act on the same principle. 1

368. In **Pump-Governors** each stroke of the prime mover to be regulated forces, by means of a small pump, a certain volume of oil into a cylinder fitted with a plunger, like a hydraulic press. The oil is discharged at an uniform rate through an adjustable opening, back into the reservoir which supplies the pump. When the prime mover moves faster or slower than its proper speed, the oil is forced into the cylinder faster or slower, as the case may be, than it is discharged, so as to raise or to lower the plunger; and the plunger communicates its movements to the regulator, so as to correct the deviation of speed. 2

The **Bellows-Governor** acts on the same principle, using air instead of oil, and a double bellows instead of a pump and a plunger-cylinder

369. In **Fan-Governors** the greater or less resistance of air or of some liquid to the motion of a fan driven by the prime mover, causes the adjustment of the opening of the regulator. 3

3