OUTLINES

OF

A NEW THEORY

OF

ROTATORY MOTION,

TRANSLATED FROM THE FRENCH OF POINSOT.

WITH

EXPLANATORY NOTES,

BY

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ADVERTISEMENT.

The following pages contain a version of the Extract which M. Poinsot has published of the Memoir presented by him to the French Institute on the 19th of May 1834.

The object of the Translator is to call the attention of those engaged in Mathematical pursuits in this country to a subject which has acquired so great an interest for the Mathematicians of the Continent from the animated discussions respecting it which have arisen out of the above Memoir.

With a view to facilitate the progress of the Reader, but more especially in the hope of rendering this publication useful to Students in the University, demonstrations of the fundamental propositions have been subjoined in the form of notes, and an Appendix containing demonstrations of the leading principles assumed, viz. the existence of an Instantaneous Axis, and of three Principal Axes, and the Conservation of Couples, has been added.

The Translator has been informed by a distinguished Member of the Institute that the Memoir will appear entire in the fourth of a succession of volumes to be published by that Society in the course of next year.

ERRATA.

PAGE	LINE	
2	5 from bottom	*for* "rotation" *read* "rotatory motion."
4	last line	*for* "rotation" *read* "rotatory motion."
14	Note (6)	*after* "$b\,b'$" *insert* "(fig. 7.)".
20	18 from top	*after* "moveable cone" *insert* "whose vertex is O."
24	22	*for* "same *furrow*" *read* "equal *furrows*."

THEORY

OF

ROTATORY MOTION.

INTRODUCTORY REMARKS.

THE following enquiry in Dynamical Science is one which has most frequently occupied my attention, and forms one of the subjects which, if I may so say, I have been most anxious thoroughly to understand.

Every one can form for himself a clear idea of the motion of a point, that is to say, of a corpuscle supposed to be infinitely small, which gives us in some degree the notion of a mathematical point. For we have only to figure to ourselves the line, straight or curved, which the point may describe, and the velocity with which it moves in this line. But if we have to consider the motion of a body of sensible magnitude and definite shape, it must be allowed that the idea which we form of it is very obscure.

At first indeed, the idea appears to become clearer by resolving itself into two others. For if we confine ourselves to the consideration of one particular point in the body, we may on the one hand follow the motion of this point, which can only describe a certain line in space, and on the other the motion of the body, which can only turn at the same time on this point as about a fixed axis. But this second motion, namely, that of a body moveable about a point, round which it is at liberty to turn in every direction, is one of which we have but a confused notion.

Not but that, by referring the different points in the body to planes or objects fixed in space, we can find *differential equations*, as they are called, of this motion, which, in the simple case of a body acted on by no external force, we have even been able to *integrate*, or at least to reduce to *quadratures*. Euler and d'Alembert (nearly at the same time and by different methods) were the first to solve this important and difficult problem: and afterwards, as is known, the illustrious Lagrange undertook to investigate anew this famous question, and to develop it in his own manner; I mean, by a series of analytical formulæ and transformations, remarkable for their symmetry and elegance. But it must be allowed that in all these solutions we see nothing but calculations, without having any clear idea of the rotation of the body. We may be able, by means of calculations, more or less long and complicated, to determine the place of the body at the end of a given time; but we do not see at all how it arrives there. We are totally unable to keep it in view and to follow it, as we might wish, with our eyes, during the whole course of its rotation.

Therefore to furnish a clear idea of this Rotatory Motion, hitherto unrepresented, has been the object of my endeavours*.

The result is an entirely new solution of the problem of the rotatory motion of a body, acted on by no force, whether it turns freely on its centre of gravity, or on any other fixed point about which it is constrained to move: a genuine solution, inasmuch as it is palpable, and enables us to follow the motion of the body as clearly as the motion of a point. And if we would pass from this geometrical representation to calculation, in order to measure all the different properties or affections of this motion, the formulæ requisite for the purpose are direct and simple, each of

* It has always appeared to the Translator, that Geometrical illustrations, whenever they can be obtained, are of very great use in enabling us to form clear and correct ideas of the meaning of analytical formulæ and operations. It may be sufficient to advert to the connexion between the singular solution of a differential equation, and the envelope of series of curves described after a given law, or between the first integrals of a partial differential equation of the second order, and the characteristics of the curved surface to which it belongs. In quadratures particularly, a large class of integrals may be made to depend upon elliptic arcs; and it may be remarked here, that a distinguishing feature of M. Poinsot's theory is the explanation of all the complications of rotatory motion by a reference to the properties of an ellipsoid, with which Analytical Geometry has made us familiar: while the calculations for determining the actual position of the body resolve themselves at once into the form of the elliptic transcendents here mentioned.

them expressing a dynamical theorem of which we have a clear idea, and which proceeds at once to its object. My analysis of the question therefore offers in addition this advantage, that every thing therein is expressed and developed in terms of the immediate conditions of the problem, without the intervention of those co-ordinates and angles which are foreign to the question, and which take their rise only in the indirect method employed to discuss it. For we may remark generally of our mathematical researches, that these auxiliary quantities, these long and difficult calculations into which we are often drawn, are almost always proofs that we have not in the beginning considered the objects themselves so thoroughly and directly as their nature requires, since all is abridged and simplified, as soon as we place ourselves in a right point of view.

I thought then that a solution so simple, and so well calculated to throw new light on the most difficult questions of Dynamics, might further the real advancement of science, and was therefore worthy the attention of Geometers: and this consideration led me to compose the Memoir, which I had the honour this day to present to the Institute.

I divide it into three principal sections. In the first I consider the actual motion of the body, and the forces which would be capable of producing it. In the second, I give the solution of the problem of the rotatory motion of a free body;

and in the third, I develope the calculations which relate to this solution.

But to give a more precise idea of this work, I shall briefly lay down the first principles of the new theory, and afterwards go hastily over the principal theorems which are the object and result of it.