

In the "Machines approuvées par l'Académie," Vol. v, No. 350, we find a machine described by M. Du Buisson, in which an inclined plane is applied to produce the required alternate rectilinear motion. The description of the machine is introduced to contribute to an history of machinery.

The small pumps which are used on board ships, and frequently for drawing liquors, in which the action is produced by an alternate motion of the operator's hand, is composed of a cylindrical tube or pipe, having a valve at its lower extremity, which is immersed in the liquid. It is sometimes indeed constructed without any valve: in this case the effect of the machine is produced by a dextrous application of the operator's thumb, at the upper extremity of the tube. These engines also belong to this division of our work.

SECTION XVII.

To convert alternate rectilinear motion, of velocity either equable, or variable by a given law, into alternate circular motion, of velocity similar to that of the original motion, either equable, or variable according to a given law, and in the same, or in different planes of direction.

A 17.

Several of the movements described in the Section III. and VII. will also be classed in this Section.

B 17. Plate 9.

In this subject, A B in the figure represents a lever, moving on the axis C as a fulcrum, which is the centre of the semicircle DEF fixed to A B. The extremities of a chain D G H K F are attached to the points D and F of the bar A B; it is adjusted by two pins so as to allow of being lengthened or shortened as occasion may require. The chain passes over two fixed pulleys G and K. Under this arrangement the alternate circular movement of the lever A B will produce the alternate rectilinear motion of the portion H of the chain, a reciprocal movement will also be afforded by this machine.

This movement has been successfully applied to a machine for cutting the tops

of piles situated below the surface of the water, and is in practice one of the most simple and effective contrivances for that purpose.

C 17.

This combination of levers in zig-zag arrangement is well known ; it is applied in the contrivance of various toys for children. It has been also applied to the construction of a machine for raising sunken vessels, by Du Vivier. See *Machines approuvées par l'Académie des Sciences*, Vol. vi. No. 429. We also find several applications of the same movement to the construction of various machines in De Besson's work, *Théâtre des Instruments de Mathématiques*, already spoken of ; but these are all of them more or less imperfect. This author proposes to communicate the alternate circular movement by a fixed adjusting screw, this is composed of two distinct screws having their respective threads in opposite directions. The screw by its rotation produces the alternate approach and separation of two nuts fixed at the extremities of the two lower or first levers of the zig-zag arrangement. This piece of mechanism is also applicable to many other practical purposes*.

The pincers generally used for lifting or drawing up heavy bodies from the bottom of the sea, is also an application of the same principle.

The common spinning reel is an highly ingenious and useful application of this combination of levers.

In the *Annales des Arts et Manufactures*, Nos. 19 and 20, we find the description of a pump on a new construction, presented to the Minister of the Marine, by M. Berger.

This is a double piston pump, which seems to embrace advantages deserving attention. A report was made to the Institute on the subject of this pump by Messrs. Borg, Monge, and Lévêque ; it is inserted at length in the abovementioned numbers of the *Annales des Arts et Manufactures*, and merits an attentive perusal. It is here asserted that the idea of this engine did not originate with

* Several applications of this machinery may be found in the first volume of Leupold's "*Theatrum Machinarum Generale.*" Leipsic, 1724.

M. Berger, (and to this M. Berger appears to assent) but that the merit of the invention belongs to Mr. Noble, an Englishman, whose pump on this construction was readily admitted into the British naval service, and was there substituted for the usual chain pumps. It appears that these pumps were first introduced to actual use in the Windsor Castle, a British seventy-four gun ship, in the year 1790, and that their use has been since continued with much benefit to the service. M. Berger proposes two methods for working this pump, one of which however, the reporters reject as impracticable: but they consider the other to be preferable to the crank movement adopted in the English service. The following description of the method they approve is extracted from their report:

“ The principal part of the arrangement is a lozenge figured combination of
 “ four iron bars or rods ab, bc, cd, da , connected at their extremities by hinges
 “ or pin-joints, so that they have free motion on them, and the component pieces
 “ or bars can either extend or close their angles, and arrange themselves in
 “ lozenges of different figure. This combination is represented in the figure as
 placed in a vertical position, so that one of the diagonals of the lozenge it com-
 poses is in an horizontal position, and supported by two bars or pillars ae, cf
 of equal height, in such a manner that the pin which attaches the contiguous sides
 of the figure shall also attach those sides to the corresponding supporting pillar.
 It is evident, that in this position, the other diagonal of the lozenge will be in a
 vertical position; this must correspond with the centre of the pump barrel and
 be situated accurately in the prolongation of its axis. To the upper extremity of
 this diagonal are attached the two rods of the lower piston, and to the other ex-
 tremity are fixed those of the higher or upper piston; and the same pin which
 attaches the contiguous bars to that diagonal also attaches them to the piston
 rods.

It will be understood that, in order to alter the angles which the bars of the figure form with each other, and so extend or reduce the dimensions of the diagonals, the two supporting pillars ae, cf must have liberty of motion. They are therefore set on a cross piece on the joints e, f to which they are attached by a pin, and have thus a movement of rotation in the same plane as that in which the bars of the arrangement are situated.

The construction and disposition of this mechanism being understood, it will appear that by causing the horizontal angles of the figure to approach each other, the upper angle is made to rise, while the lower angle is by the same action, made to descend in the same quantity; and if on the contrary, the horizontal angles of the figure are made to recede from each other or increase their distance, the upper angle will descend while the lower angle will rise; thus the action of the pistons is produced. It will be seen, that in this arrangement the maximum range of each piston is equal to the side of the figure; the action of the pump does not however require the maximum range of the pistons, nor indeed would it be possible to obtain it in practice; it being necessary for that end that the supporting pillars should vary very little from a vertical position. M. Berger considers the best dimensions for the figure to be about 22.2 inches in each side, and that 18 inches will allow sufficient range for each of the pistons.

In this action of the pistons it will be understood, that given points in the sides of the figure are elevated and depressed proportionally, so that the range of each piston, and the vertical path of any given point in one of the sides of the figure, are constantly in the ratio of the distance of the point of suspension of the piston, and the point from which it acts, from that centre of motion of the lozenge which is situated at the extremity of the contiguous supporting pillar. The horizontal line which joins the middle of the lower ends, will therefore be elevated or depressed a quantity equal to one-half the range of each piston. M. Berger transmits the action of the moving power directly to an arm placed on this line by the following arrangement:—Each lower piece of the figure is perforated in its middle by an iron axle, to these are fixed two naves of wood of about 3.75 inches in length, the projecting portion of each axis is cylindrical, and receives a small metal roller having a flanch or projecting shoulder on its outer end; these rollers support a frame which encloses the lozenge, which passes into longitudinal openings made in the side pieces of the frame, and the whole is firmly held together by nuts, as in the modes of fastening practiced in coach building. The length of the openings in the sides of the frame are determined by the range which may be required for the pistons. To the middle of each of these sides is fixed an axis: it is necessary that these should be accurately centred, or placed

accurately opposite to each other, having to perform the office of a single axis passing through the entire width of the frame; these axes pass through the sides of the frame of a set of lever handles which encompass the whole machine, nearly resembling the arrangement of working handles by which the common fire-engine is put in action. The axis of motion of the set of lever handles is supported by two upright pillars *CD*, the arm which works the lozenge is divided at one-third of its length by the arm of the enclosing frame.

It will be evident that under this arrangement, if the power of men be applied to the bars it will operate to raise or depress the frame, because the flanged rollers passing through the openings in its sides will allow the alternate opening and closing of the lozenge frame, and thus effect the required action of the pistons. It will also be seen that the extent of the motion of the frame will be but one-half that of the pistons, and that the men whose action is directed on the bars, will have to perform a motion through the same space only, as that traversed by the pistons, which will act in a perfectly equal and simultaneous manner, and in the reverse direction. Lastly, the piston rods will preserve the same vertical positions, because those points of the lozenge frame to which they are attached, describe in equal times, two equal and similar curves in vertical planes: these are placed back to back, having their concave sides in opposite directions; those points therefore can only follow the direction of their common tangent, which is vertical.

The statement respecting the range of the pistons being double that of the frame by which they are worked, should not be understood as strictly correct, for in the organization of the engine which we have described, it will become rather greater, from the effect of the rotation of the supporting bars *a e*, *c f*, because, by that rotation the horizontal diagonal constantly preserves its horizontal position, during its elevations and depressions; this circumstance is however rather advantageous to the general effect of the engine than otherwise.

In the figure *D 17*, we have omitted the frame which encloses the lozenge frame, and we have shewn but a small portion of the lever handle *g i*; upon one side of the portion shewn in the plate, a curved groove *n m k* is cut, into which the axis *n* passes, which is fixed to one of the lower sides of the lozenge; and

this axis has a small friction roller upon it, for the purpose of facilitating the motion. This arrangement appears to us to be more simple than that proposed by M. Berger. Notwithstanding the omission of the parts of the original figure above-mentioned, and the other alterations, an attentive reader will find our description perfectly intelligible. Careful reference to the articles A 7, B 7, and K 7 of this work will point out the method of describing the curve required in this instance at n m k.

E 17.

The alternate circular movement of the beam B produces an alternate rectilinear movement for the rods MN and PQ, by means of two portions of chain which are attached to each of the circular curved pieces at the extremities of the beam, and set in contrary directions. The steady motion of the rods MN and PQ is provided for by making them slide through the pieces p, q, r, s.

In Berthelot's work entitled, *La Mécanique appliquée aux Arts*, vol. i, page 13, we find the description of a mill constructed on this principle. An alternate circular lever motion is converted into an alternate rectilinear motion, by means of two chains which act on two rods, one of which is raised while the other is depressed; each rod carries a click piece which acts on the same side of a ratchet wheel, and so communicates to it a direct circular motion. Upon the axis of the ratchet wheel is fixed the large wheel which drives the lantern pinion of the mill shaft.

In volume i, page 36 of the same work, he applies the same mechanism to a pulverising engine in this instance the shaft of the ratchet wheel carries fixed projecting pieces which act to raise the pestles or beaters; and in the same volume, we find the same principle applied to raising the hammers of large forges.

Other examples of the application of this contrivance may be seen in the *Annales des Arts et des Manufactures*, vol. xii, page 83, in a memoir entitled—*Description de plusieurs Nouvelles Pompes à Feu, &c.* Some interesting observations occur in this memoir respecting the high state of improvement of our steam engines, as relates to the extensive variety of their dimensions and powers; and the consequent facility with which this important auxiliary can be adopted in operations of any given scale. The recent improvements of Woolf, as respects

the economy of fuel may also be seen in the *Annales des Arts et des Manufactures*, vol. xx. page 294; and in the *Bibliothèque Britannique*, vol. xxviii, Nos. 221—222.

F 17.

This is an engine or tool familiarly known in the practical arts, under the appellation of Drill, Trepan, or Borer.—A is the spindle or stem; BB the cord or band; CC the cross piece; D the fly; E the socket; F the drill, of which G is the cutting point.

The alternate rectilinear movement of the cross piece CC produces the alternate circular movement of the drill. This method of boring has the peculiar disadvantage of wearing the cutting tool very quickly; and which those boring engines are not subject to which have a direct rotative action, or always in the same direction.

G 17.

A B is a rod or bar, sliding within the pieces n m; the lever D F, turning on its fulcrum or axis E, communicates by means of the arm CD, its alternate circular motion to the bar A B, which has its alternate motion in a right line; a reciprocal action will also take place in this machine. This movement is applicable to pump work. Among the pumps made on this construction, and which merit particular attention, may be mentioned Franklin's double piston pump, a description of which will be found in the *Bulletin de la Société d'Encouragement*, of the 15th year—August 1816.

H 17.

This figure shews the mechanism used in the double injection steam engine, for the conversion of the alternate rectilinear motion of the inflexible rod of the piston, into the alternate circular motion of the beam, (and reciprocally). The following description of it is given by M. Prony, in the second part of his "*Nouvelle Architecture Hydraulique*, page 56."

"The parallelogram a b c d attached to the beam by the points a and c
 "which, with respect to the beam, are fixed points; but the sides of the figure
 "are at liberty to alter their inclination with respect to each other, by means of
 "their extremities being jointed together, that is to say, constructed with clips

“ or collars, which are fitted on the horizontal axes. (At page 116 of the author’s
 “ work, he gives a detailed description of the construction of this mechanism).
 “ The axes in the points a and c are in the same plane with the centre, or axis
 “ of rotation O of the beam.

“ Further, the angle d of the parallelogram is constantly retained at a de-
 “ termined distance from the fixed point f', by means of the metal rod f' d,
 “ the extremity of which has also a clip or collar which fits upon the centre or
 “ axis passing through the point d.

“ This being clearly understood, if we imagine the angle b to be urged or
 “ drawn in a vertical direction, the effort will force the sides ba and b d to
 “ assume an inclined position as in the figure; the points or centres a and c
 “ will describe circular arcs, of which the point O will be the centre, and the
 “ point d will describe a circular arc having f' d for its radius. But the curves
 “ described by the points a, c, d, being thus fixed and determined, the curve
 “ described by the point b will also be fixed and determined; and it will be
 “ easily seen by inspection of the figure, that when the motion of the beam acts
 “ to drive the point b out of a vertical line in one direction, the effect of the
 “ rotation of d about f', will be to drive it also out of the vertical line, but in
 “ the opposite direction, and therefore that these counteractions may be so com-
 “ bined and arranged that the curve described by the point b will deviate so
 “ little from a vertical right line, that in practice the difference may safely be
 “ disregarded.”

The theory of this parallel motion is also detailed in a lucid and satisfactory manner in the same work, from page 137.

I 17.

A different solution of the same problem has been given by M. de Bétancourt. M. Prony gives the following account of the method, at page 67 of the same work.

Two beams of wood ab, d O have rotatory motion about the points or centres a and O; their extremities b and d are connected by the iron rod bc' d, having joints at b and d. The lengths ab and d O from centre to centre of the joints,

arc equal: the sum of those lengths $a b + d O$ is equal to the distance of the point a from O on an horizontal projection, or measured horizontally, so that, when $a b$ and $d O$ are parallel to each other, or each horizontally situated, a right line passing through d and b will be a vertical line; and since the length of the piece $b d$ from centre to centre of the pins, is equal to the vertical distance of the points a and O , $b d$ takes a vertical position whenever $a b$ and $d O$ become horizontally situated.

By means of this arrangement, if the points b and d do not describe very considerable arcs above and below their horizontal positions, on the levels respectively of the points a and O , the middle point c of the bar $b d$ will have a sensible motion in a vertical right line. In practice it will be found that the elevation or depression of the point b , with respect to the point a , will be more nearly equal to the elevation or depression of d with respect to O , as the motion of the points b and d departs less from their horizontal position; whence it follows that the arcs described by the points b and d , may in such case be considered as equal. This hypothesis being admitted, the points b and d will be always at an equal distance from a vertical line from which the points a and O are also equidistant; therefore if c' be placed in the middle of $b d$, it will be constantly situated in the above-mentioned vertical line. This vertical line coinciding with the common axis of the steam cylinder and its piston rod $c' c$, it will only be necessary to attach an horizontal axis to the upper end c' of the piston rod, which shall turn in a collar formed in the middle point of the bar $b d$, and the conditions of the problem will be satisfied. A theoretical demonstration may be seen at page 123 of the same work.

K 17.

This figure represents the common Drill-bow—an instrument too familiarly known we presume, to need any particular description.

We shall merely remark that if the alternate rectilinear movement of the drill-bow communicates an alternate circular movement to the spindle, about which the bow-line is wound, it will not be difficult to give it a direct and continuous circular motion; for this purpose it will only be necessary to place a fly-wheel on the spindle, and by a dexterous management of the bow, to make it act on the

cylinder of the spindle in one direction only, and which may be accomplished by a little attentive practice. This contrivance has been applied by M. Raux to give motion to a thread machine, of his invention, and which received the approbation of the Institute, in the year 1806.

L 17.

In this movement, if the wheel A be made to revolve in one direction, the wheel B will revolve in the opposite direction, and they will each act on the corresponding racks of the frame which encompasses them, so as to communicate a rectilinear movement to the bar C D; this movement might consequently also be placed in the seventh range of movements in the table.

M 17.

This arrangement is a modification of the last, and is capable of several highly useful applications.

If two toothed wheels be placed upon an axis, and two racks placed diametrically opposite to each other are made to work with each of them, the alternate circular movement of the axis will communicate an alternate rectilinear movement to the racks, and the extremities of the racks will advance to the axis or recede from it, in a uniform manner. Spinning reels have been constructed on this principle; cylinders of variable diameter might also be thus constructed (see our article C 7.) by properly arranging a sufficient number of wheels and racks.

N 17.

The conversion of an alternate rectilinear motion, into alternate circular.
From the Annales des Arts, No. 43.

In the figure, a b c d represents a bar or rod, having its direction of motion confined to a groove; a second bar t v d k v' t is attached to the first by the joint r, so that the two bars are at liberty to open and shut upon the centre r, similarly to the common folding pocket rule. A plate of metal l n h m, which moves parallel to the line b d, carries the two pins p and q, which are lodged in the hollows, v t' and v t' cut in the bar t v d k v' t.

It will be seen that by this arrangement, if the piece $l n h m$ moves towards the end e of the machine the two jointed bars being supposed to be opened on the joint r , they will be shut by the action of the pin q upon the curve t/v' , and afterwards carried forwards by the continuation of the motion of the piece $l n h m$; and the reverse of this operation will take place, when the movement of this piece is made in the opposite direction. This mechanism has been applied by M. Droz, in the coining press, to the purpose of introducing and shifting the pieces.

The alternate rectilinear movement of this mechanical hand, as it may be termed, is very slow at the extremities of its course, and accelerated towards the middle of it, in order to receive the pieces as they fall from the hopper or feeder of the engine, and to place them under the press without shake, and with perfect steadiness. This movement is communicated to it by means of a pin λ , fixed to the moving plate $l n h m$, which passes into an oblong aperture formed at the lower extremity of the bar $\delta \gamma \delta$; the axis or pin y stands in an horizontal position, and perpendicular to a plane passing through the axis of the screw of the press; so that when this lever is moved either in one direction or the other, the pin λ is pushed either back or forwards, and consequently the plate $l n h m$ also. The upper extremity of the lever $\delta \gamma \delta$ moves between two metal curves which are described according to given rules, (see our article A 7). Each of these curves is attached by its extremity to the large screw of the press, and are so arranged that the pin λ recedes when the screw is lowered, and reciprocally. From this description the connexion between this piece of mechanism and the press, and the mode of its action, will be understood. The alternate circular movement of the engine being supposed uniform, it will be converted into an alternate circular movement, which shall be slow at the extremities of the oscillation, and accelerated towards the middle of its course, by means of two horizontal curves forming a groove, in which the upper extremity of the vertical lever $\delta \gamma \delta$ will move. The same will take place with respect to the lower end of the lever; in this case, the same extent of oscillation will be preserved if the axis y is placed in the middle of the lever $\delta \gamma \delta$; and will become larger or smaller, as it is shifted nearer to, or farther from the upper end of the lever. This latter irregular alternate cir-

cular motion will be converted into alternate circular of the description required by the nature and intention of the engine.

O 17. Plate 10.

In this figure, $A B B$ represents a side elevation of the beam of a steam engine; G its centre of rotation; $n m$ an iron rod which is at liberty to turn freely about an axis b placed at the extremity A of the beam, and which divides the rod $n m$ into two equal portions; the rod $n m$ is attached by the extremity n to the piston rod f , and at the other extremity m , to the rod $p q$, which turns on the fixed axis q .

Under this arrangement, we will suppose to be given—1st. The dimensions of the beam of the engine $A B B$.—2nd. The position of its centre of rotation G .—3rd. The arc $b c a$ which the extremity A of the beam will traverse at each oscillation, and which will be tangential to the direction of the piston f .—4th. The length of the rod $n m$.

From these data it is required to determine the length of the rod $p q$, and the position of its centre of rotation q , so as to ensure, as nearly as possible, the rectilinear direction of the piston.

The positions of the three points m, m', m'' , will be determined, so as to indicate the respective situation of the extremity m of the given rod $n m$, at the commencement—towards the middle—and the close of the oscillation of the beam; and so that in those three positions, the other extremity n shall be situated accurately in the direction of the piston rod f . If a circle be described which shall pass through these three points, its radius will be equal to the required length of the bar $p q$, and its centre so determined will represent the required centre of rotation q .

The curve described by the extremity n of the piston rod f , will pass through the three points n, n', n'' , and will approximate to a right line, as the arc $a c b$ described by the extremity of the beam, is smaller.

The same course of proceeding will serve to determine (in the figure H 17, plate 9.) the length of the rod $f' d$, and the position of the point of rotation f' ;

and in the figure I 17 of the same plate, will also serve to determine the length of the beam $a b$, and the position of the centre of rotation a .

P 17. Plate II.

In our article G 8, we have given a description of a piece of mechanism in general use in our cotton spinning machinery. In the operations of wool spinning, the motion of the carriage is not uniform through its entire course:—it commences by traversing a space $f k$ with an uniform motion, the distance, this generally extends is from twelve to eighteen inches, according to the quality of the wool; during this interval, the machine thickens a certain quantity of the material, which must be lengthened out to about four feet; the twisting should be increased in proportion as the thread lengthens, and the motion of the fuseses being uniform, the rate of motion of the carriage must be gradually slackened as the thread lengthens. The effective performance of this operation is the result of long practice alone; and our woollen manufactories are yet unprovided with any regular method for the purpose, depending solely on the mechanical dexterity of the operators, who regulate the motion of the carriage with the left-hand, while with the right they produce the uniform rotation of the wheel which gives motion to the fuseses. The address required for the due performance of this difficult operation is, as may be conceived, to be acquired only by a course of toilsome and attentive practice which generally occupies a long term of servitude.

These serious disadvantages arising from the misapplication of labour would be completely obviated, as well as a satisfactory general improvement effected, by the general solution of the following problem.

In the mechanism represented at G 8, plate 5, let the rotatory movement of the pulley B be uniform, and let it be required to determine the means of rendering the velocity of its movement of conversion, variable at pleasure.

The following course of proceeding will afford the required solution. A drum A (figure P 17, plate II.) is substituted for the pulley B of the figure G 8, plate 5; the figure of the drum wheel will be determined by the nature of the velocity of the required conversion. From the extremity a of the drum there projects a neck

having a spiral cut upon it which after a certain number of convolutions, terminates at its other extremity *b*. Let *fh* represent the entire course which the carriage will perform, and let *hg* be taken equal to the height *ac* of the drum wheel *A*, let a rope or band be attached to the point *f*, and a second to the point *g*; each of these must be equal in length to the distance *fg*; the other extremity of the first band is attached to the point *a*, and the second to the point *b*, after being coiled on the spiral in opposite directions. The two cords should be in contact always at the same point of the drum *A*.

It will be seen by this arrangement—1. That if the drum *A* revolves uniformly on its axis, its movement of conversion will vary as the radii of the spiral.—2. That as one of the bands is coiled by the motion of the spiral, the other is uncoiled; and by this means their tension is constantly preserved.—3. That lastly, the drum *A* will have no tendency to motion in the direction of its axis *ac*.

This piece of mechanism was communicated to us by M. Sureda, whom we have already had occasion to mention in our article O 7.

According to a series of observations made by M. Sureda, in the most careful manner for the purpose of determining the ratio of the velocities of the carriage, under the management of the most experienced operators, it appears that at the end of the first part of its course, the carriage had still to traverse a distance of four feet four lines, or 580 lines*; and which was traversed by it, while the wheel which communicates the uniform motion to the fusee performed ten revolutions the spaces respectively run through during each revolution were as follows:

* It may be necessary, for the information of the general English reader, to note that the French measure of length termed a line, is to the inch as twelve to one;—580 lines are therefore = four feet and four lines.

	Lines.
During the 1st revolution of the wheel	112
2nd ..a	88
3rd	74
4th	62
5th	53
6th	46
7th	41
8th ..e	38
9th	36
10th	30
	<hr/>
Total	580

The first part of its course of one foot and a half, was uniformly traversed in the time occupied during two revolutions of the wheel.

It will readily be conceived that notwithstanding the utmost care and intelligence on the part of the observer, these results may be subject to error; and we do not consider we exceed the probable extent of such errors in stating the following table of results in correction.

	Lines.
During the 1st revolution of the wheel	106
2nd	90
3rd	76
4th	64
5th	54
6th	46
7th	40
8th	36
9th	34
10th	34
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Total	580

In the figure N, plate XI, let the line ca be divided into nine equal parts, and through the points of division draw the perpendicular lines $y^{\text{ix}}, y^{\text{viii}}, y^{\text{vii}}, y^{\text{vi}}, y^{\text{v}}, y^{\text{iv}}, y^{\text{iii}}, y^{\text{ii}}, y^{\text{i}}, y^{\text{o}}$, which shall be relatively as the tabular numbers of the last stated series, and let a curve $bhn m$ be described through the extremities of these lines. Let y represent the ordinates of this curve, x its abscissæ from the vertex h , p the distance $cc' = c'c''$, &c. which is the measure of separation of the perpendicular lines $y^{\text{ix}}, y^{\text{viii}}, y^{\text{vii}}$, &c.—and a the distance hr from the vertex of the curve to the line ac .

The well known equation $y^2 = p^2 (x - a)$ in which $p = cc' = c'c''$, &c. and $a = y' - \frac{1}{4} = 34 - \frac{1}{4} = 33.75$ will agree with the observations, as may be shewn by substituting for x , its values $y' - a, y'' - a$, &c.; in which case we obtain successively $y = \frac{p}{2}$; $y = \frac{3}{2} p$, &c. &c.

In practice it may be arranged that the spiral shall make twelve revolutions, the two first for the first part of the course of the carriage, and the remaining ten revolutions for the second part of the course; the whole course of the spiral must be equal to the distance fg .

If the line ac of the figure N be supposed equal to the height of the drum A, and that δ represents the ratio of the diameter to the circumference, each term of the last stated series must be multiplied by $\frac{\delta}{2}$ and the curve $bhn m$ be drawn; and this will be the generating curve for the surface of the drum.

The drum A will thus be composed of two portions, one aru of a paraboloidic figure, by which the progress of the carriage will be duly regulated, during the second period of its course; and the other, $brut$, of a cylindrical figure, which will propel it during the first period of its course, with an uniform velocity.

The drum A may be placed in communication with the mover, or be detached from it, at pleasure, by the methods already explained; and thus by a suitable application of such a method, the carriage may be retrograded to the point whence it commenced its course without the necessity of reversing the action of the mover.