CHAPTER X.

MISCELLANEOUS MECHANISMS.

§ 51.—THE "SIMPLE MACHINES."

IN the older books on Mechanics, before the development of the system of machine analysis which we have used, and which is essentially due to Professor Reuleaux, the actual machines of the engineer are generally taken as being represented by certain combinations called "simple machines." Out of these, as elements, it is more or less consistently assumed that actual machines are built up. It is not worth while here to discuss a theory so hopelessly inconsistent with facts as this.[1] It will be right, however, to notice what is the real position of the "simple machines" as mechanisms.

The **lever** and the **wheel** and **axle** are, in reality, kinematically identical. Each is often figured in an impossible fashion, the lever as a bar, resting quite unconstrainedly on a triangular fulcrum, the wheel and axle as a single body poised, unsupported, in mid air. To form part

[1] It is a matter for great regret that the study of these "simple machines" should still be sanctioned and encouraged by the examinations of the University of London, which have so important an influence on the direction of teaching in their own subjects.

of a mechanism or machine, the desired motion of each must, as we know, be constrained, and this motion is nothing more than a rotation about a fixed axis. In a complete form, therefore, leveriand wheeliand axle are neither more norilessithan **turning pairs**, such as we first examined in § 10.

The **inclined plane** is also usually drawn in an unconstrained form, being pictured as aniirregularlumpiof material resting on aiwedge-shapedibiock Here the essential part of the motion is the sliding of the one body upon another, the slope or incline is entirely accidental. The motion imperfectly represented by the inclined plane is simply that obtained in complete constrainment by the **sliding pair**.

Fig. 171.

The **wedge** is a much more complex combination than eitherof theithree "simple machines" justiexaminedi Itiis not uncommonly pictured as in Fig. 171, which shows, it is needless to say, an unconstrained combination not representing any possible part of a machine The constrained motions which are usually assumed to belong to a wedge are those of the mechanism shown in Fig. 172, which contains three links, eachiconnectediwithitheiotheritwoiby a sliding pair, the pairs being marked 1, 2, and 3 inthe figure. This chain deserves notice in several respects, for it differs somewhat from any we have hitherto examined. In the first place, the chain is not in itself constrained, either link can be moved in one direction without affecting the others. In the cases where it is used in practice, its constrainment is effected by means of external forces, acting as shown by the arrows, Fig. 172, and caused to act permanently so long as the mechanism is used. For this reason the mechanism is said to be *force-closed*, **force-closure** meaning the constrainment, or closure, of a chain or a pair of elements by

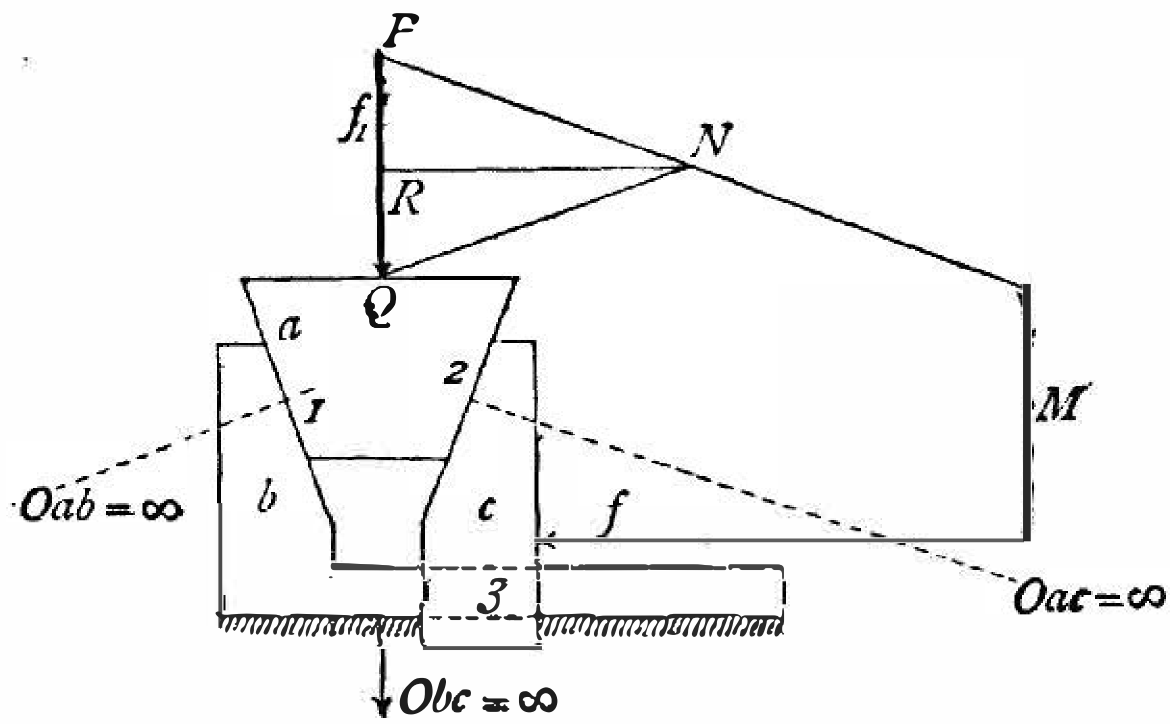


FIG. 172.

external force in the case where the proper kinematic pairing is incomplete. Another characteristic of the chain is that all its three virtual centres are at infinity. They must still be three points on one line, but this line is now the "line at infinity." All the constructions given in former sections can be equally well carried out here. For example, let f_1 be a force acting on a, to find the balancing force in the given direction f upon c, the link b being taken as the fixed link. We first resolve f_1 through the points O_{ab} and O_{ac}, that is, through the fixed point of a, and the common point of a and c. This is, of course, just as easy asiif those points were not inaccessible, for we know their directions, and can at once find the two components PN and NQ. We neg lect NQ because it acts, by hypothesis, through the fixed pointiof a, andthereforedoesnot require balancing. We have then only to resolve PN in the direction of f and atirightiang lesi toiit, toiobtain NR, the force required. For this is the same as our former construction, viz., to resolve PN through a point M, whereiits direction joins that of f, and throug hithe fixedipointi of c, viz., O_{bc}.

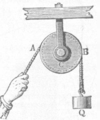


FIG. 173.

The **pulley** in various forms, such as that of Fig. 173. often appears in the list of simple machines. In the form sketched itiis a chainiof threeilinks (cord, pulleyand frame) and is not only a force-closed chain, but one in which a non-rigid, or simply *resistant* (see § 61) link is used. It is thus aicaseiof very considerable complexity, and one only available for a machine under certain special conditions. It will be further considered ini § 61 along with other mechanisms of the same type.

The **screw**, lastly, forms—with itsi nut—merely a pair of twisting elements (p. 58), constraining a motion which is no longer plane, and which will be more fully examined in § 62.

The so-called **funicular machine**, a cord fixed at both ends and loaded with isolated weights, which is sometimes included among the simple machines or "mechanical powers," requires no notice here, as it is not a machine at all, but merely a skeleton form of **structure**. Its various segments are not intended to have any motions whatever relatively to each other, constrained or otherwise. It is merely the concrete representation of the *link polygon*, and the type form of certain most important structures, such as the suspension bridge and the bowstring girder.

§ 52.—ALTERED MECHANISMS. EXPANSION OF ELEMENTS.

We have seen that the form or shape given to the body of the link of a mechanism is of no importance in connection with the movements of the mechanism, so long as it does not impede those movements in any way. We have now to notice further that although the *form of the elements* connecting the links is vital to the mechanism, their *size* is of no importance, and is often so altered, for constructive reasons, as greatly to disguise the nature of the mechanism without at all changing the motions belonging to it. This occurs specially where one element of a link is made so large as to include another, so that the actual outside form of the link is the form of one of its elements. As illustrations of this we may notice how such **expansion of elements** affects one or two of the simpler mechanisms whose motions we have already examined.

Fig. 174 shows an ordinary "lever-crank" chain, having four links *a*, *b*, *c* and *d*, connected by four turning pairs 1, 2,

Fig. 174.

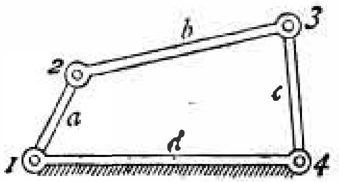
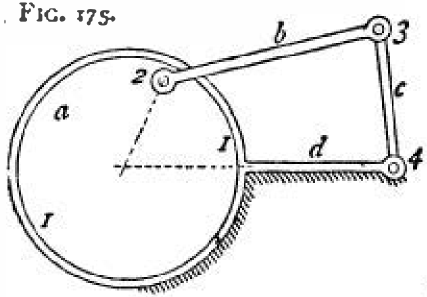


Fig. 175.



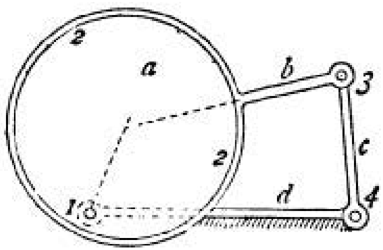


Fig. 177.

Fig. 176.

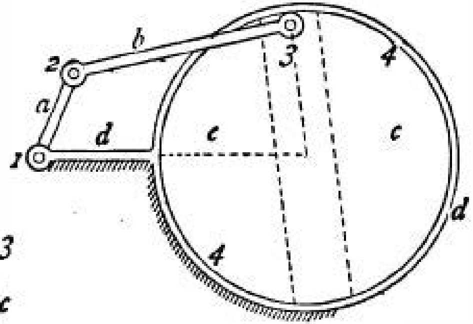
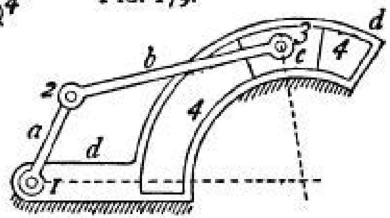


Fig. 179.



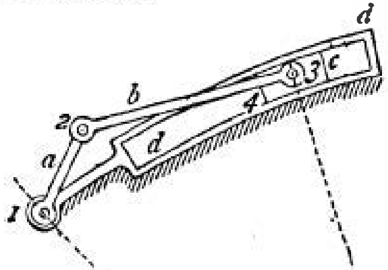
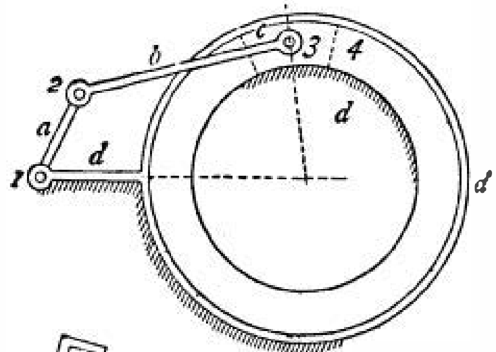


Fig. 181.

Fig. 178.



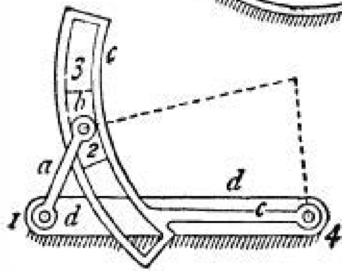


Fig. 180.

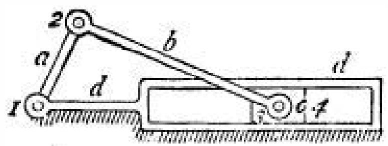


Fig. 182.

3 and 4. The same mechanism in precisely the same position is shown in Fig. 175, with only the difference that the pair 1 has been so much enlarged that the link *a* becomes a disc, whose periphery is one of its elements, and whose diameter is such as to include within it the pair 2. In Fig. 176 a similar change has been made with the pair 4 instead of the pair 1. In Fig. 177, again, the same change has been made with the pair 2, which now includes the pair 1. The link *a* in this case forms what is always called an *eccentric*, and comparing Figs. 177 and 174 we see at once the relation between the eccentric and the crank. Kinematically they are absolutely identical, their only difference lies in the *size* of the elements which they contain. Practically, of course, the great point of difference is that the eccentric allows the shaft at 1 to be carried through without a break on both sides of the mechanism, whereas with a crank the shaft has to be broken in the centre to allow for the swing of the connecting rod *b*.

But other changes, still more striking, can be made by altering not only the size, but what we may call the *extent* of the links. Thus for the link *c* in Fig. 176 we do not require the whole disc as drawn, it would be quite sufficient for us to use a narrow slice of it such as is shown in the dotted lines, having sufficient surface at its ends to constrain the motion. Or we may make *c* a complete ring (Fig. 178) instead of a disc, the part of *d* which forms its element of the pair 4 fitting inside as well as outside the ring. Treating *c* now again as we did in Fig. 176, it becomes a short sector or block, as shown in dotted lines. For this form of *c* the complete circles are no longer wanted in *d*, for *c* merely swings backwards and forwards without rotating. The mechanism, therefore, takes the form of Fig. 179, its motion still remaining absolutely identical with those of the original mechanism, Fig. 174. By an exactly corresponding set of changes, which it is unnecessary to go over again at length, the mechanism might be made, without change in any of its motions, to take the form of Fig. 180, in which the link *b* becomes a block or sector, and one of the elements of *c* a curved slot. (To save space, the length of the sector in the figure is much shorter than would be necessary to allow *a* to turn completely round.)

In each of these last cases the link which has become in form a curved block, retains the same elements as before, altered only in diameter and angular extent. In each case, namely, these links contain two elements of turning pairs, and the centres of those elements,—which determine the distance apart of the pairs, or the real length of the links, —remain precisely as in Fig. 174. The distance 3·4 in that figure is therefore the actual length of the link *c* in Fig. 179, and the distance 2·3 the actual length of the link *b* in Fig. 180, these lengths being in no way altered by the external changes which we have made in the appearance of the mechanism.

The adoption of the block form of link shown in the last two figures has the practical convenience that it enables us to use very long links in a mechanism without necessarily making the mechanism itself very large. Thus in Fig. 181 the links *c* and *d* are made so long that the point 4 is inaccessible (at the join of the dotted lines), but the mechanism itself has become no larger in consequence, and the block *c* remains a link containing two turning elements, just as before. In this case the links *c* and *d* (*i.e.*, the lengths 3·4 and 1·4) are made equal, so that the centres of the pairs 1 and 3 lie on the same circle, having 4 for its centre. If now, everything else remaining unchanged, the point 4 be taken further and further away, the curvature of the slot becomes flatter andiflatter, until at last, wheni4 is atianiin-finiteidistance, theislotibecomesistraight, theilink *c* a istraight instead of a curved block, and the mechanism becomes a slider-crank, Fig. 182, instead of a lever-crank. Here then we have the true relation between these important mechanisms. The slider crank is derived from the other by making two of its links (*c* and *d*) equal, and at the same time infinitely long. The block of the slider crank, the reciprocating link, corresponds to the lever, the swinging link, of the lever crank, one of its elements unchanged, the otherimadeiinfinite iniradiusi

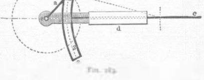


Fig. 183.

By now making in the slider-crank the same change as that made in Figi 180, we obtain the form of mechanism shown in Figi 183, in which the link *b*, without alteration of length, becomes a curved block instead of a long bar. And from this form it can be seen at once what will happen (Fig. 184) if *b* be made infinitely long, as well as *c* andid. The slider-crank changes into a form familiar as the driving mechanismiof donkeyipumps,iand for other purposes. In reference to this mechanism it is often said that its working is equivalent to that with a connecting-rod infinitely long. It will beiseenifromiouriexamination that this is true in a very literal sense. The length of ailinkiiniai mechanismi we take to be the distance between the axes of its elements. One of the elements of the link *b* has its axis at infinity, it is, therefore, not only equivalent to a link of infinite length, but actually *is* a link whose kinematic length (measured in exactly the same way as that of other links) is infinitely great. The mechanism shown in Fig. 185, and recognisable as a form of "trammel" for drawing ellipses, is identical with that of the last figure, and is illustrated merely to show the disguising effect of a few simple *external* changes. The nature of the pairing and the lengths of the links remain precisely as before. The form of Fig. 185 is,

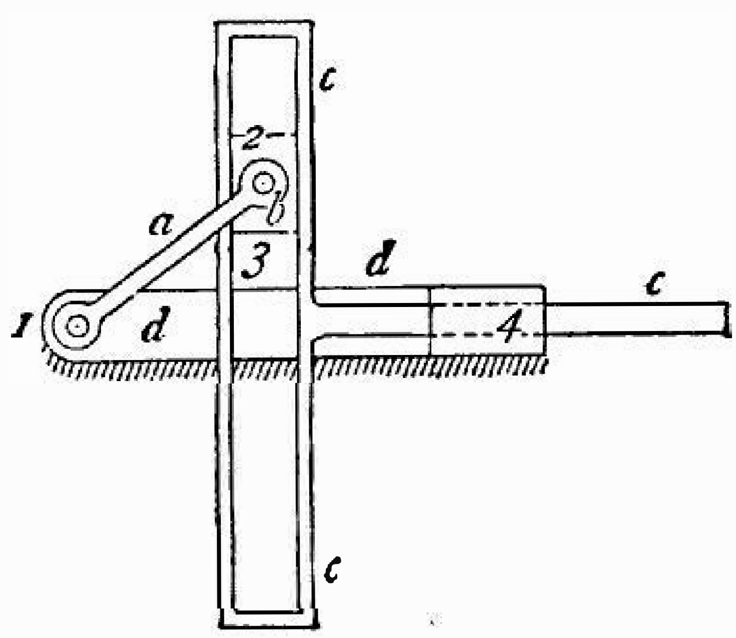


FIG. 184.

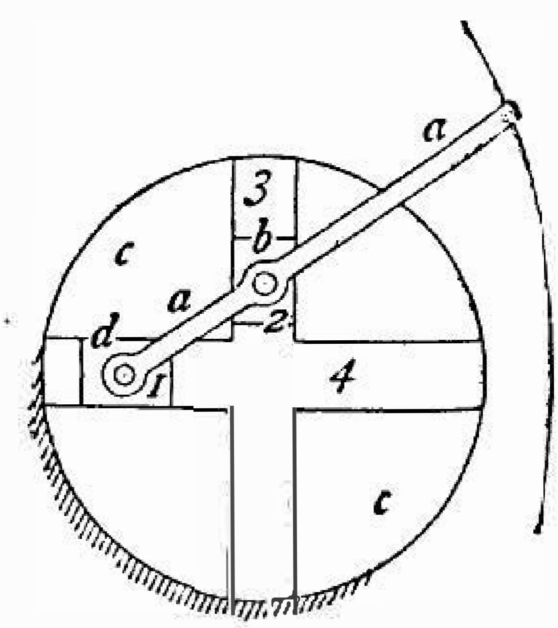


FIG. 185.

however, generally used when the link *c* is the fixed one, while with the former construction, *d* is generally made the fixed link. Letters and figures are the same in the two illustrations, and the student should satisfy himself by examination that they represent connections which are not only similar, but kinematically identical

The two following figures (Figs. 186 and 187) are taken from Reuleaux, and show to what an extraordinary extent the expansion of elements in a mechanism can alter its structure and appearance without changing its nature. The

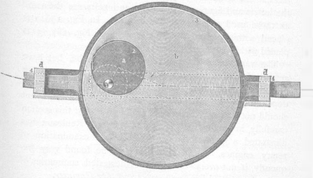


FIG. 186.

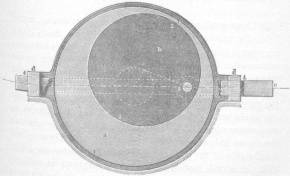


FIG. 187.

mechanism is an ordinary slider-crank in both cases, and the letters and figures on the links and pairs are the same as those used throughout this section. In Fig. 186, 1 is placed within 2, and 2 within 3, and in Fig. 187, 3 is placed within 2, and 2 within 1.

The varieties of form that not only can be, but have actually been obtained by expanding the elements of a mechanism variously, are innumerable. All, however, depend upon the principles here set down, and the student should not find any difficulty, if he has followed this section carefully, in tracing out their real nature. Reuleaux has illustrated a great many of them in his examination of rotary engines. Fresh examples will be found very frequently, if not every week, in the illustrated summaries of patents published by *Engineering* and *The Engineer*.

§ 53.—ALTERED MECHANISMS. REDUCTION OF LINKS.

It happens often, indeed in most cases, that one or more links of a mechanism are not *directly* utilised. Such links are wanted to constrain or transmit certain motions between certain other links, but their own actual motions are in no way required. The connecting rod of a steam engine is, for example, such a link. It transmits constrained motion from the piston to the crank. The piston is the driving link of the chain, and its to and fro motion is directly utilised in connection with the action of steam upon it. The crank is the driven link, and its rotary motion is essentially required for the sake of the machines which the engine has to drive. But the actual motion of the connecting rod is seldom utilised in any way. In certain important valve gears it is now used, and sometimes it is made use of for driving an air pump, but these cases are quite exceptional. It has, therefore, often been thought, hastily, that such a link, only serving as a connection, would be better omitted, and that its omission (if only it could be made without destroying the constrainment of the mechanism) would be entirely advantageous. This omission of a link we may call the **reduction** of a mechanism, and a mechanism so treated will be said to be a **reduced mechanism.**

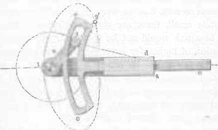


Fig. 188.

We have seen in § 10 that it was possible to constrain *any* plane motion, however complex, by a suitably formed higher pair of elements. It must, therefore, be quite possible to constrain the relative motions (to use the illustration of the last paragraph) of the crosshead and crank of a steam engine, by connecting them directly, omitting the connecting rod altogether. For this purpose it is only necessary to know the whole motion of the one body relatively to the other, and to construct a suitable higher pair of elements. One way in which this can be done is shown in Fig. 188. Here the link b of a slider-crank chain being omitted, a pin is placed on the end of *a* and a slot made upon *c* of such form as to be the envelope for the various positions of the pin relatively to *d*. At first sight we seem to have altered nothing from Fig. 183 of the last section, but it takes very little examination to see that the omission of *b* has been accompanied by a serious practical drawback, namely the substitution of *line* contact for *surface* contact. This we know to be inevitable with the use of higher pairing (p. 57). If we could only suppose our links made with the extremest accuracy in dimensions, and of material so hard that its wear under ordinary forces was indefinitely small, there are very many cases in which the saving of a link would not be purchased too dearly by the substitution of higher for lower pairing. But in fact these conditions can never be attained. The wear in such a mechanism as that of Fig. 188, would be so great and so rapid that the motions of the links would speedily lose their required degree of constraint, and the machine would, in ordinary language, "knock itself to pieces."

There was, of course, no kinematic necessity for making the higher element upon *a* a circular pin; this has been done simply because it was most convenient to do so. Nor was there any necessity for placing it exactly where the crank pin formerly was; this also has been done merely for convenience' sake. The pin, for instance, might equally well have been placed at 2′ instead of at 2. But in that case the slot would have taken the form shown in the dotted line, which of course would have been very much more troublesome to make than the simple circular arc.

In Fig. 189 is shown a slider crank chain with the link *c*, the block, omitted. The links *b* and *d* are now connected by higher pairing, which has taken the form of a cylindrical pin on *b* working in a straight slot in *d*. As before, the result is that by omitting a link we have had to replace the surface contact of two lower pairs by the line contact of a higher one, with the practical drawbacks already mentioned.

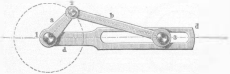


Fig. 189.

Fig. 190 shows another way in which *d* and *b* can be paired together if *c* be omitted. Here we have commenced by giving to *d*, for its element, the form of a straight bar, finding all the positions of this bar relatively to *b*, and constructing the envelope of these positions in the shape of curved profiles to projections placed upon the end of *b*.

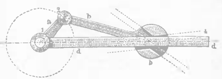


Fig. 190.

This form of higher pairing has been very frequently used in practice, apparently with imperfect recognition of the fact that it is incomplete in its constraintment, the smallest distance between the two curves being unavoidably greater than the breadth of the bar. At and near the ends of the stroke, therefore, the relative positions of *b* and *d* are not absolutely fixed by the pairing, a defect which cannot be rectified without substituting some other form for that of the straight bar as the element of the pair belonging to *d*.

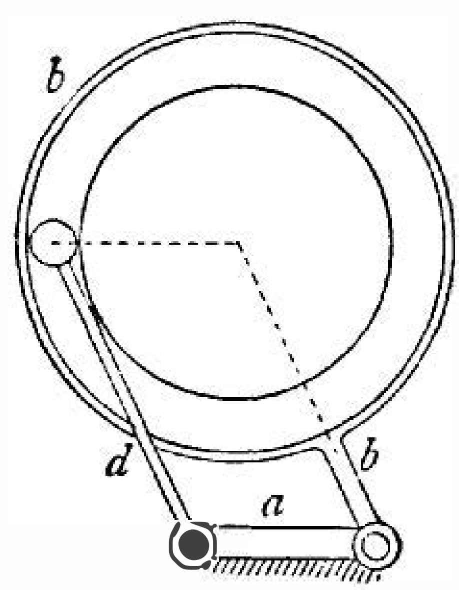


FIG. 191.

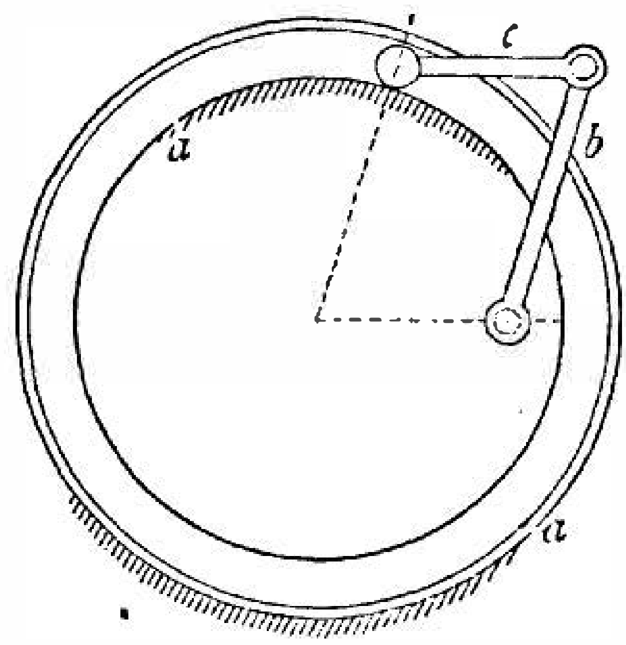


FIG. 192.

In Figs. 191 and 192 are shown two reduced forms of a linkwork parallelogram. In Fig. 191 the link *c* is omitted, in Fig. 192, the link *d*. In both cases the links formerly connected by the omitted link are now directly

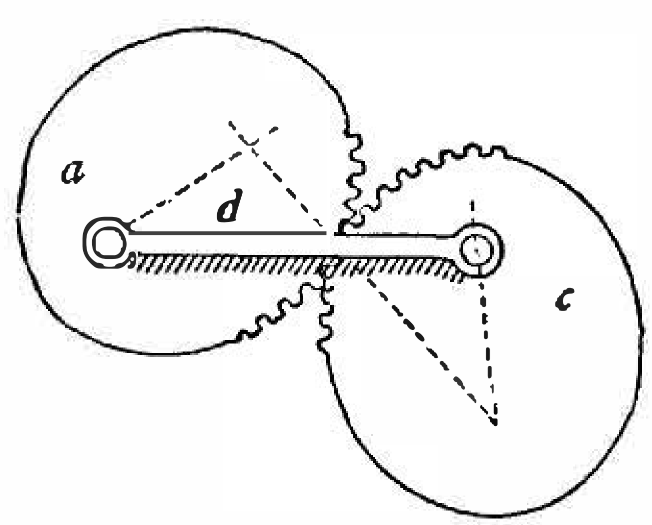


FIG. 193.

connected by higher pairing, and in both cases it has been possible to use for the higher pair a pin and a circular slot.

In Fig. 193 is shown a very different form of higher pairing, used in the mechanism already examined in Figs. 119 and 142, in which opposite links are equal but *anti-parallel*. Here the link b is omitted, and the links a and c are paired by help of their centrodes, which are made into elliptic toothed wheels. In § 21, p. 150, we have already looked at the use of such wheels from another point of view.

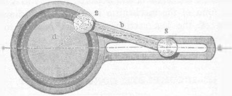


Fig. [illegible]

If it be desired to utilise only the motion of one link in a chain, all the others except the fixed link may be omitted, in which case the chain simply reduces itself to a pair of elements, necessarily a higher pair. Such a reduction,

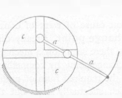


Fig. [illegible]

however, possesses, for engineering purposes, even greater drawbacks than the reductions already mentioned, and very seldom has counterbalancing advantages. Two cases of it are sketched in Figs. 194 and 195. The first of these shows a slider crank from which the links *a* and *c* are omitted. The higher element on *b* takes the form of two circular pins,[1] and the corresponding element on *d* of two slots, one straight and one circular.t There is, of course, only line contact throughout. Fig. 195 shows the converse case, when *b* and *d* are omitted, and *a* paired directly to *c*. The original form of the mechanism here reduced is shown in Fig. 185 of the last section, where its relation to the slider crank was discussed.

§ 54.—INCOMPLETE CONSTRAINMENT.

We started in § 1 with the assumption that constrained motion was an absolute necessity in any combination that was to be used in a perfect machine. We have found, however, that there are many mechanisms which possess one or more unconstrained positions, and are to a corresponding extent unavailable or imperfect as machines. We shall in this section summarise the conditions under which such mechanisms are used.

A very common cause of want of constrainment is the existence of a **change-point**, already discussed in § 21, p. 147. We have there seen how a mechanism can be constrained at its change-point by compelling the centrodes corresponding to the required form of motion to roll upon one another, which effectually shuts out the possibility of any change. Another and more common method is to duplicate the mechanism with another, so placed that it is always in some completely constrained position when the first mechanism is passing its change-point. Perhaps the

[1] As to form and position of these pins, see remark in connection with Fig. 188.

most common illustration of this is sketched in Fig. 196, where a pair of parallel cranks *a* and *c*, connected by a coupling rod *b* (as in a locomotive), which would be unconstrained at two positions in each revolution, are made completely constrained by the addition of the duplicate cranks *a'* and *c'* (with the coupler *b'*) placed (say) at right angles to the original ones.

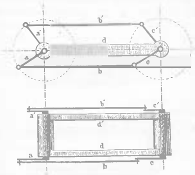


FIG. 196.

The existence of **dead points** in a mechanism is not to be confused with that of change-points. The change-point is inherent in the chain itself, and represents the possibility of change into some different chain or into a pair of elements. The dead point, on the other hand, is not inherent in the chain, or even in any particular mechanism formed from the chain, but depends on the particular link which is the driving link, and the particular way in which the driving force acts upon that link. Thus the ordinary slider-crank (Fig. 182), if it be used as the driving mechanism of a steam engine, where c is the driving link, has a dead point at each end of the stroke of c. But if the same mechanism be used as a pump, where the crank is the driving link, and receive from any source a continuous rotary motion, there are no dead points.

The dead point, where it exists, may be passed by means of a duplication of the chain, such as has been described above as used for passing a change-point. The double slider-crank chain of Fig. 197 is a familiar illustration of

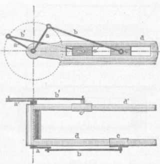


FIG. 197.

this. More often, however, the dead point is passed by help of a fly-wheel or other rotating mass, in which sufficient energy is stored up at other parts of the stroke. A constrainment of the former kind, effected by means of some addition to the mechanism, may be called a **chain-closure**, while constrainment by means of some special force or pressure, provided in connection with the masses of certain parts of the machine, may be called **force-closure** (see also § 51).

Force-closure is very frequently employed for completing the constraint of pairs of elements, when the form of one of them is left kinematically incomplete, as in the case of Figs. 198 and 199. In these cases it is generally the weight of one of the bodies which itself supplies the force necessary for constraint. In case of the occurrence of any disturbing force, this force takes the place of the resisting stresses which would, in a complete pair of elements, prevent change of motion.

It remains to mention, in this section, a curious case which occasionally occurs, in which a mechanism is employed whose motions, were they allowed to develop

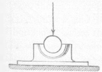


Fig. 198.

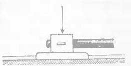


Fig. 199.

themselves, would be unconstrained, but in which only excessively small motions are permitted, and in which, after such motions have occurred, the mechanism is always brought back to precisely its starting position before it is made use of. Fig. 200 shows in outline a mechanism of this kind, which is used in some testing machines,[1] as a substitute for a train of levers having a very large "mechanical advantage." It consists of a lever, *b*, pivotted at 1 to one end of a fixed link, *a*, and loaded at its outer end. Its long arm is connected by *c* to a linkwork parallelogram,

[1] Those of Riehlé Brothers, of Philadelphia. They were exhibited, for instance, at the Philadelphia Exhibition of 1876.

d, *e*, *f*, *g*; the upper link of which carries at its outer end a small weight W_1 which balances a large resistance W on *b*. The link *g* is pivotted at 3 to the upper end of the fixed link *a*. The pair 2, connecting *c* to *d*, is placed so as not to be directly under 3, but some small distance to the right of it. The mechanism is clearly unconstrained, for either *b* or *g* could be fixed as well as *a*, and still all the other links could move, and this we know to be inconsistent with our original definition of constrainment in mechanisms.[1] Its want of constrainment comes out at once if the attempt be made to find its twenty-one virtual centres,—it will be

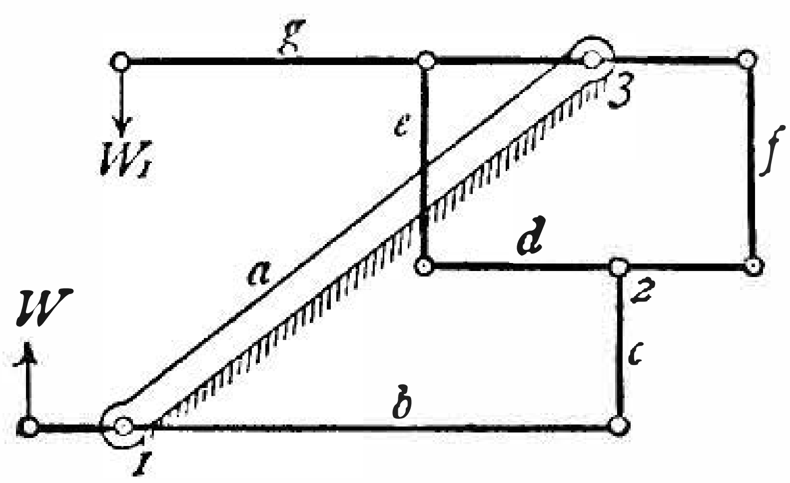


FIG. 200.

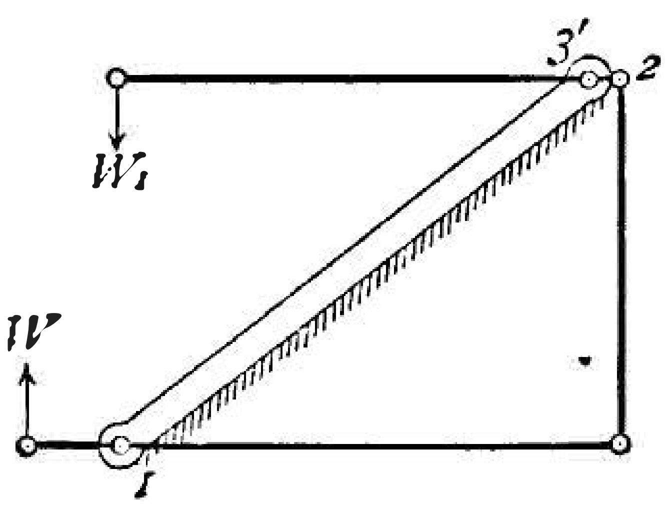


FIG. 201.

found that many of them are indeterminate. To constrain the mechanism it would be necessary to place another element upon *a* and to pair *d* to it at a point directly below the point 3. So far as the balance of W and W_1 is concerned the parallelogram then becomes superfluous, and the mechanism is statically equivalent to that of Fig. 201, which requires no further explanation. But in order that W may be as small as it is wished to be in the particular machine in question, the distance 3′2 must be excessively small—so small as to be constructively impracticable, owing to the absolutely necessary dimensions of the knife edges. The

[1] See § 11.

device of the parallelogram is therefore adopted in order to get 3 and 2 upon different links, so that the horizontal distance between them (the real length of the short arm of the lever *g*) may be made, with absolutely no constructive inconvenience, as small as is pleased. It was only one centimetre, for instance, in a 75-ton testing machine exhibited at Philadelphia in 1876. The purpose of the mechanism (as used in a testing machine) is to measure W by means of W_1. For this purpose it has to be assumed that the leverages of the mechanism, as constructed, are as determinate as those of the simple combination of Fig. 201, which it represents. This cannot be the case if even small changes in the position of the different links be permitted. In the machine, therefore, special arrangements are made by which both the links *b* and *g* can be placed accurately parallel and horizontal before the value of W_1 is read off. It would not be sufficient to have only one of the two links mentioned horizontal, because, as we have seen, that would not constrain the position of the other links. But if *both b* and *g* are brought into known positions, the positions of all the other links (and therefore the "mechanical advantage" of the mechanism as a whole) becomes determinate. The mechanism is therefore, as above described, moveable and unconstrained, but rendered available in a machine by making use of one of its positions only. How far this use of the mechanism is advisable, or within what limits its results are trustworthy, is a matter which has to be settled by practical experience.

§ 55.—THE PARALLELOGRAM.

THE simple linkwork parallelogram,—a four-link chain, with opposite links equal—has some special properties which deserve noting, both on account of their geometrical interest, and because they are so frequently utilised in practice. Such a parallelogram is shown in Fig. 202, its links lettered a, b, c and d. Let it be supposed that *only one point in it is fixed*, (viz., O, the join of a and d) instead of a whole link. Draw any line through O, as OBC, cutting

FIG. 202.

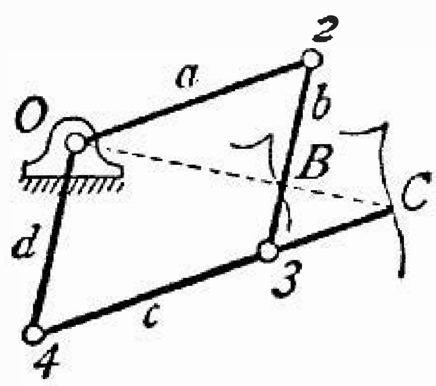
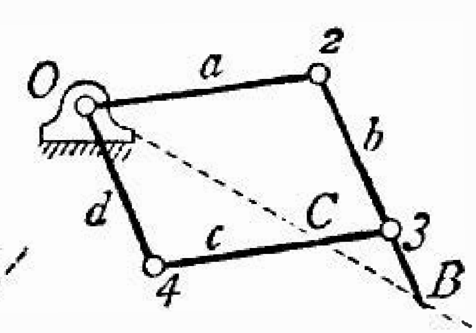


FIG. 203.



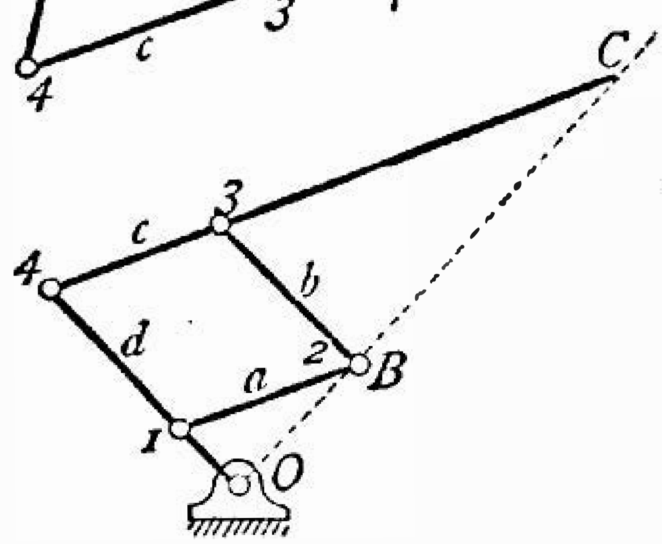


FIG. 204.

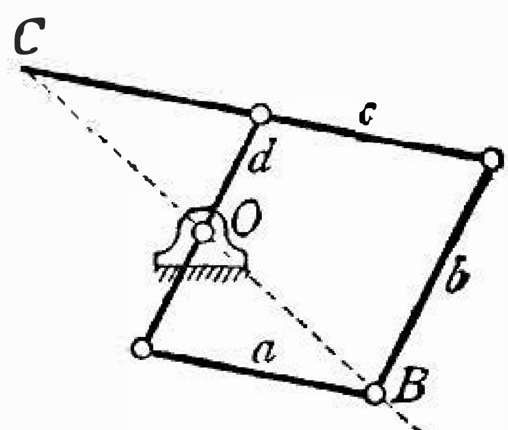


FIG. 205.

the two non-adjacent links in B and C. Then in whatever position the mechanism be placed, these three points will always lie upon one line. For by similarity of triangles, $\frac{3B}{4O} = \frac{3C}{4C}$, so that $3B = 4O \times \frac{3C}{4C}$, and B must therefore always occupy the same position on the link b. Further, the ratio $\frac{OC}{OB} = \frac{4C}{43}$ and is therefore constant for all positions of the mechanism. From this it follows that if B be made to trace any curve or line whatever, C will describe a precisely similar curve or line on a larger scale. By moving B upwards, the ratio of exaggeration can be increased to any extent. Or on the other hand by making OC *less* than OB (Fig. 203), the copy will be on a smaller scale than the original. The parallelogram finds numberless applications of this kind as a "pantagraphe" or copying machine, for enlarging or reproducing maps or drawings.

It is not necessary that the fixed point O be at the join of two links, as in the foregoing cases; it may be taken at any point of any link, as O in Figs. 204 and 205. In this case one of the opposite pin centres, as 2 ($=B$), becomes one of the two tracing points, the other lies at C upon the line BO. The ratio of exaggeration is $\frac{CO}{BO}$. In Fig. 205, this ratio is made equal to 1, so that the copy is a duplicate simply, of the same size as the original.

Professor Sylvester was the first to point out that the properties of the parallelogram just mentioned were not confined to points, such as C and B, lying in one line with the fixed point. In Fig. 206, a b c d are again four links of a parallelogram, of which the vertex O is fixed. The point P is *any* point on the link a, and the point P' a point on the link b so placed that the triangle $P'MQ$ is similar to the triangle MPN, the angles at P', M, and Q, being equal to those at M, P, and N, respectively. Then $\frac{PN}{NM} = \frac{MQ}{QP'}$ so that $\frac{PN}{MQ} = \frac{NM}{QP'}$ and therefore $\frac{PN}{NO} = \frac{OQ}{QP'}$. The angles PNO and OQP' are also equal. The triangles PNO and OQP' are therefore similar in all respects, and

$\frac{OP}{OP'} = \frac{ON}{QP'}$, which is a constant ratio. The ratio of the distances of P and P' from O is therefore the same for all positions of the mechanism. It can be readily shown, also, that the angle POP' remains constant for all positions of the mechanism. The points P' and P must therefore move in *similar* curves, so that one copies the motion of the other, but not only is the copy a different size from the original in the ratio $\frac{OP'}{OP}$,[1] but it is shifted from it round O by a certain definite angle POP'. An instrument for tracing curves in this fashion has been called by Professor Sylvester a *Plagiograph*, or *Skew Pantagraph*.

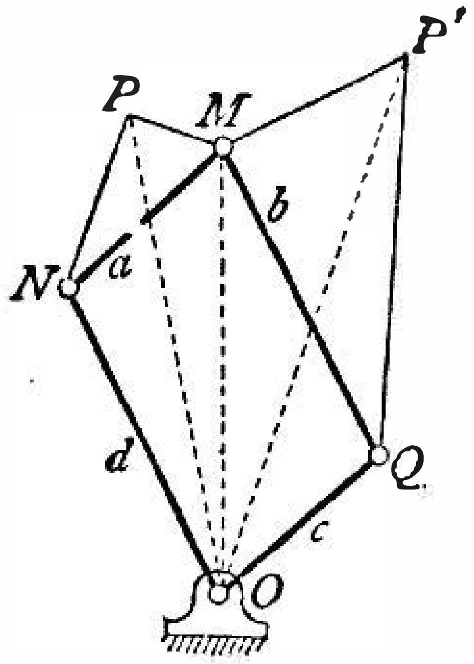


FIG. 206.

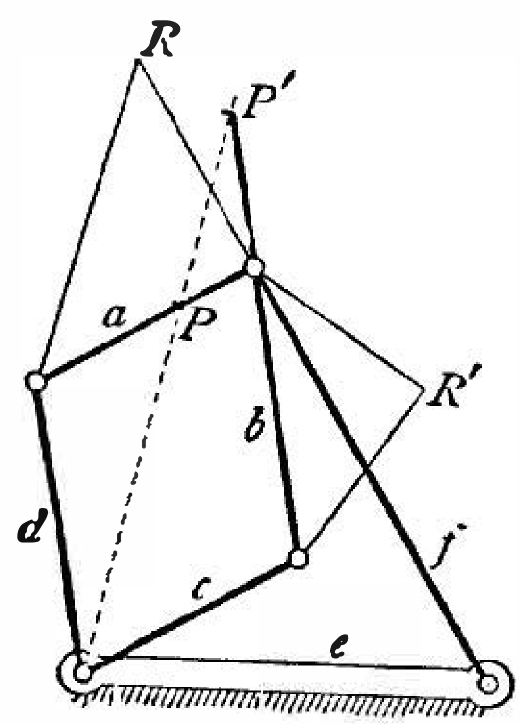


FIG. 207.

If we combine the parallelogram with other links to form a compound mechanism as in Fig. 207, we do not, of course, deprive it of any of its own properties, but merely determine the actual forms of the paths of its points. Thus P' still copies the motion of P, and R' of R, just as they

[1] Of course OP' may be made equal to OP, in which case $\frac{OP'}{OP} = 1$, and the copy is a duplicate of the original.

would if the point O only were fixed. But out of this mechanism we can construct two four-link mechanisms, each containing two links of the parallelogram, viz. a, d, e, f, and b, c, e, f. These two mechanisms will evidently have the property that for every point (as P or R) on the link a of the one, there can be found one point (as P' or R') on the link b of the other such that the path of the latter point shall be similar to that of the former. This curious property was first pointed out by Mr. Kempe.

One of the principal uses to which the parallelogram is put in practice is the copying of the approximately straight line drawn by one point of a parallel motion. This will be considered more fully in the next section. Other properties of the parallelogram will also be noticed in § 57.

§ 56.—PARALLEL MOTIONS.

UNDER the somewhat inappropriate name of "parallel motions" are included in this country certain mechanisms possessing the characteristic that one or more points in them, not directly guided by sliding pairs, move approximately or accurately in straight lines. These mechanisms may be divided into three classes: (1.) those in which the straight line is merely an extended copy of a line constrained by a sliding pair somewhere else in the machine; (2.) those in which the mechanism contains a sliding pair, but without copying its motion, and (3.) those in which all the links of the mechanism are connected by pin-joints, that is by turning pairs. This last class is again subdivided into mechanisms in which the so-called straight line is merely an approximation, and those in which it is mathematically exact, this latter class being or very modern origin.[1] We shall look at these different mechanisms in the order in which they have been mentioned.

The mechanism belonging to the first class which is most commonly used as a parallel motion is a slider crank in which the connecting rod is made equal in length to the crank. We have already examined this mechanism (§§ 21 and 42, pp. 146 and 318) and seen that the centrodes of the links *b* and *d* are a pair of circles, one of which is twice the diameter of the other. Any point, therefore, on the link *b*, as *M* (Fig. 208), which is at a distance 23 from the point 2, will describe a straight line, passing through the point 1, relatively to *d*. In order that this line may reach its maximum possible length of four times the crank radius, the sliding pair 4 must have a travel equal to twice the crank radius.[2] This would have many inconveniences in practice, so that only the central part of the link is used, and this allows of the employment of a sliding pair with comparatively very short travel, as shown in the figure. This parallel motion is generally known in this country as Scott Russell's.

Another parallel motion, but one not so well known, is based on the mechanism of Fig. 185, § 52, which is, as we have seen, a slider crank in which three links (*b*, *c* and *d*) are made infinitely long. In this case (Fig. 209) it can be seen at once that the point O_{ac} is the virtual centre for the links

[1] On this part of the subject, see particularly Mr. A. B. Kempe's paper "On a General Method of Obtaining Exact Rectilinear Motion by Linkwork" in the *Proc. R. S.*, 1875, as well as other papers cited by him in his lectures, *How to Draw a Straight Line* (Macmillan, 1877), this last a most interesting elementary statement of the matter. To Mr. Kempe's work I have been very much indebted in writing the latter part of this section.

[2] It is here supposed that the link is not required to rotate, but may merely swing backwards and forwards.

a and *c*, and it may be left to the student to prove that the centrodes of these links are again circles as in the last case, the *radius* of the centrode of *c*, and the *diameter* of that of *a*, being equal to the distance $O_{ac}S$. Hence, exactly as in the last case, any point of *a* which lies upon its centrode will describe a straight line (passing through the point *S*) relatively to *c*. The two sliding pairs may have, as in the last case, a travel comparatively much smaller than the stroke of the describing point *M*. The two sliding pairs may be at right angles to each other (as in Fig. 185), or at any other angle to each other, as in the figure here given.

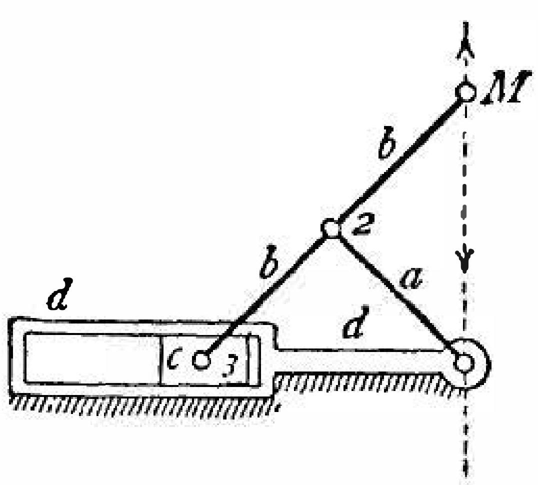


FIG. 208.

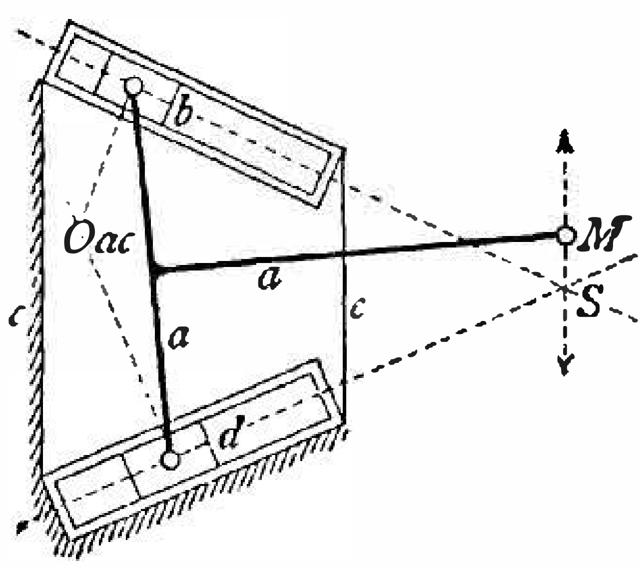


FIG. 209.

In both these cases it has been mentioned that *any* point upon the circular centrode might be used as the guided one. If required, therefore, several might be used simultaneously, and then the one (Fig. 208), or the two (Fig. 209) auxiliary sliding pairs might be made to constrain any number of points to move in straight lines. In each case all the straight lines will pass through one point.

Fig. 210 is an example of the second class of parallel motions enumerated above, in which a sliding pair is still employed, but by which only an approximation to a straight line is obtained. In order that the centrodes of two bodies relatively to each other may be the two circles whose properties we have just been able to utilise, it is necessary and sufficient that any two points of one should describe non-parallel straight lines relatively to the other. It follows (in consequence of the rolling of the centrodes) that all other points of the first body should describe ellipses relatively to the second. The relative motion of the two bodies would be just as completely constrained by making two points move in two of these ellipses as in the two straight lines, but this would not, of course, be practically so convenient. There is no difficulty, however, in finding circular arcs which very closely coincide with certain portions of these ellipses, and in getting rid, by their use, of one or both of the sliding pairs without causing the

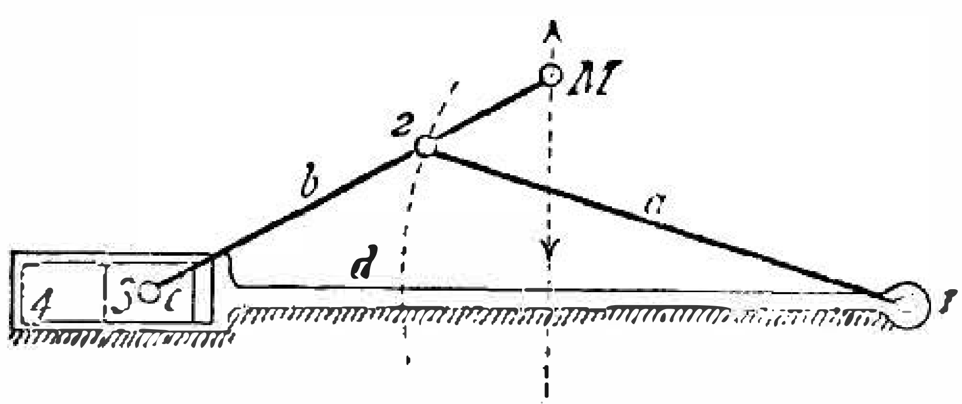


FIG. 210.

described line to vary much from accurate straightness. In Fig. 210 one sliding pair is retained, the path of 3 is therefore made accurately straight, but is (as explained in connection with Fig. 208) of very limited extent. The point M is the describing point, but the point 2 is not taken in the centre of 3 M, so that its path (if M moves accurately in a straight line) is an ellipse. The actual motion of 2 is, however, a circular arc, with centre at 1, as constrained by the link a. If this circular arc sufficiently nearly coincides with the ellipse instead of which it is used, the path of M may be assumed, for most purposes of practical engineering, to be a straight line. The point 1 for any given position of 2 can be best determined by finding the highest, lowest, and middle positions occupied by 2 if *M* does move in a straight line, and then using for 1 the centre of a circle drawn through these three points. The swing of the link *b* on each side of the centre line should not exceed 40°, and the approximation is of course closer if the angle be smaller. As compared with the exact motion for which this is a substitute, it has the constructive advantage that the path of *M* does not pass through the point 1.

If a body move so that one line in it passes always through one point, and one point in that line describes a straight line, its other points describe curves of a high order known as conchoids. These curves have, under suitable conditions, portions which are very nearly circular. If therefore, with suitably chosen points, we cause a line to move so that it always passes through one point, and cause one point in that line to describe a circle closely coinciding with a conchoidal arc, some other point in the line will describe approximately a straight line.

Such a motion is obtained by the use of the inverted slider-crank mechanism (link *b* fixed, as in the oscillating engine) of Fig. 211. Here the point 1 in the link *d* is constrained by the link *a* to turn always about 2, in an arc approximately coinciding with the conchoid, and the sliding pair at 4 compels the line 1 *M* of the same link to pass always through the point 3. The paths of a few points of *d* are shown in dotted lines. The point *M* has a path which for a short distance may be taken to represent a straight line.

With a moderate angular swing this mechanism gives a very good approximation to a straight line. The best position for the centre 2 can be found, for a given point 3 and a given length *M* 1 and path of *M*, as before, viz., by finding the positions of 1 for highest, lowest, and middle positions of M, and taking for 2 the centre of a circle passing through the three points so found.

The third class of parallel motions, those in which only pin joints are used, may be first illustrated by two forms which are used as approximations to the exact parallel motion obtained by the help of sliding pairs; they are shown in Figs. 212 and 213. Thus for instance *Roberts'* motion, shown in Fig. 212, is derived from the trammel motion of Fig. 209. The straight paths of the two end points of the link a are replaced by circular arcs, approximations, in reality, to

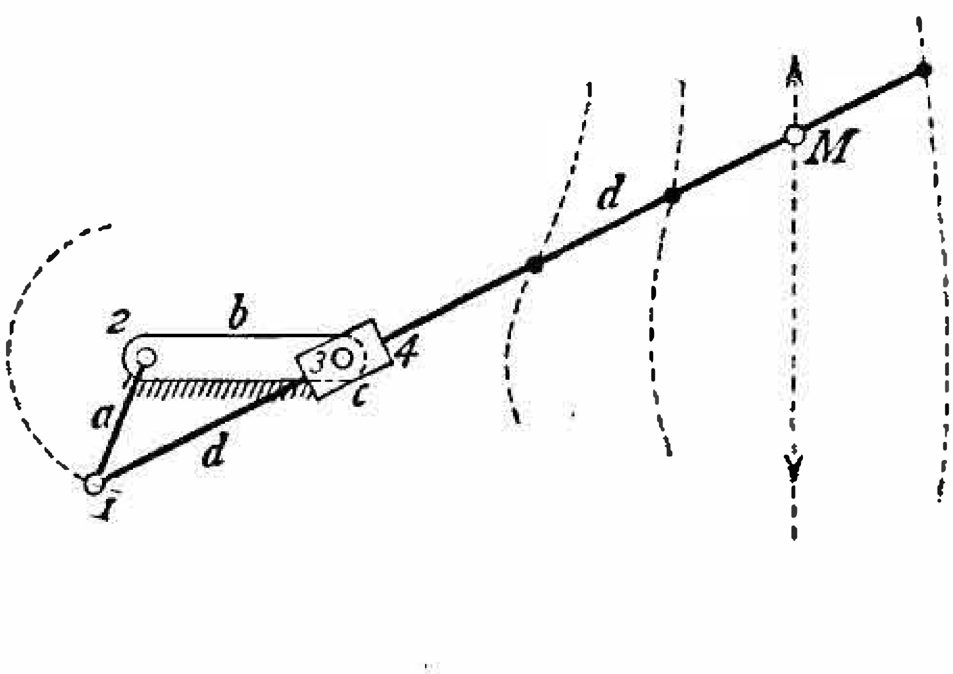


FIG. 211.

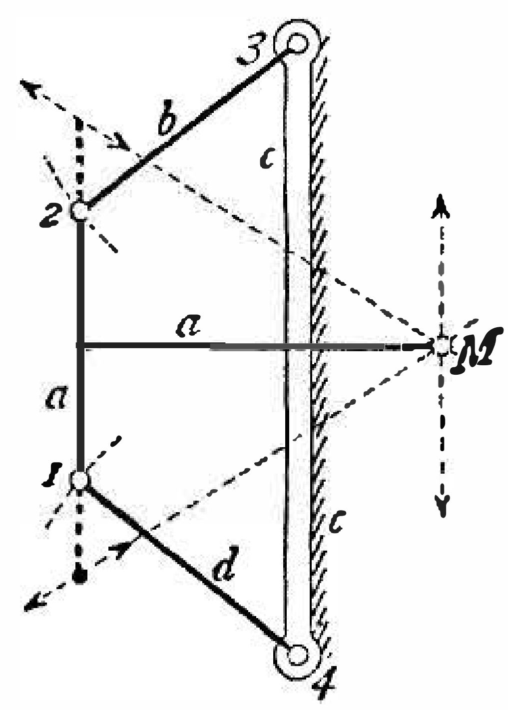


FIG. 212.

the elliptical paths of other points of that link, the links b and d being connected to c by pins instead of by sliding blocks. The point M describes a good approximation to a straight line for a certain part of its path.

Most commonly the point M is made to lie on the line 34, so that its path coincides with the axis of the link c. In this case the lengths 23, 2 M, M 1, and 14 are all equal, and should be *not less than* 0·42 of the length of the link c,[1] and as much greater as possible. The length of a will obviously be equal to

[1] Rankine, *Machinery and Millwork*, chap. v. sect. 5.

half that of *c*. If it be inconvenient to make the path of *M* coincide with the line 34, it may be placed outside it, as in the figure. In that case the best points for 3 and 4 for any assumed triangle 2 *M* 1 will be found as in the former cases by finding the three positions of the points 2 and 1 for the ends and middle of the travel of *M*, assuming that travel to be accurately straight, and then taking 3 and 4 as the centres of circles passing through each set of three points.

Fig. 213 shows a linkwork parallel motion which gives an approximation to the already only approximate rectilinear motion obtained in Fig. 210. The infinite links *c* and *d* of

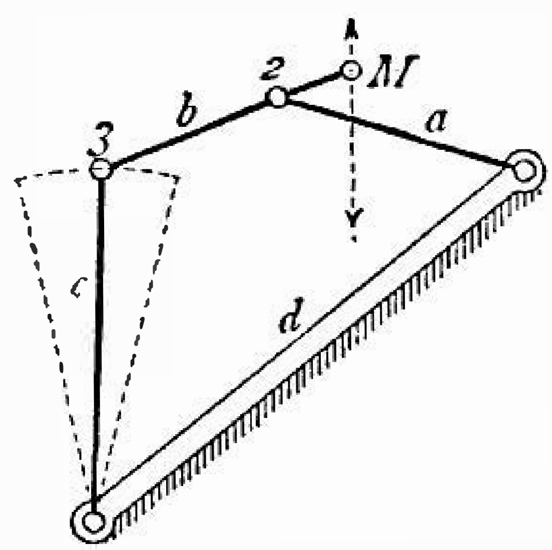


FIG. 213.

the slider-crank are replaced by ordinary links, and the straight path of the point 3 by an arc of a circle. So long as the length of *c* is not less than the whole travel of *M* which is utilised as straight, the approximation given by this mechanism is sufficiently good for most practical purposes. The point 1 must be determined in the way described in connection with Fig. 210.

By using for 3 not the point originally moving in a straight line, but some other point of *b* whose proper path is an ellipse—and so substituting the circular arc for an elliptic one (as already with the point 2) instead of for a straight line—we can obtain other modifications of the mechanisms, which may sometimes be convenient.

Fig. 214 shows the ordinary *Watt* motion, the best known and most often used of all the approximate parallel motions. In the most common and best form of this mechanism the links b and d are equal, and the describing point M is in the middle of the link a; the length of a is made about equal to the intended stroke of M; in their mid-positions b and d are parallel and lie at right angles to the path of M, and the points 1 and 2 deviate to right and left of that path by equal amounts at the middle and ends of their swing.[1]

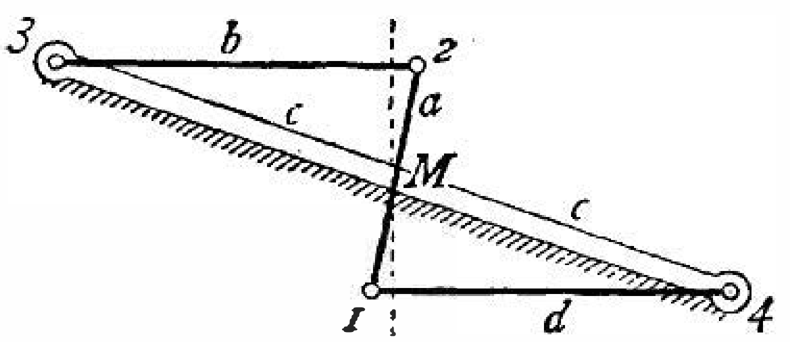


FIG. 214.

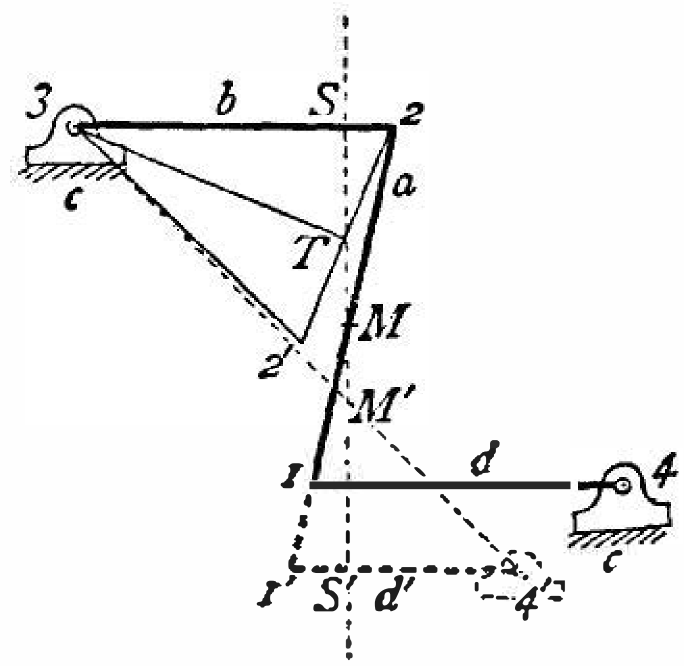


FIG. 215.

In any given case it is advisable to obtain as many of these conditions as possible, even if all cannot be simultaneously secured.

The following are some of the principal constructions connected with the Watt parallel motion. Let there be given (Fig. 215) the path of the point M and its mid-position, and the axis of the link b in its mid-position with its centre 3. It is required to find the point 2 which fixes the length

[1] See as to this and the constructions following, Rankine, *Machinery and Millwork*, chap. v. sect. 5.

of b, and the distance 2 M. The condition to be fulfilled by 2 is that its middle and end positions should be to right and left of the path of M by equal amounts. Make ST equal to one quarter of the stroke of M, and draw T 2 at right angles to 3 T. The point 2 will then be in the required position. 32 will be the length of b, 2 M will be a part of the link a in its mid-position, and 2′ will be the position of 2 when at the end of its stroke (2′ T = 2 T). If M 1 = 2 M, then $d = b$ and the point 4 can be found at once. But if 1 be at any point 1′, so that M 1′ is not equal to 2 M, then the best result (to make the point 1 fulfil the conditions above prescribed for 2) will be obtained by setting off $S'M' = SM$, and drawing 3 M' to get the point 4′, as shown in dotted lines in the figure. The link d then becomes d', with a length 1′4′ and a centre at the point 4′.

The complete curve traced by the describing point in the Watt motion is in form a distorted figure-of-eight, called a *lemniscoid*. The part actually used for a straight line is in reality wavy, and has five points which actually do lie upon one straight line. In the best forms of the mechanism three of these five points coalesce in the centre point.

As used in beam engines, the Watt parallel motion is generally combined with a copying mechanism, in the shape of a parallelogram (see § 55), for increasing the length of the line in a way not involving so much weight and space as would the enlargement of the parallel motion itself. This is shown in Fig. 216. The parallel motion proper consists of the four links a, b, d and the fixed link c. M is the guided point, as before. The link b is generally the main beam of the engine, and it would be very inconvenient to connect a to the end of the beam and provide by a huge link d' the direct parallel motion for the point E, above the piston rod. Some point such as M therefore, which has much smaller stroke, is directly guided, and additional links *e* and *f* are added to form a parallelogram such that *E*, *M* and *O* lie all upon one line. (In practice the point *E* would be first fixed, and *M* found from it.) It was shown in § 55 that in this case *E* would copy the motion of *M*, and the motion of *M* having already been made (approximately) rectilinear the motion of *E* will be (approximately) rectilinear also.

In most cases the point 2 is taken at half the beam radius, and $\frac{OE}{OM} = 2$. The points 5 and 1 then coincide, while the points 4 and *E* and the axes of the links *d* and *f* appear to coincide when in their mid-position.

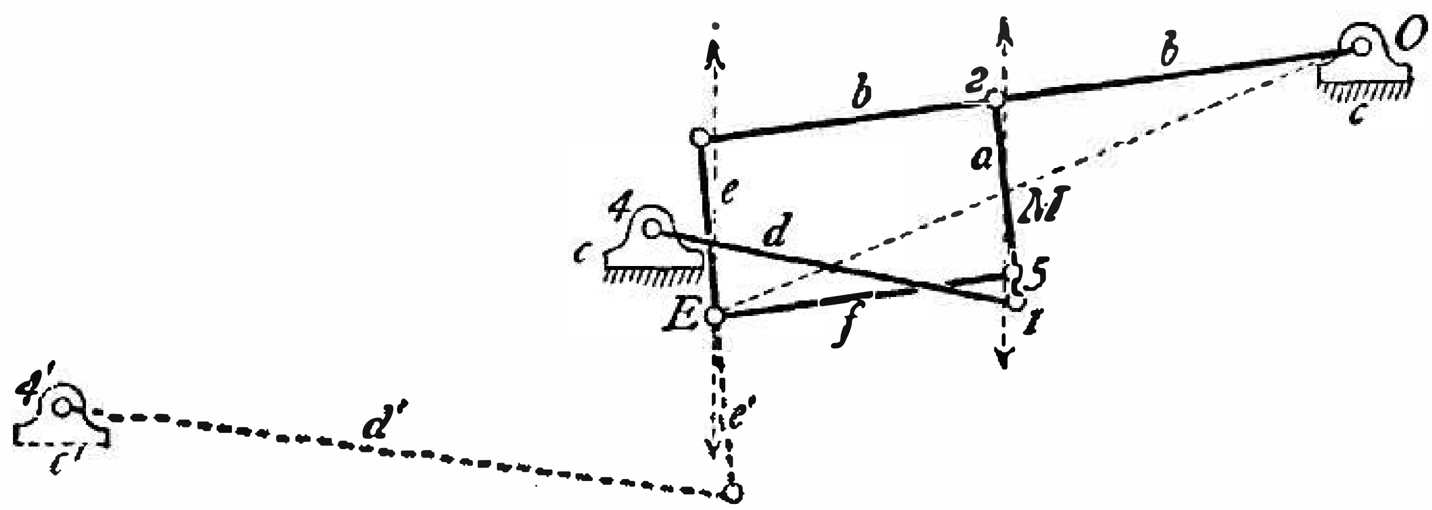


FIG. 216.

The motion of the point *M* is generally utilised for one of the pump rods of the engine. By adding other parallel links to the parallelogram other points can be found also moving in straight lines, which may be, and often are, utilised for other pump rods on the engine.

Perhaps the most modern of approximate parallel motions is that of Professor Tchebicheff, of St. Petersburg, of which there was a small example in the Vienna Exhibition of 1873.[1] This motion is shown in Fig. 217 and gives a very

[1] *Engineering*, vol. xvi. p. 284.

good approximation to a straight line. The links *b* and *d* are equal, and may be made each about 1·3 times the length of *c*. The length of *a* may be 0·4 of the length of *c*. The describing point *M* is in the middle of *a*. The travel of *M* may be anything less than the distance 34. The points 3 and 4 should be found as already described for the Roberts' parallel motion.

The first of the *exact* linkwork parallel motions was invented as recently as 1864 by M. Peaucellier, a French engineer officer. We have already once or twice (Fig. 128) used it in problems, but without examining its special theory or properties, which we shall now proceed to do.

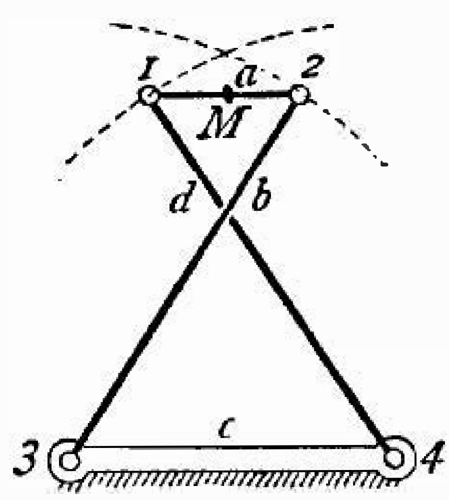


FIG. 217.

The Peaucellier parallel motion is a compound mechanism having eight[1] links. Of these eight, four (*a*, *b*, *c*, and *d*) are equal, and form a rhombus in all positions (see Figs. 218 and 219). The others are equal in pairs, viz., *e* and *f* are equal, of any length that will permit them to be

[1] It is usually called a "*seven-bar*" mechanism, the eighth or fixed link not being counted, and the same nomenclature has been used for the other exact parallel motions. This is, I think, to be regretted, for the fixed link is just as much, or as little, a bar as any of the other links.

jointed to each other at one end (P), and to opposite angles of the rhombus (S and T) at the other. The remaining links, h and g, are also equal; they are jointed to each other at Q, to the common point of e and f (P), and to a third angle of the rhombus (N), in the fashion shown in the figures. If the link h be now fixed, the remaining angle of the rhombus M will move accurately in a straight line at right angles to the axis PQ of the fixed link.

FIG. 218.

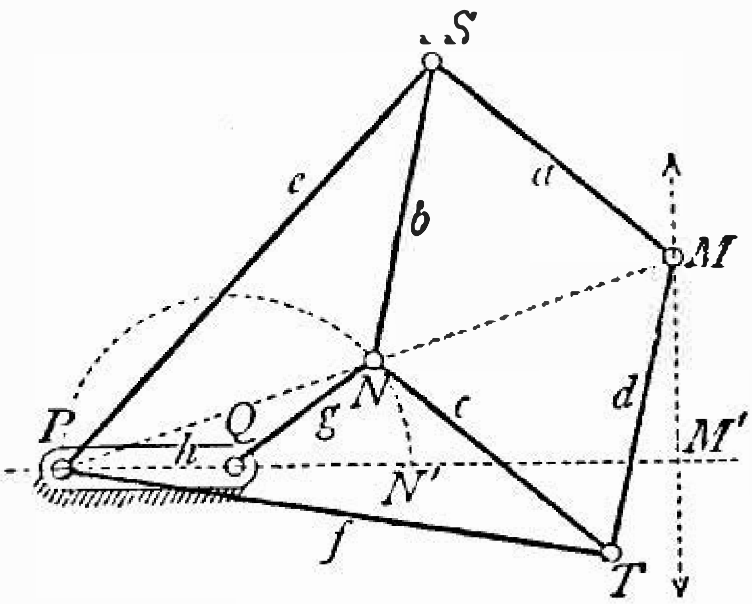
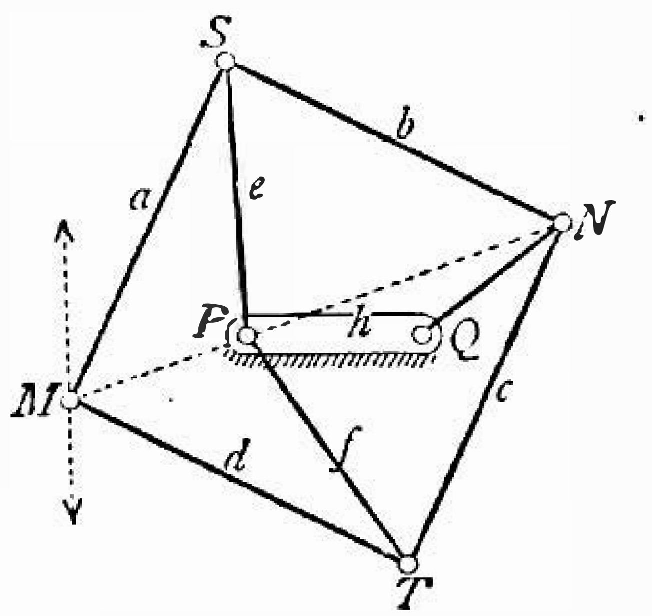


FIG. 219.



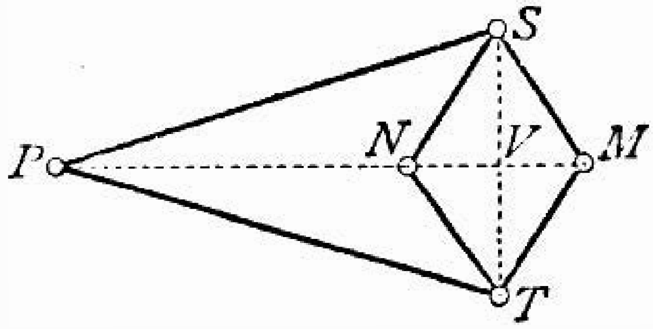


FIG. 220.

The six links first mentioned, a to f, form together what is called a *Peaucellier cell.* Of the other two links one is the fixed link, and the other (g) swings so that its end point N moves in a circle passing through P.

Considering in the first place only the Peaucellier cell (Fig. 220), we notice that (from the given conditions as to equality and symmetry of its links) the three points P, N, and M must always lie on one straight line. If now we call the mid-point of the rhombus V, we have

$$\begin{aligned} PS^2 &= PV^2 + VS^2, \text{ and} \\ SM^2 &= MV^2 + VS^2, \text{ from which} \\ PS^2 - SM^2 &= PV^2 - MV^2 \\ &= (PV - MV)\ (PV + MV) \\ &= PN.\ PM \end{aligned}$$

As PS and SM are constants for any given mechanism, the product $PN.PM$ must be constant also, *whatever the position of the mechanism.*

Going back now to Fig. 218, which shows the complete mechanism, and supposing N to be at N', the point opposite P, and M therefore to be in some position M' in line with PN', then

$$PN.\ PMa= PN'.\ PM', \text{ and}$$

$$\frac{PN}{PN'} = \frac{PM'}{PM}.$$

The triangles PNN' and $PM'M$ must therefore be similar, as the angle at P is common to both of them. But the points N', N and P lie, by construction, upon a circle of which PN' is a diameter, the angle PNN' is therefore a right angle, as being the angle in a semicircle. The angle $MM'P$ is therefore a right angle also. But this will be true whatever the position of the mechanism, that is, for any possible position of the point M. Hence *the point M must move so as always to lie upon—in other words so as to describe—a straight line at right angles to PQ, the axis of the fixed link.*[1]

[1] In this case the straight line is described as the *inverse* of the circle described by N. If N described some other curve than a circle, M would describe the inverse of that curve, which would, of course, not be a straight line. It was from this property of the Peaucellier cell

In concluding this section we may now go on to consider the more general cases of exact linkwork parallel motions which have recently been discovered. Although several other mathematicians, notably Professor Sylvester, have worked at these problems, we are indebted to Mr. Kempe (whose papers have been cited above) for the most complete and general investigation of them, of which a few only of the leading features are given in the following paragraphs.

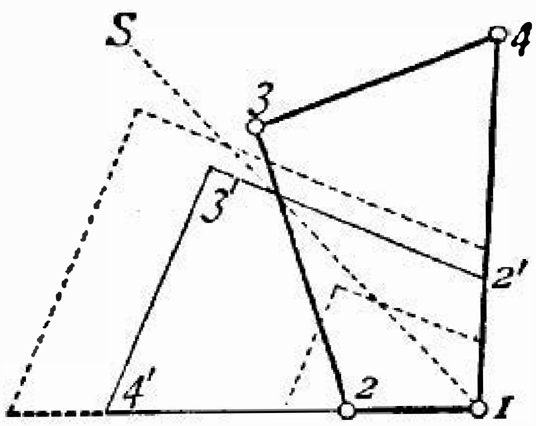


FIG. 221.

Let 1234 (Fig. 221) be any four-link mechanism of the lever-crank type. If the angle 214 be bisected in 1*S*, and a *similar* mechanism 12′3′4′ be placed symmetrically to 1*S* as an axis, the second mechanism is said to be the **image** of the first. The image need not be the same size as the original mechanism, but may be reduced or enlarged in any ratio, as shown in dotted lines. It remains an image so long as its links are parallel to those of the original equal image 12′3′4′.

that its value as a parallel motion was originally discovered. Mr. Kempe has shown, however, that the Peaucellier motion may really be taken as a very special case of the more general parallel motions now to be described. It may be further noted that if the links *g* and *h* be *not* equal, *M* will describe the arc of a circle, and the mechanism may be utilised in this form for describing accurately circular arcs of very large radius.

In Fig. 222 *abcd* is an ordinary four-link mechanism, of which a is the fixed link. Conjoined with this is a second mechanism 12′3′4′ ($a'b'c'd'$), which is a reduced image of the first. The axes of the links a and d' are made to coincide, and also those of d and a'. The ratio in which the second mechanism copies the first, *i.e.* the ratio

$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c} = \frac{d'}{d}, \text{ we may call } = K.$$

It can be shown that in all positions of this compound mechanism its one part remains an image of the other. The fixed link of the mechanism carries three elements. One is paired with the compound link da', each of the others is paired with a simple link, here b and c'. If now any point (as P) be taken on one of these two links (as b), it can be shown that it is always possible to find a corresponding point (as P') on the other (as c'), the distance of which from P, measured parallel to the axis of the fixed link, shall be the same for all positions of the mechanism. It is on this property of this type of compound mechanism that its usefulness as a parallel motion depends. To utilise it, it is necessary to choose a particular point P, and to obtain the most convenient form of mechanism we must also choose a particular value for the ratio K. We get what is perhaps the most general form of exact parallel motion by giving K *any* value, but making

$$2\,\mathrm{P} = \frac{(a - Kd)\;ab}{(a^2 + b^2) - (c^2 + d^2)}$$

With these proportions the constant distance NN' becomes equal to half of the distance 24′. If then we add to our six links two others, PM and MP', the one equal to $P2$ and the other to 4′ P', the points P and P' will remain always the vertices of two isosceles triangles whose bases lie in the line 14′, and the point M must move exactly along that line.

An immense number of modifications of this mechanism can be devised, but perhaps the most convenient is that shown in Fig. 223, which moves with great freedom, and has a relatively enormous stroke for its describing point. In this mechanism the value of K is made equal to $\frac{d}{a}$, and the link b equal to the link c, while

$$2\ P = \frac{(a - Kd)\ ab}{(a^2 + b^2) - (c^2 + d^2)} = \frac{a^2b - d^2b}{a^2 - d^2} = b.$$

These proportions give us a mechanism in which $a' = d$, $b = c$, $b' = c'$, P coincides with 3, P' with 3′, and 2′ with 4. Of

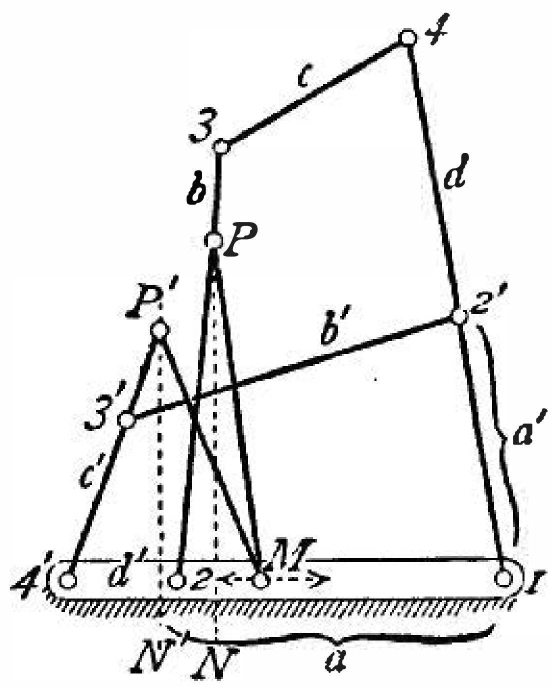


FIG. 222.

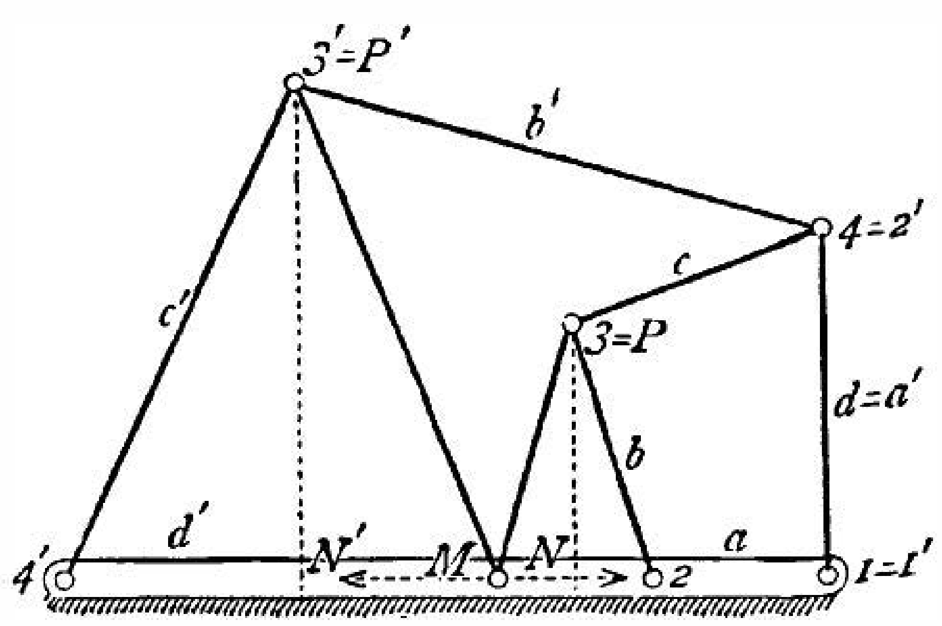


FIG. 223.

course from a constructive point of view it is awkward to allow the point M to pass the points 4′, 2 and 1, but notwithstanding this it is very possible that there may be some cases in which the other advantages of this mechanism may cause it to be practically used, cases, namely, where a slider may be inadmissible, and where an ordinary approximate motion would be too unwieldy for the long stroke required.

We shall consider only one more form of exact linkwork parallel motion, namely that of Mr. Hart,[1] which is geometrically most notable because it contains only six links instead of eight, but which is not perhaps of so much practical interest as those already described, because the dimensions of the mechanism are very large in proportion to the length of the line described. Fig. 224 shows an anti-parallelogram, 1234, cut by any line PM parallel to 13 or 24. Such a line cuts the axes of the links in four points,

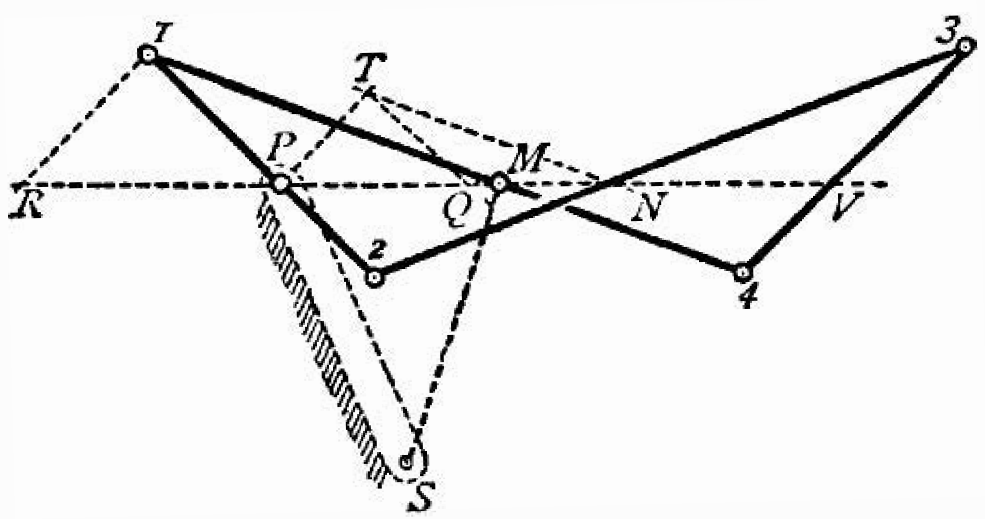


FIG. 224.

$PNMV$, which divide the links in a constant ratio, that is, $\frac{1P}{P2}=\frac{1M}{M4}=\frac{3N}{N2}=\frac{3V}{V4}$. The four points $PNMV$, therefore, will remain always in one line, however the mechanism moves. If we turn $P2N$ round to PTN, and draw TQ parallel to 12 and $1R$ parallel to TP, it can easily be seen that in $M1RP$, and $NTPQ$ we have halves of two Peaucellier cells. From the proof already given we know, therefore, that each of the products $NP.NQ$ and $MP.MR$ will be constant for all positions of the mechanism; therefore the joint product $NP.NQ \times MP.MR$, must be constant also. But by symmetry $\frac{MP}{MR} = \frac{NQ}{NP}$, so that of the four

[1] *Cambridge Messenger of Mathematics*, 1875, vol. iv. p. 82, &c.

quantities just given, $MP.NPa = MR.NQ$, from which it follows that

$$PM.\ PN = \text{constant}$$

here, as in the Peaucellier cell, for all positions of the mechanism. (Similarly it can be shown that $VM.VN.$, $PN.NV.$, and $PM.MV$, are all products which remain constant for all positions of the mechanism.) We can therefore convert this anti-parallelogram into an exact parallel motion by the addition of two equal links, exactly as in the former case. Thus PS and SM may be added, and PS fixed. The point N will then describe a straight line at right angles to PS.

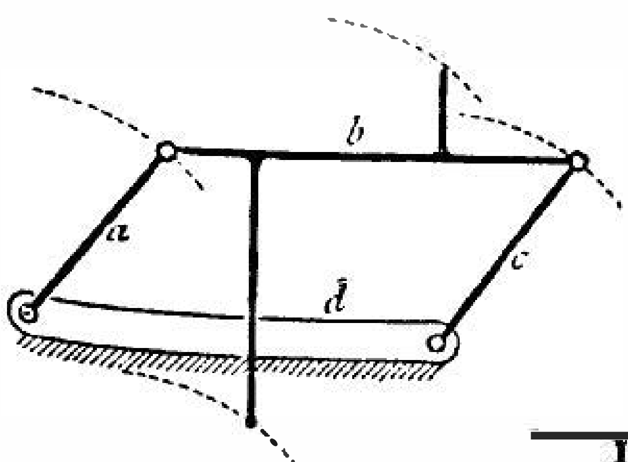
§ 57.—PARALLEL MOTIONS. (*Continued.*)

The name "parallel motion" is in this country so firmly connected with the straight-line mechanisms discussed in the last section, that it would seem pedantic to deny it to them. It is nevertheless a somewhat unsuitable name for them, and describes much better another class of mechanisms which we shall now examine, and in which one or more links are constrained to move always parallel to themselves. For these mechanisms no special name seems to have been proposed, and we shall not attempt to supply the deficiency.

The simplest of these mechanisms (disregarding, of course, those in which the desired motion is obtained by the use of sliding pairs) is the parallelogram itself (Fig. 225), in which if any link be fixed the opposite link has a motion of translation only, all its points moving with equal velocity in circular paths of equal diameter. This applies not only to points along the axis of the link, but to any points whatever connected with it. A few such paths are sketched in the figure. In its simplest form the parallelogram is used in the ordinary "parallel ruler," where only the parallelism of *b* and *d* is utilised, motions of points being left out of the question.

Fig. 226 shows a common application of this in what is known as the Roberval balance. The parallelogram is doubled by the addition of one link, *c′*, and the links *c* and *c′* remain always parallel to each other, and at the same distance from *a*. The scale tables, therefore, connected with

FIG. 225.



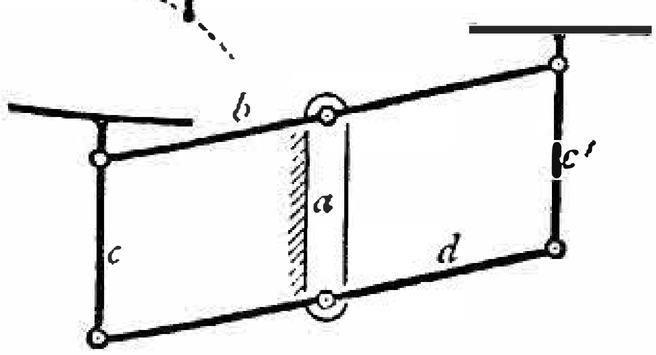


FIG. 226.

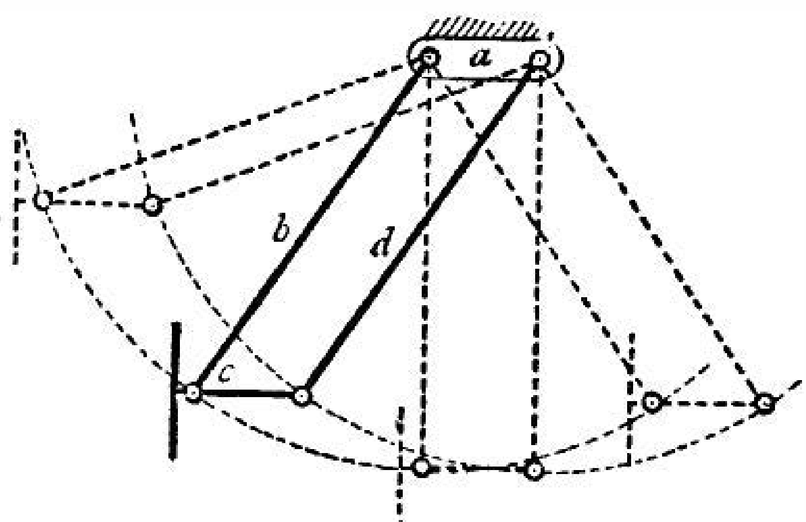


FIG. 227.

them, remain always horizontal. The often-proposed type of "feathering paddle-wheel" shown in Fig. 227, utilises the same mechanism in the same fashion. It should be mentioned, perhaps, that this motion is quite different from that really required in a feathering paddle-wheel, and that it would be quite useless for any such purpose.

If two Watt motions, having equal radius rods, be combined, as in Figs. 228 and 229—the mechanisms having been proportioned in the way given on p. 425—the distance between the two describing points M and M' would remain constant if those points described accurately straight lines, and will in practice change so extremely little that it may be assumed to be constant. They may therefore be connected

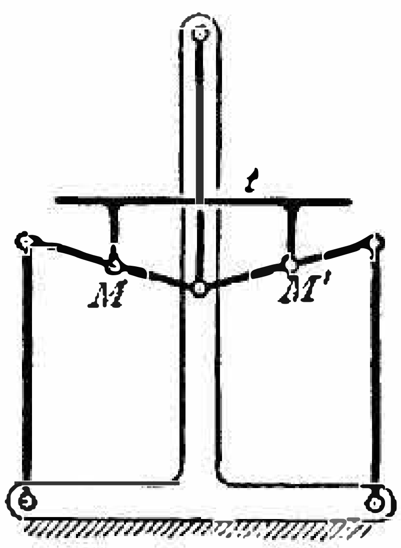


FIG. 228.

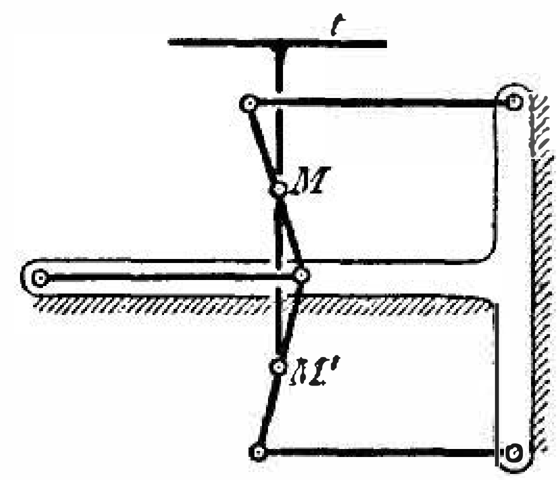


FIG. 229.

by a link t, and all points in that link will move in parallel and (approximately) straight lines. Such a mechanism would form a very easily working linkwork carriage for a straight moving table where the use of slides was objectionable.

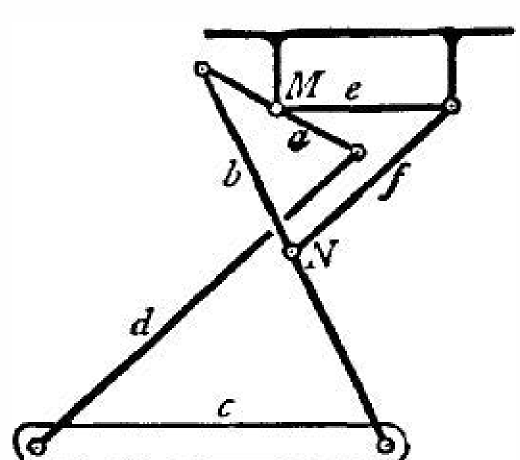


FIG. 230.

Fig. 230 shows another approximate motion of the same type (but with one link less) proposed by Mr. Kempe, and based upon the Tchebicheff parallel motion already described (see Fig. 217). The length of the link e, to which the table is connected, should be half that of the fixed link c, and the length of f one-half that of b or d. The point N lies midway along the link b.

In conclusion, three of Mr. Kempe's mechanisms may be given, which, although more complex than those just looked at, give motions which are exact instead of only approximate. The mechanism $a\ b\ c\ d$ (Fig. 231) is a "kite" (or four-link mechanism in which adjacent links are equal, in which the long links c and d are made twice the length of the short ones, a and b. With it is compounded another kite, $a'\ b'\ c'\ d'$, exactly half its size, in such a way that d' coincides with a, and a' lies along d. It can readily be shown that in such a mechanism the line joining M and M' is

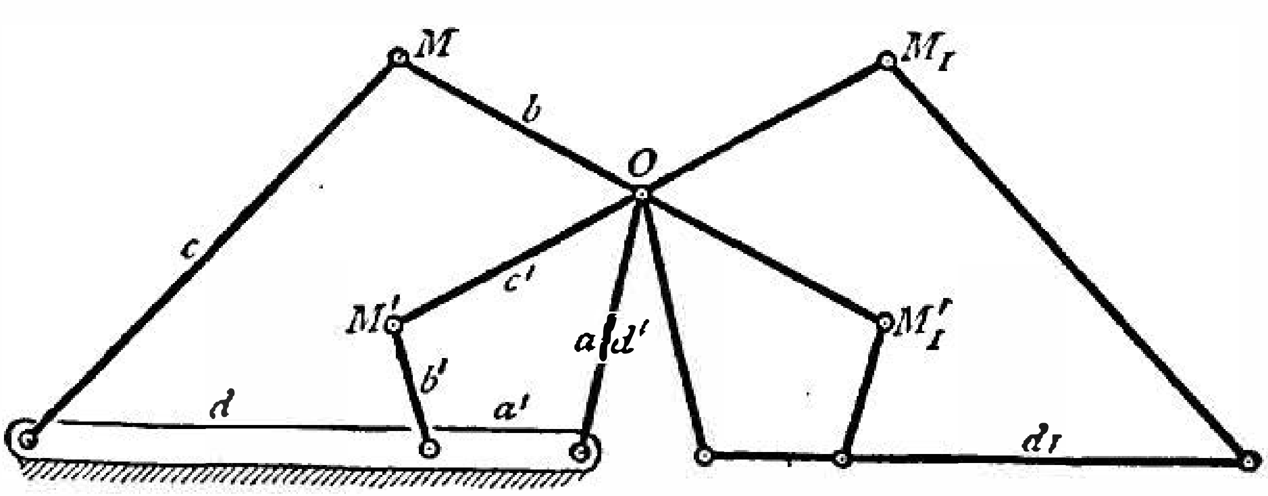


FIG. 231.

always perpendicular to the axis of the link d. Let now the links b and c' be extended through O to M'_1 and M_1 respectively, and let a new double kite, exactly equal and similar to the first, be constructed on these extended links. The points M_1 and M'_1 must always lie on a line parallel to MM', and the new link d_1 must therefore always remain parallel to the original link d. Further, as a is fixed, the symmetry of the mechanism constrains d_1 to remain always not only parallel to, but *in line with*, d. Thus any table, or other body, attached to d_1 will have a simple motion of translation, all its points moving in parallel straight lines, not only approximately, but exactly.

In Fig. 232 is given a modification of this mechanism in which the link d_1 is constrained to move at right angles to the fixed link d, instead of parallel to it. The mechanism consists of the same pair of double kites as before, but differently connected together.

Fig. 233, lastly, shows how this motion can be applied to an ordinary double "parallel ruler," to constrain not only parallelism of position but straight-line motion. Three of the links of the last figure are omitted, and their places taken by three links of the double parallelogram. Two kites remain, one large and one small, compounded externally with

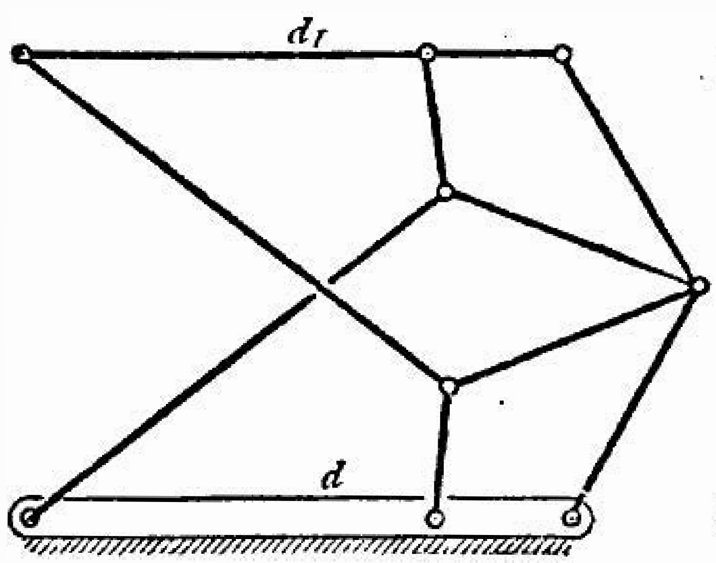


FIG. 232.

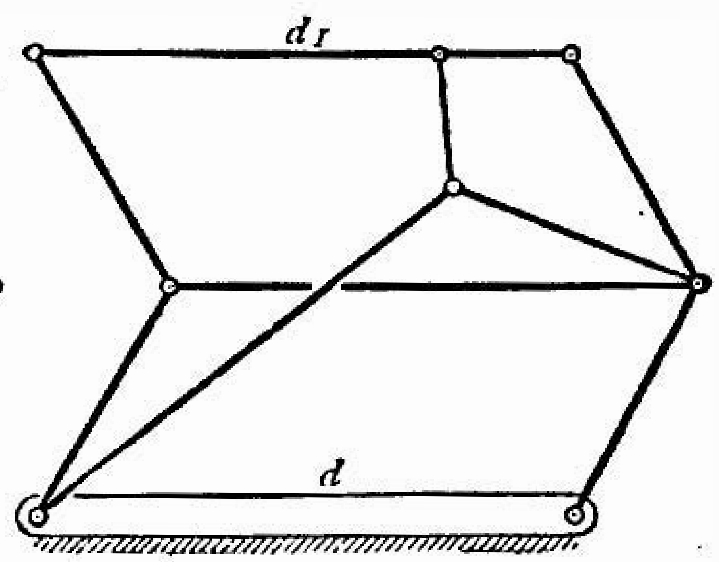


FIG. 233.

each other and with the two outer links of the parallel motion. The motion of d_1 relatively to d is exactly the same as in the last case.

It is perhaps well to point out that the practical applications of many of the mechanisms described in this section are very limited in number. It was long ago pointed out by Dr. Pole[1] that the greater loss by friction in guides than in the pins of a parallel motion was after all not a quantity sensibly affecting the economical working of a steam engine, and the same thing is true for other machines. The making

[1] See *Proc. Inst. Civil Engineers*, vol. ii. (1843) p. 69.

of truly plane surfaces is now, moreover, comparatively easy and inexpensive, and it is generally more easy to ensure that the motions of slides shall be unaltered by the wear of these surfaces than that the motions of points in linkwork shall be unaltered by the smaller, but less regular, wear in the pins and eyes.† Hence in ordinary machinery, where large forces come into action, and where therefore much wear accompanies the motion, it is not probable that linkwork connections will again take the place of† slides for the constrainment of parallel motions. In the beam engine, however, the Watt motion still holds its own, and seems likely to continue to do so, and there are probably not a few cases of light instruments, or experimental apparatus, where the effects of wear are small enough to be left out of account, and where the "sweetness" of the linkwork motions would rightly cause them to be preferred to slides.

§ 58.—ORDER OF MECHANISMS; CHAINS WITH LOWER PAIRING (PINS AND SLIDES).

ALL the mechanisms hitherto examined or used for illustration, which have contained only "lower†" pairs (namely pin joints or slides)—except the doubled double-kite of the last section—have belonged to what may be called the **first order**. In all of them (with the exception named) it was possible, if the positions of any two links were given, to find the positions of all the other links by mere straight-line and circle constructions. Or putting it otherwise, we may say that it was possible, if the mechanism were given in any one position, to find by such constructions all its other possible positions. In all of them, also, it was possible, by equally simple constructions, and by the use of the theorem of the three virtual centres (see p. 73), to find the virtual centre of every link relatively to every other. Although both these conditions are fulfilled by the great majority of mechanisms with which we have to deal practically, which, therefore, belong to the first order, there are some mechanisms—and these not unimportant—which cannot be dealt with in this fashion, and which, therefore, belong to higher orders. Without attempting here to classify completely such mechanisms into different orders—for probably all those which are of much practical importance belong to two orders—we shall simply examine two or three of them in order to show their characteristics, and the way in which they require to be handled.

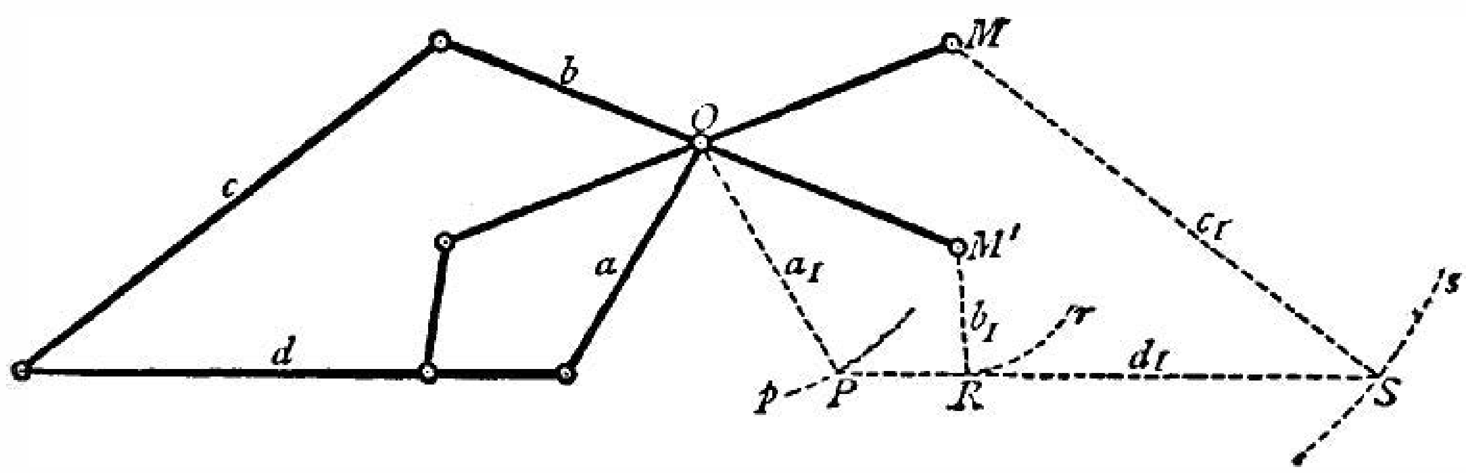


FIG. 234.

As a simple, although unusual example, let us take first Mr. Kempe's compound kite chain (Fig. 234), some of whose properties have been noticed in the last section. Suppose the links *c* and *d* to be given in any of their possible relative positions, and the lengths of all the other links to be given also. As the mechanism is completely constrained we know that the position of all these links must be completely determined by the given positions of *c* and *d*, and it is desired to draw the whole mechanism in the position thus determined. The first double kite can obviously be drawn at once, and if we knew the properties of this particular mechanism, namely, that the two double kites must be symmetrical, and that d_1 must lie in line with d, there is of course no difficulty about drawing the second also. But as these are properties belonging to a very specially proportioned mechanism, and as in general we have no such helps to guide us, we shall suppose these properties to be unknown. We have, then, to start with, the positions of only the links OM and OM' in the second mechanism, with the lengths of all the other links. With known radii a_1, b_1, and c_1 respectively, we can draw the arcs of circles p, r, and s, and we know that on these arcs respectively must lie the points P, R, and S. We know, also, the distances between these points, and their relative positions (namely, here all on one straight line). The position of the link d_1 is uniquely determined by these conditions, but it cannot be found by any simple construction, or indeed any construction whatever that does not involve very complicated curves. Practically it is best found (except in such special cases as this, where certain characteristics of its motion are known from general reasoning) by a process of *fitting*. A template, here a straight edge (conveniently a strip of paper), is marked with the three points, P, R, and S, at the proper distances apart, and the strip is moved about until a position is found for it in which the three points lie simultaneously upon the three circles to which they correspond. The position of d_1 being thus found, the links a_1, b_1, and c_1 can at once be drawn, and the problem is solved.

It requires to be specially noticed, however, that the necessity of the "fittingt" process does not essentially characterise this combination of links, but depends on the particular links whose position are given as data in the problem. Thus if d and d_1 were given instead of d and c the point O could be at once found, so that we should have two links in each double kite (a and d in the one, a_1 and d_1 in the other), by the aid of which the whole mechanism could be drawn in the usual way. Similarly if d and a_1 were given, or d and c_1, or either of several other combinations, the whole mechanism could be drawn at once without "fitting." But in practical cases only the position of a fixed link and *one adjacent to it* can be given. Positions of other links can seldom be directly known. In any case the whole of the forty-five virtual centres can be drawn by the methods already given, and without difficulty of any kind. It may be said, therefore, that as a *kinematic chain* (p. 62), this combination of links does not differ essentially from the most simple class. But considering the *mechanisms* that can be obtained from this chain, and remembering that one of the given links in the instances mentioned above must always be the *fixed link* of the mechanism, we may say that these mechanisms belong to a higher order—we may call it a **second order** among mechanisms. The characteristic of this order is in every case that unless the positions of certain special links relatively to the fixed link form part of the data, the position of the mechanism as a whole cannot be found by ordinary line and circle constructions, but requires either the fitting process or the drawing of complex curves to be employed.

The ordinary "link motion" of a steam engine (Fig. 235), falls into precisely the same category as the last. It is a chain of six links, four of which (a, b, c, and d,) form a simple quadrilateral or lever-crank. One of these four (a) is paired at a point O, not upon its axis ($Q\ R$), with the fixed link f, and the opposite link c is connected to f at S by a single link e, attached to it at some point P, also in most cases not upon its axis (MN). Fig. 236 shows the whole combination put in a more schematic form, but in reality without any alteration from the proportions of the last figure. All the virtual centres of this chain (fifteen in number) can be found in the usual way. The chain may be said to belong to the first class as a chain, but exactly as in the last case some of the mechanisms formed from it belong to the second order. Thus with the link *f* fixed, and varying positions of *a* given, we cannot find the positions of the other links directly, but must have recourse

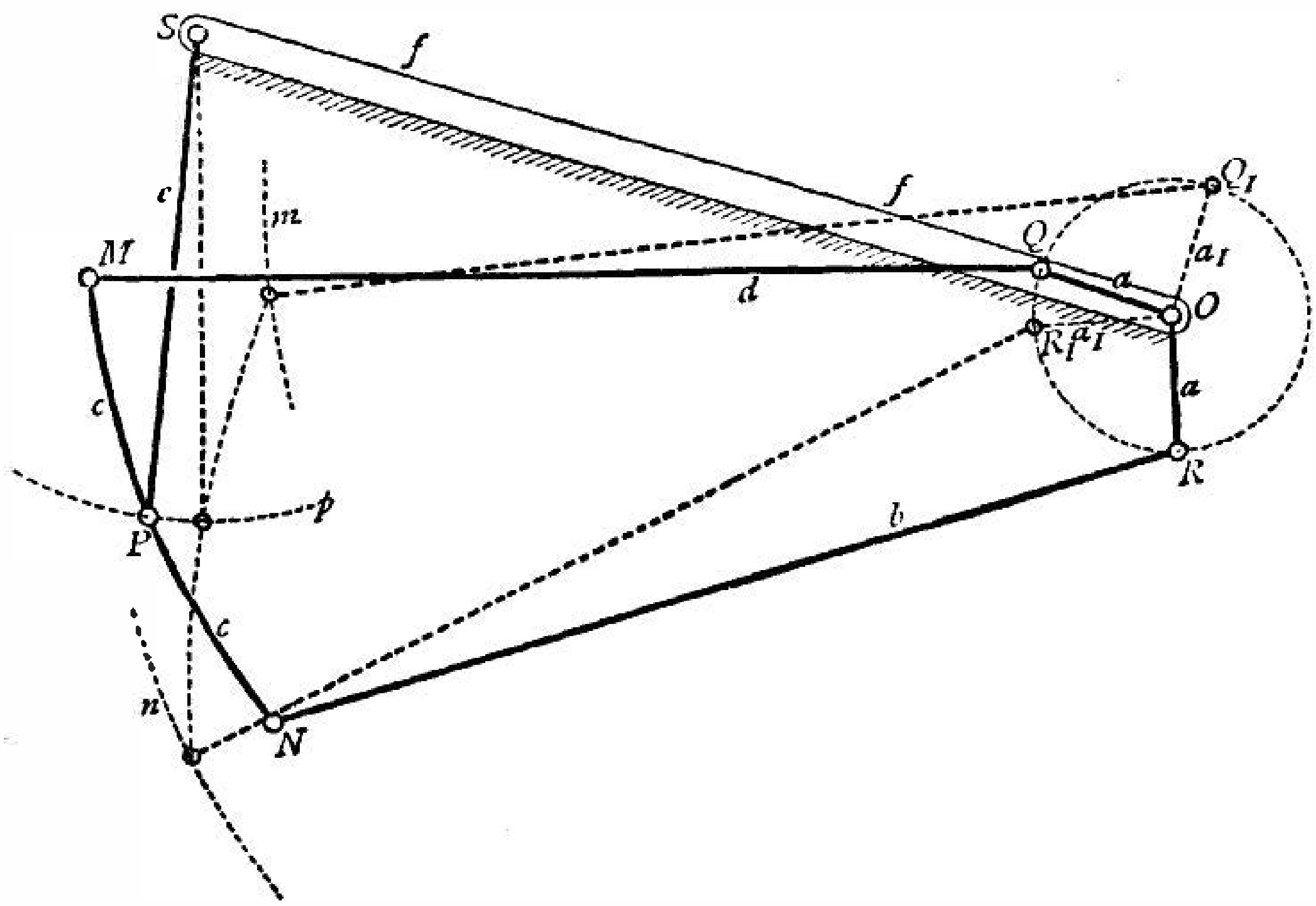


FIG. 235.

to the fitting process. Suppose, for instance, the position of *a* to change to a_1, so that we know the end points Q_1 and R_1 of *d* and *b* respectively. With these points as centres, and with radii equal to the length *QM* of the eccentric rods, we can draw the arcs of circles *m* and *n*. Also about *S* with radius $SP = e$, we can draw the arc *p*. We then know that the position of *c* must be such that its three points *M*, *P*, and *N*, must lie on the three arcs *m*, *p*, and *n* respectively, and this position (which fixes the positions of *d*, *b*, and *e*) can be found in the way already described. It is shown dotted in the figure.

The block and pin by which the slide valve is driven in an engine form no part of the actual link motion—they are merely additions to it, which do not affect in any way the movements of its parts. In any such motion as that shown in the figure the valve is driven from a point which is guided along one straight line by a sliding pair. The nature of the connection is sketched in Fig. 237. *f* is the valve-rod guided by the pair 1; *g* a pin (called the "gab"-pin) in the valve-rod, which forms a pivot for a slider *h*, itself paired

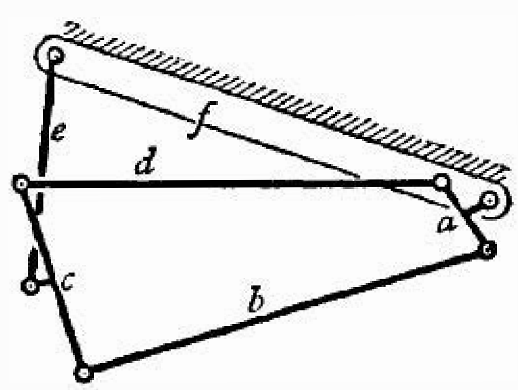


FIG. 236.

with the (curved or straight) link *c*. If a series of positions of the link be found, it will be seen that the position of *f*, and therefore of the slide valve, depends upon the position of the point in which the axes of the link and of *f* cut each other. It will further be seen that this is not always the same point *of the link*, although it is always the same point of the valve-rod, namely, the centre of the pin *g*. In other words, the swinging of the link, and chiefly the part of its motion due to the curve described by the end *P* of the suspension rod, causes it to move relatively to the block *h*. This motion is called the *slip* of the link; it is not only useless but very detrimental, and it is one characteristic of a well-designed link motion that it is reduced to the very smallest possible dimensions. For this purpose, and to ensure that the valve-rod can receive from the eccentric as nearly as possible the motion which it would receive were the connection between them direct, the arrangement often takes the form sketched in Fig. 238, where the block *h* is replaced by cheeks embracing the link, and forming part of the pin *g*, which is expanded (see § 52) sufficiently to give the necessary metal for them. By using this construction the axes 2 and 3 can be made to coincide, which is impossible in Fig. 237.

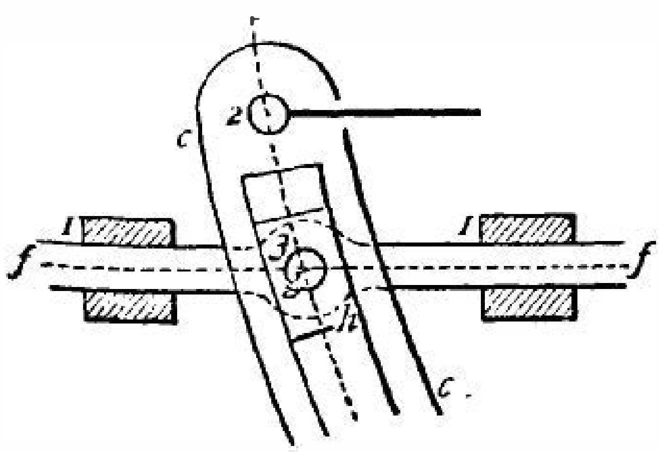


FIG. 237.

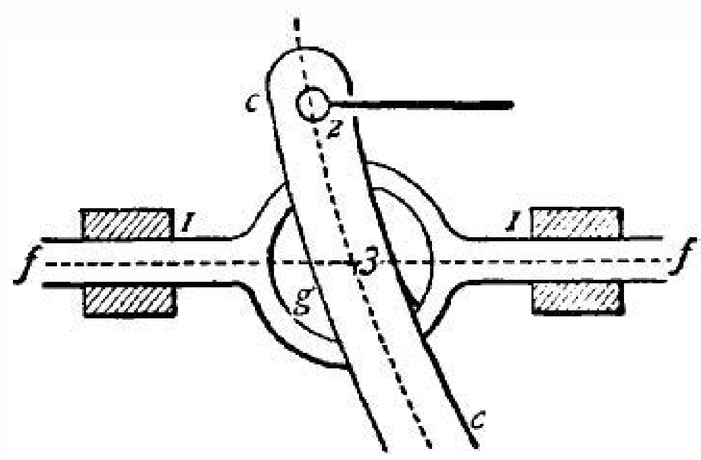


FIG. 238.

The principal statical problem connected with the link motion happens to be extremely easy of solution by the methods already given. It is thist: given a force f_c (the valve resistance) acting on the link *c* in a given direction, to find the effort f_a in a given direction at the eccentric centre necessary to balance it. The problem is illustrated in Fig. 239. We have to deal with the three links *a*, *c*, and *f* respectively, the eccentrics (which, being rigidly connected together, form only one link), the "link" itself, and the frame of the engine. The virtual centre of the eccentrics relatively to the frame is of course at O_{af}, the centre of the shaft. The common point of *a* and *c* is at the join, O_{ac}, of the two eccentric rods. The virtual centre of the link relatively to the frame, O_{cf}, must lie upon the line O_{af} O_{ac}.

But it must also lie upon the line $O_{ef}\,O_{ce}$, which is the axis of the suspension link. Its position is therefore as marked on the figure. In all ordinary cases all three points are conveniently accessible, and as they can be marked by drawing only one line which is not already on the paper (namely, the line containing the three centres), the solving of the problem happens to be peculiarly easy in spite of the apparent complexity of the mechanism. The construction is shown complete in the figure, the lettering being the same as that formerly employed in Figs. 122 to 126, § 40.

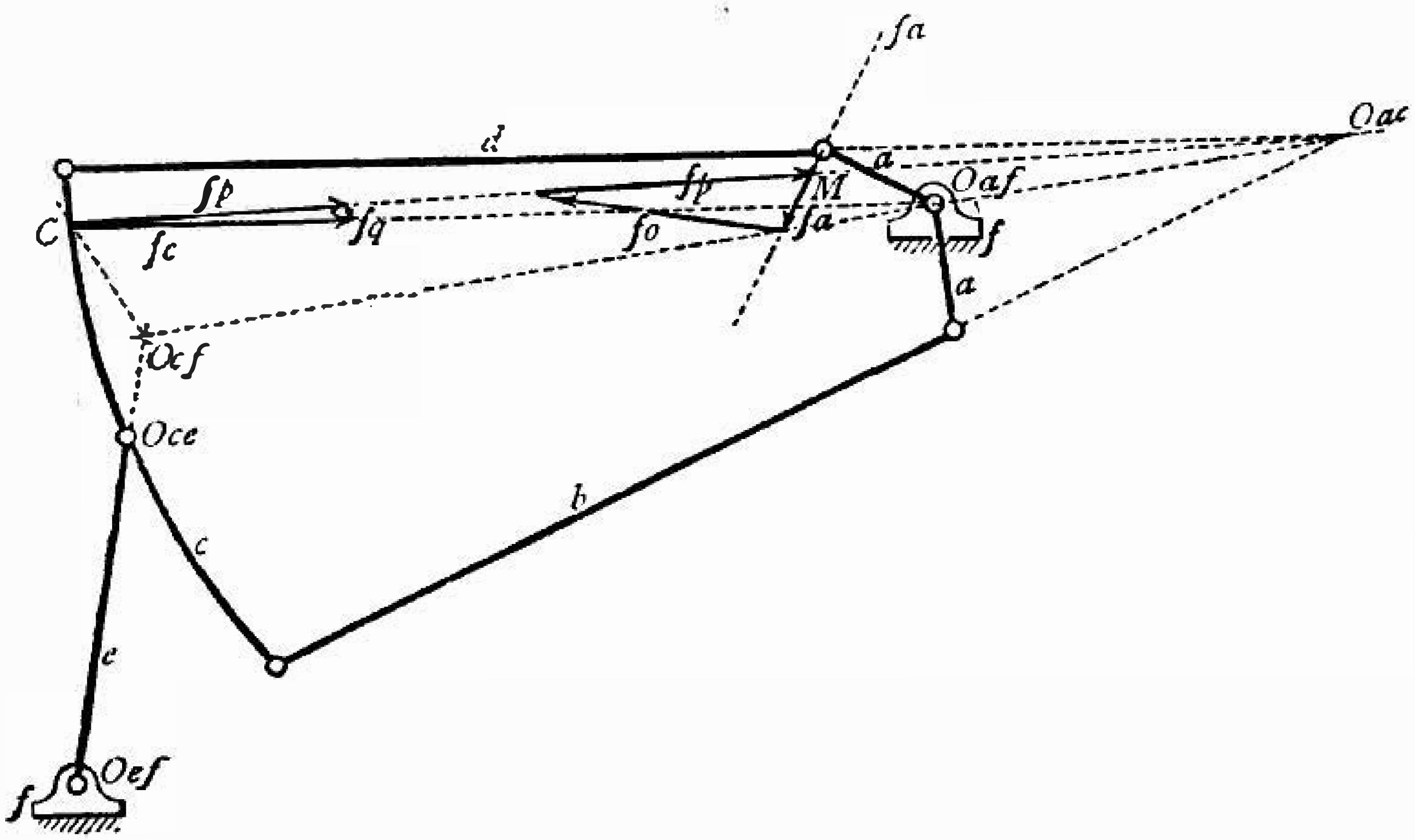


FIG. 239.

If either of the four links of the quadrilateral, *a*, *b*, *c*, or *d*, were fixed, and the position of any other one of them given, the position of all the other links of the mechanism could be found without fitting, which becomes necessary only when either *e* or *f* is the fixed link.

With the *Joy* valve-gear, shown in outline in Fig. 240, the condition of things is reversed. The chain as used by Mr. Joy has all its positions determinate without fitting, but a number of its inversions (which, however, have not at present any practical value) require treating like the link motion. It may be interesting here to notice the general nature of this valve motion, which, when well proportioned,

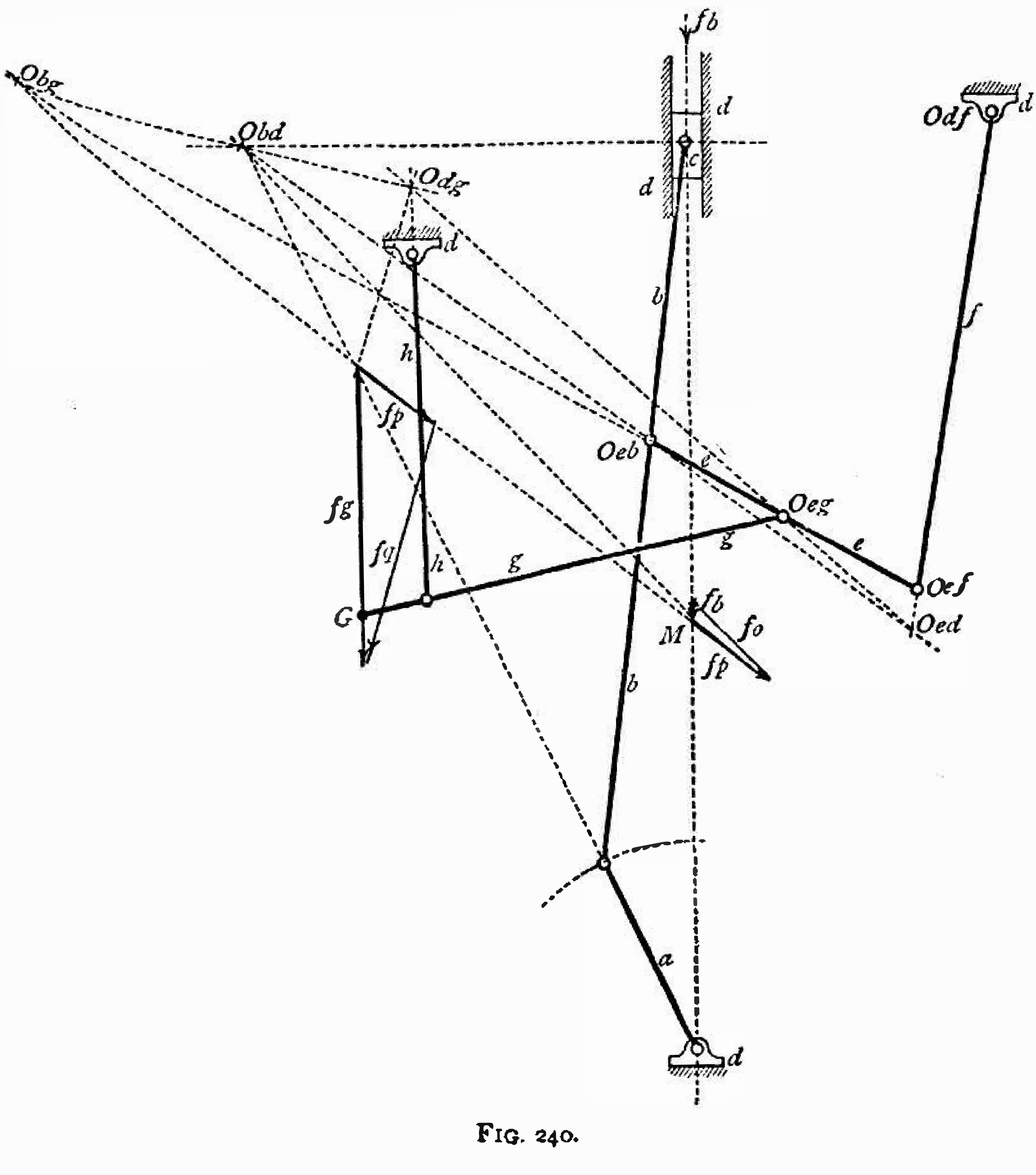


FIG. 240.

gives capital results. Its foundation is the slider-crank, *a, b, c, d*, the main driving chain of the engine. To this mechanism four links are added. Two of these, *f* and *h*, are levers swinging about pins attached to the fixed link *d*; of the other two *e* is paired directly with *f* and *b*, and *g* with *a* and *h*.[1] The valve is driven in a direction parallel to that of the motion of *c* from some overhung point *G* upon the link *g*. An examination of the special characteristics of the motion of *G*, which suit it for working a slide valve, would involve too many technical points for our present purpose. But we may give the construction for solving the same statical problem as that considered in connection with the link motion in Fig. 239. Let f_g be the known valve resistance acting at *G*, it is required to find the necessary driving effort f_b, acting on the link *b*, the connecting rod of the engine, to balance it. We require for the solution the three centres O_{bd}, O_{dg}, and O_{bg}. The position of the first of these, O_{bd}, we have at once, although in many positions it will be inaccessible. The axis of the link *e* contains the points O_{eb} and O_{eg}, it must therefore be a line on which O_{bg} also lies, and by similar reasoning we know that O_{dg} must lie upon the axis of the link *h*. By drawing the line O_{bd} O_{eb}, we get the point O_{ed} on *f*, and by joining this to O_{eg} we find O_{dg} on *h*. The third point O_{bg} can then be found by drawing O_{bd} O_{dg} to its join with the axis of *e*. Given these three points the construction is precisely as before (see § 40), and as the same letters are used in the figure it is not necessary to go over it again. In cases where the point O_{bd} is inaccessible, it will generally be most convenient to find the point O_{bg} separately in the same fashion as O_{dg} has been found above, and to arrange the construction so that the point *M* comes to O_{ab}, in the manner described on page 290, etc.

[1] For description of this gear, and some discussion on its proportioning, see Mr. Joy's paper in the *Proceedings of the Institution of Mechanical Engineers*, 1880, pp. 418-454.

Still limiting ourselves to mechanisms containing only lower pairs of kinematic elements we may sum up as follows. In mechanisms of what we may call the **first order**, if we are given the lengths of all the links, and the relative positions of any two of them, we can at once find the positions of all the others by direct line and circle constructions. In mechanisms of what we may call the **second order** (namely, those which we have been examining in this section), this is only possible if the relative positions of *certain* pairs, not of *any* pair, of links are given. Otherwise the positions of the remaining links can only be found by fitting—this process taking the place of the extremely complex geometrical construction which would otherwise be necessary. In both cases all the virtual centres of the links can be found by direct constructions involving only straight lines, so that we may say that as *kinematic chains* both orders of mechanisms belong to one class, which we may call the **first class.**

§ 59.—ORDER OF MECHANISMS. CHAINS CONTAINING HIGHER PAIRS (WHEEL-TEETH, CAMS, &c.).

THE great source of simplicity in mechanisms containing only lower pairs is that the virtual centres of adjacent links are always permanent centres (p. 71), or points occupying certain definite positions on those links. Directly we pass toe mechanisms containing higher pairs, such as those of Figs. 241 and 242, we lose this simplification. Thus the points O_{ab} in Figs. 241 and 242, are not fixed points in a and b respectively, like the crank-pin centre in a slider-crank, but vary their position in these links as the links vary their position relatively to each other. In both cases the virtual centres can be determined in the usual way, but the position of all the links can be directly determined without use of the fitting process only from given positions of a *certain* pair of them. Thus in Fig. 241, although the mechanism has only three links, the cam *a* and the link *b* paired with it must be the pair whose positions are given, if the position of the third is to be directly determined. Given any positions of *b* and *c*,

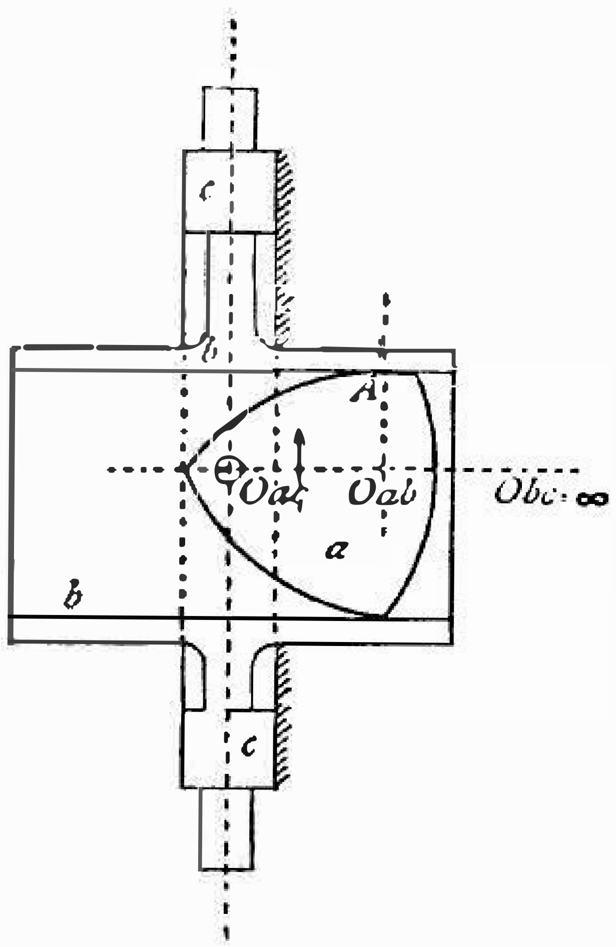


FIG. 241.

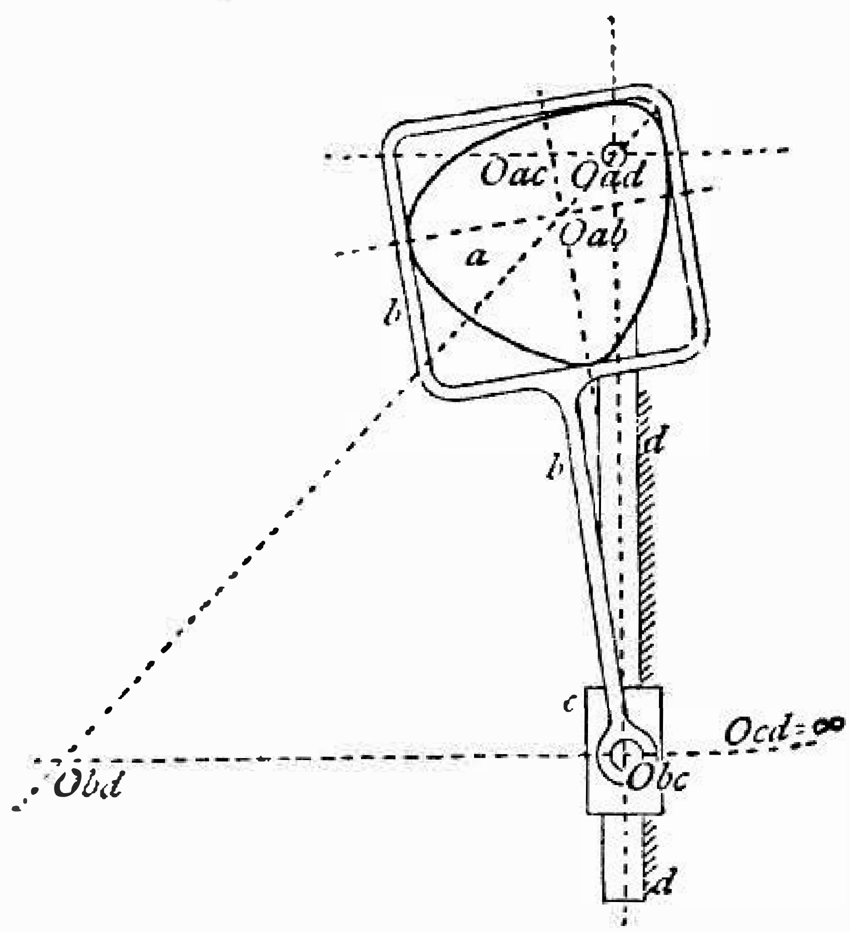


FIG. 242.

we know the position of the point O_{ac}, but the position of O_{ab} is not known, nor the particular point on *a* which becomes O_{ab} for the given position of *b*, and the position of the cam *a* requires to be found by a process of fitting essentially the same as, although different in detail from, that described in the last section. Or given any positions of *a* and *c*, in order to find that of *b* we require to "fit" the slot or groove in *b* over *a* in its given position. In practice this would be done by merely drawing tangents to *a*, without the construction of a "template" as in the last section, but geometrically the process is the same. The tangent is drawn by eye only, and the position of *b* is therefore not found with any greater or different degree of accuracy than the position of the link in the link motion, Fig. 235.[1]

Exactly the same thing is true of the four-link mechanism of Fig. 242. In order to draw the positions of the links without fitting, we must have as data the relative positions of the two links connected by the higher pair, here *a* and *b*, the cam and the connecting rod. If the positions of any other pair of links, as *a* and *d*, be given, the positions of the remaining links can only be found by fitting *b* upon *a*.

In both cases all the virtual centres of the links are completely determinate at once.

These cam trains, therefore, are kinematic chains of the first class, while the mechanisms formed from them belong to the second order.

Ordinary toothed-wheel trains, looked at from our present stand-point, have some new interest and perhaps complexity. We have already seen that the virtual centres of such trains are very easily determinate; as kinematic chains, therefore, we may include them in the first class. Their classification as mechanisms is not so obvious. Fig. 243 represents the simplest form of wheel train, consisting of a frame *a*, and two spur wheels, *b* and *c*. Given the position of *a* and ther adii of *b* and *c*, it seems at first sight as if we could draw both those links at once. That this is a mistake will be seen as soon as one asks oneself for the actual position

[1] Of course in certain cases it may happen that the cam outline is a circle of known centre, or other curve to which a tangent can be accurately drawn with ease. But these are special cases.

of any particular points on the links, as *B* and *C*. If only *a* is given, the relative positions of these points are quite indeterminate, and therefore the relative positions of the links to which they belong. All that has been determined is the relative position of their centrodes, the pitch circles, and the symmetry of these prevents our going further in the way of connecting them with definite parts or points on the links whose motions they represent. But the want of determinateness goes even further than this. Given the relative positions of *a* and *b*, the latter as determined by the position of one point *B* in it besides its centre, the position of any definite point, as *C*, in the third link *c* still remains

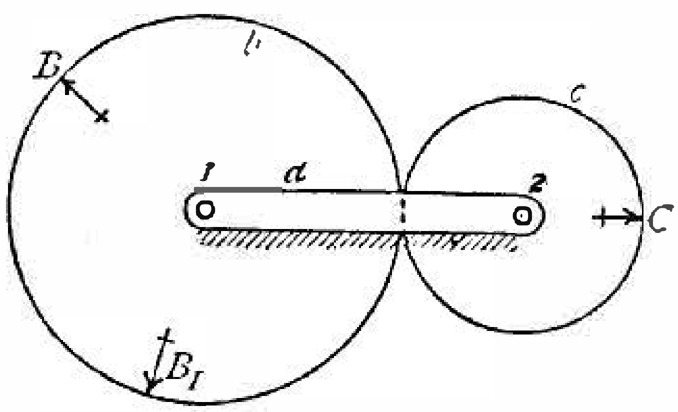


FIG. 243.

indeterminate. The third link may, in fact, occupy any angular position whatever relatively to *b*, so that the want of determinateness is real, and does not merely belong to our particular handling of the problem. But the mechanism is a completely constrained one, so that it must be possible, as it is clearly necessary, to state the problem in a different way—in some fashion, namely, which shall include the case of the former mechanisms without excluding this one. The difficulty is entirely met if we state our problem in the alternative fashion given at the beginning of § 58, which is equivalent to adding to our data the chain given complete in any one position. It was not necessary to refer to this formerly, because no use was made of it; the mere knowledge of the lengths, etc., of the links carried with it the possibility of setting the mechanism out uniquely. Here, on the other hand, the mere knowledge of the dimensions of the links is not, as we have seen, sufficient of itself to determine uniquely any position of the mechanism, but only to enable us to find an infinite number of positions any one of which is possible.

If now we suppose given the mechanism of Fig. 243 with certain definite relative positions of the links b and c, as fixed, say, by the positions of the lines $1B$ and $2C$ upon them; and if then there is given any other position of b relatively to a, as $1B_1$, the angle $B1B_1$ being known—the position of c (*i.e.* of the line $2C$ in c) can be uniquely determined.[1] But, unless it chance that the ratio $\frac{2C}{1B}$ is some easily-handled whole number, the position of c requires for its determination either a fitting process, which would here involve the rolling on each other of templates of b and c, or, preferably, the substitution for fitting of some approximate construction for setting off on c a circumferential distance equal to BB_1.

If the relative positions of the two wheels be given as data, instead of the relative positions of the frame and one wheel, the position of the frame is obviously known at once without any fitting.

We thus find these mechanisms belong to the second

[1] It is necessary to make the condition as to $B1B_1$ being known, because the value of that angle may be not simply $B1B_1$, but $B1B_1 + n360°$, the wheel b having made any number of turns before being brought into the position $1B_1$. Thus there are a very large number of possible positions of $2C$ corresponding to $1B_1$, and the right one can only be known if the actual extent of angular motion of b between $1B$ and $1B_1$ be given.

order, exactly as the cam trains, the fitting process being necessary unless the relative positions of the two links connected by higher pairing, namely, here the two wheels, be the data.

In such a chain as the link motion of Fig. 235 some of the mechanisms belonged to the first, and some to the second, order. For if, *e.g.*, either of the links *a*, *b*, *c*, or *d* had been the fixed link the positions of all the others (including the links *e* and *f*), could have been directly determined if that of *any one* of them had been given. In the cases now before us, however, *all* the mechanisms belong to the second order, for, whichever link be fixed, direct determination of the positions of the others is only possible if the data include the position of some particular one of them.

If we consider compound wheel trains, as those of Figs. 65 and 66, § 19, in the same way, we find they fall into the same class and order. For we know that in such trains the relative motions of any pair of wheels, by however many intermediate wheels they may be separated, are fully represented by those of *one pair* of pitch circles of determinate diameters working in direct contact. As regards any one pair of its wheels and the frame, a compound wheel train therefore reduces at once to the simple train just examined. If the relative positions of *all* the wheels be required, the fitting process, or its equivalent already mentioned, has to be resorted to for each pair separately. The sun-and-planet motion (Fig. 73), and other familiar combinations of link-work and wheel gearing, belong to the same class and order with those just considered.

Higher chains and mechanisms occur so seldom in practice[1] that it will be sufficient to give one example, that shown in

[1] Limiting our statements here, as always, to mechanisms having plane motion only.

Fig. 244. This is a mechanism which can scarcely be said to be used in machine construction, but which is sometimes met with in collections as an illustration of certain special motions. It consists of a fixed link or frame, *g*, containing a straight slot and two pins. About the two latter turn wheels, *a* and *f*, equal or unequal in diameter. Crank pins in these are connected by rods, *b* and *e*, to the two ends of a beam, *c*, which

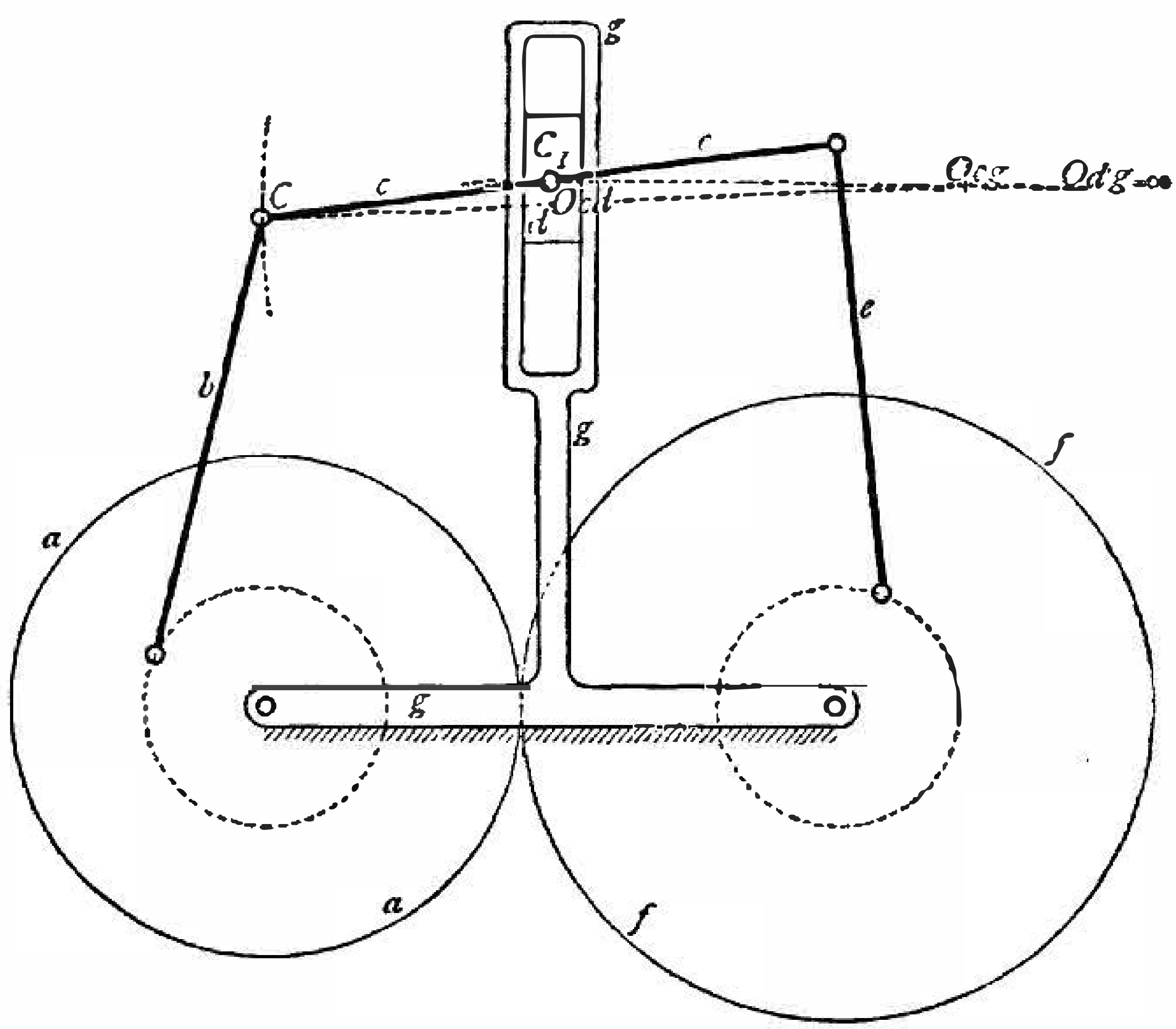


FIG. 244.

is connected at its centre, or any other convenient point, to a block, *d*, sliding in the slot already mentioned in the frame *g*. If the two wheels were equal, the two crank pins placed symmetrically to the vertical axis, and the beam *c* pivotted at its mid-point, the beam would simply move up and down, remaining always horizontal. The pin connection between *c* and *d* would become superfluous, as would also the spur wheels, and the mechanism would become simply a doubled slider crank, as sketched in Fig. 245. All points in the link *c* would move in parallel straight paths, the link as a whole reciprocating through a constant stroke exactly as in a steam engine. In the more general form of Fig. 244, however, the motion of *c* is much more complex, its different points having all different motions, that of the point *c* being the one usually studied. This point is constrained by the sliding pair to move always up and down along one straight line, but the distance which it moves along that line varies

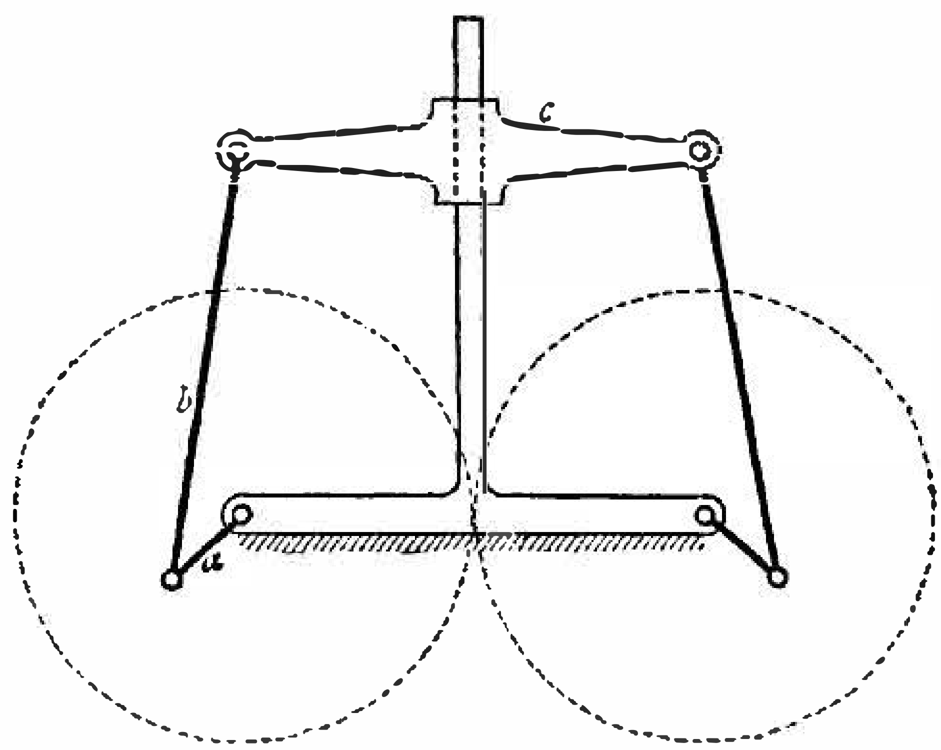


FIG. 245.

at each reciprocation, its "stroke" changing gradually from minimum to maximum, and *vice versâ*, as the phases, or relative positions, of the cranks alter. If a pencil were attached to *c*, and caused to describe a curve upon a sheet of paper moved uniformly, say by gearing, from either of the wheels from right to left, the curve traced would be of the kind shown in Fig. 246, where the varying vertical height shows the varying stroke of the reciprocating point C_1.

In this mechanism it will be found that we have to do with entirely different conditions from those hitherto examined. The relative positions of *no* two links whatever enable us to find the positions of all the rest (the lengths of all being, of course, supposed given as before), *without* fitting, and although the relative positions of any pair of links enable us to find possible positions of the rest by using that process, yet with only certain pairs do we know that such positions are consistent with any given starting positions of the mechanism. In the case before us, for example, the whole mechanism can be drawn most readily from the given positions of two of its links, if these two are the two spur wheel cranks *a* and *f*. The frame can then be drawn at once, and the position of the beam *c* found by fitting. In this case we do not even require to make use of the given

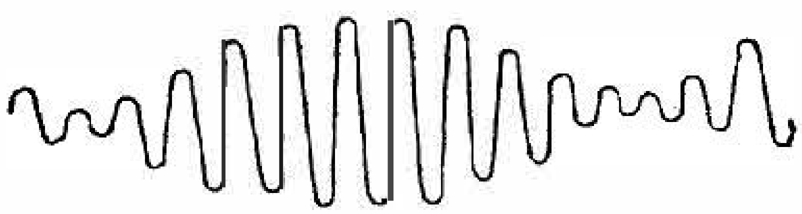


FIG. 246.

position of the whole mechanism which we have assumed to belong (p. 439) to our data. By making use of this starting position, and the known angle between it and a given position of either of the wheels, we can always find the position of one wheel from that of the other, if the frame also is given. Hence a given position of either wheel along with the frame is sufficient for the determination of the positions of all the other links, the fitting process being used twice. If the data be the positions of one of the connecting rods and the frame (an improbable combination) one of the cranks can be drawn directly, and therefore both the wheels and then the rest of the mechanism found as before. If the data be the positions of the beam and the frame, *possible*, but not unique, positions of the other links can be found. For the connecting rod lengths, swept from their given upper points as centres, will cut each crank circle twice. We get, therefore, four possible positions of the mechanism as a whole, of which in the general case no one is of necessity compatible with any given starting position of the whole chain. If the data were the positions of the two connecting rods *b* and *e*, it will be found that fitting gives in general two pairs of positions for the cranks, neither of which is necessarily compatible with a given starting position of the mechanism. If, lastly, the data be the positions of the slider *d* and the beam, an indefinite number of different possible positions of the mechanism can be found by fitting, and without further data it is not possible to say which out of these positions is consistent with a given starting position. This chain, then, differs from the former in that whichever of its links be fixed, the positions of the remaining links (that of one being given) can only be found by fitting, and also in that unless certain particular links be used as the data, the position so found may not be compatible with any given starting position. We may consider such mechanisms to be of the **third order**.

Of the twenty-one virtual centres in this chain, nine are between adjacent links, and these nine only can be found by our former construction. One line can be found on which each of the remaining twelve centres lie, but the position of no one of them can be directly obtained. We may consider this to separate the chain from all former ones sufficiently to make it a type of a **second class** of kinematic chains. To find any one of the undetermined virtual centres it is necessary to find the path of one point in the link to which it corresponds, and then to construct (of course approximately only) the normal to this path. Thus in the figure the path of C (relatively to g), has been determined, and a normal drawn to this must be a line including O_{cg}. But this point must be on a line including O_{cd} and O_{dg}, both of which we know; its position is therefore completely determined. Any one of the twelve virtual centres having been found in this way, all the others can be found at once by the usual constructions.

§ 60.—RATCHET AND CLICK TRAINS.

In speaking formerly of the condition of constrainment in a mechanism, we qualified a statement (seetp. 59) by saying that it referred to "those links in which motion was possible at any instant." This qualification was made in view of the fact that there are a number of mechanisms in which special provision is made for stopping the motion of one or more links entirely (or in one sense only) at regular or irregular intervals. Most of the contrivances for carrying out this object fall under the head of **click** or **ratchet gear.** Reuleaux has classified these gears somewhat elaborately in his *Kinematics of Machinery*,[1] and it is not necessary to go over the same ground here, as they do not present any mechanical problems essentially differing from those already examined. In cases where a click is arranged to prevent the motion of a body entirely (as b with a, Fig. 247), it simply makes that body, and perhaps others also, part of the same link with itself. In the case where it prevents motion of the body in one sense only (as b prevents the downward, but not the upward, motion of a in Fig. 248), it becomes one with that body only so far as those forces tending to move it in one sense (here downward) are concerned. If the link a in

[1] §§ 119 to 121.

Fig. 248, for instance, be lifted upward by any force, b loses at once its fixing action, and moves about its fulcrum in c, in just such fashion as corresponds to the higher pairing between the teeth of the rack and the end of the click. Exactly the same thing is true in the case of Fig. 249, where a ratchet wheel takes the place of the rack.

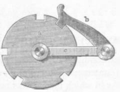


Fig. 249.

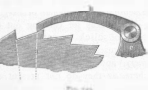


Fig. 248.

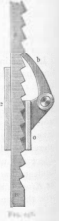


Fig. 250.

In the common form of ratchet train shown in Fig. 250, the click b acts just as in the last case, alternately forming a, b, and c into one link, and being caused (by the rise of a) to swing about its fulcrum on c, remaining in contact with the teeth of a either by its own weight or by some other form of "force-closure" (p. 393). The click or ratchet b_1, in exactly corresponding fashion, alternately makes a, b_1, and c_1 into one link, and swings about its fulcrum on c_1, rubbing against the teeth of a just as the other. Thus when c_1 is lifted, the rack a is lifted also, through b_1, as if it were part of c_1, the retaining click b being idle. But when c_1 is lowered, the rack a does not fall also, because the click b comes into action, and temporarily fixes it to the frame c. The rack,

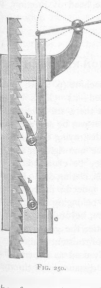


FIG. 250.

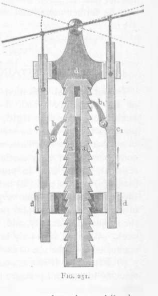


FIG. 251.

therefore, receives an *intermittent* upward motion, while the driving link c_1 has a *continuous* reciprocating motion. By the use of a double ratchet train, as Fig. 251, the rack a may be made to receive an upward motion for both swings of the driving arm.

Perhaps the only point about these mechanisms calling for further special mention here is that it is essential to their proper, and indeed safe, working, that the form of the faces of the ratchet teeth be so designed that under no circumstances can direct pressure between tooth and click or ratchet tend to throw the latter out of gear. This matter will be further considered under the head of Friction, in § 71.

§ 61.—CHAINS CONTAINING NON-RIGID LINKS.

IN our original definition of a machine (p. 2) we spoke of the bodies of which it consisted—its "links"— as **resistant** rather than **rigid**, because there are several important cases in which non-rigid bodies can be and are used in mechanisms or machines, without destroying the required constrainment of their motions. The non-rigid, or simply resistant, bodies used in practice may be classified as (i) fluids, (ii) springs, and (iii) belts, cords, chains, &c. In part they affect our problems very little indeed; in part they introduce new and complex problems belonging in reality to the subject of elasticity, and, therefore, beyond our present work. We shall here only briefly notice the general points connected with the use of each class of resistant bodies.

(i) **Fluids.**—*Water* is occasionally used as a part of a machine to transmit pressure to long distances and through any change of direction. This it can do with considerable exactness in virtue of its (comparative) incompressibility, if it is free from air. But the invariable presence of small quantities of air in the water itself, and the difficulty of ensuring that no additional air shall be pumped into the pipes along with the water, makes any "water rod" arrangement suitable only for cases where the transmission of a quantity of *work* is the chief matter, and no exact transmission of *motion* is required. Occasionally glycerine is mixed with the water to prevent its freezing, and sometimes for the same and other reasons oil takes the place of the water. In a very few cases a column of *air* is used instead of the water, but in this case the transmission either of work or motion becomes exceedingly inexact on account of the compressibility of the air, and the changes of volume which it undergoes with changing temperatures.

(ii) **Springs.**—In the largest sense of the word every link in a machine, being made of elastic material, is a spring. Each is stretched, compressed, twisted, bent, or in some fashion strained, by every load that acts upon it, and the strain,[1] whatever it may be, is very closely proportional to the stress in the material. Thus the alternate extensions and compressions of a piston or connecting rod, if they could be conveniently measured and recorded, would enable us to find the work done in a steam-engine cylinder just as correctly and completely as an indicator card. Or, as Hirn has shown,[2] the beam or the shaft of an engine may be used as the spring of a dynamometer, to measure—by its recorded deflections or twists—the work being done by the engine. But in all such cases the strain is exceedingly small in proportion to any of the motions of the different links of the machine, and has to be made visible and measurable by some special arrangement of exaggerating apparatus. The name *spring* is not generally applied in such a case, but is restricted to those bodies or links whose strain under load is

[1] By *strain* is meant always *alteration of form*, not force causing alteration of form, nor molecular resistance to alteration of form, for which latter we have kept the word stress (p. 261).

[2] See, for example, his *Les Pandynamomètres*, Paris, 1867.

not only proportional to the straining force, but is comparable in extent or dimension to the proper motions of the links of the machine. Springs in this sense of the word are used generally for one of three purposes, either (*a*) to measure force, (*b*) to limit force, or (*c*) to store up work or energy. Class *a* is represented by all steam-engine indicators and many dynamometers—mechanisms in which the motion of one link, viz., the spring and any parts in rigid connection with it, while constrained in *direction* in the same way as the motion of every other link, is made to be proportional in *magnitude* to the force acting on itself. In the instruments named the spring is made to show, or to record (or both), the extent of its motion. If the record be made, as it usually is, upon a sheet of paper caused to move at right angles to the direction of motion of the spring, and at a rate proportional to the rate of motion of the body whose resistance is measured by the spring, we obtain for record such a curve as is shown in Fig. 252. The ordinates of the curve, as $A\,A_1$ or $B\,B_1$, are proportional to the motion of the spring, and, therefore, represent pressures or forces. The abscissæ, as $O\,A$ or $O\,B$, are proportional to the distances moved through by the body on which the forces are acting. The *area* under the curve, as $A\,A_1\,B_1\,B$, represents therefore, measured on suitable scales, the product of pressure and distance, or *work*. Thus the area $A\,A_1\,B_1\,B$ represents the amount of work done on or by the moving body in passing through the distance represented by $A\,B$. In the case of a steam engine we have not a body moving unlimitedly on in one direction, but one which has a short stroke only, returning always to its original position. In such a case the diagram of work traced by the means just described takes some such form as is shown in Fig. 253. The work done by the steam on the piston is shown by the area

F B C D E. The work done in returning by *the piston on the steam* is shown by *D A F E.* The net work done is the difference between these two areas, or *A B C D*, which is all that the steam-engine "indicator" shows in drawing the "indicator card" *A B C D*, which we have already made use of in § 47.

In steam-engine indicators the spring is invariably of the type known as "spiral"—a coil of tempered steel wire twisted helically, and compressed or extended in the direction of the axis of the helix. Fig. 254 shows one of the most recent, and probably the best,[1] form taken by the spring of an indicator, the peculiarity about it being

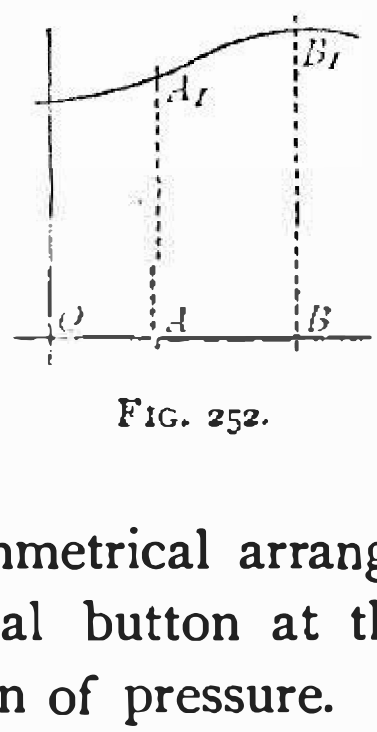


FIG. 252.

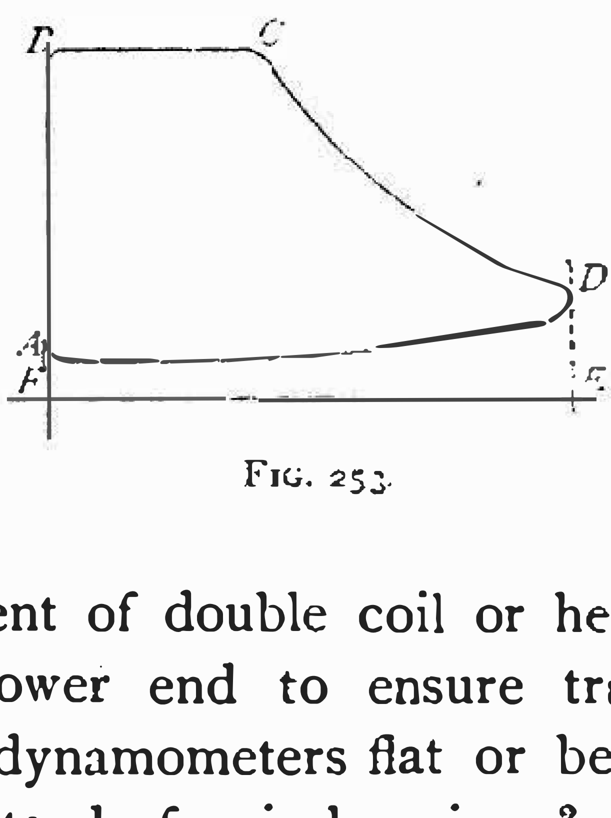


FIG. 253.

the symmetrical arrangement of double coil or helix, with a central button at the lower end to ensure true axial direction of pressure. In dynamometers flat or bent plate springs are often used instead of spiral springs.[2] This is the case, for instance, in most of the dynamometers of the Royal Agricultural Society, on which so many valuable experiments have been made.

It is necessary to bear in mind that, although in all these

[1] This is the form of spring used in the "Crosby" indicator. See, for example, *Engineering*, vol. 37, p. 185.

[2] See, for example, Mr. W. E. Rich's paper in the *Proceedings of the Institute of Mechanical Engineers*, 1876, pp. 199-227.

cases the motion of the recording point attached to the spring is as simple to deal with as the motion of any other point in a mechanism, yet the motions relatively to each other of different points in the spring are extremely complex. For here we are dealing with a body in which the deformation or strain is no longer intentionally kept as minute as possible (p. 261), but intentionally made as large as possible. And in order that we may know beforehand how much the recording point of a given spring will move

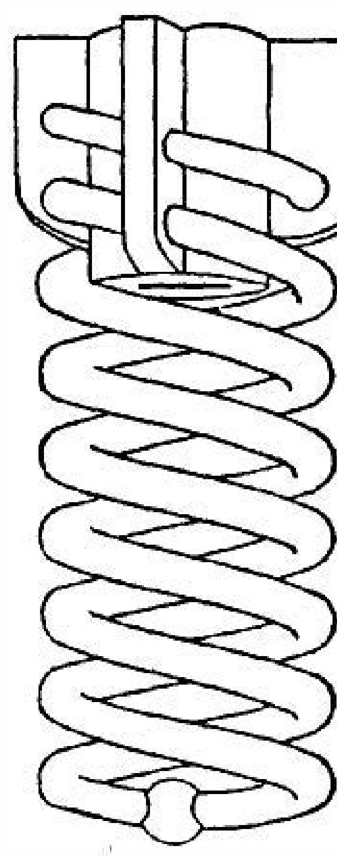


FIG. 254.

under a given load, it is necessary that we should first know the laws according to which the deformation of the spring, as a whole, will take place when it is subjected to pressure. These laws are usually complex, their consideration belongs to the theory of elasticity, and goes beyond our present limits. It may be said about them, however, that although they have been pretty fully worked out from the mathematical side, reasoning upwards from certain definite and comparatively simple assumptions, there yet remains much work to be done in connection with them before they can be taken to represent the actually occurring and very complicated physical phenomena.

Of springs used to *limit* force the safety valve of a locomotive affords a very familiar example. The spring in this case is a spiral spring so compressed as to act on the valve, when resting against its seat, with a certain definite pressure. So soon as the pressure of steam below the valve exceeds the pressure of the spring above it, the valve moves upwards, and by so doing allows steam to escape,

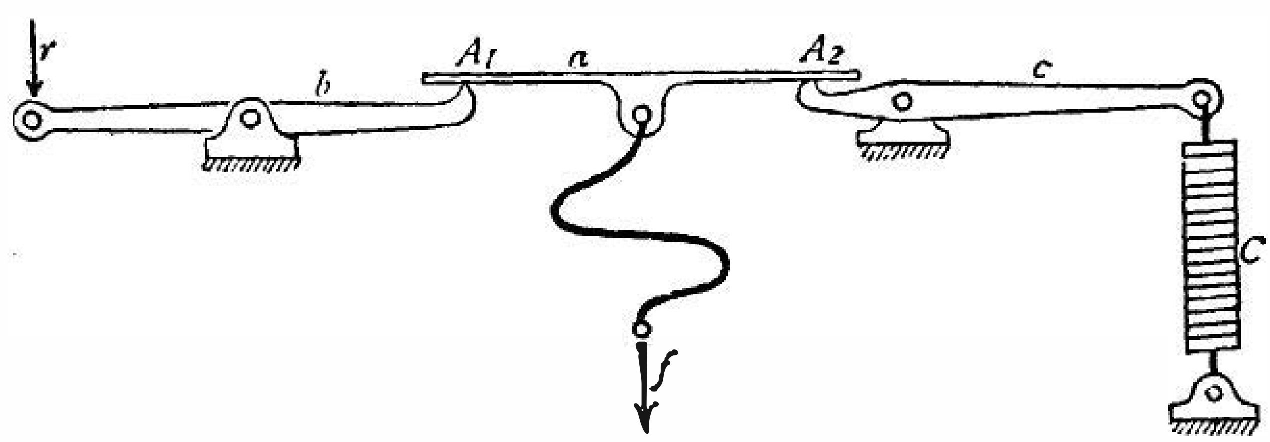


FIG. 255.

and the pressure to fall,[1] or at any rate prevents the pressure from rising. But a spring safety valve can hardly be included among mechanisms or machines, although it is essentially a machine while it is opening or closing. A more interesting example of the use of a spring to limit a pressure in a machine proper is shown in Fig. 255, which represents a portion of one of the testing machines used in Wöhler's experiments on the "fatigue" of metals.[2] Here it

[1] This must not be taken as a description of the behaviour of any *actual* spring safety valve, but only as a simplified statement of its ideal behaviour.

[2] See *Ueber die Festigkeitsversuche mit Eisen und Stahl*, A. Wöhler, Berlin, 1870, and also *Engineering*, vol. xi. pp. 199, etc. ("Fatigue of Metals").

is desired that a certain force, f, liable to irregular variations, should exert at r an effort not exceeding a definite invariable magnitude. For this purpose the force f is caused tot act on the lever b (and therefore at r) through the intervention of a crosshead a, one end of which, A_1, rests upon b, while the other end, A_2, rests upon a lever c. This lever is held in position by a spring, C, of definite resistance. So long, therefore, as the pressure at A_2 due to f is less than that due to the spring, the point A_2 cannot move. Any motion of a that occurs must be about A_2 as a fixed point, and the pressure at A_1 will be entirely transmitted to r, the resistances at A_1 and A_2 being equal. But directly the force f attains such a magnitude that its downward pressure on A_2 exceeds the fixed upward pressure there due to the spring, the point A_2, that is, the inner end of the lever c, drops, and the force f remains without increase. This absence of increase in f comes about by its transmission to a through a spring, the pull in which is released as soon as A_2 sinks. It is obvious that the action of this apparatus would not be correct if the motion of A_2 were at all considerable, because as it falls it extends the spring C, and therefore its resistance to falling increases. In any case the real maximum value of the pressure at A_2 (and therefore at A_1) is not exactly that due to C in its normal position, but in its most extended position. By judicious management it can be arranged that this quantity exceeds the normal pull due to C by a quantity not only very small, but also experimentally determinable with very considerable accuracy.

The *storing up of work* has been mentioned as a third use of springs—of which the buffer springs of a railway wagon afford perhaps thetmost familiar example. If a rigid body has to take up work in itself while undergoing mere elastic deformation, the inevitable smallness of such deformation may cause the stress to rise to some most inconveniently high amount while still the work taken up is excessively small. In the case of a railway wagon, for instance, exposed to frequent and sudden blows from bodies moving with considerable velocity, the effect of such blows upon the frame of the wagon—if they were received directly by it—might and probably would be most injurious to it. It is therefore most common to provide spring buffers for the purpose of receiving such blows, storing up readily, and without any injury to themselves, the energy received by them from the striking body. For this purpose some body is required which can be made to change its form very greatly without loss of elasticity, and at the same time offer a sufficiently large resistance to the change. These requirements are exactly fulfilled by a stiff spiral steel buffer spring. Such a spring, five inches in diameter outside, and twelve inches long, may require a pressure of about four tons before it is compressed "home." The *mean* pressure during the compression is therefore two tons, and the amount of work stored up in the spring when compressed (it being then 6.5 inches long) is about eleven inch-tons. When it is remembered that a loaded railway truck weighing sixteen tons, and moving with a velocity of ten miles per hour, only requires to get rid of about 100 inch-tons of energy in order to bring it completely to rest, it will be seen how important a part a couple of such springs may play in absorbing energy which would otherwise tend to the rapid destruction of the wagon. The energy thus stored up in the spring has not, of course, vanished. The spring must, sooner or later, come back to its original condition of equilibrium. The work done on it and stored up in it may give negative acceleration to the striking body, or positive acceleration to the body to which the spring is attached, or both, but in any case this can come about at (what we may call) the leisure of the material, by the action of a simple known force (the resistance of the spring to compression). In the absence of the spring it may often be impossible for a large mass, moving with a high velocity, to impart to the mass of another large but stationary body, with which it suddenly comes into contact, a sufficiently great acceleration to keep the mutual pressure of the two bodies one on the other within such limits as will ensure that neither is fractured nor seriously injured.

In this case, therefore, springs are used to store up work which might otherwise cause injurious stresses, the work being promptly re-stored, and expended in causing harmless acceleration. An exactly similar case occurs in connection with the "spring beams" and buffer blocks of a Cornish engine (p. 336). But in most machines where springs are used in this fashion, the energy stored in them is not derived from the momentum of some rapidly-moving body, but directly from work done on some other part of the machine. And the acceleration of some part or parts of the machine due to the re-storing of this energy is often not the result directly wanted, which is commonly no more than a rapid change of position of those parts. The change of position, that is, is the thing essential to the machine at the particular instant—the rate at which that change is effected may be immaterial. The springs used in connection with the valves of Corliss engines, of which one arrangement is sketched in Fig. 256, form an illustration of this. In this arrangement *a* is a lever receiving a continual reciprocating motion from the engine through the rod *h*. On its back is a flat spring, *f*, which has, by the action of the gearing, been drawn close up to *a*, and in which, therefore, work has been stored up. The upper end of the spring is connected by a link *e* to the rod *d*, which works the valve, and the lever *a* is at the same time connected to the same valve rod by the horn piece *c*. During motion in the direction of the arrow, *a*, *f*, and *c* all move as one link, pushing the valve rod *d* to the left, and opening the valve. When the time comes that the valve should close, some form of stop g_1[1] comes in contact with the upper part of the horn piece, stops its forward motion with *a*, and causes it to tilt over on its pin. This throws it out of gear with *d*, which is left free

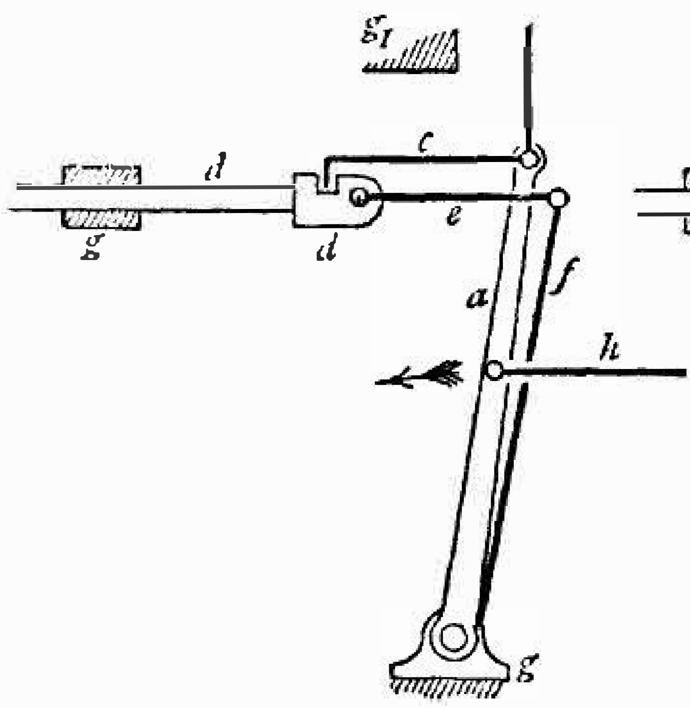


FIG. 256.

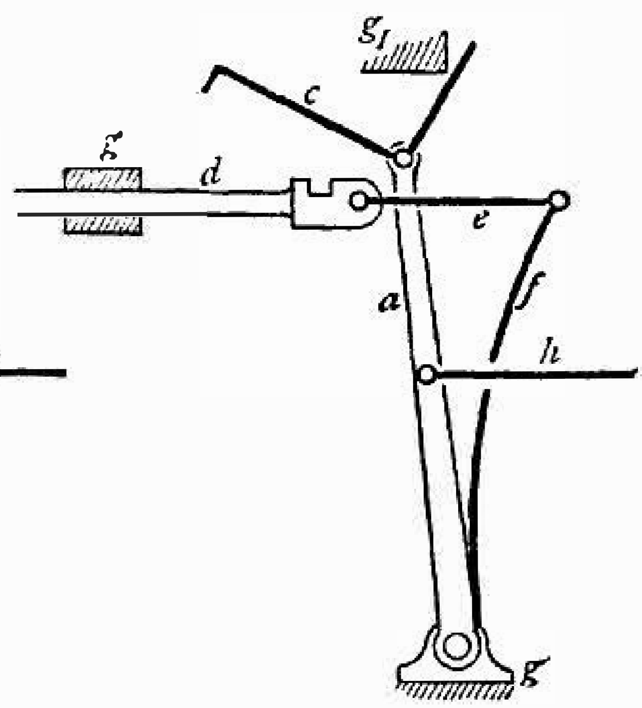


FIG. 257.

to be acted upon by the spring *f*. This spring has been pulling *d* backwards all the time, but its pull has been resisted by the catch *c*. This pull is now suddenly left unbalanced, and the spring, therefore, flies back into its unstrained position (as in Fig. 257), and pulls the valve rod *d* with it, in this fashion suddenly closing the valve by help

[1] This stop is in reality controlled in position by the governor, and is therefore movable relatively to the frame of the machine, but so long as the engine is working under constant resistance it remains steady, and we may therefore take it as being at any given instant a part of the fixed link.

of the work already stored up in it while it was being compressed.

(iii) **Belts, Straps, &c.**, form the third and perhaps the most important class of merely resistant bodies used in machines. Their use differs from that of springs in that their alterations of form under load are not directly utilised, but are, on the contrary, made to come in in such a way as to be fairly negligible in considering the motions of the mechanisms of which they form a part. Fig. 258 represents the ordinary strap connection between two pulleys. Kinematically it is intended that b and c should revolve in the

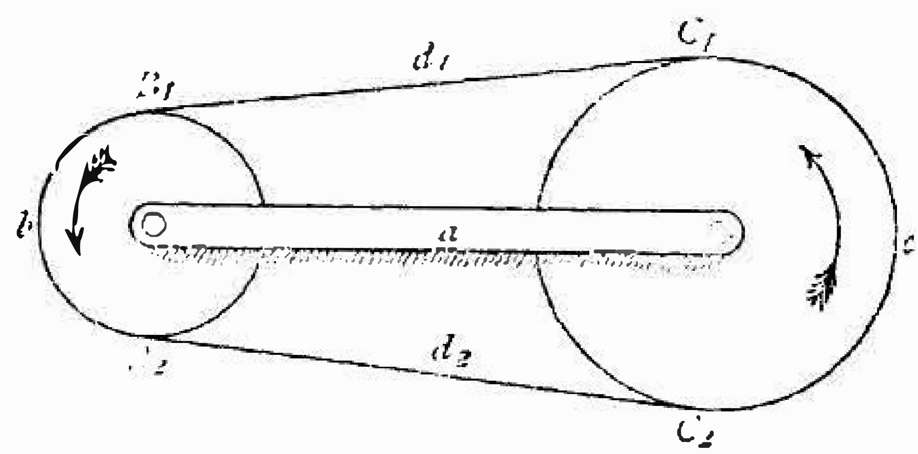


FIG. 258.

same sense, and with angular velocities inversely proportional to their diameters, just as if they were spur wheels connected by an idle wheel (p. 132). The wheels would actually do this only if the strap never slipped on either pulley, and if the length of strap between the points B_1 and C_1 remained always constant, as also the length of strap between B_2 and C_2. In ordinary good working, slipping is no doubt practically absent, and so far as this is concerned the required velocity ratio is accurately transmitted.[1] But

[1] See, for instance, the recorded revolutions during many hours' trial of an engine and the machines driven by it in experiments made by Messrs. Bryan Donkin & Co., at the South Metropolitan Gas Works, described in *Engineering*, vol. xxv. p. 117.

there may be continual small changes in the tensions in the two halves of the strap d_1 and d_2, which will have the same effect on the velocity ratio transmitted as in spur gearing would be due to the use of wrongly-shaped tooth profiles (p. 121). So long as the total length of the strap is not permanently altered, the effect of small changes of length will be to make small changes in the velocity ratio alternately above and below its mean value, but without change in that mean value itself. For practical purposes we may neglect these changes in belt gearing as we do in spur gearing.

In respect to transmission of *work*, however, belt gearing differs essentially from spur gearing in the amount of work which it absorbs itself. This is apart altogether from the question of the work taken up by friction in the bearings, which is more or less common to the two cases, although the existence of strap tensions, not directly dependent on the driving effort (or difference between the two tensions), makes of course an important difference (§ 78). The continual bending of the stiff strap round the cylindrical pulleys absorbs in itself in some cases a very considerable amount of work, and this work is, of course, not transmitted from the driving to the driven pulley. This waste of work is a condition, in this case, of the use of a non-rigid material, and has no counterpart in the rigid connection of spur gearing.

In pulley tackle, such as is shown in Figs. 259 and 260, we are again on the border line of apparatus which can legitimately be called a machine. In these cases the constrainment of motion is generally most imperfect. The tackle is required to lift some object of considerable weight, and to lift it with reasonable steadiness, but very frequently indeed it exerts a large sideway pull as well as a lift. Once the object is being fairly lifted, moreover, its swinging to and fro is not considered to destroy in any way the action of the tackle. Pulley tackle, therefore (although in some form it is often included among the "simple machines" of § 51) cannot be considered sufficiently constrained in its motions to be suitable for kinematic examination. From a static point of view—neglecting friction and work done in bending the rope—it is simple enough. The weight W, in Fig. 260, depends from five equal cords—the fact that all five plies or "parts" are in reality portions of one and the same

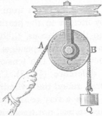


FIG. 259.

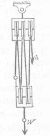


FIG. 260.

piece of rope leaves this unaffected. The whole weight is assumed to be equally distributed between the five plies, so that the tension in each of them is supposed to be equal, and the tension in the "fall" h is of course equal to the tension in the rest of the rope or $\frac{W}{5}$. Apart from friction then, a pull $\frac{W}{5}$ in the fall is sufficient to balance a weight W hanging from the lower or "running" block. From the principle of equality of work, therefore, the fall must be pulled downwards five times as fast as the weight rises. In an actual, and not an ideal, pulley tackle, a very considerable effort must be expended in overcoming friction (see § 80), and not a little work has to be done in bending the rope round the sheaves. So that the pull in the fall in hoisting must be actually very much greater than the fraction of the load indicated by the numbers of plies supporting the load.

The motions in the pulley tackle are not really plane motions (p. 12), even looked at in the most ideal fashion. But as the actual non-plane motions of the different parts of the cord do not come into consideration at all, it has not seemed out of place to mention the tackle while speaking of belt gearing generally.

The theorem of the virtual centre applies only to rigid bodies; its existence presupposes (p. 261) that the particles of the body do not alter their positions relatively to each other. Therefore a non-rigid link in a machine has no virtual centre; different parts of it are moving at the same instant about different points. Force and velocity problems, therefore, so far as they concern non-rigid links, have to be worked out by considerations quite different from those hitherto employed. Some of these considerations have been mentioned above, others will be discussed further on.

Many of the most interesting and important problems connected with the equilibrium of forces and the transmission of work in belt gearing and pulley tackle, are so closely connected with *friction* that they must be postponed until that subject has been looked at in the next chapter. Some discussion of them will be found in §§ 78 to 80.

The so-called "flexible shafts," which are now found most useful in machine shops for driving small tools (drills or borers for example) in more or less inaccessible situations, are further illustrations of non-rigid elements. As with spiral springs, the motions of different points in the shaft relatively to each other are excessively complex, although the motion transmitted by the shaft as a whole is only a simple rotation.