CHAPTER III.

THE CONSTRAINMENT OF PLANE MOTION.

§ 10. ELEMENTS OF MECHANISM.

HAVING so far investigated certain questions relating to the nature of plane motion, we have now to examine the nature of the means used to obtain such motion, *constrained*, in machines. We have already pointed out (p. 8) that the motion of any piece of a machine is determined by the *form* of its connections with the other pieces, assuming these connections to be of suitable material. This form has to be such as not only to allow the required motion, but absolutely to render impossible all other motions in the way which has been already explained in § 1. The principle of the method used to obtain this double object is as follows, supposing it applied to a perfectly general case. We first form some part of one of the two bodies whose relative motion is to be constrained, into any convenient shape, say such a projection as A on the body α, Fig. 20. Then, bringing the other body β to rest, we find all the positions of the shaped portion of the first relatively to it, and the curves bounding these positions form a figure B traced out on β by A, which is called in geometry the *envelope* of A upon β. By now removing the material of β within this figure so as (in this case) to make a curved slot or groove in β, bounded by the lines shown in the figure, we could allow the projection A to lie in the slot B, and should in this way have made a connection between the two figures which would fulfil the first condition, namely, the permitting of the required motion to take place. It would not, however, necessarily fulfil the second condition, namely, the prevention of all other motions. It is evident, in the first place, that the

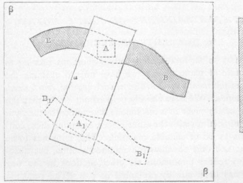

FIG. 20.

FIG. 21.

two bodies could be separated at will by being pulled right apart at right angles to the plane of motion. This disturbance is prevented by giving to A and B such a profile (or section perpendicular to the plane of motion) as may render this motion impossible; as, for instance, by carrying A right through B (as shown in section Fig. 21), and attaching a collar to its inner end. This of itself, however, does not *necessarily* constrain the motion completely, for it is quite possible that in some places the corners of A may be quite clear of B, and the motion therefore left quite uncontrolled. In order to rectify this, if it occur, either another form must be adopted for A, and therefore for B, or else some other piece, A_1, must be placed on α, with its corresponding envelope B_1 on β, in such a way that the one completely constrains the motion in every position where it is left partially free by the other. This can be done by the application of certain rules which we need not examine here.

A pair of such forms as those we have supposed to be placed on the bodies α and β, when they are arranged so as to make the motion completely constrained, are called a **pair of elements**, or more fully, a **pair of kinematic elements.** It is seen at once from their nature that these elements occur necessarily in pairs, and never singly. A single element can no more constrain motion than a single body can make a machine (§ 1); they must always go in pairs, and these pairs of elements form the lowest factor to which we can reduce a machine.

We have supposed for our illustration a very general case indeed, and one that occurs very seldom, although it does sometimes occur, in practical work. Most of the pairs of elements which we find in machines are of a very much simpler kind than that shown in Fig. 20.

Of these simpler forms the two most important are those continually employed in machines to constrain the two special forms of plane motion which we examined in § 2, rotation and rectilinear translation. These may be called, on account of the motions which they constrain, the **turning** and the **sliding pairs** respectively. The former takes the shape of some solid of revolution, having such a profile as to prevent axial motion; in its commonest form it is the cylindric pin and eye of Fig. 22, where the collars upon the pin prevent the axial motion. The sliding pair is in form essentially *prismatic*, that is, it is a solid having plane sides, parallel to the direction of motion. It commonly takes in machines some such form as that shown in

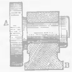

FIG. 23.

Fig. 23—a bar and a guide, or a slot and a block. The profile of the elements in each case is such as to prevent any motion *across* the required direction, just as in the turning pair.

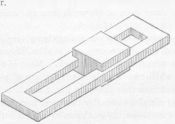

FIG. 24.

Two characteristics of these particular pairs render them specially valuable to engineers. *Firstly*, they are very easily made. The production of circular surfaces is probably the most easy operation with which the engineer has to do, and the lathe, the machine in which this operation is carried on, is the most common of all machine tools. Next to the production of turned or bored surfaces, that of flat surfaces is the operation most readily performed—the planing or shaping machines used for the purpose are always at hand. *Secondly,* the contact between the two elements in each pair is a *surface* contact. In the general case (shown in Fig. 20) the element AA_1 only touched BB_1 along, at most, three or four lines, but in the turning and the sliding pairs contact exists over a considerable surface in each element. From a geometrical point of view the constrainment is equally good in both cases; but, looked at as part of a machine, we have to keep in mind that the surfaces will wear, and we must consider that constraint the most perfect which is likely to be least disturbed by the wearing away of the constraining forms. From this point of view that form of element is best in which the pressure is distributed over the largest area, and contact over a surface is always more advantageous than contact only along a line or a few lines. Pairs of elements working with *surface* contact are called **lower pairs**; all others having *line* contact may be distinguished as **higher pairs.**

There are only three classes of surface with which this surface contact, during motion, is possible. These are (1) plane surfaces, (2) surfaces of revolution, and (3) cylindric screw surfaces. The first is utilised in the sliding and the second in the turning pair, the third (Fig. 24) is utilised in a **twisting pair** of elements (of which the common screw and nut form the most familiar example) of which we shall have to say something further on; the motion constrained by the latter is not *plane* and therefore does not fall to be considered here. The only plane motions, therefore, which can be *constrained directly* by elements having surface contact, are rotation and rectilinear translation. For all other plane motions we must have recourse either to higher pairs of elements, with line contact only, or to indirect constraint

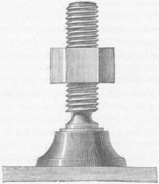

Fig. 24.

with lower pairs in the way indicated in the following section.

§ 11. LINKS, CHAINS, AND MECHANISMS.

We have seen how, in order to constrain the motion of one body relatively to another, it is necessary to connect them by a suitably-formed pair of elements. Two bodies thus connected form the simplest combination which we can treat as a machine,—but by our definition two such bodies may actually, as they often do, form a machine. We have now to look at the way in which more complex machine forms are built up from this very simple beginning. In the case supposed each body carried one element only; and with this limitation nothing more can be obtained. To go further, that is, to combine more than two bodies into a machine, each one must have at least two elements forming part of it, and the number can be increased indefinitely. For the present let us see what can be done with bodies each containing not more than two single elements.

One essential condition of the motions in any machine, and therefore in the combination of bodies, which we now wish to make, is that at no instant shall it be possible for any one of the bodies which form it to move in more than one single way. If any alternative motion were possible at any instant, the particular motion occurring would be determined by the direction and magnitude of the particular forces causing motion at that instant. This condition is impossible—that is, it must be made impossible in a machine, in order that its fundamental conditions as to constraint may be complied with. The same condition may, for convenience sake, be stated somewhat differently, namely, thus— **It is essential that among all the bodies which form a machine, and in which motion is possible at a given instant,[1] no one should be able to move without all the others undergoing certain definite changes of position also.** For if at any instant the bodies *a*, *b*, *c*, etc. in some machine are movable, and if *a* and *b* can either move or not move while *c* is moving, it is only a question of the nature of the moving force whether *c* move alone or whether it carry *a* and *b* with it. But the relative position of *c* to *a* and *b* is of course quite different if the latter move, to what it would be were

[1] This limitation is necessary because in many machines there are certain bodies which can only move periodically, being held fast by special contrivances when they are not required to move. (See § 60.)

they to remain stationary, and, under the circumstances supposed, the position of *c* at a particular instant relatively to *a* and *b* would depend entirely upon the forces acting on *c*, and could be altered altogether by a change in those forces. By definition, therefore, the motion of *c* relatively to *a* and *b* would have ceased to be constrained, and a combination such as has been supposed could not form part of a machine.

The motion of every body which forms part of a machine must be constrained relatively to all the other bodies constituting the machine. This is a proposition so obvious that we may simply state it without proof.

Bearing in mind these conditions, we can now go on to examine the way in which a machine can be built up of bodies each containing not more than two elements. The question comes at once, Can a machine contain bodies of single as well as of double elements? It cannot; for a body having only one element can only have its motion constrained to the one body to which it is paired. Such a body cannot therefore form a part of a machine containing more than one other body, for its motion relatively to any other bodies would be quite unconstrained. Our present work, then, may be limited to an examination of the ways in which bodies containing two elements each can be combined into machines.

Bodies, such as we have now to consider, which are arranged to form part of a machine by being provided with two or more suitably-formed elements, are called, when they are looked at merely in reference to the motions of the machine, **kinematic links**, or simply **links**. In order that a series of links may be combined into a machine it is necessary, of course, that the elements which they carry should be such as, when connected in pairs, give the required motions. In order, further, that the proper pairing may take place, the two elements on any one link must not belong to the same pair, but the links must be so arranged that (calling them *a*, *b*, *c*, &c.) one element on *a* shall pair with one on *b*, the second on *b* with one on *c*, the other on *c* with one on *d*, and

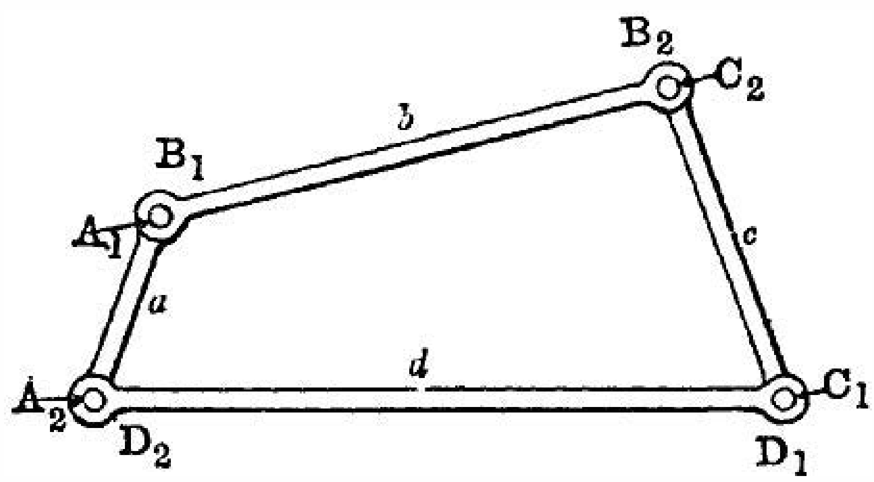

FIG. 25.

so on. On the last link there will then be one element left unpaired, while only one of those on the first (*a*) has been paired. These two elements must then be paired together, and the arrangement is complete. Figs. 25 and 26 show this for two very simple cases where the pairs used are all either

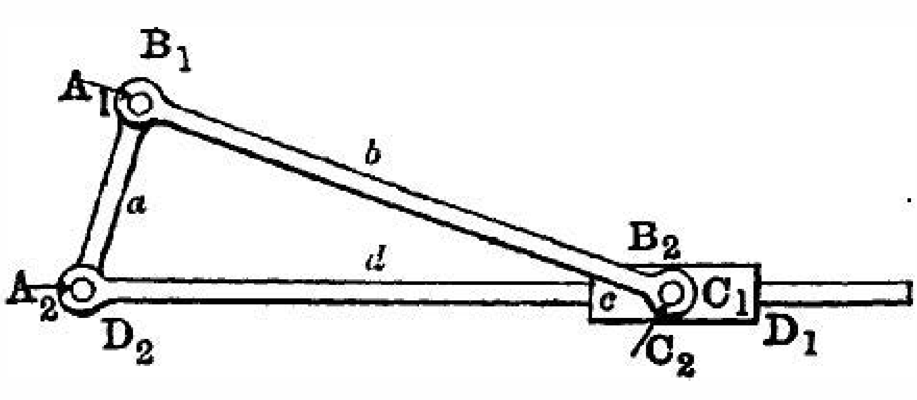

FIG. 26.

turning or sliding pairs. Four links *a*, *b*, *c*, and *d*, are used in each case, carrying respectively the elements[1] A_1A_2,

[1] Here and elsewhere the collars or flanges necessary for preventing cross motion in the pairs (see p. 54) are omitted in the figures wherever their insertion might tend to impair the clearness of the illustrations.

B_1B_2, C_1C_2, and D_1D_2; a is connected with b by the pair, A_1B_1, b with c by the pair B_2C_2, c with d by C_1D_1, and then there are left D_2 and A_2 to form a fourth pair. These being connected, the combination is finished.

A series of links completely connected in this way—connected, that is, so that no element is left single, but each one paired with its partner,—is called a **closed kinematic chain**, or simply a **chain**. Each link in the chains of Figs. 25 and 26 is paired directly with two others, its **adjacent** links. Its motion relatively to each of these is therefore completely constrained by the pair of elements

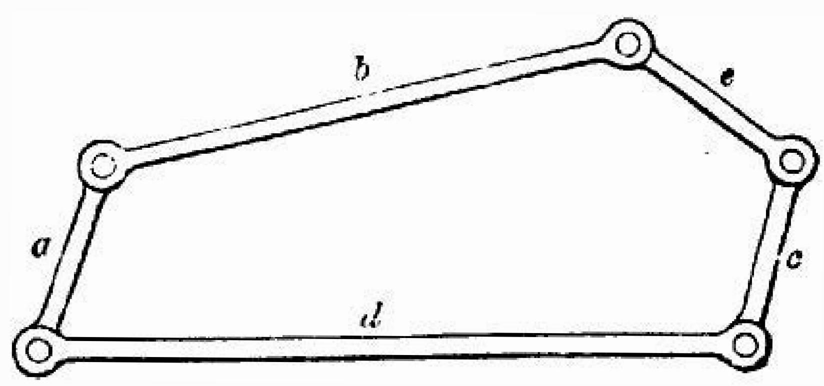

FIG. 27.

which connect them. But its motion relatively to non-adjacent links must equally be constrained, and that this is the case cannot be assumed without proof. For although the constrainment is really complete in the particular chains shown, it would have been just as easy to construct a chain in which it would not have been complete. Figure 27 shows such a chain. It does not differ much in appearance from that of Fig. 25, and has been put together by exactly the same method, but yet it is a totally useless combination of links, while the other is among the most familiar chains in existence. To prove that the (plane) motion of a body is constrained we know that we need only to prove the constrainment of two of its points (p. 34).

The motion of the whole body is that of any line in it, so that if one line have its motion constrained the whole body must also have constrained motion. If, on the other hand, it can be shown that even one point of the body is unconstrained the motion of the whole body must also be unconstrained. It is not always easy to prove that no point in a particular body, or that not more than one of its points, is constrained. But there is seldom much difficulty in showing either that *two* points *are* constrained, or that *one* point is *not*, or else that two of the moveable links might be made into one, in any of their relative positions, without destroying the movability of the rest of the mechanism. Any of these three conditions would be sufficient to settle the matter

In the cases before us let us take first the chain of Fig. 25. Can it be proved, for example, that the motion of the link *b* is constrained relatively to the non-adjacent link *d*? We know that the motion of every point in the links *a* and *c* is constrained relatively to *d*; but *b* has one point *in common* with each of these links, viz. its virtual centre relatively to each of them (p. 45). If these points be *permanent centres*, *i.e.* if they retain always the same position on *a* and *c* respectively, then their motion would be constrained, and hence the motion also of *b*, as they both belong to that body, would be also constrained. In Fig. 25 this is the case—*b* moves relatively to *a* about the centre of the pair A_1B_1, and relatively to *c* about the centre of the pair B_2C_2. Both centres are permanent, and the motion of *b* relatively to *d* is therefore constrained. The motion of *d* relatively to *b* is therefore constrained also. By precisely the same reasoning it can be proved that the relative motions of *a* and *c* are also constrained. Here, however, in Fig. 26, there is the difference that the virtual centre of *c* relatively to d is not a point on the pair—the centre of a turning pair—as in the last case. It is the point at infinity (p. 43) in the direction perpendicular to the motion of c on d. It is, however, *the same point* for all positions of c on d, and must therefore be treated as a permanent centre just as fully as the visible centre of the pair B_2C_2. Figs. 25 and 26 therefore show *completely constrained* kinematic chains.

In more complex chains, such as that shown later on in Fig. 28, the proof of constrainment is not so simple, but can be handled in exactly the same way. In such a case the points corresponding to the centres A_1B_1 and B_2C_2, are not always themselves moving about permanent centres, but about points whose constrainment has first to be proved. We shall find that this is always very easy to do.

The case is quite different with the chain shown in Fig. 27, in which a fifth link, e, is added between c and b. Examining the motion of b relatively to d, we see at once by reasoning similar to that given above, that *one* point is constrained, namely the centre point of the pair connecting a and b. The virtual centre of b relatively to c is, however, no longer a permanent centre, but a moving point whose constrainment has to be proved. This will be found impossible, however, without the assumption that either a or c is fixed as well as d. It will be found further that either a or c *could* be fixed as well as d, while still the remaining three links would remain movable, the chain, in fact, becoming identical with that of Fig. 25. This contradicts the fundamental condition (p. 59), that it shall not be possible for any link to be moved without all the other movable links moving also. Having thus proved that the relative motion of one link relatively to any other is unconstrained, it is unnecessary to examine the motions of other pairs of links—we may say at once that the chain is not a constrained one, and cannot therefore, in its present form, become part of a machine.

We have now before us a kinematic chain completely constrained; in other words, a combination of bodies so connected that every motion of each relatively to every one of the others is absolutely determinate, independent of external force. The step from the chain to the machine is a very simple one. The chain in itself only constrains the motions of its links relatively to each other; the motions of the different parts of a machine must be constrained relatively to some definite standard, as, for instance, the earth (see § 3). To convert the chain into the machine, one of its links must therefore be fixed relatively to the earth or other standard. The motions of the remaining links are then constrained relatively to the same standard, and the problem is solved.

Any chain having one link fixed might be called a machine, and essentially it is one. But it is convenient to have some word to distinguish the ideal machine, such as is shown in our engravings, with its straight bars and regularly shaped blocks, from the actual machine of the engineer with its complex masses of iron and steel. In their motions the ideal and the actual machines are identical, in all dynamic problems also the one can represent the other, but still there is so great an apparent difference between them, that in common usage the former is called generally a **mechanism**, and the word **machine** is reserved exclusively for the latter. Using, then, this established nomenclature, we can put down the conclusions at which we have arrived in the form of the following propositions:—

We obtain the simplest combination having the nature of a machine by connecting two bodies of

suitable material by such geometric forms as completely constrain their relative motions:—

These constraining forms are called elements, and can only occur in pairs. If contact between the two elements of a pair exist only along a line or a limited number of lines, it is called a **higher** pair, while pairs which have *surface* contact are called **lower** pairs. Two kinds of lower pairs only are available for the constrainment of plane motions, these pairs being called **turning** and **sliding** pairs respectively, from the nature of the motion which is constrained by them :—

Where a constrained combination is made of more than two bodies, each one must carry at least two elements, belonging to different (although possibly congruent)[1] pairs. Such bodies are called links. Lastly,—

A series of links so connected that each element in each is paired with its partner in another, and further so that the motion of every link is constrained relatively to that of every other, is a kinematic chain, and by fixing one of the links of such a chain relatively to the earth (or other standard) it becomes, finally, a mechanism. A mechanism is the ideal form of a machine, and represents it fully and absolutely for all our problems.

The form and position of the elements of any link determine its motions; the shape of the body of the link itself is quite immaterial, so long as it is not such as to foul any of the other links during its motion. In practice links of similar machines take the most widely different forms in different cases, and very frequently this form bears no resemblance to the

[1] See p. 17.

skeleton form of the corresponding link in the mechanism, the elements in the corresponding links being, however, identical. The mechanism of Fig. 26 is that which appears, for instance, in an ordinary horizontal steam engine. The link *a* becomes the crank of the engine, which in form it resembles: the link *b* becomes the connecting-rod, not quite so similar in form: the link *c* of the mechanism becomes in the machine the crosshead, piston-rod, and piston: and the link *d* the cylinder, the frame or bedplate with its crosshead guides, and the plummer block for the main bearing. The two last links have, therefore, in their ideal form, scarcely any resemblance to their counterparts in the actual machine. In another case, as we shall see, the link *b* is used as a cylinder and *d* as a piston; in another *a* becomes a fixed cast-iron framing, and so on; it is unnecessary to multiply examples. In every case the motions of the bodies forming the machine are determined by the nature of the elements connecting them, and are unaffected (under the limitation stated above) by the form of the bodies themselves.

Of course this holds good equally for the case where only two bodies are used, connected by a single pair of elements. The form of the bodies may be anything whatever, provided it is not such as to hinder the motion, so long as the elements themselves are correctly designed; the motion is determined by the latter only, and is quite unaffected by the former.

We have seen that a mechanism is formed from a chain by fixing one link of it. But *any* link of the chain may be fixed; no one link is in this respect different from the others. Hence **we can obtain from any chain as many mechanisms as it has links.** In such a case as Fig. 26 the four mechanisms which could be thus obtained would all be different; but this is not always the case, two or more of them may be similar or identical. Their total number, however, must always be equal to that of the links in the chain. The alteration by which we change one mechanism into another, using the same chain, the change, that is, in the choice of the fixed link, is called the **inversion** of the chain.

We have spoken in this section exclusively of chains whose links each contain only two elements. Such chains are called **simple chains**, and include very many of the most important mechanical combinations existing. But what

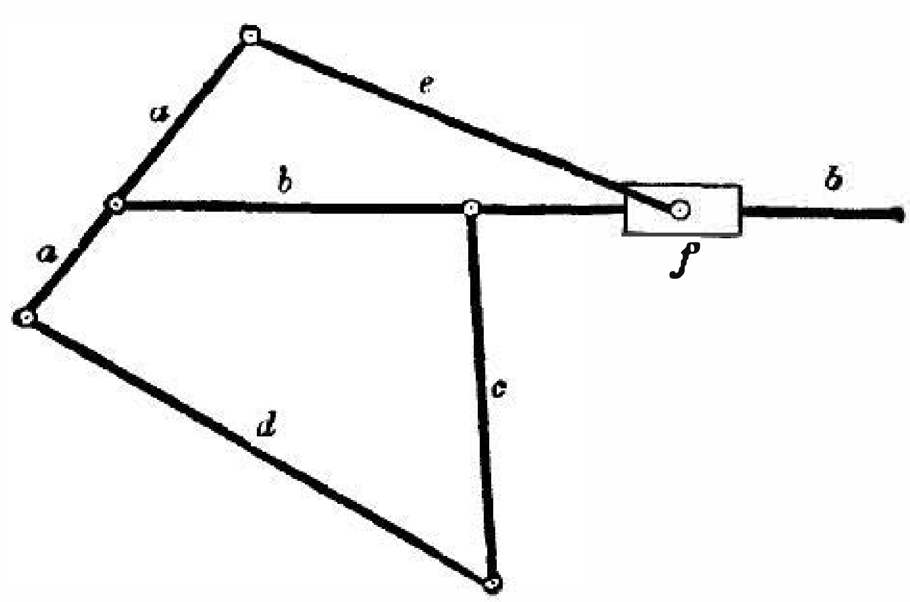

FIG. 28.

has been said about them applies, with only verbal alterations, to **compound chains**, or those chains which have some links containing more than two elements. Such a chain is shown in Fig. 28, which is a chain containing six links, two of which, *a* and *b*, have each three elements. Compound chains, and the mechanisms formed from them, do not differ essentially from simple chains and mechanisms. Naturally they are a little more difficult to deal with,—nothing more. We shall have occasion to examine several of the more important of them further on.

We must now proceed to apply to the mechanisms whose nature we have been examining the principles of "virtual" motion, which we sketched out in former sections.