LECTURE II.

We shall in the present Lecture examine in some detail a few of the results which can be obtained by treating mechanisms upon the plan which Reuleaux has proposed, and which is illustrated by his models; that is to say, by the analytical treatment of which we have already seen the general nature.

We have seen how kinematic chains are built up from pairs of elements and links. The pairing and the linkage renders the relative motions in the chain absolutely determinate, and the determinate relative motion exists equally whether or not any link of the chain be fixed relatively to the earth or to any portion of space that we choose to treat as stationary.

We have now to consider in more detail the effect of fixing one link of the chain. In practice, of course, one link is always fixed, or in other words, its motion relatively to the earth, to a locomotive or whatever it may be, is made zero. A chain with one link fixed is simply what we know as a mechanism.

In examining pairs of elements we saw that we could fix either element of the pair with lower pairs, the relative motions remaining unaltered; with the higher pairs the inversion gives us a totally different motion. We have seen also that we can fix any one link of a kinematic chain just as we can fix either element of a pair. We therefore can get as many mechanisms from any chain as it has links. From any such chain as Fig. 3, for instance, which has four links, we can get four mechanisms. The fact that a kinematic chain gives us as many mechanisms as it has links appears, looked at from this point of view, a mere matter of course. It has, however, never been hitherto distinctly recognized, so far as I know, and it can hardly be realized too distinctly, the consequences which result from it being most important, as we shall see. All that I shall attempt to do in this lecture will be to look at some of the mechanisms obtained from the particular chain just mentioned, and various modifications of it.

We have already noticed that the chain has four links. We see further that it is a chain in which all the motions are conplane, each of its four pairs being simply a cylinder pair, and the four cylinder pairs having parallel axes. It is so proportioned that by causing one link to swing, another one can be made to revolve. In order that we may refer more easily to the links, a letter is attached to each in the engraving.

For convenience sake we may also use a short symbol for this chain (the one used by Reuleaux) namely, (C''_4).* The C_4 within brackets stands for the four cylinder pairs, the symbol for parallel being added to indicate their relative positions. This is the symbol for the *chain*, no link being fixed. To distinguish the four mechanisms formed from it, we shall put the letter which stands

* In words "C parallel 4."

for the fixed link in the position of an index after the formula. Thus we can denote the particular mechanism shown in Fig. 10, in which the link d is fixed, by the formula $(C''_4)^d$. We have here then, the first of the four mechanisms we can get from this chain. You will recognize it easily enough as exactly

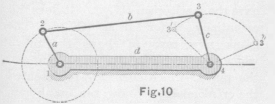

Fig.10

similar to the beam and crank of a beam engine. The link c is half the beam, a the crank and b the connecting rod. The whole mechanism is an excellent illustration of what I said in my last lecture, that the *form* of the links is indifferent. If you think of the mechanism as forming part of a beam engine, for instance, you will see in the link d the abstract form of what is generally a most complex structure, a bed plate with its bearings, an entablature and plummer block, cast iron columns, and in some cases even brick and masonry. All these are represented by the fixed link *d* so far as their kinematic relations are concerned.

If now we fix the connecting rod *b* instead of fixing the link *d* as before, we have the mechanism $(\mathrm{C}''_4)^b$. It does not essentially differ from $(\mathrm{C}''_4)^d$. The crank now revolves about the pin 2 which was formerly the crank pin, and the pin 1, which formerly represented the crank shaft, is now the crank pin, but there is nothing changed in the nature of the mechanism. By this inversion therefore we have got nothing new.

Let us now fix the link *a*, which was formerly the crank (Fig. 11). We have now the mechanism $(\mathrm{C}''_4)^a$; it contains precisely the same elements as before, and the relative motions of the links are unaltered, but as a mechanism it is entirely different. It is now a combination frequently enough used in mills and elsewhere, known by the name of a "drag-link coupling." The links b and d have become cranks, and one drives the other by means of the link c.

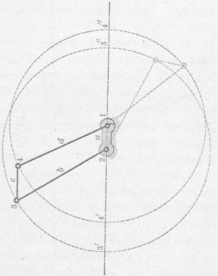

Fig. 11

By fixing the remaining link of the chain, the link c, (which we have supposed to be longer than a), we have the entirely different mechanism $(C''_4)^e$, (Fig.

12). The two arms no longer revolve but only swing, and the link a turns right round once in every double swing

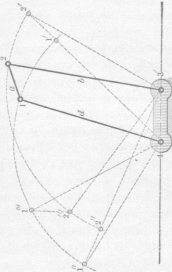

Fig. 12

of d and b. This mechanism is occasionally used in part of its stroke in parallel motions with some modifications, but is not so well known as the others.

The four inversions of this one chain, therefore, give us three different mechanisms. Looked at separately it is hard to see the relation in which these stand to each other; from the point of view which we have taken their mutual relationship has become at once evident.

Without altering the pairing at all, we can greatly alter the chain by changing the relative length of its links. If we made all the links equal we should have a square, of which all four inversions would give similar mechanisms. If we make $b=d$ and $c=a$ we get a mechanism which is perfectly familiar in the couplings of locomotives and many other cases. All four mechanisms are again similar, each one consisting of a pair of cranks revolving with equal velocities and connected by a link which moves always parallel to itself.

These mechanisms are among those which have the peculiarity to which I alluded yesterday, that in one of their positions their motions are not determinate. This occurs at the "dead points" when a and c are both standing in the direction of the axis of d or b. If no means be taken to prevent it, it is then possible to move the crank either in one direction or the other, and the two cranks may go on revolving in the same direction, or may revolve in opposite directions according to circumstances.

Such an indeterminateness is, of course, inadmissible in machinery, where we therefore adopt the well-known method of combining two mechanisms of the same kind, and placing them with their cranks at right angles, so that they do not cross the dead points at the same time. The motions are thus made determinate and the cranks revolve in similar directions. We might, however, wish them to revolve in *opposite* directions, as in the mechanism shown in Fig. 9. It may be worth our while to look for a moment at the means which may be used in this case to secure the determinateness of the motions in the mechanism. To distinguish between the two cases we may call the former "parallel cranks" and represent it bythe formula $(C''_2 \| C''_2)$, and the latter "anti-parallel cranks," $(C''_2 \mathrel{\underline{\smash{\angle}}} C''_2)$. This chain, with the link d fixed, is shown in Fig. 13e

In the case of the parallel cranks all points of the centroids of b and d are at infinity, for they are at the intersections of the parallel links a and c. We

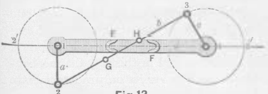

Fig.13

have already seen, however, that in the mechanism Fig. 9 the centroids are quite different, those of b and d being hyperbolæ. If, therefore, when the mechanism is brought into either dead point, where the cranks might change from the anti-parallel to theparallel position, we can only make certain that the right centroids roll upon each other, we shall get the motion that we want. Fig. 13 shows an arrangement by which, just in that position of possible change, a tooth made on one link and a recess upon the other link gear together at a point corresponding to the point of contact of the centroids. The teeth G and H are virtually formed upon the centroid of *b*, and the recesses E and F upon that of *d*. At the points where these come into gear, the two centroids are compelled to roll upon one another, just as the pitch circles of two toothed wheels are compelled to roll on one another, and in this way the mechanism is carried over its only indeterminate point, and the cranks remain continuously anti-parallel and revolve in opposite directions.

This anti-parallel chain gives us two different mechanisms. Fig. 13 shows us $(C''_2 \mathsf{Z}\, C''_2)^d$. In the other mechanism $(C''_2 \mathsf{Z}\, C''_2)^a$ the two cranks revolve in the *same* direction with very varying velocity ratios.

Returning again to the chain (C''_4), it will be seen at once that we may substitute for the pair of elements at 4 a slot and a sector concentric with it, as in Fig. 14. The motions remain entirely unaltered. By adopting this contruction, however, it becomes possible to construct the mechanism without covering with it the center of the pair 4, *i. e.*, the point of intersection of the links *c* and *d*. We

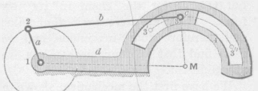

Fig. 14

can therefore lengthen these links without making the mechanism inconveniently largee The only constructive alteration is that the slot becomes flatter as the links are lengthened. If we lengthen them little by little until they become infinitely long, the curved slot becomes straight, and its center line will pass through the point 1. The mechanism modified in this fashion takes the extremely familiar form already shown in Fig. 8. It now contains three cylinder pairs with parallel axes; the fourth cylinder pair has become a straight slot with a block working in it, namely, a prism pair. The axis of the prism pair is normal to the axis of the three cylinder pairs, and we may therefore use the symbol $(C''_3 P^{\perp})$ for the chain in its new form. There are here again four links, and therefore four inversions, and we shall find that all four mechanisms are now different.

We have first the mechanism shown in Fig. 8, and familiar by its continual use in direct-acting engines $(C''_3 P^{\perp})^d$. Next, following the same order as before, we may fix the link *b*, the connecting rod of Fig. 8. The mechanism thus obtained, $(C'' P^{\perp})^b$, is quite different from the former, but equally familiar (Fig. 15). To make it more recognizible, the prism pair 4 is *reversed* in the figure, that is the link *c* is made to carry the open prism and *d* the full one. The motion is obviously unaffected by the change. The mechanism can be easily seen to be that of the oscillating engine. The link c corresponds to the cylinder, swinging on fixed trunnions at 3, and the link d to the piston rod and piston of the steam engine. We see, then, that the relation between the mechanisms which are familiar to us as the driving trains of the

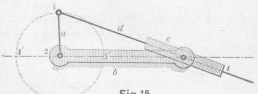

Fig.15

direct-acting and oscillating engines, is simply that they are different inversions of one and the same chain.

Let us now suppose the chain fixed upon the link which was the crank in the last two mechanisms (Fig. 16). This gives us a third mechanism which entirely differs from either of the two former ones. It is quite familiar as a "quickreturn" motion in some machine tools, for which purpose also the mechanism last mentioned has sometimes been used.

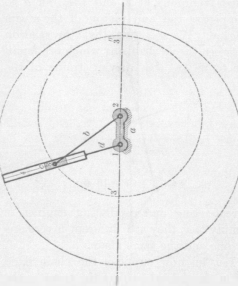

Fig. 16

Fixing, lastly, the link c, we get the less familiar mechanism shown in Fig. 17 $(C''_3 P^{\perp})^c$. The link b swings about 3.

and the crank a rotates in space somewhat as in the mechanism $(C''_4)^c$, which we have already seen This train has some

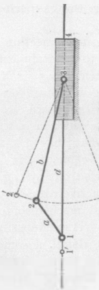

Fig.17

practical applications in machinery, but is not very often used.

Here, then, we have obtained from one and the same chain four entirely different mechanisms, all of them more or less familiar. The method we have adopted has again been successful in making the real relations of these apparently dissimilar things perfectly obvious.

We have seen in connection with Fig. 14 that we can to a certain extent alter the size and extent of a pair of elements without altering its nature or changing the motion of the chain to which it belongs. This alteration in the size of elements, or what may be called the "expansion" of elements, is a process continually carried out by engineers for practical constructive reasons; and often gives to identical mechanisms extremely different forms. It is impossible here to go into this in detail, a somewhat extreme case of it is shown, for the sake of illustration, in Fig. 18. Here we have the mechanism shown already in Fig. 8, $(C''.P\perp)^d$. The pin of the pair 2 is so enlarged as to include altogether the pair 3, the connecting-rod *b* being simply a circular disc, with an eccentric cylindric hole in it. The pin 1 again (the "crank shafte'), is made large enough to include

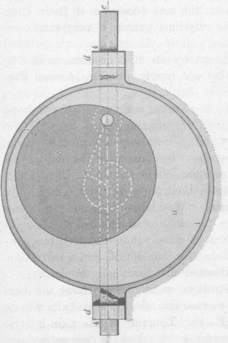

Fig. 18

the whole of 2. We have therefore 3 within 2, and 2 within 1. We have one very common illustration of the extent to which this expansion of pairs is carried practically in the link motion. The curved link and block are in their kinematic relations simply a very much expanded pair of cylindric elements reduced in extent by use of a process similar to that by which we got Fig. 14 from Fig. 10.

We have seen what results have been obtained by making two links of the chain (C''_4) infinitely long. The same process can be carried still further. In the familiar chain shown in Fig. 7, for instance, *three* links, *d*, *c* and *b*, are made infinite. We have therefore another prism pair in it, and its formula becomes ($C''_2 P\perp_2$). It gives us the two mechanisms already mentioned and a third one, all of which are practically applied.

We must pass over without mention many other modifications and alterations of the chain, and mention only one other form in which it occurs, a form which has some special interest. The condition of movability of a chain that contains four cylinder pairs is not that their axes should be parallel, but that they should *meet in one point.* The

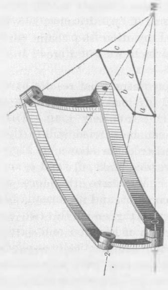

Fig. 19

axes are parallel only in the special case where this point is at an infinite distance. Fig. 19 is an illustration of the more general, although less familiar, case when the point of intersection is at a finite distance. This chain, which may be indicated by the formula $(C\angle_4)$* has again its four inversions, and furnishes us with three different mechanisms as in the case of $(C''_4)e$ I cannot here go into these; I mention the chain partly because of its theoretic interest, and partly because—although it looks so unfameliar, it is not unfrequently applied in machinery. If instead of subtending only a small angle, as in Fig. 19, three of the links were made to subtend an angle of 90°, we should have the common Hooke's or universal joint. In these "conic" chains, links which are quadrants take the place of the infinite links, so that a chain having three quadrants corresponds to the chain of Fige 7, in which there are three infinite links. The formula of the universal jointis $(C\perp_3 C\angle)^a$; it corresponds exactly to $(C''_2 P\perp_2)^a$ and is analogous to $(C''_4)^a$ and $(C''_3 P\perp)^a$, Figs. 11 and 16 respectively. I have already mentioned in passing that the mechanism $(C''_2 P_2 \perp)^a$

* " C four oblique."

is that of Oldham's coupling. We see then clearly the close relationship in which this coupling (for parallel shafts) stands to the Hooke's joint or coupling for inclined shafts. It is another illustration of the important results which follow naturally from Reuleaux's simple method of analysis, and which hardly seem attainable, certainly not with equal directness, by any other method.

We may notice one other way in which the chain (C''_4), which we have seen in so many forms, may be modified. In some cases we may not wish to utilize the motion of all the four links, but (say) only three of them; in that case we may omit the fourth link, if we carry out a proper pairing between the two links thus left unconnected. If, for instance, we omit the link b we must make a proper pairing between a and d. This pairing will always be higher; the chain becomes a *reduced* chain. Such a chain, reduced by b, is shown in Fig. 20. The higher pairing is carried out by placing upon a a suitable element, here a circular pin at 2, and giving to c the form of the *envelope* of the motion of the pin relatively to it. This envelope is

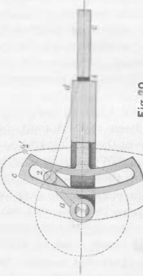

Fig. 20

the curved slot shown in the figure. It was not necessary to take the new element at 2, but if it had been in any other place, as 2′, the form of the slot would have been more complex, as is shown by dotted lines. For such a chain we may use the formula $(C''_3P^{\perp}) - b$. The process of reduction can be carried on in this way until only two links are left, which then become really a pair of higher elements. It is constantly employed in machinery, mostly in the case of compound chains, or chains in which some links contain more than two elements.

The chain which we have been examining has been applied more often than any other to the leading trains of engines and pumps. We shall in conclusion look at a few of these machines, in order to notice the constructive disguises which often appear in them and render their kinematic identity almost unrecognizable.

Fig. 21 shows a rotary engine which has been patented several times, and which is founded on the same mechanism, $(C''_3P^{\perp})^d$, as the common direct-acting engine. The letters and figures placed upon it correspond to those on Fig. 8, so that the identity of the mechanisms may be the more easily traced. The extraordinary change of form under-

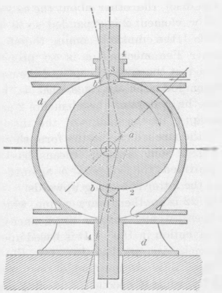

Fig.21

gone by the connecting rod b is worth special notice. It has become a bar having a cross section like a half moon.

It still consists however, kinematically, of two cylinders or portions of them, one described about the axis of 2 (the center of the disc), the other about the axis of 3. The element 2 is expanded so as to include 1, the crank *a* becoming therefore a disc. The mechanism is so proportioned that the virtual length of the connecting rod—the distance, that is, between the centers of the elements 2 and 3, is equal to the radius of the disc *a*. With the mechanism in the form shown in Fig. 21 some separate means has to be provided for keeping *b* against *a* when the latter is moving upwards.

Fig. 22 is another rotary engine, which has been patented a dozen times since its first invention in 1805. It is based upon the mechanism of Fig. 16 $(C''_3 P^{\perp})^a$. The fixed link *a* is here made the steam cylinder, while *d* becomes a moving piston. The reference letters are the same as before. It will be noticed that the cylindric element of the link *c* is expanded to include its prismatic element; in all the cases formerly noticed the latter had been the larger.

In order to illustrate this part of his subject Reauleux examines (in the work I have already mentioned) some forty or fifty rotary engines and pumps all de-

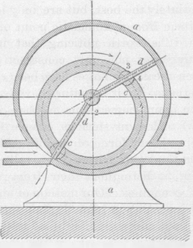

Fig.22

rived from the (C''_4) chain and such modifications of it as we have been looking at. Models of a number of these are now on the table before you. Many of them bear scarcely any external resemblance to the common steam engine, to which they are, notwithstanding their dissimilarity, so closely related. We are not now concerned as to which form is absolutely the best, but are only looking at them from a kinematic point of view. But it is worth noticing that in very many cases not only constructive but mechanical advantages have been claimed for them. Their inventors have over and over again claimed some mechanical gain more or less mysterious, compared with the ordinary form of engine. Most of them, moreover, have been called "rotary" to distinguish them from reciprocating engines. Our method of analysis, although only kinematic, has shown us not only that there can be no mechanical advantage possessed by one over the other, but that the word "rotary" is essentially a misnomer, if it be supposed to indicate that there is any more or different rotation in them than in ordinary engines.e

We shall now only notice one more of

these engines. It is one which has puzzled people a great deal, and with very good reason, for its motions are very strange, and its analysis apparently very complex.

The engine I refer to is shown (in one form) in Fig. 23. It is known as the "disc engine." It was brought a good deal into notice in 1851, and for a few years was used in the *Times* office without any ultimate success. Analysis shows that it is based upon the same chain as the Hooke's joint, the conic chain, namely, in which three out of four links subtend right angles, and of which the formula is $(C^{\perp}_{3}, C^{\angle})^{a}$. The fixed link is, however, *d*, while in Hooke's joint *a* (the acute-angled link) is fixed. Patents have been taken out also for disc engines in which *a*, and others in which *b*, is the fixed link.

I may say, in conclusion, that while it may be impossible for many educational institutions in this country to possess themselves of models so perfect in execution, and therefore so expensive, as

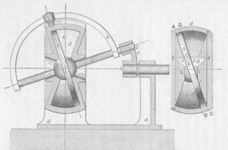

Fig.23

those which have been sent to the Loan Collection, it is yet quite practicable to construct many of them in a form which, while quite cheap, is yet well adopted for educational purposes. I have some made in this way of hard wood with brass pins, which are very serviceable for col lege purposese It is just the simpler models, which are the most easily made, which are the most useful in instruction.

I venture to hope that the treatment of the theory of mechanism illustrated by the models now in the South Kensington Collection, and of which I have here endeavored to set forth some leading principles, may prove valuable in aiding the study and the comprehension of this branch of machine science both to the student and the engineer.